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On constraining electroweak-baryogenesis with inhomogeneous primordial nucleosynthesis

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Abstract

Primordial nucleosynthesis calculations are shown to be able to provide constraints on models of electroweak baryogenesis which produce a highly inhomogeneous distribution of the baryon-to-photon ratio. Such baryogenesis scenarios overproduce ⁴He and/or ⁷Li and can be ruled out whenever a fraction $f \leq 3 \times 10^{-6} (100 \text{ GeV}/T)^3$ of nucleated bubbles of broken-symmetry phase contributes $\geq 10\%$ of the baryon number within a horizon volume.

In this letter we discuss how the sensitivity of big bang nucleosynthesis to the baryon-to-photon number (η) and its spatial distribution could be utilized to probe electroweak physics in a new manner. Models of electroweak baryogenesis have so far concentrated on producing the presently observed value for η . In this paper, we point out a new constraint on models that result in a highly inhomogeneous distribution of η . We will show that micro-physical processes that generate fluctuations in η on sub-horizon scales for epochs corresponding to temperatures $T \leq 1.5 \text{ TeV}$ may be subject to nucleosynthesis constraints.

A long-standing problem in astrophysics is the explanation for the apparent baryon number asymmetry in the universe. Ref. [1] provides an overview of this problem and the attempts to solve it. Ever since the work of Sakharov [2] an explanation has been sought for the baryon number asymmetry in C , CP , and baryon-number violating processes in environments associated with departures from thermal and chemical equilibrium in the early universe. However, any net

baryon number generated at very early epochs in the history of the universe (e.g., via C and CP violating, nonequilibrium baryon number violating decay of heavy X and Y bosons associated with Grand Unification) will probably, though not necessarily inevitably, be erased by subsequent anomalous electroweak processes [3]. Regeneration of baryon number could then occur during a first order cosmic electroweak symmetry-breaking phase transition [3]. It is not clear if adequate baryogenesis could be achieved with a minimal Weinberg–Salam model without implying a Higgs mass below the present experimental lower bound (see, however, Ref. [4]). Several plausible extensions of the minimal standard model, such as multi-Higgs models or supersymmetric models, could lead to significant baryon number generation at this epoch [5–7]. A review of baryogenesis associated with first-order electroweak phase transitions is given in Ref. [8].

Since a temperature dependent nucleation rate is a generic feature of first order phase transitions, we expect some supercooling in a primordial electroweak

transition and the concomitant generation of distinct bubbles of low temperature phase. These bubbles of broken phase grow until they coalesce. As the bubble walls propagate toward coalescence the universe is out of thermal and chemical equilibrium in the vicinity of the walls. These nonequilibrium conditions, together with baryon number violating anomalous electroweak interactions and C and CP violation, provide all the necessary ingredients for baryogenesis. The necessity of this baryogenesis occurring in the inhomogeneous environments engendered by bubble nucleation and coalescence ultimately may lead to an inhomogeneous distribution of η and, hence, entropy-per-baryon.

In most of these baryogenesis scenarios the final distribution in η is probably too homogeneous to affect nucleosynthesis. However, one can speculate on models in which significant inhomogeneities in η , ($\Delta\eta/\eta \geq 1$), may occur. Such inhomogeneities, for example, might arise in nonadiabatic (thin wall) models [9] whenever the velocity of an expanding broken phase bubble varies during the transition. In nonadiabatic scenarios the rate of baryogenesis depends strongly upon the velocity of the wall. We note that this velocity may change at only about the 10% level during the course of the transition. However, there remains considerable uncertainty in the determination of the wall velocity in these models. As recently pointed out, this effect may occur in adiabatic models as well [10].

Another possibility might be the formation [11] of distinct domains of baryon-number and anti-baryon number. After annihilating they could leave behind a small number of baryon bubbles containing all of the net baryon number. Finally, strong spatial inhomogeneities may result from any scenario in which most of the generated net baryon number is associated with the collisions of bubble walls at the end of the transition. Although such models are speculative it is nevertheless interesting to investigate the constraints which might be placed on such scenarios from primordial nucleosynthesis.

