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Statistical Rate Event Analysis with Elite Sample Selection Scheme

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To my parents, Wendy and Emma.

ABSTRACT OF THE DISSERTATION

Statistical Rate Event Analysis with Elite Sample Selection Scheme

by

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Master of Science, Graduate Program in Electrical Engineering
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Dr. Sheldon X.-D. Tan, Chairperson

Accurately estimating the failure region of rare events for memory-cell and analog circuit blocks under process variations is a challenging task. As the first part of the thesis, author propose a new statistical method, called *EliteScope* to estimate the circuit failure rates in rare event regions and to provide conditions of parameters to achieve targeted performance. The new method is based on the iterative blockade framework to reduce the number of samples. But it consists of two new techniques to improve existing methods. First, the new approach employs an elite learning sample selection scheme, which can consider the effectiveness of samples and well-coverage for the parameter space. As a result, it can reduce additional simulation costs by pruning less effective samples while keeping the accuracy of failure estimation. Second, the *EliteScope* identifies the failure regions in terms of parameter spaces to provide a good design guidance to accomplish the performance target. It applies variance based feature selection to find the dominant parameters and then determine the in-spec boundaries of those parameters. We demonstrate the advantage of our proposed method using several memory and analog circuits with different number of

process parameters. Experiments on four circuit examples show that *EliteScope* achieves a significant improvement on failure region estimation in terms of accuracy and simulation cost over traditional approaches. The 16-bit 6T-SRAM column example also demonstrate that the new method is scalable for handling large problems with large number of process variables.

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Chapter 1

Introduction

1.1 Introduction of Statistical Rare Event Analysis

As VLSI technology scales into the nanometer regime, chip design engineering face several challenges in maintaining historical rates of performance improvement and capacity increase with CMOS technologies. One profound change in the chip design business is that engineers can't put the design precisely into the silicon chips. Chip performance, manufacture yield and lifetime become unpredictable at the design stage. Chip performance, manufacture yield and lifetime can't be determined accurately at the design stage. The main culprit is that many chip parameters – such as oxide thickness due to chemical and mechanical polish (CMP) and impurity density from doping fluctuations – can't be determined precisely, and thus are unpredictable. The so-called manufacture process variations start to play a big role and their influence on the chip's performance, yield and reliability becomes significant.

As a result, how to efficiently and accurately assess the impacts of the process variations of interconnects in the various physical design steps are critical for fast design closure, yield improvement, cost reduction of VLSI design and fabrication processes. The design methodologies and design tools from system level down to the physical levels have to embrace variability impacts on the VLSI chips, which calls for statistical/stochastic based approaches for designing next generation VLSI systems.

Moreover, the performance uncertainties related to the process variation have become a major concern for IC development [2]. Many IC components such as SRAM bit-cells need to be tremendously robust as they are duplicated in the millions [8]. Such modules require accurate statistical failure analysis in rare event region. However, the traditional Monte Carlo (MC) based statistical analysis method faces a challenge as it may require millions of simulations [14].

In this regard, it is imperative to develop new design methodologies to consider the impacts of various process and environmental uncertainties and elevated temperature on chip performance. Variation impacts and thermal constraints have to be incorporated into every steps of design process to ensure the reliable chips and profitable manufacture yields. The design methodologies and design tools from system level down to the physical levels have to consider variability and thermal impacts on the chip performance, which calls for new statistical and thermal-aware optimization approaches for designing manometer VLSI systems.

1.2 Motivation

To mitigate this problem, a number of statistical analysis algorithms have been developed in the literature [4, 15, 19, 20, 14, 9, 1, 11, 12]. One way is by means of importance sampling (IS) [4], which consist of two steps. First, it shifts the mean value of the initial performance distribution and places it on interested failure region. The standard deviation based on the shifted mean is recalculated by considering samples only placed on the failure region. The new probability density function (PDF) is generated based on updated mean and standard deviation so that more samples in the failure region can be drawn. Work in [12] applied the mixture of IS for cross-validation of multiple failure regions due to disjointed process parameters.

However, these approaches can only estimate single performance metric. Multiple important samplings are required to estimate more metrics. Also, it is difficult to calculate the failure probability for a generated distribution by IS.

The statistical blockade (SB) is another effective approach for improving the performance of MC method [14]. The idea of this approach is using a threshold bound to separate an interested failure region from the whole distribution so that it can *block* some unnecessary sampling and simulation for efficiency improvement. This method builds a supervised learning model with the threshold bound and initial simulation data, which is known as "classifier", to recognize failure samples. Later samples that tend to be placed in the failure region can be captured without simulation.

This approach was improved by using the recursive statistical blockade (RSB) scheme to locate the rare event failure region in an iterative way [15]. This iterative method

can improve the accuracy of the classifier by increasing the number of samples in the failure region of interest. However, this method can incur the significant extra cost as it needs more samples for the simulation. Wu *et al.* [20] applied a nonlinear SVM (Support Vector Machine) classifier to model nonlinear and multiple disjoint failure regions of circuits. This method applies the generalized Pareto distribution (GPD) fitting for tail distribution to model failure probability in each iteration. However, this method cannot further investigate failure region without rerunning the whole algorithm. The reason is that the pruned parameters depending on the initial samples cannot be remained as important ones in the failure region, which keeps changing. Also, the previous approaches cannot provide the design guideline in terms of the design parameters to explicitly avoid performance failure, which are important for improving the yield of circuit.

Recently, Sun *et al.* proposed scaled-sigma sampling (SSS) [16] and subset simulation technique (SUS) [17] as better solutions to estimate rare failure rates with high-dimensional variation space. Unlike traditional importance sampling (IS), SSS applied soft maximum theorem to construct an analytical model, which is insensitive to dimensionality, for rare failure rate estimation. Subset simulation (SUS), on the other hand, tends to express rare failure probability as a product of conditional probability of intermediate failure events, which is similar to the recursive statistical blockade concept. These intermediate terms are then accurately estimated using MCMC (Markov Chain Monte Carlo) algorithm.

1.3 Contributions and Outline of the Thesis

In this thesis, the author proposes a new statistical failure region diagnosis method. The new method, called *EliteScope*, is based on recursive statistical blockade method to reduce the sample counts while still maintaining estimation accuracy. But, it consists of two new techniques to improve existing methods. The main contribution of this paper article are as follows:

Firstly, in the recursive statistical blockade method, more samples will be generated when the failure region is redefined gradually. To mitigate these problems, the new approach applies an elite sampling scheme, which considers both effectiveness of samples and well-coverage for process parameter search space, to reduce the number of generated samples after the failure regions are relocated.

Secondly, our approach provides safe boundaries or in-spec boundaries of process parameters to satisfy the design specification and manage yield variability of circuit. The new method first applies variance based feature selection to find the dominant parameters. A quasi-random sampling with dominant parameters is then used to quickly determine proper boundaries of those parameters.

The presented method has been tested on several types of digital and analog/mixed-signal circuits: 4-gate logic circuit with 48 process parameters; a charge pump operation failure diagnosis in a PLL circuit with 81 process parameters; 6T-SRAM reading failure diagnosis with 27 process parameters; 16-bit 6T-SRAM column reading failure diagnosis with 432 process parameters. Experimental results show that given the same computing costs, the proposed method in general can be more accurate than all the existing methods.

For instance, for the 16-bit 6T-SRAM column, with the similar number of samples used, *EliteScope* can deliver about 3-10X accuracy improvement over existing method. Furthermore, the 16-bit 6T-SRAM column example also shows that the new method can easily handle the statistical analysis problems with large number of process variables.

The rest of this paper is organized as follows. Chapter 2 discusses essential background for sample-based failure analysis and revisits some important techniques for improving MC performance. In Chapter 3, we first introduce an overall flow of the proposed method and review of the mathematical framework for recursive statistical blockade based method. We then present the proposed elite sampling method to reduce the number samples for efficiency improvement. We then introduce new guidance technique of parameters to meet target performance. Chapter 4 shows the experimental result for verifying the accuracy and efficiency of the proposed method. Computation complexity and convergence performance analysis of *EliteScope* is also discussed. Chapter Conclusion will highlight the author‘s contribution wrap-up this thesis.

