Title

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Abstract

A *longitudinal stability model for a* **2** *vehicle platoon yields a pair of linearly coupled equations that model the dynamics of the tow bar and the vehicles. Using reasonable approximations, the vehicle separation distance obeys a second order non-linear differential equation with constant coefficients. Spring force and damping are provided by a special shock absorber link. The towbar linkage can be made to serve as a low pass mechanical system. The design spring constant of 60 lbf/in and a damping rate of 200 lbf/ft/sec yield a low-pass system with sufficient damping and a cut off frequency of 0.78* **Hz.** *Steady state spring extensions are of the order of several inches for vehicle spacing from 0.2 to 1.0 vehicle lengths (200.24 inches).*

Keywords: Vehicle aerodynamics, aerodynamic drag force, non-linear equations, dynamical systems, experiments.

Executive Summary

In order to design a tow-bar system for measuring the aerodynamic drag forces experienced by each member of a two-vehicle platoon, a longitudinal stability model was conceptualized for the dynamic system formed by the two vehicles and the tow-bar. Considering the platoon as a system of two masses linked by a complex structure of spring - shock absorber type, the model can be linearized such that a pair of linearly coupled equations describing the behavior of the system is obtained. Using established models for the drag force and the rolling resistance force, as well as reasonable approximations for the extensions in the double-arm tow-bar system, the vehicle separation distance obeys a second order non-linear differential equation with constant coefficients. Spring force and damping are provided by a special shock absorber link in each arm of the tow-bar. While the double-arm design is chosen to ensure stability a high speed testing, the main purpose of the design is to use the tow-bar linkage as a low-pass mechanical system, such that high frequencies are eliminated from the data acquisition. The calculations made based on the linearized model lead to a design spring constant of *60* lbf/inch and a necessary damping rate of 200 lbf/ft/sec. These physical characteristics then yield a low pass system with sufficient damping and a cut off frequency of **0.78 Hz.** The steady state spring extensions during testing are of the order of several inches for vehicle spacing from **0.2** to 1.0 vehicle lengths. The vehicles used for the experiments are Ford Windstar minivans with a vehicle length of 200.25 inches.

Field Tests

It has been shown in the wind tunnel that the platooning of vehicles significantly reduces the drag experienced by all vehicles in a platoon of vehicles (1). **A** full scale field test is planned to support the wind tunnel tests. The testing utilizes towed Ford Windstar **vans** to simulate the platoon. In the wind tunnel tests, models of the Chevrolet Lumina **LPV** were used. The use of the Ford Windstar **in** field tests should not greatly effect the comparison because the vehicle shapes are similar.

The field tests begin by looking at a **2** vehicle platoon. The two Windstars are connected using the tow bar system sketched in Figure 1. The two arm design shown in the figure provides lateral stability during testing at high speeds, a feature which cannot be achieved using a single-arm design.

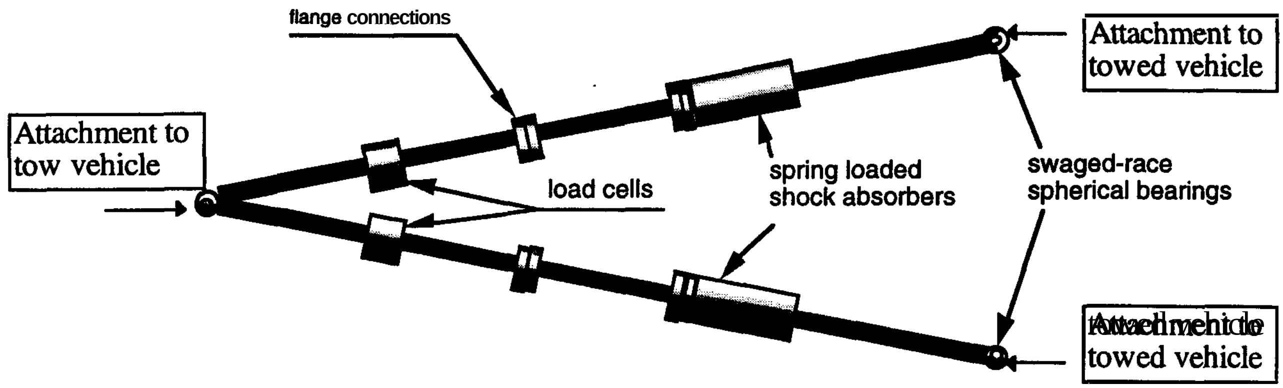


Figure 1: Schematic of the Tow bar

A load cell in each arm of the tow bar measures the corresponding axial force, consisting of the aerodynamic drag and the rolling resistance. The transmission losses will be eliminated for the trail vehicle by the use **of** free wheeling hubs which fit between the drive wheels and the axles. The signals from the tow bar will be fed into a computer onboard the lead Windstars.

Tow Bar Dynamics Model

A longitudinal stability model is developed to study the dynamics and interaction of the two vehicles and the tow bar. The results of this analysis are used to calculate the required physical characteristics **of** the springs and the shock absorbers, thus the dimensions **of** the tow-bar system. It is assumed that the tow bar incorporates a spring-damper device, **as in** figure **2.** The spring-damper helps to remove unwanted road noise from the load cell signals, thus providing a cleaner drag measurement.

