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Self-consistent flows in subsonic interstellar medium fluids

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Abstract

Two dimensional numerical simulations of a subsonic interstellar medium fluid, characterized by a low turbulent Mach number ($\tilde{M} < 1$), demonstrate a self-consistent generation of nonlinear turbulent flows in a hydrodynamic regime. Dynamically weak compressive effects are associated with passively convected thermal fluctuations and these enhance the rate of selective decay. Turbulent relaxation leads to self-organization in thermally dominated velocity fluctuations with the formation of large-scale steady state coherent flows via an inverse cascade mechanism. In agreement with theoretical predictions, thermal and density fluctuations are found to be anti-correlated.

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One of the most remarkable features of the interstellar medium is perhaps its ubiquity in exhibiting a Kolmogorov-like $k^{-5/3}$ density fluctuations spectrum over a wide length-scale. For instance, recent power spectral measurements obtained from the Voyager 2 data report turbulent fluctuation level of density turbulence from 1 to 60 AU, corresponding to the period 1977 to 1999 [1]. The corresponding density spectrum was shown to obey a Kolmogorov-like $k^{-5/3}$ spectrum. Similarly, observations of interstellar scintillations at radio wavelengths reveal a Kolmogorov-like scaling of the electron density spectrum with a spectral slope of $-5/3$ over many decades in wavenumber space in the interstellar medium [2] (from an outer scale of a few parsecs to scales of 200 km or less). Solar wind turbulence also exhibits a Kolmogorov-like spectrum in velocity and magnetic field, despite possessing a weakly compressive component [3]. There also exist proxy evidences that are suggestive of (Kolmogorov-like) turbulent density fluctuations being driven by weak compressions in the solar wind plasma in a low turbulent Mach number regime [4]. It is to be noted that the Kolmogorov-like spectrum is a characteristic of incompressible hydrodynamic fluids. It is then intriguing that the solar wind and interstellar medium which predominantly are compressible magnetofluids behave as though they are incompressible. The origin and characteristic of Kolmogorov-like turbulent density fluctuation spectrum is therefore an enigma in astrophysical plasmas that continues to pose one of the major obstacles in our understanding of (magneto) fluid turbulence. Three dimensional, high resolution simulations of Shaikh and Zank [5] propose the origin of a Kolmogorov-like density fluctuation spectrum as a consequence of turbulent damping of the compressive component of velocity fluctuations in the interstellar medium which convect density in the field of nearly incompressible turbulent velocity fluctuations. Lithwick and Goldreich, on the other hand describe the density fluctuations as arising possibly due to slow and the entropy modes [6]. The relationship between the Kolmogorov-like turbulence spectrum, compressibility and incompressibility is however elucidated in a rigorous manner by Zank et al [7–9].

The theory of nearly incompressible (NI) fluids proposed by Zank et al. [7–9] possesses weak compressional effects in the fluid density fluctuations, unlike purely incompressible (IN) hydrodynamics, and are included by first order corrections to the leading order purely incompressible fluctuations in the low turbulent Mach number hydrodynamic fluid equations. The compressible density fluctuations are coupled through thermal fluctuations which are passively advected by incompressible velocity fields. The resulting equations, thus, com-

prise the familiar incompressible hydrodynamical (and MHD) equations at leading order, together with a modified set of compressible hydrodynamical equations. The initial motivation for NI theory was, as described above, to understand compressible effects such as density fluctuation spectra in the solar wind and interstellar medium [2]. Of perhaps equally compelling interest is how “turbulence affects the structure and motions of nearly all temperature and density regimes in the interstellar gas” [10]. Since density and temperature are included self-consistently within the NI models, we address this question directly in this Letter. Furthermore, the basic nonlinear aspects of NI hydro (and MHD) remain completely unexplored. This letter, therefore, presents a fully self-consistent investigation of thermally dominated NI hydrodynamics turbulence which is applicable to large β plasmas (β is ratio of thermal to magnetic pressure). Our simulations indicate the emergence of steady-state coherent structures in the decaying NI turbulence through a turbulent relaxation mechanism in which selective decay (as defined by ratio of enstrophy and energy) rates are enhanced dramatically by compressive effects. Thermally induced compressible density fluctuations are found to be anti-correlated with the thermal fluctuations. This is in agreement with theoretical prediction of Zank & Matthaeus [7, 8].

