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Sensory Differentiation for Equitable Inclusion: Designing for Balance as the Nexus of Sensory Regulation and Embodied Mathematics Learning

by

Sofia Tancredi

A dissertation submitted in partial satisfaction of

the requirements for the degree of

Joint Doctor of Philosophy
with San Francisco State University

in

Special Education

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Dor Abrahamson, Chair
Professor Kimiko Ryokai
Professor Gloria Soto

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Abstract

Sensory differentiation for equitable inclusion: Designing for balance as the nexus of sensory regulation and embodied mathematics learning
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Doctor of Philosophy in Special Education

University of California, Berkeley

San Francisco State University

Professor Dor Abrahamson, Chair

Stimulation of the senses plays a fundamental role in regulating arousal to support engagement with the world. Research on neurodiversity reveals a range of sensory regulation needs amongst humans, including need for different intensities of vestibular (balance) sensory stimulation. In parallel, the embodiment turn in the cognitive sciences challenges the separation of bodily experiences from cognition, problematizing the presumed irrelevance of sensory regulatory movements like fidgeting to cognition. In this design-based research dissertation, I explore the integration of embodied cognition and sensory regulation perspectives for understanding movement in learning as it informs inclusive design. I propose and evaluate a design, Balance Board Math (BBM), that explicitly integrates sensory regulatory movement into embodied design to serve conceptual learning, focusing on the bodily sense of balance. BBM is a digital platform for discovery-based math learning that establish the vestibular-stimulating action of bodily rocking as the basis for exploring various mathematical concepts. To serve sensory regulation inclusively, BBM introduces vestibular sensory differentiation, offering learners adaptive vestibular stimulation at their preferred intensity.

This dissertation contains four papers. The first describes the BBM design, rationale, and design principles, as well as results from an in-depth semi-structured BBM task-based interview study with 6 school-age children showing that children used their sense of balance to explore and discuss mathematical ideas, and that doing so changed how they oriented to mathematical graphs. The second elaborates on the BBM design with regards to how it transforms mathematical thinking, illustrated with examples from multiple empirical BBM studies. The third paper is a conceptual paper outlining an embodied rethinking of sensory-regulatory stimming as epistemic activity, using the BBM design as an example to discuss implications for pedagogical design. The fourth paper is a hybrid conceptual–empirical paper discussing sensory differentiation, evaluated through a study of how children stimmed within 20 BBM task-based semi-structured interviews. Overall, this dissertation contributes to learning theory by studying sensory regulation as an interactive and epistemic resource, expanding models of embodied cognition.

Dedication

To the students, colleagues, and friends who have shared their sensory worlds with me.

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1 Introduction

Sensory stimulation is theorized to play a key role in attention and self-regulation (e.g., Dunn, 1997; Dunn & Bennett, 2002). In parallel, recent developments in cognitive science suggest that moving in new ways forms the basis for cognitive structures (Varela et al., 1991). Despite the importance of sensorimotor activity from these theoretical perspectives, traditionally sedentary classroom norms and activities provide limited stimulation to several sensory systems. Furthermore, these two bodies of literature have remained disparate due to the assumption that regulation and cognition are two separate processes. This project questions this assumption, asking: How might sensory regulatory activity interact with or even *participate in* cognitive processes? What if students' spontaneous regulatory activity, such as rocking and fidgeting, were invited to further participate in conceptual exploration? How might learners' sensory regulation needs and learning goals be served through such novel forms of instruction? This design-based research project aims to investigate these questions through creating and empirically evaluating an instructional design that makes the sensory regulatory behavior of rocking central to mathematical exploration: Balance Board Math.

Design Problem

Research suggests that the optimal sensory stimulation range varies by individual. Thus, whereas the sensory majority may be able to regulate and attend with a given amount of stimulation, this level of stimulation may be insufficient for some learners. Sensory differences are associated with a number of special-education profiles including learning disabilities, attention deficit hyperactivity disorder (ADHD), and autism (Dove & Dunn, 2008; Little et al., 2018), suggesting learners in special education may be especially vulnerable to this kind of environment–need sensory mismatch. Sensory processing has been linked to academic learning outcomes: for example, sensory differences predict 47% of variance in academic performance for children on the autism spectrum (Ashburner et al., 2008), with a significant correlation found between sensation seeking and academic performance, as well as teacher ratings of attention. Additionally, studies of children with ADHD found atypical sensory processing to be correlated with lower daily function (Mimouni-Bloch et al., 2018), whereas self-directed movement such as fidgeting was positively correlated with working memory performance (Sarver et al. 2015). More broadly, research with special education students has found that motion-based learning games support short-term memory development and positive affective experience (Kosmas et al., 2018). Taken together, these results suggest that sensory experiences impact students' academic outcomes.

One sensory group that stands to be disadvantaged in the sedentary classroom is *sensory seekers*: learners who need more movement stimulation than is typically offered in the classroom in order to regulate and attend (Dunn, 1997). These students are expected to spend their days in sedentary learning environments ill-matched to their sensory needs. In particular, the vestibular system in the inner ear, which governs balance and orientation, is minimally stimulated when sedentary. The vestibular system has been implicated in cognitive development (Hitier et al., 2014; Wiener-Vacher et al., 2013) and even abstract conceptual reasoning (Antle et al., 2013). Furthermore, self-initiated sensory-seeking movements that could help these students to better regulate, such as pacing or rocking, are generally read as disruptive. Discipline of such bodily activity is often compounded by racialized readings of students' bodies (Annamma et al., 2013;

Hilton, 2017). Children are confined to a vestibularly sparse learning environment that may not meet their sensory regulation needs.

Historically, the onus of resolving sensory mismatch has fallen on students, who are asked to conform to bodily classroom norms at the expense of comfort (Annamma, 2017) and allocate their cognitive resources to navigating ill-fitted sensory environments. This project shifts the onus of sensory harmony onto the design of learning tasks and environments that can meet all learners' sensory regulation needs.

Prior Solutions

Prior solutions have looked to address sensory needs separately from academic instruction. Many schools offer recess breaks where a greater range of sensory behaviors are permitted. In classrooms, sensory seekers may be granted accommodations such as alternative seating arrangements on the floor or on a ball chair (Bundy et al., 2019), permission for intermittent exercise, or access to sensory tools like chewing gum or lap pads (Critz et al., 2015; Singer, 2015). These solutions offer sensory input in parallel to academic instruction and at times may conflict with classroom activities, as when a student needs to step away from group work to do exercises. Beyond the classroom, children with atypical sensory processing may participate in sensory therapies with occupational therapists (Ayres, 1974; Bundy et al., 2019).

Some mathematics instructional designs do offer a greater degree of sensorimotor engagement, such as activities with manipulatives. More recently, instructional designs inspired by embodied cognition theory (described in the subsequent section) often incorporate gesture or whole-body activity. These designs differ in how movement is deployed, presenting a spectrum from explicit instruction of specific movements (e.g., Nathan & Walkington, 2017) to creative movements (e.g., Vogelstein et al., 2019). Between these poles, some interventions prompt specific movements, such as gestures, for learners within traditional activities such as problem-solving (e.g., Broaders et al., 2007), while others create situations that participants engage through movement, such as a gym number line or a walking-scale torus (Kelton & Ma, 2018, 2020). Design framework *embodied design* also falls between these poles (Abrahamson, 2014). In *action-based embodied design*, learners are provided with a set of resources and a motor control problem and figure out for themselves how to move in a new way to satisfy task specifications. Whereas these more movement-based pedagogical approaches may offer more sensory stimulation than traditional instruction, they do not consistently stimulate the vestibular sense, nor explicitly accommodate various sensory profiles.

Prior work in HCI has explored movement and regulation, from embodied self-regulation in the margins of computing space (Karlesky & Isbister, 2016) to the effect of physical exertion on enjoyment (Mueller et al., 2003). Separately, embodied interactions have also been leveraged to support math learning (e.g., Gelsomini et al., 2020). Balance boards have previously been explored in recreational (Byrne et al., 2016) and rehabilitation contexts (Albiol-Pérez et al., 2012), with the Wii Balance Board being a popular commercial gaming input device. Antle et al. (2013) also examined the impact of a balance-based input device on abstract reasoning about questions of justice. I seek to build upon these lines of research by integrating regulatory and conceptual affordances of embodied interactions.

Theoretical Framework

Theoretically, this dissertation seeks to integrate research on the relationship between bodily movement and learning in the learning sciences, psychology, and occupational therapy. On the one hand, embodied cognition theory views the body as playing a central role in cognition. Embodied cognition is far, as of yet, from a singular cohesive theoretical stance, enfolding a range of perspectives from more conservative (Barsalou, 1999) to more radical (e.g. Hutto & Myin, 2013) and spanning a range of foci, notably the "four Es" of cognition that view the mind as embodied, enacted, embedded, and/or extended (Newen et al., 2018). Of particular prospective explanatory value for looking at the intersection of regulatory and conceptual movement is the enactivist strand of embodied cognition theory (Varela et al., 1991), which views cognition as built from processes of perception and action. According to enactivism, cognitive structures arise from repeated patterns of perceptually guided activity. Enactivist views of mathematics analyze mathematical thinking as overtly and covertly embodied, highlighting how even purportedly "abstract" symbols are manipulated as if they were objects and objectify what are in fact multimodal notions (for example, the "+" symbol denotes the process of grouping together) (Hutto et al., 2015). Mathematical expertise is associated with differences in covert or overt bodily engagement: for example, skilled graphers have been found to describe graphs with larger gestures than unskilled graphers, as if "being the graph" not merely "seeing the graph" (Gerofsky, 2011). From an enactivist perspective, sensorimotor activity forms the basis of math thinking and learning.

On the other hand, psychologists and occupational therapists have identified individuals' needs for sensory regulatory movement, such as swinging, tapping, flapping hands, or pacing, to regulate and sustain attention. Occupational therapy research traditions originating in the work of Anna Jean Ayres (1974) have identified movement as playing a sensory-regulatory function in and beyond learning contexts. *Sensory regulation* is the maintenance of optimal levels of alertness through engaging sensory stimuli that are of an adaptive intensity for one's neurological sensory processing profile (e.g., Dunn, 1997). In a similar vein, psychology research has underscored affective dimensions of spontaneous movements, such as fidgeting, as reflective of learners' stress (Farley et al., 2013).

The embodied cognition and sensory regulation perspectives each offer distinct and prospectively conflicting implications for how to understand and foster learning. Embodied cognition inspired approaches emphasize movement as a means of exploring concepts (for example, walking along a number line); sensory regulation approaches emphasize movement as a means of regulating one's arousal state to support ongoing engagement (for example, bouncing on an exercise ball). Potential conflict between movements serving these dual functions has not yet been addressed in the literature. Do sensory regulatory movements add to cognitive load (Sweller, 1994) by competing for learners' physical and attentional resources, and if so, in what circumstances? If cognition is sensorimotor activity, and regulation through sensorimotor activity is also always ongoing, how do these processes interact? Are sensory regulatory and cognitive processes indeed distinct, or might regulatory activity interact with or even participate in cognitive processes? Together, these siloed theoretical perspectives stand to mutually inform each other towards a more holistic understanding of movement in learning, knowing, and thinking. This project launches from the observed necessity of integrating these dual-movement functions that might enter into either conflict or synergy, particularly in the learning of sensorially minoritized learners. Implicit to disciplinary divides on this subject, I argue, lies an epistemological and theoretical treatment of sensory regulation and learning as disparate facets

of cognition and action. Informed by critical disability studies, this work dialogues with research on inclusive design and *Universal Design for Learning* (UDL) (Rose & Meyer, 2002). UDL aims to proactively accommodate students' diverse learning needs and preferences by presenting multiple modes of representation, action/expression, and engagement. Recent work expands and reimagines UDL from an embodied perspective (Abrahamson et al., 2019), in particular the *Special Education Embodied Design (SpEED)* design-based research framework, which embraces learners' embodied resources as the starting point of instructional design (Tancredi, Chen, et al., 2022). Inspired by the neurodiversity movement and paradigm (Bertilsdotter Rosqvist et al., 2020; Walker, 2021; Yergeau, 2018), BBM is a case of SpEED, focused specifically on sensory neurodiversity.

Overview

The core methodology of this dissertation is *design-based research (DBR),* an approach to educational research that is oriented towards utility to both theory and practice (Bakker, 2018; Collins, 1992). Theory and design mutually inform one another through iterative design cycles, with theory and solutions co-emerging. DBR responds to limitations of other research designs by both furnishing innovative tools and developing "humble theory" (Cobb et al., 2003) in a way that is fundamentally contextualized. DBR outcomes include novel or revised theoretical constructs as well as empirically evaluated educational designs (diSessa & Cobb, 2004). The DBR approach makes it possible to study types of learning that do not yet exist (Bakker, 2018). This makes DBR a particularly apt method for work that seeks to move beyond historic barriers posed to minoritized learners.

The BBM DBR project seeks to imagine beyond current tensions between learners' sensory needs and instructional demands to the fostering of synergistic sensory–cognitive activity. It looks to integrate the historically disparate uses of movement as movement-for-sensory-regulation and movement-for-learning, engaging users in sensory-regulating movement as a means of math conceptual exploration. The central design conjecture is that directly incorporating vestibular stimulation into learning activities will improve their efficacy both by supporting sensory regulation and by offering new concrete ways of developing mathematical concepts. Sensory regulatory behaviors stand to serve as a fruitful conceptual resource when integrated into math learning activities.

The first paper in this dissertation introduces the BBM design and rationale as a case of sensory-responsive design that integrates embodied cognition and sensory regulation functions of movement. The paper describes BBM and evaluates whether and how BBM fulfills its design principles through an empirical design-oriented study, finding that learners discover and control dynamical properties of graphs by rocking, thus grounding functions and graphing concepts in experiences of bodily rocking, even as they self-regulate by adapting their actions according to their sensory profile and needs. This paper was published in the proceedings of the *Interaction Design and Children* conference (Tancredi, Wang, et al., 2022).

The second paper elaborates upon BBM as it pertains to mathematical thinking. Using examples from BBM participants, it illustrates how balance-based, sensory-differentiated instruction can change how students learn and think about concepts, spanning examples from arithmetic to functions and graphing. It proposes that rendering concepts explorable through the balance modality is a process of *restructuration* (Wilensky & Papert, 2010), whereby the structure of knowledge is transformed, so that learners' experience of disciplinary forms such as number lines and graphs can become imbued with conceptually generative multimodal

experiences. The paper discusses how balance-based concepts can expand accessibility. This paper was published as a design snapshot, illustrating how digital technologies can transform mathematical thinking and learning, in *Digital Experiences in Mathematics Education* (Tancredi, 2024).

Building from ideas developed through the BBM project, the third paper is a critical-pedagogy conceptual paper that re-theorizes peripheral sensorimotor activity such as rocking and fidgeting from a post-cognitivist perspective as *part of* embodied, enacted, embedded, extended, emotional, ecological cognition. It argues for stimming as intrinsic to adaptive functioning, interaction, and cognitive dynamics. This paper uses the BBM design as an example to discuss implications of an embodied view of stimming for inclusive pedagogical design. At present, this paper has been conditionally accepted as part of a topical collection on human movement and learning at *Educational Psychology Review* (Tancredi & Abrahamson, under review).

The fourth paper is a conceptual–empirical paper that argues the need for pedagogical design to offer differentiated somatosensory stimulation to accommodate learners' sensory diversity. This conceptual argument is elaborated through an empirical study of neurotypical and neurodivergent children's stimming when using BBM. The paper offers an analysis of stimming as epistemic actions within mathematical discourse.

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Thank you to Dor Abrahamson for advising and supporting me in truly innumerable ways. Your discerning perspective on theory and design has inspired and sharpened this work from its inception, and I truly can't imagine having done it anywhere else. Thank you to Yue-Ting Siu for your excellent advising, and for continually grounding me in how research can inform practice and meet today's accessibility needs. Thank you to Kimiko Ryokai for your generous design input and guidance in taking BBM from a first prototype to a digital interaction system, even amidst a pandemic, and for bringing BBM into dialogue with human-computer interaction research. Thank you to Gloria Soto for feedback that improved this dissertation, and for your steady mentorship during my final year in the joint doctoral program.

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I have benefitted from some wonderful academic communities. Special thanks to the Embodied Design Research Lab for surrounding me with inspiring ideas, and for testing and reflecting on BBM as it evolved, from brainstorms to test demos to data sessions to recording everyone's stims. Thank you to SpEED cofounders Rachel Chen and Christina Krause for deep dives into inclusion and design in the BWW hexagon– collaborating with you has been instrumental. Rachel, thanks also for so many enriching conversations about stimming. Thank you to SpEED collaborators and associates near and far. I am also grateful to colleagues at the RITMO center at University of Oslo, the Embodied Underground, the Gesture and Multimodality Group, and the Human-Computer Relations at Work network for feedback on the BBM project and much wonderful dialogue about design, fidgeting, cognition, the senses, and inclusion. Thanks to PhET Interactive Simulations and the CHROME lab for thinking and working together on multimodality and accessibility.

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I am so grateful to the friends and family that supported me on this journey. Thank you to my husband, Lukas, for the unwavering support. Thanks to my son, Jules, for showing me something new about learning every day, and for sometimes even allowing me to use my laptop. Thanks to my siblings Anna and Alexander, my parents, and esteemed aliens Sarah, Hannah, Tierney, and Solene for their care and wise counsel. Thanks to the friends that climbed, hiked, read, puzzled, and camped with me, keeping me connected to the physical world even during deep writing mode.

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1 Balance Board Math: "Being the Graph" Through the Sense of Balance for Embodied Self-Regulation and Learning

This first paper was co-authored by myself, Julia Wang, Helen Li, Carissa Yao, Genna Macfarlan, and Kimiko Ryokai. It introduces Balance Board Math (BBM): a new balance-based interface for math instruction. BBM integrates disparate work on embodied cognition and on sensory regulation to offer learners integrated opportunities to both self-regulate through movement and to use their sense of balance as a resource for conceptual understanding. This approach imagines beyond common views that self-initiated background activity, such as fidgeting, is unproductive for education. With a sensor-equipped balance board and dynamic real-time display, BBM's Balance Graphing activities offer users opportunities to playfully explore and embody different aspects of functions and graphs such as frequency and amplitude. This paper describes results from an in-depth study with 6 school-aged children examining how their movement and personal sense of balance were used to both self-regulate and make sense of mathematical concepts through BBM. By inviting learners' regulatory movements to serve as an interaction resource for exploring mathematical concepts, I propose that BBM offers a new genre of sensory-responsive design that could better serve instructional differentiation.

1 Introduction

Many children crave movement, yet traditional classrooms demand they sit still. Research in occupational therapy and psychology suggests that movement such as rocking and fidgeting can have positive effects on self-regulation and sustained attention by supporting an underlying need for sensory stimulation [1]. What if instead of suppressing spontaneous movements, we made them part of mathematics instruction? What if children were empowered to engage their own sense of balance in exploring mathematical concepts? Balance Board Math (BBM) aims to cultivate a context where sensory regulation and learning activity are integrated. With the BBM system, children generate dynamic graphs on a large display by rocking on a wooden balance board (Figure 1). BBM offers a variety of activities where children can playfully explore the relationship between their physical movements and graphical visualization of their movements in real time. By controlling mathematical forms through movement, BBM users can reflect and refine their thinking about how different parameters of graphs such as amplitude, frequency, and function work as a system. We contribute our design and evaluation of BBM and discuss implications for sensory accessibility, investigating the balance sensory system as an untapped regulatory and conceptual resource in educational research.

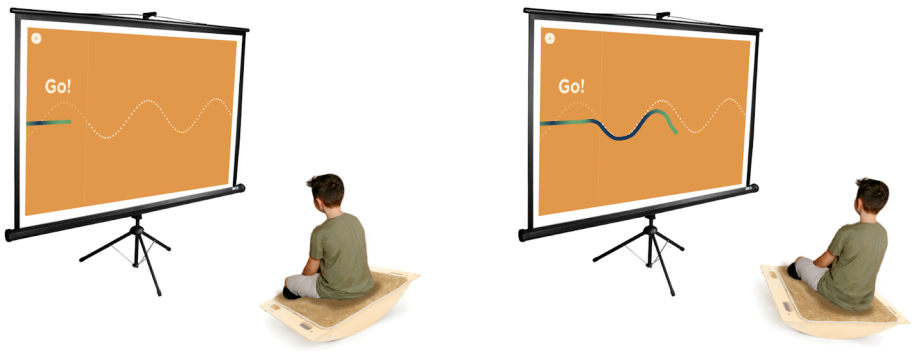


Figure 1: The Balance Board Math system. The child shifts their balance on the balance board, and these shifts dynamically change the height of a graph being generated on a large display in front of them. The display highlights different features of their graph in different activities such as amplitude, function, and frequency explorations, leading the child to explore different parameters of their graphing.

1.1 Background

From different theoretical perspectives, movement in learning contexts is modeled as functioning for cognition and regulation. Embodied cognition theory views the body as playing a central role in cognitive processes. Far from one cohesive theoretical stance, embodied cognition enfolds a range of perspectives from more conservative [2] to more radical (e.g., [3]), spanning a range of foci, notably the "four Es": mind as embodied, enacted, embedded, and/or extended [4]. Embodied perspectives have been fruitfully applied to education [e.g., 5], notably to the role of gesture in thinking and learning [6, 7]. Gesture research in mathematics has found gesture to reflect students' thought processes [8] and even their mathematical expertise: skilled graphers were found to describe graphs from a different perspective than unskilled graphers, immersively "being the graph" with their bodies, not merely "seeing the graph" and tracing it as if on a page [9]. Enactive perspectives conceptualize cognition as emerging from sensorimotor activity: per [10], "(1) perception consists in perceptually guided action and (2) cognitive structures emerge from the recurrent sensorimotor patterns that enable action to be perceptually guided" (p.173). Enactivist views of mathematics analyze mathematical thinking as overtly and covertly embodied, highlighting how even purportedly "abstract" symbols objectify what are in fact multimodal notions (for example, the "+" symbol denotes the process of grouping together), and are manipulated as if they were objects [11]. From an enactivist perspective, sensorimotor activity forms the basis of math thinking and learning.

Separately, theories of sensory modulation and integration highlight the role of movement in self-regulation. Prominent models in occupational therapy assert that individuals require different intensities of sensory stimuli to support optimal functioning [1], with neurodivergent populations more likely to fall at the poles. One individual's felt experience of something might differ from another's due to the sensitivity of their neurological thresholds, yielding different sensory preferences and requiring different amounts of stimulation for self-regulation [1]. Sensation seeking, or self-regulatory movement describes active attempts to meet one's sensory needs through activities such as rocking, pacing, and fidgeting that fall outside of the direct scope of the task at hand [1]. The implications of this model for education are that mismatch between sensory needs and task/environment can adversely affect learning. Research on neurodivergent populations offers some evidence of this: sensory processing differences in sensation seeking and auditory processing explain 47% of academic achievement variance among children on the autism spectrum [12], and among children with ADHD, higher levels of gross motor activity has been correlated with improved working memory performance [13]. From a sensory regulation perspective, sensorimotor activity forms the foundation of self-regulation that supports academic performance.

Bringing together embodied/enactive cognition and sensory regulation perspectives, and with an eye towards the full sensory spectrum, especially those at the most sensory seeking end who are at odds with common educational practices, we ask: If cognition is a sensorimotor activity, and regulation through sensorimotor activity is ongoing, how might these processes interact? Are sensory-regulatory and cognitive processes indeed always distinct, or might

regulatory activity interact with or even participate in cognitive processes? A prime context for exploring movement for regulation and cognition is the vestibular sensory system in the inner ear, which detects balance, orientation, and acceleration. The vestibular sense has been implicated in both self-regulation [14] and cognitive development, including spatial and numerical cognition and arithmetic [15], and has been largely neglected in educational research. In this work, we seek to integrate regulatory and conceptual views of sensorimotor activity in learning and explore their intersections through a design that offers learners vestibular activity with both conceptual and regulatory affordances.

1.2 Related work

Prior work in Human-Computer Interaction has explored movement as a regulatory and learning resource, separately. Work on movement-for-regulation examines embodied self-regulation in the margins of computing space [16] and the effect of physical exertion on enjoyment (e.g., [17]). Self-stimulatory movements such as hand-flapping have been explored as inputs for game locomotion with autistic participants [18]. The sense of balance has been highlighted as a design resource [19], drawing upon balance boards for exercise (e.g., [20, 21]), therapy, rehabilitation [21, 22, 23, 24], co-regulation [25], and recreation [26, 27]. With regards to movement and conceptualization, [28] examined the impact of a a balance-based input device abstract reasoning about questions of justice, applying theory on body-based conceptual metaphors as the foundations for abstract thought [29]. [30] used a balance board input device for seesaw physics problems. We build upon these related works by integrating regulatory and conceptual affordances of embodied, balance-based interactions in the context of math instruction.

Common solutions in schools to accommodate learners' sensory needs offer stimulation outside of, or in parallel to instructional activities: breaks, recess, sensory therapies [31], and/or specialized tools such as fidget toys or alternative seating [32]. We propose that these solutions fall short of their full potential for supporting classroom conceptual learning in that there is limited attention to how completing regulatory activities in parallel might affect the cognitive load [33] imposed on students, or to the regulatory impact of instructional activities themselves. Looking to the sensory qualities of math instructional designs, work inspired by embodied cognition has expanded upon a history of bodily involvement in early education [34, 35] to call upon the body as a learning resource across ages and contexts (e.g., [36, 37, 36, 38, 39]). Recent technologies such as motion sensors have further expanded whole-body interaction possibilities [40, 41]. The types and intensities of sensory stimulation offered in embodied design activities range from small movements such as finger-gestures (e.g., [42]) to immersive whole-body activities such as gym-scale number lines [37]. Very few offer substantive vestibular stimulation. The field does not yet expressly attend to sensory dimensions of these designs as impacting regulation.

Movement-based instructional designs present a spectrum from explicit instruction of specific movements (e.g., [44]) to creative movements (e.g., [45]). The action-based embodied design genre falls between these poles [46]: learners figure out for themselves how to move in a new way given a task and set of resources. An example design is the Mathematics Imagery Trainer for Proportion (MIT-P), wherein learners control the height of two points on a display and try to figure out how to make the display turn green, where green is associated with their two points standing in a target ratio (for example, 1:2). With exploration and practice, users learn to move their hands continuously while maintaining the target ratio, generating a new movement pattern wherein the gap between the two points grows as the points move upwards, a basis for

proportional reasoning [47]. The present project instantiates action-based embodied design with a focus on actions that stimulate the vestibular sense.

2 Balance Board Math

2.1 Design Principles

To integrate regulatory and conceptual views of sensorimotor activity in mathematics learning, we defined the following design principles, described further in subsequent section 2.1.1-2.1.4:

1. Foster movements that ground mathematical concepts in experiences of balance
2. Support learners' discovery and control of dynamical properties
3. Invite embodied self-regulation during and through instructional activities
4. Inclusively adapt to different sensory profiles

2.1.1 Foster Movements that Ground Mathematical Concepts in Experiences of Balance.

Following action-based embodied design [28], we look to ground mathematical concepts by leveraging children's capacity to enact complex movements (see also [46, 47]). BBM aims to invite children to explore graphs through their movement and sense of balance. A balance board input device stimulates the vestibular sense as part of multimodal experience. The child's movement on the balance board is translated into a real-time sinusoidal graph on a large display in front of them so that the child creates a mathematical representation of their board movement. During one round, the graph progresses steadily across the screen in the x axis, and the board's angle controls the graph's y value. Rocking left/back dynamically increases the y value; rocking right/down decreases it. This real-time relationship between the children's whole-body rocking and the digital visualization in front of them creates a dynamic environment within which to investigate whether children can truly embody and, therefore, "be the graph" by rocking its form [9]. The goal is to encourage multimodal exploration of graphs, mapping different aspects of rocking to graphical features.

2.1.2 Support Learners' Discovery and Control of Dynamical Properties.

In action-based embodied design, instructional designs create a context of goal-directed activity within which focal concepts are detectable by children as consistent dynamical properties [43]. This entails inviting children to enact continuous, exploratory movement in an environment where feedback is consistently associated with a property of interest. In the example discussed in section 1.2 of the MIT-P, children are invited to try to "make green," where green color is associated with the invariant property of ratio [43]. This approach shares some foundations with constructivist [48], discovery-based [49], and constructionist pedagogies [50] that view learning as learner-driven exploration.

In BBM, participants receive open-ended prompts, such as "try to draw as much green as possible." In each activity, children figure out how to dynamically conserve a focal property such as the amplitude of their rocking. Conservation of these properties is reflected in consistent color and sound feedback. To make green, the child's movements must generate a graph that aligns with either a focal function, amplitude, or frequency, each of which can be modified (Table 1). For videos of each activity in action, see Appendix A.1. While maintaining the focal parameter, children can vary other aspects, such as varying amplitude while maintaining frequency. After Function Exploration, the other activities introduce a grid and numbers to support children in

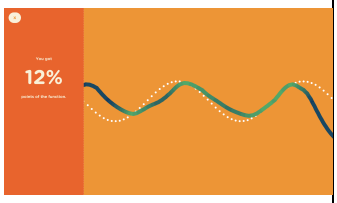
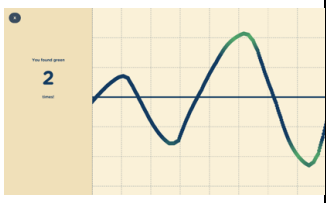
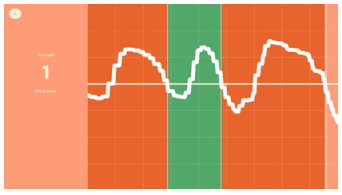
refining and discussing rocking. For example, the gridlines in the Frequency Exploration activity can help evenly subdivide the x-axis of the graph, a prospective support for rocking at a consistent frequency (Table 1). This follows action-based embodied design guidelines whereby cultural tools are introduced as resources that children can appropriate for their situated action [43].