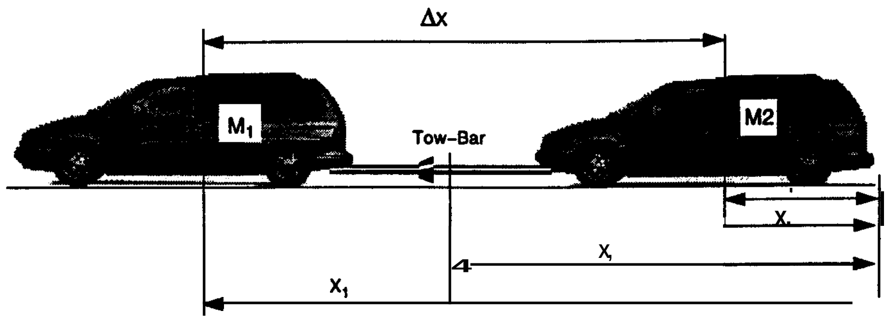


Figure **2:** Longitudinal Stability Model

The longitudinal stability model begins by defining the forces and accelerations between the vehicles and the tow bar using Newton's second law. For the first and second vehicle the forces and accelerations are defined **as**

$$M_1 \ddot{X}_1 = \sum F_1 = F_{p1} - F_{R1} - D_1 + F_{1TB} \qquad (1)$$

$$M_2 \ddot{X}_2 = \sum F_2 = -F_{R2} - D_2 + F_{2TB} \qquad (2)$$

where M_1, M_2 are the masses of the respective vehicles, F_{P1} **is** the propulsive force, F_{R1}, F_{R2} are the two rolling resistances of the vehicles, D_1, D, are the drag terms, and F_{1TB}, F, **are** the tow bar forces, assumed positive. The negative **(-F_{1TB})** force is shown in Figure **3, as** are the other defined quantities.

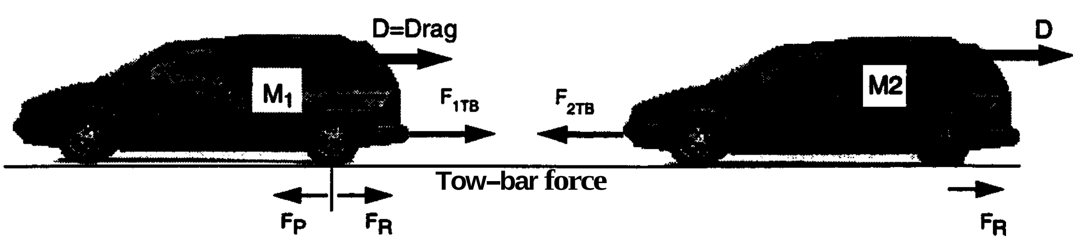


Figure **3:** Forces and Accelerations on Vehicles 1 and **2**

After balancing the forces through the tow bar and defining the mean position, $X_o = \frac{X_1 + X_2}{2}$ **, and** spacing between the vehicles, $\Delta X = X_1 - X_r$, a pair of non-linear coupled equations emerges.

$$2M\ddot{X}_o = F_{P1} - F_{R1} - F_{R2} - D_1 - D_2 \tag{3}$$

$$M\Delta\ddot{X} = F_{P1} - F_{R1} + F_{R2} - D_1 + D_2 + 2F_{1TB} \tag{4}$$

The masses of the two vehicles are assumed to be identical, and equal to **M.** After linearizing the coupled equations, three relations result **(A** complete derivation **is** presented in appendix **A).** The first equation determines the equilibrium spring extension ΔX_{EQ}.

$$k_{TB}\Delta X_{EQ} = \frac{F_{R2} + k_2 U_o^2}{2\cos\theta} \tag{5}$$

In equation *(5)* for the equilibrium spring extension, F_{R2} represents the rolling resistance, and $k_2 U_o^2$ is the drag **of** the trail vehicle (Together they make up the total load in the tow-bar). U_0 is the average speed **of** the vehicles and θ is the angle between the towbar legs and the centerline. Equations *(6)* and (7) below describe the dynamics.

$$2M\ddot{X}_o'' + 2U_o(k_1 + k_2)\dot{X}_o' = F_{P1}' + 2U_o(k_2 - k_1)\Delta\ddot{X}' \tag{6}$$

$$M\Delta\ddot{X}' + \left[4c_{TB}\cos\theta + U_o(k_1 + k_2)\right]\Delta\dot{X}' + 4k_{TB}\cos\theta\Delta X' = F_{P1}' + 2U_o(k_2 - k_1)\dot{X}_o' \tag{7}$$

The fluctuation of the average velocity of the two vehicles is $\dot{X}_o'$, and the fluctuating axial spring extension between the two vehicles is $\Delta X'$. The coupling term $-(k_2 - k_1)$ is small and will be neglected. Equation *(6)* is of no interest for drag measurement. Small fluctuations in vehicle spacing obey equation (7) which can be written **as** $\Delta\ddot{X}' + 2\zeta\omega_n\Delta\dot{X}' + \omega_n^2\Delta X' = F_{P1}'$ with the definitions

$$\omega_n^2 = \frac{k_{TB}}{\frac{M}{4}} \cos\theta \tag{8}$$

$$\zeta = \frac{\dfrac{\left[c_{TB}\cos\theta + \dfrac{U_o}{4}(k_1 + k_2)\right]}{2}}{\sqrt{k_{TB}\dfrac{M}{4}}} \tag{9}$$

ω_n is the natural frequency of the tow bar, ζ is the damping ratio, c_{TB} is the force rate (lbf/ft/sec) in the spring-damper and k_{TB} is the spring rate(lbf/ft) (Again, see Appendix **A** for details)

Analysis of the Model

The anticipated equilibrium tow bar force (for each leg separately) is shown in figure **4 as** a function of vehicle speed and for various separation distances. This force limits the range of various design parameters. Figure **4** shows that the force varies with vehicle speed vehicle separation. The velocity range investigated is to be between **40** mph to **70** mph. Under these conditions the force in the tow bar does not exceed **90** lbf and varies only moderately with separation distance. The force in the tow bar dictates the size of the load cell necessary to obtain accurate measurements while supporting the expected load, and in part determines the equilibrium spring extension (the ΔX_{EQ} in equations ***5).*** The damping ration, expression (9), contains a contribution from the spring-loaded shock absorber in the tow bar, and a contribution arising from the drag of the two vehicles. This latter term, $\frac{U_o(k_1 + k_2)}{4}$, is shown plotted in figure ***5*** as a function of vehicle speed and vehicle spacing It is seen that this term varies only moderately with vehicle spacing.

