Title
Sea Ice Dispersion Driven by Fluctuating Wind and Ocean Currents

Author
Shaddy, Bryan C.

Publication Date
2021-08-23

Data Availability
The data associated with this publication are within the manuscript.

SEA ICE DYNAMICS IN RESPONSE TO ENVIRONMENTAL FORCING

By

Bryan Chandos Shaddy

A capstone project submitted for Graduation with University Honors

May 4, 2021

University Honors
University of California, Riverside

APPROVED

Dr. Bhargav Rallabandi
Department of Mechanical Engineering

Dr. Richard Cardullo, Howard H Hays Jr. Chair
University Honors

ABSTRACT

The motion of sea ice is driven by wind and ocean currents and comprises both a steady drift and a fluctuating component. Here, we systematically describe the relation between sea ice dispersion and environmental noise starting from a Lagrangian description of non-interacting ice floes. We quantify the nonlinear dynamics of sea ice through stochastic simulations, accounting for noise in wind and ocean currents, in addition to Coriolis forces. The ice follows dispersive behavior on time scales on the order of days, consistent with observations. We find that the dispersion coefficient of the ice depends strongly on the wind fluctuation size and the correlation time of fluctuations. We also examine the cross-stream velocity fluctuations of the ice using a probability density function. Finally, we look at the autocorrelation function for the cross-stream sea ice velocity to quantify the randomness of the system. Our results are useful in quantifying sea ice properties under known environmental conditions, or alternatively as a way to use wind data and sea ice images to infer ocean statistics.

ACKNOWLEDGEMENTS

I would like to start by acknowledging my immense gratitude to my advisor Dr. Bhargav Rallabandi who has played a crucial role in my development as a researcher and student. Not only has Professor Rallabandi supported my research endeavors, but he has also helped me to pursue my interests academically here at UC Riverside and beyond. If it were not for the mentorship I have received, I would not be in the place I am or have the ambitions that I do. Having a place in his research group has provided an excellent community to be a part of, especially during the COVID-19 pandemic, and for that I am extremely appreciative; I wish them all continued success in what they do.

TABLE OF CONTENS

1. INTRODUCTION

Ice floes are sheets of floating ice formed from frozen ocean water located in the Arctic Oceans and their motion plays a crucial role in our understanding of the global climate system. They have sizes ranging from 10 meters to 10 kilometers across and 1 to 3 meters thick. Ice floes are also observed to move around the ocean due to winds and ocean currents.

Traditionally sea ice is considered as a continuum on the climate scale, as shown in Figure 1(a), but to be accurate these types of models require an understanding of the small-scale interactions that occur, as shown in Figure 1(b). These small-scale interactions act as inputs for climate scale models, and it is therefore important to understand the floe scale behaviors. This is what our model aims to do, it helps us further understand the small-scale floe dynamics that can be used to inform larger scale models.

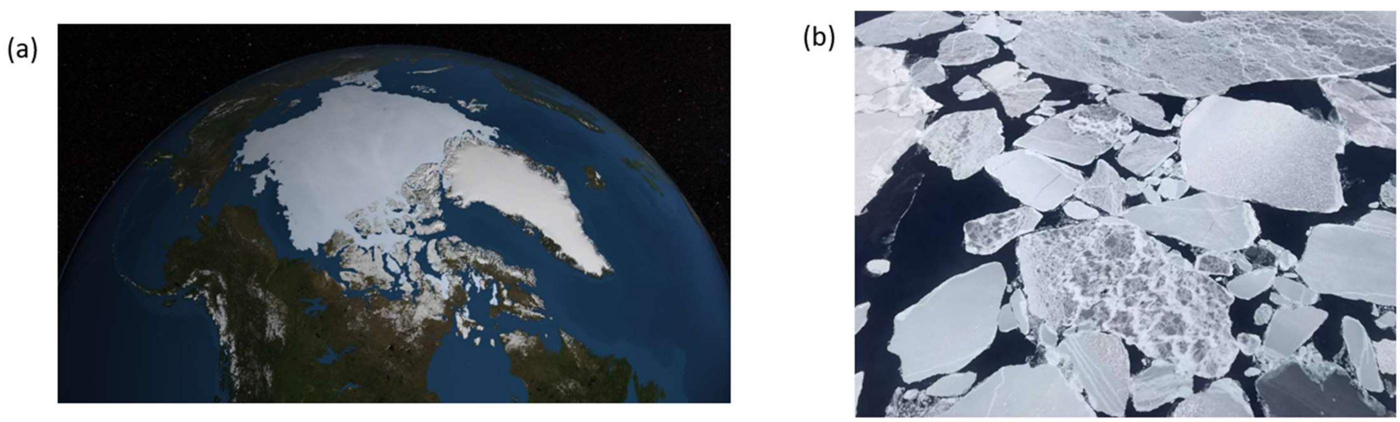

Figure 1 Sea ice dynamics at the climate scale, as shown in (a), are influenced by the interactions of sea ice that occur at the floe scale, shown in (b) [1][9].