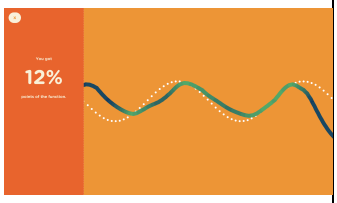
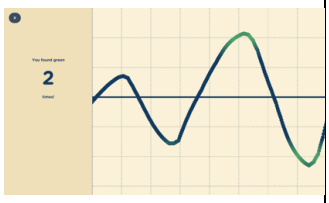
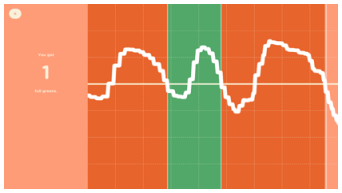
Children are invited to attempt activities multiple times–once reaching the end of the screen, they receive feedback on what percent or number of times they made green and can reflect and try again. Using this feedback, children can generate and test their own hypotheses for how to maximize the target feedback. Drawing inspiration from ethnomethodological conversation analysis perspectives on inclusive instruction [51], we see collaboration as a context for children to find shared ways to make sense of and communicate about their movements together. Collaborative modes in BBM include taking turns on a balance board and 2-board split-screen mode.

2.1.3 Invite Embodied Self-Regulation During and Through Instructional Activities.

Meeting one's sensory needs underlies learners' capacity to self-regulate and attend [14]. Children often do so through moving their bodies, as with fidgeting and rocking [1]. HCI work suggests that embodied self-regulation is ubiquitous in the general population and that self-regulation tools are a fruitful design space in their own right [16]. We are committed to welcoming children's regulatory activity into the learning space. Unlike previous work, BBM invites self-regulation directly into the learning process. Rocking, a common regulatory activity, especially for sensory-seeking learners, directly participates in the learning process because BBM uses rocking as the central form of interaction, in contrast to traditional input devices such as a keyboard, mouse, or touchscreen. The activities are designed to offer the kind of repetitive, stimulating, and self-modulated dynamics prevalent in self-regulatory movements like fidgeting. Additionally, by sitting on a balance board, a common regulatory device, children are implicitly welcomed to rock on the board during and between activities as a means of self-regulation. In this way, rocking serves multiple purposes in BBM: it allows for children to physically embody sinusoidal graphs while also allowing for self-regulation.

Table 1: BBM Base Activities

Activity	Function Exploration	Amplitude Exploration	Frequency Exploration
Visual Output			
Conceptual mapping	A sinusoidal function is experienced as stable and symmetrical rocking.	A function's amplitude is experienced as degree of lean to the left or right	A function's frequency is experienced as the speed of rocking and number of rocks in a given time interval.

Activity	Function Exploration	Amplitude Exploration	Frequency Exploration
Visual Output			
Goal	To trace a given function (white dotted line) by rocking	To rock at a certain amplitude	To rock at a certain frequency
Color feedback	A blue-green line color gradient corresponds with the user's accuracy or inaccuracy in tracing the predefined function.	The parts of the user's function that meet the target maximum and minimum turn green, while the rest of the function remains blue.	A green-red background color gradient for each period corresponds with how closely that period matches the expected rocking frequency.
Numerical feedback	Percent match between their graph and the function sin(x).	Number of times reaching the target amplitude.	Number of times matching the target frequency.
Sound feedback	Presence/ absence of a harmonic tone corresponds with the user's accuracy in tracing the predefined function.	Changes in pitch correspond with changes in the user's tilt. A "counter" noise plays each time the user meets the target maximum or minimum.	Different tones play for each local maximum, minimum, and passage through the x-axis. A chord plays each time the user completes a period, which is progressively more major and less minor as the user's period matches that of the target frequency.

2.1.4 Inclusively Adapt to Different Sensory Profiles

BBM follows inclusive design [52], seeking to expand the spectrum of sensory modulation profiles accommodated by instructional designs. We offer amplified stimulation opportunities for *sensory-seeking* students who crave movement. At the same time, we aim to design learning contexts where students across the sensory spectrum can tune to their preferences. To do so, the activities are programmed with adjustable graphing speed and sensitivity so that participants can rock in ways commensurate with their sensory profile. BBM also offers feedback through vision and sound. Additionally, BBM allows flexibility of body position: children may plant their feet on the floor for a more controlled rock, or sit with their legs crossed for more range, may rock left-to-right or front-to-back, and may hold the board's rim or interior handles.

2.2 Technical Implementation

Children interact with BBM primarily via a wooden balance board. We use Southpaw's Rocker Balance Board (28" L x 25" W x 7.5" H), which is made of wood and has a carpeted mat on top (Figure 2). The large surface and gently curved base allow children to rock on the board with stability and comfort. We attached foam balls to the board's sides to prevent learners from injuring their fingers and cushion extreme rocking. A wooden compartment affixed to the bottom of the balance board houses a WitMotion Inclinometer sensor to sense board angle and an Arduino Mega

2560 to send sensor data to a custom BBM web application via USB-connection. The BBM web application is projected in front of the user.

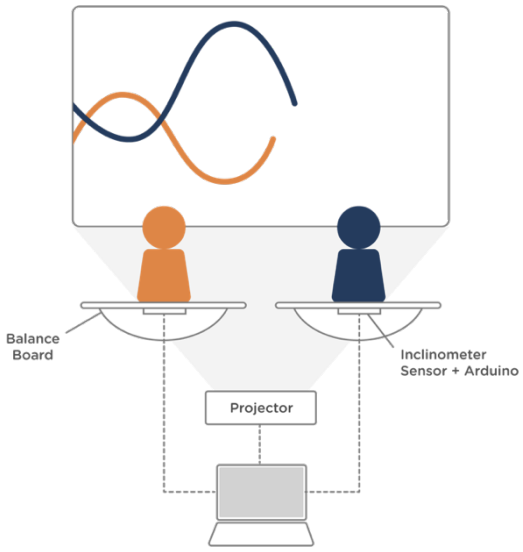
3 Study

3.1 Research Questions

We evaluated whether children's interactions with BBM fulfilled its core design principles by investigating the following research questions:

1. How do BBM's unique interaction design features facilitate children's whole-body embodied explorations of functions and graphs?
2. How do children engage in discovery-based learning of math concepts when using BBM?
3. How do children engage in embodied self-regulation when they are invited into the BBM learning space?
4. How do children's different sensory modulation profiles interact with the BBM platform?

Figure 2
BBM System Diagram



3.2 Participants

Children in kindergarten through grade 12 were eligible for this study. 6 participants in grades 2 through 6, two girls and four boys, participated. Interviews were conducted in a lab setting and at a learning center. In the lab setting, only the participants, the interviewers, and the participants' guardians were present. The interviews at the learning center took place in a partitioned cubicle within a larger, multipurpose indoor space. Following Covid-19 guidelines at the time of data collection, masks were worn by all present. Two participated individually (pseudonyms: Maria and Ben), and four participated in pairs (pseudonyms: Kyle and Cory, Lyla and Arthur). Four participants stayed for post interviews. We ran participants until at least 3 students identified as sensory-seeking and 3 not were included. Learning center instructors with training in sensory profiles identified three participants as sensory seeking (Maria, Ben, and Arthur). We asked parents and students informal sensory questions to corroborate that the other three participants did

not exhibit sensory-seeking tendencies. Participants' special education status was not collected in this study.

3.3 Methods

We conducted task-based, semi-structured interviews centered around the different Balance Board Math activities. The interview began with a calibration phase where participants were invited to rock on the board. Children were invited to rock fast and slow, big and small, and to try out different positions of their bodies and the boards until they felt comfortable. Next, for each activity, the on-screen prompt such as "see how much green you can find on the screen" was stated aloud, and children were told that they could try each activity as many times as they liked. Between tries while seated on the balance board, when the screen stopped at the graph from their previous attempt, the children were asked questions about their thinking such as "what do you think makes green?". In later tries, children were also asked to reflect and perhaps iterate on their brainstormed solutions. Children in pairs were also encouraged to take turns on the board and discuss their ideas with each other. These sessions were analyzed as a dyad in interaction. In each session, participants tried out the Function Exploration, Amplitude Exploration, and Frequency Exploration activities, except for Cory, who did not try the Frequency Exploration activity. All participants started out with Function Exploration, differing in whether they did Amplitude Exploration or Frequency Exploration next. Each session lasted between 14 and 57 minutes with an average length of 28 minutes. The interviews were audio and video recorded when approved by children's guardians and children (five of six participants), and the research team took field notes during interviews. In the lab, one video camera was set up behind the participants to capture the digital output on the screen as well as participants' backs, and one in front of participants to capture their gestures and facial expressions. For sessions in the learning center, only the front camera was functional. All recorded interviews were transcribed.

Before and after participants engaged with the platform, we conducted structured pre and post interviews at a desk in which children were invited to describe three sinusoidal graphs printed on paper using only gesture, then only language. We used questions like "Can you show my friend who can't see it what the graph is like using your hands?" and "how would you describe it to her in words?" We adapted this approach from prior work on children's conceptions of graphing that prompted children to gesture visual graphs [9], and added the secondary verbal description to accommodate kids' desire to verbalize. When children combined both gesture and words, we asked them to repeat only in gesture, then only in words. Pre and post graphs featured one reference function (sin(x) or cos(x)), one higher amplitude function, and one higher frequency function (Appendix A.2). Participants spent between 14 and 49 seconds on each pre interview graph with the average length of 33 seconds per graph. Participants spent between 26 and 71 seconds on each post interview graph with the average length of 38 seconds per graph. All but one of the participants were seated for pre and post interviews. One participant, Ben, preferred to remain on the balance board for his post interview. Children who participated as dyads were split up and interviewed separately.

Video recordings of children's completion of and reflections on the activities were iteratively coded for instances of different movement functions such as *activity-related, regulatory, expressive,* and *cognitive* using the ELAN annotation software. Following fidgeting literature, movements that exhibited no direct salience to the activity at hand, such as tapping feet while waiting, were coded as regulatory. Each movement sequence could receive multiple codes. We compared types of board engagement among participants, and between sensory-seeking and

non-sensory-seeking subgroups. Interview transcripts were coded for children's perceptual orientation towards the task using inductive coding. Video recordings of children's pre and post graph interviews were coded deductively using 6 hallmark "being-the-graph" criteria from Susan Gerofsky's 2010 paper (Table 2), which we then compiled into one index variable to compare pre to post. Coding consensus was reached using two videos, then two coders coded all videos. Inter-rater agreement was 86.4%. Videos were blinded for pre/post order to mitigate potential biases. Being-the-graph pre and post index scores were square-transformed to meet the normal distribution assumption, checked with the Shapiro-Wilk test. An F-Test Two-Sample for Variances found no significant difference in variances between pre and post scores, so a paired, two-tailed t-test at the 5% level assuming equal variances was used to compare the mean being-the-graph index pre and post intervention.

Table 2
"Being-the-graph" codes for pre/post graph gestures

Category [53]	Range	Description
being-the-graph index	0-100	An equally weighted average was taken for all 7 "being-the-graph" variables below and converted to a percentage.
placement of the x-axis against the gesture's body	0-1	X-axis placement was binary coded with "throat and above" marked as 0 and "below throat" marked as 1.
acceleration/ deceleration in gestural movement	0-2	Gestural movement acceleration was coded as 0 (no acceleration/deceleration), 1 (some acceleration/deceleration), or 2 (significant acceleration/deceleration).
presence or absence of eye tracking	0-2	Eye tracking was coded as absent (0), partial (1), or consistent (2).
engagement of the spine in movement	0-2	Spinal engagement was coded as absent (0), partial (1), or significant (2).
distal or proximal nature of the gesture	1-3	Gestures were coded as (1) further than elbow distance, (2) within elbow distance from the body, and (3) close to the body (less than elbow distance).
hand usage	0-1	Hand usage was coded 0 if the participant traced with a single finger and 1 if they used their full hand.
parts of the body used to create the gesture	0-8	Each body part type involved in creating the graph's shape counted as 1 point from the following list: hands, wrists, elbows, shoulders, torso/spine, head, hips, legs.

3.4 Results

3.4.1 Grounding Mathematical Concepts in Experiences of Balance: Towards Being-the-Graph

In BBM, to achieve desired graphical outcomes, children must coordinate rocking the board with the resulting dynamic graph. We observed that children controlled and transformed their rocking to achieve desired graphical effects. In talking about the activities, all children spoke explicitly about features of their rocking, their produced graphs, and display feedback, frequently making connections between them using deictic gestures (Figure 4). For example, Kyle described the result of a Frequency Exploration round as follows: "I went a lot slower and I only have like three [...] hills, like cycles." Here, he connects his felt experience of rocking *slower* with the *frequency* of his generated graph. Over the course of the activities, children gradually developed more coordinated ways of interacting with and talking about their bodily movement and graphical features.

Figure 3

Children pointing to features of interest in the graphs they have generated

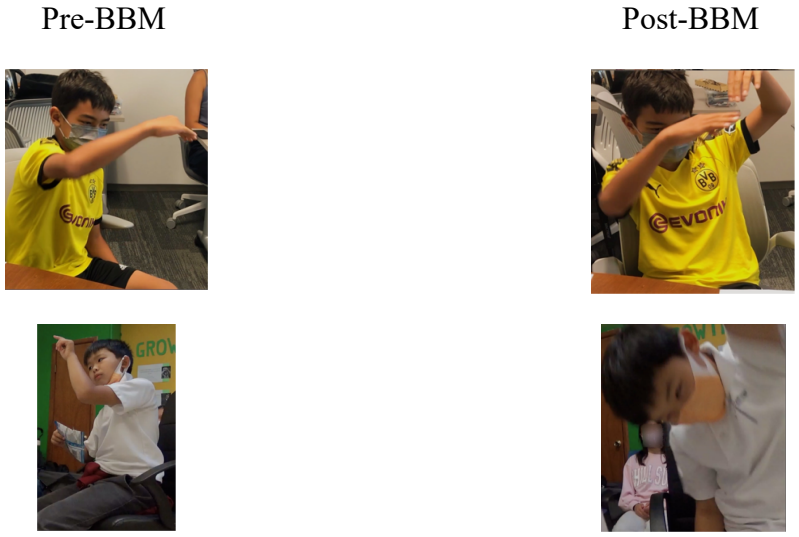


Through coordination of board and display, the children came to experience the graphs themselves multimodally. Their explanations of how to make *green* often came to combine both board-oriented notions such as *speed of rocking* or *angle*, and graph-oriented notions such as *period* or *y-axis location*. For example, Lyla described her Amplitude Exploration strategy as: "try to find the greens but keep the balance." Here, she coordinates the display's color feedback with her embodied experience of rocking evenly. Her solution indicates that she has integrated her personal sensorimotor experiences of rocking evenly in each direction with the graphical form of a sinusoid with a given amplitude. She enters into the concepts multimodally, drawing upon her active sensorimotor experiences. Our findings suggest that by bringing together their vestibular and proprioceptive experiences with the board and the graphical display, children came to draw upon their bodily experience to think about mathematical properties of graphs.

We found that actively "being the graph" on the board transferred to how children made sense of static, printed graphs afterwards. For example, Kyle, who had connected his rocking speed with graph frequency earlier during the Frequency Exploration activity, described pre-given graphs on paper using movement-based language: a lower frequency graph was "slower" and a higher frequency one "quicker;" a high amplitude graph was "rocking a lot farther." As he spoke, Kyle physically rocked his body in his chair in ways that mirrored his descriptions. This suggests that the coordination developed between rocking and features of graphs became a resource for sense-making of static 2-dimensional representations. Each child showed different idiosyncratic changes from pre to post. For example, Kyle, who initially gestured with only one hand from right

to left in his pre interview, gestured with two hands from left to right after the intervention (Figure 4, upper). Kyle's gesture direction changed to follow mathematical convention of reading graphical data left to right. His two-handed gesture distinguished motion above the x-axis, gestured with one hand, from motion below the x-axis, gestured with the other, reflecting a newfound attention to vertical symmetry. Ben went from raising his hand for high amplitude graphs to standing up and reaching skywards (Figure 4, lower). The scale of Ben's graph expanded after participation, making more use of his whole body. Ben's expression of high amplitude became more prominent, and the scale of his gestural graph expanded. Despite the variety of graphing gestures, looking across children, there was a significant increase in mean being-the-graph gesture index scores from pre to post ($t = 2.37$, $d.f.=11$, $p < 0.05$). This suggests that overall, children's gestures shifted from *seeing* towards ***being*** the graph, consistent with the gestures of more expert graphers [9].

Figure 4
Examples of two children gesturally depicting graphs pre (left) and post (right) participation in the BBM activities.



Note: Kyle (upper) moved from gesturing with one hand (pre) to producing larger gestures with two hands (post). Ben (lower) moved from a small deictic gesture to a large movement involving his entire body.

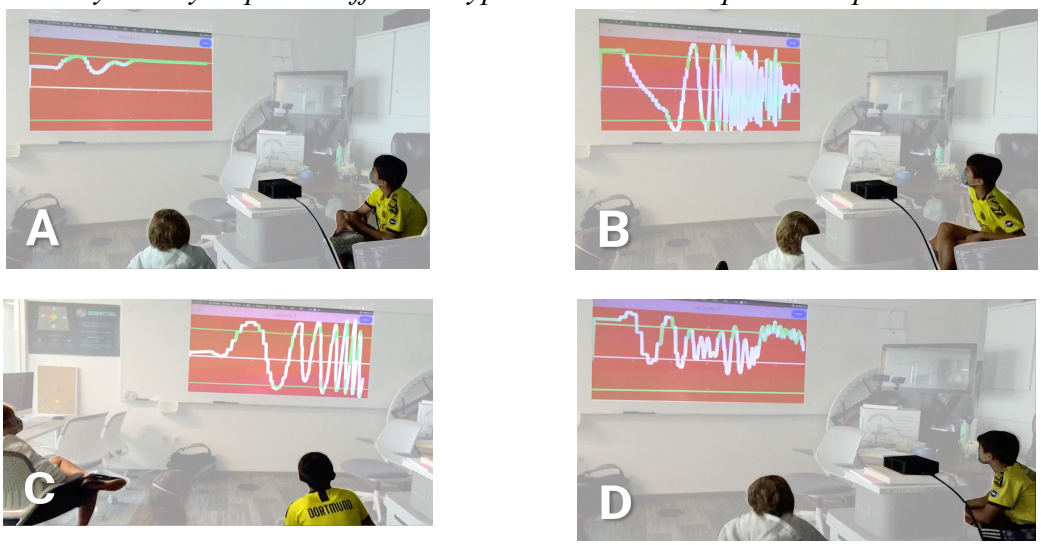
3.4.2 Discovery of Focal Concepts through Dynamical Conservation.

BBM highlights aspects of graphs with color and quantitative information. Children took up the display's color and quantitative feedback to formulate, test, reject, and refine their hypotheses about each activity. All children connected observations about their graph with reflections on why certain parts of their graphs were *green*. Kyle and Cory's initial engagement of Amplitude Exploration exemplifies children's open-ended progression of reasoning within activities. First, Cory explores the activity by leaning to one side without returning to the x axis (Figure 5a). He discovers that his graph stays flat and green. However, because Cory's graph never completes a full period, BBM displays, "you met the amplitude 0 times." Cory comments at this stage, "If you stay on the line, it's all green. I don't know if it is the *amplitude*." While he has identified the y=3 reference line, he notices that simply making green in this way does not increase amplitude count

Cory then proposes that "you have to *go back and forth between lines*" which he and Kyle decide to test out with another round. This time, Cory rocks from y=3 down to y=-3 (Figure 5b). Cory confirms from this that "every time I hit the bottom it turned green, and every time I hit the top it turned green;" "it makes green if you go straight *from line to line*." This time he reaches the amplitude 32 times. He proposes that the count reflects "it go[ing] *to the top and bottom* as many times as possible." Kyle then takes a turn (Figure 5c), which sparks a new idea about getting a greater number of greens by going "faster with less movement." This idea begins to highlight graphical frequency ("faster") and amplitude ("less movement") (tested in Figure 5d). Observing the resulting graph, Kyle offers that "I feel like it's kinda just the same thing except it's smaller." Here, Kyle is coming to recognize similarity in the overall shape of the graph (still roughly sinusoidal) even as the amplitude differs ("smaller"). Through exploration, Kyle and Cory progressively differentiate features such as amplitude and frequency that draw them closer to math disciplinary practice.

Figure 5

Kyle and Cory Jointly Explore Different Hypotheses in the Amplitude Exploration Activity



After further explorations, Kyle and Cory analyze another round (Figure 6), blocking the projector light with their fingers to point out, "every peak when you go up and down, there are 13. Every spike going up and down." At this stage, Kyle and Cory have identified the vertically symmetrical nature of amplitude and have tuned into each local maximum and minimum. In this sequence, Kyle and Cory generated hypotheses from analyzing color and numerical feedback, informing each attempt with new ideas to explore, and arriving at a way of seeing that more closely reflects math disciplinary practice. BBM's open-ended prompts and visual feedback on the children's personally generated graphs supported their discovery-based math conceptual learning.

Figure 6

Kyle and Cory highlight features of a graph by blocking the projector light with their finger



3.4.3 Embodied Self-Regulation During and Through Instructional Activities

Unlike a desk chair, BBM's balance board celebrates the child's own sense of bodily balance and movements by making readily available means of vestibular stimulation, a common aspect of sensory self-regulation [14]. The experience of self-guided vestibular stimulation sparked joy for sensory-seeking 2nd-grader Maria, who commented: "I wish I had one of these [balance boards] at home!" Consistent with a sensory regulation perspective (e.g., [1]), most participants also rocked on the board in between BBM activities in ways that serve a regulatory function. These instances occurred most frequently during conversation, when peers or interviewers were speaking, during quiet reflection, and sometimes when the child him or herself was speaking. During activities like listening that entail less inherent vestibular stimulation, the children were able to seek sensory stimulation with the board to support their engagement. Sensory seeking participants engaged in a higher frequency and intensity of regulatory rocking both during and between activities, suggesting that BBM indeed offers a learning space where vestibular self-regulation is always implicitly welcomed as an accessible means to seek regulatory vestibular stimulation within and beyond the activities.

In the BBM setting where rocking is part of both activities and background regulatory activity, we found that these two processes sometimes converged. Participants' regulatory rocking was influenced by and fluidly blended with their thinking and talking about the learning activities. For example, Kyle switched from using the board in the front/back configuration (leaning forwards and backwards) to using it in the left/right configuration (leaning left to right). Prior to the switch, when sitting in a swivel chair, his regulatory rocking was consistently a rhythmic front/back rocking. However, after the switch, Kyle's regulatory movement in the chair switched to twisting left to right. In another instance, Kyle rocks side to side, apparently for regulation. As the interviewer comments, "you mentioned earlier something about the distance between each hill," Kyle's rocking suddenly shifts to slow, sweeping rocks, corresponding to how he had earlier created spaced out features on the graph. Kyle's bodily movements reacted to the interviewer's speech before he had yet verbalized a response. This moment is consistent with embodied cognition literature suggesting that language comprehension is a spatial, sensorimotor process [54] and work on gesture as part of conceptualization [55]. Kyle acted out the conjecture raised by his interlocutor. Movement-for-regulation fluidly transformed into movement-as-thinking, intertwining and interacting rather than unfolding as separate processes.

3.5 Accommodating Diverse Sensory Modulation Profiles

The children exhibited a range of sensory preferences with regards to rocking on the board, which BBM accommodated through custom speed and sensitivity settings and the multiple solutions possible within each activity. For example, Lyla rocked gently with limited amplitude (Figure 8, left), whereas sensory seeking Arthur and Ben rocked intensely, using the board's full rocking range (Figure 8, right). Other participants like Cory, Maria, and Kyle varied their intensity across different activities. Participants also chose to position their feet differently, with participants who preferred gentle rocking tending to keep their feet grounded on the floor, and participants who preferred intense rocking tending to pull their legs onto the board, crossing their legs or kneeling. BBM's flexibility enabled all participants with diverse sensory modulation profiles to successfully explore all activities and "find green." Lyla and Ben, a pair of participants with opposite sensory preferences, were able to communicate effectively with each other about their hypotheses and enact learnings from observing each other's activity. As predicted, different sensory profiles led to different engagement styles, all of which led to successful interactions with the platform.



Figure 7: Examples of two participants' different rocking styles. Left: Lyla rocks gently with feet planted on the floor. Right: Arthur rocks heavily with legs crossed on the board.

4. Discussion

Our findings enter into dialogue with embodied learning and sensory regulation literatures and chart novel research directions at their intersection. With regards to embodied learning, we found that children were able to form new mathematical ideas through the achievement and analysis of dynamical consistencies in their movements in an action-based embodied design environment. These findings brought children closer to mathematical professional vision [56] in that they came to attend to graphs in ways that reflect disciplinary practice, such as identifying one full cycle above and below the midline (mathematical period) as a meaningful unit and attending to the amplitude of sinusoidal graphs. BBM corroborates principles of embodied design, demonstrating that they obtain effectively in a new content area (see also [58]).

Prior work in embodied mathematical cognition has highlighted that experts view static graphs dynamically. From an enactivist pedagogical perspective, "one objective of education is to enculturate students into understanding static images as offering opportunities for action" (p.7) [11]. This study sheds light on one pathway towards such enculturation: we found that dynamic, whole-body graphing experiences led even static, 2d images of graphs to evoke dynamic experiences for children, reflected in a greater degree of "being-the-graph" in their gestural expressions. By interacting with designed pedagogical tools, the children changed their perception [57]; the participants came to draw upon their own sense of balance and movement to think about the mathematical meanings of graphs' abstract features.

That children used their balance experiences to conceptualize is particularly novel, as vestibular input has not historically been the explicit basis for mathematics instruction. Whereas vestibular

research has implicated the system in skills affecting math such as spatial reasoning, and conceptual metaphor work has suggested a role for balance experiences in abstract reasoning, this study explicitly introduces mathematical concepts through vestibular experiences. That students can draw upon these experiences to perceive and think about math concepts suggests the epistemological viability of different perceptual modalities in mathematics, offering means to challenge the discipline's oculocentric history that has marginalized learners with disabilities [51, 59]. BBM offers a proof of concept for engaging senses traditionally muted in classrooms, particularly the sense of balance, in mathematical learning.

With regards to sensory regulation literature, our findings are consistent with models of sensory modulation that suggest that children are tuned to require different stimulation intensities [1]. As expected, children identified as sensory seeking exhibited more intense and frequent movements. These movements occurred not just in the background of activities. Also as expected, children showed spontaneous rocking activity during tasks that were not as sensorially stimulating, such as listening or waiting. Our findings extrapolate beyond sensory regulation models: firstly, we found that when offered the chance to select the sensory intensity of a learning task, children differentiated according to their sensory profile. Second, we found that participants with opposite sensory preferences were able to communicate effectively with each other about their activity despite notable differences in the actual movements they enacted.

This study offers some preliminary glimpses at how sensory regulation and embodied learning theory might interact. Contrary to prevailing research assumptions that these two processes are separate, we found instances where activity traditionally interpreted as sensory regulatory behavior was dynamically taken up as a structure to think with, influenced by the discussion of different proto-mathematical ideas, and where activity movement affected ensuing regulatory movements. Our findings suggest that movement for exploration and for regulation may affect each other. At minimum, we identify opportunities to take up regulatory forms such as fidgeting as resources for richer, more integrative learning experiences within which these actions take on new meanings. More boldly, we suggest that background activity might be thought of as part of ongoing cognitive activity, participating ongoingly, if generally covertly, in embodied cognition.

4.1 Limitations and Future work

The present study is limited by its small sample size. We plan to run more participants in future studies and investigate two-board, collaborative interactions. We are also working with classroom teachers to design classroom interaction models and activities for further concepts such as function addition, and plan to use written pre/post math assessments to assess learning outcomes. We are still working to expand BBM's accessibility through sound/sonification and visual design to improve accessibility for colorblind and visually impaired participants. We hope to investigate inclusive learning by working with and across participants with different sensory profiles. Methodologically, we noted that having children sit during their pre/post interviews prospectively limited the range and bodily engagement possible for them and will conduct pre/post interviews in a standing position for future sessions. The informal identification of sensory profiles in this study is not highly replicable, so we plan to collect more structured sensory profiles using questionnaires, as well as continuous regulation data (EDA).

Conclusion

We presented the design and evaluation of Balance Board Math, which offers children integrated opportunities to self-regulate through movement and reach conceptual understanding using their sense of balance. Our study showed that BBM fostered exploratory movements offering a new entry point into mathematical practice of graphing. BBM activities impacted how children made sense of graphs, eliciting an increase in markers of ***being**-the-graph* as opposed to ***seeing**-the-graph*. BBM celebrates using movement to explore activities, to support self-regulation, to communicate ideas, to reflect, and to comprehend, with these different forms and functions of rocking fluidly combining and intertwining. Our findings suggest that sensory self-regulation and cognition interact and thus, should not be treated as entirely distinct aspects of activity. BBM's flexible balance board interface accommodating a broad range of physical movements combined with a dynamic display of sinusoidal graph visualizations invited children with different sensory preferences to interact naturally and playfully with the activities, suggesting that sensory-flexible instructional resources can support expanded instructional accessibility. We offer BBM as one such promising technology as an instructional and research tool.