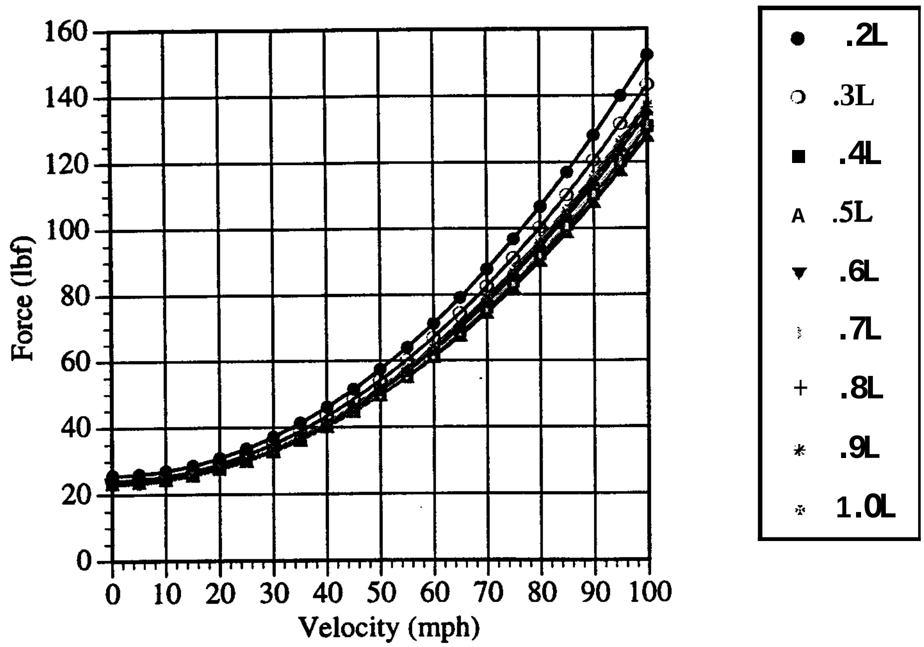


Figure 4: Tow Bar Forces

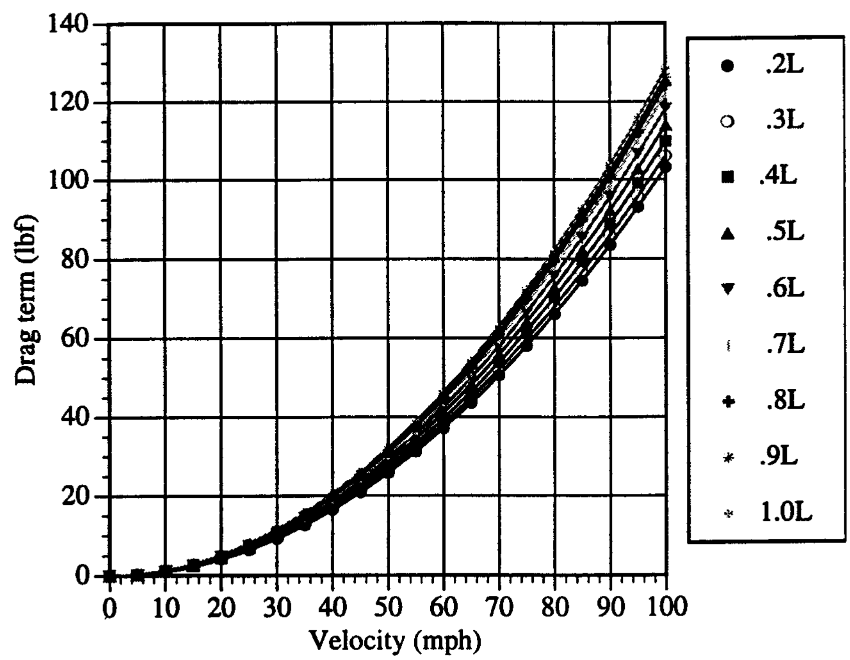


Figure 5: Drag term variation with velocity

Using the results from Figures **4** and **5** and equations (5)-(9), a FORTRAN code is run to define the different design possibilities for **a** broad range of ζ and ω_n. Several of the results from the FORTRAN analysis are summarized in Figures *6,* 7, and 8. Values of the damping ratio **are** required to be in the range ζ = (0.5 - 1.0) and ω_n is allowed to range from 1.0 rad/sec to **2.2** rad/sec (0.16Hz-0.35Hz respectively). Figure 6 gives the maximum value of the spring rate, k_n, for each choice of ω_n, assuming $\theta = 0$. Figures **7** and **8** give the tow bar extensions from equation (5), for the two vehicle spacings of **0.2** vehicle length and 1.0 vehicle length, respectively. The extensions depend heavily on the speed of the platoon and vary on the order of one inch for every 10 mph.