Sea ice motion is characterized by very interesting dynamical behaviors including stochastic trajectories and dispersion on the order of days [2][7]. To give a sense of what this chaotic behavior looks like, Figure 2 shows a path followed by sea ice as it moves through the ocean. As seen, the paths sea ice follows are random and nearly unpredictable. In addition to the randomness of these paths, they are also dispersive on timescales on the order of days, which is to say the floes spread from each other in a way that is linearly proportional to time.

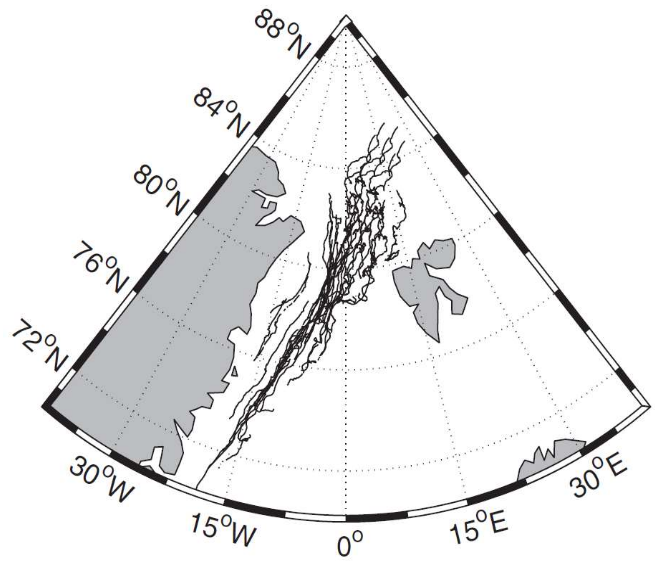

Figure 2 Stochastic sea ice trajectories through the Fram Straight measured for the years 2002, 2003, 2007, 2008, and 2009 using buoys deployed via aircraft on the ice, adapted from [2].

Knowing that sea ice is observed to follow a random path and disperse leads to the question of where this behavior comes from. Our hypothesis is that the random motion of sea ice is tied to noise in the wind and ocean currents which drive its motion. The goal of this work is to quantify the dispersion of sea ice due to environmental noise. By doing this we hope to gain an understanding of the connection between the stochasticity of sea ice motion and the specific conditions of the environmental noise present. To address these issues, I have built a mechanistic description of sea ice motion.

2. THEORY FOR STOCHASTICS SEA ICE DYNAMICS

To begin understanding how sea ice is led to move around the ocean we first perform a momentum balance for an individual floe. We analyze the forces that act on a single floe, considering wind and ocean currents, in addition to Coriolis forces. Additional forces may arise due to ocean tilt, turbulence, ice fracturing, or collisions between neighboring floes, but they are not considered here. Depicted in Figure 3 is a momentum balance for a single floe, with stresses on the top surface due to winds, stresses on the bottom surface due to ocean currents, and a Coriolis force that acts at the floe's center of mass.

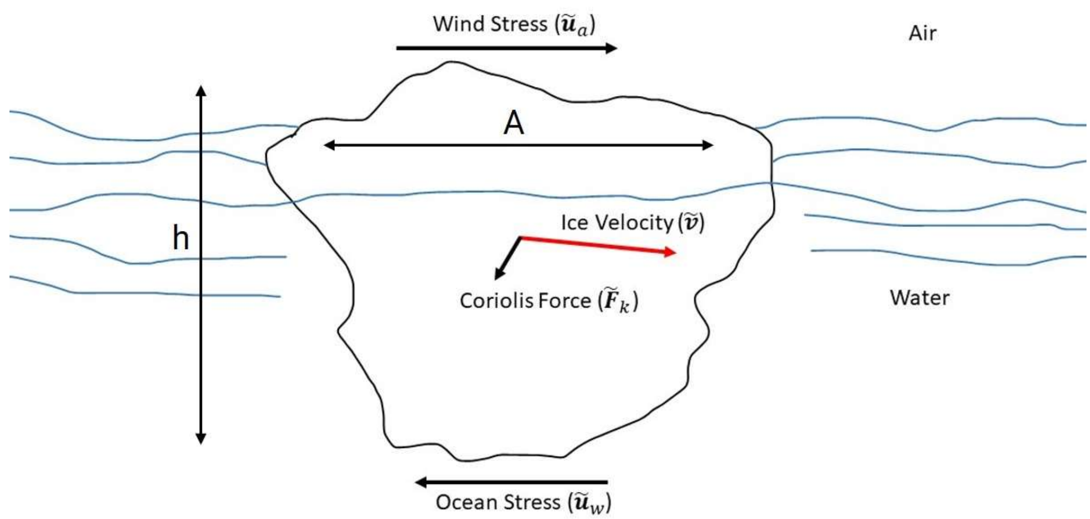

Figure 3 Momentum balance for a single floe with wind stresses, ocean stresses, and Coriolis forces indicated.

