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ABSTRACT

By use of a formula given by Perkins for the distribution of secondary particles produced in high-energy collisions, we calculate the neutrino flux to be expected if a beam of high-energy protons impinges on a target. The resulting simple formula for the flux is used to estimate the flux of neutrinos that may be obtained from a 200-BeV AGS accelerator, operated at various beam energies. Because of the smaller amount of shielding and a greater pulse rate at lower energies, it is seen that the energy of the neutrinos to be studied will dominate the choice of operating energy.

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I. INTRODUCTION

From the time of the interpretation of beta-decay phenomena in terms of the neutrino hypothesis, there has been strong interest in using neutrino beams to induce nuclear reactions. These reactions are of interest not only to dramatically demonstrate the existence of the neutrino[1] but also to explore the host of other questions[2] that have arisen in the ensuing years as a result of the discovery of the general area of the weak interactions.

The feasibility of performing high-energy experiments using neutrinos, pointed out by Schwartz,[3] and the subsequent demonstration of the existence of two neutrinos,[4] has stimulated considerable interest in assessing the performance of various particle accelerators in providing neutrino beams adequate for the exploration of the multitude of other vital questions concerning the weak interactions.[5]

This note is concerned with an estimate of the neutrino flux from the decays of pions and kaons produced by proton bombardment of an external target. Our estimate is based on a formula for the production of high-energy mesons suggested by D. H. Perkins.[6] By using this formula, a simple formula is deduced in Sec. II for the neutrino flux. This formula can easily be applied to experiments at various energies using a high-energy accelerator, and in Sec. III the formula has been used to obtain numerical estimates of the neutrino fluxes expected from a 200-BeV AGS accelerator operated at various proton-beam energies.

II. NEUTRINO FLUX

The assumed experimental arrangement is shown in Fig. 1: neutrinos would be produced by using an external proton beam to bombard the target, T , yielding pions and kaons. These subsequently decay into muons and neutrinos as they proceed along the flight path, L . Muons and other unwanted background are removed from the beam in shielding of length S . The shielding length must be chosen large enough to remove all high-energy muons, and is determined by the energy of the proton beam, being approximately proportional to the energy. The pure neutrino beam is then used to study the desired reactions in the detector D . Once S is chosen, the maximum beam intensity at D for high-energy neutrinos is obtained by choosing L = S , as is easily seen from the subsequent analysis.

The number of mesons produced in the solid angle $d\Omega_M$ and in the energy region dE_M per proton collision in the target may be written as

$$\frac{d^2N_M(E_M, \Omega_M)}{dE_M\, d\Omega_M}\, dE_M\, d\Omega_M \quad . \tag{1}$$

Of these particles, a fraction will decay in the region between the target and the shield to produce high-energy neutrinos at the detector. We treat this decay process using the idealization of the geometry of Fig. 1 as shown in Fig. 2. The meson is produced at the point T and then proceeds a distance r before emitting a neutrino at angle θ_ν relative to its line of flight. The neutrino then passes through point D after travelling a distance R . Thus the number of neutrinos produced per unit energy by meson decay between r and r + dr is

$$\int_{\Omega_M} \frac{d^2N_M}{dE_M\, d\Omega_M} \quad d\Omega_M \quad e^{-r/\lambda} \quad \frac{dr}{\lambda} \quad , \tag{2}$$

where the mean path length λ is given by

$$\lambda = \frac{1}{(\gamma \ v \ \tau_0)} \quad . \tag{3}$$

Here, γ is the time dilation factor, v is the meson velocity, and τ_0 is the proper lifetime. For the cases of interest, namely, high energy neutrino production, θ_M is very small, and for $L \approx S$ the exponential in Eq. (2) is found to be close to one so that only a small fraction of the mesons produce useful decays. (By increasing L , this fraction would be increased, but the solid angle of the detector as seen at the target would be reduced to produce an over-all reduction.) In the rest system of the decaying particle, the spectrum of neutrinos per decaying particle is

$$I'(p'_\nu, \Omega'_\nu) = \frac{\delta(p'_\nu - p^o_\nu)}{4\pi(p^o_\nu)^2} , \qquad (4)$$

where I' is the number of neutrinos produced per unit volume in momentum space, δ is the Dirac delta function, Ω'_ν is the direction of the neutrino in the rest frame, p'_ν is the neutrino energy, and p^o_ν is the single value of p'_ν allowed in the rest system;[7] we have

$$p^o_\nu = \frac{1}{2} \frac{(M^2 - \mu^2)}{M} , \qquad (5)$$

where M is the mass of the decaying particle and μ is the mass of the muon. For pion decay this value is 30.6 MeV, while for kaon decay it is 236 MeV.

