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GALACTIC CHEMICAL EVOLUTION WITH LOW AND HIGH PRIMORDIAL LITHIUM

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ABSTRACT

We discuss two scenarios for the galactic chemical evolution of lithium. In one, the primordial ${}^7\text{Li}$ abundance is taken to be that presently observed in Population II stars. This value is subsequently enriched to the Population I value by stellar sources. In the other scenario, the primordial ${}^7\text{Li}$ abundance is taken to be as high as that suggested by baryon number inhomogeneous cosmological models in which the universe is closed with baryons. This value is then depleted to the present-day Population I value by astration. The Population II value is obtained by gradual main-sequence destruction. Our models include the results of new calculations of lithium destruction and production in stars. The models are constrained by observations of deuterium, lithium, and beryllium as a function of galactic age and by the observed lack of correlation between Fe and Li for metal-poor stars. We find that these constraints can be satisfied by either scenario. We discuss the possibility that the recently derived upper limit to the lithium absorption along the line of sight to SN 1987A may provide a means to distinguish between these two scenarios.

Subject headings: abundances — cosmology — galaxies: evolution — interstellar: abundances — nucleosynthesis

1. INTRODUCTION

The galactic evolution of light element abundances has been studied in a number of papers over the past two decades (Truran and Cameron 1971; Mitler 1972; Reeves *et al.* 1973; Audouze and Tinsley 1974; Mathews 1977; Reeves and Meyer 1978; Audouze *et al.* 1983; Delbourgo-Salvador *et al.* 1985). The unique cosmic-ray origin (Reeves, Fowler, and Hoyle 1970; Meneguzzi, Audouze, and Reeves 1971; Mitler 1972) for ${}^6\text{Li}$, ${}^9\text{Be}$, ${}^{10,11}\text{B}$ (and, to some extent, ${}^7\text{Li}$), along with the big bang production of ${}^7\text{Li}$, allow for important constraints on galactic evolutionary history and an opportunity to understand better the yields and conditions of primordial nucleosynthesis. Several recent developments in this field, however, warrant a reexamination of light element galactic chemical evolution.

One development is the recent suggestion (Applegate, Hogan, and Scherrer 1987, 1988; Alcock, Fuller, and Mathews 1987; Fuller, Mathews, and Alcock 1988; Malaney and Fowler 1988) that a first-order phase transition from quark-gluon plasma to hadronic matter may naturally lead to isothermal baryon number density fluctuations which will remain during the epoch of primordial nucleosynthesis and modify the resulting nucleosynthesis yields from those of the standard homogeneous big bang. Such inhomogeneous cosmologies are intriguing as a possible means of satisfying the abundance constraints in a universe which is closed by baryonic dark matter. On the other hand, such inhomogeneous models tend to produce an abundance of ${}^7\text{Li}$ which exceeds the value observed in the solar neighborhood and in Population II stars. Another new theoretical development is the possibility of energetic hadron showers produced by the decays of long-lived particles during the keV era (Dimopoulos *et al.* 1988), which could initiate a new phase of primordial nucleosynthesis. Such scenarios could also lead to a high primordial lithium abundance, in this case resulting from the copious production of ${}^6\text{Li}$.

At the same time, there are now available lithium abundances (Hobbs and Pilachowski 1988) for stars in five Population I open clusters which have ages ranging from $<10^8$ yr $\sim 10^{10}$ yr. These data seem to indicate that the ${}^7\text{Li}$ abundance has not changed significantly from the Population I value of $\text{Li}/\text{H} \sim 10^{-9}$ for the past 10^{10} yr (although there is a controversy over the age of the oldest cluster studied, NGC 188, which may only be $\sim 5 \times 10^9$ yr; Twarog and Twarog 1990). These new data serve as an important constraint on models of lithium evolution in the solar neighborhood. Furthermore, a recent search for interstellar ${}^7\text{Li}$ absorption lines in the direction of SN 1987A by Sahu, Sahu, and Pottash (1988) yielded a null result. The authors have inferred an upper limit for Li/H which is consistent with the lowest values observed on the surface of Population II stars (Spite and Spite 1982; Rebolo, Molaro, and Beckmann 1988). Although there is some controversy as to the validity of this upper limit (see Malaney and Alcock 1990), it is nevertheless useful to make a study of lithium production and destruction during galactic evolution to deduce whether a high or low primordial lithium abundance is consistent with this new datum.

Another important motivation for the present study is that in the past, such studies have been limited by the use of simplified models for lithium destruction in stars. Previous assumptions ranged from total lithium destruction in the ejecta of all stars (Mitler 1972; Audouze and Tinsley 1974) to ${}^7\text{Li}$ destruction occurring only in stars with $M < 2 M_{\odot}$ (Audouze *et al.* 1983). For the present study, we utilize the results of a recent theoretical study (Dearborn and Hawkins 1990) of ${}^7\text{Li}$ and ${}^9\text{Be}$ destruction as a function of initial stellar mass. This allows for a more realistic picture of lithium astration during the history of the Galaxy.