Using quadratic drag laws, the wind stress acting on the top surface of the floe is represented by the following equation:

$$\tilde{\boldsymbol{F}}_w = A\rho_a C_a(\tilde{\boldsymbol{u}}_a - \tilde{\boldsymbol{v}})|\tilde{\boldsymbol{u}}_a - \tilde{\boldsymbol{v}}|. \quad (1)$$

Here, A is the projected area of the floe, ρ_a is the density of air, C_a is the drag coefficient of air across the floe's top surface, $\tilde{\boldsymbol{u}}_a$ is the wind velocity vector, and $\tilde{\boldsymbol{v}}$ is the velocity vector of the floe.

Similarly, the ocean currents acting on the bottom surface of the floe create a force following quadratic drag laws as represented by

$$\widetilde{\boldsymbol{F}}_o = A\rho_w C_w(\widetilde{\boldsymbol{u}}_w - \widetilde{\boldsymbol{v}})|\widetilde{\boldsymbol{u}}_w - \widetilde{\boldsymbol{v}}|, \quad (2)$$

where ρ_w is the density of air, C_w is the drag coefficient of air across the floe's top surface, and $\widetilde{\boldsymbol{u}}_w$ is the ocean current velocity.

Lastly, the Coriolis force, which arises due to the floe's placement near one of the poles on the rotating Earth is represented as

$$\widetilde{\boldsymbol{F}}_k = -\rho Ah f \boldsymbol{k} \, x \, \widetilde{\boldsymbol{v}}, \quad (3)$$

where ρ is the density of the floe, h is the floe's mean thickness, f is a frequency factor tied to the rotation of Earth, and $\boldsymbol{k}$ is a unit vector normal to the Earth's surface pointed upwards.

It is important to note that although ocean water is much denser than air and the submerged portion of the floe on which ocean stresses act is larger and rougher, in most cases wind stresses remain the dominant force in driving sea ice motion due to the wind's greater speed. There are some exceptions to this rule however, such as in straights where ocean currents may be magnified and ice moves at an increased rate due to geographic features [3].

2.1 MODEL SETUP

With all major forces acting on sea ice represented mathematically, we can now sum them and use Newton's Second Law to obtain a governing differential equation for sea ice motion, shown by the following equation:

$$\rho Ah \frac{d\widetilde{\boldsymbol{v}}}{d\tilde{t}} = A\rho_a C_a(\widetilde{\boldsymbol{u}}_a - \widetilde{\boldsymbol{v}})| \, \widetilde{\boldsymbol{u}}_a \, - \widetilde{\boldsymbol{v}}| + A\rho_w C_w(\widetilde{\boldsymbol{u}}_w - \widetilde{\boldsymbol{v}})|\widetilde{\boldsymbol{u}}_w - \widetilde{\boldsymbol{v}}| - \rho Ah f \boldsymbol{k} \, x \, \widetilde{\boldsymbol{v}}. \quad (4)$$

Once (4) is obtained it can be simplified through rescaling. To begin, the dimensional time scale $\tilde{t}$ is nondimensionalized by dividing by t^* to obtain t, where t^* is a characteristic time scale for sea ice dynamics and represents the time needed for the dynamics of a floe to react to a change in forcing. The dimensional wind velocity $\tilde{\boldsymbol{u}}_a$ is then rescaled by dividing by the characteristic wind speed u_a^* to obtain the unitless $\boldsymbol{u}_a$, where the characteristic wind speed u_a^* is simply the mean wind speed which acts on the ice. Next, the sea ice velocity $\tilde{\boldsymbol{v}}$ is rescaled by dividing by the characteristic free-drift ice speed v_d^*, giving us the dimensionless ice velocity $\boldsymbol{v}$. The characteristic free-drift ice speed v_d^* is the ice speed once steady state has been reached and the wind and ocean drag have balanced. v_d^* is equal to $(\frac{\rho_a C_a}{\rho_w C_w})^{1/2} u_a^*$, where the prefactor, known as the Nansen number N, is typically ~2% and represents the ratio of free-drift ice speed to wind speed. Next, the dimensional ocean velocity $\tilde{\boldsymbol{u}}_w$ is rescaled by dividing by the characteristic ocean speed u_w^*, producing the dimensionless ocean velocity $\boldsymbol{u}_w$. The characteristic ocean speed u_w^* is equal to βv_d^*, where β is a dimensionless variable representing the ratio of ocean speed to ice speed. Finally, the Coriolis forcing term is nondimensionalized using the new parameter κ, which depends on ice thickness and is equal to $\frac{\rho h f}{u_a^* \sqrt{\rho_w \rho_a C_w C_a}}$.