The decay spectrum in the laboratory system may be found by making a Lorentz transformation. If the intensity in the laboratory is denoted by I then

$$I = I'J , \qquad (6)$$

where J is the Jacobian of the transformation. One readily finds[8]

$$J = \gamma(1 - v \cos \theta_\nu) \quad . \qquad (7)$$

The resulting neutrino flux[9] at D is found by taking the product of Eqs. (2), (4), (6), and (7), dividing by a factor of R^2, and carrying out integrations over the meson distribution and flight path. One finds

$$\frac{d^2 N_\nu}{dA\, dp_\nu} = \int_\Omega d\Omega_M \int_0^\infty dE_M \int_0^{L/\cos\theta_M} dr \times \left\{ \frac{e^{-r/\lambda}}{\lambda} \frac{d^2N_M}{dE_M\, d\Omega_M} \frac{\delta[\gamma\, p_\nu(1 - v\cos\theta_\nu) - p^o_\nu]}{4\pi(p^o_\nu)^2} \frac{\gamma(1 - v\cos\theta_\nu)}{R^2} \right\}, \tag{8}$$

where $d^2N_\nu/dA\, dp_\nu$ is the number of neutrinos per unit area at the detector per unit neutrino energy, per nucleon interaction in the target.

The integrals in Eq. (8) have been evaluated using the aforementioned formula suggested by Perkins for the production of secondary particles in high-energy nucleon-nucleon collisions. This formula is in reasonable agreement with available very-high-energy data obtained from cosmic-ray experiments and with the 30-BeV results from accelerators. Perkins' formula is probably accurate in the forward direction to within a factor of 2 for pion production above 10 BeV and for kaon production at somewhat higher energies. As will be shown, only forward mesons are of importance for high-energy neutrino production so this expression is only important in Eq. (8) where it is expected to be the most trustworthy. Perkins' expression for the number of pions of energy E_M and momentum p_M produced at angle θ_M in a nucleon-nucleon collision per unit energy per steradian is

$$\frac{d^2N_M}{dE_M\, d\Omega_M} = \frac{n\, p_M^{\,2}}{2\pi\, T p_\perp^{\,2}} |\cos\theta_M| \exp[-E_M/T] \exp[-p_M \sin\theta_M/p_\perp] , \tag{9}$$

where $T = 0.313\, E_0^{3/4}$, $n = 1.28\, E_0^{1/4}$, $p_\perp = 0.2$ BeV and E_0 is the laboratory energy of the incident proton in BeV. The pion multiplicity is n . This formula is also approximately valid for kaons if the multiplicity is reduced by a factor of 10. Upon combining Eqs. (8) and (9) one finds for the neutrino flux

$$\frac{d^2N_\nu}{dA\, dp_\nu} = \frac{n\,L}{2\pi\, T\, p_\perp^{\,2}} \int_{\Omega_M} d\Omega_M \int_0^\infty dE_M \int_0^1 dx \left\{ \frac{\exp[-xL/\lambda \cos\theta_M]}{\lambda}\, p_M^{\,2} \right.$$

$$\left. \times \exp[-E_M/T]\, \exp[-p_M \sin\theta_M/p_\perp\,]\, \frac{\delta[\gamma p_\nu(1 - v\cos\theta_\nu) - p^o_\nu]}{4\pi\, p^o_\nu{}^2}\, \frac{\gamma(1 - v\cos\theta_\nu)}{R^2} p_\nu^{\,2} \right\}. \tag{10}$$

The integral over Ω_M may be evaluated by noting the relations $(D = L + S)$

$$R^2 = r^2 + D^2 - 2r\,D\cos\theta_\pi \quad ,$$

and (11)

$$D^2 = r^2 + R^2 + 2r\,R\cos\theta_\nu \quad ,$$

which leads immediately to

$$\frac{d(\cos\theta_\pi)}{R^2} = \frac{d(\cos\theta_\nu)}{D(D^2 - r^2\sin^2\theta_\nu)^{\frac{1}{2}}} \quad . \tag{12}$$

