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Research Note

Binaries in Open Clusters: The Effects of Rotation on Determinations of Frequency and Mass Ratio

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Summary. A re-examination of the HR diagrams for 175 A and F stars in four young clusters (Coma, Alpha Persei, Praesepe and the Pleiades) shows that the number of stars that appear to be binaries because they fall significantly above the main sequence is a very sensitive function of where the MS is drawn, as are the mass ratios deduced for the binaries. In particular, the fact that single stars are typically more rapid rotators than close binaries raises many of them into the region of the HR diagram populated by the binaries. Thus, a main sequence fitted to the rotating single stars will conceal some binaries and make the others appear to have artificially low mass ratios; while a main sequence fitted to non-rotating single stars will make the rotating ones look like binaries, and thereby give an upper limit to the number of binaries.

Various plausible main sequences yield apparent binary frequencies of 7–87%, with a most probable value of 45–50% as an upper limit to the percentage of the stars that are binaries with $M_2/M_1 \geq 0.5$. At least a few known binaries are missed with all but one of the MS's tried. The mass ratios implied by distances above the main sequence also vary with choice of MS. The distribution may be either monotonically decreasing from the smallest mass ratio detectable, as reported by Jaschek, or bimodal with a second peak near $M_2/M_1 = 1.0$, as reported by several authors for field populations.

Thus, rotating stars and binaries cannot be distinguished without data in addition to the HR diagram, and it is not possible to say on the basis of available information that the open cluster stars and the field stars differ in binary frequency or distribution of mass ratios.

Key words: binary stars — open clusters — stellar rotation

I. Introduction

More than 200 years after the realization that there are physically associated pairs of stars (Michell, 1767), we

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still do not know how binary systems form. The collection of data which may lead to, or at least test, theories of binary formation nevertheless continues. Statistical data are an important part of this, addressing questions like: what fraction of all stars are in binary or multiple systems; what is the distribution of periods, eccentricities, total masses, and mass ratios among the systems; and how do these things vary from one stellar population to another? This paper deals with the problem of determining binary frequencies and mass ratios in young open clusters.

A. Binaries in Clusters

Hertzsprung first suggested identifying binaries from their positions above the main sequence in HR diagrams (Atkinson, 1937), and Haffner and Heckmann (1936, 1937) applied the method to Praesepe, finding that there were indeed stars typically about 2/3 magnitude above the main sequence, suggesting binaries with roughly equal components.

More recently, Bettis (1975) and Jaschek (1976) have searched for binaries in this way in the Hyades, Pleiades, Praesepe, α Persei, and Coma Berenices, using modern photoelectric photometry, the former for the range A0–K4 and the latter for A0–F6. They concur in finding a binary incidence of about 30% (as lower limits, of course, since binaries with sufficiently faint secondaries will look just like single stars), which is not very different from results of direct searches by spectroscopic and other methods (see also Batten, 1973). In addition, the mass ratios implied by the distances of stars above the main sequence (assuming that the secondary is also a MS star) are consistent with a distribution of secondary masses not significantly different from that associated with a van Rhijn (1936) luminosity function.

B. Binaries in the Field

These cluster results are somewhat different from those found for various populations of field stars. Abt and Levy (1976a, b; 1977) have searched complete samples of

F3–G2 and B2–B5 IV–V stars in the solar neighborhood. They find that at least 50% of the B stars and 90% of the F–G stars probably have companions. The method discussed here is sensitive to systems with $M_2/M_1 \geq 0.5$ and separations $\lesssim 5''$ and would have found 20–30% of their B and F–G stars to be binaries (if they were at the distances of our clusters and in the absence of rotation and other sources of confusion). In addition, in both samples, there is evidence for two populations of binaries. In the systems with periods greater than 10–100 years, the secondary masses follow a van Rhijn distribution; but the closer systems show a distribution which falls off much less steeply, about as $M^{-1/4}$. That is, the short-period systems typically have mass ratios not very different from one. Abt and Levy suggest that the wide and close populations may reflect formation from separate protostars and from fission of single protostars respectively. Wolff (1977) studying late B spectroscopic binaries (SB's) also finds two populations, which manifest themselves as a bimodal distribution of mass ratios, peaked at $M_2/M_1 \lesssim 0.2$ and ~ 1.0 . She suggests that these are the fission binaries before ($M_2/M_1 \sim 1.0$) and after ($M_2/M_1 \lesssim 0.2$) evolution by mass exchange, the low-mass companions being degenerate dwarfs, as a rule.