After rescaling (4) as described and introducing the new variables mentioned, the new version of the governing differential equation is represented as

$$\frac{d\boldsymbol{v}}{dt} = (\boldsymbol{u}_a - N\boldsymbol{v})|\boldsymbol{u}_a - N\mathrm{v}| + (\boldsymbol{v} - \beta\boldsymbol{u}_w)|\boldsymbol{v} - \beta\boldsymbol{u}_w| - \kappa\boldsymbol{k}\ x\ \tilde{\boldsymbol{v}}. \tag{5}$$

The rescaled equation is further simplified by noting that the Nansen number N has been found empirically to be equal to roughly 0.025, allowing $N\boldsymbol{v}$ to be dropped from the wind forcing term. With this we now have the simplified governing equation shown here:

$$\frac{d\boldsymbol{v}}{dt} = \boldsymbol{u}_a|\boldsymbol{u}_a| + (\boldsymbol{v} - \beta\boldsymbol{u}_w)|\boldsymbol{v} - \beta\boldsymbol{u}_w| - \kappa\boldsymbol{k}\ x\ \tilde{\boldsymbol{v}}. \tag{6}$$

Using the simplified governing equation (6) for sea ice dynamics, inputs for wind and ocean currents may be inputted, and outputs of floe position and velocity can be obtained through integration. We integrate (6) numerically using a backward Euler approach, using the wind speed and ocean speed as noisy environmental inputs, to obtain ice velocity [5]. From the sea ice velocity obtained by integrating (6) numerically, we then integrate through time again to gather sea ice position. The noisy environmental inputs used are further quantified below in section 2.2. Additionally, the numerical integration schemes used to obtain floe velocities and positions are shown here, respectively:

$$v^{(n+1)} = \frac{v^{(n)} + \left(u_a^{(n+1)}|u_a|^{(n+1)} + \beta u_w^{(n+1)}|v - \beta u_w|^{(n)}\right)\delta t}{1 + (|v - \beta u_w|^{(n)} + i\kappa)\delta t}, \tag{7}$$

$$x^{(n+1)} = x^{(n)} + \frac{v^{(n)} + v^{(n+1)}}{2}\delta t, \tag{8}$$

where all variables continue from (6) but now the variables $x, v, u_a,$ and u_w have a superscript denoting the time step from which they were obtained. Additionally, a time step size δt now appears as part of the numerical integration. (7) has also been created to take exact inputs for u_a and u_w, where both are stochastic currents modeled based on the environment.

2.2 ENVIRONMENTAL MODEL INPUTS

Now our focus turns to modeling the noisy wind and ocean currents which drive the motion of sea ice and will be the inputs to the numerical sea ice dynamic equations (7) and (8). To accurately understand the tie between environmental noise and the stochastic dynamics of sea ice, we must model the noise of the acting wind and ocean currents based on what is observed empirically. To model these noisy wind and ocean currents, we are treating them as containing both a steady mean component and a fluctuating component. For the wind currents, the mean component has also been rescaled to have a magnitude of 1 acting in a steady, unchanging direction. Alternatively, the mean ocean current has been modeled to act at some angle ϕ from the mean wind direction.

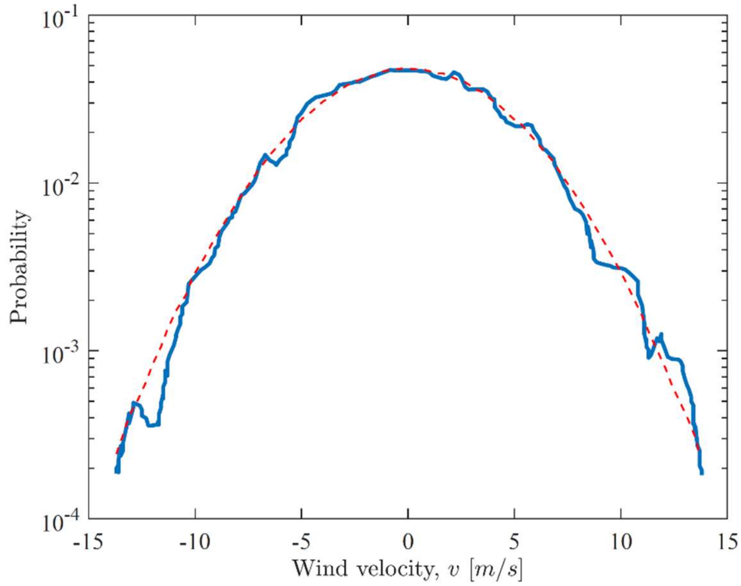

Figure 4 Probability density function for wind velocities measured over the Beaufort Sea (solid blue curve) showing excellent agreement with a Gaussian distribution (red dashed curve) [10].

To model the environmental noise most important to sea ice motion, we chose focus on the fluctuating component of the wind due to its greater effect on ice dynamics as mentioned previously. Shown in Figure 4 above is a probability density function for wind velocities over the Beaufort Sea in the Arctic Ocean, with a Gaussian distribution overlaid in red for reference. As shown in Figure 4, a Gaussian distribution typically represents the wind noise data well. Using this distribution of noise, we have designed out model to incorporate a normal distribution for the fluctuating component of the wind current.

The wind noise in our model has also been designed to have a single correlation time τ, which is on the order of 6 to 24 hours, consistent with wind speed observations in the Arctic [6].

This wind correlation time represents how often the fluctuating wind component shifts. What this means is that the wind no longer has any memory of or tie to its previous orientations after a length of time τ has passed.