Using Eq. (12) to conveniently handle the delta function in Eq. (10) one obtains:

$$\frac{d^2 N_\nu}{dA\, dp_\nu} = \frac{n M^2 L}{4\pi T p_\perp^2 D^2 p^c_\nu \tau_0} \int_0^\infty dE_M$$

$$\times \int_0^1 \left\{ \frac{dx \exp[-xL/\lambda \cos \theta_M] \exp[-E_M/T] \exp[-p_M \sin \theta^o_M / p_\perp]}{[1 - (r^2/D^2) \sin^2 \theta^o_\nu]^{\frac{1}{2}}} \right\} . \quad (13)$$

In this expression, θ^o_ν is given by

$$\cos \theta^o_\nu = \frac{1}{v} \left[1 - \frac{p^o_\nu}{\gamma p_\nu} \right] , \quad (14)$$

which for large p_ν becomes, approximately,

$$\theta^o_\nu \approx \left[\frac{2 p^o_\nu}{\gamma p_\nu} - \frac{1}{\gamma^2} \right]^{\frac{1}{2}} . \quad (15)$$

Similarly, θ^o_M is obtained by noting that

$$x L \sin \theta_\nu = D \cos \theta_M \sin(\theta_\nu - \theta_M) . \quad (16)$$

Since for high-energy neutrinos $p_M \gg p_\perp$, only small values of θ_M will contribute to the flux and θ_M , θ_ν may be treated as small. Then θ^o_M is approximately given by

$$\theta^o_M \approx (1 - \frac{xL}{D}) \theta^o_\nu . \quad (17)$$

We may also note that for high-energy neutrinos, the lower limit on the integral over the pion-energy distribution, E_1 , is to the same degree of approximation

$$E_1 = \frac{M}{2} \frac{p_\nu}{p^o_\nu} \quad . \tag{18}$$

This follows from setting $\theta^o_\nu = 0$ in Eq. (15). In order to further simplify the evaluation of Eq. (13) advantage may be taken of the fact that very few pions decay in the region between the target and shield;[10] i.e.,

$$\frac{x\,L}{\lambda} \ll 1 \; . \tag{19}$$

The neutrino flux is therefore

$$\frac{d^2 N_\nu}{dA\, dp_\nu} \approx \frac{n\, M_m^{\;2}\, L}{4\pi\, T\, p_\perp^{\;2}\, D^2\, p^o_\nu\, \tau_0} \int_{E_1}^{\infty} dE_M \int_0^1 dx\, \exp[-E_M/T]$$

$$\times \exp[\, - \frac{p_M}{p_\perp} (1 - \frac{xL}{D})\, \theta^o_\nu \; . \tag{20}$$

The integral over x may now be carried out to give

$$\frac{d^2 N_\nu}{dA\, dp_\nu} = \frac{n\, M^2}{4\pi\, T\, p_\perp\;\; D\, p^o_\nu\, \tau_0} \int_{E_1}^{\infty} \frac{dE_M}{p_M\, \theta^o_\nu} \exp[-E_M/T]$$

$$\times \exp[-\frac{p_M}{p_\perp} \frac{S}{D}\, \theta^o_\nu] - \exp[-\frac{p_M}{p_\perp}\, \theta^o_\nu\,] \quad . \tag{21}$$

To proceed further, it is convenient to introduce the variable

$$y^2 = \frac{E_M - E_1}{T} \quad . \tag{22}$$

In addition we make the approximation $p_M \approx E_M$ for the energies of interest, so that Eq. (21) becomes

$$\frac{d^2 N_\nu}{dA\, dp_\nu} = \frac{n M^2}{4\pi T p_\perp D p^o_\nu \tau_0} \left(\frac{2p_\nu T}{M p^o_\nu} \right)^{\frac{1}{2}} \exp[-E_1/T]$$

$$\times \int_0^\infty dy\ e^{-y^2} \left\{ \exp\left[- \frac{S}{D} \left(\frac{2 p^o_\nu M T}{p_\nu p_\perp^2} \right)^{\frac{1}{2}} y \right] - \exp\left[- \left(\frac{2 p^o_\nu M T}{p_\nu p_\perp^2} \right)^{\frac{1}{2}} y \right] \right\}. \qquad (23)$$