Chapter 2

Background Knowledge of Statistical Rare Event Analysis

2.1 Existing Sampling Approaches

2.1.1 Importance Sampling

In sample-based statistical analysis, importance sampling (IS) is a general technique to estimate properties of rare event region using the samples generated from the initial distribution. Fig. 2.1 shows the generated distribution $g(x)$ by IS with two parameters. As we can see in the figure, the property of the failure region can be captured as more samples are obtained in the failure region. Therefore, the proper sampling scheme is needed to build the right distribution representing the rare event region. One of IS-based approaches is focusing on quasi-random sampling to explore the parameters space more uniformly. The samples can be selected by the initial MC sampling, so that more regular

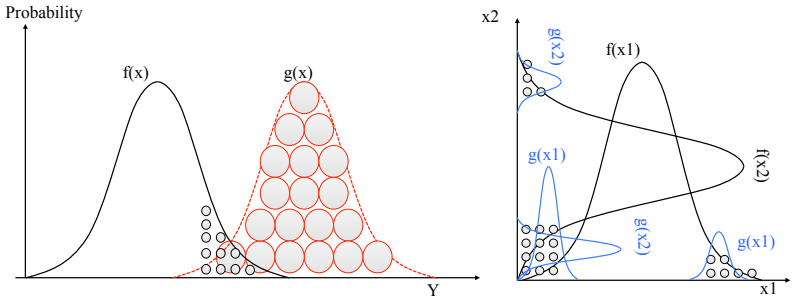


Figure 2.1: Generated PDF by importance sampling in 2-D random variables

space filling makes the initial samples to cover large variety combinations of parameters [3]. Another well-known approach is the mean shifting and variance reconstructing, which the initial distribution is centered around the failure region [12, 11]. However, all these approaches assume a linear relation between the reconstructed and the initial distributions, so the generated samples cannot reflect the nonlinear rare event region correctly.

2.2 Statistical Blockade

We first briefly review the concept of the statistical blockade (SB) approach for fast estimation of properties of rare event region. A general framework of SB is shown in Fig. 2.2. This method starts with drawing initial samples with uniform or normal distribution to capture a crude shape of performance distribution by circuit simulation. The classifier can be built by training with initial simulation data. Once we obtain the classifier, samples that tend to fall into the failure criteria can be identified without actual simulation. With these filtered samples, SB calculates the probability of failure region by fitting samples in

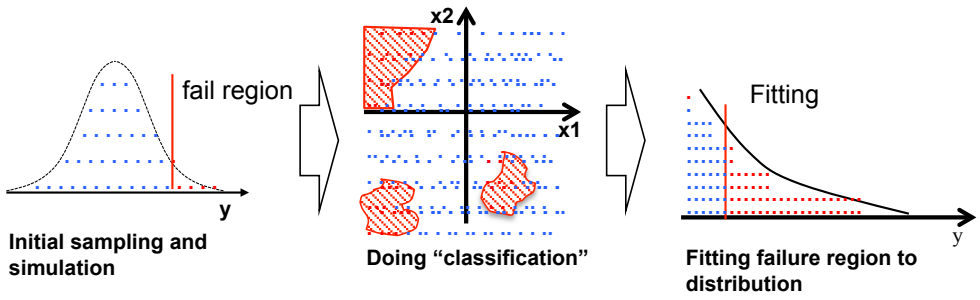
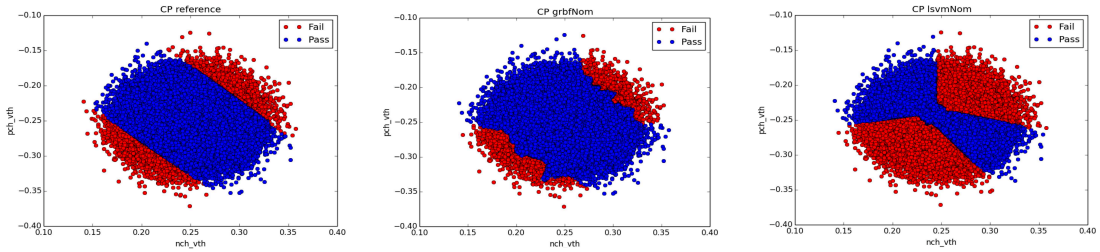


Figure 2.2: General flow of statistical blockade approach

proper distribution model. Thus, "classification" and "failure probability calculation" are both key steps in the SB method. The rest of this section describes these two steps.

2.2.1 Classification

The classification is a step that the samples can be classified into likely-to-fail samples for circuit simulation. Building a classifier needs a training step with initial samples to render real shapes of the failure region. The classifier can shrink the number of samples, thus the simulation cost is reduced. However, it is not capable of fully replacing the simulator due to its accuracy. So, a marginal filtering approach is used to improve the accuracy of classification [15, 19, 20, 14]. This method uses relaxed threshold bounds instead of a real failure criterion to capture more samples to minimize classification error. Meanwhile, it is not sufficient to use a simple and linear classifier due to the nonlinearity of the failure region [15, 19, 14]. So, Gaussian radial basis function kernel (GRBF) and neural network methods are available for nonlinear classifiers [20, 18]. Fig. 2.3 shows the accuracy of classification in a 2-dimensional search space example. Even the solution space is separated with



(a) Reference (b) GRBF (c) LSVM

Figure 2.3: The classification accuracy of two different methods

nonlinear relation, GRBF can recognize patterns properly while the LSVM draws a wrong boundary between two categories.

2.2.2 Failure probability calculation

The failure samples should be fitted to a particular distribution form in order to calculate the probability of the failure region. Suppose that simulation results for the certain performance metric Y can be fitted to the Gaussian distribution. The PDF of the result distribution can be represented as

$$f(y, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \tag{2.1}$$

where parameters μ and σ are the mean and standard deviation in this distribution. We define $F_Y(y)$ as the cumulative density function (CDF) of the performance metric Y. If we know the threshold value t which separates a tail region from the whole distribution $f(y)$, the conditional CDF of this region can be written as follows:

$$F_t(y) = P(Y \geq y \mid Y \geq t) = \frac{F_Y(y) - F_Y(t)}{1 - F_Y(t)} \tag{2.2}$$

where $F_t(y)$ means the failure probability decided by y. Once we have a suitable fitting model for CDF of the failure region with a failure bound y, the failure probability with given values can be calculated as:

$$P(Y \geq y) = [1 - F_Y(t)] \cdot [1 - F_t(y)] \tag{2.3}$$

In the several generalized extreme value distributions, GPD is one of the most accurate model to describe tail distribution corresponding to failure region [7]. With the location parameter μ, the scale parameter σ and the shape parameter ξ, CDF of the failure region can be formulated by GPD fitting.

$$\begin{aligned} F_t(y) &= G_{(\xi,\mu,\sigma)}(y) \\ &= \begin{cases} 1 - \left(1 + \dfrac{\xi(y-\mu)}{\sigma}\right)^{-1/\xi} & \text{for } \xi \neq 0 \\ 1 - e^{\frac{-(y-\mu)}{\sigma}} & \text{for } \xi = 0 \end{cases} \end{aligned} \tag{2.4}$$

The location parameter μ means a starting point of GPD and it corresponds to the threshold t of the tail distribution. Consequently, the failure probability with given threshold t and failure bound y can be computed as follows.

$$P(Y \geq y) = [1 - F_Y(t)] \cdot [1 - G_{(\xi,t,\sigma)}(y)] \tag{2.5}$$

To approximate the rest of parameters for GPD fitting, we use the maximum likelihood estimation [6].

2.3 Recursive Statistical Blockade

It is a typically difficult task to choose the right threshold bound in the failure analysis for an extreme rare event region. For instance, the failure region is decided by the failure criterion t_c and the probability of this region is around 99.9999%. Suppose that we use single threshold method, then we can choose a very loose threshold t as $P(Y > t)$ around 99% to safely cover whole failure region even though the threshold can be quite far away t_c. Moreover, the number of MC samples for filtering will be determined at once. If the number of MC samples is relatively enormous, a classifier will select too many likely-to-fail samples, which will significantly increases simulation cost. Meanwhile, if the selected likely-to-fail samples is every small, this will cause inaccurate estimation of the failure region and thus the failure probability.

To mitigate this problem, one idea is to gradually locate the failure region in an iterative way based on RSB scheme [15]. Unlike the single threshold method that calculates a failure probability at once, our approach updates a failure region $Y_1(> 99\%)$ and the probability by GPD fitting after the first iteration. With this updated failure region, the threshold bound is re-computed for a newly updated region $Y_2(> 99.99\%)$. The GRBF classifier is trained by failure samples in the first iteration so that it can capture likely-to-fail samples more precisely in the updated failure region. In the second iteration, the number of MC samples increases from $10n$ to 10^2n as the increasing ratio is 10, so the new classifier will capture more likely-to-fails samples thus better estimating the updated failure region.