If fluctuations occur, the bubble size at coalescence will probably provide a typical length scale of fluctuations in η , but fluctuations can occur on larger scales than that. However, it is difficult to quantify that length scale. Thermal and/or quantum nucleation is especially difficult to follow at the electroweak epoch because the nucleating action may be dynamically renormalized by the presence of bubbles of broken phase [12]. Another

complication may be hydrodynamic instability of phase boundaries [13].

Despite these caveats it is nevertheless instructive to consider simple models of homogeneous nucleation of phase in the small supercooling limit [14]. In these models the nucleation rate per unit volume is assumed to be

$$p(T) \approx CT^4 e^{-S(T)}, \quad (1)$$

where $S(T) = a(T)(T_c/(T_c - T))$ is the nucleating action, with $a(T)$ a monotonically increasing function of temperature, and where C is a scale factor of order unity. Integrating the nucleation rate through the end of the phase transition, and assuming that bubble walls move at the speed of light, yields an estimate for the time required for bubbles to coalesce. We can express this coalescence time (or bubble size at coalescence) as a fraction δ of the Hubble time (or horizon scale) H^{-1} [13]:

$$\delta \approx \left[4B \ln \left(\frac{m_{\text{pl}}}{T_c} \right) \right]^{-1}, \quad (2)$$

where B is the logarithmic derivative of the nucleating action S , in units of H^{-1} at the epoch of the phase transition. The value of B depends on calculable details of models for the electroweak transition and is within one or two orders of magnitude of unity. The horizon size is

$$\begin{aligned} H^{-1} &\approx \left(\frac{90}{8\pi^3} \right)^{1/2} g^{-1/2} \frac{m_{\text{pl}}}{T^2} \\ &\approx (1.45 \text{ cm}) \left(\frac{g}{100} \right)^{-1/2} \left(\frac{T}{100 \text{ GeV}} \right)^{-2}, \end{aligned} \quad (3)$$

where $g = \sum_b g_b + \frac{7}{8} \sum_f g_f$ is the total statistical weight in relativistic bosons (g_b) and fermions (g_f) at temperature T , and m_{pl} is the Planck mass. In the standard model, $g \approx 100$ for an electroweak transition at $T = 100$ GeV. The total statistical weight is slightly uncertain due to the unknown top quark mass and extra degrees of freedom associated with extensions of the standard model.

The average bubble size at coalescence δH^{-1} is a result of competition between the nucleation rate and the very slow expansion of the universe. Most bubbles will have size δH^{-1} at coalescence. This follows on noting that larger bubbles would have to be nucleated early, near T_c , where the nucleation rate is exponen-

tially suppressed. Smaller bubbles would have to be nucleated near the end of the phase transition where the effective nucleation rate is again small, since very little unbroken phase would remain. A nucleation/coalescence epoch which approximates the homogeneous nucleation scenario will leave a nearly regular lattice of bubbles at coalescence.

Even though several electroweak scenarios have been proposed [8,9,15] not much is known about the actual nucleation scale δH^{-1} and the expected coexistence temperature T_c in these models. It has been argued that the minimal standard model gives $\delta \sim 10^{-3}$ [16]. These models have not been investigated in sufficient detail to ascertain the relationship between the nucleation scale δH^{-1} and the scale of separation between centers of fluctuations in η , which we shall denote $\delta_\eta H^{-1}$. However, it is possible that $\delta_\eta \geq \delta$, corresponding to less than, or equal to, one fluctuation produced per nucleated bubble.