Blaauw and van Albada (1967, 1973), in an extensive search for binaries among the OB stars in five associations, found that the distribution of semi-major axes was bimodal and that there were far fewer low mass stars among the secondaries of the SB's than one would expect from the van Rhijn function or the Salpeter (1955) birthrate function. Finally, Trimble and Cheung (1976; Trimble, 1974) found evidence for two populations (large separations with relatively small mass ratios, and smaller separations with roughly equal component masses) for both evolved and unevolved systems, in studies of the mass ratios and semi-major axes of the binaries in the *Sixth Catalogue of the Orbital Elements of Spectroscopic Binary Systems* (Batten, 1967) and its first two supplements (Pedoussant and Ginestet, 1971; Pedoussant and Carquillat, 1973). Thus, these field studies differ from the cluster studies in finding more binaries and a larger number of binaries with mass ratios near one.

C. Problems

It is, of course, possible that field and cluster binaries are genuinely different; though if the field is populated largely by stars from disrupted clusters, this would be surprising. The alternative is that there is something wrong with one or more of the studies. The *Catalogue* sample, for instance, must be biased by observational selection effects that are impossible even to describe, let alone allow for; but the more systematic work of Abt and Levy, Wolff, and Blaauw and van Albada should be relatively free of discrimination against close (as op-

posed to wide) unequal pairs, mass ratios of 0.6 to 0.8 and so forth.

There may also be difficulties with the cluster studies. Bettis (1975) defined the "single star main sequence" by a least-squares fit through the observed points, thereby immediately classing the 40% of the stars below the fitted line as single, according to Dabrowski and Beardsley (1977). They have looked again at the same data for the same clusters, but using a previously determined ZAMS (Johnson, 1964). Not surprisingly, this lowers the single-star main sequence by $\sim 0.1-0^m2$, which is large compared to the errors of the observed points and therefore a meaningful change. This, in turn, increases both the total number of binaries found by the method (from $\sim 30\%$ to $\sim 50\%$) and the values of the mass ratios, so that the presence of an $M_2/M_1 \sim 1.0$ peak can no longer be excluded in the clusters.

Jaschek (1976) took as his single-star main sequence the lower envelope of the observed MS (except in the Pleiades, where differential reddening is important) so that his results should be free of the problem found by Dabrowski and Beardsley. But he has considered stars higher on the main sequence ($B-V \sim 0.0-0.6$; spectral type $\sim A0-F5$), for which rotation may not be negligible, especially in young clusters. It is important to recall that SB's often rotate synchronously with their orbits, and so much more slowly than single stars of the same spectral type.

II. The Effects of Rotation

Collins and Sonneborn (1977) have calculated emergent fluxes at 53 wavelengths from model atmospheres of rotating B0–F8 main sequence stars, seen both pole-on and equator-on, as a function of w , the ratio of rotation velocity to critical velocity. They have also convolved these fluxes with the response functions of the Strömgren four-color, $ubvy$, photometric system to obtain zero age main sequences in the M_v , $b-y$ plane. The position of the main sequence is a steep function of w , but depends very little on aspect angle (although the mass of a star at a particular position on a main sequence will depend on the angle from which it is viewed). The rotating main sequences fall above (because stars are moved to the right in the HR diagram) the non-rotating ones by amounts that can be approximated by

$$\Delta M_v = 0.1(1 - (b - y) \sqrt{3}/2)(1 - \exp(3w^2)).$$

Thus, at $b - y = 0$, the main sequence is raised by $\Delta M_v = 0.03, 0.11, 0.34, 1.04$, or 1^m91 for $w = 0.3, 0.5, 0.7, 0.9$, or 1.0 respectively.