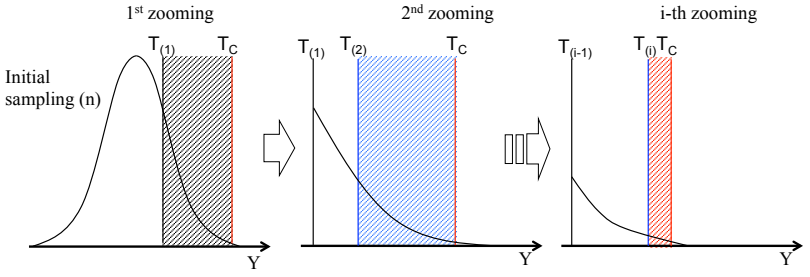


Figure 2.4: Iterative locating of failure region by changing thresholds

As the algorithm iterates, the failure region is scoped continuously close to the given failure criterion t_c based on the re-computed t_i and likely-to-fail samples are converged on the updated failure region. Therefore, the proposed method can achieve accurate failure analysis than a single threshold method with relatively less total simulation cost. Fig. 2.4 shows an iterative locating procedure for finding the failure region.

Mathematically, as discussed in Section 2.1, the CDF with given a threshold t and a failure criterion t_c can be calculated as

$$\begin{aligned} P_{IS}(Y \geq t_c) &= P_{MC}(Y \geq t) \cdot P(Y \geq t_c | Y \geq t) \\ P(Y \geq t_c | t \geq t) &= \frac{P(Y \geq t_c, Y \geq t)}{P(Y \geq t)} \end{aligned} \tag{2.6}$$

The conditional probability part in (2.6) can be estimated by GPD fitting using simulated failure samples. Therefore, (2.6) can be rewritten as

$$P_{IS}(Y > t_c) = P_{MC}(Y \geq t) \cdot P_{MIS}(Y \geq t_c | Y \geq t) \tag{2.7}$$

where P_{MIS} represents the conditional probability in updated distribution by GPD fitting. If the proposed method iterates twice with t_1 and t_2 as threshold bounds, the second failure probability can be calculated based on the first failure region. So, the failure probability in

each step can be calculated as

$$
\begin{aligned}
&P_{IS}(Y \geq t_c) = P_{IS(2)}(Y \geq t_c) \\
&P_{IS(1)}(Y \geq t_2) = P_{MC}(Y \geq t_1) \\
&\qquad\qquad \cdot P_{MIS(1)}(Y \geq t_2 | Y \geq t_1) \\
&P_{IS(2)}(Y \geq t_c) = P_{MC}(Y \geq t_1) \cdot P_{MIS(1)}(Y \geq t_2) \\
&\qquad\qquad \cdot P_{MIS(2)}(Y \geq t_c | Y \geq t_2) \\
&P_{MIS(i)}(Y \geq t_c | Y \geq t_i) = \frac{P_{MIS(i)}(Y \geq t_c)}{P_{MIS(i)}(Y \geq t_i)}
\end{aligned}
\tag{2.8}
$$

Without the loss of generality, we can formulate the iterative failure probability calculation as

$$
\begin{aligned}
&P_{IS(i)}(Y \geq t_c) \\
&= \begin{cases}
P_{MC}(Y \geq t_i) \cdot P_{MIS(i)}(Y \geq t_c | Y \geq t_i) & \text{for } i = 1 \\
P_{MC}(Y \geq t_i) \cdot \prod_{i=1}^{k-1} (P_{MIS(i)}(Y \geq t_{i+1}) & \\
\qquad\qquad \cdot P_{MIS(k)}(Y \geq t_c | Y \geq t_k)) & \text{for } i > 1
\end{cases}
\end{aligned}
\tag{2.9}
$$

where k is the number of iterations. Finally, the failure probability can be obtained by combining all calculated probabilities in each iteration.

Chapter 3

Proposed Elite Sample Selection Scheme Method

3.1 The Overall Analysis Flow of *EliteScope*

In this subsection, we first present the overall analysis flow of the proposed *EliteScope* method, which is illustrated in Fig. 3.1. Then we present the mathematical framework for the iterative computing of failure probability. Then we explain our three major contributions of the proposed method: (1) Iterative computing of failure probability (2) Elite learning sample selection (3) Parameters guidance for performance targeting.

Our algorithm starts with given data, such as process variations and some parameters for failure region determination. The failure criteria t_c denotes the reference value of failure and the percentile bound p to calculate the threshold in each i_{th} iteration. The first step is to perform initial MC sampling and simulation to capture overall circuit performance

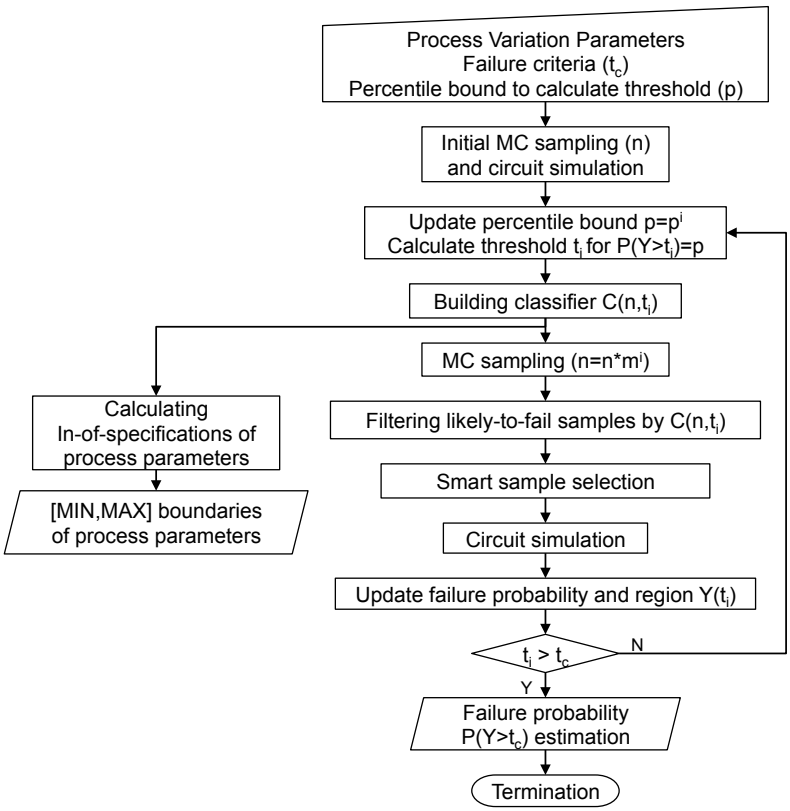


Figure 3.1: The proposed iterative failure region diagnosis flow

metrics. After this, the relaxed threshold t_i can be obtained to separate a failure region from main PDF and the probability of this region is $P(Y > t_i) = p$. The classifier can then be modeled with n simulation result of the initial samples. In the classification step, the GRBF nonlinear classifier is used for accurate sample filtering. With the simulation result and the classifier, the new method can calculate the in-spec conditions of process parameters to achieve targeted yield in i_{th} iteration. At the same time, the algorithm generates $n * m^i$ (m is a constant number) MC samples, which will be filtered by the classifier C_i to likely-to-fail samples based on t_i. Then, the elite sample selection can be employed to further reduce

the number of samples for actual simulation. After the simulation, the failure probabilities $P(Y > t_i)$ are updated by GPD fitting. Our approach iterates the above whole procedure with the updated threshold bound t_i by percentile bound p, and the increased number of MC samples to calculate the failure probability $P(Y > t_i)$. Finally, it finishes when the threshold bound meets the given failure criterion t_c.

3.1.1 Elite Learning Sample Selection

The simulation cost is a major bottleneck in the statistical analysis of the circuit. The proposed iterative failure diagnosis method can lead to an extra simulation cost in each iteration. To mitigate this problem, we propose the elite learning sample selection scheme, which significantly reduces the number of samples required. The elite sample selection process is represented in the box named "Smart sample selection" in Fig. 3.1. Effectiveness of the sample group is the first factor. Each sample consists of the combination of process parameters, which affect differently on simulation results. Therefore, the sensitivity of each parameter should be considered for the sample selection.

Suppose we have a set of n samples, which are represented by the parameter vectors x_i, $i = 1, ..., n$. Each sample has m process variables (m dimensions). Together they form a process parameter matrix $X = [x_1 \quad x_2 \quad ... \quad x_n]$ such that each column indicates a x_i. It is not difficult to see that each row of X is the n samples of a single parameter. Denote X^j as the vector formed from the jth row of X (jth process variable). A scalar vector $y = [y_1, y_2, ..., y_n]^T$ contains all the corresponding n simulation results. σ_{X^j} and σ_Y are variances of X^j and y, respectively. The proposed selection method calculates correlation coefficients between parameters and simulation results for the sensitivity analysis as follows:

$$
\begin{aligned}
&X^j, y \in R^n \\
&\rho_{X^j,y} = \frac{cov(X^j, y)}{\sigma_{X^j}\sigma_y} \qquad j = 1, 2, ..., m \\
&\rho_{X,y} = [\rho_{X^1,y} \quad \rho_{X^2,y} \quad ... \quad \rho_{X^m,y}]^T \in R^m
\end{aligned} \tag{3.1}
$$

Where $\rho_{X,y}$ means the co-variance coefficient of simulation results and m process parameters.