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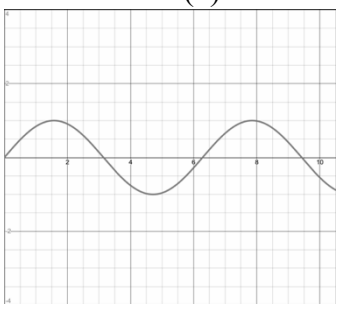
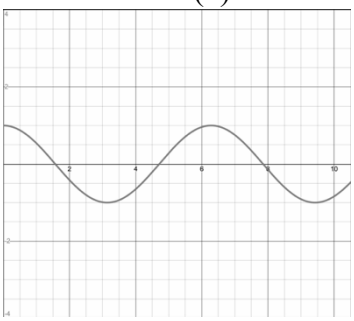
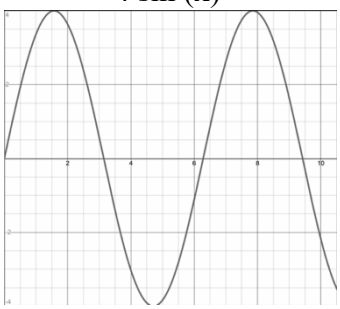
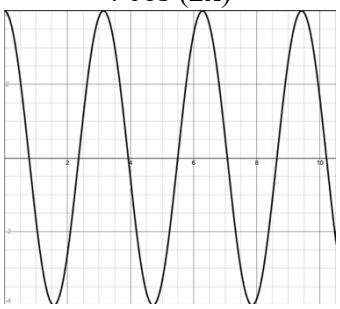
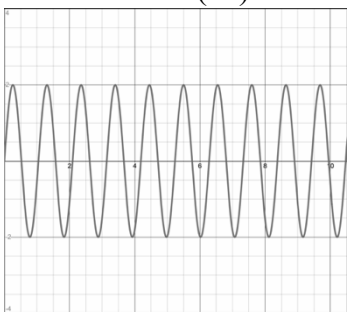
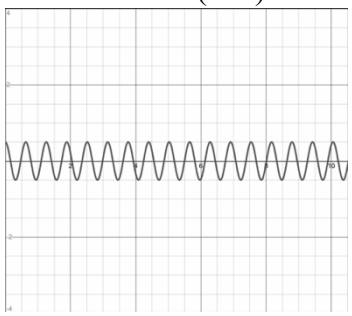
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A Chapter Appendices

A.1 Demonstration Videos of Exploration Activities

See IDC published version of this paper for demo videos of activities.

	Pre	Post
Basic sinusoid	sin(x) 	cos(x) 
High-amplitude sinusoid	4 sin (x) 	4 cos (2x) 
High-frequency sinusoid	2 sin (6x) 	½ cos (10x) 

Graph Description Prompts

Balance Board Math: Exploring the Sense of Balance as a Basis for Functions and Graphing and Number Line Concepts

In this paper, I explore the question: can math concepts be experienced through the sensory modality of balance? I highlight BBM as a set of pedagogical math activities designed to instantiate mathematical concepts through stimulation to the vestibular sense, an organ in the inner ear that detects our bodily balance and orientation. I describe how BBM establishes the different ways children spontaneously rock and move as the basis for inclusively exploring mathematical concepts together across diverse sensory profiles. In particular, I describe two activity sets where students explore focal concepts by shifting their balance on rockable balance boards: "the Balance Number Line," using analog materials to foster understandings of the number line and negative numbers; and "Balance Graphing," using sensors and a digital display to foster exploration of functions and graphing concepts, including the parameters of trigonometric functions and function addition. I outline proposed ways that engaging with concepts through balance-activating movement can change learners' mathematical thinking and learning.

1. Introduction

You've seen sinusoids, but have you felt one? The concept of balance gets evoked regularly in mathematics to teach concepts such as balancing equations, yet rarely do we engage learners' bodily sense of balance in the classroom. What if we learned concepts not just through seeing diagrams, touching manipulatives, or hearing language, but also through experiencing different balance sensations? How might our mathematical thinking change? What new avenues for inclusive learning might balance-based thinking open up for learners with diverse sensory experiences and needs?

Balance Board Math (BBM) explores the vestibular (balance and orientation) sensory system as a basis for mathematical conceptualization within several conceptual domains. For example, learners use rockable wooden boards to experience the frequency of a sinusoidal function as a rate of rocking their bodies (Fig.1), or a negative number as the counterbalance to its positive counterpart that brings their body back into balance. This snapshot describes how using the sense of balance to explore mathematical concepts, as enabled by digital technology, can prospectively change how learners enter into and think about mathematical concepts. BBM is inspired by the intersection of two design problems: one sensory, and one cognitive.

Figure 1
A Pair of Learners Exploring Balance Board Math Activities



1.1 Sensory Accessibility

People, especially children, often spontaneously fidget and move in ways that stimulate their balance sensory system: rocking, jumping, pacing, and twisting. Research on sensory processing and integration suggests that such movements are part of *sensory regulation:* seeking out optimal levels of sensory stimulation to control one's affect and arousal levels (e.g., Dahl Reeves, 2001). We crave different intensities of stimulation based on their neurology, with many neurodivergent individuals, including those on the autism spectrum and those with ADHD, especially likely to fall at extreme ends of the sensation-seeking spectrum (Dunn, 1997). For example, some people may need a high level of stimulation to the vestibular sense of balance and pursue this by rocking, bouncing, or pacing. Even if learners' sensory stimulation needs are identified and accommodated, these tend to only be addressed through offering learners peripheral sensory tools such as fidget toys. Pedagogical activities themselves often do not offer the sensory stimulation people crave, and may even be incompatible with people's own spontaneous movements. *What if instead, we made people's spontaneous movements such as rocking their bodies part of mathematics instruction itself?* BBM seeks to offer learners differentiated vestibular stimulation as fundamentally integrated into instructional activities. In so doing, the motivation is to render math instruction more accessible and engaging to all learners, especially those with higher needs for vestibular stimulation not met by current practice. Furthermore, the sense of balance offers an untapped shared resource across learners with different sensory profiles from each other, such as blind and sighted learners, to engage in learning experiences together.

1.2 Embodied Cognition and Instructional Design

In recent decades, a novel paradigm in cognitive sciences has emerged that highlights the centrality of bodily experience to cognition: *embodied cognition*. In contrast to computational models of cognition that characterize thinking as an amodal process of input, processing, and output, embodied cognition suggests that cognition evolved for and arises through multimodal experiences through our bodies with the environments. This perspective sheds new light on mathematical thinking and knowing, highlighting the multimodal nature of concepts, wherein

even apparently abstract processes, such as solving algebraic equations, make use of our sensorimotor experiences of manipulating objects in the world (i.e., Hutto et al., 2015). Empirical work in education inspired by this perspective has found bodily activity to influence and express thought, from gesture to background movements (for a summary, see, for example, Shapiro & Stolz, 2019). One particular strand of embodied cognition, enactivism, highlights how cognitive structures emerge from patterns of perceptually guided action in the world (Varela et al., 1991). *Action-based embodied design* concretizes this theoretical perspective, seeking to foster new perceptuomotor coordinations as proto-instantiations of mathematical concepts (Abrahamson, 2014; Alberto et al., 2021). In this view, teaching can be made more effective by explicitly cultivating sensorimotor activity as the core of conceptual thinking.

1.3 The Unsung Sense of Balance

The notion of balance has long been leveraged in mathematics to support reasoning about mathematical equations (i.e., MacGregor, 1991). Furthermore, research on the bodily sense of balance (the *vestibular system*) reveals its importance to cognitive development, spatial reasoning, and numeracy (e.g., Mast et al., 2014). Despite the vestibular sense's importance to foundational math competencies, it is not intentionally stimulated by extant math pedagogical designs. Could explicitly engaging this sense support math skills and conceptual learning?

To date, balance-based conceptualizations have been explored in the context of abstract reasoning about social justice issues, wherein it was found that using a balance-driven input device rather than a handheld one to make comparisons led participants to be more deeply affected by injustice (Antle et al., 2013). Balance-based input devices have also been explored in physics learning about seesaws (Pouw et al., 2016). BBM represents a first foray into vestibular-driven conceptualizations of mathematical concepts. BBM seeks to pose sensorimotor challenges wherein learners figure out new patterns of balance and disbalance, drawing upon mathematical tools such as graphical representations and number lines to refine their control and explanation of these patterns.

2. The BBM Design

BBM is driven by the conjecture that integrating sensory-regulatory and cognitive affordances of movement can render conceptual learning more accessible. The design principles guiding BBM (Tancredi, Wang, et al., 2022) are as follows:

1. **Foster movements that ground mathematical concepts in experiences of balance.** This entails the congruence of balance-activating movements with focal mathematical concepts.
2. **Support learners' discovery and control of dynamical properties.** Learners discover feedback from the digital platform and/or their sensory experiences that guides them to explore, refine, and think about their movements.
3. **Invite embodied self-regulation during and through instructional activities.** Learners are welcomed to move their bodies in ways commonly used for sensory regulation.
4. **Inclusively adapt to different sensory profiles.** The design is built to enable learners to rock at any intensity to accommodate a wide range of sensory stimulation preferences.

For more detail on the design and rationale of BBM, see (Tancredi, Wang et al., 2022).

The aim of the present snapshot is to illustrate how balance-based approaches create opportunities for new ways of conceptualizing math, drawing upon brief examples from multiple studies wherein BBM has been tested with hundreds of children and adults in semi-structured

interviews, classrooms, and informal learning settings. These include a Balance Number Line case study with a middle schooler on the autism spectrum (Tancredi, 2020), semi-structured interviews with elementary and middle schoolers with a range of sensory profiles (Tancredi, Wang et al., 2022), and a classroom study with four high school classes using the Function Addition activity (Tancredi et al., 2023). Sensation-seeking neurodivergent learners, including children on the autism spectrum, have been focal participants throughout the project to examine how the design interacts with their sensory needs and practices.

To illustrate the pedagogical potential of balance-driven math experiences, I will describe two BBM configurations and some mathematical explorations possible with each: the Balance Number Line, and Balance Graphing. Both of these configurations by default use a balance board with a flat surface and semicircular prism base, rocked from a seated position (i.e., Fig.1), (see Appendix 2 for a discussion of other prospective balance board experiences). The board is rocked left-to-right for Balance Number Line, and front-to-back or left-to-right for Balance Graphing.

2.1 The Balance Number Line

A first set of BBM activities, the Balance Number Line (Table 1), establishes balance-based conceptualizations of absolute value and negative numbers. Learners sit on a balance board and slide their hands along a number line in front of them (Fig. 2). 0 on the number line is aligned with the middle of the balance board and the child's body. When one moves their hands, the shift in weight yields a resulting tilt in the balance board. For example, moving one's hand to -2 results in a slight lean to the left, whereas 5 results in a large lean to the right. This also applies when the two hands are placed at different points on the number line: -3 and 3 are experienced as being in balance, whereas -3 and 4 yield a slight lean to the right. Black magnetic sheet coats the upper half of the board to allow for affixing magnetic arrows that can be used to keep track of a sequence of movements.

Figure 2

The Balance Number Line

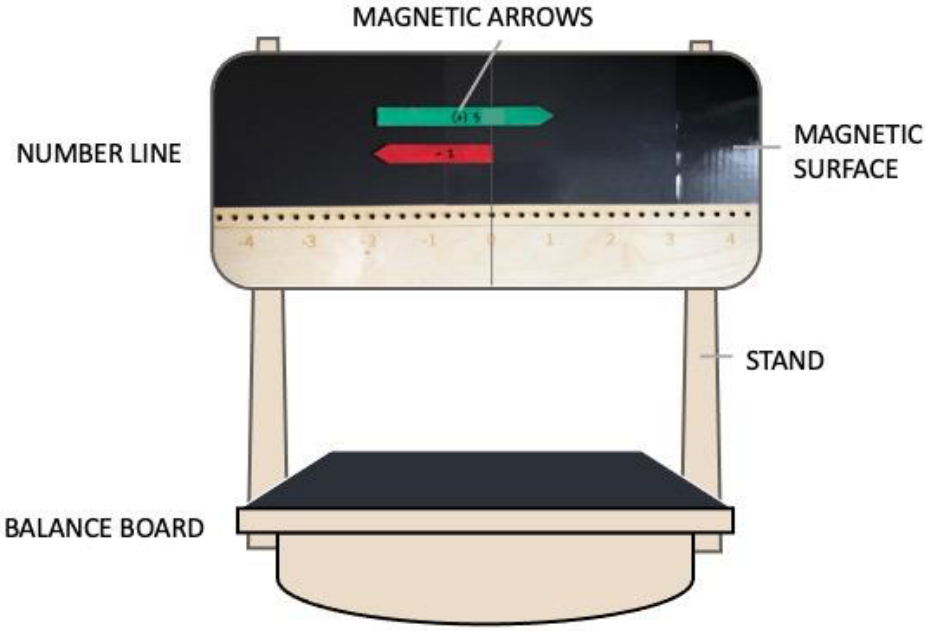


Table 1
Balance Number Line Activities

Activity	*Purpose*	*Balance Experience*
Move-in-balance	Provide a balance-based definition of negative numbers.	Learners experience positive numbers and their negative counterparts as perfectly balancing one another on a continuous spectrum by moving their hands symmetrically outwards from 0.
Expressions and equations	Compare different equations and their solutions, differentiating between sign and operation.	Learners experience arithmetic as shifting states of balance elicited by sliding their hands along a number line, and solutions as a specific final angle of tilt.

2.2 Balance Graphing

A second set of BBM activities, Balance Graphing (BG), establishes balance-based conceptualizations of functions and graphing concepts. BG joins other dynamic graphing activities (Duijzer et al., 2019) that use sensors to enable learners to generate their own graphical representations in real-time. In BG, users create graphs on a digital display by rocking a balance board (Fig. 3). The display varies by activity, always offering graphical representations of the board angle. The board's angle, detected by an inclinometer sensor affixed to the board, is relayed via Arduino to a computer program and graphed (Fig. 3). After a graph is drawn, it remains visible on the screen until a new graphing round is initiated.

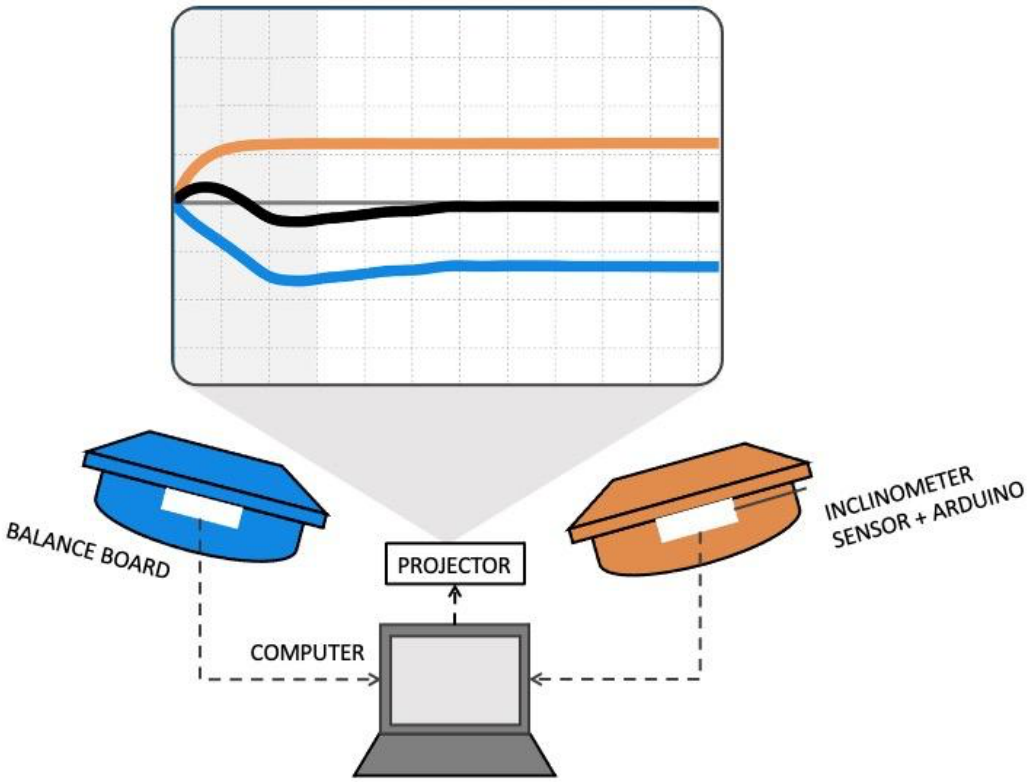
BG activities (Table 2) annotate or augment user's graphs with feedback to foreground different mathematical properties. This includes real-time feedback from the digital display as they are graphing, and summary feedback at the end of each graphing round. Feedback information is conveyed through visual features and sound (*sonification*). For example, in one activity, Function Addition (described further in section 4.3), the movement of each board generates one of the colored graphs, and the sum of these two functions is displayed in black. Fig. 3 shows one such graph. Here, the left balance board (blue) tilts to the right and the right board (orange) tilts the same degree left. Holding these positions, they trace lines at $y = -1$ and $y = 1$, respectively. A black line shows the sum of these two graphs, here roughly tracing $y = 0$. As we will see in later examples, BBM activities can be used alone, in pairs, or in teams with one or two balance boards.

BG activities (Table 2) target specific areas of trigonometry highlighted as challenging for students by teacher partners. These include the parameters of trigonometric functions, including understanding the relationship between frequency and period, as well as the symmetrical nature of amplitude, the correspondence between the sine and cosine functions, and the unit circle. BBM uses experiences of balance and discovery-based learning to foster understanding of these concepts. For example, in the BG Amplitude Exploration activity, learners experience the symmetrical nature of amplitude as rocking to the same angle left and right. In the BG Frequency Exploration activity, learners can relate their felt sense of rocking speed to how many periods they generate on the screen in a given time period. In the BG Unit Circle activity, learners reconstruct the unit circle by modulating sine and cosine. The

exploratory and movement-based nature of these activities offers new ways into these challenge areas. See the supplementary video for some examples of learners exploring BG activities.

Figure 3

Balance Graphing Configuration



As context for all BG activities, I will describe here the precise way inclinometer data is translated to graphical output. A grid is visible on the display, without labels for x and y. The x-axis corresponds to time (in seconds); graphs are generated at a constant rate. The y-axis corresponds to board angle (in degrees of tilt). The *angle* of the board's inclinometer sensor is translated to specific *y values* for the graph. Angles on one side of flat correspond to positive y values and the other, to negative values. For example, when rocking front-to-back ("roller coaster" style), lifting the front of the board by leaning backwards generates positive values, and lowering the front of the board by leaning forward, negative values. When rocking left-to-right ("surfing" style), the right edge of the board is treated as the front since it faces the direction the graph progresses. Thus, to move one's graph upwards into the positives, one creates an "uphill" angle with the board, and to move it downwards into the negatives, one creates a "downhill" angle. Holding the board in a given position yields a constant y value. (See Appendix 3 for a discussion of alternate mappings).

To accommodate learners' rocking preferences and compatibility with different displays, graphing rate and sensor sensitivity are adjustable to adapt the graph's scale and rate of appearance to a child's preferred range and speed of rocking, with default units being about 10 degrees for y and about 2 seconds for x.

Table 2

Balance Graphing Activities

Activity	*Purpose*	*Balance Experience*	*Key Display Features*
Function Exploration	Experience "being the graph"	A graph's slope and contour is experienced by rocking as if surfing or riding over the surface of the graph	Color, sound, and percentage feedback expresses how closely they have followed the target graph
Amplitude Exploration	Identify and control the amplitude parameter of trigonometric functions (as a symmetrical phenomenon, and as distinct from frequency)	The amplitude of a function is experienced as the degree of tilt in either direction, and can be analyzed in relation to the y coordinates of graphical minima and maxima	Real-time stars (appearing at local minima and maxima that meet target amplitude) and a summary count express how consistently they have rocked at a focal amplitude
Frequency Exploration	Identify and control the frequency parameter of trigonometric functions; perceive one period as a unit and relate period to frequency	Frequency is experienced as rocking rate, and can be analyzed in relation to the observed period length of a graph	Color and sound feedback convey how closely they matched a focal frequency for each graphical period
Function Addition	Develop graphical intuition around how the sum of two functions behaves (including when functions are on the same or opposite sides of the x axis)	Function addition is experienced as the moment-to-moment relationship between one's own board's angle relative to a peer's	Each player's graph and the sum of their graphs appears in real-time as they rock
Unit Circle	Build understanding of the relationship between the unit circle and the sine and cosine graphs	Sine and cosine are experienced as a distinct rocking coordination pattern of alternating movement and varied rate of change that enable tracing the perimeter of the unit circle	The display simultaneously each players' graphs, and traces their location on the coordinate plane (i.e., sine player controls x and cosine player controls y, like an etch-a-sketch)

3. Example Learning Trajectory With BBM

BBM offers a digital environment that, through feedback, invites learners to move in new ways, such as rocking at a given amplitude. Let us track the thinking of a pair of middle school girls, Molly and Annette, as they generate a series of graphs with BBM in the Amplitude Exploration activity.

To contextualize this example, the BBM Amplitude Exploration activity (Fig. 4) is designed to foster attention to the amplitude parameter of a sinusoidal functions. Learners are invited to "find green stars", which appear when they rock at a particular (modifiable) amplitude[1]. Finding stars below the x-axis generates a low audio tone, and above x, a higher one. Through rounds of graphing and reflection, learners come to recognize and modulate the amplitude of their graphs by symmetrically controlling the angle of their rocking.

Figure 4

A Round of Amplitude Exploration In Progress



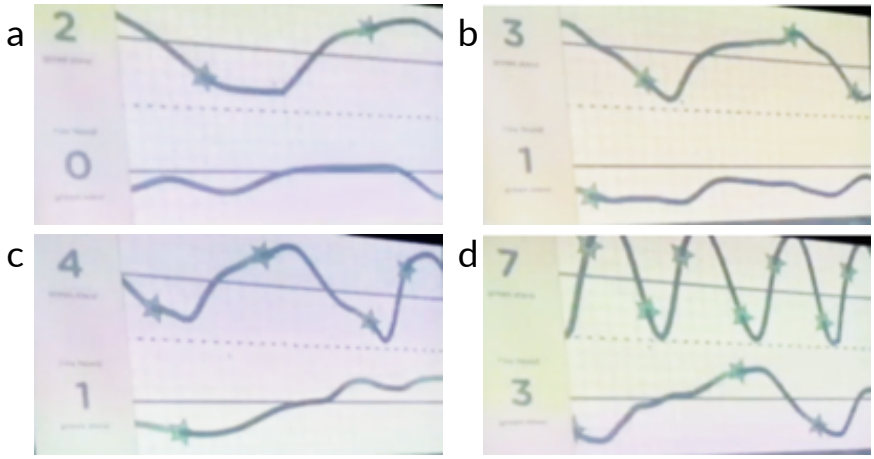
When first introduced to the Amplitude Exploration activity, Molly and Annette are invited to "find as many green stars as possible", and informed that they can try as many times as they would like. In the first round, Annette (Fig. 5a, upper portion of screen) finds 2 green stars, one below the x-axis, and one above. Molly mostly graphs below the x-axis (Fig. 5a). Towards the end of the round, her graphing line begins to turn green. She does not find any stars during

[1] This activity presently focuses on sinusoidal functions centered on the x axis because having the function centered at 0 (board in balance) facilitates perception of symmetry in rocking to the same degree to either side. However, in this activity, I have observed learners to also explore generating sinusoids centered on one side of the x-axis, suggesting that with some modifications, this activity might also offer means to explore sinusoids centered elsewhere.

the round. Invited to reflect on their graphs, Annette comments: “Hmmm. I see, it's like that” and gestures a wave form with her arm. When prompted to explain further she says that to get green stars, one must use a “wave pattern.” She asks the researcher if she is allowed to know how many stars there are total, a question the researcher invites her to think about during subsequent rounds. Molly, meanwhile, comments that her graph was “almost there,” since her line turned green but a star did not appear. Molly and Annette share that next round, they plan to “do the wave thing again” (Annette), and “try [to] follow the green line” (Molly).

Figure 5

Sequence of Amplitude Exploration Graphs Generated by Molly and Annette



The pair completes another round of graphing (Fig. 5b). This time, Annette finds 3 stars, and Molly finds one. Annette gestures the sinusoidal shape of her graph with her hand and arm again and shares, “I feel like there should be more stars, like I feel like, maybe it should go like up-down-up-down faster?” When asked to describe where the stars are that she found, she notices their y-coordinates: “oh yeah! Maybe it's like, ‘cause they are- all of them were second to bottom, about second to bottom, second to top square.” Molly, meanwhile, notes that while she also got the first star, the star’s locations for each of them are “not in the same exact spots”- that is, she notes the flexibility of the stars’ x-coordinates. Thus, through exploration, Molly and Annette have arrived at a proto-notion of amplitude as a feature of a sinusoidal function, specified by y-coordinate rather than x-coordinate.In their next round (Fig. 5c), Annette completes two cycles in the graphing time period, resulting in 4 stars. Molly finds a first star at a local minimum and almost reaches a second star, but is slightly shy, her line turning green. They try graphing again immediately. Finally, Annette finds 7 stars, and Molly finds 3 (Fig. 5d). Annette comments: “I guess mine is like very high frequency, or, kind of high frequency,” noting she achieved this by “mov[ing] more up and down so it made shorter waves.” Molly comment on how she “kept going in the same direction” when the line started turning green to achieve the stars. The researcher then repeated Annette’s question from earlier: how many stars could one get in this activity? Annette initially expresses that since the stars are evenly spaced, perhaps she has reached the maximal number, but Molly reasons, “I think if you go faster up and down, then maybe there's more.” Annette agrees, noting, “maybe if you go like double as fast then you will find almost double the amount of stars.” Thus, Molly and Annette have begun to parameterize their graphical creations according to both amplitude and frequency. In the next section, we will look more closely at how using BBM transforms learners’ mathematical thinking.

4. Transforming Math Knowledge

BBM transforms mathematical thinking and knowing in several ways:

1. *Concepts beyond vision*: Restructurating concepts through the balance sensory modality
2. *Being-the-graph*: Cultivating whole-body experiences of disciplinary forms
3. *Coordinated action:* Coordination dynamics as distributed thinking
4. *Embodied design:* disciplinary tools as resources for action

I elaborate each below using an example BBM activity and how learners have interacted, or can interact, with that activity.

4.1 Concepts Beyond Vision: Restructurating Concepts Through the Balance Sensory Modality

Wilensky and Papert (2010) coined the term *restructurations* to describe a shift in the encoding of knowledge in a domain based on a shift in representational infrastructure, for example, a shift from roman numerals to hindu-arabic numerals. Restructurations can be evaluated in terms of how they expand access to and capacity within a discipline, including by accommodating diverse learners, enhancing engagement, and enhancing fit with human evolutionary-cognitive dispositions.

BBM restructurates mathematical thinking through the balance sensory modality, rendering concepts explorable and expressible using experiences of balance and tilt. For example, in the BBM Balance Number Line activities, negative numbers' opposition to their positive counterparts is conceptualized as counterbalance. Learners establish this conceptualization through different exploratory activities on the board. One such activity is the "move-in-balance" challenge. A learner is asked, "do you think you can find a way to move both hands simultaneously along the number line while maintaining the board in consistent balance?" The solution to this problem entails moving the hands apart symmetrically such that they are always equidistant from 0 (for example, passing through -1 and 1 simultaneously) (Fig. 6). Through problems like this one, learners come to conceptualize a number and its opposite as in balance with each other. For example, a middle school student on the autism spectrum experienced placing his left hand on -1 and his right on 5 as tilting him "way to the right," whereas -2 and 2 felt "zen." Based on these two experiences, he spontaneously generated and tested the prediction that -5 and 5 would also feel "zen." These activities offer a view of negative and positive numbers as not just equidistant from 0, as is visible on a visual number line representation, but as bearing mutually neutralizing displacements in opposing directions.

Figure 6

Moving-in-Balance, Solved By a Middle School Student



Note: see published version for moving .gif of this image

4.2 Being-the-graph: Cultivating Whole-Body Experiences of Math Representational Forms

Research on gesture suggests that those with greater graphing expertise show differences when asked to represent graphs gesturally: they tend to gesture as if *"being the graph"* (using more dynamic, whole-body movements, even taking a perspective of being inside the graph) rather than merely *"seeing the graph"* (gesturing as if sketching the graph on paper) (Gerofsky, 2011). The sense of balance offers a prospective means to realize a "being-the-graph" experience by generating a graph's contours with their body. Using a balance board to graph, generating a high-slope graph feels steep; a high-frequency graph feels rapid. Learners can imbue the graphical representation with meanings from these embodied experiences.

Let us take the example of the BG "Frequency Exploration" activity, designed to orient learners to sinusoidal graphs' period and frequency. This activity links felt experiences of symmetrical rocking to periods on the graph, and rocking speed to frequency. As they rock, each local maximum and minimum and each passage through the x-axis generates a sound: low for the minimum, medium for the x-axis, and high for the maximum. Each time a child completes one period (rocking in each direction and back to center), they receive color and sound feedback reflecting their proximity to a target frequency (a modulable parameter). The background behind each period on their graph turns green for matching the target frequency, yellow for almost matching, and red for being farther off.[2] For example, in Fig 7, the child on the left has generated three different periods in the upper graph. Based on proximity to the focal frequency, each has turned a different color: red, yellow, and green. They also hear one of three chords: highly consonant for green, somewhat consonant for yellow, and dissonant for red.

Fig. 7 A Round of Frequency Exploration In Progress.

[2] The sensitivity for these thresholds can be adjusted.

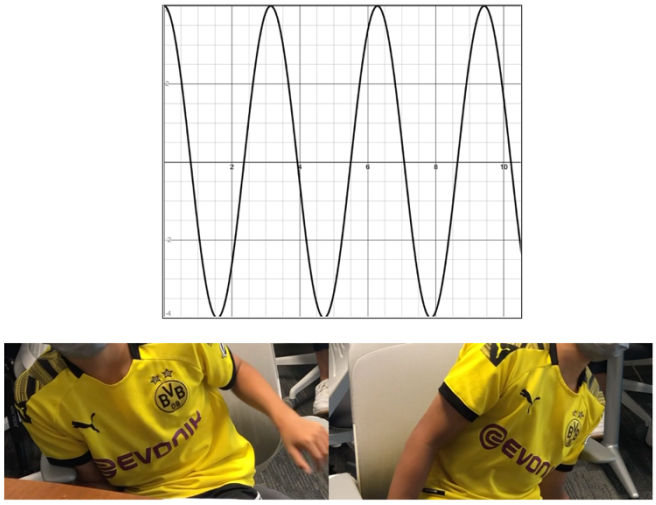


To illustrate, middle-schooler Kyle was invited to explore Frequency Exploration and "try to turn the whole screen green" (see Tancredi, Wang et al., 2022). Across rounds of exploration, Kyle came to connect the contours of graphs, including the graphical period, with his felt sense of rocking speed. He recognized an inverse relation between the number of periods ("hills") drawn and their width: "maybe if they're like, the hills are further apart [they will be green]- but then that doesn't make as many hills." He also related his felt experience of rocking speed to the number of periods drawn in a given timespan: "I went a lot slower, and I only got three cy- hills, like cycles."