In this paper, we also consider some new possibilities for ${}^7\text{Li}$ formation which have been proposed recently. In addition to the possible injection of ${}^7\text{Li}$ from carbon stars and the infall of

extragalactic primordial material, we study the consequences of ${}^7\text{Li}$ production (Dearborn *et al.* 1989) by the ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ reaction in the shock-heated hydrogen envelopes of Type II supernovae, and neutrino inelastic scattering in core-bounce supernovae (Woosley and Haxton 1988; Woosley *et al.* 1990).

For the present purposes, we study two possible scenarios for the galactic evolution of ${}^7\text{Li}$. In one case, the ${}^7\text{Li}$ abundance is presumed to begin with a low primordial abundance (${}^7\text{Li}/\text{H} \sim 10^{-10}$) as given by the standard homogeneous big bang with a baryon to photon ratio of $\eta \sim 3 \times 10^{-10}$. This value is then enriched to the present Population I value ($\text{Li}/\text{H} \sim 10^{-9}$) by stellar sources as mentioned above. In the other scenario, we begin with a high primordial ${}^7\text{Li}$ abundance ($\text{Li}/\text{H} \sim 7 \times 10^{-9}$) with no stellar sources and gradual main-sequence depletion of lithium to the present Population II value. Although the first scenario is the most popular and is supported by standard models of halo star evolution (Deliyannis, Demarque, and Kawaler 1989), the latter scenario is at least made plausible by the predictions of baryon number inhomogeneous models as a source for a high primordial ${}^7\text{Li}$ abundance and speculations of lithium destruction on the surface of low-mass main-sequence stars owing to rotation-induced turbulent mixing (Vauclair 1988), magnetic bubbles (Hubbard and Dearborn 1980), mass loss (Wilson, Bowen, and Struck-Marcell 1987), uncertainties in opacities (Stringfellow, Swenson, and Faulkner 1987), or diffusion (Michaud 1986).

II. GALACTIC CHEMICAL EVOLUTION MODEL

The rate of change of the local mass per unit area, m_i , of a light element species in the interstellar medium can be written

$$dm_i/dt = P_i^{\text{cr}} + P_i^{\text{s}} + E_i + X_i(0)f_i - Bm_i, \quad (1)$$

where P_i^{cr} is the rate at which the species, i , is being produced in the interstellar medium (ISM) by cosmic-ray interactions, P_i^{s} is the rate of stellar production (such as carbon stars or supernovae as a source of lithium), $E_i(t)$ is the rate at which stars

return whatever is left of the original abundance of light element, i , at the end of their lifetime, and B is the fractional rate of loss of mass from the ISM by star formation. The quantity f_i is the rate at which primordial material may be infalling into the disk, and $X_i(0)$ is the primordial mass fraction of species i . For simplicity, we shall take f_i to be constant in time for all models.

Most of the quantities in equation (1) can be defined in terms of standard separable galactic chemical evolution models (Audouze and Tinsley 1976) based upon an initial mass function, $\phi(m)$, and a stellar birthrate function, $f(t)$. In Table 1, we summarize the functional forms used for $\phi(m)$ and $f(t)$. These are taken from Miller and Scalo (1979) corresponding to their constant, maximum exponentially increasing, or maximum decreasing star formation rates. These models span a broad range of possible scenarios. The use of other star formation rates, such as a bimodal function (Larson 1986) or scaling the birthrate function with the ISM mass, should produce similar results.

The fractional birthrate, B , is

$$B = \int dm m \phi(m) f(t) / M_G(t), \quad (2)$$

where $M_G(t)$ is the mass of the interstellar medium. The total rate of change of the mass of the interstellar medium is given by

$$dM_G/dt = -BM_G + P(t) + f_i(t), \quad (3)$$

where P is the total rate of mass ejection from stars,

$$P(t) = \int dm (m - m_r) \phi(m) f[t - \tau(m)], \quad (4)$$

where m_r is the remnant mass for a star of initial mass m which dies at an age $\tau(m)$. We take stellar main-sequence ages from Rana (1987) scaled by a factor of $T_0/15$, where T_0 is the galactic

TABLE 1
PARAMETERS FOR THE MODELS

PARAMETERS	$f(t) \text{ Gyr}^{-1}$		
	$2.6 \exp(-2.4t/T_0)$	$\frac{1}{T_0}$	$0.41 \exp(+1.6t/T_0)$
	T_0	T_0	T_0
$\phi(m) (\text{pc}^{-2})$:			
$0.1 \leq m/M_\odot \leq 1$	$43m^{-1.4}$	$18m^{-1.4}$	$14m^{-1.4}$
$1 \leq m/M_\odot \leq 10$	$43m^{-2.5}$	$18m^{-2.5}$	$14m^{-2.5}$
$10 \leq m/M_\odot$	$243m^{-3.3}$	$104m^{-3.3}$	$78m^{-3.3}$
$M_G(0) (M_\odot \text{ pc}^{-2})$	56	24 31	21 27
$M_{\text{infall}} (M_\odot \text{ pc}^{-2})$	24	15 9	12 5
$M_{\text{tot}} (M_\odot \text{ pc}^{-2})$	80	36 40	33 32
$M_G(T_0) (M_\odot \text{ pc}^{-2})$	7.5	7.5	7.5
Present gas fraction	0.09	0.21 0.19	0.23 0.23
$X_C (\times 10^{-8})$	1.5	1.0	0.6
$X_{\text{SN}} (\times 10^{-8})$	6.2	3.3	1.7