In Refs. [17], two of us (hereafter referred as JF) have studied in detail the evolution of fluctuations from $T \approx 100$ MeV to $T \approx 1$ keV taking into account neutrino, photon, and baryon dissipation processes. JF's results indicate that fluctuations generated at the electroweak epoch may survive through the epoch of primordial nucleosynthesis. If fluctuations with particular characteristics produced at an early epoch did survive, their presence could alter the nuclear abundance yields emerging from primordial nucleosynthesis [18,19]. If these abundance yields do not agree with observationally inferred primordial abundances, then we can conclude that these fluctuations could not have existed. This, in turn, would allow us to constrain the fluctuation generation mechanism.

We follow JF and define the amplitude of fluctuations $\Delta(x)$, in terms of the spatial distribution of baryon-to-photon number, $\eta(x)$, and its horizon average, $\bar{\eta}$, by $\eta(x) = \bar{\eta}[1 + \Delta(x)]$. The corresponding distribution in entropy-per-baryon is then $s(x) = \bar{s}[1 + \Delta(x)]^{-1}$, where the average conserved entropy-per-baryon in units of Boltzmann's constant is $\bar{s} \approx 2.63 \times 10^8 \Omega_b^{-1} h^{-2}$. In this expression Ω_b is the fraction of the closure density contributed by baryons and h is the present Hubble parameter in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. In this paper $\Delta(x)$ always refers to the initial amplitude of the fluctuations.

The primary criterion for fluctuation survival is that the scale associated with the separations of the centers

of fluctuations ($\delta_\eta H^{-1}$) be comparable to, or exceed, the comoving proton diffusion length (d_{100}) at the beginning of the nucleosynthesis epoch [17–19]. Here d_{100} is the comoving proton diffusion length referenced to the epoch of $T = 100$ GeV (see for example Eq. (4)). Were this condition not satisfied, baryon diffusion would erase fluctuations in η prior to nucleosynthesis. The proton diffusion length is actually a fairly sensitive function of amplitude $(1 + \Delta)$. In Fig. 1 we give the comoving proton diffusion length d_{100} at the epoch $T = 500$ keV as a function of $(1 + \Delta)$. This temperature very roughly corresponds to the epoch of weak freeze-out, where the neutron-to-proton interconversion rate from lepton capture falls below the free neutron decay rate. Note that higher baryon density implies a smaller diffusion length for baryons. Whenever $(1 + \Delta) \leq 10^2$ the baryon diffusion length corresponds to $d_{100} \sim 0.1$ cm.

We also can describe fluctuations by their separation length scale, l_{100} , where we express a length scale comoving with the Hubble expansion in terms of its proper length at an epoch where $T = 100$ GeV. The corresponding proper length at any epoch where the temperature is T is then

$$l = l_{100} \left(\frac{R}{R_{100}} \right) = l_{100} \left(\frac{g_{100}^{1/3} T_{100}}{g^{1/3} T} \right), \quad (4)$$

where R and R_{100} are the scale factors at an epoch of temperature T and 100 GeV, respectively, $T_{100} = 100$ GeV, and where g and g_{100} are the statistical weights in relativistic particles at an epoch of temperature T and 100 GeV, respectively. In this expression we have assumed that the co-moving entropy density is conserved.

In order for a fluctuation to affect the outcome of nucleosynthesis $l_{100} \geq l_{100}^{\text{min}} \approx d_{100}$. This scale is found from detailed nucleosynthesis calculations to be roughly the scale of the proton diffusion length at the nucleosynthesis epoch. Physically, the origin of this limiting length is that any fluctuation scale smaller than the proton diffusion length will be damped out by baryon diffusion prior to nucleosynthesis. Therefore, the minimum fluctuation scale for inhomogeneous nucleosynthesis effects can be expressed in terms of a fraction of the horizon scale H^{-1} at any epoch as