Kyle and other children's gestures showed increased hallmarks of being-the-graph in post-interviews after BBM. Compared to a pre-interview, when asked to gesturally show a set of sinusoidal graphs, the children used larger scale, more dynamic gestures (Tancredi, Wang, et al., 2022). For example, Kyle leaned left and right to different angles reflecting graphical amplitudes (Fig. 8) whereas his torso remained static in the pre-interview. He also used language related to speed, such as "slower, but rocking a lot farther." Thus, even static, 2D representations of graphs appear to evoke multimodal experiences for children through BBM.

Figure 8

Static Graph and A Child's Gestures When Describing It After Using BBM



4.3 Coordinated action: Coordination dynamics as distributed thinking

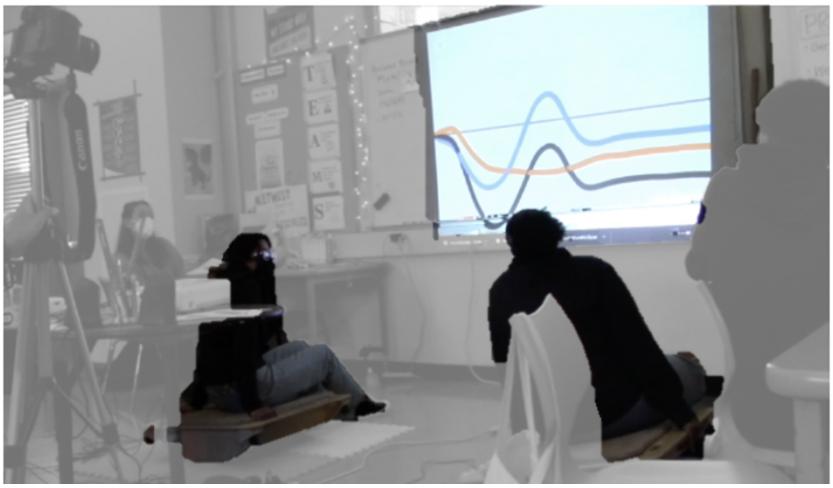
Through multi-player interactions, BBM players can experience the relationship between two quantities as the relation between their own movements and that of a peer. For example, in the BBM Function Addition activity, two boards generate graphs on the same graphing area, each with a different color (blue or orange), and the sum of their two graphs is shown in black in real-time (Fig. 8). Function Addition is designed to build learners' graphical intuitions around the meaning of adding functions, including how a sum function relates to its component functions, how the sum of two functions relates to their value at any given x, and how a given sum function can be achieved with different combinations of functions. To achieve desired functions, learners must attend to the coordination of their joint activity, attending to features such as frequency, amplitude, and slope.

When invited to explore this activity at a science fair, even pre-primary school children frequently explored rocking in opposite directions to bring the sum line towards the x-axis, and rocking in the same direction to drive it to the extremes of the screen.

Let us follow a class of high school students as they explore Function Addition (Tancredi et al., 2023). Students began by exploring a few rounds of graphing to observe the relationship between the players' graphs and the black graph. Next, they were given a focal function such $y = 0$ (tracing the center line), a high-amplitude sinusoid such as $y = -\left(\frac{5\pi}{6}\right) sin(x)$ (a wave that reaches the very top and very bottom of the screen), a line with a specific slope such as $y = -2$, and complex functions such as $y = sin(x) + x$, and invited to brainstorm and sketch ideas to generate these sum graphs, taking turns testing out their ideas on the boards in front of the class (Fig. 9).

Figure 9

Two High School Students Explore Function Addition

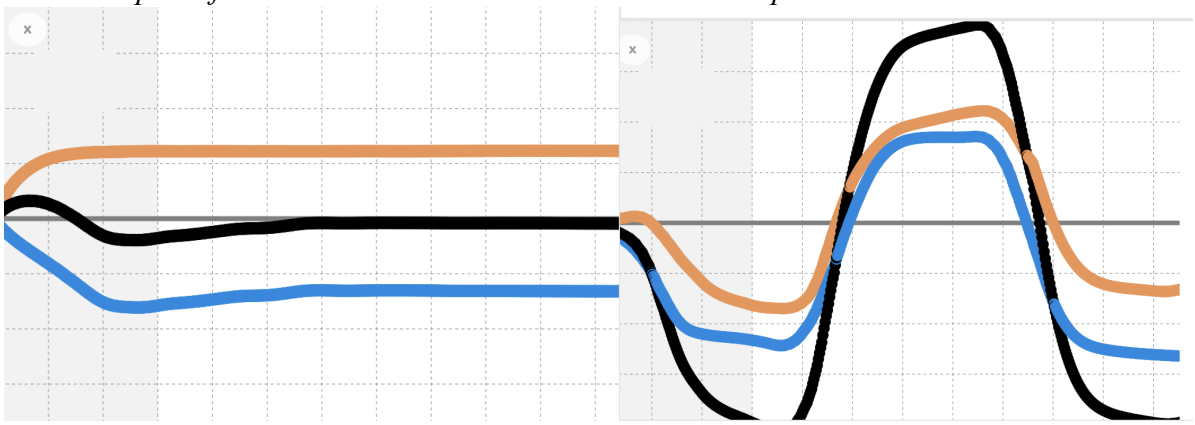


Student teams generated multiple solutions to each challenge. For example, one student solution to generate a sum function of $y = 0$ (Fig. 10a) was for orange and blue to start at $y = 0$, then have orange increase to $y = 1$ while blue simultaneously decreases to $y = -1$, holding these positions. Other student solutions to this challenge included both staying perfectly still, and drawing opposite phase sinusoids. An example student solution to generate a high amplitude sinusoid was for both blue and orange to draw sinusoids that were in phase with one another at a shared frequency and amplitude of about 2 (Fig. 10b).

As they tested and refined solution ideas, the students began by focusing on their direction of movement, attending with increasing specificity to their angles, and eventually the boards' relative positions to one another. For example, one group initially planned by pointing in a direction of travel ("let's go this way first"). After a couple more rounds, they start to use counting aloud ("one, two, three" on each side) to synchronize their rocking. Next, they attend to the specific angles of the boards (one tilts "all the way left," the other "tilt[s] all the way to the right"), even rehearsing these angles prior to graphing. Function Addition enabled different bodily coordinations in time and space to become the basis for understanding the relation between two complex quantities. The class showed improvements on pencil-and-paper function addition problems following these explorations.

Figure 10

Two Examples of Student-Generated Function Addition Graphs

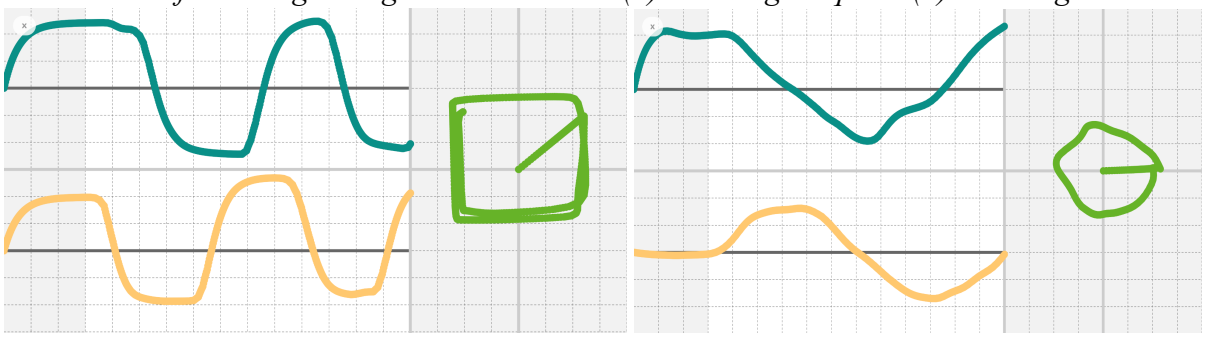


Note: (a) Goal: sum function $y = 0$. (b) Goal: a high amplitude sinusoidal sum function

The BBM Unit Circle activity provides another example of coordinated action as a basis for conceptualization. Unit Circle is designed to support learner's understanding of the relationship between the unit circle and the sine and cosine functions. One board controls sine, and the other controls cosine. In addition to displaying the graphs of their rocking, as in other activities (sine in teal on the upper left, cosine in yellow on the lower left), Unit Circle displays a second visualization: a trace of their position in a cartesian grid with sine controlling y and cosine controlling x, analogous to an etch-a-sketch (Fig. 11, shown in green on the right).

Figure 11

Screenshots of Drawings using Sine and Cosine (a) Drawing a Square (b) Drawing a Circle



One pedagogical trajectory with Unit Circle, applicable for both those somewhat familiar and wholly unfamiliar with trigonometry, is to first invite users to generate horizontal or vertical lines (wherein one board moves and the other stays still), followed by a diagonal line (requiring simultaneous, coordinated movement). Subsequently, they are invited to generate a square, requiring timed alternation of movement. From here, they are invited to try to trace a circle. This requires a process of smoothing the functions they made to generate a square, slowing down through the peaks and troughs while maintaining the same phase relation with one another. When perfected, the users have drawn the sine and cosine functions. The activity is designed such that the canonical forms of sine and cosine constitute the enacted solution to the presented movement problem. Learners experience the phase relation of sine and cosine, and the relationship between each point along these functions and each point on the unit circle, through their continuous coordinated action. Thus, learners come to know these functions' form and phase relations, as they have reinvented them to trace the perimeter of the unit circle. Learners come to understand these representations through coordinated action.

In these activities, learners are distributed, each controlling one of interacting variables in a system. In so doing, they can leverage core human competencies of concerted action (Melser, 2004) to instantiate and understand the relations among components through coordinated joint action.

4.4 Embodied design: Disciplinary tools as resources for action

In the embodied design framework (Abrahamson, 2014) for enactivist pedagogical design, learners explore a *field of promoted action* (Reed & Bril, 1996) wherein through feedback, they figure out how to move in a new way to think in a new way. Disciplinary tools and artifacts are introduced as resources for controlling and communicating about actions, prompting reflection and discourse that establishes these new ways of moving as reflecting disciplinary concepts.

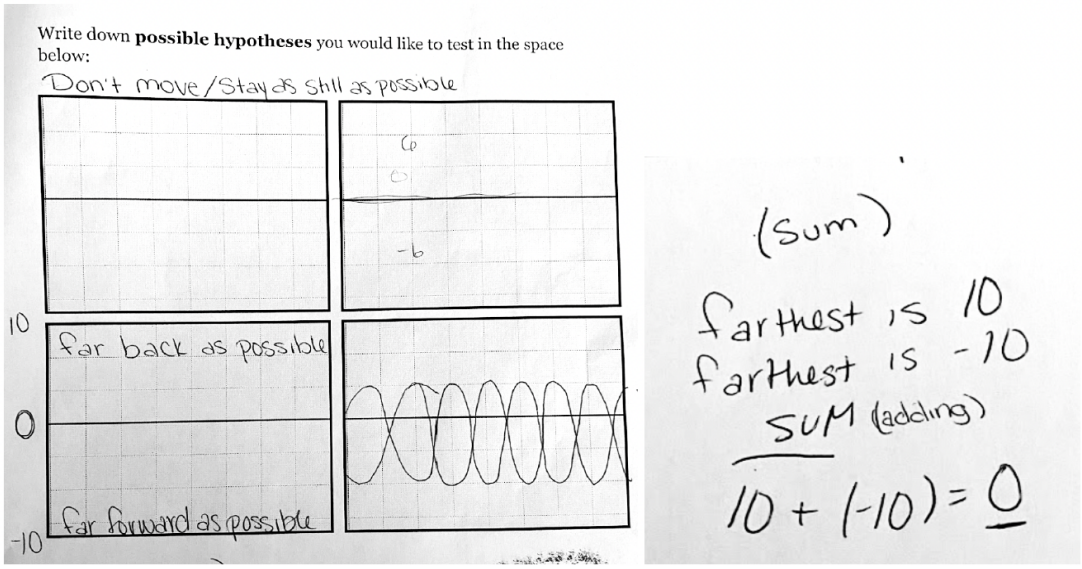
BBM instantiates this design approach; grids, number lines, and numbers become resources for enacting and communicating about ways of moving: for example in Frequency

Exploration, learners might discuss how many units across corresponds to green and calculate how many total greens are possible.

In a high school classroom exploration of Function Addition, learners progressed from qualitative thinking ("[rock] slower") towards more discrete and quantitative thinking ("go 2 down [on the grid])." The learners began to quantify to reason about and plan their movements, for example jotting down "10 + (-10) = 0" (Fig. 12). BBM thus instantiates the design principles of action-based embodied design (Abrahamson, 2014; Alberto et al., 2021) by rendering mathematical disciplinary tools as resources for refining their enactment of solutions to motor control problems.

Figure 12

Students' Written Brainstorming to Generate a Sum Graph of y=0

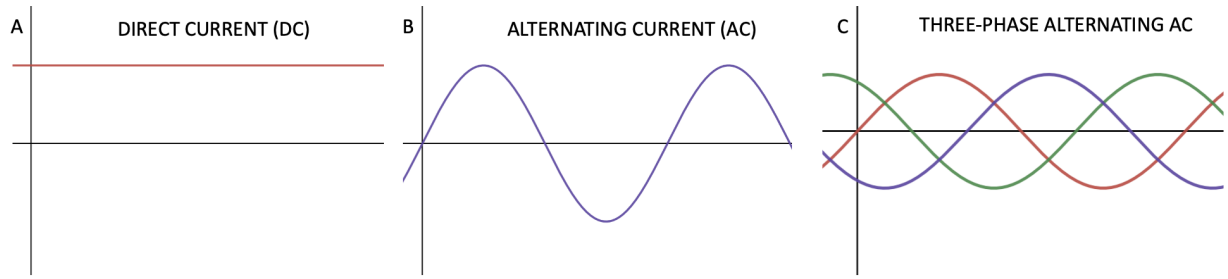


5. Discussion

The concepts explored here reflect the potential of technology to foster balance-activating movements as a resource for thinking about a range of math concepts at different levels. Beyond the concepts discussed here, balance-based models could shed light on other areas within and beyond mathematics. For example, the board offers a whole-body experience of slope, of interest for relating a graph to its derivative (Appendix 3). In physics, Balance Graphing activities could be developed to contrast voltage and current in alternating current (in which voltage switches periodically in a sinusoidal function) and direct current (in which voltage maintains a constant direction), as well as relations among different signals in contexts such as the three-phase alternating current waves sent out by power plants (Fig 13). Multi-board configurations also enable embodied understandings of coordinated action, which may be of interest for activities exploring the coordination in biological systems, such as that of the contractions of valves of the heart.

Figure 13

Types of Current



The presented design uses a ubiquitous occupational therapy and productivity tool, the balance board, to engage the sense of balance. Balance boards enable controlled stimulation of the vestibular sense. However, they are by no means necessary to stimulate this foundational sensory system. Pedagogical designs can stimulate the vestibular sense through everyday objects such as rocking chairs, or simply inviting rocking, tilting, spinning, or whole-body movement. Vestibular stimulation is an axis of any activity; however, most activities provide extremely limited stimulation to this system and do not leverage it as a resource for conceptual thinking. BBM offers an exploratory case of vestibular-centric, vestibular-differentiated mathematical tools.

6. Concluding Remarks

BBM is built to foster balance-driven mathematical conceptualizations. These constitute restructurations that reformulate traditionally graphical disciplinary representations. For example, negative numbers are restructured as counterbalances to positive numbers. Furthermore, enacting coordinated actions with peers constitutes a means to conceptualize the relations among multiple evolving quantities. Through BBM activities, children can cultivate multimodal mathematical notions, drawing upon these to animate and reason about 2-dimensional representations.

BBM activities set disciplinary representations such as graphs and number lines up as resources for controlling and communicating about dynamic balance experiences, with the balance experiences in turn informing learners' experiences of these representations. In so doing, BBM instantiates the embodied design process of drawing upon children' embodied experiences, in dialogue with disciplinary resources and practices, as the basis for math instruction (Abrahamson, 2014) and inclusive education (Abrahamson et al., 2019; Tancredi, Chen, et al., 2022).

BBM offers a different entry point into supporting learners' comprehension of mathematical concepts- one that makes use of learners' histories of sensorimotor experiences in the world and skill for learning new ways of moving. Additionally, experiences of balance may especially offer an additional channel of common ground for joint meaning-making across learners with different sensorial access to the world. This could have implications for creating inclusive opportunities across learners of different profiles, including nonspeaking, blind, and deaf learners. Further accessibility features, such as revised sonification and voicing and additional physical configurations such as rocking chair options, are necessary for BBM to realize its inclusive potential. Beyond BBM, explorations of balance-based conceptualizations may also open onto new somatic-driven mathematical epistemologies, towards a more sensory-expansive and sensorially inclusive mathematics.

Tutorial and Code

A tutorial for setting up Balance Graphing and an open-access version of the activity code can be found at tinyurl.com/BBM-Balance-Graphing.

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Appendix 1: Balance Board Math Activities

For interested readers, I elaborate here on additional BBM activities not described above.

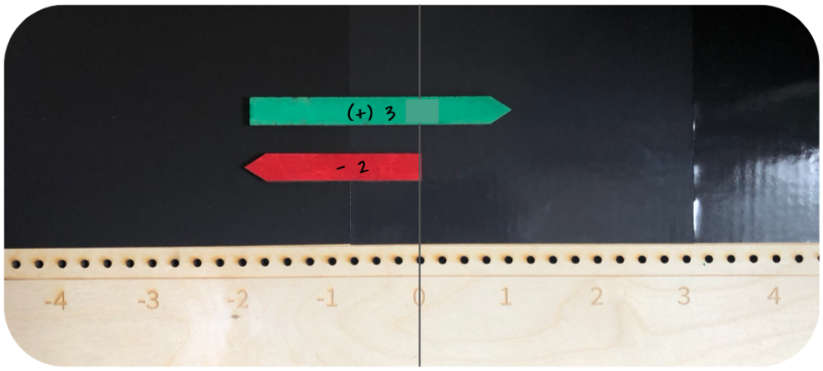
Balance Number Line (BNL)

An additional dimension of the BNL is the use of magnetic arrows. Magnetic number arrows are available to document, plan, and compare movements on the number line. The purpose of these arrows is to help learners make sense of a common sticking point with equations: parsing numbers' signs and operations.

Vector arrows correspond to a specific number, such as -2 or 3. Each arrow has a length (for example, the $+3$ arrow in Fig. 14 has a length of 3 number line units), a color (red for negative numbers and green for positive numbers), and a default direction indicated by the head of the arrow. Each arrow is labeled with its number, including its sign, such that when the point faces its default direction, the label is upright. When adding, arrows are placed in their default orientation such that the flat end is at the existing cumulative sum (0 for a first addend) and the pointed end indicates the new cumulative sum. When subtracting, arrows are flipped horizontally before adding. The negative signifier is an embedded quality of the number arrow, while the operation of subtraction corresponds to the action of flipping an arrow, grounding the difference between the operation of subtraction and the negative sign. In practice, once introduced to placement guidelines, learners can be invited to translate expressions and equations using the magnetic arrows.

Figure 14

Example BNL Number Arrows for 0 + (-2) + 3



To enact an expression through balance, the learner then moves their hands along each arrow, pausing at their endpoints. Arrows and balance convey different aspects of a mathematical expression. Balance conveys only the relative distance to 0 at each step of an expression. Through challenges such as finding all arrows that can restore balance when starting at a given point on the number line, learners can reflect on what is the same across procedures such as adding 2 and subtracting negative 2.

BNL users are invited to codify their movements using the arrows, and later translate the arrows to written expression and equations. For example, upon solving the moving-in-balance challenge, a child might be asked to identify some specific pairs of hand positions that balance, such as having the left hand at -1 and the right hand at 1 (position 1), or the left at -3.5 and the right at 3.5 (position 2). They are then asked to compare what arrows can get their hands from position 1 to position 2. For example, to go from -1 to -3.5, they need to subtract 2.5 (or add (-

2.5)), whereas to go from 1 to 3.5, they need to add 2.5 (or subtract (-2.5)). By analyzing movement between sets of balanced pairs (positive and negative counterparts), learners can reconcile their movement solution with their prior knowledge of symbolization, establishing symmetrical arithmetic transformations. Once familiar with the board and arrow set-up, learners can be invited to invent original balance-movement sequences by moving their hands along the number line. Symbolic expressions become a useful means for the child to encode these unique moves for other learners to be able to decode and reenact.

Balance Graphing (BG) – Function Exploration

In addition to the activities described above, learners also explore an introductory activity, Function Exploration. In Function Exploration (Fig. 14), users attempt to trace a focal function, for example, sin(x), by rocking on the board. The purpose of this activity is to introduce the basic mapping of board movement to a graph, and to be able to compare two functions drawn by two peers at the same time (for example, $sin(x)$ vs. $cos(x)$, $sin(x)$ vs. $sin(2x)$, or $y = x$ vs. $y = 0.5x$). As with all BG activities, the digital display is programmed to draw a graph in real-time of their board's angle. If the learners' graph follows the dotted line on the screen (within some tolerance), the line they are generating turns dark blue, and they hear a resonant sound. If they stray beyond the tolerance, their line turns green, and the sound fades to silence. When the graphs reach the end of the screen, each player's total percentage of green points is displayed on the left side. By default, the target function is displayed as a dotted line on the screen, although it can also be hidden.

For example, in Fig. 15, the child on the left is trying to trace the sine function. She leans forward as she traces a downtrending section of the function. The girl on the right tries to trace the cosine function. Her board is mostly flat as she anticipates leaning backward for an upward-trending section.

Figure 15

A Round of Function Exploration In Progress



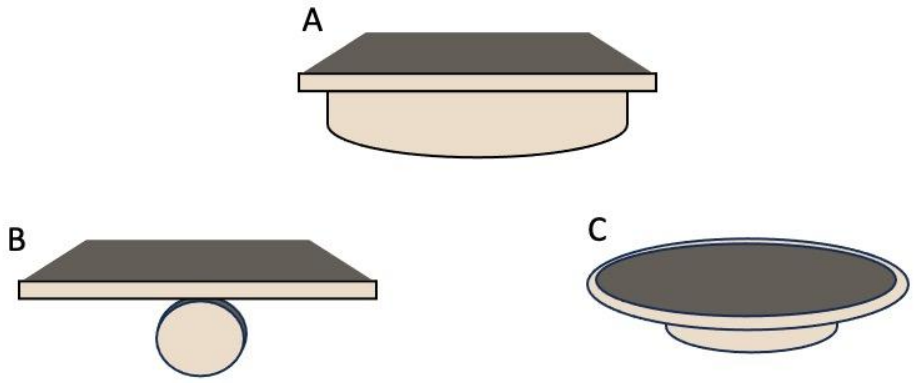
Appendix 2: Discussion of Balance Board Types

Balance boards designs vary by their application, including physical therapy and productivity. The single rocking axis of the BBM board (Figure 16a) simplifies the vestibular stimulation provided by the activity and mirrors the kinds of spontaneous rocking exhibited by learners when rocking and fidgeting. In BBM, the experience of being in balance is important. As such, boards where resting in a balanced state is highly difficult, such as boards with a cylindrical base (Fig. 16b), are not desirable. I acknowledge, however, that a semisphere-base boards (Fig. 16c) could open additional possibilities of interest for further math concepts, not explored here.

Common usage of balance boards involves standing on a balance board and attempting to maintain the board in balance. In BBM, in contrast, the goal is for users to experience different angles and rocking patterns with ease. As such, rather than stand on the board, learners sit on the board's surface. In this position, rather than maintaining the head vertical, the core and head move together with the board, such that rocking the board generates consistent vestibular sensory stimulation. To support this differential usage position, I default to using a very large balance board with handle cut-outs and added protective bumpers to cushion extreme rocking. Balance Graphing activities are also compatible with other single-axis rockable objects, such as rocking chairs, and have been successfully used with assorted rocking and balancing objects.

Figure 16

Types of Balance Boards



Note. Balance boards involve a flat surface with a rounded base. (a) Board with a gradually sloped base: can rock in two directions, used for BBM activities. (b) Board with a cylindrical base: rocks in only two directions. More difficult to achieve balance. (c) Board with a half-spherical base: can tilt in any direction

Appendix 3: Two Mapping Modes of Board Angle to Graph

Inclinometer angle data can be mapped onto graphical output in different ways. Two options of mathematical interest are angle-based and slope-based mappings (Table 3). In angle-based mapping, a given level of board tilt *(angle)* maps onto a given y position when graphing. In slope-based mapping, the *slope* of the surface of the board relative to the ground controls the local slope of the graph moment-to-moment. Whereas angle-based mapping is the default setting used in Balance Graphing activities, slope-based mapping may be of particular interest in teaching other math concepts such as derivatives. For example, in future activities, a learner could use angle-based vs. slope-based mapping to translate between a function and its first-order derivative. If a learner traces a function f(x) according to a slope-based mapping (generating a

graph of the angle of their board as they go), they will draw that function's first-order derivative, f'(x) (Fig 17). For example, at the first point shown in Fig 17, the slope of f(x) is 0, so the learner would make their board flat (angle 0). Thus f'(x) is 0. At the second point on f(x) highlighted in Fig 17, the slope of f(x) is negative, so they lean right to match this slope. They lean two units to the right, so f'(x) is -2. Thus, tasks where learners are challenged to coordinate action in angle-based and slope-based mode could develop their understanding of the relation between functions and their derivatives.

Figure 17

Angle vs. Slope-Based Mapping of a Function; Slope-Based Mapping Supports Exploring Derivatives

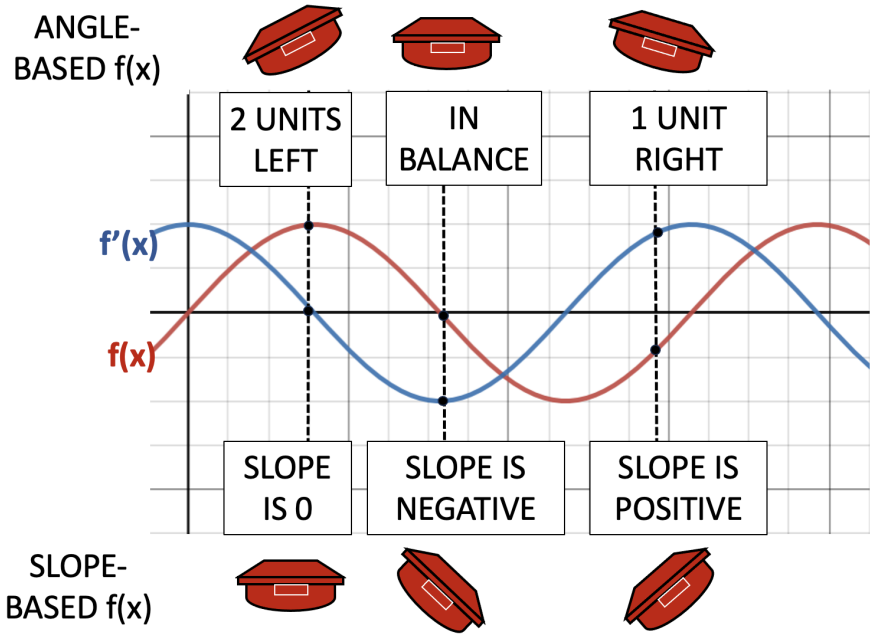


Table 3

Comparison of Angle vs. Slope Based Board Mapping

	Angle-Based Mapping	**Slope-Based Mapping**
Overall	Y position corresponds to rocking angles: a given angle is a given y value	Each moment, the next y value is determined relative to the last one by the current slope of the board
Lean back (or left)	Y value increases	Y value increases (because board slope is positive)
Lean forward (or right)	Y value decreases	Y value decreases (because board slope is negative)
Board is in balance	Y value is 0	Y value remains the same as prior

	Angle-Based Mapping	**Slope-Based Mapping**
		value
Holding board still while not in balance	Y value remains constant	Y value increases or decreases per the slope of the board's tilt

Stimming as Thinking: A Critical Reevaluation of Self-Stimulatory Behavior as an Epistemic Resource for Inclusive Education

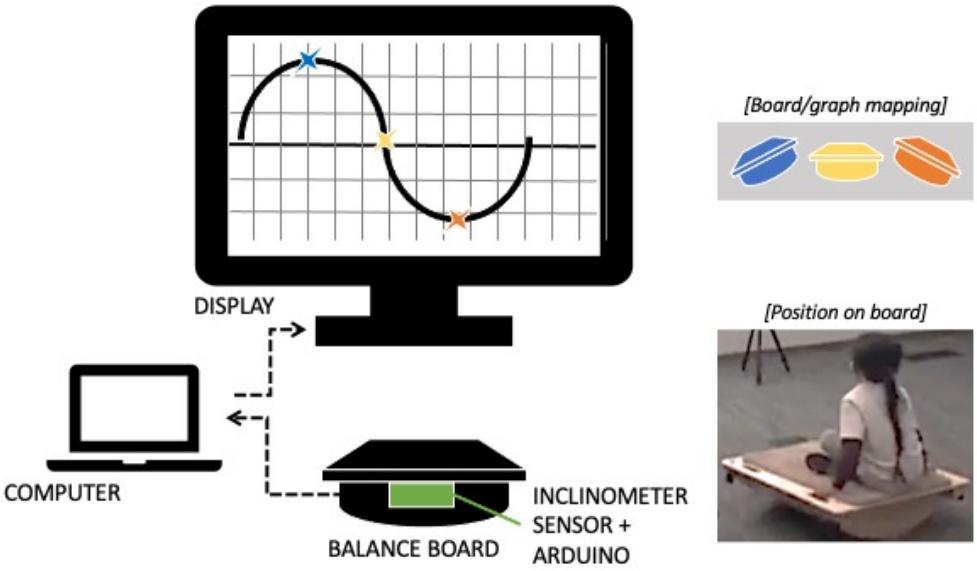
This critical–conceptual paper, authored by myself and Dor Abrahamson, challenges the assumption that peripheral sensorimotor activity, such as rocking and fidgeting, is irrelevant to and distracting from learning. We argue that stimming is an intrinsic part of adaptive functioning, interaction, and cognitive dynamics. We propose that when cultural resources build from students' own sensorimotor dynamics, rather than subjugating them to hegemonic corporeal norms, learners' sensorimotor behaviors may be embraced and empowered as intrinsic mental activity. This call for transformative inclusive pedagogy is of particular importance for neurodivergent children whose sensorimotor engagements have historically been ostracized as disruptive. Following a conceptual analysis that builds on a range of neuro-cognitive empirical studies, we imagine inclusive educational futures that disrupt sedentary instructional design to elevate minoritized sensorimotor behavior profiles. As proof of concept, we exemplify these imagined educational futures by presenting an inclusive embodied activity, Balance Board Math, a technological tool designed to elicit stimming-as-thinking. We put forth a tentative set of design heuristics for realizing stimming's pedagogical potential.