In the NI model, the background incompressible fluid can be described by the usual equations of incompressible hydrodynamics,

$$\partial_t \mathbf{U}^\infty + \mathbf{U}^\infty \cdot \nabla \mathbf{U}^\infty = -\nabla p^\infty + \mu \nabla^2 \mathbf{U}^\infty, \quad \nabla \cdot \mathbf{U}^\infty = 0. \quad (1)$$

Here, the superscript ∞ indicates that the velocity, $\mathbf{U}^\infty$, and the pressure, p^∞ , variables satisfy the incompressible fluid equations Eq. (1). The incompressible pressure satisfies the nonlinear Poisson equation $\nabla^2 p^\infty = -\nabla \cdot (\mathbf{U}^\infty \cdot \nabla \mathbf{U}^\infty)$. The nonlinear fluid equations describing the dynamical evolution of the compressible fluctuations in the NI hydrodynamical description [7, 8] contain the compressible fluid velocity, $\mathbf{U}_1$, the compressible density ρ_1 , and the temperature T_1 , and satisfy

$$\partial_t \mathbf{U}_1 + \mathbf{U}^\infty \cdot \nabla \mathbf{U}_1 + \mathbf{U}_1 \cdot \nabla \mathbf{U}^\infty - \rho_1 \nabla p^\infty = \nabla^2 \mathbf{U}_1 + \left(\xi + \frac{1}{3} \right) \nabla (\nabla \cdot \mathbf{U}_1), \quad (2)$$

$$\partial_t \rho_1 + \mathbf{U}^\infty \cdot \nabla \rho_1 + \nabla \cdot \mathbf{U}_1 = 0, \quad (3)$$

$$\partial_t T_1 + \mathbf{U}^\infty \cdot \nabla T_1 = \frac{1}{P_r} \nabla^2 T_1, \quad (4)$$

$$\nabla \cdot \mathbf{U}_1 = \frac{1}{P_r} \frac{\gamma - 1}{\gamma} \nabla^2 T_1. \quad (5)$$

The above equations are normalized, and correspond to their respective unnormalized variables as follows; $\mathbf{U}_1/M_s = \bar{\mathbf{U}}_1, \gamma^{1/2}\rho_1/\rho_0 = \bar{\rho}_1, \rho_0 C_p T_1/(\gamma^{1/2}p_0) = \bar{T}_1$. The time, and space co-ordinates are normalized by characteristic time and length scales respectively $L\nabla = \bar{\nabla}, u_0 t/L = \bar{t}$. The bars are absent from all the normalized variables for the sake of convenience. Here $P_r = \rho_0 C_v/\kappa$ is the Prandtl number, C_v is the specific heat at constant volume, $M_s = u_0/C_s$ is the fluid Mach number, $\epsilon^2 = \gamma M_s^2$ an expansion parameter, γ the ratio of the specific heats, $C_s^2 = \gamma p_0/\rho_0$ is the acoustic speed associated with the sound waves, and ξ is the normalized viscosity. Eqs. (1) to (5) describe the dynamical evolution of the decaying NI system with no linear instabilities. The compressional velocity fluctuations in Eq. (2) modify the effective dissipation processes and are generated explicitly through thermal fluctuations. The latter are governed by the passive scalar temperature convected by IN velocity fluctuations. Thus compressional effects play a significant role in the relaxation mechanism amidst complex nonlinear interactions and are explored below. The NI model Eqs. (1) to (5) has been used to explain the existence of observed anti-correlated fluctuations in the solar wind and has been suggested as an explanation for the observed interstellar density spectrum [8].

It is worth noting here that the results, presented in this paper, concentrate specifically on a high plasma- β limit. The high plasma- β assumption has largely been utilized in studies of the ISM. Such an assumption can also be justified in the solar wind, since, within $\sim 6 AU$, the solar wind plasma beta ~ 1 , but beyond the ionization cavity where pickup ions dominate, β (the plasma beta) easily exceeds 1 (see [11]; for a comprehensive review and the references therein). In a hot ISM region, the typical local interstellar parameters are; plasma density $n_0 \approx 0.07 - 0.1 cc$, plasma temperature $T \approx 6500 \text{ }^\circ K$, and interstellar magnetic field $B_0 \approx 0.2 \mu G$. This essentially yields a very high value of plasma $\beta \simeq 10 - 100$ where thermal pressure in an interstellar gas dominates the magnetic pressure. This essentially serves as a justification of using hydrodynamic fluid model to describe the interstellar processes. Moreover, we restrict ourselves to a two-dimensional (2D) NI model due primarily to the radially expanding solar wind plasma interacting with the interstellar medium often leads to an axis-symmetry situation where the variations in the z -direction can be ignored (see Zank 1999). Secondly, 2D simulations are relatively affordable over 3D computations. Although turbulence in 2D and 3D possesses distinct spectral features characterized essentially by the number of the inviscid quadratic invariants, our two-dimensional NI hydrodynamics

simulations offers a plausible approach to the understanding of the ISM turbulent processes.