As has been remarked earlier in this note the maximum flux of high energy neutrinos is obtained when $S = L$.[11] If this relation is now used, it is convenient to introduce the variable

$$\beta = \left(\frac{M T p^o_\nu}{2p_\perp^2 p_\nu} \right)^{\frac{1}{2}} = \frac{M}{2p_\perp} \left(\frac{T}{E_1} \right)^{\frac{1}{2}} , \qquad (24)$$

in terms of which Eq. (23) becomes

$$\frac{d^2 N_\nu}{dA\, dp_\nu} = \frac{n M e^{-E_1/T}}{4\pi p^o_\nu \tau_0 L} \left(\frac{E_1}{T} \right)^{\frac{1}{2}} \int_0^\infty dy\ e^{-y^2} \left[e^{-\beta y} - e^{-2\beta y} \right]. \qquad (25)$$

The integral may be represented in terms of the error function

$$\mathrm{erf}(\alpha) = \frac{2}{\sqrt{\pi}} \int_0^\alpha e^{-y^2}\, dy \quad . \qquad (26)$$

The result is

$$\frac{d^2 N_\nu}{dA\, dp_\nu} = \frac{n M \, e^{-E_1/T}}{4\pi \, p^o_\nu \, \tau_0 \, L \, p_\perp} \left(\frac{E_1}{T} \right)^{\frac{1}{2}} \phi(\beta) \ , \tag{27}$$

where

$$\phi(\beta) = \frac{\sqrt{\pi}}{2} \left\{ e^{\beta^2/4} \left[1 - \mathrm{erf}\left(\frac{\beta}{2} \right) \right] - e^{\beta^2} \left[1 - \mathrm{erf}(\beta) \right] \right\} . \tag{28}$$

It is worth noting that for small β, the geometry of the decay is essentially fixed by θ_ν, while for large β the angular distribution of the mesons is dominant. From the Perkins' formula one sees that

$$\begin{aligned} \theta_M &\lesssim p_\perp / p \\ &\lesssim \frac{2 p_\perp \, p^o_\nu}{M \, p_\nu} \equiv \theta_M^{\ m} \ . \end{aligned} \tag{29}$$

On the other hand, from the relation between the neutrino energy and the decay angle, we find for small decay angles

$$E_M \approx \frac{M \, p_\nu}{2 p^o_\nu} \left(1 + \frac{1}{4} \, \frac{p_\nu^{\ 2} \, \theta_\nu^{\ 2}}{p^{o\,2}_\nu} \right) . \tag{30}$$

The angular distribution of the neutrinos is thus dominated by the distribution in meson energies. Thus we see that

$$\theta_\nu \lesssim \left(\frac{8 \, p^{o\,3}_\nu \, T}{p_\nu^{\ 3} \, M} \right)^{\frac{1}{2}} \equiv \theta_\nu^{\ m} \tag{31}$$

and the ratio of these two bounds, $\theta_\nu^m/\theta_M^m = 2\beta$.

In the limit $\beta \ll 1$ (high-energy neutrinos), the neutrino distribution is particularly simple:

$$\frac{d^2 N_\nu}{dA\, dp_\nu} \approx \frac{n M^2 \exp(-E_1/T)}{16\pi\, p^o_\nu\, \tau_0\, L\, p_\perp^2} . \tag{32}$$

In this limit, we need only the forward production of mesons from the Perkins' formula, and here it is believed to be the most accurate.

III. THE PERFORMANCE OF A 200-BeV ACCELERATOR

To illustrate the results obtained from Eq. (27), the neutrino flux has been estimated for a 200-BeV AGS accelerator. We assume that when operated at energy E_0 it will produce a current of

$$(1.0 \times 10^{13}) \times \left(\frac{200}{E_0}\right) \text{ protons/sec.} \tag{33}$$

The figure of 1.0×10^{13} protons/sec represents a scaling derived on reasonable assumptions from the expected performance of the CERN-BNL accelerators.[12] To use the factor $200/E_0$ we assume that the machine can be operated at a higher pulse rate and correspondingly larger average current when operated at less than full energy. Thus we obtain a neutrino flux (per m^2 BeV sec at the detector) of

$$\frac{d^2 N_\nu}{dp_\nu\, dA\, dt} = \frac{1.0 \times 10^{15}\, n\, M\, G}{6\pi\, p^o_\nu\, \tau_0\, p_\perp\, L\, E_0} \left(\frac{E_1}{T}\right)^{\frac{1}{2}} e^{-E_1/T}\, \phi(\beta) . \tag{34}$$

A factor of one-third has been inserted to represent target efficiency. In addition to this, the factor G represents the fraction of the total particles produced that lead to neutrinos. For pions this is two thirds. For kaons we used a factor of 10% as the relation between kaon and pion production for Perkins' formula, a factor of 1/2 for the ratio of charged-meson production to the total, and finally a factor of 65% for the $K_{\mu 2}$ branching ratio. Finally, in the calculations we have assumed a shielding length of 1 meter per BeV operating energy.