NOTES.—In rows 2, 3, 4, and 6 the first number corresponds to models with low primordial lithium, and the second number is for the high primordial lithium models. M_G is the mass of the interstellar medium in the solar neighborhood. M_{infall} is the local time-integrated mass due to infalling extragalactic material. M_{tot} is the present total mass in the solar neighborhood. X_C is the average mass fraction of ${}^7\text{Li}$ ejected from 1 to $5 M_\odot$ stars which is necessary to increase the ${}^7\text{Li}$ abundance from the Population II value to the solar system value. X_{SN} is the average mass fraction of ${}^7\text{Li}$ required per supernova.

age. By introducing this scaling, the resultant galactic evolutionary models become nearly independent of the galactic age, so that all evolutionary abundance curves can be represented as a function of t/T_0 . For the remnant masses, we take $m_r = 0.15 + 0.38m$ from Iben and Renzini (1983), assuming a mass-loss parameter of unity.

When solving equation (3), we set the initial value of $M_G(0)$ for each model such that M_G at a present time has a local value for the solar neighborhood (Tinsley 1977) of $M_G = 7.5 \pm 1.5 M_\odot \text{ pc}^{-2}$. See Table 1 for a summary of model parameters. The total surface density for the disk derived in this way is most consistent with a recent determination of $M_{\text{tot}} = 46 \pm 9 M_\odot \text{ pc}^{-2}$ (Kuijken and Gilmore 1989) for models with a constant or exponentially increasing star formation rate. We note that there is some recent evidence (Kulkarni and Heiles 1987) that the interstellar medium surface density is greater ($\sim 13 \pm 3 M_\odot \text{ pc}^{-2}$) than the commonly used value from Tinsley (1977). These two data would imply a present fraction of the local galactic mass in the interstellar medium of 0.28 ± 0.09 . Since the total galactic mass in the constant and increasing star formation models is smaller than the Kuijken and Gilmore (1989) value, we obtain about the same present gas fraction as implied by these new measurements (see Table 1) and therefore the correct amount of processing of the interstellar medium. However, for the exponentially decreasing star formation rate, a higher value for the gas fraction at the present time would imply less stellar processing than that inferred from our models and therefore less destruction of primordial ^7Li .

The specification of the cosmic-ray production term in equation (1) is complicated by the fact that the dominant mechanisms for cosmic-ray acceleration are still not well defined. Proposed mechanisms include supernova shock interactions with the interstellar medium (Krymsky 1977; Axford, Leer, and Skadron 1977; Blandford and Ostriker 1978; Bell 1978), hydrodynamic acceleration in supernovae (Colgate and Petscheck 1978), acceleration from supernova remnants (Scott and Chevalier 1975), or winds from massive stars (Meyer 1985). Most of the proposed mechanisms are associated in one way or another with the rate of formation of massive stars. Hence, we take the cosmic-ray activity to be proportional to the SN II rate,

$$P_i^{\text{cr}} = a_i \int_{m_1}^{m_u} dm \phi(m) f(t), \quad (5)$$

where the limits of integration are taken to be from 9 to $62 M_\odot$. The constants, a_i , are essentially proportional to the integral of the cosmic-ray production cross sections times an appropriate cosmic-ray energy spectrum (Viola and Mathews 1987). The light isotopes, ^6Li , ^9Be , and $^{10,11}\text{B}$ are at present thought to be produced solely by cosmic rays (Arnould 1986; Viola and Mathews 1987). In the present study, the absolute normalization for the a_i constants are determined by the requirement that ^9Be be produced in its solar abundance ($\text{Be}/\text{H} = 1.4 \times 10^{-11}$; Reeves and Meyer 1978) when the solar system condensed 4.6 Gyr ago.

The ratio of the observed abundances of $^7\text{Li}/^9\text{Be}$ in cosmic rays with energies above $\sim 100 \text{ MeV/nucleon}$ ranges from 3–5 (Simpson 1983). However, values for a_7/a_9 for equation (5) may vary from ~ 5 to 50 (Reeves and Meyer 1978; Walker, Mathews, and Viola 1985), depending upon the choice of a postulated low-energy component to the cosmic-ray spectrum, which is added to a power law in total relativistic cosmic-ray energy. The purpose of this low-energy component is to repro-

duce the $^{11}\text{B}/^{10}\text{B}$ abundance ratio (Meneguzzi, Audouze, and Reeves 1971). However, neglecting this low-energy component best reproduces the $^6\text{Li}/^9\text{Be}$ abundance ratio. Furthermore, it now appears that ^{11}B can be made in supernovae (Dearborn *et al.* 1989) rather than in low-energy cosmic rays (even if ^7Li is not produced in supernovae). Therefore, we use $a_7/a_9 = 7$ from Walker, Mathews, and Viola (1985), which corresponds to neglecting the low-energy component.