The second factor is the coverage ratio of parameters search space by selected samples. The diversity of samples can be calculated by Euclidean distances with the reference sample, which is the median from simulation results. Samples around the median can be chosen as the median is located on the highest probability region in the distribution of simulation results. Simultaneously, samples found in the boundary region of the search space can be selected as these samples represent the maximum and minimum conditions of parameters. Thus, the proposed sampling method can calculate two distance factors of a given sample that covers both central and boundary regions of the search space as follows:

$$
\begin{aligned}
&\tilde{y} = median(y) \\
&x_{ref} = \{x | \tilde{y} = f(x), x \in R^m\} \\
&D_{central}(x) = \frac{1}{\left|\frac{x - x_{ref}}{Range(X)}\right|} \in R^m \\
&D_{boundary}(x) = \left|\frac{x - x_{ref}}{Range(X)}\right| \in R^m
\end{aligned} \tag{3.2}
$$

where $Range(X)$ is a normalization term such that the jth element of vector $x - x_{ref}$ is normalized by $|max(X^j) - min(X^j)|$, the value range of jth row in X . In (3.2), $D_{central}$ increases when the sample is closer to the reference sample. On the other hand, $D_{boundary}$ increases. Since $D(x)$ and $\rho_{X,y}$ means the distance and correlation coefficient in same dimensions, we can obtain the sample's weight by the inner product of $\rho_{X,y}$ and $D(x)$ of

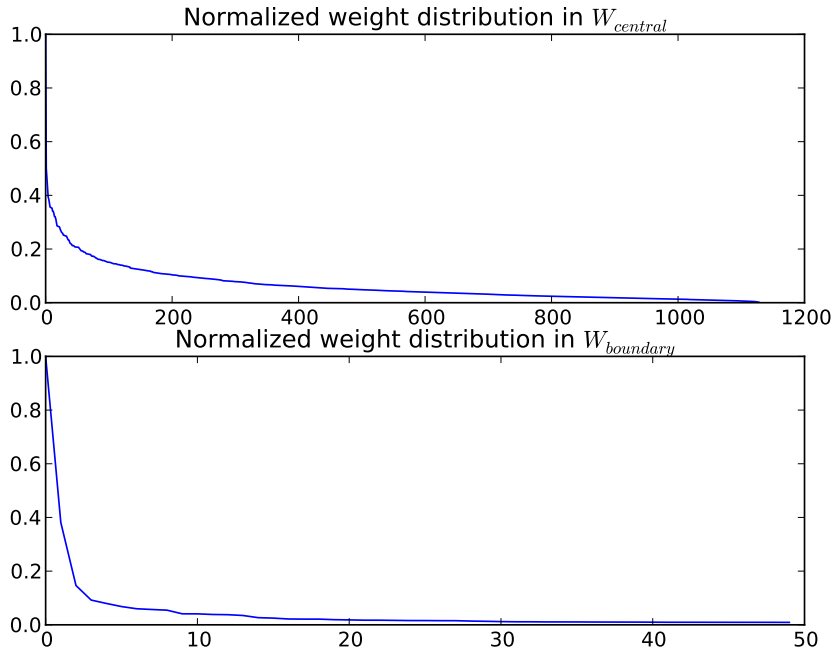


Figure 3.2: Sample candidates weight Distribution of a single-bit SRAM test

each sample.

$$\begin{aligned} W_{central}(x) &= {\rho_{X,y}}^T \cdot D_{central}(x) \\ W_{boundary}(x) &= {\rho_{X,y}}^T \cdot D_{boundary}(x) \end{aligned} \tag{3.3}$$

According to the selection ratio r, which determines the number of selected samples, The final set of samples can be chosen in the following way.

$$E(n, r) = S(\frac{nr}{2}, W_{central}(x_n)) \cup S(\frac{nr}{2}, W_{boundary}(x_n)) \tag{3.4}$$

where $S(n, W(x_n))$ is the set of n samples sorted by $W(x_n)$.

An example of normalized sample candidates weight distribution is shown in Fig. 3.2. Nearly 1200 sample candidates are filtered out by the non-linear **SVM** classifier. By employing Elite Learning Sample Selection scheme, the weight concerning both

central and boundary distance are calculated sorted in descending order. All weights in the same set are then normalized by sets maximum. For the normalized weight distribution in the set $W_{boundary}$, only first 50 samples are listed since rest samples‘ weight are very close to zero and thus negligible. It is very clear to see that weight values decease dramatically. According to previous discussion, sample with larger weight leads to more significant contribution in constructing the failure region. Unlikely to traditional **RSB** [15] method which directly simulates all sample candidates, we only need to utilize samples with larger weight to perform actual simulation to estimate failure region with great efficiency.

3.1.2 Parameters Guidance for Performance Targeting

In order to improve yield of a circuit, designers need to know good ranges of process parameters with regards to the circuit performance specification. However, applying all possible combinations of parameters is impossible due to exponential possibilities with a large number of parameters.

The proposed method ranks priorities of process parameters based on its variances. The parameter guidance operation is represented by the two left boxes in Fig. 3.1. Since the parameters with huge variance mainly lead to spread samples in search spaces, these parameters must be handled to avoid certain failure regions. In Fig. 3.1, the variances of parameters can be calculated from simulation data of updated failure region in our iterative framework. Given n samples with m process parameters as denoted by $X = [x_1 \quad x_2 \quad ... \quad x_n]$, the variance of samples can be written mathematically as follows:

$$Var(X) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2 \tag{3.5}$$

where x_i is a ith X and μ is the mean vector of n samples of X. Next, our method redraws samples with only considering the distributions of high ranked parameters. Nominal values are assigned for not chosen parameters. We use SOBOL sequence [5] to redraw these samples. It uses a quasi-random low-discrepancy sequence, so these samples can cover the search spaces of parameters more uniformly than the previous samples for simulation. Suppose that l is the number of redrawn samples and first k high ranked parameters are chosen. Redrawn samples can be formed as

$$\begin{aligned} x_i &= [x_{i,1}, x_{i,2}, ..., x_{i,k}, x_{i,(k+1)}, ...x_{i,m}]^T \in R^m \\ [x_{i,1}, x_{i,2}, ..., x_{i,k}] &= SOBOL[x_{1,...l}, MIN(x_{1,...,l}), MAX(x_{1,...,l})] \\ [x_{i,(k+1)}, x_{i,(k+2)}, ...x_{i,m}] &= NOMINAL(x_{1,...l}) \end{aligned} \tag{3.6}$$

where $i = 1, 2, ..., l$, $x_{i,k}$ denotes the kth element of x_i, terms $MIN(x_{1,...,l})$ and $MAX(x_{1,...,l})$ mean the minimum and the maximum values of l vectors in X, respectively. We assign nominal values for rest $m - k$ parameters of redrawn samples. As a result, we can generate samples with not only reduced dimensions but also well-coverage of the failure region. The classifier with the updated threshold can filter out these samples to determine pass or fail condition of process parameters. With the classification result of samples, the proposed method induces *if-then* rules from the highest-ranked parameters so that all failure conditions of parameters can be filtered. The overall steps of the new in-spec guidance method are explained in Fig. 3.3.

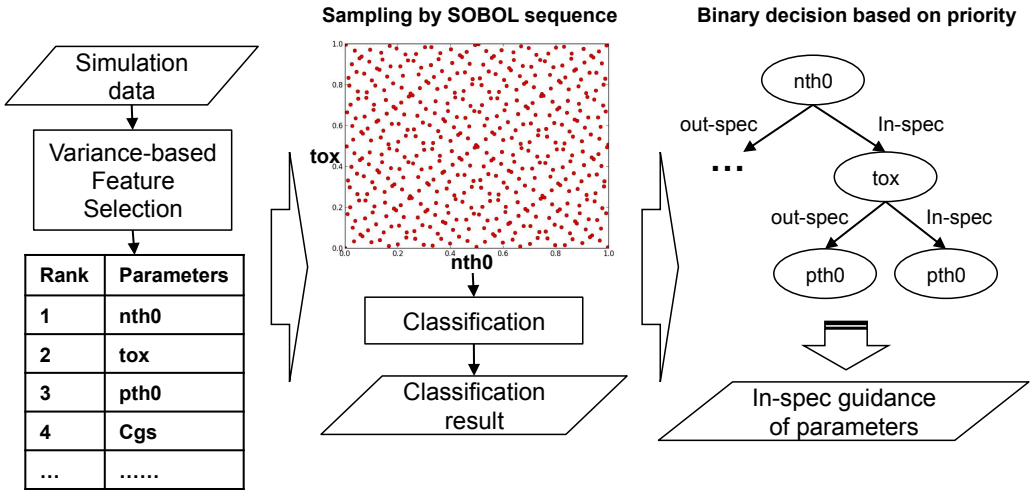


Figure 3.3: Overall flow of parameters guidance for performance targeting 1) Feature selection 2) Sampling and Classification 3) Calculate the boundaries for in-spec conditions

Chapter 4

Numerical Experimental Results

4.1 Experiment Configuration

The proposed method (*EliteScope*) has been implemented in Python 2 and tested on a Linux workstation with 32 CPUs (2.6GHz Xeon processors) and 64GB RAM.