Introduction

A symphony of stimming, from twirling pens to tapping toes inside shoes, takes place around and through children's learning, mostly hidden in plain sight. At best, educational research and practice assumes this ongoing flow of spontaneous activity to be irrelevant, and at worst a harmful distraction. But are such assumptions sound, given growing evidence for the role of bodily activity in mental activity? What would it mean for learners, especially children currently marginalized for their stims, if their stimming were considered relevant to learning? What could teaching look like that endorses stimming as central and contributing to curricular content learning?

This paper was motivated in the context of an ongoing pedagogical design-based research project, Balance Board Math (BBM) (Fig. 1), for which the central design conjecture is *leveraging stimming as conceptual cognition*. Analyzing our data led us to speculate more broadly about potential implications of our findings for cognitive science theory, as well as the practice of pedagogical intervention. In this essentially conceptual paper, we put forth a revisioning of repetitive motor behaviors commonly known in education as stimming. We invite you to rethink with us the axiological (Section 1), theoretical (Section 2), and practical (Section 3) facets of stimming. In Section 3, we briefly overview the BBM project as an example context to think about realizing stimming's pedagogical potential.

Figure 1
Diagram of BBM Graphing Configuration



Note. BBM's Balance Graphing activities constitute a motion-graphing learning environment controlled by the common stim of bodily rocking. Inclinometer sensors enable children seated on balance boards to generate real-time graphs of their rocking movement, projected on a screen in front of them. Graphs are generated at a steady speed, with the *x*-axis measuring time and the *y*-axis capturing the board's angular tilt moment-to-moment. The board may be used in a front/back configuration (as if riding a roller coaster over the graph being drawn) or a left/right configuration (as if surfing over the waves of the graph being drawn), such that rocking one direction yields an increase in *y*, and the other yields a decrease.

We argue that educational psychology requires a theory of stimming that:

a) recognizes neurodivergent testimony of their sensory experiences and practices;
b) models the actual and possible functions of stimming, including self-regulative, communicative, and cognitive functions;
c) informs transformative pedagogical reimaginings of how stimming could be endorsed and centered in learning activities.

Grounded in this theoretical perspective, we illustrate how stimming might be *instrumented* (i.e. functionally extended) through furnishing opportunities for stimulation, *interactional* (socio-ecologically) as a productive resource for communication and shared sense-making, and *instrumentalized* (i.e. contextually applied) as epistemic actions in direct interactions with digital education resources.

1. The Current State of Stimming

We begin with an overview of how the sociobiological phenomenon of stimming is understood, first in education norms and pedagogical practices, and then within research literature.

1.1 Questioning the Status Quo

1.1.1 Classroom Norms.

It is a widely accepted norm that the vast majority of academic learning today is a sedentary affair. One might characterize the status quo (a literal latin status, or position) to be sitting with both feet on the floor, hands at rest, gazing frontwards. Such posture is imagined to reflect attentive studenthood, and indeed, is often used as a measure of engagement in learning

analytics (e.g., De Carolis et al., 2019). We problematize this status quo for all people, attending in particular from the experiences of neurodivergent[3] people (Walker, 2021) whose neurology leads them to act in ways that openly flaunt such norms. Whereas the neurotypical majority are able to get by under status quo, neurodivergent experience reveals possibilities for educational practice that could open new possibilities for everyone.

Anthropologist David Howes (2022) writes, "[P]eople are positioned differently—sensorially, socially—in accordance with the prevailing sensory regime" (p. 327). Individuals' sensory neurology, including their sensitivity to sensory stimuli (Dunn, 1997) and sensory integration profile (Lane et al., 2019), impacts their experience of academic environments and norms. For example, low sensitivity to sensory input is associated with *sensation-seeking* behavior: actively pursuing greater levels of sensory stimulation through activity such as stimming (Dunn, 1997). Notably, many learner groups currently tracked into special education exhibit divergent sensory profiles: those on the autism spectrum, and those with ADHD, Traumatic Brain Injuries, Fragile X Syndrome, and Down Syndrome (Smith Roley et al., 2001). Indeed, greater sensory atypicality has even been found to be associated with lower academic performance amongst children on the autism spectrum (Ashburner et al., 2010).

We propose that these challenges are a reflection of the unsuitability of common cultural infrastructures, practice, teacher-training, and norms to accommodate the full diversity of sensory profiles, rather than some inherent pathology. By analogy, some lefthanded people, including one of the authors, struggle to cut through paper with standard scissors, not due to any inherent motor challenges, but rather due to the design features of scissors designed by right-handed people that assume right-handed use. The blade angle of right-handed scissors *creates* apparently motor-challenged lefties. If the environment accommodated lefties' bodily engagement, as with neutral or left-handed scissors, we could engage with the same acuity. This trivial instance exemplifies the way exclusionary design can generate challenges. At a systemic level, neurodivergent students find that classrooms do not meet their sensory needs. Furthermore, when students engage in sensory activity such as stimming to meet these needs themselves, these bodily expressions are subject to discipline (i.e., Annamma, 2017). Indeed, autistic individuals often report the need to suppress or substitute their stims for social acceptability (Charlton et al., 2021). If stimming is frequently defined as task-irrelevant, and ill-suited to the setting, perhaps it is the academic task and setting that must change.

Thus, in mainstream education, which by-and-large is managed by neurotypical individuals, neurodivergent individuals are stigmatized for their inclination towards sensory exploration through movement. The dominant sensory regime is reifieds sedentary studenthood through environment and tool design, language (Nolan & McBride, 2015), and norms that collectively desensitize us to other forms of sensorial exploration. What might education look like if instead, the full range of sensorial explorations were embraced?

1.1.2 Epistemology and Instructional Practices.

We turn, now, to the questions of disciplinary knowledge. Historically, stimming has been seen as unrelated to the disciplinary practices that shape thinking and knowing. Our present cultural resources evolved through the collective, distributed, and iterated efforts of individuals who had privileged status, power, leisure, and access to engage in academic disciplinary practices such as mathematics. In so doing, each adapted environments and artifacts to their

[3] Analogous to biodiversity, variation in human neurology is referred to as *neurodiversity* (Walker, 2021). The terms neurodiversity, neurodivergent, and neurotypical originated from the autism rights movement and have been taken up in discussion of other neurotypes such as those with ADHD and dyslexia.

cognitive preferences, just as historical urban architecture features such as stairs privilege the ambulatory individuals who created them in their image. The resulting disciplinary approaches implicitly privilege dominant groups' skills and orientations, such as the cognitive styles and skills associated with males in the field of computer science (Turkle & Papert, 1990). Whereas neurodivergent people have always been present, due to their minoritized status, their profiles and likelesses did not always enter the equation in formulating the arena for what became normative disciplinary praxis. The world as we know it was created largely for neurotypical folks, including academic disciplines. We must surface ableist prejudices resulting from a history of sequestered discourse, where sensorially pluralist perspectives have been covert or all too absent.

How might sensory experience shape epistemology? Let us begin with the example of a student who has been blind from birth's understanding of triangles. She has never seen a triangle, but she knows what they are. Her way of knowing triangles reveals the tendency in mathematics to implicitly define geometric shapes' features visually (Abrahamson, Flood, et al., 2019). The implicit oculocentrism of the mathematics field yields pedagogical practices that make geometry less accessible for her: triangles are taught with images such as diagrams, which are created visually, then described to her. Special Education Embodied Design (SpEED), a design-based research framework for inclusive design grounded in embodied cognition, calls this an instance of *modalism* (Tancredi, Chen, et al., 2022) in that privileging the dominant modality of vision changes how our learner gets to interact with the activity. SpEED differentiates the sensory and motor systems engaged, such as vision (*modality),* materials, such as printed textbooks *(media),* and system of meaning making, such as diagrams and language (*semiotic mode*) of instructional designs and highlights their interrelation. Rather than build from the blind student's experiences in modalities she has access to, such as touch as proprioception, she is presented an onslaught of verbal language describing experience in a modality to which she does not have access. SpEED calls for equitable instruction to offer her experiences rooted in modalities through which she explores the world, such as touch and proprioception, for example, by teaching with tangible manipulatives (i.e., Lambert et al., 2022). Analogously, highly sensation-seeking learners require greater stimuli intensity to register themselves and the world, and stimming is a more explicit part of their exploration, yet our teaching methods and norms present concepts in static formats and penalize spontaneous movement.

Thinking and stimming have historically been categorized as fundamentally separate processes. However, we propose that if stimming appears incongruous to ongoing constructive/cognitive engagement, this is not a biological necessity. Rather, the artifacts and activities of neurotypical people have precluded the explicit blending and hybrid functioning of these neurocognitive systems. It is insufficient, and indeed, rooted in ableist assumptions, to propose that stimming be addressed such that learning can occur. Instead, the underlying intertwinement of stimming and thinking, masked by our modern, western, neurotypical-centric practices, must be reclaimed.

1.1.3 Research orientations

As with classroom norms and instructional practices, education research's theorization and treatment of stimming is infused with sociohistorical positionalities we argue are problematic in that they were developed largely by neurotypical researchers who brought to bear their own normative sensory assumptions. Historically, work on stimming and learning has characterized stimming as deleterious and studied means of its extinction. Grounding in behaviorism, researchers used punishment, including such measures as "slapping the subject

briskly on the hands when he began to engage in self-stimulatory behavior" (Koegel & Covert, 1972, p.383) to suppress stims in favor of learning outcomes such as pressing a button at desired times. Such work has been critiqued as neurotypical conversion therapy, ignoring the purpose and lived experience of stimming, and suppressing intrinsic autistic ways of being (Wilkenfeld & McCarthy, 2020), yet is still cited today to describe the relationship between stimming and learning (for example, in Shafer et al., 2017)[4]. Psychology has evolved beyond behaviorism— it is ethically and theoretically imperative that educational models and approaches to stimming evolve, too.

1.2 A Stim by Any Other Name

We aim to reevaluate the constitution, function, and normativity of stimming, reinterpreting and building upon prior perspectives within and beyond the cognitive sciences. We begin by summarizing the many ways spontaneous background activity has been theorized across different research contexts and clinical fields (Box 1). We highlight overlaps between the construct of stimming with the constructs of stereotypies and hyperactivity (psychiatry), fidgeting and displacement behavior (psychology), embodied self-regulation (human-computer interaction), sensation seeking (occupational therapy), and nonverbal adapters (communication-studies) (Box 1).

Box 1

Spontaneous Background Activity Terms

Stimming: Psychiatry term used in diagnostic criteria for autism and autistic community term, abbreviation for "self-stimulation": repetitive actions, such as physical movements or vocalizations.
Hyperactivity: Psychiatry diagnostic term denoting "excess movement that is not fitting to the setting" as part of the criteria for ADHD.
Stereotypies, tics, punding: Medical terms for patterned, repetitive movements.
Sensory/sensation seeking: Occupational-therapy term for the activity of obtaining sensory input for self-regulation through actions such as bodily movement. Also used to describe individuals who engage in such activity to a high degree.
Embodied self-regulation: Human-computer interaction term for managing attention and emotion through physical activity.
Fidgeting: Psychology term for repetitive body movements, generally characterized as task-irrelevant, involuntary, and superfluous.
Displacement behavior: Psychology term for activity such as face touching, scratching, and lip biting, associated with feelings of anxiety and stress.
Adapters: Communication-studies term for a category of nonverbal communication kinesics wherein actions first learned as part of need satisfaction, managing emotions, interpersonal interaction, or instrumental learning are partially or fully completed.
Stimming semiosis: Semiotics term to describe stereotypic movement as autistic semiosis, wherein the body is both index and sign for sensory significations.

[4] We acknowledge that some forms of stimming such as head banging can cause harm to an individual. Nevertheless, we argue that understanding what an individual is experiencing when engaging in such stims, and understanding the broader functions of stimming in context, are necessary to appropriately respond to such stims.

These constructs differ along several key axes, the first of which is focal population. Medical constructs such as stereotypy and hyperactivity are applied specifically to those with medical diagnoses and are often used as defining features of a given diagnosis. Medical research categorizes patterned, periodic movements and utterances as *tics* (variable-frequency), *stereotypies* (clustered, repetitive rhythmic sequences), or *punding* (prolonged, complex stereotypy). Thus the behaviors in question are defined in contrast to an imagined norm and presumed to be, to some degree, pathological and, often, in need of remediation. In psychiatry, stereotypy is part of the psychiatric diagnostic criteria for autism: "stereotyped or repetitive motor movements" (American Psychiatric Association, 2013), theorized as a form of self-stimulation, or *stimming. Hyperactivity*, defined as excess movement not fitting the setting, is part of diagnostic criteria for Attention Hyperactivity Deficit Disorder (ADHD). In response to this pathologization, critical scholars and neurodiversity advocacy groups (e.g., Loud Hands Project & ASAN, 2012) have reclaimed the medical term "stimming," celebrating stimming as an expression of autistic embodied experience and identity, not only self-regulatory but also political, aesthetic, and communicative (Felepchuk, 2021; Nolan & McBride, 2015). Other repetitive motor action constructs are studied in the general population. For example, psychology defines *fidgeting* as task-irrelevant, peripheral, involuntary, and superfluous repetitive bodily movements (see, for example, Mathis, 2019; Ricciardi et al., 2019).

Constructs in other fields presume different functions for peripheral motor behaviors. In human-computer interaction, it has been theorized as a form of *embodied self-regulation* to manage attention and emotion (Karlesky & Isbister, 2016). Such a view resonates with occupational therapy models, which conceptualize stimming as *sensation seeking*: pursuing sensory stimulation for the purpose of self-regulation. For example, rocking back and forth offers stimulation of the sense of balance, theorized as a means of modulating arousal by either calming or alerting the nervous system (e.g., Lane et al., 2019). As such, the mind and body are viewed as spontaneously acting to achieve desirable levels of stimulation. Other functional views of repetitive motor action include *adapters* in communication-studies: nonverbal communication kinesics theorized to express internal states such as arousal and anxiety, similar to psychology's construct of *displacement behaviors* (Mohiyeddini et al., 2015). Stimming has further been positioned as semiotic, constituting autistic sensory utterances in which "the body and its sensory apparatus function as both index and sign systems that hold these together" (Connolly, 2008, p. 242, as cited in Nolan & McBride, 2015).

Another key distinction in the literature is that some constructs (hyperactivity, fidgeting) are explicitly limited to overt motor behavior whereas others (stimming, embodied self-regulation) include stimulatory behaviors and experiences more broadly, including such activity as humming or wearing heavy clothing. Furthermore, some presume that behaviors are subconscious (such as fidgeting) whereas others (such a stimming semiosis) include conscious activity as well.

Given these differences among constructs, we elect the term "stimming" in this paper due to its conceptual congruence with embodied perspectives and axiological resonance with inclusive education. In our usage, stimming is spontaneous, repetitive, often rhythmic, activity that provides sensory stimulation. While stimming is often overt motor action, it situates these on a spectrum that includes covert behaviors as well (we discuss this further in light of our theoretical framework in section 2). The term stimming spans both subconscious and conscious behaviors, relevant as we probe stimming's pedagogical potential within contexts of reflection on action. Additionally, the term stimming foregrounds the function of seeking sensory

stimulation, which we find highly relevant to an analysis of embodied action. Finally, this term is used to depathologize a broad range of behaviors within neuroinclusive dialogue, resonating with a goal of inclusive education. Although we take as our focal population the full spectrum of the general population, we propose that building from work on those who are presently marginalized for their stims bolsters more robust theory, towards more inclusive practice.

2. An Embodied View of Stimming

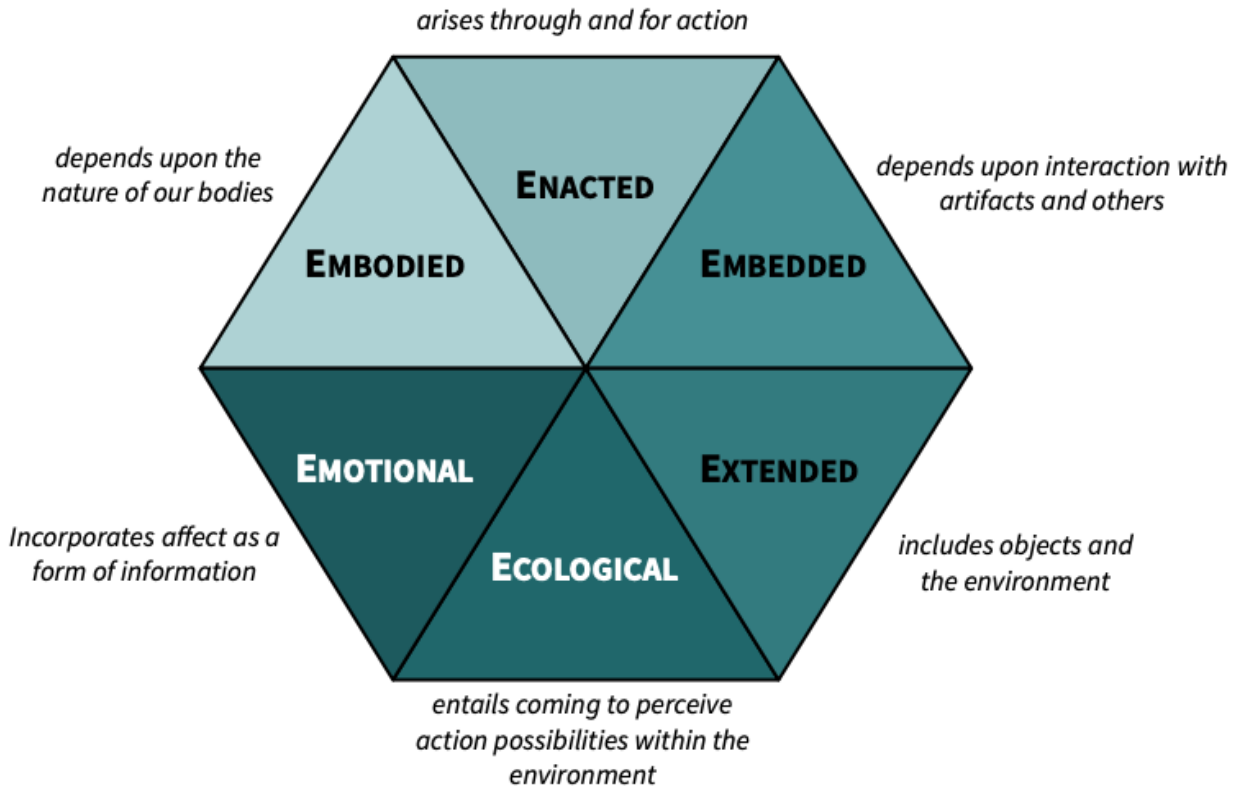
2.1 4(ish) E Cognition

Beginning in the latter portion of the 20th century, thinking was conceptualized in the cognitive sciences as the mental processing of information. Thinking, whether in theoretical scholarship or empirical research, was modeled as the logical manipulations of amodal symbolic propositions, couched within a Cartesian mind-body dualism. This view, whereby thinking is completed in the head, offers little purchase on the exploratory function of stimming described by autistic scholars. A different view garnering evidence in recent decades, *embodied cognition,* posits instead that thought is constituted in ongoing sensorimotor adaptation to the environment in ways that serve our organismic and cultural needs. This paradigm foregrounds the role that the nature and activity of our bodies plays in cognitive processes. With its focus on interaction, post-cognitivist theory offers an alternative conceptualization of stimming as part of the cognitive system's dynamic interactions, through the body, with the physical and social environment.

Post-cognitivism is often characterized with a set of interrelated E adjectives referred to as "4E cognition" (Fig. 2): cognition as *embodied, enacted, embedded, and extended* (Newen et al., 2020); sometimes further specified as *ecological* and *emotional* (i.e., Troncoso et al., 2023)[5].

Fig. 2 Qualities of cognition, per postcognitivist perspectives.

[5] There remains some contention among post-cognitivist theorists, who endorse Es to different extents and disagree on topics such as mental representations. And yet, these perspectives cohere in highlighting that cognition arises through an agent's interaction, through a body, with their physical and social environment.



In the following subsections, we review and reinterpret existing literature on stimming from a post-cognitivist perspective, introducing each E adjective in sequence and drawing evidence from research on stimming and related constructs (Box 1) to analyze how this dimension of cognition may inform an integrative picture of stimming in cognition. We use this structure to circumscribe stimming from a post-cognitivist perspective, acknowledging the inherent overlap of the respective Es.

We put forth five precepts for a 4E model of stimming as: (1) an *embodied* cognitive processes that impact thought and cognitive load (Section 2.2); (2) an *enactive* substrate for complex actions and concepts (Section 2.3); (3) *embedded* and *emotional:* reflecting and regulating cognitive–affective system dynamics (Section 2.4-5); (4) *ecological:* responding to properties of the physical and cultural environment (Section 2.6); and (5) a *extended* cognitive resource that functions as socially communicative (Section 2.7).

2.2 Stimming as Embodied: Influencing Thought and Lightening the Load

"Knowing is bound to the world through the body" (Smith, 2005, p. 279). Embodied cognition, inspired by Phenomenological philosophy, foregrounds the inherent sensorimotor qualities of cognitive activity and, in so doing, melts the classical ontological boundary between overt and covert sensorimotor activity. Cognitive processes, such as language comprehension, covertly implicate the motor system: for example, hearing the words "kick," "pick," and "lick" triggers neural activity in and adjacent to the motor and premotor areas activated when moving the leg, arm, and face respectively (Hauk et al., 2004). Just because another person cannot hear or see something doesn't mean it's not a subjectively real—even loud!—sensorimotor experience. From this perspective, *overt* forms of stimming (fidgeting with a pen, pacing, humming),

discreet forms (twisting a ring, biting one's cheek, tapping toes inside a shoe), and entirely *covert* forms (audiating a Mahler symphony in one's head) might all bear similarities in the stimmer's phenomenological experience.

Bodily processes, down to our very breathing, affect cognition (Varga & Heck, 2017). Awareness increases when breathing in, such that stimuli presented during an inhale are recalled more accurately than those presented during an exhale (Perl et al., 2019). Our perception of time expands and contracts in keeping with our heartbeat (Arslanova et al., 2023).Additionally, background motor activity impacts speech: when asked to move marbles while telling a story, people are more fluent when the direction of movement aligns with their speech, such as moving them upwards while describing rising temperatures, than when background movement and speech are incongruent (Casasanto & Lozano, 2007). Furthermore, right-hand squeezing a ball or walking while thinking increases creativity (Goldstein et al., 2010, Oppezzo & Schwartz, 2014). Even imagined actions can affect perception: for example, imagined body motions can induce nystagmus eye movements and changes in vestibular stimuli perception (Mast et al., 2014). All of these sub-conscious, semi-conscious, backgrounded or covert bodily phenomena influence thought. Stimming ought to be presumed relevant as well.

Perhaps, as assumed in many classrooms, the impact of stimming on thinking is counterproductive. Adding parallel physical or mental tasks or activities indeed adversely affects cognitive performance, theorized to add to an individual's *cognitive load*. Cognitive load (Sweller, 1994) denotes the working memory resources taken up by a given task. Even acts as simple as standing can compete for cognitive resources (Ruffieux et al., 2015). One might suppose, then, that stimming adds to cognitive load, adversely affecting performance of a primary task. However, if this were the case, the finding that stimming actually *increases* in the face of high cognitive load may be surprising. Fidgeting for example, increases with sustained effort on a task (Farley et al., 2013) and when reading more boring texts (Witchel et al., 2016). Does stimming indeed compete for learners' physical and attentional resources? If so, why might such a deleterious process have persisted throughout evolutionary history?

We might find a fruitful analogy in work on another bodily process: gesture. In contrast to other parallel activities, gesture has actually been found to reduce cognitive load. Inhibiting gestures actually leads people to perform worse on memory tasks (Goldin-Meadow et al., 2001), suggesting that the act of limiting gesture itself may even require cognitive resources. Gesture has a facilitatory role in tasks such as problem-solving: children told to gesture when solving math problems generate more problem-solving strategies, and ultimately solve more problems (Broaders et al., 2007). Working memory generally exhibits fewer limitations when learning "biologically primary information" of relevance in our evolutionary past, such as movement, than when learning cultural knowledge, such as reading (Paas & Sweller, 2011). Bodily activity such as stimming, then, may present means to leverage evolutionarily foundational human capacities for the learning of cultural knowledge such as mathematics.

Might stimming, too, lighten the load? Several findings suggest the plausibility of such a phenomenon. Analogous to findings with gesture, autistic adults self-report that the act of *suppressing* stimming requires cognitive resources (Charlton et al., 2021). Additionally, in boys with ADHD, increased hyperactivity during working memory testing is associated with improved performance (Sarver et al., 2015). In neurotypical adults, stims like doodling improve auditory recall (Andrade, 2010). How might stimming achieve such an impact? A conservative explanation is that stimming's impact on arousal mediates its impact on cognitive performance. A bolder explanation is that, just like gesture, stimming is part of cognition itself.

2.3 Stimming as Enactive: A Substrate for Developing Complex Actions and Thoughts

A view of cognition as *enactive* highlights that cognition evolved for action. As such, perception and action unfold together. Repeated sensorimotor patterns of perceptually-guided action give rise to cognitive structures. In education, this view of cognition as enactive has been applied to show that students can ground mathematical notions by learning to move in a new way that consequently takes on conceptual significations (Abrahamson & Bakker, 2016).

Given that stimming is part of an individual's action manifold, we submit that enactivism suggests the relevance of stimming to the development of new perceptual orientations. That is, stimming constitutes a form of action that might impact cognitive structures that are in turn the basis for conceptual learning. Or reciprocally, if perception guides all motor activity (Mechsner, 2004), then stimming, too, must be affected by, and affect, an individual's perception. Consistent with this view, features such as the presence of a visual music video impact how much people stim when listening to a song (Witchel et al., 2016), suggesting that sensory features of a task shape concurrent stimming. We will focus, in developing this view, on two areas where the relationship between stimming and cognitive structures is especially clear: stimming's function in infancy, and rhythm in cognition.

2.3.1 Stimming as the Foundation of Functional Motor Behavior.

Infant studies have found that stereotyped movements play a key role in early motor skill development. Simple repetitive behaviors, such as arm waving and body-rocking, are observed to precede goal-directed complex motor behaviors that use the same body segments (reaching, crawling) (Thelen, 1979). For example, repetitive, patterned arm movements precede effective infant reaching; the amplitude and timing of these movements is modulated towards accurate reaching patterns (Thelen et al., 1993). Additionally, infants who received less stimulation to the vestibular sensory system from caregiver actions, such as rocking or bouncing, engage in more stereotyped movement themselves (Thelen, 1980). These ontogenic findings suggest a view of stimming as core to developing and refining skilled action: "simple, stereotyped motor behavior in infants is the foundation on which complex, functional behavior is built" (Shafer et al., 2017, p. 2). Stimming can thus be characterized as foundational to functional motor action.

Stereotypy's functional role is characterized as specific to early development (Shafer et al., 2019). However, this is based on a neurotypical developmental trajectory within the current designed environment. Nolan and McBride (2015) argue that science must move beyond assumptions of *sensory periodization*, whereby sensory exploration is valued in early childhood but expected to be outgrown and replaced with social/cultural signs and symbols. They highlight the ongoing role of stimming in autistic ways of being and making sense throughout development. Stimming could ostensibly continue to offer a basis for developing complex motor development throughout the lifespan, if embraced as such.

2.3.2 Rhythm and Thinking.

> "The better we understand the biological basis of rhythm, the better we will be able to employ rhythm—in all its guises—to improve communication and to better understand ourselves." (Kraus, 2021, pp. 218–219)

Surveying the field's state-of-the-art understanding of neural oscillatory activity, Buzsáki (2006) prefaces, as follows, his book titled *Rhythms of the Brain*:

> [B]rains are foretelling devices, and their predictive powers emerge from the various rhythms they perpetually generate. At the same time, brain activity can be tuned to become an ideal observer of the environment, due to an organized system of rhythms. … The notion that oscillators or "central pattern generators" are responsible for the coordination of motor patterns, such as breathing and walking, is old and well accepted in neuroscience. But the tantalizing conjecture that neuronal oscillators can be exploited for a plethora of other brain-generated functions, including cognition, is quite new and controversial. (pp. vii–viii)

Buzsáki (2006, p. 114) proceeds to demonstrate how multiple neural oscillators coordinate operations within and across neuronal networks to form a hierarchical system in the cerebral cortex. Even consciousness itself, scholars surmise, could be theorized as emergent from the rhythmic neural activity. More recently, a *rhythmic theory of attention* has been proposed by Fiebelkorn and Kastner (2019), who cite their neuroscientific experimental studies to evidence how rhythmic neural activity shapes sensory and motor aspects of goal-oriented engagement with the environment, such as in searching for particular features. As Basso et al. (2021, p. 2) sum this up: "...brain-body connectivity is bidirectional: oscillatory rhythms in the brain drive movement and movement drives oscillatory rhythms."