In principle, a_7/a_9 should increase at early times, when ^7Li can be produced by the $^4\text{He} + ^4\text{He}$ reaction, while the synthesis of Be must await the production of C, N, and O nuclei to spallate. However, by choosing a flat cosmic-ray spectrum at low energies, the importance of the $^4\text{He} + ^4\text{He} \rightarrow ^7\text{Li}$ reaction is diminished relative to the spallation reactions on C, N, and O nuclei, so that to a good approximation the a_7/a_9 ratio can be taken as constant over the history of the galaxy. In any event, cosmic rays contribute only a small fraction of the observed ^7Li abundance ($\sim 10\%$). Therefore, large variations in a_7/a_9 do not significantly affect the results presented here.

For ^7Li , there are possible stellar sources from carbon stars (Cameron and Fowler 1971) and/or supernovae (Dearborn *et al.* 1989; Woosley and Haxton 1988) in addition to production by cosmic radiation and an initial (primordial) abundance. The stellar source term in equation (1) can be written

$$P_i^s = \int_{m_1}^{m_u} dm (m - m_r) \phi(m) f[t - \tau(m)] X_i(m), \quad (6)$$

and X_i is the mass fraction of species, i , in the ejecta. The mass fraction of ^7Li produced in C star ejecta is taken in the models discussed below to have one of two values: either 0 (if the lithium once formed is subsequently destroyed by convection) or a mass fraction adjusted to give a present Population I value of $^7\text{Li}/\text{H} \sim 10^{-9}$. The produced lithium abundance is constrained to be less than that observed in super Li-rich C stars, $\text{Li}/\text{H} \sim 10^{-7}$ (Boesgaard 1976; Smith and Lambert 1989). The mass range of progenitor stars which become C stars is taken to be from 1 to $5 M_\odot$ (Lattanzio 1988).

Similarly, the ^7Li yield from supernovae is constrained to be either zero or that which is necessary to reproduce the solar system value. An upper limit to the allowable mass fraction of ^7Li produced in supernova ejecta can be obtained by assuming that all of the primordial ^3He in the hydrogen envelope is converted to ^7Li ; this gives $X_7 < 5 \times 10^{-5}$. For both carbon stars and supernovae, the inferred yields necessary to reproduce the present Population I ^7Li abundance are significantly less than these upper limits (see Table 1).

For the ejection rate, $E_i(t)$, from stars which have not produced light elements, the expression is the same as equation (3), but with the X_i given by the star's initial mass fraction and the limits of integration taken over all stellar masses. The fraction of the initial ^7Li which survives destruction during stellar evolution as a function of progenitor mass is taken from Dearborn and Hawkins (1990). The essential result of that work is that the ejected lithium is substantially depleted for stars of all masses and metallicities by at least a factor of 200. This depletion occurs at the lower boundary of the convective layer during the main sequence. As the star ascends the giant branch, the deepening of the convective zone homogenizes the composition of the star, and the surface lithium abundance drops. During the second dredge-up phase in stars more massive than $3 M_\odot$, the abundance drops even further.

There is little direct evidence for the necessity of an infall term in equation (1), although it is generally considered to be

important for understanding light element chemical evolution (see Reeves and Meyer 1978; Clayton 1987). It has been supposed that the Oort clouds may represent such material and would place an upper limit on the infall rate at the present time of $f_I < 2 M_\odot \text{ pc}^{-2} \text{ Gyr}^{-1}$ (Tinsley 1977). On the other hand, these clouds can also be interpreted as the result of a galactic fountain effect (Corbelli and Salpeter 1988) from material being ejected and returning to the disk. In what follows, we include the effects of constant extragalactic infall, because both the standard model and the inhomogeneous model require some replenishment of astrated deuterium to account for the presently observed ^2H abundance. The deduced infall rates are, in fact, larger for the standard homogeneous model because of a lower primordial deuterium abundance. In all cases, except for models with an exponentially decreasing star formation rate, the required infall rates are less than the limit deduced from the high-velocity clouds. For that case, the infall rate was taken to be at the upper limit of $2 M_\odot \text{ pc}^{-2} \text{ Gyr}^{-1}$. The required rate for these models would otherwise be $\sim 3 M_\odot \text{ pc}^{-2} \text{ Gyr}^{-1}$.

III. MODEL CALCULATIONS

With the above assumptions and input data equation (1) can be solved numerically. We wish to consider the evolution of ^7Li with and without the possibility of stellar sources and with both high and low primordial abundances. In this way, some scenarios may be found to be more consistent with observations than others. Before studying lithium, however, it is first necessary to consider deuterium and beryllium to ensure that both the destruction of primordial material and the cosmic-ray history are consistent with observations.

a) Deuterium

Deuterium is the most fragile of the light elements. Essentially all of the initial deuterium is destroyed in the inner convective region before a star contracts to the main sequence (Bodenheimer 1966; Mazzitelli and Moroetti 1980). Therefore, deuterium is the best indicator of the degree to which light elements have been destroyed. At the present time, this destruction could have been as little as a factor of 2 (Audouze and Tinsley 1974) or as large as a factor of 50 (Delbourgo-Salvador, Audouze, and Vidal-Madjar 1987).