Early-type single stars in young clusters are known to be rapid rotators. Abt (1970) for instance gives average values of $v \sin i$ as a function of M_v for single stars in the Pleiades and Alpha Persei as 235, 210, 150, and 100 km s^{-1} for $M_v = 0, +1, +2$, and $+3$ respec-

tively. These correspond to values of w ranging (non-monotonically) from 0.55 to 0.95 over the range A0–F5, in the Collins-Sonneborn models. The average w is about 0.7, and it must drop quite rapidly with spectral type, (though not as rapidly as among older stars) beyond F5. Thus, both rotation and a binary companion raise a star's position in the HR diagram, and the MS for rotating single stars should be somewhat tilted relative to that for non-rotating stars.

The next section will explore the effects of this and other uncertainties in the position of the single-star ZAMS on the frequencies and mass ratios of binaries derived from open cluster HR diagrams.

III. The Data and Its Analysis

A. M_v and $b-y$ for Cluster Stars

Since Collins and Sonneborn (1977) calculated the effects of rotation in the M_v , $b-y$ plane, it makes sense to look at the clusters in the same way. There is four-colour data for stars bluer than $b-y = +0.4$ in the literature for all four clusters studied by Jaschek (1976): Pleiades (Crawford and Perry, 1976), Alpha Persei (Crawford and Barnes, 1974), Coma (Crawford and Barnes, 1969a) and Praesepe (Crawford and Barnes, 1969b). These papers also give color excesses, $E(b-y)$, for each star in the Pleiades and Alpha Persei (the reddening is negligible in Coma and Praesepe), from which individual absorption values, A_v , can be derived, and distance moduli for the clusters. Several sets of apparent V magnitudes exist for each cluster. Those used here come from the sources cited by Crawford and his colleagues: Pleiades, Johnson and Mitchell (1958); Alpha Persei, Mitchell (1960); Coma, Johnson and Knuckles (1955); and Praesepe, Dickens et al. (1968) and Johnson (1952).

As soon as the data were plotted, several things became clear. First, evolutionary effects limit the analysis to stars redder than $b-y = 0.05$ in the Pleiades and Alpha Persei, and redder than $b-y = 0.17$ in Coma and Praesepe. Second, the theoretical (non-rotating) main sequence was more sharply curved than any of the cluster ones. A theoretical main sequence with w changing gradually from 0.7 at A0 to 0.0 at F8 was a slightly better fit, but still could not convincingly be put through the points (see Fig. 1). Both curves are also, on average, too bright. And third, the four main sequences could be made to line up better if the distance moduli were changed slightly from those derived by Crawford and his colleagues. In Figure 2, the relative distance moduli have been adjusted such that, if that of the Pleiades is taken to be 5.54, then after corrections for reddening and absorption, $V_0 - M_v = 6.1$ for Alpha Persei, 5.95 for Praesepe, and 4.60 for Coma.

B. Choosing a Single-Star, Zero Age Main Sequence

It will shortly become clear that all other errors and uncertainties in an analysis of this sort are small com-