Curves of the neutrino flux from pion decay have been plotted in Fig. 3 for various operating energies, E_0. Similar curves for the neutrino flux from kaon decay are presented in Fig. 4. It is of interest to note that all of the neutrinos so far considered are of the μ type. To obtain electron-associated neutrinos of high energy, one must either depend on subsequent μ decays or on such decays as K_{e3}. In either case, the ν_e intensity will be very small compared to the ν_μ.

IV. CONCLUSION

The curves of neutrino fluxes, Figs. 3 and 4, lead us to the following conclusions:

(a) The maximum neutrino flux obtainable from an accelerator capable of very high energy is a strongly varying function of the energy at which the machine is operated. For example, if one wishes to study neutrino reactions in the region below 2 BeV, an operating energy of

10 BeV is strongly favored over higher energies such as 30 or 100 BeV. This effect is principally a result of a reduction in pulse rate and the additional shield length required at higher energies. (The precise optimum operating energy will depend on the details of the pion-production spectrum. Since the Perkins' formula is only expected to be valid for very high energies, the detailed results are not entirely trustworthy. Nevertheless, qualitatively, the situation should be as given by the curves.)

(b) Operation at higher energies will only be indicated if higher energy neutrino interactions are of interest. It is of course clear that high energy neutrinos can only be produced by a high energy accelerator.

(c) Neutrino production by kaon decays is not of great significance for most of the energy range; however, for an accelerator of limited energy capability it might be of importance in achieving maximum energy neutrinos.

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FOOTNOTES AND REFERENCES

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1. F. Reines and C. L. Cowan, Phys. Rev. 92, 830 (1953).
2. T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960).
3. M. Schwartz, Phys. Rev. Letters 4, 306 (1960).
4. G. Danky, J.-M. Gaillard, K. Gowlianos, L. M. Lederman, N. Mistry, M. Schwartz, and J. Steinberger, Phys. Rev. Letters 9, 36 (1962).
5. Proposal for a 10-BeV High-Intensity Accelerator, Midwestern Universities Research Association, March 30, 1962.
6. G. Cocconi, L. J. Koester, and D. H. Perkins, Particle Fluxes from Proton Synchrotrons in The Berkeley High-Energy Physics Study, June 15 through August 15, 1961, UCRL-10022, 167, April 1963 (unpublished).
7. Throughout this note we choose units in which $c = 1$.
8. As is well known, the neutrino spectrum from a two-body decay is constant in energy in the laboratory system and for the decay of one particle is given by

$$\frac{dn_\nu}{dp_\nu} = \frac{1}{2\gamma v p^o_\nu} .$$

The spectrum extends over the range

$$p^o_\nu \left(\frac{1 - v}{1 + v}\right)^{\frac{1}{2}} \leqslant p_\nu \leqslant p^o_\nu \left(\frac{1 + v}{1 - v}\right)^{\frac{1}{2}} .$$

These follow immediately from Eqs. (4), (6), and (7).

9. The term "flux" is used in a slightly unconventional sense since it does not mean the current perpendicular to the target; however, the individual angular contributions correspond to the flux in that particular direction. The total integral is thus directly related to the total rate of events in the detector from all angles.

10. For example, for 1-BeV neutrinos produced from secondary pions from a primary beam of 10-BeV protons, the shield length must be of the order of 10 m. Since we have $p_\nu < 2\gamma\, p^o_\nu$, then we obtain $\gamma > 16$ and $\lambda > 120$ meters. Thus we have $L/\lambda \ll 1$. A higher operating energy would require a longer shield length and hence a greater L, but λ becomes larger also.

11. This is readily seen from Eq. (23). For high p_ν, the exponentials may be expanded and the dependence of the flux on L becomes

$$\frac{d^2 N_\nu}{dA\, dp_\nu} \propto \frac{L}{(L+S)^2} .$$

The flux is thus maximized by choosing $L = S$.

12. David L. Judd and Lloyd Smith, private communication.

FIGURE CAPTIONS

Fig. 1. The experimental arrangement.

Fig. 2. Idealized decay geometry.

Fig. 3. Neutrino flux from pion decay at various operating energies.

Fig. 4. Neutrino flux from kaon decay at various operating energies.