The bidirectionality of motor–neural rhythmic induction may explain observations by learning scientists that students solving movement-based mathematics tasks spontaneously organize their physical actions into repetitive spatial–temporal patterns, which Palatnik and Abrahamson (2018) characterize as tacit goal structures within their interactional inquiry. Put simply, if we are expected to do some new repetitive job with stuff in the world, we try to encapsulate the task as a recurring sequence of perceptuomotor actions, presumably so as to minimize the allocation of cognitive resources (see also Schack & Mechsner, 2006). Such observations have turned the attention of educational scientists to the rhythmicity of mathematical activities (Bautista & Roth, 2012; Radford, 2015; N. Sinclair, 2018). More generally, sensorimotor activity is inherently spatiotemporal. Time serves as an implicit organizing modality of sensorimotor bio-anatomical interaction with the environment—the very patterns that, per Varela et al. (1991) enable action to be perceptually guided. This principle of learning as *developing spatiotemporally patterned sensorimotor activit*y should obtain whether actions are performed overtly on material features of the environment or covertly on imagined objects (Ingram et al., 2022), whether these actions are the performance itself or a form of simulation (Kirsh, 2010), whether the objects in question are material or symbolic (Landy & Goldstone, 2007), and whether the agent's intentionality is enacted directly or semiotically mediated (Shvarts & Abrahamson, 2023).

Beyond its intrapersonal organizational function, rhythmicality emerges spontaneously as regulating interpersonal joint action—when two or more agent are to coordinate their behaviors, it is advantageous to fall in step with each other (Sebanz & Knoblich, 2009; in mathematics education, Nemirovsky et al., 2013; Abrahamson et al., 2022). Basso et al. (2021) conjecture, therefore, that rhythmicality spans the biological gamut of cellular, neural, sensorimotor, cognitive, and social behavior. Interpersonal generation of rhythmic joint actions could serve also as a semiotic resource for prospectively indexing common referents, enriching a milieu's multimodal linguistic repertory (see, for example, Co-Operative Action, per Goodwin, 2013).

What might all this mean for a functional interpretation of stimming? From an enactivist perspective, our ecological inclination to perform rhythmically repetitive motor actions as a form of regulating our grip on the world suggests that stims, as temporally iterated actions, themselves

afford meaningful engagement, even if these meanings are obscured from an observer. That is, rhythmic activity of stimming constitutes a means of making sense of the world: *thinking.*

2.4 Stimming as Embedded: Cognition as a Dynamical System

The theoretical notion of embedded cognition disrupts the presumed functional hierarchy between peripheral and central components of the cognitive mechanism. In particular, *complex dynamic systems theory (DST)* models cognition as a self-organized (rather than centrally controlled) dynamic system in flux that is dynamically reconfigured as it adapts to act within the ecology (Richardson & Chemero, 2014). In this view, the dynamically stable cognitive system can be perturbed by even low-level microinteractions, pushing it to transition into a new dynamically stable state, requisite for motor and cognitive development (Smith & Thelen, 2003). From a complex dynamical systems view, the cognitive system is made up of numerous elements that are embedded within and open to the environment (Smith & Thelen, 2003). Through interaction, these elements self-organize to form emergent coordinations.

From this perspective, the peripheral bodily activity of stimming can be construed as an interactive element of the dynamical cognitive system. In complex systems, it is the interactions among all the elements that give rise to emergent dynamics: "no single element has causal priority" (Smith, 2005, p. 278). As such, stimming is as much a part of emergent cognitive coordinations as central elements like the brain. The chaotically repetitive patterns of stimming should be endorsed as an analytically insightful constituent of the cognitive system's overall dynamics.

Consistent with this perspective, research shows that higher-level cognitive processes are reflected in bodily activity: mind wandering is associated with an increase in non-instrumental movements (Carierre et al., 2013, Dias da Silva et al., 2022). Mind wandering has been reevaluated from an embodied perspective as enriching cognitive processes underlying task efficacy (Trasmundi & Toro, 2023). For example, in the context of reading, the most effective approach may not be to read at a constant fluent rate, but rather to multi-actionally draw upon goal-oriented reading and complementary mind-wandering processes (Trasmundi & Toro, 2023, p. 6):

> Increasing or lowering the cognitive control over thoughts and modifying the environment in ways in which seems fit to the demands of the activity at hand are embodied processes that encompass the nervous system, the body, and the environment in which a person is embedded.

Stimming, then, can express and impact the state of the overall cognitive system as it dynamically couples and recouples with the environment.

2.5 Stimming as Emotional: Reflecting and Regulating System Dynamics

Recasting stimming as part of a complex dynamical cognitive-affective system, we recognize the prospective functionality of stimming in supporting and maintaining that system within a desirable homeostatic, dynamically stable attractor state (e.g., Kelso, 2000). Such a view is consistent with occupational therapy models of stimming as seeking an optimal level of sensory stimulation to regulate arousal (e.g., Dunn, 1997). Autistic[6] and non-autistic adults self-report that stimming/fidgeting helps them to release excess energy, maintain balance and concentration

[6] We use identity-first language here in respect of preferences expressed by a majority of autistic adults (J. Sinclair, 2010; Taboas et al., 2023), but acknowledge and respect that language preferences are not homogeneous.

(Charlton et al., 2021), and support focus and optimal arousal (Karlesky & Isbister, 2016). Supporting this view, stimming increases when access to sensory stimulation is limited, such as when an infant is confined to a playpen or chair (Thelen, 1980), and is reduced in contexts providing ample stimulation, such as exercise (Liu et al., 2015). Animals in captivity have likewise been observed to stim, particularly when understimulated by their environments or lacking their regular dosage of exercise (Davis et al., 2004). Furthermore, stimming affects the balance of activity in the sympathetic "fight or flight" nervous system and the parasympathetic "rest and digest" nervous system (Lane et al., 2019). In addition to alertness, stimming may regulate emotional state. Sensation activates brain networks that produce emotional responses (Rodriguez & Kross, 2023). In particular, displacement behavior is inversely related with stress, suggesting stimming may help with stress regulation (Mohiyeddini & Semple, 2013), consistent with self-reports of stimming supporting calm and pleasure (Karlesky & Isbister, 2016).

Stimming, then, interacts with the nervous system and brain to modulate arousal and emotion. These, in turn, affect cognitive processes such as learning, judgment, and memory (Storbeck & Clore, 2008). In sum, stimming expresses cognitive dynamics and dynamically supports adaptive human cognitive system function.

2.6 Stimming as Ecological: Elicited by the Physical and Cultural Environment

A view of cognition as *ecological* (rooted in *ecological psychology*, Gibson, 1966) conceptualizes perception in terms of *affordances*: patterned perceptual information in the environment experienced as opportunities for action (Gibson, 1966). Motor actions respond to perceived environmental invitations (Heft, 1989). This perspective can shed light on the presumed-peripheral activity of stimming. This perspective suggests a view of stims as solicited by qualities of the environment presented to an individual. We propose that, given stimming's functional role, agents can perceive stim affordances in their environments. For example, a hard surface might afford rhythmic tapping, whereas a malleable object might afford squeezing. The type of sensory stimulation offered by interaction with an object or environmental feature would determine its efficacy in supporting self-regulation.

From early life, different types of movement differentially affect arousal: for example, rocking infants intermittently and vertically promotes bright-alert behavior, whereas rocking them continuously and horizontally promotes drowsiness (Byrne & Horowitz, 1981). Additionally, input to the somatosensory senses including proprioception, tactility, and vestibular function, as well as rhythmic movement, play a crucial role in nervous system regulation (Dahl Reeves, 2001; Lane et al., 2019). The objects available in the immediate environment, such as a pen, and their sensory features, such as the pen's weight and length, invite possible movements, such as twirling, clicking, or tapping (Karlesky & Isbister, 2016). For example, the rhythm of background noise might affect that of a fidget, or a recent experience, such as hearing the word "beetle," might elicit a given stim, such as humming "Love Me Do." Stimming is elicited by features of the environment that offer up opportunities for regulatory sensorimotor activity.

Stim forms are further shaped and constrained by an individual's personal and cultural history and intrinsic dynamics. Individuals favor different forms of fidgeting contingent upon their emotional state: squeezing when angry, versus clicking, pressing, or tapping when bored (da Câmara et al., 2018). Furthermore, two individuals presented with the same stimuli may stim differently. For example, people engage in more non-instrumental movement when listening to a favorite song than to other music (Witchel et al., 2016). Personal meaning adds a dimension to stims: for example, fidgeting with a wedding band may also carry the reminder of a loved one

(Karlesky & Isbister, 2016). Cultural norms can act as additional constraints upon stim possibilities and add layers of meaning to repetitive action. For example, repetitive motor actions, such as rocking and touching beads, have a long history within spiritual and contemplative practices like prayer. Stim forms are thus imbricated in *embodied semiosis* (Nolan & McBride, 2015), wherein the body is the nexus of lived and cultural experience.

2.7 Stimming as Extended: an Intercorporeal Resource for Interaction and Communication

The idea of extended cognition highlights cognition as inclusive of objects (Clark & Chalmers, 1998) and even other people (Gallagher, 2013). Stimming has been historically approached as an individual behavior serving an intrapersonal regulating function (if, indeed, granted a function), whereby repeated vocalizations and/or motor actions constitute a multimodal soliloquy. Yet, stimming in the presence of other people may also serve an interpersonal expressive function, whether this function is premeditated or emergent. Advocates in the autistic community and critical autism scholars, grounding within autistic sociality (Milton et al., 2012), propose stimming to be an intrinsic expression of embodied autistic semiosis (Nolan & McBride, 2015). Stimming thus offers a prospective way of being with others that is communicative and socially valuable (Bakan, 2014).

Per the *ethnomethodological approach to conversation analysis* (EMCA, e.g., Mondada, 2014), conversing individuals use a variety of culturally established multimodal utterances and kinesics, including speech, gesture, face movements, posture, eye contact, and a range of semiotic actions in any medium (e.g., doodling), to manage turn-taking while maintaining an apparent interpersonal consensus of common ground. We put forth stimming as one such multimodal utterance. Researchers of non-speaking autistic individuals have drawn on conversation-analysis and Co-Operative Action methodologies to demonstrate that stimming can serve as an interactive resource for engaging in collaborative joint action (Chen, 2016; Chen, 2022; Dickerson et al., 2007). For example, Dickerson et al. (2007) found that autistic children responding to math questions effectively used repeated tapping on focal objects such as flashcards to display their active engagement and to indicate upcoming talk. Chen (2022) found that autistic children's stims could become the substrate for co-constructed, improvisational interaction with caregivers. Conn (2015), in analyzing autobiographies of autistic authors, identifies a sensory and body based *autistic intersubjectivity* whereby "people are physically attuned to each other, act in synchronicity and enjoy each other's presence, but carry out interaction in non-face-to-face and largely non-verbal ways" (p. 1202). As such, stimming could be viewed as a broadly valuable means for social participation in cultural practices, wherein two or more individuals partake in coordinated intercorporeal enactments.

Indeed, some forms of stimming are morpho-dynamically isomorphic to *beats*, one of four types in a broadly endorsed gesture taxonomy (along with iconics, emblems, and metaphorics, McNeill, 1992). Co-speech beats, such as emphatically hitting the air in time with vocalized utterance, serve as semantic augmentors of the spoken content itself, but they can also serve the pragmatic function of maintaining our conversational turn while we recall a word or formulate an idea. The kinesiological resemblance of some neuro-atypical stims to neurotypical individuals' routine conversational practices suggests the plausibility that stims could be interpreted and endorsed more broadly as communicative actions, paving pedagogical pathways for normalizing stimming as part of interaction within communal activities. By conceptualizing

stimming as a potentially interactive resource (Chen, 2021, 2022), we hope to shift the perspectives of both the theoretical and pragmatic fields of educational psychology.

In sum, from an embodied perspective, stimming expresses and regulates the dynamics of the cognitive-affective-bodily system, influencing thinking and participating in the development of cognitive structures, elicited by the physical and cultural environment and poised to contribute to social interaction. Such a view of stimming is congruent with empirical work across the cognitive sciences. Wth this revised model of stimming, we turn our attention to educational practice.

3. Tapping the Untapped: Implications for Instructional Design

> "If 'stimming' was an acceptable and valued aspect of social and cultural behaviour, how might it be incorporated into design or social practices?" (Nolan & McBride, 2015, p. 1075)

Given our revised model of stimming, how might we adapt learning contexts to embrace and leverage stimming as part of the learning process? We recall that currently in mainstream classrooms, physical movement—be it idiosyncratic stimming or conceptual enactment—is neither appreciated nor legitimized, let alone encouraged and leveraged as cognitive activity (Montessori, 1967). Consequently, thinking-by-moving is suppressed into covert processes such as minute gestures or imaginary actions. We propose that inviting thinking and stimming into the classroom discourse as overt physical behaviors offers new avenues for instruction that are both more inclusive and more effective (see Feucht 2010, on classroom *epistemic climate*). Such an invitation would occur through *instrumenting* (Vérillon & Rabardel, 1995) stimming with media that facilitate stimming activity. First, depathologizing and normalizing stimming could spare students the cognitive resources currently allocated to suppressing their need to stim and enhance stimming's impacts on arousal and mood. Further, considering *the functional superimposition of stimming and thinking* as cognitively allied processes could open new horizons for instructional design, where the two processes are integrated within meaningful action (i.e., *instrumentalized*, per Vérillon & Rabardel, 1995). This section will elaborate on the pedagogical conceptualization of stimming-as-thinking, demonstrating its potential implementation with an exploratory design for learning mathematical functions: BBM (described in Section 3.1). We draw examples from the BBM project and data to propose and elaborate upon three guiding heuristics for pedagogical design: offering stim affordances (Section 3.2), legitimizing stims as a discursive resource (Section 3.3), and leveraging stims as epistemic actions (Section 3.4).

3.1 Illustrative Empirical Context: Balance Board Math

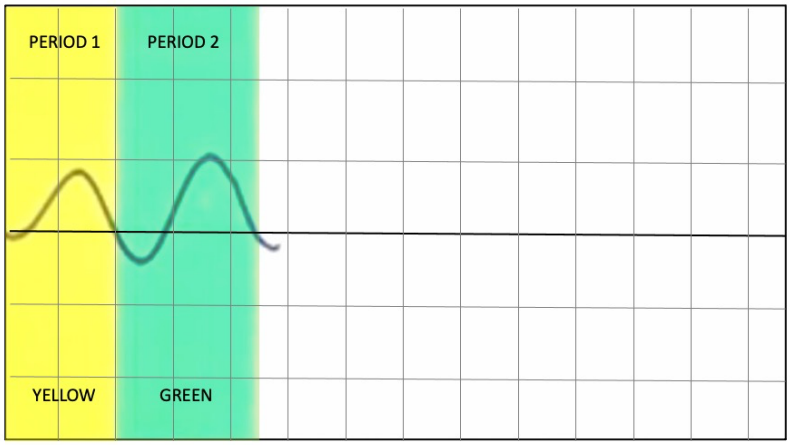
We draw from multiple iterations and studies within a larger design-based research project, BBM, to ground and illustrate our conceptual proposal. The BBM project investigates the design conjecture that movement for learning and movement for self-regulation can be integrated to strengthen instructional efficacy. BBM focuses on the vestibular (balance) modality as an untapped resource for both regulation and learning. BBM activities (across design cycles) invite children to rock on large wooden balance boards to explore mathematical concepts. Expressed in terms of SpEED parameters (Tancredi, Chen, et al., 2022), BBM designs novel *media*: a sensor-equipped balance board and interface facilitate and capture rocking movements, to leverages the vestibular *modality*, commonly engaged in stimming but uncommonly engaged in learning

contexts, towards bringing the stimming form of rocking into discourse as a type of the gestural *semiotic mode.* In so doing, BBM draws thinking and stimming processes, typically separated and covert, both together and into the open.

Working with BBM, children iteratively generate graphs within different digital environments, projected on the screen in front of them, that facilitate their discovery and control of different mathematically salient graphical properties, such as the frequency and amplitude of sinusoidal functions (see dissertation paper 1). For example, in one activity, Frequency Exploration, each time the board rocks left and right and returns to center (one period), the screen shows how the background behind that period changes color depending on the period length (Fig. 2). Graphers also hear distinct tones each time they reach a local maximum or local minimum and each time they pass through the *x*-axis; and different chords corresponding to different colors when they complete one graphical period. Children work to understand the colors and ultimately, to control the period (and consequently frequency) of their rocking to generate a consistent color, through rounds of graphing and reflection.

Figure 2

Example of a Graph Being Generated in the BBM Frequency Exploration Activity



Note. The focal period length here was set to 2.5 graph units. The graph is generated from left to right. First, the grapher rocked forward (in yellow, rise to a peak) and then backwards (still in yellow, descent toward a trough), completing one cycle (Period 1) in two time units on the *x*-axis (note the gridlines). Upon completing this period, the area behind that portion of the graph turned yellow because it was produced by rocking at a slightly higher frequency than the target frequency. Next, they rocked forward and backwards once again, this time taking longer to complete the movement (just under 2.5 units on the *x*-axis) (Period 2), corresponding to the focal frequency set for this round. This time, the screen turned green.

BBM is a proof-of-concept case of inclusively leveling the playing ground through creating enactive-inquiry resources that harness stims as content-oriented epistemic actions.BBM follows SpEED principles, namely: (1) learning happens through the body's sensorimotor engagement with the world; (2) learning begins from learners' existing embodied resources; and (3) instruction must flexibly adapt to learners' sensorimotor diversities (Tancredi, Chen, et al., 2022). As such, BBM *leverages stimming as an embodied resource for learning*, recognizing differential stimming as an axis of diversity to be accommodated in inclusive instruction. In

outlining implications of an embodied model of stimming for pedagogical design, we share a discussion of the BBM implementation reflective of each heuristic below.

3.2 Instrumenting Stimming: Offering Stim Affordances

We propose that it is the work of pedagogy to recognize the *stim affordances* that we present to children, attending to the sensory qualities of instruction, including the type of sensory stimulation (rhythm and intensity) and the sensory systems engaged (visual, auditory, etc.). Somatosensory systems including proprioception and balance are often the focus of stimming, so these are of particular importance here (see dissertation section 4). Objects by virtue of their design may invite or inhibit rhythmic, repetitive stim movement. Some in human-computer interaction have designed fidget widgets to offer stim affordances in the margins of computing spaces (Karlesky & Isbister, 2016). We call for such offerings to be not only in the margins but also part of the central pedagogical activity. It is also important to recognize that surrounding environments may present unpleasant, overwhelming, or impoverished stimuli. Although offering stim affordances may help make such environments more tolerable, the surrounding environment, too, must be addressed to truly offer children sensorially equitable learning experiences. Instructional design must create a context of sensory comfort as a starting point, offering stim affordances as one avenue for fulfilling such sensory comfort.

BBM's physical and digital features as well as usage protocol were iteratively designed to instrumentalize stimming by offering the affordance of rocking, a common stim (Table 1). The media configuration was adapted to match the sensory stimulation to the inner ear elicited by stim-rocking. For example, rather than have users stand on balance boards, wherein the head typically maintains a fixed orientation, users are invited to sit, providing rhythmic shifts in head movement. The board's base, curved in a single axis rather than multiple axes, facilitates consistent, rhythmic back-and-forth rocking movement commonly engaged for stimming. The use of a board with embedded handles and the addition of bumpers under the surface of the board allows users to safely rock to a range of angles all the way to about 45 degrees each way. Users can adjust the intensity of vestibular stimulation by self-modulating their rocking angle, ranging from slight leans left to right to sweeping rocks all the way to the board's bumpers. The BBM protocol includes a calibration procedure whereby users rock at rates and intensities comfortable to them, with graphing speed and sensor sensitivity parameters set accordingly. Thus, users can modulate the intensity of vestibular stimulation according to their sensory comfort to participate effectively. Additionally, users are welcomed to remain on the board as desired between activities during reflection and discussion, implicitly endorsing their freedom to engage in rocking movements anytime. As such, BBM offers one instantiation of how a climate of stim acceptance, found to support open stimming (Kapp et al., 2019), might be encoded into instructional materials and practices.

Table 1
BBM Design Features Offering Stim Affordances

BBM Design Feature	**Stimulatory affordance**
Seated board position	Vestibular-activating changes in head position
Board curved in single axis	Rhythmic back-and-forth movement

BBM Design Feature	Stimulatory affordance
Bumpers, handles	Safe access to broad range of vestibular stimulation intensities
Calibration protocol with adjustable graphing sensitivity and speed	User-led stimulation intensity
Continuous board access	Implicit endorsement of rocking movement throughout activities and discussion

3.3 Interactional Stimming: Legitimizing Stims as a Discursive Resource

When welcomed in the pedagogical sphere, stimming is an overtly observable feature of interactions. Stimming's instrumentalization for thinking in turn makes stims a prospective resource for learners and educators to make their thinking visible to peers and to communicate multimodally. Prior work has shown how designing for joint stimming is generative for inclusive, creative interaction (Chen, 2022). In the BBM project, the interactional dimension of stimming came to the fore when participants in an early iteration took turns on the board. The children spontaneously engaged in dialogue to share, refine, and test their graphing hypotheses. This led, in later BBM iterations, to the inclusion of a second balance board and collaborative activities for simultaneous and joint graphing, enabling peers to each have continuous and shared access to rocking. Thus, each child's movements are overtly observable to their peers, and rocking is available as an ongoing interactional resource for joint reasoning. As seen in autistic play (Conn, 2015), BBM shows one way children could be invited to engage in joint stimming.

3.4 Instrumentalizing Stimming: Leveraging Stims as Epistemic Actions

An embodied view of stimming suggests that stimming is not independent of enactive thought. As such, we propose a pedagogy wherein stimming is embraced as an active part of learning itself: *stimming-as-thinking*. In such an approach, pedagogical activities themselves are stimmy, offering means to engage stimming not merely for regulation but as *conceptual* exploration.

The design conjecture that the stim of rocking can be instrumentalized for learning mathematical concepts has guided BBM's iterations. BBM offers children the ability to observe their rocking activity through generating a graphical trace of their activity in real time. Graphs remain observable after they are generated, allowing children to reflect on their qualities. These characteristics of BBM allow children to bring the often backgrounded process of rocking to the fore of their attention. By way of analogy, imagine a person walking on the beach, who looks back to reflect on their evenly spaced footprints in the sand—in BBM, one looks at one's graphical trace to consider mathematical properties of one's rocking. Artifacts and tools from the discipline of mathematics, such as a cartesian grid and numbers, become, in this context, resources for refining and discussing one's rocking movements, consistent with the *embodied design* principles (Abrahamson, 2014). Additionally, BBM ensures the *congruence* (Johnson-Glenberg & Meganow-Romanowicz, 2017) between stim movements and focal concepts. For example, children explore the concept of amplitude as the degree of tilt in either direction of flat (0), frequency as their rate of rocking, and slope as the literal slope of the board's surface. Rocking movements are not merely a stimulating input device, but rather a multimodal experience of both the focal concept and the mathematical representation of that concept (i.e., the

slope of the graph line). In this context, learners can modulate the amplitude and timing of their rocking to develop new movement patterns as the basis for emerging graphing concepts, just as the infants in Thelen et al.'s studies (1993) modulated their stereotyped arm movements in amplitude and timing to become refined, effective movement patterns.

We put forth BBM as an example design where stims become available as an interactional resource, dynamically enfolded into not only children's thinking, but also their communication, supporting the coordination of joint action and preceding, illustrating, or complementing the ideas they express verbally. When instrumentalized, stimming can play a functional role in facilitating children learning to move in new ways that bear import for mathematical thinking.

4. Conclusion and Implications

A 4E analysis of the historically marginalized activity of stimming yields precepts for a new model of stimming as (a) reflective and regulatory of the cognitive system, (b) influential upon thinking, (c) forming a substrate for complex actions and concepts, (d) responsive to properties of the physical and cultural environment, and (e) socially communicative. Stimming offers an essential means to organize coherent sensorimotor experience. Stimming actions, articulated across time, space, and people, can structure the flow of internal and external sensory input and harmonious interaction with the environment. We have highlighted preliminary empirical substantiation for this view across multiple fields of cognitive science.

This theoretical analysis reveals stimming to be an interactional resource with not only social but also epistemic potential when endorsed as such. It is but an artifact of cultural-historical factors that this form is presently unrealized. When we center the cognitive needs of neurodivergent learners in instructional design, we stand to enrich learning opportunities for everyone. We distill the implications of this perspective, as demonstrated by our research excerpts, into the following set of tentative pedagogical design heuristics for inclusive learning environments:

(1) ***Instrument*** **stimming: offer stim affordances**. This entails designing environments and tools with an eye towards their stimulatory possibilities: the opportunities that they present to tap, bounce, spin, and move, ideally catering to a range of possibilities to accommodate the diversity of user's sensory profiles. For example, the BBM balance boards enable children to rock at different intensities, self-differentiating for their own sensory needs.

(2) ***Interactional*** **stimming: legitimize stims as a discursive resource**. This entails attending to learner's stims in learning contexts with the assumption that these stims are part of nonverbal communication and can serve as a window into the dynamics of a learner's thought. For example, children in BBM can rock in ways that make visible their engagement with others' ideas.

(3**)** ***Instrumentalize*** **stimming: leverage stims as epistemic actions**. This entails teaching in such a way that children's stims are made relevant to the concepts being taught: for example, using rocking angle to teach slope. By providing children with means to document, modulate, and reflect on their stims, they can become means to explore pedagogical concepts. For example, in BBM, children can generate graphs of their rocking stims, enabling them to ultimately use rocking as a way to reflect on graphing concepts such as frequency and amplitude.

We have offered axiological, theoretical, and empirical arguments for depathologizing stimming across special and general education. Therefore, we challenge prevalent perceptions of

stimming as nonessential, involuntary, and task-irrelevant. Instead, we theorize stimming as maintaining the dynamic equilibrium of the cognitive–affective system and, when enabled, intrinsically driving problem solving and social interaction. Actualizing the inclusive affordances of stimming in educational practice will depend on their further exploration and evaluation.

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Rocking the Sensory Foundations of Instructional Design: Adaptive Balance Stimulation for Inclusion of Sensory Neurodiversity

This conceptual-empirical paper introduces the rationale and first demonstration of *sensory-adaptive embodied design.* I begin with an analysis of classroom sensorium and how current and emerging norms, practices, and technologies affect learners' sensory regulation, focusing on the vestibular, proprioceptive, and tactile senses. I propose that the intersection of embodied cognition and sensory regulation perspectives compels a revised view of the relationship between students' bodies and their learning, wherein sensory regulation must be recognized as an ongoing dimension of learning, to be nourished and leveraged *within* instruction. I investigate this genre through analysis of BBM data focusing on the vestibular sensory-regulatory activity of bodily rocking as a resource for conceptual learning in mathematics. I found that when using BBM, neurodivergent and neurotypical learners engaged in overt vestibular activity within and beyond structured activities and discourse. Rocking was instrumentalized as epistemic actions that became part of their mathematical discourse. This study describes a proof-of-concept somatocentric conceptual learning activity. Findings are consistent with sensory-adaptive design supporting inclusive instruction.

> "I remember as a child spinning all the time and loving spinning and loving swinging and feeling that movement all the time, but then I also realised that there was a point where it wasn't acceptable to be spinning anymore … so it actually still feels glorious if there's nobody around and I can skip or I can spin and it's like I'm breaking the rules."
>
> – anonymous autistic[7] adult (Kapp et al., 2019, p. 1788)

1. Introduction

Ellie sits in her 4th grade classroom, in a hard plastic chair with metal legs she's been in for an hour. It's the late morning, and she and her peers have been assigned to complete a lab on the computer. Her peers a couple feet away are having a hushed conversation about their weekend birthday party. The overhead lights emit a steady hum. The light from her peers' screen flashes as they advance through the steps. Beyond them, Ellie catches a glimpse of motion through the window as someone walks by outside. Ellie leans backwards, tilting her chair onto its back legs. The feeling wakes her up. She rocks forward and does it again. The lights don't sound as annoying when she's rocking. "Ellie, stop that," her teacher admonishes. Ellie checks the digital clock on her screen: still 45 minutes till lunch.

From rigid seats to clacking keyboards, classrooms create distinct sensoria for learners like Ellie, further circumscribed by behavioral norms (sit facing forward—do not lean backwards in your chair; use a pen to write—do not tap it against the desk). Certain sensory channels, primarily vision and audition, are considered epistemically relevant: a visual diagram, a teacher's voice. Other sensory channels—touch, proprioception, and balance—are presumed irrelevant, as reflected in the minimal stimulation to these channels provided by most formal learning contexts (Miller Kuhaneck & Kelleher, 2015). For the purposes of this paper, we will call these senses *regulatory senses*, because stimulation to these channels regulates learners' arousal and affective state (Lane et al., 2019). Think, for example, of a parent soothing a child by rocking them (Korner & Thoman, 1972), a child getting excited by jumping on a trampoline (Budzynski-

[7] I use identity-first rather than person-first language here in keeping with the source article. For more on language preferences in the autistic community, see Taboas et al. (2022).

Seymour et al., 2019), or a friend offering a comforting hand (Rodriguez & Kross, 2023). People spontaneously engage in *sensory regulation*: moving in spontaneous ways, such as fidgeting, that provide sensory stimulation to regulate their attention and arousal. The forms and intensities of sensory regulation vary because people's sensory neurology varies: for example, swinging on a hammock may make one child queasy and another energized; a hug may soothe one child and overwhelm another. As embodied cognition theory (Varela et al., 1991) sheds light on the importance of bodily activity to cognition, and inspires new sensorimotor-based instructional approaches, the regulatory senses deserve new attention (Tancredi & Abrahamson, under review). I propose that sensory regulation draws into question: (1) the neglect of the regulatory senses, and particularly the vestibular sense, in educational practice; and (2) the viability of unitary sensory norms and practices for equitable inclusion of all children.

This conceptual–empirical design-based research paper motivates and investigates whether and how regulatory sensory activity can play a role in learning. Motivated by the intersection of work on embodied cognition and on sensory neurodiversity, I study learners' sensory regulation activity in the context of a design-based research project focused on the pedagogical potential of sensory stimulation to the vestibular sensory system: Balance Board Math (BBM) (Tancredi, Wang et al., 2022; Tancredi, 2024). In BBM, learners rock on sensor-equipped balance boards to experience key features of graphs through their sense of balance, such as experiencing slope as steepness. BBM activities are designed to foster the perception of mathematically-relevant aspects of graphs, such as the frequency, amplitude, and phase of a sinusoidal function, through movement. BBM is a first proof-of-concept for *sensory-adaptive embodied design*: instructional design that enables learners to self-adjust the sensory stimulation they experience as part of math learning activities.