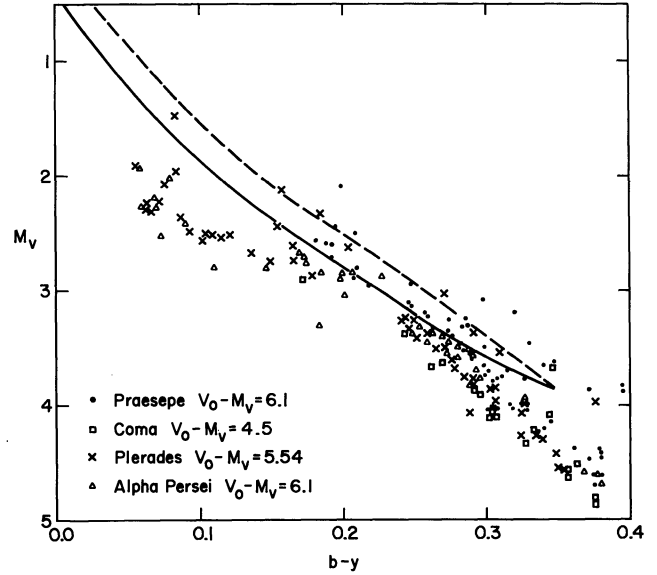


Fig. 1. 175 stars redder than $b-y = 0.05$ (in the Pleiades and Alpha Praesepe Persei) or 0.17 (in Coma and Praesepe) in the M_v , $b-y$ plane, using the $ubvy$ photometry and distance moduli of Crawford and his colleagues. The solid curve is the non-rotating theoretical main sequence of Collins and Sonneborn (1977); the dashed curve is that MS modified to allow for rotation, i.e. ratio of rotation velocity to break-up velocity, $w = 0.7$ for A0 to F2, changing gradually to 0.0 at F8, in approximate agreement with the observations of single stars in the Pleiades and Alpha Persei, reported by Abt (1970)

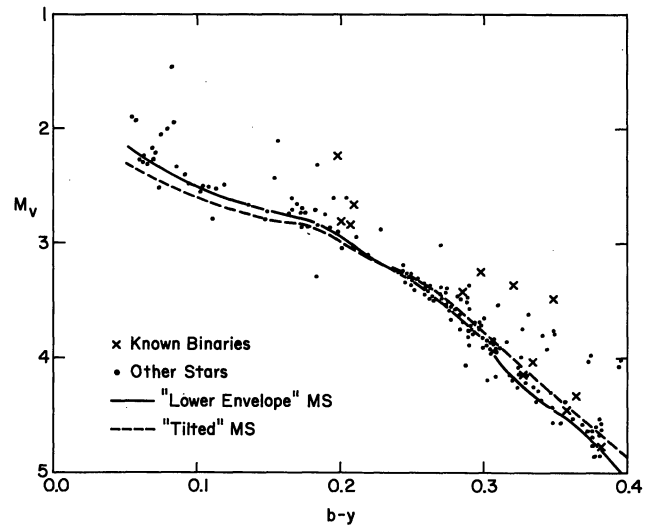


Fig. 2. The 175 stars of Figure 1 with the cluster distance moduli adjusted to bring the main sequences more nearly on top of one another. The X's are known binaries and the dots all other stars. The solid line is the "lower envelope" main sequence given in Table 1. The dashed line in the "tilted" (pseudo-rotating) main sequence discussed in Section II.B. Numbers and mass ratios of binaries derived using these main sequences are given in Table 1

pared to those associated with where one draws the main sequence. The theoretical MS's are not much help, being clearly the wrong shape, and probably too bright as well. The frequency of binary stars and distributions of mass ratios derived from them are, however, given in the last columns of Table 1 for reference.

Table 1. Numbers of binaries as a function of mass ratio found among 175 A and F stars in four young clusters, assuming that positions above the main sequence are due to light contributed by companions. The calibration of $M_2/M_1 = \alpha$ in terms of distance above the MS, δm , is discussed in Section II.C. The consequences of eight possible main sequences are given: (1) Lower envelope to densely populated region, as shown in Figure 2. (2) Same raised by 0^m1 . (3) Same lowered by 0^m1 . (4) Lower envelope tilted, as shown in Figure 2, to mimic the effect of rotation of single upper MS stars. (5) Same raised by 0^m1 . (6) Same lowered by 0^m1 . (7) Collins and Sonneborn's (1977) non-rotating MS as shown in Figure 1. (8) Collins and Sonneborn's MS for rotation at 0.7 of critical speed at A 0—F 2, declining to no rotation at F 8. The total detectable binary frequency for each MS is also given. Numbers in parentheses are binaries known to Crawford and his colleagues at the time the photometry was done. All but one of the MS's misses at least a few known binaries.