Using this observed distribution for wind noise and this correlation time τ, we can construct a model for the noisy wind currents acting on sea ice. To do this we think of the wind as represented by an Ornstein Uhlenbeck process, which is described by the stochastic differential equation shown here:

$$\frac{du_a}{dt} = \frac{1}{\tau_a}(1 - u_a) + \sqrt{\frac{2\sigma_a^2}{\tau_a}}\frac{dW^{\mathbb{C}}}{dt}, \tag{9}$$

where τ_a is the wind correlation time, σ_a is the wind speed standard deviation, and $\frac{dW^{\mathbb{C}}}{dt}$ is the white noise term.

We can then solve (9) numerically at each time step throughout our simulation to obtain the noisy wind currents [8]. The numerical scheme used to solve (9) is as follows:

$$u_a^{(n+1)} = 1 - \left(1 - u_a^{(n)}\right)e^{-\frac{\delta t}{\tau_a}} + \sigma_a\left(1 - e^{-\frac{2\delta t}{\tau_a}}\right)^{\frac{1}{2}} N^{\mathbb{C}}(0,1), \tag{10}$$

where $N^{\mathbb{C}}(0,1)$ gives random numbers used for the stochastic portion of the wind current.

3. RESULTS AND DISCUSSION

Once we have mathematical models representing the dynamic behavior of sea ice, (7) and (8), and the environmental conditions influencing the ice, (10), we can now focus on the tie between them that leads to the dispersive behavior of sea ice. To do this, we first must understand what exactly sea ice dispersion looks like. Traditionally, to understand the dispersive behavior of sea ice, one will look at the cross-stream position of sea ice as it moves through time. The cross-stream position is defined as the perpendicular position of an ice floe relative to a mean path which is found by averaging the position of many floes. Figure 5 below depicts exactly this, with a mean floe path defined and a perpendicular deviation indicated.

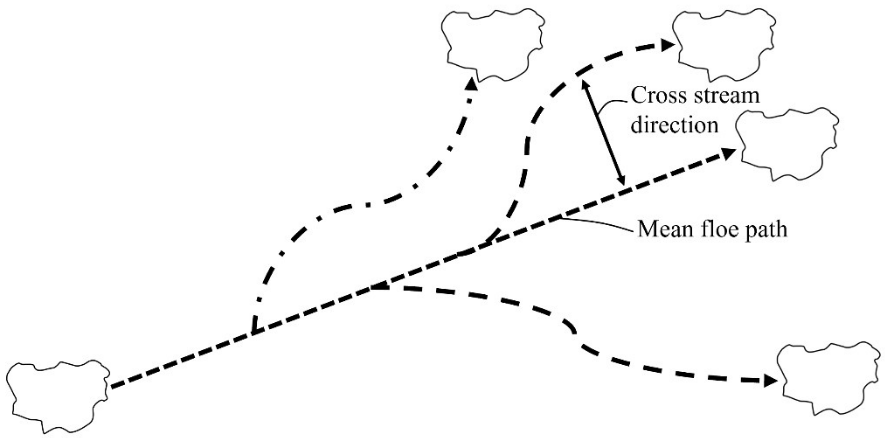

Figure 5 Sea ice cross-stream position relative to a mean path.

To obtain the statistical information necessary to perform this analysis, we use our numerical schemes described in (7) and (8) to predict the stochastic paths of many floes simultaneously. These paths are found by integrating the sea ice acceleration to obtain velocity, and then integrating through time once more to obtain positions. The paths obtained through this are all different due to the random noise added to the forcing term (10) and allow us to examine their dispersion similarly to what is done empirically. Once these trajectories are obtained, we can find a mean path like what is shown in Figure 5 and then use this to find the perpendicular position of each floe relative to this mean trajectory.

Using the perpendicular sea ice positions described, we are now able to analyze how these change through time. Specifically, we study how the cross-stream position of the floes from their mean path increases as they are transported by wind and ocean currents over time scales of days due to environmental noise.

3.1 MEAN SQUARED POSITION

To mathematically study the dispersion of floes, we analyze the cross-steam mean squared position of the many floes modeled as they change through time. The mean squared position is a way to define statistically the variance of the many floes' positions relative to the mean. Based on the observed dispersion of sea ice, it is expected that the mean squared position will increase as the floes move through time. Particularly, it has been observed that the mean squared position scales linearly with time once the floes have entered their dispersive regime. With this understanding, we analyze the mean squared position of the modeled floes as a function of time to compare with what has been observed.

As stated previously, in the sea ice system winds are the dominant driving force due to their large magnitudes and the quadratic relation between drag force and velocity. This means that sea ice's stochastic dynamic behavior can be recreated by simply considering the acting winds, neglecting ocean currents and Coriolis forces as mentioned before. Following this simplification, we consider only the wind force input into our model. From this we can obtain the needed statistics to analyze the mean squared position and compare it to empirical results. Figure 6 below depicts the mean squared position of our modeled sea ice in blue as a function of time, with empirical results shown in red and black [2].

From the results presented in Figure 6 we can see that in the dispersive regime, which is to say on time scales on the order of days, a linear relation between the mean squared position and time is found from simulations. These results match the empirical observations shown in red and black on Figure 6 within an order of magnitude and validate our model's ability to accurately capture the dynamic behavior of sea ice. There are some cases where observations have found a relation between mean squared position and time that deviates from linear, but our model did not reproduce this behavior. We believe that this deviation could be due to turbulence, collisions, or ice fracturing, but it remains a topic for future investigation.