The nonlinear evolution of Eqs. (1) to (5) is investigated using a 2/3 de-aliased Fourier spectral code (our **H**eat **F**luctuation **D**ominated Hydro, HFD-code) with 256^2 Fourier modes in a two-dimensional box of size $10\pi \times 10\pi$ for periodic boundary conditions along the x and the y -directions. The time integration uses a second order predictor-corrector method. All the fluctuations are initialized with a gaussian random number generator to ensure all the Fourier modes are spatially uncorrelated and randomly phased. The initially normalized energy spectrum, peaked at $k_{\min}$, is chosen to lie within the wavenumber band $k_{\min} < k < k_{\max}/2$. This is shown in Fig 1(a) for the x component of the NI velocity field in configuration space. During the evolution, turbulence eventually decays through vortex-merging in which like-signed smaller length-scale fluctuations merge to form relatively large-scale fluctuations. The process continues until all the merging has occurred to finally form the largest scale coherent vortex dominated by the minimum allowed k in the simulation. Various stages in the simulations are shown in Fig 1 with the final stage being a large-scale (comparable to computational box) coherent vortex. A similar configuration for the y -component of the NI fluid velocity is observed (not shown in Fig). The passively convected temperature, the density and the background incompressible velocity fluctuations, however, remain turbulent (not shown here) during the entire course of the simulation time and their evolution depends critically upon the magnitude of the Prandtl number (P_r). We shall return to this point later.

Close inspection of kinetic energy (K.E.) as the NI and IN fluids relax (shown in Fig 3) reveals that the K.E. of the NI fluid $E_{ni} = \frac{1}{2} \sum_{\mathbf{k}} |\mathbf{U}_1(\mathbf{k}, t)|^2$ (solid-curve) decays more rapidly than $E_{in} = \frac{1}{2} \sum_{\mathbf{k}} |\mathbf{U}^\infty(\mathbf{k}, t)|^2$ (the IN fluid, dashed-curve). The decay rate ($dE_{ni}/dt > dE_{in}/dt$) is very large during the initial phase in which almost all the K.E. of NI turbulence is dissipated, while there is a small change in the K.E. of the IN fluid. Correspondingly, the selective decay rate in NI turbulence ($\sqrt{\Omega_{ni}/E_{ni}}$) is faster than that ($\sqrt{\Omega_{in}/E_{in}}$) in IN turbulence, where $\Omega_{ni} = \frac{1}{2} \sum_{\mathbf{k}} |\nabla \times \mathbf{U}_1(\mathbf{k}, t)|^2$ and $\Omega_{in} = \frac{1}{2} \sum_{\mathbf{k}} |\nabla \times \mathbf{U}^\infty(\mathbf{k}, t)|^2$ are respectively NI and IN turbulence vorticities. This is shown in Fig 2. The decay rates in NI and IN turbulence imply that the characteristic time associated with the decay of enstrophy is much shorter than that of energy throughout the simulation. A comparison of the two curves, (dashed and solid lines) in Fig 3, shows that compressional effects in NI turbulence enhance decay rates significantly and lead to the formation of coherent vortices on much

faster time scales when compared with IN turbulence. Rapid decay rates observed here in low Mach number NI turbulence are consistent with the perturbative expansion that is used to obtain the high frequency component i.e., NI modes, against the slow IN modes [7]. These fast-time-scale compressional modes, dominated by thermal fluctuations, accelerate nonlinear selective decay processes to consequently generate coherent vortical structures on much faster time scales (when compared with the background IN turbulence) through turbulent relaxation.

An analytic understanding of the emergence of coherent vortices observed in our fluid simulation can be achieved by considering exact nonlinear wave solutions of the NI and IN velocity fields i.e Eqs. (1) & (2). The underlying mechanism on which such an analysis is based is that the coherent structures emerge from the nonlinear interaction processes to eventually become stationary in space and time thereby forming long-lived stable entities. This is what we observe in our simulations. We may first simplify Eqs. (1) & (2) by expressing the IN and NI velocities in terms of their respective fluxes $\mathbf{U}^\infty = \hat{z} \times \nabla\phi$ and $\mathbf{U}_1 = \hat{z} \times \nabla\phi + \nabla\psi$. The representation of the two flux functions in this fashion is not an arbitrary choice, but is motivated by several factors. Firstly, the flux functions describing the potentials of IN and NI fluids strictly obey the respective conditions of incompressibility and weak compressibility. Secondly, the governing nonlinear equations describing the evolution of NI and IN velocity fluctuations can be transformed formally into the respective vorticity equations, and finally, the nonlinear interaction terms can be expressed in a Poisson bracket form to yield the exact nonlinear solutions from the vorticity equations. The $\hat{z}$ components of the NI vorticity equation then reads $\partial_t \nabla^2 \psi + [\phi, \nabla^2 \psi] = 0$, where $[\cdot, \cdot]$ is the poisson bracket. The small scale viscous terms (ineffective on vortex length scales) and gradients associated with the IN vorticity terms (small compared to corresponding NI terms) in Eq. (2) are omitted (consistent with the simulations). In a moving (with velocity u) vortex frame, $x = x', y = y' - ut$, the NI vorticity can be written in terms of the Poisson bracket as $[\phi - ux, \nabla^2 \psi] = 0$. This then readily yields $\nabla^2 \psi = G(\phi - ux)$ due to the vanishing of the Poisson brackets. G is an arbitrary function of its argument and is continuous and differential within and outside the region of vortex boundaries. For G linear, we obtain the vortex solution $\psi = AK_1(r)y$ for $r > r_0$ and $\psi = (B/rJ_1(\alpha r) + C/rJ_1(\beta r) + D)y$ for $r < r_0$ where ϕ was determined from IN vorticity equation. Here J_1, K_1 are respectively the Bessel function of the first kind and the modified Bessel function of the second kind,