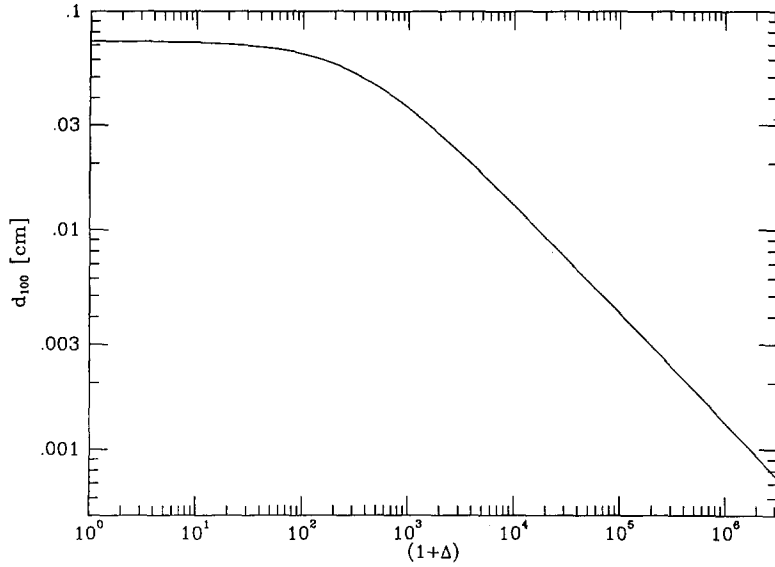


Fig. 1. The co-moving proton diffusion length, d_{100} , at temperature $T = 500$ keV as a function of fluctuation amplitude $(1 + \Delta)$. The calculation assumes $\Omega_b h^2 = 0.0125$.

$$\delta_{\min} \equiv \frac{l_{\min}}{H^{-1}} \approx l_{100}^{\min} \left(\frac{8\pi^3}{90} \right)^{1/2} g_{100}^{1/3} g^{1/6} \frac{T T_{100}}{m_{\text{pl}}}, \quad (5a)$$

$$\delta_{\min} \approx (6.9 \times 10^{-2}) \left(\frac{g_{100}}{100} \right)^{1/2} \left(\frac{g}{g_{100}} \right)^{1/6} \times \left(\frac{l_{100}^{\min}}{0.1 \text{ cm}} \right) \left(\frac{T}{100 \text{ GeV}} \right). \quad (5b)$$

Note that $\delta_{\min} \leq 1$ for $T \lesssim 1.45$ TeV. We conclude that micro-physical, subhorizon-scale fluctuation-generating processes operating at epochs for which $T \lesssim 1.45$ TeV conceivably could have constrainable nucleosynthesis signatures. Fluctuations in η on initially superhorizon scales l which satisfy $l \geq l_{\min}$ are similarly at risk of running afoul of primordial abundance constraints. We note, however, that even fluctuations with $l \lesssim l_{\min}$ may yet survive to affect nucleosynthesis if they have amplitudes large enough that the length scales of their high density regions, l_{100}^{H} , exceed d_{100} . In this case baryons would be unable to diffuse out of the high density cores of fluctuations prior to nucleosynthesis.

In any scheme for baryogenesis associated with an electroweak symmetry breaking epoch at temperature

T we must produce the average proper baryon number density within the horizon

$$\bar{n}_b = \frac{S}{\bar{s}} \approx 0.167 \text{ GeV}^3 \left(\frac{g}{100} \right) \left(\frac{T}{100 \text{ GeV}} \right)^3 \Omega_b h^2, \quad (6)$$

where the entropy per unit proper volume is $S \approx (2\pi^2/45)gT^3$. Homogeneous and inhomogeneous standard big bang nucleosynthesis calculations together with observational abundance constraints imply that $\Omega_b \approx 0.01 h^{-2}$ [1,18].