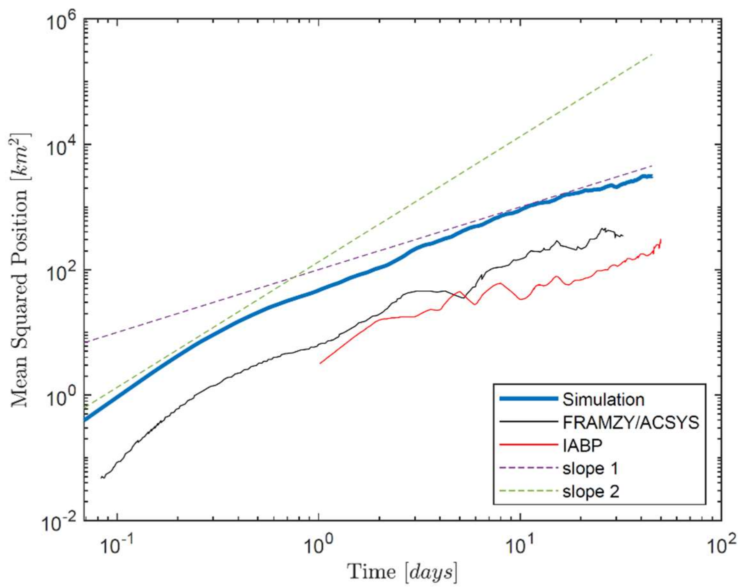

Figure 6 Floe mean squared position as a function of time from simulations shown in blue and empirical results in red and black [2].

3.2 COEFFICIENT OF DISPERSION

Using the linear relation between mean squared position and time that is found from Figure 6, we are now able to determine a coefficient of dispersion that ties the cross-stream mean squared positions to time explicitly. To do this we use an equation representing the results shown in Figure 6, once a linear relation with time has been found, which is shown here:

$$< r^2 > = 2Dt \, , \tag{11}$$

where $< r^2 >$ is the mean squared position, D is the dispersion coefficient, and t is time.

From (11) we can easily find the coefficient of dispersion D that we are interested in. To add physical dimensions to this problem, for comparison to empirical results, we chose an input of 7 m/s for the mean wind speed based on the average summer (August and September) wind speed in the Arctic from 1996 to 2015 [4]. We have also selected to use a wind fluctuation speed equal to 2.8 m/s based on what has been measured in the Arctic over an 11-month period during 1997-1998 [10]. Lastly, we have chosen a wind correlation time of 6 hours based on what has been observed [6]. Future work remains to obtain more consistent environmental data for use as an input into our model, but for now we use these values.

Using these inputs, we find a dispersion coefficient of roughly 70 m^2/s, which is on the order of what has been found empirically. Shown in Figure 7 on the following page is the dispersion coefficient as it varies in time over a span of roughly 32 days. Figure 7 depicts empirical results for the dispersion coefficient in black, with results obtained from modeling overlaid in blue [2]. From this figure we can see that the empirical results and simulations agree quite nicely, though it is somewhat difficult to compare them at a finer level due to the fluctuation of the dispersion coefficient that is found empirically as time progresses. Roughly speaking however, the two results appear to be on the same order, adding weight to the results obtained from the model.

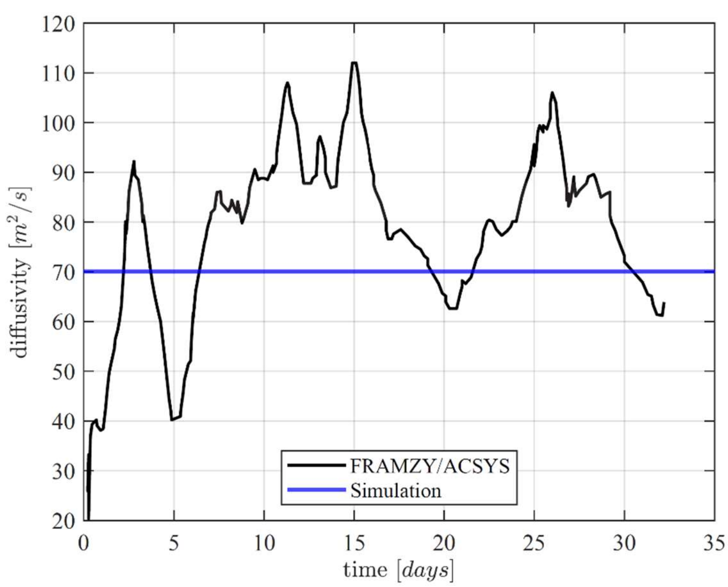

Figure 7 Model prediction shown in blue provides a good estimate for the measured dispersion coefficient shown in black as it varies through time for 32 days [2].

3.3 CROSS-STREAM VELOCITY FLUCTUATIONS

The next result we will consider is the cross-stream velocity fluctuations of sea ice relative to the mean velocity obtained across all simulated trajectories. Like how the cross-stream position has been analyzed, we look at the velocity component of individual floes perpendicular to their mean velocity. We do this by finding a unit vector perpendicular to the mean floe velocity and taking the dot product of the individual floe velocities with this unit vector. From this we obtain the velocity component exactly perpendicular to the mean velocity.