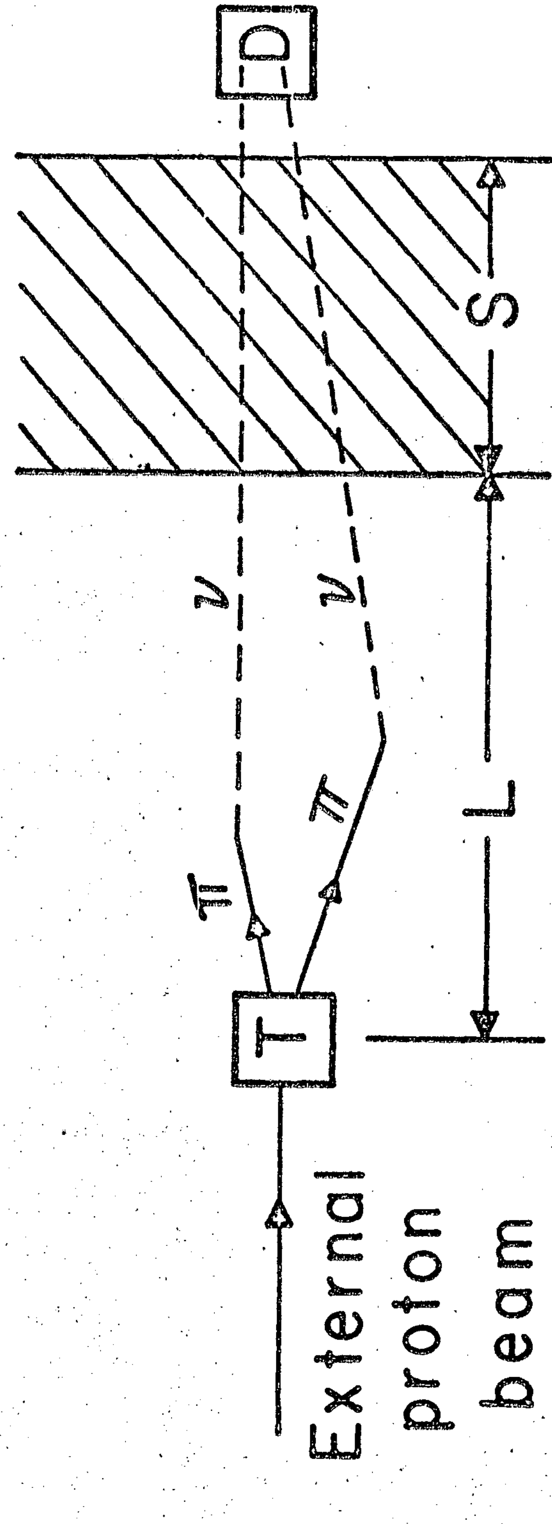

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Fig. 1.