Deuterium has not been detected on stellar surfaces. The best indicator of the present interstellar deuterium abundance is a result of absorption in the wings of Lyman ($L\beta$, $L\gamma$, $L\delta$, $L\epsilon$) features along the line of sight to O and B stars (Boesgaard and Steigman 1985). In addition, there exists the solar system deuterium, which was present when the Sun condensed 4.6 Gyr ago. Due to the effects of low-temperature fractionation, the deuterium abundance in the Earth and meteorites is probably higher than the initial interstellar value. The best value for the solar system is obtained from the analysis of deuterated molecules in the atmospheres of the giant planets (Kunde *et al.* 1982; Gautier and Owen 1989). These two values are summarized in Figure 1. Also shown for comparison are the predicted values of D/H for the different star formation rates discussed above and two initial values of D/H.

The two different values for the initial deuterium which we use correspond to (1) the standard homogeneous big bang with $\Omega_b = 0.07$ ($\eta_{10} = 5$, $h_0 = \frac{1}{2}$) for which $\text{D}/\text{H} = 3.6 \times 10^{-5}$ (Boesgaard and Steigman 1985) and $^7\text{Li}/\text{H} = 1.0 \times 10^{-10}$, and (2) a multizone inhomogeneous big bang model in which the neutron diffusion is coupled with the nucleosynthesis (Mathews *et al.* 1988). This model yields a deuterium abun-

FIG. 1.—Observed ^2H abundance (points) for the giant planets and the present day ISM compared with models for ^2H destruction with different star formation rates, f , and primordial values. Star formation rates are from Miller and Scalo (1979) (see Table 1). The infall rate of extragalactic material is adjusted in each model to reproduce the lower limit to the present ^2H abundance. For the decreasing star formation-rate models, the infall rate is fixed at a maximum value of $2 M_\odot \text{ pc}^{-2} \text{ Gyr}^{-1}$.

dance of $\text{D}/\text{H} = 5 \times 10^{-5}$ when the average separation distance between fluctuations is chosen to give the minimum ^4He and ^7Li abundance. For either of these initial conditions and the standard star formation rates, too much deuterium is destroyed to be consistent with the present-day lower limit to the ISM abundance of $\text{D}/\text{H} > 0.8 \times 10^{-5}$ (Boesgaard and Steigman 1985). Therefore, we adjust a constant infall rate (up to the limit of $2 M_\odot \text{ pc}^{-2} \text{ Gyr}^{-1}$) of unastrated primordial material in each model in order to obtain the minimum present-day observed D/H as shown in Figure 1. In this way, the models are completely specified. With these initial values, the required infall rates range from $f_I = 0.5\text{--}2.0 M_\odot \text{ pc}^{-2} \text{ Gyr}^{-1}$ (see Table 1).

b) Beryllium

It has been pointed out (Reeves and Meyer 1978; Arnould 1986) that Be is an important constraint on galactic chemical evolution, because numerous observations of the Be abundance as a function of stellar age indicate that Be has probably not changed by more than a factor of 2 for the past 10 Gyr. This behavior is somewhat different from that expected for deuterium. Beryllium is also particularly useful because it may not be as easily destroyed as lithium, so that the surface beryllium abundance for low-mass stars may be a more reliable indicator of the initial interstellar value than lithium. Furthermore, there are presently no competing stellar sources that are thought to have contributed to the abundance of Be, so that observations of Be are probably a good indicator of the cosmic-ray production history. We use Be, therefore, to determine the cosmic-ray contribution to ^7Li and as a test of the chemical evolution models specified above.

Figure 2 shows the Be/H evolution compared with the stellar data given in Reeves and Meyer (1978). The stellar age scale tends to be larger than the white-dwarf age (Winget *et al.* 1987) or the cosmochronometric age (Fowler 1987). For purposes of comparison, we scale the stellar ages by a factor of $T_0/15$ (except for the Sun). The cosmic-ray production is normalized to a solar value of $\text{Be}/\text{H} = 1.4 \times 10^{-11}$, as described

FIG. 2.—Calculated models for Be/H vs. time compared to observations summarized in Reeves and Meyer (1978). Upper curve at $t/T_0 < 0.6$ for each star formation rate corresponds to models with low primordial ^2H and ^7Li (homogeneous big bang) and therefore higher infall rates of extragalactic material. Lower curves are for models with high primordial ^2H and ^7Li (inhomogeneous big bang). Curves are normalized to the solar abundance at $t/T_0 = 0.62$.

above. Although the uncertainties in the data are large, the models yield a consistent description of the cosmic-ray production rate. It appears that an exponentially decreasing or constant star formation rate is favored. This is consistent with studies of the galactic chemical evolution of heavier elements (e.g., Mathews, Bazan, and Cowan 1989).