The performance and accuracy of proposed method have been evaluated on a number of circuits: (1) The critical path delay of the 4-gate logic circuit, (2) Failure rate of 6T-SRAM single-bit cell, (3) Failure rate of 6T-SRAM 16-bit column and (4) Charge pump circuit in PLL, which are highly replicated instances for system-on-chip (SOC) designs. All circuits were designed with the BSIM4 transistor model and simulated in NGSPICE [10]. Table. 4.1 shows 9 major process parameters of MOSFETs. To demonstrate the advantage of the proposed method, we compare the proposed *EliteScope* against three other methods, Monte Carlo (MC), REscope [20], and the Recursive statistical blockade (RSB) method [15] in terms of their accuracies and performances. The three other methods are also implemented in Python 2 and tested on the same workstation. As the last part of the

Table 4.1: Process parameters of MOSFET

Variable name	**Std(σ)**	**Unit**
Flat-band voltage (V_{fb})	0.1	V
Gate oxide thickness(t_{ox})	0.05	m
Mobility (μ_0)	0.1	m^2/Vs
Doping concentration at depletion N_{dep})	0.1	cm^{-3}
Channel-length offset (ΔL)	0.05	m
Channel-width offset (ΔQ)	0.05	m
Source/drain sheet resistance(R_{sh})	0.1	Ohm/mm^2
Source-gate overlap unit capacitance(C_{gso})	0.1	F/m
Drain-gate overlap unit capacitance(C_{gdo})	0.1	F/m

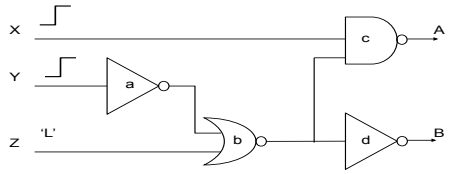


Figure 4.1: The schematic of the 4-gates logic circuit

section, we will discuss algorithm and classifier complexity as well as the issue concerning the convergence performance of *EliteScope*.

4.1.1 The Critical Path Delay of the Simple Logic Circuit

The test logic circuit consists of four gates (2 INVs, 1 NOR, and 1 NAND) as shown in Fig 4.1. The critical path delay in the circuit is $max(fall\_A, fall\_B)$ (X,Y is rising and Z is 0). Two critical paths can be found, and the total number of process parameters is 48. The failure criterion is set to be P(Y> t_c)=0.000125, which indicates the 4-sigma range in the distribution of the critical path delay. Two iteration threshold bounds

Table 4.2: Comparison of the accuracy and efficiency on the 4-gates circuit

	Failure probability	**# Sim. runs**	**Speed -up(x)**	**Error (%)**
Monte Carlo	1.25E-04	600K	-	-
Rare Event Microscope (REscope)	6.00E-04	4531	132.4	380.16
Recursive Statistical Blockade (RSB)	1.49E-04	12369	48.5	19.2
Proposed method (EliteScope)	1.51E-04	5620	106.8	20.8

are P(Y> t_1)=0.07 and P(Y> t_2)=0.0049, respectively. As we can see from Table 4.2, both

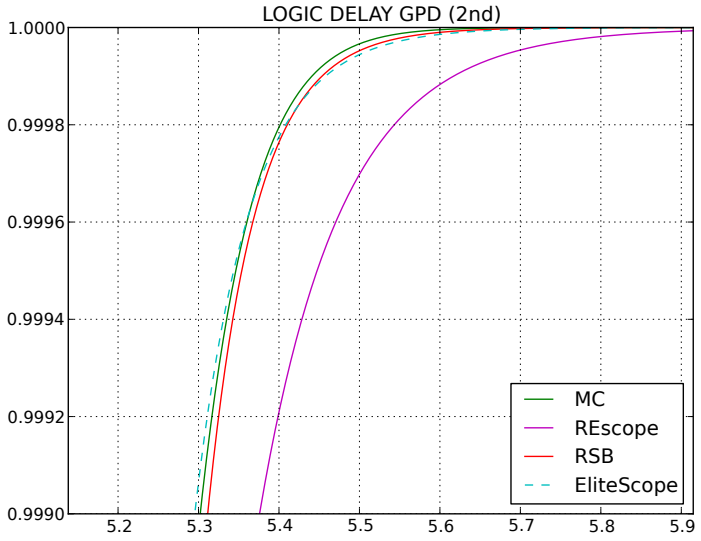


Figure 4.2: The failure distribution P(Y< t_c)=0.999875 of the critical path delay of the simple circuit

EliteScope and RSB have similar accuracies for failure region estimation. But RSB takes 2.20X more simulation time. By applying the elite learning sample selection, *EliteScope* only use a small amount of samples, which are filtered by classifier, for simulation and further tail distribution fitting while RSB just directly simulates all of them.

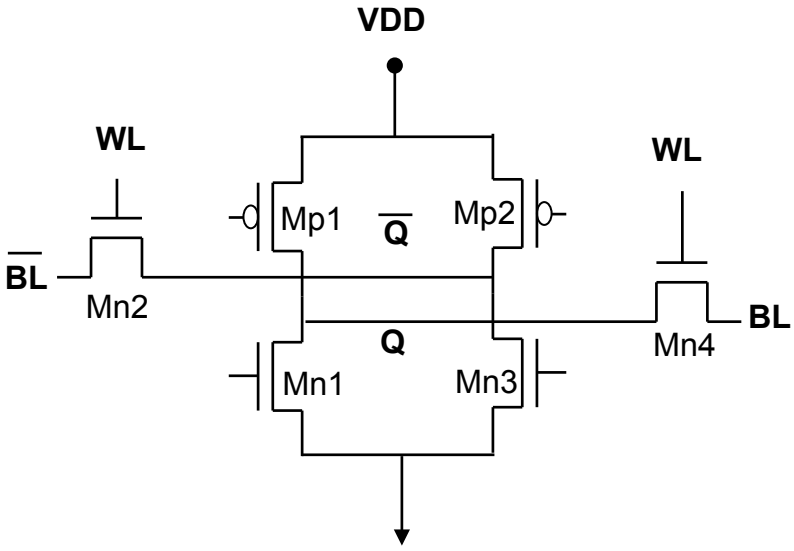


Figure 4.3: The schematic of the 6T-SRAM single-bit cell

In this case, REscope does not deliver very accurate estimation. There is about $19X$ accuracy difference between REscope and *EliteScope* even though their simulation cost difference is only about 20% [1].

4.1.2 Failure Rate Diagnosis of Single-bit 6T-SRAM Cell

The second example is a single-bit 6T-SRAM circuit. The schematic design of the single-bit 6T-SRAM cell using BSIM4v4.7 MOSFET Model is shown in Fig. 4.3. 6T-SRAM fails when the voltage gap between BL and $\overline{BL}$ is not large enough to be determined by sense amplifiers in certain period. We measure the delay of discharging $\overline{BL}$ as the failure criterion. The experimental setup for the initial conditions are: $\bar{Q} = 1$, $Q = 0$, BL and $\overline{BL} = 0$. When WL turns on, $\overline{BL}$ is discharged by MN2 and MN1 and BL charged by MP2.

[1] We note that it is difficult to make the simulation samples exact same for both methods as we do not control them directly.

For process variables, we use the 9 model parameters in Table 4.1. To guarantee unbiased behavior, transistors on left hand side should be totally identical to their corresponding transistors on the right (e.g. Mp1 and Mp2 share identical process parameters). So, the number of the process parameters is $27(3*9)$ in this reading operation. The initial number of samples for capturing the circuit behavior is 2,000.