$M_2/M_1 = \alpha$	δm	MS (1)	MS (2)	MS (3)	MS (4)	MS (5)	MS (6)	MS (7)	MS (8)
<0.5	< 0^m07	87 (2)	130 (5)	22 (0)	98 (3)	132 (5)	53 (3)	146	162
0.5—0.6	0.07—0.13	35 (2)	10 (3)	49 (2)	27 (2)	12 (4)	40 (0)	8	4
0.6—0.7	0.13—0.23	18 (4)	12 (1)	51 (2)	19 (4)	3 (0)	32 (2)	9	4
0.7—0.8	0.23—0.37	15 (2)	7 (1)	23 (5)	5 (0)	12 (1)	21 (4)	9	3
0.8—0.9	0.37—0.55	8 (0)	7 (1)	13 (1)	13 (2)	7 (1)	8 (1)	1	2
0.9—1.0	0.55—0.75	7 (2)	7 (3)	8 (2)	10 (2)	8 (3)	13 (1)	1	0
> 1.0	> 0.75	5 (2)	2 (0)	9 (3)	3 (1)	1 (0)	8 (3)	1	0
Apparent binary fraction:		88/175 =0.503	45/175 =0.257	153/175 =0.874	77/175 =0.440	43/175 =0.246	122/175 =0.710	29/175 =0.166	13/175 =0.074

The ZAMS derived by Crawford (1975, for the F stars) is simply a weighted fit to the stars in these clusters (plus the Hyades and a few others) neglecting only a few known binaries which fall conspicuously above most of the other stars. It therefore resembles the least-squares fitted MS's used by Bettis (1975) and suggested by Dabrowski and Beardsley (1977) to give artificially low binary frequencies for the lower main sequence.

The standard sources of main sequences having failed, six plausible MS's were chosen and their consequences explored. Number one (the solid curve in Fig. 2) is a lower envelope to the most highly populated region of the observed HR diagram. It should represent single, non-rotating stars. Numbers two and three were arrived at by raising and lowering the "lower envelope" MS by 0^m1 . These shifts represent the maximum likely error in the photometry (plus the authors' slight uncertainty in just where the most highly populated region is) and fulfill a criterion (suggested by an anonymous referee) that a reasonable MS should allow some points below, because of unavoidable observational errors, but not as many as half. MS number four (the dashed curve of Fig. 2) is an attempt to allow realistically for rotation. A main sequence was fitted through the middle of the observed points. Then the blue end was lowered by 0^m3 , corresponding to removing the effect of rotation at $w=0.7$ among the A stars and lesser amounts at later types. MS4 is therefore tilted relative to MS1, though it has about the same numbers of stars above and below it as MS1. MS's five and six are this "tilted" MS displaced up and down by 0^m1 .

This attempt to allow for rotation cannot really work, of course. The effect of rotation is to move single stars into the same regions of the HR diagram as are occupied by binaries of various mass ratios, and there is

no way to tell the phenomena apart without additional data. The most that can be said is that, if no allowance is made for the rotation of single stars, binary incidence and mass ratios will surely be underestimated; whereas if the upper MS is moved down by the entire 0^m3 introduced by rotation at $w=0.7$, the binary frequency and mass ratios will surely be overestimated. The tilted MS is, therefore, intended as a plausible alternative to be used in exploring the consequences of small changes in MS position and shape.