Assume that baryons are distributed in high density regions with baryon number density n_b^{H} , which in total occupy a fraction f_v of the horizon volume, and in low density regions with baryon number density n_b^{L} . In this case, we can write

$$\bar{n}_b = f_v n_b^{\text{H}} + (1 - f_v) n_b^{\text{L}}. \quad (7)$$

We define $\Lambda_{\text{H}} \equiv n_b^{\text{H}}/\bar{n}_b$ and $\Lambda_{\text{L}} \equiv n_b^{\text{L}}/\bar{n}_b$, so that the density contrast between high and low density regions is $\Lambda \equiv \Lambda_{\text{H}}/\Lambda_{\text{L}}$. If the horizon is filled with a regular lattice of fluctuation cells whose centers are separated by l_{100}^{s} , then the length scale of high density regions is $l_{100}^{\text{H}} = f^{1/3} l_{100}^{\text{s}}$.

In Ref. [18], three of us have calculated in detail the outcome of primordial nucleosynthesis with inhomogeneous initial conditions. In these calculations the nuclear reaction rates were coupled to all significant fluctuation dissipation processes: neutrino heat transport, baryon diffusion, photon diffusive heat transport, and hydrodynamic expansion with photon–electron Thomson drag. The light element abundance yields are found to be inconsistent with observations for all but a very narrow range of fluctuation characteristics. This is why nucleosynthesis is so powerful in constraining primordial inhomogeneities.

A representative case of these calculations is displayed in Fig. 2. In this figure we show the ${}^4\text{He}$ mass fraction and number fraction of ${}^7\text{Li}$ emerging from an inhomogeneous big bang with fluctuation separations l_{100}^s . Dotted lines indicate l_{100}^{\min} (BDL) and the electroweak horizon scale (EWH). Though we show results for particular values of fluctuation amplitude and gaussian width α_{100} (roughly, $f_V \approx (\alpha_{100}/l_{100}^s)^3$), the figure illustrates some general trends for abundance yields as a function of length scale. In particular, we note that when $l_{100}^{\min} \leq l_{100}^s \leq \text{EWH}$ the abundances of ${}^4\text{He}$ and/or ${}^7\text{Li}$ always exceed observational limits¹. This is a general feature of inhomogeneous primordial nucleosynthesis whenever l_{100}^s is below the electroweak horizon scale. It is, however, intriguing that the electroweak horizon is close to the minimum in ${}^4\text{He}$ and ${}^7\text{Li}$ (the “helium dip”).

We have also explored inhomogeneous nucleosynthesis yields for the light elements as functions of initial density contrast and volume filling fraction, Λ and f_V , respectively. Fig. 3 shows the results of numerous numerical calculations for $l_{100}^s = 0.5$ cm. In this figure the parameter space of Λ and f_V laying to the right of the shaded line gives ${}^4\text{He}$ overproduction (${}^4\text{He}$ mass fraction $> 24\%$). In general, we find that ${}^4\text{He}$ and/or ${}^7\text{Li}$ are overproduced whenever 10% or more of the baryons reside in high density regions and either $l_{100}^s \geq l_{100}^{\min}$ or $l_{100}^H \geq d_{100}$.

This constraint can be put in the context of electroweak baryogenesis with a simple model. Assume that an electroweak phase transition has produced a regular