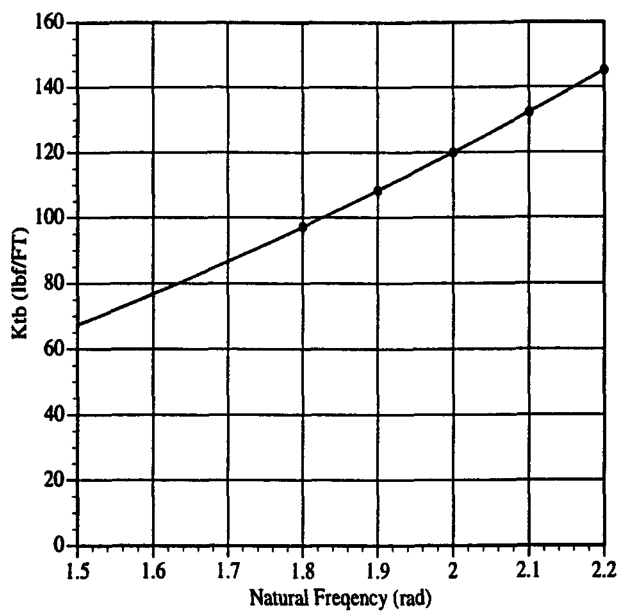


Figure *6:* Tow bar spring coefficient dependence on ω_n

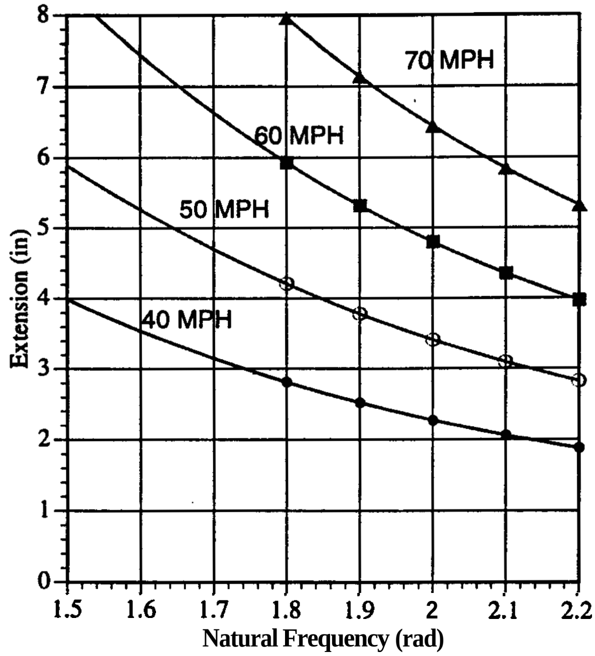


Figure 7: Tow bar extensions, $\Delta X'_{EQ}$, at spacing 0.2 vehicle lengths

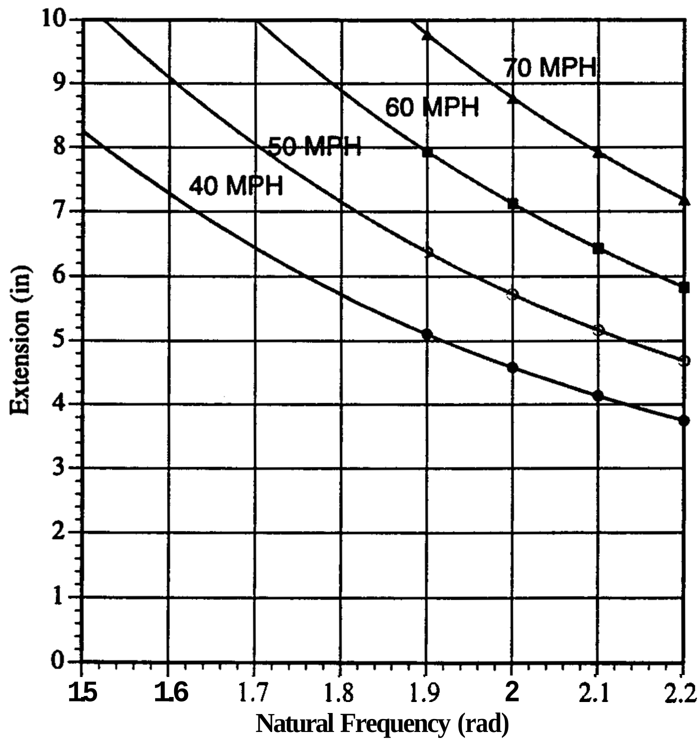


Figure 8: Tow bar extensions, $\Delta X'_{EQ}$, at spacing 1.0 vehicle lengths

Utilizing these plots, one may now size the spring-loaded damper to give the desired cut-off frequency and damping ratio. For example, if the cut-off frequency, ω_c, is desired to be 0.35 Hz (2.2 rad/sec), the spring rate must be about 145 lbf/ft (figure 6). Spring rate immediately determines the equilibrium extension to be 5-7 inches at 70 MPH (figures 7 and 8). Use of figure 5 and expression (9) suggest a required damping rate of $c_n = 70$ lbf/ft/sec to give ζ in the range 0.65-1.0.

The Final Tow Bar Design and Fabrication

Final Design Values

Based on the tow bar dynamics model, the force in each leg of the tow bar does not exceed 90 lbf with vehicles traveling between 40 mph to 70 mph, resulting in a total of maximum axial load of 180 lbf. The two load cells, each with maximum capacity of 250 lbf, are purchased from Sensotec, Inc. in Columbus, Ohio. The 250 lbf rating on the load cell ensures a safety factor of at least 2.5 when the tow bar is loaded.

The two spring-loaded shock absorbers are made by Works Performance Shocks in Northridge, California. The spring rate in the production shock absorber is 120 lbf/in (60 lbf/in in each unit), with the damping rate at approximately 200 lb/ft/sec (100 lb/ft/sec in each unit). The damping rate is achieved by using silicone oil with the appropriate viscosity. Oil is stored in a simple sleeve-type accumulator surrounding the spring-loaded shocks. With these values, the design cutoff frequency is approximately 0.78 Hz, and the equilibrium spring extension is between 1-2 inches for vehicle spacings in the range of 0.2 - 1.0 vehicle length. A second set of (interchangeable) springs at 30 lbf/in brings the cut off frequency down to 0.55 Hz at 2-4 inches equilibrium extension.