We note in passing that the Be evolution depicted in Figure 2 is essentially identical to the evolutionary curve for ^6Li , because this isotope in these models is also taken to be of cosmic-ray origin. The only difference with ^6Li evolution is in the normalization of the curves. All the present models would predict a solar system ^6Li abundance consistent with the meteoritic value.

c) Lithium

The cosmic-ray contribution for ^7Li is generated by scaling the production rate relative to ^9Be as described above and by allowing for the slightly different astration factors (Dearborn and Hawkins 1990). With the models and cosmic-ray production defined, it is then possible to examine the evolution of ^7Li . As in the case of deuterium, we consider two possibilities: the standard big bang, in which the initial $\text{Li}/\text{H} = 10^{-10}$ is increased to the Population I value of 10^{-9} via stellar sources (e.g., carbon stars or supernovae), and the possibility that the initial ^7Li is as high as that given in baryon number-inhomogeneous big bang scenarios. This high primordial ^7Li abundance is then astrated to the present-day Population I value in the ISM and depleted on stellar surfaces (e.g., by gradual mixing with the interior) to the presently observed Population II value.

For the ^7Li abundance in the high-primordial lithium models, we use the minimum value of Li/H obtained (Mathews *et al.* 1988) in a model in which neutron diffusion and nucleosynthesis are solved simultaneously on a multizone spherical grid. We note, however, that in other calculations which model the coupling of diffusion and nucleosynthesis somewhat differently (e.g., Terasawa and Sato 1989) or consider different geometrical shapes and amplitudes for the fluctuations (e.g.,

Mathews *et al.* 1988; Kurki-Suonio and Matzner 1989), the calculated $^7\text{Li}/\text{H}$ varies from 2×10^{-9} to 10^{-7} for conditions which satisfy the ^2H abundance constraint. There are also published calculations based upon simpler schematic models for neutron diffusion with $^7\text{Li}/\text{H}$ ranging from $\sim 10^{-8}$ (Applegate, Hogan, and Scherrer 1987; Alcock, Fuller, and Mathews 1987; Audouze *et al.* 1988) to $\sim 10^{-10}$ (Malaney and Fowler 1988).

Further uncertainties in the ^7Li abundance from inhomogeneous models arise from the possibility that hydrodynamical effects (Alcock *et al.* 1989) during the epoch of primordial nucleosynthesis may reduce the ^7Li yields, and that in a more realistic model in which the fluctuation distances and amplitudes are distributed randomly, the ^7Li and ^4He abundances may be higher. For the present purposes, we take our calculation using $^7\text{Li}/\text{H} = 7 \times 10^{-9}$ as representative of the possible lithium production in an inhomogeneous big bang.

Figures 3 and 4 show examples of $^7\text{Li}/\text{H}$ evolution for models with low and high primordial abundances and different star formation rates. These are compared with the inferred initial abundances for the five clusters studied in Hobbs and Pilachowski (1988) as well as the solar system meteoritic value (Boesgaard and Steigman 1985). The values for the youngest three clusters are from an average of the highest measured lithium abundances. The observational error is taken to be 40% for these clusters. The uncertainty in Li/H for M67 is taken to be 50% because of added uncertainty of extrapolating to the initial abundance. For NGC 188, the initial Li abundance could range from 10^{-9} to 10^{-8} or more, depending upon the age of this cluster and how one wishes to extrapolate to the initial abundance. The age uncertainties of these clusters are guessed to be $\sim 20\%$, except for NGC 188, which could have an age from 5 to 10 Gyr (Twarog and Twarog 1990; Van den Bergh 1985).

FIG. 3.—Models for Li/H as a function of time (starting from $\text{Li}/\text{H} = 10^{-10}$) compared with the meteoritic Li abundance and the inferred initial abundances for five clusters observed by Hobbs and Pilachowski (1988). Dotted line represents an exponentially decreasing star formation rate. Solid line represents a constant star formation rate, and dashed line represents increasing star formation (Miller and Scalo 1979).

For each star formation rate, the upper curve at early times corresponds to stellar ^7Li production in supernovae. The lower curve is for stellar ^7Li production in carbon stars. Also shown is the upper limit to Li/H along the line of sight to SN 1987A inferred from the work of Sahu, Sahu, and Pottasch (1989) and Malaney and Alcock (1990). The time for this point corresponds to $[\text{Fe}/\text{H}] \sim -0.6 \pm 0.3$ (see Fig. 5).

FIG. 4.—Same as Fig. 3, but for a high primordial ${}^7\text{Li}$ abundance

Note in Figure 4 that the depletion of ${}^7\text{Li}$ from a high primordial value to the present Population I value is achievable within the constraints on these models. Thus, the deuterium and ${}^7\text{Li}$ abundances do not in themselves rule out the possibility for high primordial ${}^7\text{Li}$. On the other hand, scenarios in which the ${}^7\text{Li}$ abundance begins at the Population II abundance and then is produced in stars up to the present Population I abundance also satisfy these constraints (at least for a decreasing star formation rate) as shown in Figure 3. Thus, it is difficult at the present time to distinguish between high or low primordial values based only upon the stellar and solar system observations of Li/H . The distinction between these models would require a measurement of Li/H for material which is substantially less processed.