Once we have the cross-stream velocity components we can compute the probability density function. Depicted in Figure 8(a) is a probability density function obtained from simulations in blue, as described. Overlaid on this plot is a Gaussian distribution which we use as a reference for the linearity of the sea ice dynamics. From Figure 8(a), we see that the cross-stream velocity fluctuations appear to have a Gaussian distribution and linear behavior.

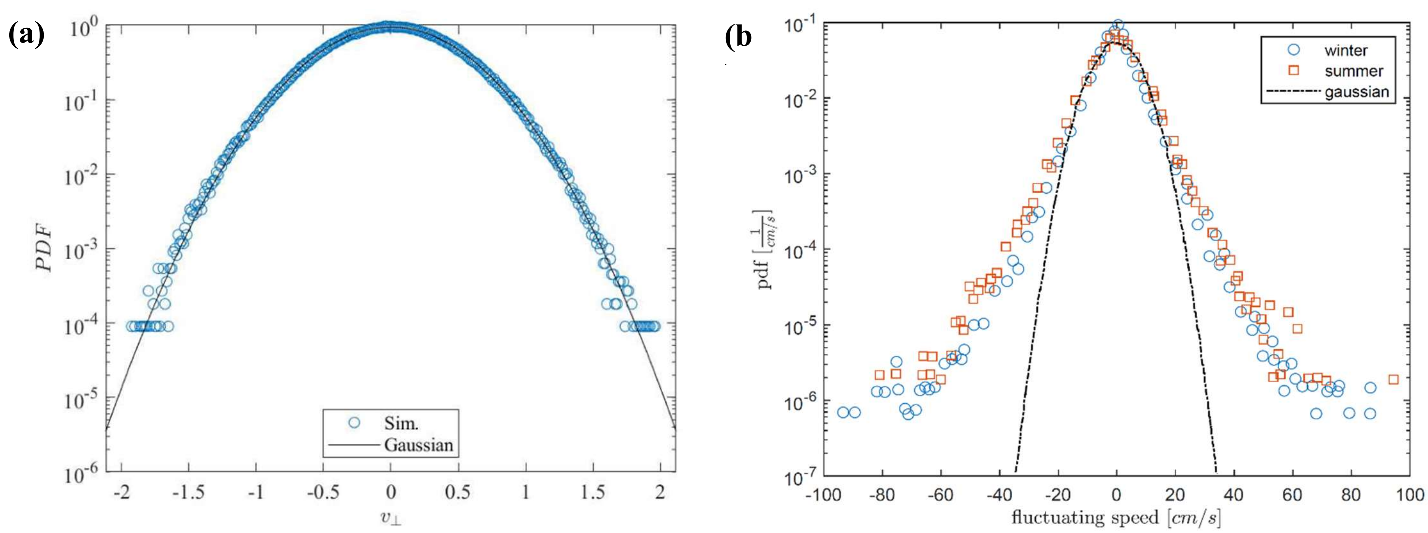

Figure 8 (a) Cross-stream sea ice velocity PDF from simulations showing Gaussian behavior. (b) Empirically derived cross-stream sea ice velocity PDF showing non-Gaussian behavior near the tails of the distribution [7].

The result from Figure 8(a) is of note as it deviates from what has been observed empirically. Shown in Figure 8(b) is the same analysis from an empirical study, again with a Gaussian distribution overlaid. Empirical results show that, while for small cross-stream velocity fluctuations the probability density function appears Gaussian, for larger fluctuations the probability density function deviates from a Gaussian distribution, with tails near the edges. This deviation is not reproduced by our model and we are unsure of its origins. Some have reported that this behavior may be due to fracturing of the ice during its transport, but this remains uncertain.

The non-linear behavior found from empirical results are quite intriguing as they do not arise due to the external forces on the system as we have modeled, but rather due to some other unknown phenomenon. In future work, this behavior should be given particular attention to be able to model and reproduce it consistently. By doing this an understanding of the origins of this non-linear behavior may be obtained.

3.4 CROSS-STREAM VELOCITY AUTOCORRELATION FUNCTION

The last result examined is the cross-stream ice velocity autocorrelation function. The autocorrelation function tells us how similar the dynamics of sea ice at one instant in time are to the dynamics of sea ice after some time lag τ from the starting time. From this we can gain a quantitative understanding of how random the dynamics of sea ice are and on what time scales they are correlated. To complete this analysis, we first collect the cross-stream sea ice velocity fluctuations as we did for the cross-stream ice velocity PDF. With the cross-stream ice velocity time series we can now find the autocorrelation function over a range of time lags τ using the definition shown here:

$$C_{v_\perp v_\perp}(\tau) \equiv < \overline{v_\perp(t)} v_\perp(t+\tau) >, \tag{12}$$

where C is the correlation function, $v_\perp$ is the cross-stream ice velocity fluctuation, and τ is the time lag.