Table 4.3: Comparison of the accuracy and efficiency on the 6T-SRAM circuit

	Failure probability	**# Sim. runs**	**Speed -up(x)**	**Error (%)**
Monte Carlo (MC)	2.300E-04	1 million	-	-
Rare Event Microscope (REscope)	3.79E-04	5009	199.6	64.78
Recursive Statistical Blockade (RSB)	2.78E-04	29260	34.2	20.86
Proposed method (EliteScope)	2.85E-04	15730	63.6	24.00

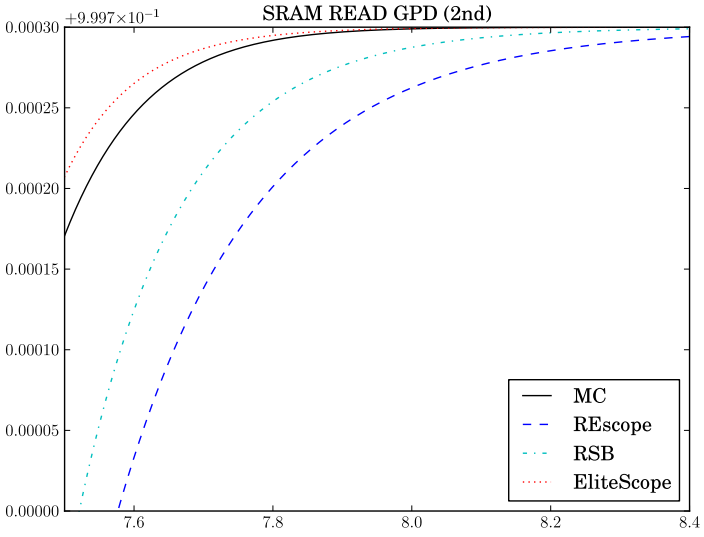


Figure 4.4: Estimating the CDF of the 6T-SRAM read time around 3-sigma region

Table 4.4: Estimated in-spec guidance of parameters on the 6T-SRAM circuit

Rank	**Parameter @MOSFET**	**Initial condition** (μ, σ)	**In-spec Guidance [MIN,MAX]**
1	vfb@MN1	(-5.5E-01,0.1)	[-7.31E-01,-3.78E-01]
2	vfb@MP2	(5.5E-01,0.1)	[3.54E-01,7.15E-01]
3	$ndep$@MN1	(2.8E+18,0.1)	[1.84E+18,3.76E+18]
4	$ndep$@MN2	(2.8E+18,0.1)	[1.70E+18,3.84E+18]
5	$ndep$@MP2	(2.8E+18,0.1)	[1.89E+18,3.78e+18]
Failure probability	$0.0009(= t_2)$	**Estimation Error (%)**	**1.21**

We set the failure criterion t_c as P(Y$\geq t_c$)=0.00023, which means 3-sigma in terms of the yield level. The proposed method iterates twice with 97% percentile bound for each iteration to separate the failure region from initial distribution. Hence, threshold bounds t_1 and t_2 are calculated as P(Y$\geq t_1$)=0.03 and P(Y$\geq t_2$)=0.0009 ($0.03 \times 3\%$), respectively. Table 4.3 shows the accuracy and performance of failure analysis performed by different approaches.

As we can see, compared to REscope, EliteScope obtains better accuracy with the similar computing costs. Compared to the RSB method, which gives better accuracy, but taking almost $2X$ computing time. Fig. 4.4 shows that our proposed method is more accurate than previous methods since the tail of CDF depicting the 3-sigma failure region is more correlated to the golden reference (MC).For the estimated specification guidance for parameters, we find that only 1.2% of samples, which meet the in-spec guidance, are determined as the misclassification samples by the classifier in Table 4.4.

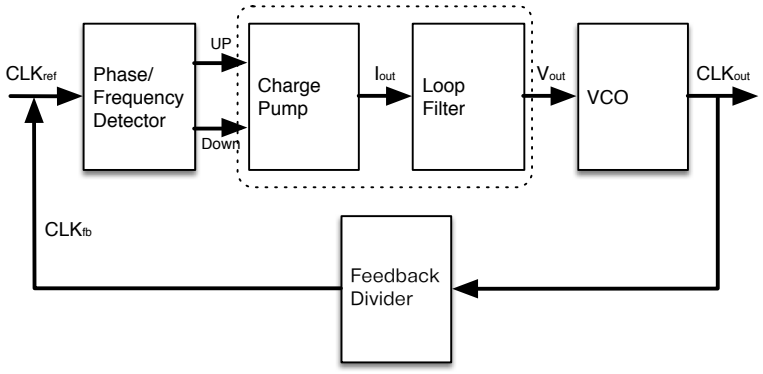


Figure 4.5: A functional diagram of the PLL circuit

4.1.3 Charge Pump Failure Rate Diagnosis

The third example is a charge pump circuit. In a large logic circuit, a clock is frequently distributed to several sub-clocks, so frequencies of sub-clocks are prone to be inaccurate due to propagation delays. A PLL is frequently used to adjust the phase of clock. The functional block diagram of PLL is shown in Fig.4.5. After comparing the output clock (CLK_{out}) with the reference clock (CLK_{ref}) by phase detector, a charge pump circuit adjusts the frequency of clock signal by charging and discharging capacitors controlled by input signals (UP and DN). The mismatch of MOSFETs in a charge pump can cause the unbalanced timing and phase jitters between two different operation modes. Hence, we measure the timing ratio of charging and discharging operations, which can be formulated mathematically as $r_{min} \leq \frac{t_{discharge}}{t_{charge}} \leq r_{max}$ ($r_{min,max}$ represents the minimum and maximum ratio to determine failures). A charge pump circuit consists of 9 MOSFETs as shown in Fig. 4.6. The total number of process parameters is $81(9*9)$, so the dimension of parameters is much higher than 6T-SRAM case. We initially perform 3,000 sampling

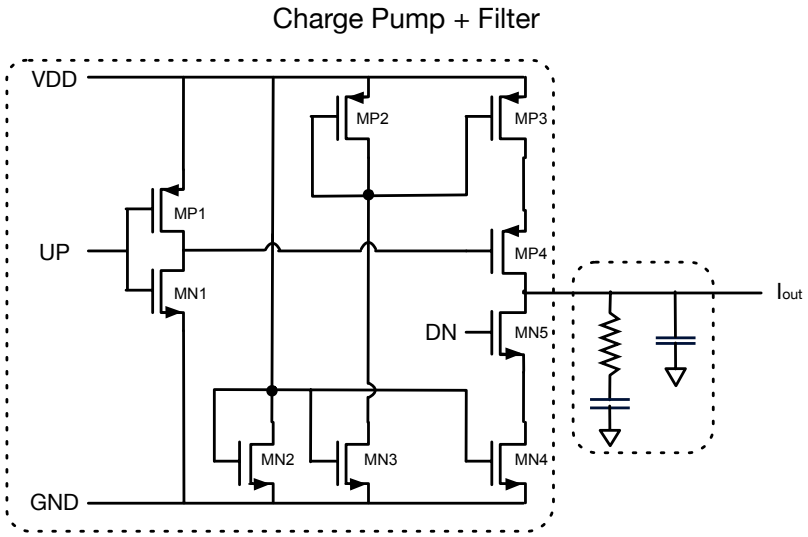


Figure 4.6: Schematic representations of the charge pump and filter

and simulation to model the initial performance distribution accurately. Similar to the 6T-SRAM case, we perform our algorithm twice with 97% percentile bound (P(Y$\geq t_1$)=0.03, P(Y$\geq t_2$)=0.0009).

The result is summarized in Table 4.5.*EliteScope* approach requires only 6263 Spice simulation runs for estimating the failure probability of 3-sigma region with 10.78% relative error compared to traditional Monte Carlo method. Even though RSB achieves better accuracy with only 2.39% error, it runs nearly 4000 more simulations than *EliteScope*. REscope requires 4875 simulation runs, but it gives significant large errors compared to the Monte Carlo method.

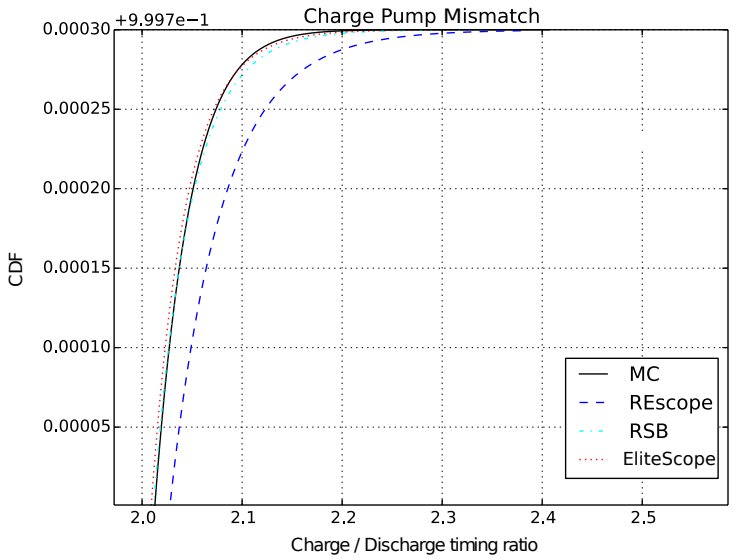


Figure 4.7: Estimating the CDF of charge pump mismatch around 3-sigma region

Table 4.6 shows that the proposed method makes the decision for in-spec conditions of process parameters with 98% confidence level by managing only the first 5 ranked parameters of 81. The tail distribution of *EliteScope* in 3-sigma failure region is much closer to MC than REscope as we can see in Fig. 4.7.