Additional Considerations

The final tow bar design has to meet several additional criteria as follows:

1) The connections to the trail cars must be simple and non-destructive,

2) The tow bar linkage must have minimal weight and be unobtrusive,

3) The tow bar length must be adjustable ***so*** vehicle spacing can be varied between **0.2** to 1.0 vehicle length,

4) The tow bar must remain horizontal during towing and must transmit moment free horizontal forces to the load cells.

There are no commercially available products to meet the above criteria; **thus,** a customized design of attachments and linkages are necessary. The triangular shaped tow bar design is broken down into three major components as shown figure 9a below, and Figure 9b next page. First are the attachments to the towing vehicle, second is the tow bar linkage itself, and third are the attachments to the trailing vehicle. All of these fixtures are fabricated in-house at the University of Southern California Engineering Machine Shop.

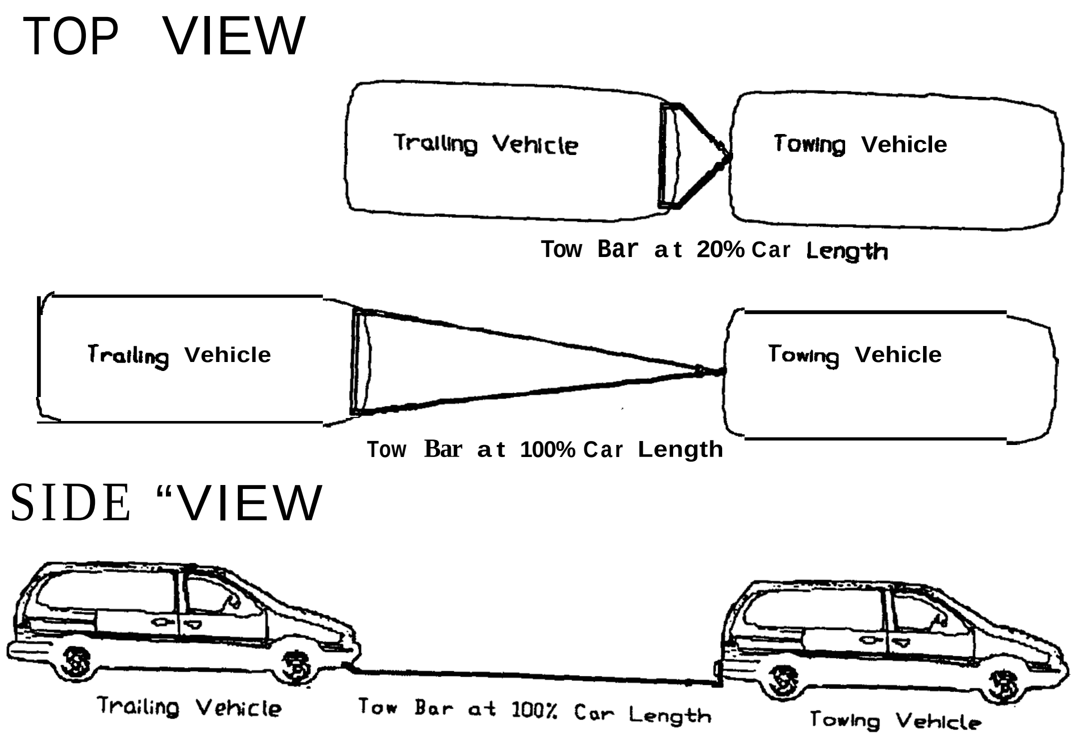


Figure 9a: Overall road test configuration

The attachment to the towing vehicle utilizes a U-shaped frame, bolted to the chassis **of** the towing Windstar. At the center of the frame, there is a tow pin for a hitch that connects the tow bar linkages to the towing car. The hitch is height adjustable to ensure that the linkages remain horizontal. The connection point for the tow bar is moved inwards underneath the towing car **as** far **as** possible to meet the minimum spacing requirement of 0.2 vehicle length.