A possibility for such data has recently been reported by Sahu, Sahu, and Pottash (1988). They obtained an upper limit to the Li/H ratio along the line of sight to SN 1987A shortly after maximum light. Such data should indicate the lithium abundance in the relatively less processed material of the LMC and galactic halo. They quote an upper limit of $\text{Li}/\text{H} < 1.6 \times 10^{-10}$, which they interpret also to be the upper limit to the primordial lithium abundance. Such a low number argues in favor of homogeneous big bang nucleosynthesis with low baryon number. However, Malaney and Alcock (1990) have recently reanalyzed this upper limit and have derived a value which is more than an order of magnitude higher. Furthermore, such a low value for the Li/H ratio is puzzling if this corresponds to material in the LMC. The LMC is commonly presumed to be enriched to at least $[\text{Fe}/\text{H}] \sim -0.6$ and is probably as high as $[\text{Fe}/\text{H}] = -0.3$. Since the enrichment of iron should be delayed by the time scales for Type I and II supernovae (see discussion below), the enrichment of lithium resulting from stellar sources should proceed as fast or faster than Fe (see Figs 3 and 5). Therefore, the LMC value of Li/H should be $\geq 2.5\text{--}5.0 \times 10^{-10}$, significantly higher than the Sahu, Sahu, and Pottash (1988) upper limit.

For purposes of comparison with the galactic evolutionary models for lithium, Figures 3 and 4 show limits to LMC lithium (labeled SN 1987A) from both the analysis of Malaney and Alcock (1990), and Sahu, Sahu, and Pottash (1988). We have plotted the inferred upper limits at a time ($t/T_0 \sim 0.2 \pm 0.1$) corresponding to $[\text{Fe}/\text{H}] \sim -0.6 \pm 0.3$ from the locally observed age-metallicity relation (Twarog 1980; see also Fig.

5). We note, however, that the age-metallicity relation in the solar neighborhood shows intrinsic scatter and appears to depend upon location as well, so that this comparison with the LMC may be even more uncertain than indicated in Figures 3 and 4.

This is a potentially useful upper limit. However, the SN 1987A upper limit does not yet rule out scenarios with high primordial lithium unless one accepts the Sahu, Sahu, and Pottash limit, which causes difficulty for some of the low primordial lithium scenarios as well. It is also difficult to reconcile the Sahu, Sahu, and Pottash (1988) limit with the Hobbs and Pilachowski (1988) data in any of the models, even allowing for the large uncertainties in the stellar ages and abundances. Nevertheless, because of its potential value, one should look critically at the derivation of this upper limit.

The relationship between Li I column density and total Li column density involves two correction factors. The first is to derive the $\text{Li II}/\text{Li I}$ ratio, which is obtained by a comparison with the $\text{Ca II}/\text{Ca I}$ ratio. This correction factor contains an unknown uncertainty. A more important correction is the depletion factor, which accounts for the condensation of lithium onto grains. The procedure which is used (Field 1974; Snow 1975) is based upon comparisons of depletion factors for potassium and lithium in the local interstellar medium. It should be pointed out that the argument is somewhat circular, because the local depletion factors were derived from presumed known total lithium and potassium abundances. Furthermore, at low temperatures, the equilibrium vapor pressures of potassium and lithium are very different, so that their comparison can be misleading. While it is clear that interstellar lithium could be an important probe of the history of lithium in the universe, much further study of the depletion onto grains will be needed before one should have confidence in the interpretation of these data. For these reasons, Malaney and Alcock (1990) deduce a much higher upper limit of $\text{Li}/\text{H} < 4.4 \times 10^{-9}$ along the line of sight to SN 1987A.

The correlation of lithium with iron (see Rebolo, Molero, and Beckmann 1988) is another possible indicator of lithium galactic evolution. The fact that there is no correlation for low Fe abundance has been taken as evidence (e.g., Reeves 1988) that the Population II stellar value is the true primordial abundance. This certainly is an important indicator of the fact that

FIG. 5.—Models (lines) of $[\text{Fe}/\text{H}]$ vs. time for different star formation rates compared to the values (points) derived in Twarog (1980).

there is indeed primordial Li. However, this is not proof that there has been no main-sequence destruction of the original primordial Li (Vauclair 1988). The reason is that the growth of the iron abundance in the disk may occur over a very short time interval (Twarog 1980). The variation of $[\text{Fe}/\text{H}]$ from -4 to -2 probably occurs in less than the first 10^9 yr (Twarog 1980; Matteucci 1986). Therefore, any gradual lithium destruction process would appear to vary little over the time interval sampled by most of the $[\text{Li}/\text{H}]$ versus $[\text{Fe}/\text{H}]$ correlation. It is also true that if the ^7Li abundance starts out at the Population II value, any galactic source of ^7Li would not affect the correlation until the accumulated galactic yield exceeded the primordial contribution ($\sim 10\%$ of the Population I value). This would happen at $[\text{Fe}/\text{H}] \sim -1.0$ if the stellar source of ^7Li tracks the stellar source of iron. Thus, the correlation of lithium with iron probably does not distinguish between models with low or high primordial lithium abundances.