C. The Analysis

Dabrowski and Beardsley (1977) use the convenient notations (a) mass ratio $M_2/M_1 = \alpha$, (b) magnitude difference between two components of a binary system $= \Delta m$, (c) vertical displacement of an observed point above a particular MS $= \delta m$, and introduce the relationships (one approximate and one exact)

$$M_2/M_1 = 10^{(-0.1\Delta m)}$$

and

$$\Delta m = -2.5 \log(10^{(0.4\delta m)} - 1)$$

for binaries with both components on the main sequence. These conventions are followed here. The assumption that the secondary is on the main sequence is a reasonably safe one, because white dwarf secondaries would not produce a detectable δm (provided that the primary is earlier than about K4, according to J. P. Dabrowski, private communication, 1977), and giant secondaries would raise the system into the giant regions of the HR diagram which are not included in the present

analysis. The resulting calibration of α in terms of δm is indicated in Table 1.

Table 1 shows the numbers of binaries of various mass ratios found among the 175 stars in the four clusters, using the six different MS's described in section B. Numbers of known binaries (as given by Crawford and his colleagues) are shown in parentheses. Note that several known binaries fall in the column $\alpha < 0.5$ for every main sequence but one, and are therefore classified as "single" by the present method of analysis. Although additional binaries have subsequently been found in some of the clusters, they typically do not have photometry on the relevant system, so we cannot tell whether or not they would have been detected as double in our analysis.

The fraction of stars found to be detectably (more than $0^m.07$, corresponding to $\alpha=0.5$) above the MS ranges from 0.25 to 0.87, with the intermediate positions of the "lower envelope" and "tilted" MS's producing frequencies of 0.50 and 0.44 respectively. These are, in effect, upper limits to the real binary population. Although a few known binaries have been missed (and therefore undoubtedly some previously unknown ones as well), some rotating stars have also necessarily been included that should not have been counted as binaries. If the real incidence of binaries with $M_2/M_1 \gtrsim 0.5$ is like that in the Abt and Levy (1976a,b; 1977) populations, then our 45–50% of stars above the main sequences must include roughly equal numbers of binaries and fast rotators.

The distributions of mass ratios deduced also vary somewhat with choice of main sequence. The "lower envelope" MS and its displacements produce $q(\alpha)$ distributions that decline monotonically from peaks at the lowest detectable mass ratio. The "tilted" MS and its displacements, on the other hand, all produce distributions with two peaks, one at relatively low and one at relatively high mass ratio.

IV. Conclusions

A reanalysis of the HR diagrams of four young, open clusters indicates that the number of binaries revealed by their positions above the main sequence is an exceedingly sensitive function of the precise position of the zero age main sequence that is assumed.

In particular, since close binaries typically rotate synchronously and so more slowly than single stars of the same color, rotation introduces an uncertainty into the position of the MS among A and early F stars in these clusters, which affects both the numbers and mass ratios of the binaries found. Plausible main sequences can result in apparent binary frequencies of 7–87%, with most probable values of 45–50%. These stars above the main sequence include some rapid rotators (perhaps about half of them) and so provide an upper limit to the

number of binaries with $M_2/M_1 \geq 0.5$. The distribution of mass ratios found also varies significantly with choice of MS. Main sequences chosen as lower envelopes to the observed points produce distributions that decline monotonically from a peak at the lowest mass ratio to which the method is sensitive while main sequences tilted to mimic the effects of rotation produce bimodal distributions, with peaks near the lowest and highest values of mass ratio. Thus it is not possible to say that this cluster binary population differs significantly in either binary frequency or in distribution of mass ratios from the field and association binary populations studied by Abt and Levy (1976a,b; 1977), Blaauw and van Albada (1967, 1973), Trimble and Cheung (1976) and Wolff (1977).

Although the effects of a binary companion cannot be distinguished from those of rotation in the HR diagram, the two cases do produce different trajectories in, for instance, a $c_1 - (b - y)$ diagram, rotation yielding larger c_1 values at a given $(b - y)$ color. The reason for this is that, in an unevolved binary system, the red (cooler) area of the less massive star is always smaller than the blue (hotter) area of the more massive star, while in rotating stars, the red (equatorial) area is larger than the blue (polar) area. The possibility of separating the binary and rotating stars in young clusters using *ubvy* data will be explored in a later paper.