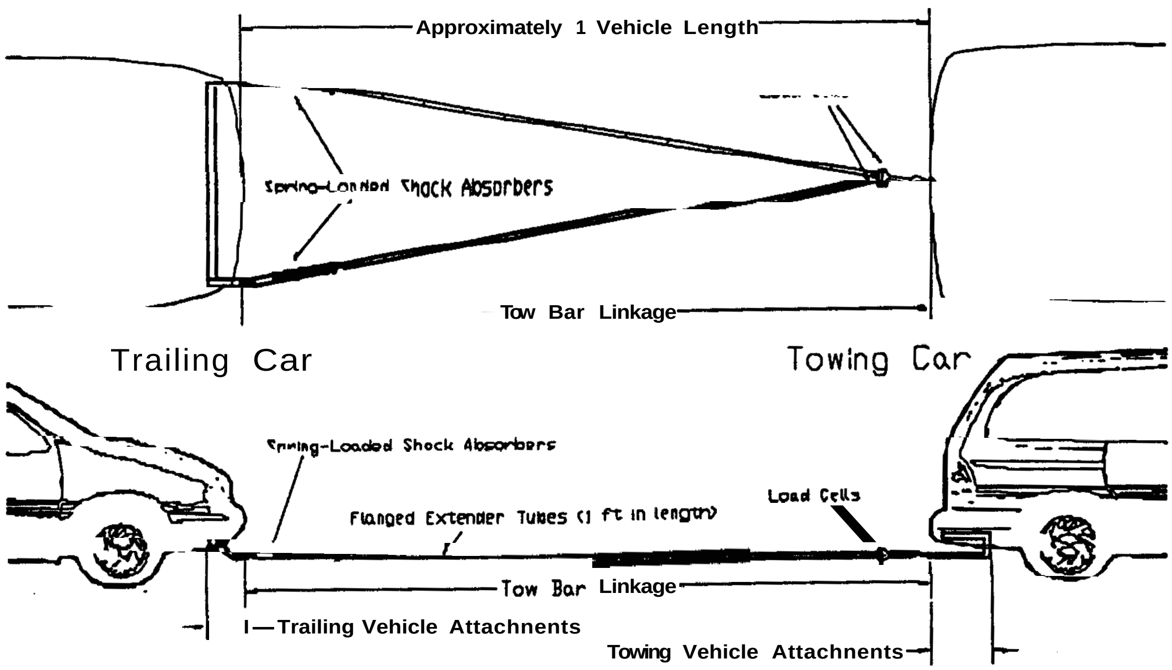


Figure 9b: Overall tow bar configuration

The first link in the tow bar contains the two load cells (one on each side). These links are connected to the hitch through a spherical bearing to ensure that no unwanted moments are transmitted to the load cells. The final links to the trailing car contain the spring-loaded shock absorbers. The tow bar ***extender linkages*** are simple flanged tubes one foot in length that can be connected or disconnected according to the desired spacing between the vehicles. The tubes are able to withstand a buckling load of 300 lbf, resulting in a total buckling threshold (both legs) of 600 lbf. The buckling threshold is a factor of 3.3 greater than the maximum expected towing load.

The trailing car attachments are designed to be easily installed on the Windstar. The trailing support tube runs the width of the car, in front of the bumper, and is supported by brackets that are connected to the chassis just below the front bumper. The tube supports a mounting fixture for direct connection on either end (through spherical bearings) to the final tow bar linkage. The tube support design also for lateral adjustment of the tow bar linkage. The tow bar extender links and attachments to the vehicles are secured by standard, commercially available bolts. The connections between the three major components of the tow bar employ removable steel pins for easy installation. (See appendix B for the detailed drawings of the complete tow bar design.)

Calibration

The tow bar (load cells, spring-loaded shock absorbers, and the first and final tow bar linkages) is calibrated by mounting the tow bar in a vertical position within a secure frame. Vertical loads are applied by means of calibrated weights. (See appendix B for detail design.) The calibration cart is a simple rectangular frame structure that provides attachments for the tow bar and dead weight loading up to several hundred pounds. A trailing support tube identical to the tube mounted on the trailing car is mounted permanently on the top of the calibration cart. Loads are applied by means of a cable and pulley arrangement. (See Figure 10 on next page.)

A static calibration procedure is employed in order to establish the force response of the load cells-tow bar system. A dynamic calibration procedure using smaller weights is **also** employed in order to determine the frequency response of the system. Both procedures are performed using a PC laptop running the LabView Data Acquisition software package. The programs (written by P. Hong) will determine the relationship between the output signals from the load cells and the corresponding load on the tow bar. (See appendix C for a sample of the program.)

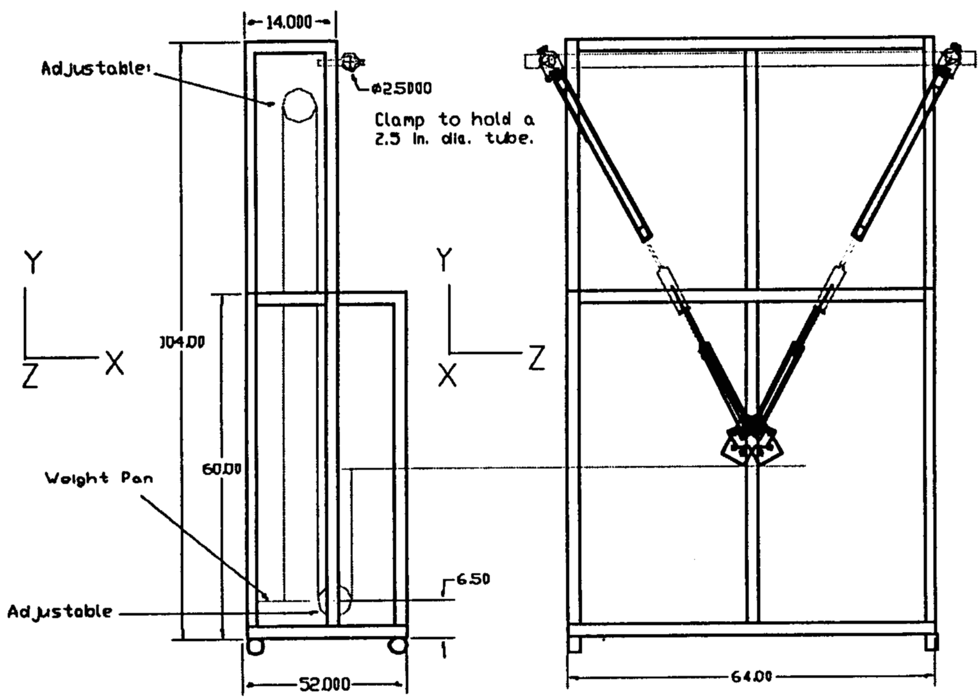


Figure 10: Calibration Cart Configuration

References

[1] Zabat, M., Stabile, N., Frascaroli, S. & Browand, F.; 1995; "Drag Forces Experienced by 2, 3 & 4-Vehicle Platoons at Close Spacings", SAE Paper No. 950632

APPENDIX A

Derivation of Longitudinal Stability Model

The model is based on Figures 1 and 2 and starts with a force balance using Newton's second law.