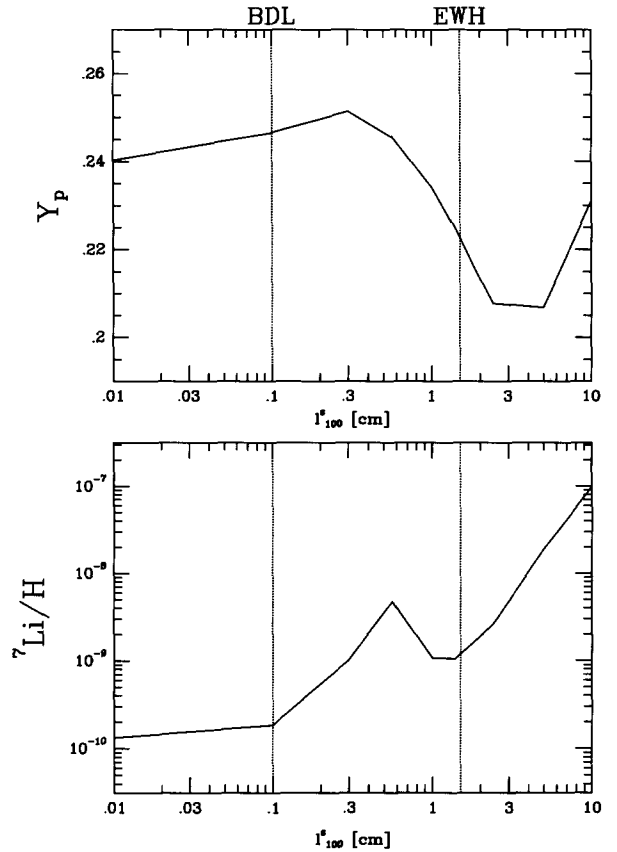


Fig. 2. The ${}^4\text{He}$ mass fraction (upper panel) and ${}^7\text{Li}$ number fraction relative to hydrogen from inhomogeneous nucleosynthesis calculations are shown as functions of the co-moving fluctuation separation l_{100}^s . These calculations employ an initial density contrast $\Lambda = 1.25 \times 10^6$, Gaussian width $\alpha_{100} = l_{100}^s/20$, and $\Omega_b h^2 = 0.0125$. The dotted lines indicate the minimum scale above which fluctuations survive baryon diffusion, BDL, and the electroweak horizon scale, EWH.

lattice of bubbles at coalescence, all of equal size. In fact, we expect a distribution of bubble sizes, but for now assume all bubbles have size equal to the nucleation scale δH^{-1} . Since we expect $\delta \sim 10^{-3}$ there will be roughly $\delta^{-3} \sim 10^9$ bubbles within the horizon. As they expand toward coalescence, these 10^9 bubbles must produce the horizon-averaged baryon number density \bar{n}_b . It is likely, however, as noted above that not all bubbles will contribute equally to this average. Assume that a fraction f of the bubbles contributes a substantial fraction b of the total baryon number in the horizon. As far as the net baryon distribution is concerned, this scenario approximates a regular lattice of

¹ The mass fraction of ${}^4\text{He}$, Y_p , must satisfy $Y_p \leq 0.24$. It is believed that the primordial ${}^7\text{Li}$ abundance is the Population II result ${}^7\text{Li}/\text{H} \approx 10^{-10}$, but it is certainly smaller than the Population I value of $\sim 10^{-9}$. See discussions in Refs. [20,18].

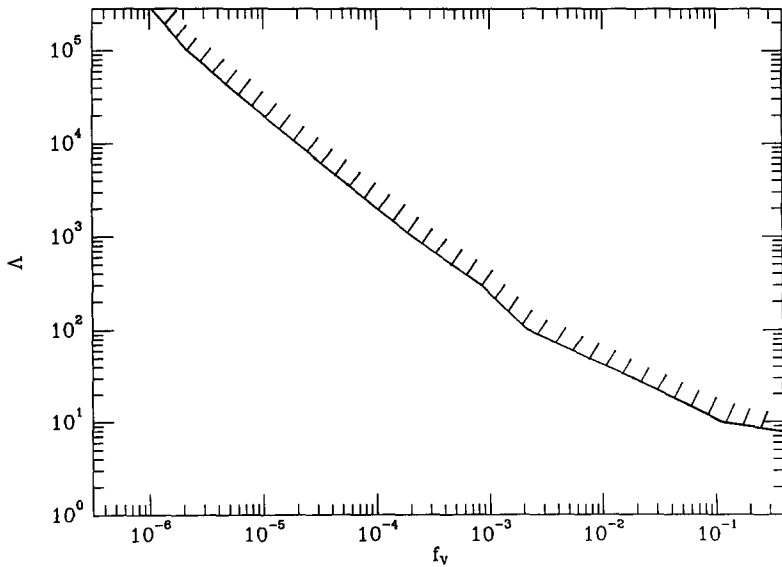


Fig. 3. Constraints from inhomogeneous big bang nucleosynthesis on models with a high-density region volume filling fraction f_v and density contrast $\Lambda = n_b^H/n_b^L$. Any models with parameters falling to the right of the shaded line will overproduce ${}^4\text{He}$ relative to observational limits. For this calculation we assumed spherical square-wave fluctuations with fluctuation mean separation $l_{100}^H = 0.5$ cm and $\Omega_b h^2 = 0.0125$.

fluctuations with effective horizon-fraction separations $\delta_\eta \approx f^{-1/3} \delta$.