Table 4.5: Comparison of the accuracy and efficiency for the charge pump circuit

	Failure probability	**# Sim. runs**	**Speed -up(x)**	**Error (%)**
Monte Carlo(MC)	2.300E-04	1 million	-	-
Rare Event Microscope (REscope)	3.337E-04	4875	205.1	45.09
Recursive Statistical Blockade (RSB)	2.245-04	10432	95.9	2.39
Proposed method (*EliteScope*)	2.052-04	6263	159.7	10.78

Table 4.6: Estimated in-spec guidance of parameters for the charge pump circuit

Rank	**Parameter @MOSFET**	**Initial condition** (μ, σ)	**In-spec Guidance [MIN,MAX]**
1	*ndep*@MN1	(2.8E+18,0.1)	[1.74E+18,3.78E+18]
2	*ndep*@MP2	(2.8E+18,0.1)	[1.68E+18,3.73E+18]
3	*ndep*@MP4	(2.8E+18,0.1)	[1.78E+18,3.81E+18]
4	*ndep*@MN5	(2.8E+18,0.1)	[1.78E+18,3.77E+18]
5	*ndep*@MP3	(2.8E+18,0.1)	[1.88E+18,3.80E+18]
Failure probability	$0.0009(= t_2)$	**Estimation Error (%)**	**2.00**

4.1.4 16-bit 6T-SRAM Column Failure Rate Diagnosis

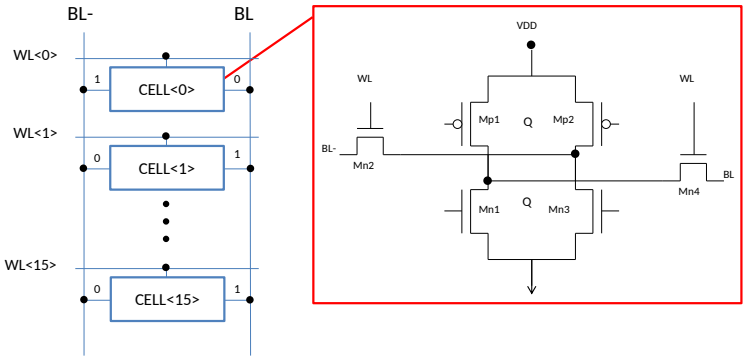


Figure 4.8: The schematic of a 16-bit 6T-SRAM column

To illustrate the scalability of the proposed method on large analog circuits, we perform comparison on one large 16-bit 6T-SRAM column circuit (one-bit line) as shown in Fig. 4.8. In this example, we treat the delay of discharging $\overline{BL}$ as the failure criterion. To mimic the worst-case scenario, in which the impact of leakage current can be maximized,

logic 0 is stored in *cell* $< 0 >$ and the rest of the cells stores logic 1. In the reading operation, only *cell* $< 0 >$'s word-line is turned on while all other word-lines are turned off. We choose the same process parameters used in one SRAM cell experiment. The model parameters are independent in different cells. As a result, we have 432 random variables (16 cells * 27 random variables) that make this case a good example for scalability study. We run 6,000 samples to capture the circuit behavior. The same failure criterion is set as $P(Y \geq t_c)$=0.00023, which is about the 3-sigma in terms of the circuit yield. The proposed method iterates twice with 97% percentile bound as a slope guard to separate the failure region from the initial distribution. Hence, threshold bounds t_1 and t_2 are calculated as $P(Y \geq t_1)$=0.03 and $P(Y \geq t_2)$=0.0009, respectively.

We set the same number of tail fitting samples for REscope, RSB, and *EliteScope* (the actual samples used to fit the tail distribution) so that we can fairly compare their accuracy. The estimated failure probability and their errors obtained from the three methods are shown in Table 4.7. All results are compared against the results from the Monte Carlo method with one million runs, which give the failure probability as 2.3×10^{-4}, the golden reference for all the other methods.

In Table 4.7, the first row indicates the number of tail fitting samples each approach uses. For each column, two terms are given for each method, the first term is the absolute failure probability (**FP**) obtained by the different methods, the second term is the relative failure probability error rate against the Monte Carlo method.

When a small number of tail fitting samples are used (only a few hundred), all methods result in large errors since the small number of samples cannot build a reliable

Table 4.7: The accuracy comparisons for 16-bit 6T-SRAM column case

# of Tail Fitting Samples		248	1392	6346
Monte Carlo Reference(MC)	**FP**	2.3E-04	2.3E-04	2.3E-04
	Error(%)	0	0	0
Rare Event Microscope (REscope)	**FP**	11.4E-04	6.29E-04	2.61E-04
	Error(%)	395.65	173.48	13.48
Recursive Statistical Blockade (RSB)	**FP**	7.705E-04	5.569E-04	2.39E-04
	Error(%)	235.00	142.13	3.91
Proposed Method (EliteScope)	**FP**	6.47E-04	3.385E-04	2.33E-04
	Error(%)	181.30	47.17	1.30

model for the tail distribution. By using more tail fitting samples, overall performance will be naturally improved. But still, the proposed method presents good performance with the lowest estimation relative error among all approaches.

We note that by applying the elite learning sampling selection, we select 1392 samples with higher effectiveness out of 6961 samples generated for tail distribution fitting. Furthermore, the selected elite samples is quite effective for capturing tail distribution quite precisely. Compared to the REscope method, the proposed method achieves $3.67X$ improvement in accuracy using the same simulation costs.

When 6364 tail fitting samples are generated to fit the tail distribution, all the methods obtain a better approximation to golden reference (very close to t_c) while *EliteScope* achieves the lowest error – 1% error compared to the standard MC simulation. Note that all results are obtained based on GPD fitting in this case. In this case *EliteScope* is about $10X$ more accurate than the REscope.

4.1.5 Classifier Computation Complexity and Performance Convergence Analysis

In our implementation part,we use the Nu-Support Vector Classification ($NuSVC$) as our classifier. It is a built-in classifier function in the $scikit-learn$ Python machine learning package. It is a non-linear C-support vector machine classifier. The computation complexity of the SVM is typically more than quadratic ($O(n^2)$), where n is the number of training samples. Depending on the testcase and parameter selection, the computation time spent on classifier training varies. The total computation time of *EliteScope* is mainly spent of two parts: 1) classifier training and 2) NGSPICE circuit simulation. In low dimensional test cases, NGSPICE simulation is fast due to the simple circuit netlist. Thus, classifier training dominate the time cost since *EliteScope* would run fewer NGSPICE simulation (usually 15% to 30% of sample candidates) by applying Elite Learning Sample Selection scheme. But for 16-bit SRAM circuit, which is a high-dimensional variable case, both classifier training cost and NGSPICE simulation costs drastically increase. One reason is the non-linear computational complexity of $NuSVC$. As we use more training samples to better capture the circuit behavior in high-dimensional space, more classifier training time is consumed . On the other hand, performing one single NGSPICE simulation costs 4 second due to the large circuit size. The total NGSPICE simulation consumption time required by RSB is $3X$ than classifier training time, while*EliteScope* further sort out those samples, which are more worthy to simulate. The proposed method can save over 50% of total computation time which makes it even more time efficient in high-dimensional case while keeping a acceptable accuracy level.

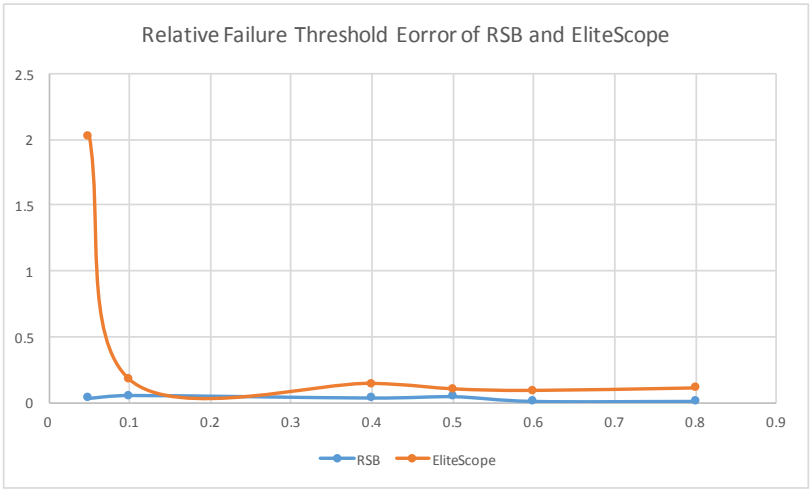


Figure 4.9: Estimated Failure Threshold Error of RSB and EliteScope

To further prove the feasibility of *EliteScope*, we repeatedly perform single-bit SRAM tests by using different values of selection ratio r. Failure region threshold estimated by RSB and *EliteScope* are compared. We perform two separate tests for a given selection ratio and take the average of failure threshold estimation value as the data in the figure.