In section 1, I outline the design problem of why extant educational theory and practice need a sensory-regulation infusion. The focus of this paper is on the general design problem of integrating sensory seeking into content learning; the math content addressed by the design offers a context for this primarily sensory investigation. In section 2, I conduct a domain analysis of how current solutions accommodate sensory diversity, including both common classroom sensory accommodations and *embodied design* work in the learning sciences that incorporates movement into instruction. I highlight the value and limitations of these approaches. In section 3, I describe BBM's core conjecture: that regulatory and cognitive aspects of instructional design can be integrated. I describe the BBM design as a case of sensory-adaptive instruction centering a regulatory sense. In section 4, I describe data collection and analysis methods within the broader BBM project. In section 5, I describe mixed-methods findings from BBM participants' sensory activity on whether and how children leverage the sense of balance in the case of BBM. I conclude with implications for the potential of sensory-adaptive embodied design to advance and inform inclusive instruction.

1.1 Design Problem

In this section, I describe the sensory systems at play in sensory regulation, with a focus on the under-researched sense of balance. I highlight how neurodivergent students' sensory regulation needs are often not met in classrooms, and how this reveals a broader issue with sensory dimensions of current educational practice.

1.1.1 Beyond and Beneath the "Five" Senses: Sensory Regulation and the Sense of Balance.

Western sensory discourse favors the five senses of vision, hearing, touch, taste, and smell, but these are far from the only human senses. *Sensory regulation* (described further in section 1.2.2), depends upon the senses of proprioception (sense of body-in-space), vestibular function (rotation and acceleration), and tactility (pressure), as well as interoception (respiration, heartbeat, hunger and thirst, and autonomic nervous system responses) (Lane et al., 2019, Wilbarger & Wilbarger, 1991). The regulatory senses form the foundation for our experience of the world and our affective experience in it.[8] The pathways by which regulatory senses impact arousal are well-documented: for example, the tactile system has two types of pathways: anterolateral pathways that process information about light touch and pain, and dorsal pathways that carry information about touch pressure and vibration (McGlone & Reilly, 2010). The former pathway feeds into the reticular formation, a brain structure that drives high alertness and arousal. The latter pathway, meanwhile, actually inhibits activity in the reticular formation. Occupational therapy research has studied the implications of these pathways for supporting sensory regulation. For example, given the calming effect of dorsal tactile stimulation, deep pressure has been found to reduce aversive reactions in highly tactile-sensitive individuals (Dahl Reeves, 2001). Other features found to impact sensory regulation include the variability, duration, rhythmicity, and intensity of sensory inputs (Dunn, 2008).

Perhaps most neglected among the regulatory senses in education is the sense of balance. To date, there has been 10 times more research on the visual system than on the vestibular system (Mast et al., 2014). Our sense of balance and self-movement depends upon the vestibular system in the inner ear. Displacement of hairs by a membrane in two otolith organs detect linear acceleration (Khan & Chang, 2013). Fluid in three orthogonally positioned semicircular canals senses movement in three orthogonal planes of rotation: pitch, yaw, and roll. The vestibular system supports appropriate orientation of the eyes and body in response to head motion. It integrates with the brain at multiple levels including the brainstem and cerebellum as well as the cerebral cortex. The vestibular sensory system is implicated in both regulation (Lane et al., 2019) and cognition (Hitier et al., 2014; Wiener-Vacher et al., 2013). Neurocognitive work has found that otolith stimulation affects mental object manipulation, semicircular canal stimulation impacts mental transformations, and vestibular stimulation improves spatial and verbal memory, and that vestibular function affects spatial and numerical cognition, arithmetic, and reading (Mast et al., 2014). Balance skills and experiences have even been specifically shown to impact abstract conceptual domains such as spatial and proportional reasoning (Frick & Möhring, 2016) and social justice (Antle et al., 2013). Beyond the vestibular sense itself, the concept of balance is sometimes evoked in discussion of mathematical concepts such as balancing equations (Lakoff & Nunez, 2000), as well as in visual design (Dondis, 2023). Given its affective and cognitive importance, the paucity of work on the vestibular sense in learning is a notable gap.

[8] This is not to say that the other senses do not impact learners' affective state and performance; indeed, stimuli to other sensory channels such as vision and audition can likewise be regulatory (soothing music; a sunset) or dysregulating (a sudden bang; flashing lights).

1.1.2 Sensory Neurodiversity and Minoritization.

Whereas sensory regulation is always ongoing for all people, its impact in today's classroom is not felt equally. This is because learners have different sensory regulation needs, which may be more or less fulfilled by current environments and practices. Neurologically, students experience the same sensory input differently (Dunn, 1997). For example, the flicker of fluorescent light may be imperceptible for most students, but for some, it is intolerable. The extremely limited vestibular stimulation offered by sitting still for all but 3 minutes a day, as expected in some classrooms (Dunn et al., 2012), only moving when walking from class to class or during physical education (Miller Kuhaneck & Kelleher, 2015), may be feasible for many children, but stifling for others. Those with higher or lower neurological thresholds for sensory input than the sensory majority may find themselves at odds with their learning tasks and environment. As Meryl Alper writes, "though each body will experience some sensory misfitting, certain bodies are more likely than others to consistently misfit and face significant consequences for doing so in public and private spaces" (2023, p. 174). I propose that our typical classroom instructional environments and practices cause the *sensorial minoritization* of learners whose sensory needs are not met. This process of minoritization is likely exacerbated for students already minoritized along other axes of their identity.

Students whose sensory profiles are not average are especially vulnerable to sensorial minoritization. Sensory atypicalities are associated with a number of neurodivergences, including autism (Little et al., 2018), learning disabilities (Dove & Dunn, 2008), and attention deficit hyperactivity disorder (ADHD; Dunn & Bennett, 2002; Little et al., 2018). However, these labels do not denote homogeneous sensory profiles; indeed, four distinct sensory sub-profiles have been documented amongst autistic people alone (Tomcheck et al., 2018), and Sensory Processing Disorder appears in children without special education diagnoses, and is estimated to be present in 5-10% of children in the United States (Ahn et al., 2004). Amongst children with ADHD, atypical sensory processing is correlated with lower daily function (Mimouni-Bloch et al., 2018). Amongst children on the autism spectrum, sensory differences have been found to explain 47% of variance in academic performance (Ashburner et al., 2008). Amongst the general population, sensory profiles predict depression, resiliency, and externalizing behavior (Dean et al., 2018). These findings support the idea that sensorial minoritization has a pervasive impact on wellbeing and learning outcomes.

We have identified both under-researched sensory systems and vulnerable students who warrant further attention in educational research and practice. Next, we will mobilize and integrate theories concerning the body and learning to generate hypotheses about the regulatory senses and sensory diversity in educational spaces.

1.3 Theoretical Framework: Embodied Cognition and Sensory Regulation

The function of movement is modeled separately in research on embodied learning and on sensory regulation, yet both functions are at play simultaneously during conceptual learning.

1.3.1 Embodied Cognition.

Cognitive science's embodiment turn highlights the body's role in cognition (Newen et al., 2018; Shapiro, 2014). Embodied cognition theory posits that our bodies' interactions with the environment are centrally constitutive of how we think (Gallese & Lakoff, 2005; Varela et al., 1991) and consequently, how we learn (Hall & Nemirovsky, 2012; Macrine & Fugate, 2022; Shapiro & Stolz, 2019), including in the domain of mathematics (Abrahamson & Sánchez-

Garcia, 2016; Hutto et al., 2015; Lakoff & Nuñez, 2000; Nuñez et al., 1999; Roth, 2001). Empirical work inspired by this perspective has found that even basic physiological processes such as respiration impact cognitive processes (e.g., Varga & Heck, 2017).

In education, embodied cognition takes up and extends cognitive developmental psychology lineages theorizing the sensorimotor roots of conceptual learning (e.g., Piaget, 1936; Steffe & Kieren, 1994; Thelen & Smith, 1994). A key conjecture driven by this line of theory is that

> the actions one engages in mathematical work, such as writing down an equation, are as perceptuo-motor acts as the ones of kicking a ball or eating a sandwich; elements of, say, an equation-writing act and other perceptuo-motor activities relevant to the context at hand are not merely accompanying the thought, but are the thought itself as well as the experience of what the thought is about. (Nemirovsky, 2003, p. 109)

Empirical evidence suggests bodily activity impacts thinking: for example, research on gestures has found that they facilitate spatial visualization (Chu & Kita, 2011), guide problem-solving and idea generation (Broaders et al., 2007), and support retention (Cook et al., 2008). Embodied cognition suggests that how learners engage with learning artifacts, environments, and tasks, including the sensory modalities at play, changes their learning. Furthermore, it implies that the sensorimotor diversity of learners, including learners with disabilities, matters for teaching them accessibly (Tancredi, Chen, et al., 2022). If knowledge depends upon bodily experience, what does it mean for a sensorially diverse body of learners to learn using sensorially monolithic designs? I propose that in treating only limited sensory activity as useful and epistemically relevant (Abrahamson et al., 2018), extant sensory norms pose an issue of *epistemic injustice* (Fricker, 2007).

1.3.2 Sensory Regulation.

Sensory regulation denotes the impact of sensory experience on the nervous system's capacity for adaptive physiological responses. In sensory integration theory (Ayres 1974), sensory input is theorized to impact self-regulation neurally and behaviorally. For example, self-directed self-regulatory behaviors, such as sucking one's thumb or exercising, can reduce distress and control arousal (Dahl Reeves, 2001). Arousal, in turn, mobilizes resources (neuronal activation, neurotransmitter release, and cerebral blood flow) to support alertness that is commensurate with the task at hand. Thus, sensorimotor activity serves as nourishment that both supports arousal regulation and organizes the bodily-cognitive system to produce adaptive behavior (Schaaf & Miller, 2005). The regulatory senses, and particularly the vestibular sense, are integral to sensory regulation (Lane et al., 2019). For example, vestibular stimulation from rocking can decrease the stress hormone cortisol through inhibiting brainstem pathways (Sailesh et al., 2018).

Within this framework, people's sensory regulation needs are acknowledged to differ due to differences in their neural response to stimuli. Sensory modulation shapes how the sensory environment registers in their brain and nervous system, affecting self-regulation. The existence of such differences have been correlated with brain differences in sensory tracts (Chang et al., 2016; Green et al., 2015) and associated with different balance of activity in the sympathetic "fight or flight" nervous system vs. the parasympathetic "rest and digest" nervous system (Lane et al., 2019). In particular, differences in sensitivity to sensory input can lead to under- or overstimulation, driving responses such as withdrawal, distraction, or compensating with self-

generated stimuli to alert or calm the nervous system to an optimal state (Dunn, 2002). The latter includes *stimming*: spontaneous sensorimotor activity theorized to serve sensory regulation, such as rocking back and forth, fidgeting, tapping objects, or tearing paper. *Sensory seeking* activity such as fidgeting has been positively correlated with performance in some contexts, for example in working memory performance for children with ADHD (Sarver et al., 2015). Additionally, individuals, even as children, appear accurate in judging when a sensory experience is regulatory for them (Bagatell et al., 2010; Sutton et al., 2013). Thus, work on sensory regulation highlights the diversity of learners' experiences within the same classrooms, and the importance of sensory experience to their adaptive functioning.

Sensory regulation suggests that sensory dimensions of classroom tasks, environments, and norms regarding movement are always impacting what neuronal and physiological resources are made available to students in regulating their attention and emotions. The way students are invited to move in classroom settings affects their self-regulation. This suggests that instructional designs may inadvertently minoritize learners by enforcing a sensorially stunted, monolithic learning experience. Given the sensory diversity present in classrooms and the relationship between sensory regulation and performance, sensitivity to the full range of learners' sensory experiences and needs, institutionally and instructionally, is critical to inclusive instruction.

The regulatory senses, and the vestibular sense in particular, are a site of discrepancy between learners' sensory regulation needs and current educational practice, contributing to sensorial minoritization. At the same time, embodied cognition suggests that learners' sensory regulation practices, such as vestibular stimming, may also be an overlooked part of their *sensemaking during conceptual learning*. Next, we will look at how embodied learning and sensory regulation have informed practice to date, bringing each to bear on the other to highlight limitations and possibilities for informing inclusive pedagogical design.

2. Domain Analysis: Sensory Siloes

> "My hands are an automatic feedback loop, touching and feeling simultaneously. I think I understand the whole world when I rub my fingertips together.
>
> When I'm brought to a new place, my fingers tap out the walls and tables and chairs and counters. They skim over the paper and make me laugh, they press against each other and remind me that I am real, they drum and produce sound to remind me of cause-and-effect. My fingers map out a world and then they make it real."
>
> – Julia Bascom, Loud Hands: Autistic People Speaking, 2012

2.1 Sensory Interventions

Children affected by sensory-mismatched classrooms are not entirely without recourse. Some are granted specialized services, such as sensory therapies (Ayres, 1974; Bundy et al., 2019). Some are offered sensorially-rich, regulatory "sensory diet" activities (Wilbarger & Wilbarger, 2002) or access to sensory rooms (Botts et al., 2008) at prescribed times of day. Some receive accommodations: access to fidget toys, alternative seating such as an exercise ball chair (Bundy et al., 2019), permission to chew gum, or lap pads (Critz et al., 2015). I propose that when viewed from an embodied cognition perspective, these solutions do not grant students full access. These solutions leave academic instruction as sensorially restricted as ever. Rather than instruction that itself nourishes a child's sensory needs, sensory experience is offered as an add-on. Ellie might be invited to bounce on an exercise ball and continue a worksheet, or jump on a

trampoline for 3 minutes, then sit still and listen to the teacher describe a concept later. Embodied cognition suggests that sensory regulation should not be considered as occurring independently from cognition (Tancredi & Abrahamson, under review). If sensorimotor experience shapes cognitive structures, then this suggests that the form of surrounding activities implemented for regulatory purposes are not incidental to learning. Sensory stimulation cannot be tacked on as a supplement, like taking vitamins. Instead, sensory stimulation is inherently part of the unfolding cognitive process.

Sensory regulation theory suggests that it is important to differentiate instructional activities according to the types and intensities of sensory engagement they offer learners, attending specifically to the foundational regulatory modalities of vestibular function, proprioception, and tactility. The implication of sensory regulation theory is that sensory modality and intensity are critical axes that must adapt to each child in the moment. Alignment between a learner's sensory profile and the degree of sensory activation they experience stands to support attentive arousal: for example, vestibular-seeking students responded well to therapy ball seating, while non-vestibular-seeking students did not (Bagatell et al., 2010). Additionally, instructional designs would need to attend to the features that support nervous-system regulation, including rhythmicity (Dahl Reeves, 2001). I hypothesize that to optimally serve regulation and learning, instructional designs must be sensory-adaptive: able to meet each child's dynamic sensory regulation needs.

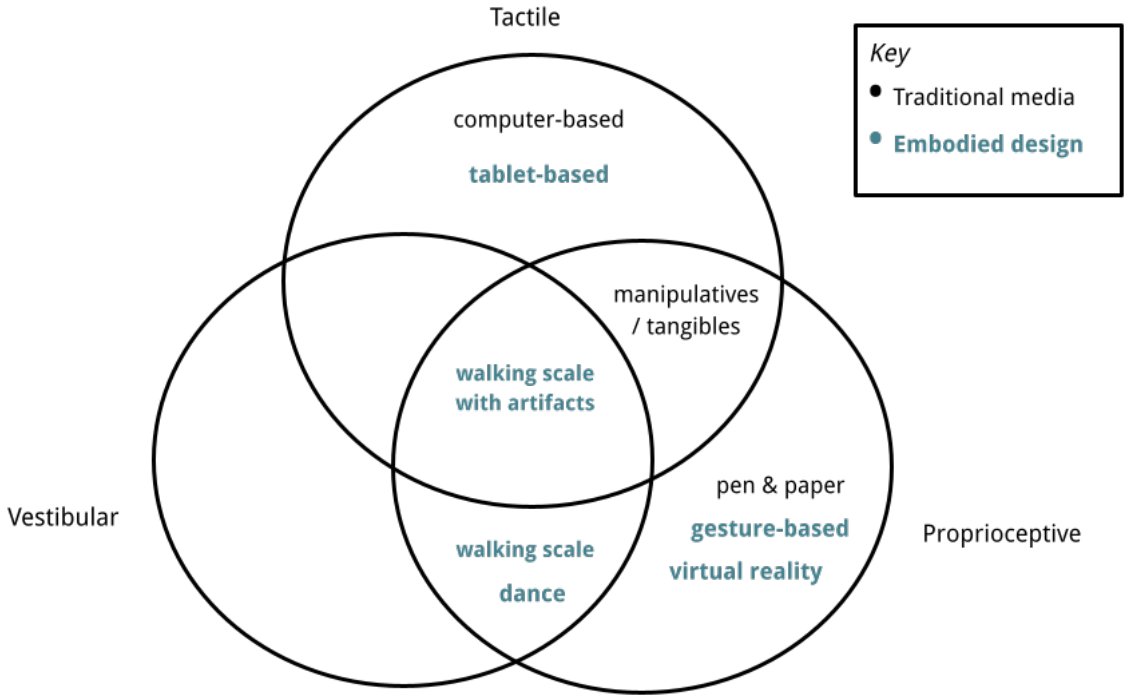
2.2 Embodied Design

Math classroom sensoria offer varied types and intensities of sensory stimulation: for example, different wall décor, lighting, and ambient noise (Kuhaneck & Kelleher, 2015). Instructional activities, too, vary in the degree of sensory stimulation they incorporate. Common math instructional activities, such as lectures, discussion, and pencil-and-paper practice problems, generally provide minimal opportunities for stimulation of the foundational sensory systems (Figure 1). Other techniques, such as the use of manipulatives, offer some degree of tactile stimulation. By and large, some senses—notably vision and hearing—are foregrounded as epistemically relevant in instructional practice, whereas others, such as balance, are generally treated as mere background. In turn, the majority of content is expressed through these modalities, further stoking such assumptions.

Learning sciences research inspired by embodied cognition is under way exploring more active sensorimotor engagement as the basis for learning. For example, *embodied design* (Abrahamson, 2014; Abrahamson et al., 2020; Shvarts et al., 2021), an enactivist math instructional design framework, poses perceptuomotor coordination problems to teach mathematical concepts. In general, embodied-inspired math designs more substantively and intensively incorporate foundational sensory modalities than traditional instructional methods (Figure 1), from touchscreen actions (e.g., Duijzer et al., 2017; Ottmar et al., 2015; Shvarts et al., 2021; Sinclair & Heyd-Metzuyanim, 2014) to gestures (e.g., Broader et al., 2007; Cook et al., 2008; Georgiou & Iannou, 2020; Nathan & Walkington, 2017) to manipulating objects like tape and string or 3d pens (e.g., Nemirovsky et al., 2012; Ng et al., 2018; Palatnik & Abrahamson, 2021) to gross-motor activity such as manipulating virtual-reality graphs (Chatain et al., 2022), dance (Vogelstein et al., 2019), navigating walking-scale number lines (Kelton & Ma, 2018), constructing collaborative geometric shapes on a floor mat (Durán López et al., 2017), and exploring or constructing 3d geometries (Kelton & Ma, 2020; Palatnik & Abrahamson, 2021).

These meaningful expansions make embodied design a promising avenue for more sensory-forward instruction.

Figure 1. Sensory systems at play in traditional and embodied design math activities.



Special Education Embodied Design (SpEED) (Tancredi, Chen, et al., 2022) is an extension of embodied design focused specifically on designing instructional media that expand the modalities and semiotic modes of instruction to mitigate *modalism*. Modalism is the process of granting specific sensorimotor configurations and modes of interaction supremacy over others (Tancredi, Chen et al., 2022). To date, SpEED has been taken up to imagine instruction for blind and visually impaired students (Abrahamson et al., 2019), Deaf learners (Krause & Wille, 2021) and learners on the autism spectrum (Chen et al., 2022), towards a model of inclusive design that offers robust multimodal, multimedia, multisemiotic experiences (Lambert et al., 2022). Simply engaging a given set of sensorimotor modalities does not in itself suggest parity of learning experience; learning is shaped by how a learner interacts through that set of sensory *modalities* with physical instructional *media* and *semiotic modes* of meaning-making (Tancredi, Chen, et al., 2022). For example, in some math instructional activities, sensory activation is incidental, as when typing on a computer provides some tactile experience that is unrelated to the letters being typed. In others, the sensory stimulation is *congruent* with the concept, as when turning one's arm to turn a virtual gear (Johnson-Glenberg & Megowan-Romanowicz, 2017). There remain, however, numerous unexplored opportunities in creating activities that more substantially engage these foundational sensory systems, particularly the vestibular system.

Embodied design emphasizes movement as a means of exploring concepts (for example, walking along a number line); sensory regulation approaches emphasize movement as a means of regulating one's arousal state to support ongoing engagement (for example, bouncing on an exercise ball). An embodied cognition perspective challenges the independence of these two movement types (Tancredi & Abrahamson, under review). Furthermore, these two movement

forms may conflict. For example, Ellie might be asked to wear a VR headset and generate graphs with arm movements, and she cannot bounce on her exercise ball simultaneously. How might instructional design holistically integrate these two functions to foster movement that serves *both* regulation and learning?

2.3 A Starting Point: Embracing Neurodivergent Sensory Practices

Nolan and McBride query: "[I]f 'stimming' was an acceptable and valued aspect of social and cultural behaviour, how might it be incorporated into design or social practices?" (p. 1075). I propose that to integrate sensory regulation and embodied cognition theoretical perspectives is to imagine a theory and pedagogy capable of building from each learner's way of inhabiting their body in the world. --Neurodivergent people may move in distinct ways that flow from their sensory experience of the world. Those on the autism spectrum, for example, often stim: they engage in repetitive physical movements, such as hand-flapping or bodily rocking. These ways of moving are driven by sensory seeking, engaging the regulatory senses in rhythmic ways known to regulate the nervous system. Study of neurodivergent interaction and phenomenology has shown that in addition to its sensory regulation function, stimming can also be expressive, communicative, and exploratory (Bakan, 2014; Chen, 2016; Chen et al., 2022; Dickerson et al., 2007), an "expression of embodied autistic semiosis that communicates sensory significations" (Nolan & McBride, 2015, p. 1070). This suggests that sensory practices such as stimming not only impact regulation but have the potential to be part of thinking (Tancredi & Abrahamson, under review). I argue that stimming constitutes an untapped resource for learning.

Current educational practice falls short of accommodating all learners in several ways. Sensory accommodations typically treat sensory regulation as separate from cognition, without rendering focal content more sensory stimulating. Movement-based learning such as embodied design offers greater sensory stimulation, but does not accommodate a range of sensory needs and can give rise to conflict between pedagogical and regulatory activity. Neurodivergent phenomenological testimony suggests that activity that stimulates the regulatory senses can also be exploratory. What if we designed for sensory stimulation to serve both regulatory and epistemic functions simultaneously, in an integrated way? We will explore this possibility through the BBM project and data in the coming sections.

3. Balance Board Math: Vestibular-Activating Math Instruction

BBM is designed to offer ways to interact with mathematical concepts using the sense of balance through activities on a balance board. Balance boards are a common sensory regulation tool that enable vestibular stimulation through bodily rocking. Balance boards are typically used in parallel to cognitive activity, for example, as a place to stand while working at a standing desk. BBM, in contrast, establishes bodily rocking on a balance board as itself a resource for exploring mathematical concepts (Tancredi, Wang et al., 2022). The design establishes the sensory regulatory activity of rocking as relevant to learning activities. In this section, I describe BBM's central conjecture and design with a focus on sensory stimulation and outline this study's research questions.

3.1 Design Conjecture and Research Question

BBM's design conjecture, following a conceptual analysis of sensory regulation from an embodied perspective (Tancredi & Abrahamson, under review), is that incorporating balance stimulation into math instruction can: (1) offer affordances for sensory regulation; and (2)

legitimize regulatory-sense explorations as epistemic actions. In the following analysis, I examine each of these qualities in the context of BBM to answer the research questions:

1) Do children stim-rock when using BBM? If so, is the intensity of their stim-rocking correlated with their vestibular sensory profile?
2) Does vestibular-stimulating activity relate to students' exploration and learning of mathematical concepts, for children across the vestibular sensory spectrum? If so, how?

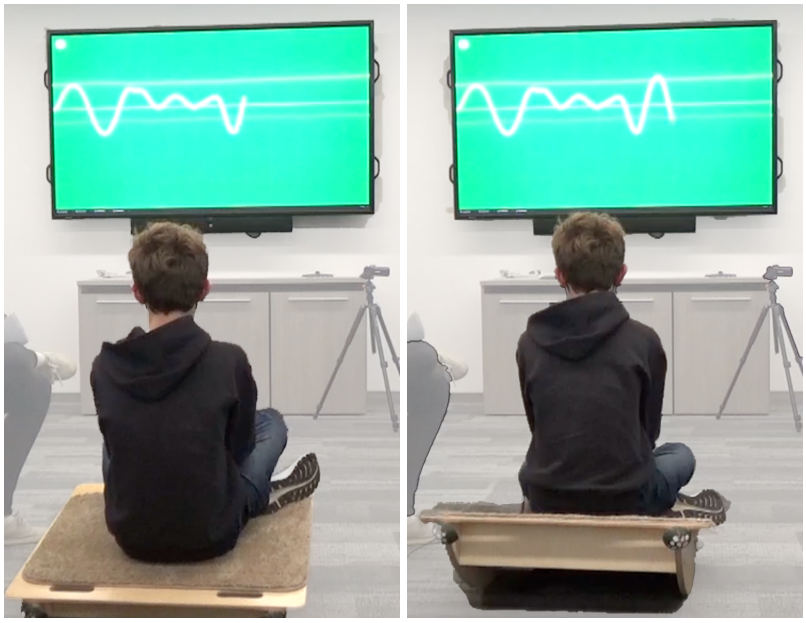
The focus of the present study is on the function of stimming during learning. For evaluation of learning outcomes with BBM, see prior work (Tancredi, Wang, et al. 2022; Tancredi, Wang et al., 2023).

3.2 BBM Design

Two BBM *configurations,* each with their own sets of activities, were developed through iterative design and testing: the Balance Number Line, with activities on the number line, absolute value, and arithmetic; and Balance Graphing, with activities on functions and graphing concepts (Tancredi, 2024). This study focuses on the Balance Graphing configuration (Figure 3). Graphing has shown to be a rich environment for studying embodied interaction (Chatain et al., 2022; Duijzer et al., 2019; Gerofsky, 2011; Nemirovsky et al., 1998). Here, we take dynamic graphing as a first context to evaluate the design problem of leveraging vestibular sensory-seeking for conceptual learning.

Amongst motion-graphing embodied learning environments (Duijzer et al., 2019), Balance Graphing is unique in its use of vestibular stimulation. In Balance Graphing, shifts in the board's balance are detected with a sensor. A dynamic graph of the angle of the board is drawn in real time on a display in front of the user. Rocking right and left or forward and back generates a sinusoidal graph. BBM users engage with different *activities:* discovery-based movement problems designed to guide them to attend to and control specific parameters of their graph relevant to mathematical concepts, such as the amplitude or phase of a sinusoidal graph. Balance Graphing is designed with a high-ceiling low-floor approach, enabling productive exploration of functions and graphing for learners preschool to adult. In this study, we limited participants to school-aged children (grades K-12).

Figure 3. Basic mechanism of Balance Graphing



BBM offers learners the ability to self-differentiate the intensity of vestibular stimulation they experience in exploring movement-based learning activities. One way to do so is to modulate the frequency or amplitude of one's rocking. Any given activity offers flexibility in one of these parameters to allow users to achieve a comfortable overall level of vestibular stimulation for their preferences. For example, the girls in Figure 4 are both working to rock at a similar amplitude, yet they have the flexibility to rock at different frequencies. The girl on the left (upper graph) rocks faster than her peer on the right (lower graph). To cater to a maximal range of sensory profiles, the interface's sensor sensitivity and graphing speed can be adjusted for each user. In sum, BBM activities introduce options that can offer the sensory stimulation craved by vestibular seekers and provide another entry point into concepts for all learners.

Figure 4. Peers Rocking at Different Frequencies but Similar Amplitude



3.2.1 Design Process

BBM was iteratively designed by an interdisciplinary team with expertise in special education, learning sciences, human-computer interaction, and engineering, as well as input from public school classroom teachers. In addition to the research participants described in section 4.1, BBM was also tested by over 100 people of all ages at a science fair and design showcases, and by over 50 researchers of mathematics education, learning sciences, engineering, and movement science during research meetings. Key iterations of the design include moving towards activities that enable rhythmic rocking, recognizing the importance of rhythm in sensory regulation. Another key iteration was the expansion from a single-user setup to a dual-user configuration, creating a collaborative task context that could better support rocking-based discourse. Based on analysis of how students oriented to graphs throughout activities, changes were also made to visual and sound feedback to better support discovery of focal concepts.

3.2.2 Example BBM Activity

I will describe here in detail an example BBM activity from the Balance Graphing configuration, Frequency Exploration, to illustrate how the tool is used. In Frequency Exploration, one or two players, each on their own balance board, rock to each generate a graph on the screen. They are given the directive to "figure out how to make the whole screen green" and invited to explore this directive through rounds of graphing and reflection. When graphing in this activity, the background behind their graph line highlights each time they complete one full periodic cycle (rocking any amount left, right, and back to balance) with a color and sound (Figure 5). The activity is tuned to a specific frequency: any periods that correspond to that frequency create a pleasant chord and green background color. Periods that are a bit longer or shorter create a slightly dissonant chord and a yellow background. Periods that are much longer or shorter create a dissonant chord and red background. Users iteratively explore this feedback to figure out how to modulate their rocking to get green for the entire graphing period. Through exploration, they

come to treat one period of a sinusoidal function as one unit, to connect the length of a period to their rocking rate, and to reason about how many periods of a given frequency fit within a timeframe. For more details on the BBM Balance Graphing design, see Tancredi, Wang et al. (2022), and for the full set of digital and analog BBM math activities, see Tancredi (2024).