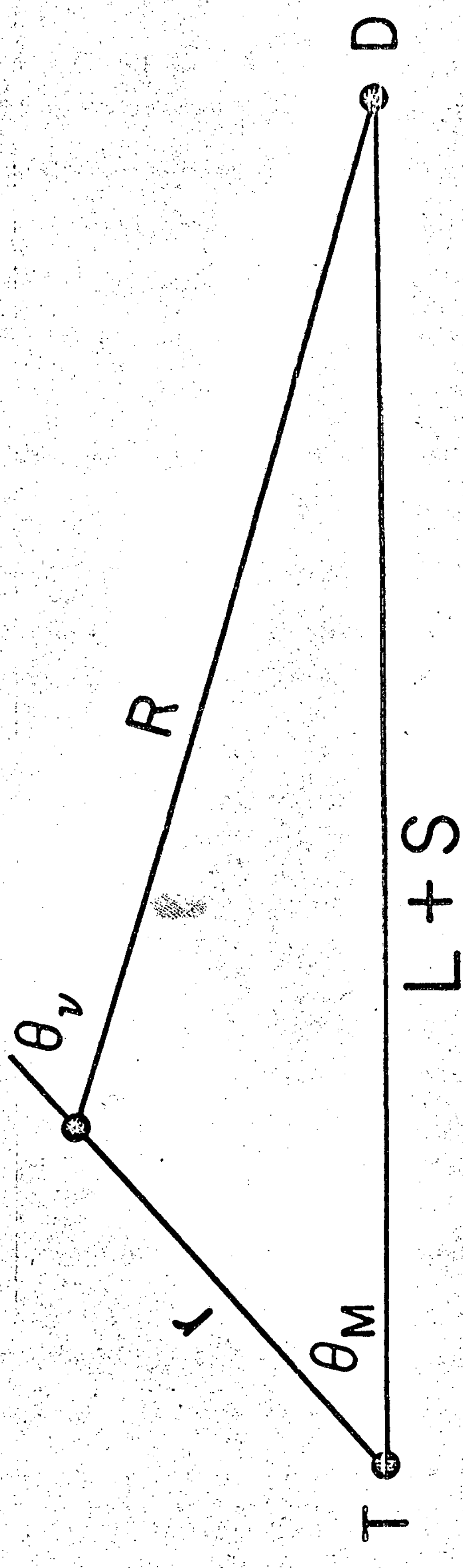

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Fig. 2.

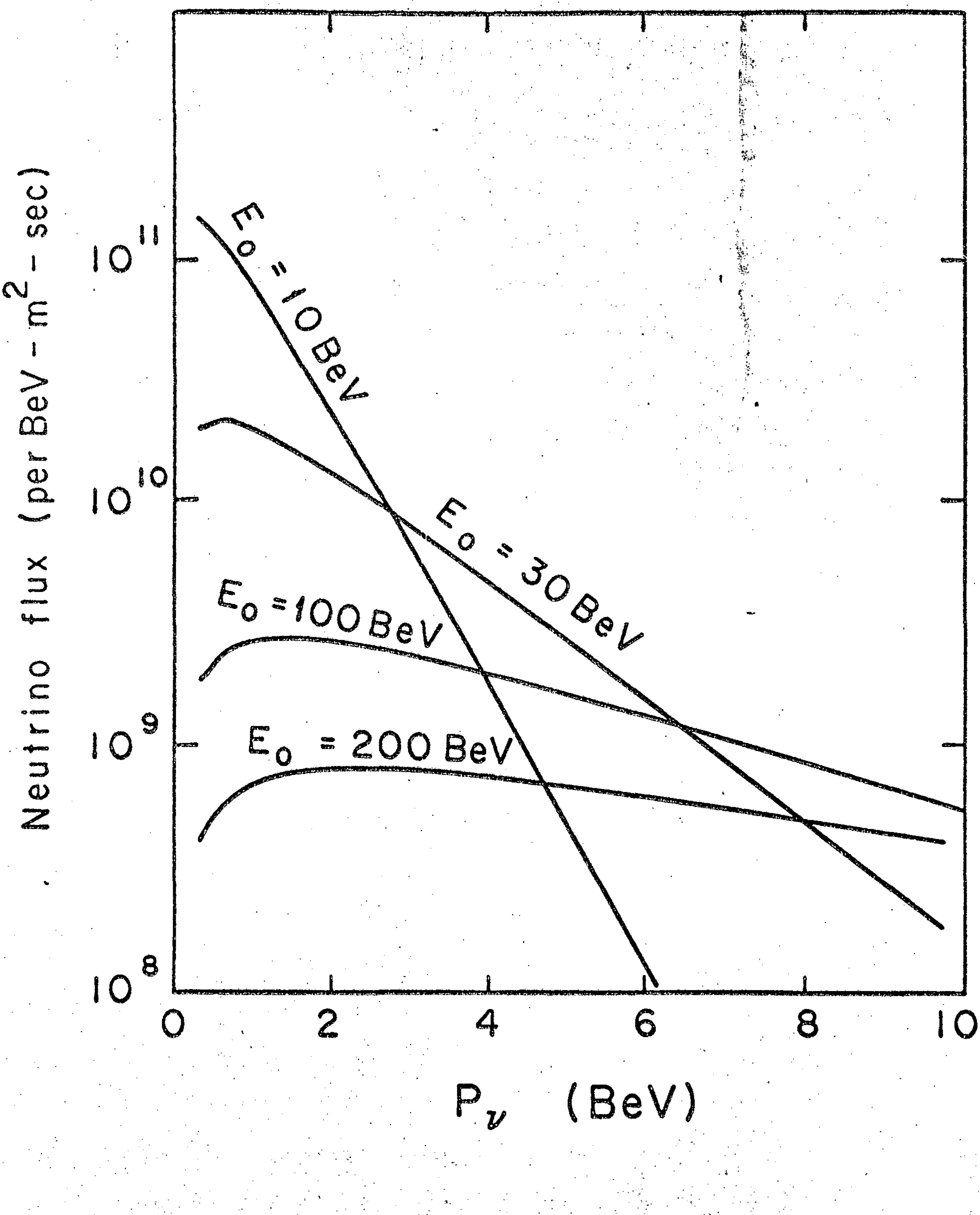
Neutrino flux (per BeV - m^2 - sec)
10^{11}
10^{10}
10^{9}
10^{8}
E_o = 10 BeV
E_o = 30 BeV
E_o = 100 BeV
E_o = 200 BeV
0
2
4
6
8
10
P_ν (BeV)
MU-31457

Fig. 3.