Fig. 4.9 shows the absolute estimation error of failure threshold of RSB and *EliteScope*. Even though *EliteScope* encounters over 200% error when selection ratio equals to 0.05, it is acceptable since sample are still out of number andsome highly-weighted samples are not considered. After 10% samples are used, the estimation error of *EliteScope* quickly converge to RSB but still exists due to its nature limitation. Fig. 4.10 illustrates the relative error of *EliteScope* compared to RSB method with the selection range between 0.1 to 0.8. One observation is that the relative error *EliteScope* do decrease as more and more sample are used, but only a maximum of 50% decrease of relative error, at a cost

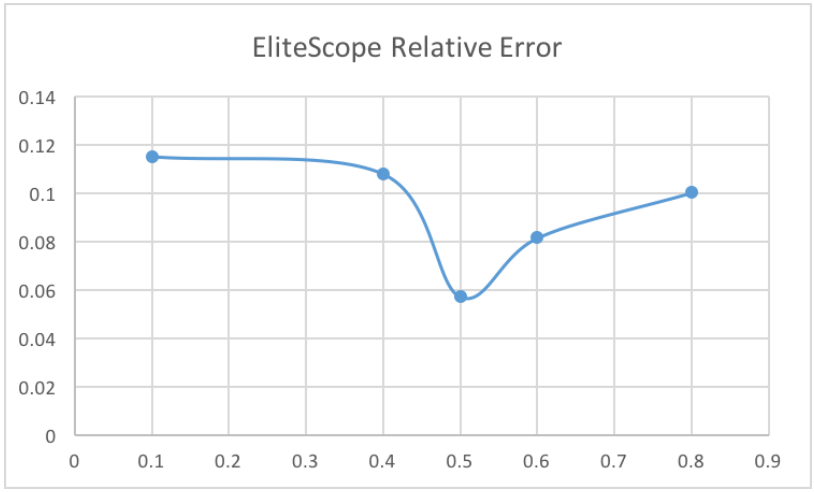


Figure 4.10: Relative Failure Threshold Error of RSB and EliteScope

of $5X$ simulation consumption, is received. Those low-weight samples provide very limited contribution of estimating failure threshold. The result further support that Elite Learning Sample Selection do play a smart role in selecting samples with great efficiency in failure region estimation.

Chapter 5

Conclusions and Future Research

In this thesis, the author present a novel statistical diagnosis method for rare failure events. The proposed method introduced two new techniques to speed up the failure analysis while providing the in-spec guidance of process parameters. First, the proposed method applies the smart sample selection method to reduce the additional simulation cost during iterative failure region locating process. Second, the new approach can provide safe design space of parameters, which can help design to improve the yield and meet the target performance of design. Experiments on four circuit cases show that *EliteScope* achieves a significant improvement on failure region estimation in terms of accuracy and simulation cost over traditional approaches. The 16-bit 6T-SRAM column example also shows that the new method is salable for handling large problems with large number of process variables.

One conference paper[21] and one journal paper [22] are published based on the thesis‘s work. As a co-author, another conference paper[13] is published when participated the full-chip thermal estimation project.

Future research direction for this topic can be derived into two aspects. Sophisticated machine learning theory and implementation will play a better role on rare event tail distribution estimation for large-size logical circuit with high dimension. Secondly, process parameters of VLSI circuits are also affected by circuits‘ own reliability (e.g. Electromigration, thermal circle, TDDB) during time being used. So a more comprehensive characterization and simulation considering life- time reliability criteria using different reliability model is also a promising topic.

Bibliography

[1] K. Agarwal and S. Nassif. Statistical analysis of sram cell stability. In *Proc. IEEE/ACM Design Automation Conference (DAC)*, pages 57–62, 2006.

[2] B.H. Calhoun, Yu Cao, Xin Li, Ken Mai, L.T. Pileggi, R.A Rutenbar, and Kenneth L. Shepard. Digital circuit design challenges and opportunities in the era of nanoscale cmos. *Proceedings of the IEEE*, 96(2):343–365, Feb 2008.

[3] D.Montgomery. *Design and Analysis of Experiments*. Wiley, 2013.

[4] D.E. Hocevar, M.R. Lightner, and Timothy N. Trick. A study of variance reduction techniques for estimating circuit yields. *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, 2(3):180–192, July 1983.

[5] Stephen Joe and Frances Y. Kuo. Remark on algorithm 659: Implementing sobol's quasirandom sequence generator. *ACM Trans. Math. Softw.*, 29(1):49–57, March 2003.

[6] J.R.Hosking. Algorithm as 215: Maximum-likelihood estimation of the parameters of the generalized extreme-value distribution. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 34(3):301–310, 1985.

[7] J.R.Wallis J.R.Hosking. Parameter and quantile estimation for the generalized pareto distribution. *Technometrics*, 29(3):339–349, 1987.

[8] H. Masuda, S. Ohkawa, A Kurokawa, and M. Aoki. Challenge: variability characterization and modeling for 65- to 90-nm processes. In *Proc. IEEE Custom Integrated Circuits Conference (CICC)*, pages 593–599, Sept 2005.

[9] S. Mukhopadhyay, H. Mahmoodi, and K. Roy. Statistical design and optimization of sram cell for yield enhancement. In *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, pages 10–13, Nov 2004.

[10] Paolo Nenzi and V Holger. Ngspice users manual. *Version 25plus*, 2010.

[11] M. Qazi, M. Tikekar, L. Dolecek, D. Shah, and A Chandrakasan. Loop flattening & spherical sampling: Highly efficient model reduction techniques for sram yield analysis. In *Proc. Design, Automation and Test In Europe. (DATE)*, pages 801–806, March 2010.

[12] S. Nassif R. Kanj, R. Joshi. Mixture important sampling and its application to the analysis of sram designs in the presence of rare failure events. In *Proc. IEEE/ACM Design Automation Conference (DAC)*, pages 69–72, 2006.

[13] S. Sadiqbatcha, Y. Zhao, J. Zhang, H. Amrouch, and J. Henkeland S. X. . Tan. Online full-chip heatmap estimation using lstm neural networks. In *2020 25th Asia and South Pacific Design Automation Conference (ASP-DAC)*, 2020.

[14] A Singhee and R.A Rutenbar. Statistical blockade: Very fast statistical simulation and modeling of rare circuit events and its application to memory design. *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, 28(8):1176–1189, Aug 2009.

[15] A Singhee, Jiajing Wang, B.H. Calhoun, and R.A Rutenbar. Recursive statistical blockade: An enhanced technique for rare event simulation with application to sram circuit design. In *International Conference on VLSI Design*, pages 131–136, Jan 2008.

[16] S. Sun, X. Li, H. Liu, K. Luo, and B. Gu. Fast statistical analysis of rare circuit failure events via scaled-sigma sampling for high-dimensional variation space, 2015.

[17] Shupeng Sun and Xin Li. Fast statistical analysis of rare circuit failure events via subset simulation in high-dimensional variation space. In *Proc. Int. Conf. on Computer Aided Design (ICCAD)*, pages 324–331, Nov 2014.

[18] T.Hastie. *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. Springer, 2009.

[19] Wei Wu, Fang Gong, Gengsheng Chen, and Lei He. A fast and provably bounded failure analysis of memory circuits in high dimensions. In *Proc. Asia South Pacific Design Automation Conf. (ASPDAC)*, pages 424–429, Jan 2014.

[20] Wei Wu, Wenyao Xu, Rahul Krishnan, Yen-Lung Chen, and Lei He. Rescope: High-dimensional statistical circuit simulation towards full failure region coverage. In *Proc. IEEE/ACM Design Automation Conference (DAC)*, pages 1–6, June 2014.

[21] Y. Zhao, H. Shin, H. Chen, S. X. . Tan, G. Shi, and X. Li. Statistical rare event analysis using smart sampling and parameter guidance. In *2015 28th IEEE International System-on-Chip Conference (SOCC)*, pages 53–58, Sep. 2015.

[22] Yue Zhao, Taeyoung Kim, Hosoon Shin, Sheldon Tan, Xin Li, Haibao Chen, and Hai Wang. Statistical rare-event analysis and parameter guidance by elite learning sample selection. *ACM Transactions on Design Automation of Electronic Systems*, 21:1–21, 05 2016.