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References

- Abt, H.A.: 1970, in *Stellar Rotation*, A. Slettebak (ed.), NY: Gordon and Breach, p. 193
 Abt, H.A., Levy, S.G.: 1976a, *Astrophys. J. Suppl.* **30**, 273
 Abt, H.A., Levy, S.G.: 1976b, *Bull. Am. Astron. Soc.* **8**, 521
 Abt, H.A., Levy, S.G.: 1977, Preprint
 Atkinson, R.d'E.: 1937, *Observatory* **60**, 299
 Batten, A.H.: 1967, *Publ. Dominion Astrophys. Obs.* **13**, No. 8
 Batten, A.H.: 1973, *Binary and Multiple Systems of Stars*, Oxford: Pergamon
 Bettis, C.L.: 1975, *Publ. Astron. Soc. Pac.* **87**, 707
 Blaauw, A., van Albada, T.S.: 1967, in *The Determination of Radial Velocities and Their Applications*, A. H. Batten and J. F. Heard (eds.), NY: Academic Press, p. 215
 Blaauw, A., van Albada, T.S.: 1973, Preprint
 Collins, G.W., Sonneborn, G.H.: 1977, *Astrophys. J. Suppl.* **34**, 41
 Crawford, D.L.: 1975, *Astron. J.* **80**, 955
 Crawford, D.L., Barnes, J.V.: 1969a, *Astron. J.* **74**, 467
 Crawford, D.L., Barnes, J.V.: 1969b, *Astron. J.* **74**, 818
 Crawford, D.L., Barnes, J.V.: 1974, *Astron. J.* **79**, 687
 Crawford, D.L., Perry, C.L.: 1976, *Astron. J.* **81**, 419
 Dabrowski, J.P., Beardsley, W.R.: 1977, *Publ. Astron. Soc. Pacific* **89**, 225
 Dickens, R.J., Kraft, R.P., Krzeminski, W.: 1968, *Astron. J.* **73**, 6
 Haffner, H., Heckmann, O.: 1936, *Naturwiss.* **24**, 635

- Haffner, H., Heckmann, O.: 1937, *Göttingen-Nachrichten* 2, 77 =
Veröff. Göttingen No. 55, 77
- Jaschek, C.: 1976, *Astron. Astrophys.* **50**, 185
- Johnson, H.L.: 1952, *Astrophys. J.* **116**, 121
- Johnson, H.L.: 1964, *Bol. Tonantzintla y Tacubaya Obs.* **3**, 305
- Johnson, H.L., Knuckles, C.R.: 1955, *Astrophys. J.* **122**, 209
- Johnson, H.L., Mitchell, R.I.: 1958, *Astrophys. J.* **128**, 31
- Michell, J.: 1767, *Phil. Trans. Roy. Soc.* **57**, 234
- Mitchell, R.I.: 1960, *Astrophys. J.* **132**, 68
- Pedoussant, A., Carquillat, J.M.: 1973, *Astron. Astrophys. Suppl.* **10**, 105
- Pedoussant, A., Ginestet, N.: 1971, *Astron. Astrophys. Suppl.* **4**, 253
- van Rhijn, P.J.: 1936, Publ. Kapteyn Astr. Lab. Groningen, No. 47
- Salpeter, E.E.: 1955, *Astrophys. J.* **121**, 161
- Trimble, V.: 1974, *Astron. J.* **79**, 967
- Trimble, V., Cheung, C.: 1976, in *Structure and Evolution of Close Binaries*, P. Eggleton, S. Mitton, and J. Whelan (eds.), IAU Symp. No. 74, Dordrecht: D. Reidel, p. 369
- Wolff, S.C.: 1977, Preprint