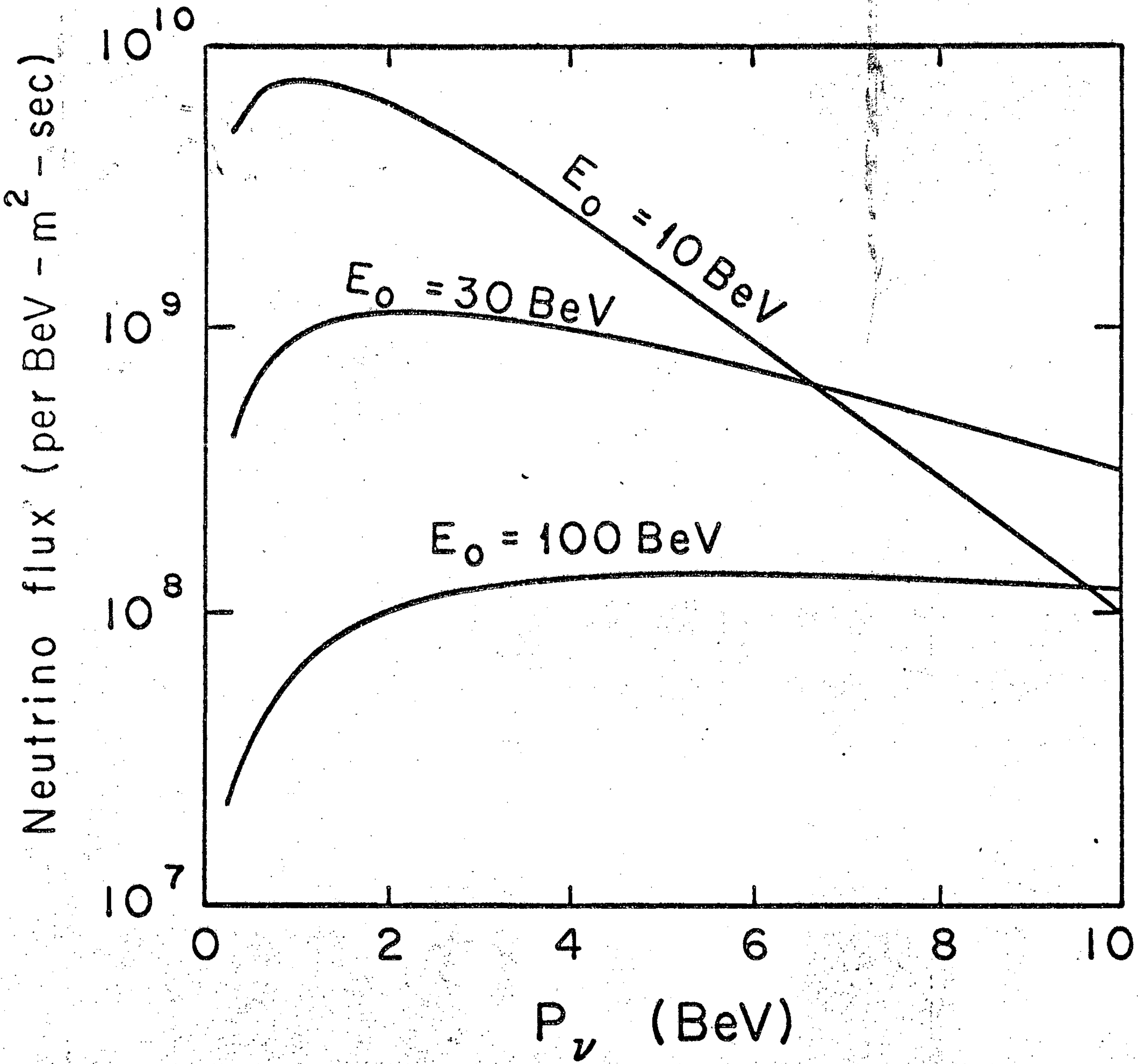

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Fig. 4.