With (12) we can now compute the autocorrelation function for many different floes and over many different starting times to obtain statistically robust results. The autocorrelation function computed from our simulations is presented below in Figure 9.

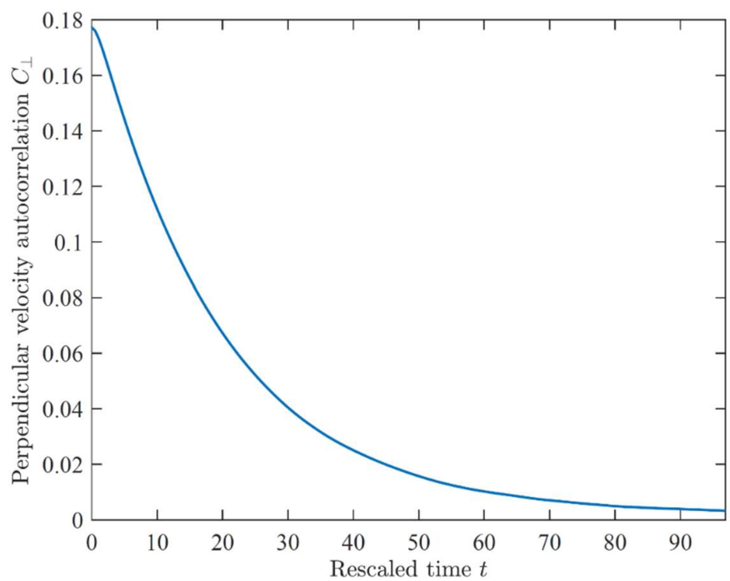

Figure 9 Cross-stream sea ice velocity autocorrelation function showing exponential decay.

From our results for the autocorrelation function we find the behavior of sea ice in our simulations to be very similar to what has been found empirically. The most significant of these results is that during short times, the autocorrelation function can be modeled by an exponential decay, matching results that have been measured directly [7]. Further, our model found the correlation time for sea ice dynamics to be roughly 7.4 hours for the set of inputs described in section 3.2, however this result is tied very closely to the correlation time used to model the wind and as such can vary.

4. CONCLUSION

In all, we have built a mechanistic model which makes predictions about the stochastic dynamics of sea ice resulting from noisy wind and ocean currents. From our model we have obtained predicted sea ice statistics consistent with what has been observed. Results for the mean squared position as a function of time were examined and compared with empirical results, with both finding a quadratic relation between mean squared position and time during short times and a linear relation on the order of days once ice has entered the dispersive regime [2]. The dispersion coefficient of sea ice was also explored, with the model giving predictions of about 70 m^2/s, close to what has been measured [2]. A cross stream ice velocity PDF was also presented in comparison to observed results, finding that our simulations do not reproduce some of the non-linear behavior found elsewhere [7]. Understanding these non-linear empirical results remains a focus for future work. Lastly, the autocorrelation function for the sea ice cross stream velocity was presented in comparison to empirical results, with both finding that the autocorrelation function can be modeled by exponential decay during short times [7]. Future work may involve fine tuning our model to incorporate collisions between floes and ultimately incorporate our results with large scale climate models.

REFERENCES

[1] Blumberg, S. (2017, October 30). IceBridge Launches Two Sets of Antarctic Flights. Retrieved April 29, 2021, from https://www.nasa.gov/feature/goddard/2017/icebridge-launches-two-sets-of-antarctic-flights

[2] Gabrielski, A., Badin, G., & Kaleschke, L. (2015). Anomalous dispersion of sea ice in the FRAM Strait region. *Journal of Geophysical Research: Oceans, 120*(3), 1809-1824. doi:10.1002/2014jc010359

[3] Kwok, R. (2005). Variability of Nares Strait Ice Flux. *Geophysical Research Letters, 32*(24). doi:10.1029/2005gl024768

[4] Liu, Q., Babanin, A. V., Zieger, S., Young, I. R., & Guan, C. (2016). Wind and Wave Climate in the Arctic Ocean as Observed by Altimeters. *Journal of Climate, 29*(22), 7957-7975. doi:10.1175/jcli-d-16-0219.1

[5] Moin, P. (2011). *Fundamentals of Engineering Numerical Analysis*. New York: Cambridge University Press.

[6] Monahan, A. H. (2012). The temporal Autocorrelation Structure of Sea Surface Winds. *Journal of Climate, 25*(19), 6684-6700. doi:10.1175/jcli-d-11-00698.1

[7] Rampal, P., Weiss, J., Marsan, D., & Bourgoin, M. (2009). Arctic sea ice velocity field: General circulation and turbulent-like fluctuations. *Journal of Geophysical Research, 114*(C10). doi:10.1029/2008jc005227

[8] Scott, M. (n.d.). *Applied Stochastic Processes in Science and Engineering*.

[9] SVS: Summer Arctic Sea Ice Retreat: May - August 2013. (n.d.). Retrieved April 29, 2021, from https://svs.gsfc.nasa.gov/4096

[10] Weiss, J. (2013). *Drift, Deformation, and Fracture of Sea Ice a Perspective Across Scales*. Dordrecht: Springer Netherlands