Figure 5. Rocking at different frequencies.



4. Methods

4.1 Participants

Across two design iterations, 20 children participated individually (N=6) and in pairs (N=14) in semi-structured task-based clinical interviews using BBM activities. Given the focus on inclusive instruction and sensory neurodiversity, to ensure representation of at least 3 sensation-seeking and 3 non-sensation-seeking children, participants were recruited both in the community through publicly available postings, and at two sites supporting neurodivergent students: an afterschool learning center, and an alternative school. Four participants were on the autism spectrum, per parent report. To capture learners' sensory profiles, during the first round of data collection, participants' afterschool instructors[9] were asked to identify whether a child was more highly

[9] The afterschool site in question was a site where children participated in sensory and academic interventions, including sensorimotor integration exercises. As such, instructors had conducted prior sensory assessments with students as part of the intake process. At this stage, details on learners' sensory profiles beyond an informal identification as seeking or non-seeking was not requested.

sensation-seeking than average. In the second iteration (N=14), parents of participants completed a sensory questionnaire, the Sensory Processing Measure 2 (SPM-2) (Glennon et al., 2017). The SPM-2 is a likert-scale questionnaire designed to measure sensory integration and processing difficulties in children and adults. Questions include, "This child is fearful of movement, such as riding swings or slides" and "This child rocks, sways, or squirms when seated." Participants' SPM-2 balance-motion percentiles (range: 18–99, mean: 48, sd: 27) and overall sensory percentiles (range: 16–99, mean: 56, sd: 33) covered a broad range. 5 participants' SPM-2 overall sensory scores and 2 participants' balance/motion subscores were in the range clinically described as "moderate to severe difficulties," suggesting high likelihood that sensory mismatch impacts their daily activities. To check if age could be a confounding factor in analyzing sensory profile, we checked for a correlation between grade and balance-motion percentile ($r(12)=-0.28$, $p = 0.33$) and between grade and percent of time regulatory rocking ($r(12)=-0.10$, $p=0.73$) and did not find any statistically significant correlation.

The overall BBM participant population included 5 students from an alternative school, 4 students from an afterschool learning center, 11 K–12 children recruited from the community with flyers posted on local forums such as NextDoor, 10 families (23 people) at a science fair, and 25 public school students (analyzed elsewhere in Tancredi et al., 2023). 38% were identified by their parents as "Caucasian", 31% "Hispanic," "Latinx," or "Mexican-American," 27% "Asian," 13% "Black," and 2% "Native American" (not mutually exclusive), and 60% as female. Compared to the full sample of participants who were part of the BBM design-based research study, interview participants had lower representation of Black and Hispanic students of color (13 participants' parents included "Caucasian" in their racial and ethnic background, 10 "Asian," 1 "Black," and 1 "Native American), relatively fewer female students (6 female, 14 male) and younger ages (K–8, average grade: 4, sd: 2). Structural issues of support-need identification and access may have contributed to the lower representation of female students and students of color at the alternative school and afterschool sites where interviews took place.

4.2 Data Collection

Each interview lasted 14–48 minutes (mean: 31). Interviews began with a calibration process: children were invited to sit on a balance board and rock fast and slow, big and small, and sensitivity and speed settings were adjusted to their preferred rocking speed and amplitude. Children then explored 3–4 activities in a randomized order, each focused on a different functions-and-graphing concept: Function Exploration, Amplitude Exploration, Frequency Exploration, and Function Addition. Children were invited to try each activity as many times as they liked. After each graphing round, children were prompted for their thoughts on how the activity worked with questions including, "Did it work how you expected?" and "What might you try doing differently next time?" Children remained seated on the balance board during discussion unless taking turns on the board with a peer, in which case they sat on a nearby swivel chair. Interviews were audio and video recorded.

4.3 Data Analysis

Two coders[10] first identified all instances of children rocking in the data in ELAN coding software. These instances were then coded as either *instrumental* rocking that generated a graph, or *non-instrumental* rocking that did not (Table 1). Analysis focused on non-instrumental

[10] A note on sensory positionality: one of the coders was themselves a vestibular sensory seeker.

rocking. The structure of each instance of rocking was analyzed in relation to concurrent, recent, and consequent graphical products, utterances, and movements by the participant, peer, or researcher to identify congruence. Rocking that was not identified as congruent was presumed *regulatory*, per sensory-regulation theory. Inter-rater reliability was calculated with Krippendorff's alpha (Krippendorff, 2013) for a set of 25 randomly selected 10-second clips (instrumental: 0.87, congruent: 1.00, regulatory: 0.83). Inductive coding of the function of congruent rocking episodes identified rocking as sometimes *epistemic* (part of exploring a concept or idea), and sometimes *dialogic* (expressing an idea to an interlocutor).

Table 1. Hierarchy and description of qualitative codes.

Code	**Sub-codes**	**Definition**	**Example**
Instrumental rocking		Rocking that generates a graph	Rocking left and right at different frequencies and observing the resulting graph during the Frequency Exploration activity
Non- instrumental rocking	Regulatory	Peripheral, rhythmic rocking for sensory regulation	While waiting for an activity to load, rhythmically rocking and saying "whee"
	Congruent	Rocking that explicitly shares structure with graphical products, utterances, or movements	Rocking slowly while reviewing a graph with a low frequency; demonstrating a rocking pattern to a peer

Descriptive statistics were calculated for all participants whose parents completed an SPM-2 questionnaire (N=14). We used two quantitative approaches to examine the relationship between sensory profile and movement during the task-based interviews. Firstly, we identified a subgroup of students as "highly vestibular sensation seeking," defined as those whose parental SPM-2 responses indicated a tendency to rock, sway, or squirm in their seat at least occasionally (N=4 students). We compared these students' regulatory rocking to those not identified as highly vestibular sensation seeking using summary statistics. Secondly, we tested the correlation between regulatory movement and balance-motion percentile score using a Pearson's correlation test. A scatterplot of regulatory movement by balance motion percentile was generated. Two outliers, determined graphically (Figure 7), were omitted. One had a ceiling value for balance-motion percentile (in the "severe difficulties" range for balance-motion on the SPM2), and the other had the peak percentage of regulatory movement (72.8%) in the sample. These two outliers participated in an interview together. We also ran the correlation test again including the two outliers.

5. Results

All participants were able to use balance boards to generate graphs and discovered and controlled some focal parameters of the graphs they generated. We will begin with a quantitative overview of how learners rocked and whether this related to their sensory profiles before delving into representative episodes that illustrate how rocking functioned for learners.

5.1 Overview: Descriptive Statistics

Frequency of Rocking Codes.

Learners rocked during the interview both during graph generation and between rounds of graphing. They exhibited a range of rocking intensities, from subtle tilts to rocking all the way to the bumper. Participants rocked to generate graphs an average of 17.6% of the task-based interview (range: 0.2–26.8, median: 18.2). Non-instrumental rocking that was congruent with discourse about graphs occurred in every participant, an average of 6.7% of the time (range: 2.1–13.1, median: 6.7). Additional regulatory rocking occurred an average of 31.0% of time (range: 9.8–72.8, median: 26.5). Rocking types were intermittent throughout the course of the task-based interview, as shown for an example participant in Figure 6.

Figure 6. Density plot of codes for an example participant.

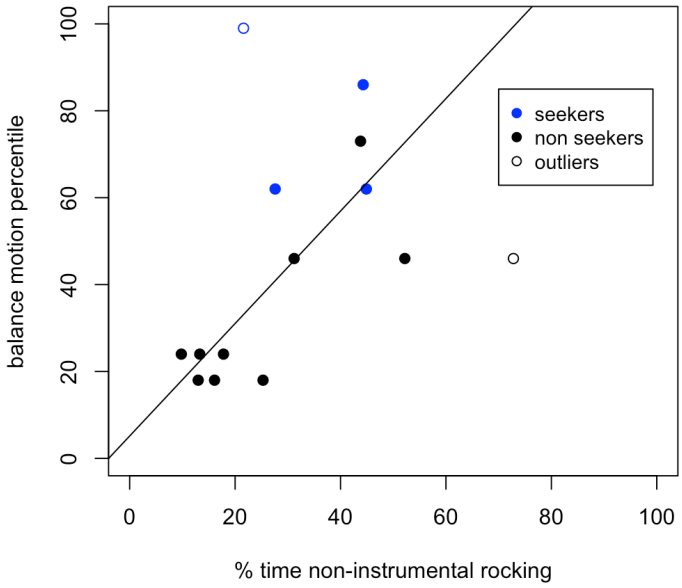


Note: The y-axis on this image is time, a total of 30 minutes 53 seconds. Clusters of congruent rocking follow instrumental rocking rounds, interspersed with longer bouts of regulatory rocking.

Rocking Frequency and Sensory Profile.

Participants who were more highly vestibular-sensation seeking (N=4) rocked more of the time during BBM than others (N=10) on average (seekers: mean 34.60, sd 11.80; non-seekers: mean 29.53, sd 20.69). Omitting the outlier student pair, there was a strong correlation between percent of time regulatory rocking and SPM-2 balance-motion percentile $r(10)=0.79$, $p = 0.002$, 95% CI: [0.40, 0.94]. Including the two outlier participants, there was a moderate positive correlation between percent of time regulatory rocking and SPM-2 balance-motion percentile, which was not statistically significant $r(12) = 0.43$, $p=0.122$, 95% CI: [-0.12, 0.78] (Figure 7).

Figure 7. Scatter Plot of % of time regulatory rocking versus balance-motion percentile.



Note. Vestibular seekers are highlighted in blue. Outliers are displayed using the empty circles. The best-fit line when omitting outliers is shown with the solid line.

5.2 Rocking as Sensory Regulation

Non-instrumental rocking frequently exhibited the characteristics of sensory regulation: spontaneous, rhythmic sensory stimulation. To illustrate the form of stim-rocking, we share a brief description of the rocking of one participant, Ian[11], identified as highly sensation seeking, during discussion immediately following one of his graphs as he explores the concept of amplitude:

Ian: [Rocking at the same frequency as when graphing.]
Now I learned how to do it.
[Pauses rocking and points at graph on screen in front of him with right index finger.]
You start on- uhh, was it [squints and touches forehead, rocking very slightly left and right] uhh, you start on-
[resumes rocking at prior rate] white, and then you have to go down and try to follow the green [rocks twice as quickly]

Researcher: So, where is green? How do you find it?

Ian: [Returns to prior rocking speed] On the bottom.

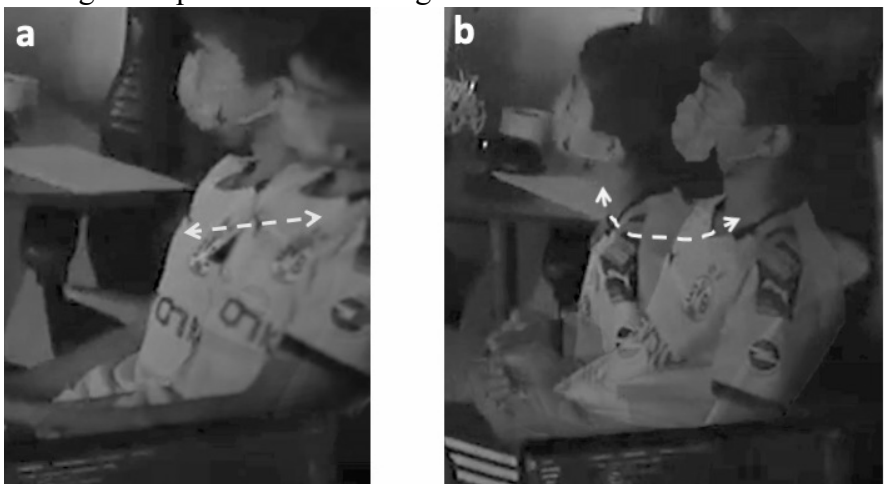
Ian continued to rock at the same rate as he clarified his hypothesis about "follow[ing] the green" (the focal amplitude) at the bottom (several units below the *x*-axis) through further dialogue. When he had finished speaking, he continued rocking at the same rate, then vocalized, "Whee! Whee! Whee!" Ian dynamically and ongoingly exercised the rocking affordance of the balance board. Between task-related comments, he spontaneously annotated his stims with exclamations, suggesting the movements brought him joy.

Notably, the spontaneous, peripheral stims that arose in BBM data were not arbitrary stimulation to the vestibular system, as might be predicted by sensory regulation theory alone.

[11] All names are pseudonyms.

Instead, they indicated dynamic changes related to children's active task and thinking. As an example, we share the peripheral bobbing head movements of one 5th grader, Kyle, who was taking turns on the balance board with a peer, Cory. Kyle used the board in two different orientations: initially, he used it in a front-to-back orientation, and later, he used it in a left-to-right rocking orientation (as Cory had left the board in that orientation). When off the board, he sat in a swivel chair to watch Cory. Early in the interview, after having rocked front-to-back, Kyle sat in the swivel chair and bobbed his body in a front-to-back orientation (Fig. 8a). In contrast, after trying a round of rocking in the left-to-right configuration, Kyle's bodily movements shifted from front/back bobbing to primarily left/right turns (Fig. 8b). Kyle was not on a balance board during these movements, and yet his stimming carried cardinal vestiges of his recent task activity. This sequence highlights how non-instrumental peripheral movement that would be traditionally interpreted as task-irrelevant sensory regulation may actually be contextually shaped by the active task: here, generating and thinking about graphs with a balance board. Indeed, these movements could be epistemically productive enactments of relevant functional actions. Perhaps the BBM context has made bodily rocking semiotically significant. That is, the specific act of rocking has been imbued with mathematical significance through its use as a proximal activity impacting the distal form of graphs (see Abrahamson & Bakker, 2016). Or perhaps, what we observe here is not a specific case of stimming related to thinking elicited by the design, but rather an instance of rendering visible the relationship between stimming and thinking. In this view, stims are never independent from ongoing cognitive and motor activity, and yet only contexts that instrument stimming make the relationship visible.

Figure 8. Kyle changes the plane of his rocking.



Note: These illustrations were generated by layering screenshots of the end points of the child's movement trajectories. The same child (Kyle) is featured twice in each of the two images, sitting in a swivel chair, while his peer Cory (not seen in these images) sits on the board. (a) After using the board in a front-to-back configuration, Kyle rocks front to back in his chair. (b) Later, after using the board in a left-to-right configuration, Kyle swivels left and right in his chair. Note the different orientation of Kyle's shoulders in each pair of still images, as he bobs front-to-back or spins left-to-right.

In summary, in a context where stimming was implicitly permitted and regulatory sensory stimulation was readily accessible, children stimmed openly and spontaneously. The

timing and forms of stims reflected aspects of the immediate and imminent sensorimotor environment.

5.3 Rocking as Epistemic Action

Participants' rocking patterns were congruent with concepts that they explored and expressed. One way this manifested is that when discussing features of graphs, children spontaneously rocked to reanimate focal aspects of the features. For example, 1st-grader Marco, reviewing his first graph in calibration mode, rocked as follows:

Researcher:	You drew that graph from rocking.
Marco:	Oh.
Researcher:	Can you see your rocking? [tracing gesture in the air]
Marco:	Yeah. That means [lifts forefinger of both hands twice] I think first I went this way [slight rock left] that way [slight rock right], so that equals one dip? [points to graph]
Researcher:	Mhmm. So when you went to the right, that's when it went down, and when you went to the left it started going back up, and then you went right and it went down, and then left back up, right down again. [gesturing graph movement throughout with flat palm and arm]
Marco:	So you mean I went deeper [leans far to the left] on the second try, so at least it went [slight rock right and to center] like this [right lean] and then like this [left lean]? [continues slight rocking]
Researcher:	Yes, it looks like you went a little bit deeper on one side than the other.

Through using the balance board as an instrument, Marco has come to perceive graphs as affording rocking within his BBM utilization scheme (Vérillon & Rabardel, 1995).

Learners did not just rock in ways that instantiated graphs they had produced; they also rocked in ways that reflected unfolding graphical ideas. For example, kindergartener Kay's rocking diverged from his speech as he reflected on a round of graphing in the BBM Amplitude exploration activity, wherein rocking at a focal amplitude reveals green stars at the local maxima and minima of one's graph. In his last graphing round, Kay had found two stars by traveling far enough below the *x*-axis:

Researcher:	Ok, so you found two that time!
Kay:	Because I did biiiig rocks! [starts slight rocking, then starts quick small rocks.] I think I wanna do another one.
Researcher:	You wanna do another one, see if you can find some more?
Kay:	Yeah. but this one might be [starts small quick rocks] not as… Can you start it now?

Kay goes on to try out using quick small rocks during the subsequent graphing round. Rather than rocking in a way that aligns with his description of his past action of "big rocks," Kay rocks in a new way during his reflection. Treating his rocking as a gesture, this instance is reminiscent of the hallmark gesture–speech mismatch of a transition in thinking (Church & Goldin-Meadow, 1986). Kay's new rocking form is not arbitrary; it signifies his idea for an upcoming strategy. Rocking here appears to reflect the edge of his thinking, beyond what he is yet able to verbalize.

Kay later goes on to produced a Frequency Exploration graph, in which every period on the graph elicited yellow color feedback (per a mechanism described in section 4.1). Thus, Kay had just rocked at a consistent frequency (higher than the target for the round) to generate a series of periods of a consistent length. Earlier, the researcher had asked him about the maximum

number of greens a hypothetical robot could get in that activity if it had complete control of its rocking. Kay spontaneously revisited this idea as he reflected on his new graph:

Researcher:	Yeah, so you were making yellow really consistently when you were going faster. [Kay starts rocking.]
Kay:	Yeah, so you have to go big, [Kay rocks far left, (Fig. 9a)] but sloooow. [slowly shifting from left to right (Fig. 9b)]. So I think if the robot did it [slightly faster pace] it would go so fast [accelerating, jerky rocking] that it made [sic] the screen all red.
Researcher:	Yeah, maybe it would even make it all red from going [Kay's rocking starts to accelerate] so fast. [Kay rocks with very quick small movements (Fig. 9c-d)]
Kay:	Yeah but when you do it super little but so fast, then it gets it all white. [Kay returns to moderate, rhythmic rocks (Fig. 9e-f)]

Figure 9. Kay modulates his rocking as he imagines a robot generating a graph.



Note: Kay rocks during discussion of his latest graph. Images a-b: Kay rocks far slowly. Images c-d: Kay switches to short, quick rocks as he describes a hypothetical robot's graph. Images e-f: Kay returns to the larger, slower rocking pattern.

Kay began and ended this sequence rocking repetitively in a consistent rhythm, with no overt relationship between his rocking and the conversation at hand. However, Kay's rocking transformed as he discusses his own and a hypothetical robot's graphing strategies. First, as he described his strategy of rocking "big but slow," he modulated the speed and amplitude of his rocking to match, looking down at the board as he initiated this change. Next, Kay brought up

the robot. As he verbally evoked this imagined figure, his rocking accelerated (reducing in amplitude to further increase his frequency). Then, as the researcher spoke, Kay continued to extrapolate upon how rocking would change at higher and higher speeds. These frequency- and amplitude-modulated changes in rocking preceded Kay speaking about the robot's speed. Once he had expressed this thought, his rocking returned to the initial pattern. Kay, a kindergartener, was able to reflect on the frequency of sinusoidal functions, using rocking as a resource for his reflections. Kay's rocking accompanied and illustrated his speech, taking the form of depictive gestures, then preceded his speech, perhaps reflecting an emergent thought not yet expressed in words. The interplay of rocking and speech here suggests a *growth point* (McNeill & Duncan, 2000) wherein thinking and speech dynamically evolve together. This growth point not only binds imagistic and linguistic activity, but also regulatory activity. Rocking is not merely expressive, nor merely regulatory, but plays an active role in the thinking under way, enfolded in a fundamentally affective-cognitive-expressive process. Rocking here might be seen to become the substrate for his thought and expression, dynamically recruited beyond regulation as a tool for mathematical reasoning.

5.4 Rocking as a Discursive Resource

In BBM interviews, rocking played not only an epistemic role, but also an interactional one. For example, participants rocked in ways congruent with an idea expressed verbally by themselves, a peer, or a researcher. In one instance, Kyle had generated a graph, and he and his peer Cory were consequently asked what they thought elicited green color feedback in the graphing activity.

Researcher:	What do you think made green here?
Kyle:	Still not sure.
Cory:	When it went up to the top and it went down, it turned green. [Kyle rocks slowly to the left (which corresponds to up on the graph) then back to center.] If you just move faster with less movement [Kyle rocks quickly with low amplitude] like up and down like at the top, and at the bottom, you can just go back and forth. [Kyle stops rocking and looks at his last graph still on display, then begins rocking slowly once again]
Researcher:	Actually, we have whiteboard markers, maybe you can just show me on here what you mean. [Kyle repeats the rapid low-amplitude rocking.]

Within the interaction, Kyle's change in rocking rate signaled engagement with his peer's idea, consistent with research on stimming as an interactional process (Dickerson et al., 2017). The BBM context enables rocking to become an interactive resource tacitly bearing frequency-modulated conversational ethnomethods. The role of Kyle's rocking here is not merely an acknowledgment of his peer's words; it also revealed Kyle's thinking about these words, part of his multimodal thinking-for-speaking (Slobin, 1987). That is, the change in rocking rate made available an aspect of Kyle's prospective thinking about the graph that became observable to his interlocutors, much as gesture can reveal aspects of thought not yet verbalized (Church & Goldin-Meadow, 1986). Indeed, his stimming could itself here be interpreted as a gesture.

Learners recognized and responded to information conveyed in their peers' rocking. John, a neurotypical 5th grader, and his peer Peter, an autistic 2nd grader, participated together, each sitting on their own balance board. After a round of the BBM Amplitude Exploration activity, John rocked in the background. The researcher asked Peter and John whether it was

possible to get more stars than they had done that round:

John: [Rocking in the background.]
Researcher: You think it's possible to get even more stars than last time?
John and Peter: Yeah.
Researcher: How are you gonna do it?
John: Like- [swings board faster and further left to right, vocalizing during each rock] heeh, heeh, heeh, heeh, heeh.
Peter: Like this. [rocks rapidly left to right, not far each way]
John: No, then you're gonna get little lines. [pinches fingers together.] Like this: [rocks fast and far].
[John continues rocking in the background, more quickly now.]

John's initial rocking was backgrounded, as is typical of sensory regulatory movements. However, when the conversation turned to discussing how to generate a given graphing outcome, he recruited this ongoing movement to show the activity that would produce this outcome. He modulated the speed and amplitude of his rocking action. Peter then dialogued with John through this medium, expressing his understanding of the needed rocking pattern. John recognized that Peter was not attending to the amplitude of his rocking in order to maximize his speed. He multimodally expressed the relationship between Peter's rocking and the graph that would result, showing the reduced amplitude of the graph with his finger gesture, and highlighting the amplitude of his own rocking. After this exchange, as the conversation moved on to other ideas, such as staying completely still, John continued rocking in the background. His background rocking carries forward the higher speed that appeared as part of the dialogue.

The children's taking up of one another's multimodal rocking utterances might be seen as an ethnomethod (Mondada, 2014) to play a "yes and" function in conversation, demonstrating understanding of a peer's idea as a basis for consequent elaboration. Furthermore, from a mathematical perspective, the rocking of both Peter and Kyle reflects their engagement with mathematical ideas of frequency and amplitude not fully expressed in words. Vestibular stimming here constitutes an interactional and semiotic resource for facilitating joint action and joint meaning making about mathematical ideas.

6. Discussion

In a somatocentric instructional design, we observed children across the sensory spectrum engage with vestibular-stimulating sensory activity. Learners modulated the amplitude and timing of their rocking to develop new movement patterns that, in turn, became the basis for their emerging graphing concepts. In this context, rocking became available as an interactional resource, dynamically enfolded not only into children's thinking, but also their communicating, supporting the coordination of joint action and preceding, illustrating, or complementing the ideas they expressed verbally. Perhaps instructional designs that welcome regulatory-sensory exploration render visible not only children's covert sensory regulation processes but also the relationship between these stim forms and a child's central task. Perhaps further, these accommodating instructional designs legitimize a ubiquitous form of multimodal reasoning that, once welcomed and liberated, could enhance all students' mathematical learning.

There are several notable limitations of this analysis. Firstly, the limited sample size prohibits modeling the relative impacts of multiple factors, such as both age and sensory profile. Given the strong influence of outliers on the correlation test, replication with a larger sample is critical. The presence of two outliers in one dyad is particularly notable: one participant

exhibited an exceptionally high SPM-2 balance and motion percentile, and the other exhibited exceptionally frequent non-instrumental rocking (72.8% of the time). In review of this dyad, the high incidence of rocking may have been influenced by peer interaction, as these two students engaged each other in interactional stimming. Another limitation of this analysis is that the racial and gender demographics of the present sample are somewhat skewed. This limitation has been addressed in other work with the BBM design with more robust representation of minoritized populations (Tancredi et al., 2023). In future work, to focus on the aspects of vestibular sensory profile most relevant to instructional differentiation, it would be appropriate to select or develop a sensory profile measure that norms vestibular sensory *seeking* rather than overall level of vestibular atypicality.

BBM offers a proof of concept for sensory-adaptive instruction as a proactive, inclusive education approach. In alignment with the Universal Design for Learning framework, sensory differentiation instantiates proactively accessible design that can meet the needs of students with and without disabilities learning together. Similar to accessibility features such as a volume slider that enables each user to adjust settings to their personal needs, BBM offers each child the opportunity to adjust the intensity of vestibular stimulation on offer to their personal sensory-regulation needs. For children whose vestibular needs are at odds with standard instructional practice, BBM normalizes their vestibular stimming, recognizing, endorsing, and elevating their sensorimotor activity as relevant to disciplinary discourse. For children whose vestibular needs are met within standard instructional practice, BBM offers them more fine-tuned sensory regulation opportunities, as well as another way of thinking about concepts and sharing their thinking with peers across the sensory spectrum.

7. Conclusion

When vestibular stimulation is made inherent to math instructional design, children move in ways reflective of sensory regulation, thinking, and communication. This is an example of synergistic stimulation of a regulatory sense (here, the vestibular sense) integrally for both conceptual exploration and sensory regulation. The findings suggest that instructional design can render visible and facilitate stimming as regulatory, epistemic, and interactional. This challenges the presumed independence of two bio-epistemic processes pertaining to educational research, theory, and practice—the sensory-regulatory and the cognitive. Instead the findings suggest that sensory-regulatory and cognitive processes may interact, or indeed that the very definition of cognition may need to expand to *incorporate* regulatory activity (Tancredi & Abrahamson, under review). Notably, learners across the sensory spectrum were able to make use of multimodal reasoning for their mathematical learning in a context that legitimizes such explorations.

I have argued that lack of access to sensory regulation is a question of accessibility. The empirical findings suggest that integration of sensory regulation into instructional design is possible. Without sensitivity to sensory experience, embodied design could unwittngly institute a new set of restrictive bodily norms that perpetuate or exacerbate accessibility and equity barriers for learners with non-neuromajority sensory profiles. However, if developed to be sensory-adaptive, embodied design, with its focus on the body as a vehicle of interactive sensorimotor learning, could lead the way towards more sensorially-equitable instruction.

Although this paper has focused on the educational domain of mathematics, analysis of sensory regulation is warranted across academic disciplines, as embodied approaches burgeon in the sciences (e.g., Anderson & Wall, 2016; Champion, 2021; DeLiema et al., 2019; Lindberg &

Enyedy, 2021; Lindgren et al., 2019; Scherr et al., 2013; Solomon, 2021, Zohar et al., 2018), the humanities (e.g., Glenberg et al., 2004; Hall et al., 2020; Ioannou & Ioannou, 2020; Kaschak et al., 2017; Sakr et al., 2016), special education (e.g., Meckin & Bryans-Kinns, 2013; Valente et al., 2021), and the maker movement (e.g. Blikstein, 2013; Eisenberg 2002; Simpson et al., 2021). Attending to the regulatory senses in instructional designs and ensuring sensory diversity among learning-sciences research-participant populations offers a pathway towards more inclusive academic instruction.

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6 Conclusion

This dissertation demonstrates that learners can engage with mathematical concepts through their bodily sense of balance, and that movement forms that serve sensory regulation can become part of embodied conceptual exploration. Findings from the first paper shows that bodily rocking can serve sensory regulation and learning in an integrated way. The second paper describes how such integration can impact how learners explore and think about mathematical concepts. The third paper puts forth a new post-cognitivist theory of stimming as sensory regulatory, interactional, and cognitive. The fourth paper shows vestibular-stimming-as-thinking and offers a proof of concept for sensory differentiation of instruction.

This research forges connections between previously disparate research on sensory regulation and embodied math learning. It contributes theory and evidence elaborating the relation between the purportedly embodied nature of cognition and the process of sensory regulation. I propose a model of stimming as part of embodied cognition, with potential to play a role in sensory regulation, cognition, communication, and ultimately learning that can be supported and actualized through pedagogical design, as exemplified through findings with learners using the BBM design. Additionally, this dissertation extends the notion of *epistemological pluralism* (Turkle & Papert, 1990) to examine how diverse sensory profiles shape how people explore the world through their senses, impacting the modalities that we might recognize as participating in knowledge construction.

BBM offers an initial proof-of-concept for sensory differentiation, indicating the need for broader work ahead: systematic research on each of the somatosensory modalities and on multimodality, for learners with and without disabilities. Future work will investigate use of the vestibular sensory modality, as well as proprioception, across topics of different complexity levels and academic disciplines, and in inclusive education settings with learners of different sensory profiles, such as blind and sighted children. Experimental work will test how features of vestibular and proprioceptive stimulation, such as user control of sensory stimulation, impact regulation and learning, and compare these designs to standard practice. Given the prevalence of sensory atypicalities amongst numerous special-education populations, the first findings presented in this dissertation suggest that sensory differentiation may offer a pathway towards more equitable pedagogy to include learners currently in the special-education system, and ultimately an adaptive pedagogy that can serve all learners.

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