In Figure 5, we show an example of a calculation of Fe/H as a function of time in a simplified model for Fe production which reproduces the essential features of more sophisticated models (Matteucci 1986; Matteucci and Greggio 1986; Mathews, Bazan, and Cowan 1990). The essential features of the models are taken from Iben and Tutukov (1987) and Tornambè and Matteucci (1987). Type Ia and Ib supernovae are assumed to occur with rates proportional to the rate at which $1-8 M_\odot$ stars evolve off the main sequence as a member of a binary system which includes a CO white-dwarf He star remnant as the other member (with a correction for the time scale for orbit decay by gravitational radiation after the common envelope stage). The average yield of iron from Type II supernovae is presumed to be about half of the yield from Type I supernovae per event. The relative rates of Type I and Type II supernovae are normalized at the present time to the rates given in Van den Bergh, McClure, and Evans (1987). The Fe yields are normalized overall to reproduce the meteoritic value when the solar system formed at $t = T_0 - 4.6$ Gyr. This normalization results in $\sim 0.5 M_\odot$ of iron per Type I supernova. The calculated Fe evolution is compared with derived Fe/H versus galactic age from Twarog (1980). The stellar ages in Twarog (1980) are scaled by a factor of $T_0/15$. All such models for the evolution of iron give essentially the same result: that the Fe abundance grows nearly linearly in time. Thus, the logarithmic abundance increases rapidly for the first 10^9 yr and at a slower rate thereafter.

Figures 6 and 7 show comparisons of the $[\text{Fe}/\text{H}]$ versus $\log N(\text{Li})$ [for $\log(\text{H}) = 12$] correlation from the models discussed above for each of the different star formation rates and lithium evolution scenarios. For the models which begin with high primordial ^7Li (Fig. 7), an exponential decay constant (Vauclair 1988) for surface lithium has been adjusted such that the stars with ages as old as the halo have depleted their lithium abundance from the primordial value to the present Population II value of $\text{Li}/\text{H} \sim 10^{-10}$. Stars born later, when the interstellar lithium abundance was different, are presumed to deplete their lithium with the same decay constant. The decay constant chosen in this way happens to be quite close to the value derived in Vauclair (1988) on the basis of a model with turbulent mixing induced by rotation. The data on Figures 6 and 7 are from the sources summarized in Rebolo, Molaro, and Beckmann (1988). It appears that the data do not distinguish among scenarios with high or low primordial lithium. If anything, the gradual destruction scenarios fare slightly better than the stellar source models. Also, carbon

FIG. 6.— $[\text{Fe}/\text{H}]$ vs. $\log N(\text{Li})$ correlation for various star formation rates compared with the observations summarized in Rebolo, Molaro, and Beckmann (1988). These models begin with a low primordial ^7Li abundance which is subsequently enhanced to the Population I value by production in supernovae (upper curves) or carbon stars (lower curves).

stars appear to agree slightly better than supernovae as a source for ^7Li .

Another possible constraint on the models for gradual main-sequence destruction of lithium can be obtained from the direct measurements of Hobbs and Pilachowski (1988) before extrapolating to the initial stellar value. The constraint is the maximum observed surface lithium abundance for stars in a given cluster. If there is gradual main-sequence destruction of lithium, then the directly observed lithium abundances as a function of galactic age should be comparable to that predicted by slow main-sequence destruction. A comparison of the average of the highest measured lithium abundance for stars in each cluster with the exponential depletion model described above is summarized in Figure 8. It is apparent that such exponential depletion of lithium is at least consistent with these data.

IV. CONCLUSION

We have demonstrated that most of the observed abundances of light elements from various sources are equally con-

FIG. 7.—Same as Fig. 6, but with a high primordial ^7Li abundance and gradual exponential main-sequence destruction of ^7Li .

FIG. 8.—Comparison of the maximum observed Li/H (Hobbs and Pilachowski 1988) as a function of stellar age (points) with models of high primordial lithium followed by gradual exponential main-sequence destruction (lines) for different star formation rates.

sistent with chemical evolution models, starting with a low primordial lithium abundance followed by galactic sources to produce the Population I abundance, or models with a high primordial lithium abundance followed by gradual main-sequence depletion of lithium to reproduce the presently observed stellar Population II abundance. Among the models,

it appears that those with an exponentially decreasing star formation rate best reproduce the available constraints. This is consistent with studies of the evolution of heavier elements.

The only way to distinguish between the models with high and low primordial lithium abundances will be to obtain a good measure of the lithium abundance for unprocessed extragalactic material. The recent analysis of lithium absorption along the line of sight to SN 1987A goes part of the way toward making this test. However, we note that the derived upper limit is subject to large uncertainties (Malaney and Alcock 1990), because of the substantial correction factor for the depletion of lithium onto grains. Thus, more and better measurements, as well as a more detailed understanding of lithium production and destruction in stars, may be necessary to ultimately establish which of these scenarios is most consistent with observations.

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