$x = r \cos \theta, y = r \sin \theta, r = \sqrt{x^2 + y^2}, \theta = \tan^{-1} y/x, r_0$ is radius of vortex. The constants A, B, C, D, α and β can be determined by matching the functions ψ and its derivatives across the vortex radius r_0 . In brief, we find that the nonlinear structures in NI turbulence, observed in our fluid simulations, are influenced by the background IN turbulence and that they are generated by nonlinear interaction processes.

Dynamically finite compressional effects in the NI fluid are induced through thermal fluctuations which depend upon the Prandtl number. Thus the passively convected thermal transport and density fluctuations mutually influence each other in a rather subtle manner and both depend critically on the Prandtl number [see Eqs. (2) to (5)]. The dependence can be determined by computing the energy associated with the density and the temperature fluctuations for different values of P_r (shown in Fig 4). Clearly, larger P_r implies a smaller thermal fluctuations dissipation rate (Fig 4a). Correspondingly, the density fluctuations (shown in Fig 4b) grow as rapidly as thermal fluctuations dissipate. This relationship thus suggests that density and thermal fluctuations are anti-correlated in thermally driven NI hydrodynamics and supports Zank & Matthaeus' prediction [7],

$$-\rho_1 = \left(\frac{\gamma - 1}{\gamma} \right) T_1.$$

In summary, we have investigated NI hydrodynamics in the so-called heat-fluctuation-dominated regime. Such a regime has been observed in the solar wind and has been suggested as an explanation for the observed ISM density spectrum. Three important results have emerged from our simulations. Firstly, beginning with arbitrary initial incompressible and NI data, the compressible velocity fluctuation organized themselves into coherent structures forming eventually a single vortex. The corresponding density and temperature fluctuations remained turbulent (due to turbulent convective flow). We showed theoretically that the vortices emerged from nonlinear driving by the incompressible turbulent velocity field. The second important result is that the NI energy and selective decay rates are higher than the IN case (as might be expected with the inclusion of nonlinear compressive modes), which leads to the more rapid formation of coherent vortices for NI components. Finally, the theoretical NI model in the HFD regime, Eqs. (2) to (5), predict the existence of anti-correlated density and temperature fluctuations. Our simulation demonstrate this explicitly

and we find that it is true for all the Prandtl number simulated here.

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FIG. 1: Decaying turbulence in a NI fluid coupled to an IN fluid through nonlinear interactions. (a) Initial condition ($t = 0$) specified on the x -component of NI velocity field shows random fluctuations in a two dimensional box. (b), (c) and (d) show fields at $t = 5, 15, 40$ respectively. Box size is $10\pi \times 10\pi$. $k_{\min} = 0.2, k_{\max} = 17.06$. Other constants are $\gamma = 5/3, P_r = 1000., \mu = 1.e - 3, \xi = 1.e - 5$.

FIG. 2: Time evolution of ratio of kinetic energy in NI and IN fluids. The NI energy decays more rapidly, due to compressional effects.

FIG. 3: Evolution of mean Fourier mode associated with selective decay rates. Dashed and solid curves are $k_{in} = \sqrt{\Omega_{in}/E_{in}}$ and $k_{ni} = \sqrt{\Omega_{ni}/E_{ni}}$ rates for IN and NI turbulence respectively. Decay rates are stronger in NI turbulence than in IN hydrodynamics.

FIG. 4: Energy associated with temperature and density fluctuations in NI turbulence are shown respectively in (a) and (b). Shown here are solid ($P_r = 10^2$), dashed($P_r = 10^3$), and dashed-dot($P_r = 10^4$) curves. The decay rates depends critically upon the Prandtl number P_r . The density and the temperature fluctuations are clearly anti-correlated in agreement with the prediction of Ref. [7].