Eq. (7) shows that the baryon distribution is characterized by two independent quantities, which we take to be f_v and Λ . In the above hypothetical scenario it is clear that $f_v \leq f$, with equality obtaining when the baryon number produced in a bubble is uniformly distributed over the volume swept out by the bubble wall. The density contrast will be

$$\Lambda \geq \left(\frac{1-f_v}{f_v} \right) \left(\frac{b}{1-b} \right), \quad (8)$$

assuming that each of the $f\delta^{-3}$ bubbles contributes to $b\bar{n}_b$. Equality in Eq. (8) obtains in the limit of uniform baryon distribution across each of the ‘‘significant’’ $f\delta^{-3}$ bubbles. Note that we also assume that significant bubbles are uniformly distributed in space. In this model, $b = (f_v n_b^H) / \bar{n}_b = f_v \Lambda_H$. Since $\Lambda_H = 1 + \Delta$, we may conclude that $(1 + \Delta) \approx b/f_v$. The total fraction of baryons in high density regions is $f_v \Lambda$, which is just b in the limit where $\Lambda f_v \ll 1$. Our nucleosynthesis constraint will apply whenever more than 10% of the baryons are in high density regions, $b \geq 0.1$, and either the fluctuation separation exceeds the minimum value required for nucleosynthesis effects, $f^{-1/3} \delta > \delta_{\min}$, or

the high density region length scale exceeds the proton diffusion length, $l_{100}^H \geq d_{100}$.

For example, in the context of this model assume that only 1000 bubbles out of the total of 10^9 bubbles in the horizon produce 90% of the baryon number. This implies that $f_v \approx f \approx 10^{-6}$ and $\delta_\eta \approx f^{-1/3} \delta \approx 10^2 \delta \approx 0.1 > \delta_{\min}$. This last comparison follows on assuming that $\delta \approx 10^{-3}$. Eq. (8) implies that $\Lambda \approx 9 \times 10^6$. Since $b = 0.9$ exceeds 0.1 and $\delta_\eta > \delta_{\min}$ we would conclude that this scenario is incompatible with nucleosynthesis constraints and, therefore, ruled out. In general, whenever $f < (\delta/\delta_{\min})^3 \sim 3 \times 10^{-6}$ ($100 \text{ GeV}/T$)³ and $b > 0.1$ the nucleosynthesis constraint will be violated. In other words, any scenarios where fewer than about 3000 bubbles contribute more than 10% of the baryon number can be ruled out.

In some models of electroweak baryogenesis the significant bubbles for baryon production might be those which are nucleated earliest. We would then expect these bubbles to be larger at coalescence than the average bubble (δH^{-1}). If the ratio of significant bubble size to the average bubble size is r , and there are $f\delta^{-3}$ significant bubbles, then the effective volume filling fraction for the higher density regions of the baryon distribution is roughly $f_v \leq r^3 f$. Note that in this case,

however, the effective horizon-fraction separation is still $\delta_\eta \approx f^{-1/3} \delta$.

Though the detailed relationship between the fluctuation scale and amplitude and such model parameters as the Higgs particle mass, the top quark mass, and the critical temperature T_c are as yet poorly understood, they are in principle calculable. The parameters of electroweak baryogenesis models must not lead to a violation of the constraints on fluctuation characteristics as describes above. This is probably readily attainable for some models, but may represent a restriction for others.

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