$$M_1 \ddot{X}_1 = \sum F_1 = F_{P1} - F_{R1} - D_1 + F_{1TB} \tag{1}$$

$$M_2 \ddot{X}_2 = \sum F_2 = -F_{R2} - D_2 + F_{2TB} \tag{2}$$

where M_1 and M_2 are the masses of each vehicle respectively, F„ is the propulsive force of the lead vehicle, F_{R1} and F_{R2} are the rolling resistances of the vehicles, D_1 and D_2 are the drag terms, and F„ and F_{2TB} are the forces on vehicles 1 and 2 due to the tow bar, assumed to act in the positive x-direction. A force balance of the tow bar gives (assuming the tow bar has negligible mass)

$$F_{1TB} + F_{2TB} = 0 \quad \Rightarrow \quad F_{2TB} = -F_{1TB}.$$

Let the reference point and the space between the vehicles be defined as follows.

$$X_{/} - \frac{X_{/} + X_{/}}{2}$$

$$\Delta X = X_1 - X_{/}$$

where X, and X, are the distances of the vehicles from a reference line. The reference point $X_{/}$ is defined as the point located in between the two vehicles at the middle of the distance between the their centers of gravity .

Rearrange the definitions to get:

$$X_{/} = X_{/} + \frac{Ax}{2}, \quad X_{/} = X_o - \frac{Ax}{2}$$

Then,

$$M_1 \ddot{X}_o + \frac{M_1}{2} \Delta \ddot{X} = (RHS)_1$$

$$M_2 \ddot{X}_o - \frac{M_2}{2} \Delta \ddot{X} = (RHS)_2$$

Let $M_1 = M_2 = M$. This is true, and it simplifies the analysis as follows.

$$M\ddot{X}_o + \frac{M}{2} \Delta \ddot{X} = F_{P1} - F_{R1} - D_1 + F_{1TB} \tag{3}$$

$$M\ddot{X}_o - \frac{M}{2} \Delta \ddot{X} = -F_{R2} - D_2 - F_{1TB} \tag{4}$$

Adding and subtracting the above equations yields:

add: $$2M\ddot{X}_o = F_{P1} - F_{R1} - F_{R2} - D_1 - 4 \qquad (5)$$

subtract: $$MAX = F_{P1} - F_{R1} + F_{R2} - D_1 + 4 + 2F_{1TB} \qquad (6)$$

These two equations are coupled through the drag terms D, and D_2. The drag terms can be modeled **as:**

$$D_1 = k_1\dot{X}_1^2 \text{ and } D_2 = k_2\dot{X}_2^2$$

where **k,** and k_2 are shorthand for $c_{d_1}\frac{\rho}{2}S$ and $c_{d_2}\frac{\rho}{2}S$ (which depended upon AX). Here, c_d stands for the aerodynamic drag coefficient for each of the vehicles, ρ is the air density, and S is the frontal area of the vehicle's body., Utilizing the definitions for X, and $\mathbf{X_2}$ give the following expressions.

$$X_1 = X_o + \frac{\Delta X}{2} \qquad X_2 = X_o - \frac{AX}{2}$$

$$\dot{X}_1 = \dot{X}_o + \frac{\Delta\dot{X}}{2} \qquad \dot{X}_2 = \dot{X}_o - \frac{\Delta\dot{X}}{2}$$

$$\dot{X}_1^2 = \dot{X}_o + \dot{X}_o AX + \frac{\Delta\dot{X}^2}{4} \qquad \dot{X}_2 = \dot{X}_o^2 + \dot{X}_o AX + \frac{\Delta\dot{X}^2}{4}\ .$$

Substitute the drag terms into equations 5 and ***6***.

$$2M\ddot{X}_o = F_{P1} - F_{R1} - F_{R2} - k_1\left[\dot{X}_o^2 + \dot{X}_o\Delta\dot{X} + \frac{\Delta\dot{X}^2}{4}\right] - k_2\left[\dot{X}_o^2 + \dot{X}_o\Delta\dot{X} + \frac{\Delta\dot{X}^2}{4}\right] \qquad (7)$$

$$M\Delta\ddot{X} = F_{P1} - F_{R1} + F_{R2} - k_1\left[\dot{X}_o^2 + \dot{X}_o\Delta\dot{X} + \frac{\Delta\dot{X}^2}{4}\right] + k_2\left[\dot{X}_o^2 + \dot{X}_o\Delta\dot{X} + \frac{\Delta\dot{X}^2}{4}\right] + 2F_{1TB} \qquad (8)$$

The state of equilibrium in the platoon motion is defined **as** the state when the reference point X_0 is moving with the constant velocity U_0 (i.e. the platoon moves with constant velocity under a constant traction force F_{P1}) while the distance ΔX is constant and corresponds to the spacing between the cars where the aerodynamic drag forces experienced by the two vehicles are equal. This state of equilibrium is perturbed by small fluctuations in the speed **of** the reference point $\dot{X}_o$ **,** the spacing ΔX**,** and in the propulsive force F_P. These two equation can therefore be linearized about the equilibrium state:

$$\dot{X}_0 = U_0 + \dot{X}'_0 \qquad \dot{X}' << U_0$$

$$\Delta\dot{X}' = \Delta\dot{X}' \qquad \Delta\dot{X}' << U_0$$

$$F_{P1} = \overline{F}_{P1} + F'_{P1}$$

where U_o is the average velocity of the platoon. Here, the terms $\dot{X}'$, $\Delta\dot{X}'$, F'_P denominate fluctuating terms. Introducing the above expressions into (7) and **(8)** one can expand and separate the terms of the two equations according to their order of magnitude. The lowest order equation is:

$$0 = \overline{F}_{P1} - F_{R1} - F_{R2} - k_1 U_o^2 - k_2 U_o^2$$

or

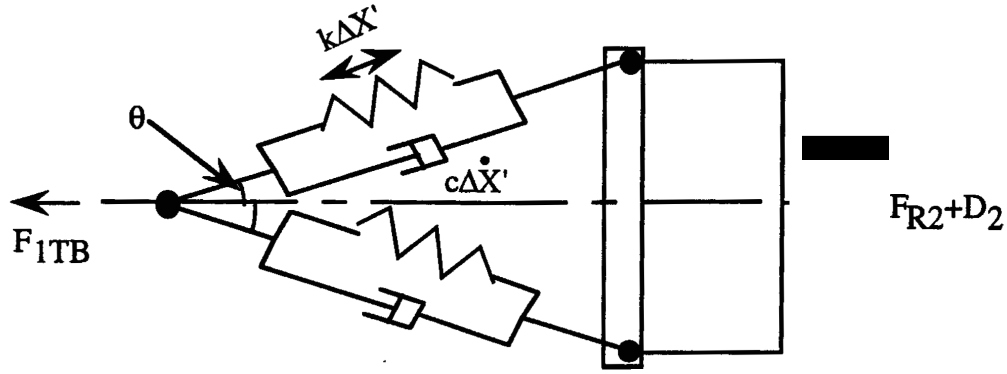
$$\overline{F}_{P1} = F_{R1} + F_{R2} + k_1 U_o^2 + k_2 U_o^2$$

Thus the propulsive force is the summation of the total rolling resistance and the total drag at U_o. The next order is:

$$2M\ddot{X}_o{}' + 2U_o(k_1 + k_2)\dot{X}_o{}' = F_{P1}{}' + U_o(k_2 - k_1)\Delta\dot{X} \qquad (9)$$

$$M\Delta\ddot{X} = F_{P1}{}' + 2F_{R2} + 2k_2U_o^2 + 2U_o(k_2 - k_1)\dot{X}_o{}' - U_o(k_1 + k_2)\Delta\dot{X} + 2F_{1TB} \qquad (10)$$

Now define the vehicle separation ΔX to be the **sum of** a fixed portion ΔX_F, a smaller equilibrium spring extension AX, and a fluctuating spring extension AX. Remember **also** that k_1 and $\mathbf{k_2}$ are functions **of** ΔX_F



The tow bar force can be modeled as a spring-damper as sketched.

$$F_{1TB} = \left[2k_{TB}\Delta X + 2c_{TB}\Delta \dot{X} \right] \cos\theta \cdot$$

Thus, substituting back into equation (10)

$$M\Delta X = F_{P1}' + 2F_{R2} + 2k_2U_o^2 + 2U_o(k_2 - k_1)\dot{X}_o' - U_o(k_1 + k_2)\Delta \dot{X} - \left[4k_{TB}\left(\Delta X_{EQ} + \Delta X'\right) + 4c_{TB}\Delta \dot{X}' \right] \cos\theta \quad (11)$$

The equilibrium extension is given by:

$$\Delta \ddot{X}' = \Delta X' \equiv 0 \quad \Delta X' = 0 \qquad F_{r1}' = 0$$

which yields

$$2F_{R2} + 2k_2U_o^2 - 2\left[2k_{TB}\Delta X_{EQ}\right]\cos\theta = 0. \qquad (12)$$

Equation (12) can be solved for the equilibrium force in each arm of the tow bar.

$$k_{TB}\Delta X_{EQ} = \frac{F_{R2} + k_2U_o^2}{2\cos\theta} \qquad (13)$$

The rolling resistance can be approximated as:

$F_{R2} = r_o Mg$ where $r_o \cong 0.012$ and $M = 120.0$ slugs

The drag is a function of the velocity and the vehicle spacing while θ is a function of only the vehicle spacing.

$$D = D(U_o, \Delta X_F)$$
$$\theta = \theta(\Delta X_F)$$

The quantity $k_{TB}\Delta X_{EQ}$ has been calculated and is presented in Figure **3**. With (12), equations (9), (10) simplify to:

$$2M\ddot{X}_o' + 2U_o(k_1 + k_2)\dot{X}_o' = F_{P1}' + U_o(k_2 - k_1)\Delta\ddot{X}' \tag{14}$$

$$M\Delta\ddot{X}' + [4c_{TB}\cos\theta + U_o(k_1 + k_2)]\Delta\dot{X}' + 4k_{TB}\cos\theta\Delta X' = F_{P1}' + 2U_o(k_2 - k_1)\dot{X}_o' \tag{15}$$

These are two linear coupled equations for $X,'$, the fluctuation of average velocity(of the two vehicles) and for $\Delta X'$ the fluctuating axial spring extension between the two vehicles. The coupling term $-(k_2 - k_1)$ is small. We will consider equation (15) in this limit:

$$M\Delta\ddot{X}' + [4c_{TB}\cos\theta + U_o(k_1 + k_2)]\Delta\dot{X}' + 4k_{TB}\cos\theta\Delta X' = F_{P1}' = A\cos\omega t \tag{16}$$

where $F_{P1}' = A\cos\omega t$ is any small perturbation. Dividing by M gives

$$\Delta\ddot{X}' + \left[\frac{c_{TB}\cos\theta}{M/4} + \frac{U_o(k_1 + k_2)/4}{M/4}\right]\Delta\dot{X}' + \frac{k_{TB}}{M/4}\cos\theta\Delta X' = \frac{A}{M}\cos\omega t \tag{17}$$

which can be put in normal form by defining:

$$\omega_n^2 = \frac{k_{TB}}{M/4}\cos\theta \tag{18}$$

$$\zeta = \frac{[c_{TB}\cos\theta + U_o/4(k_1 + k_2)]/2}{\sqrt{K_{TB}M/4}} \tag{19}$$

Then,

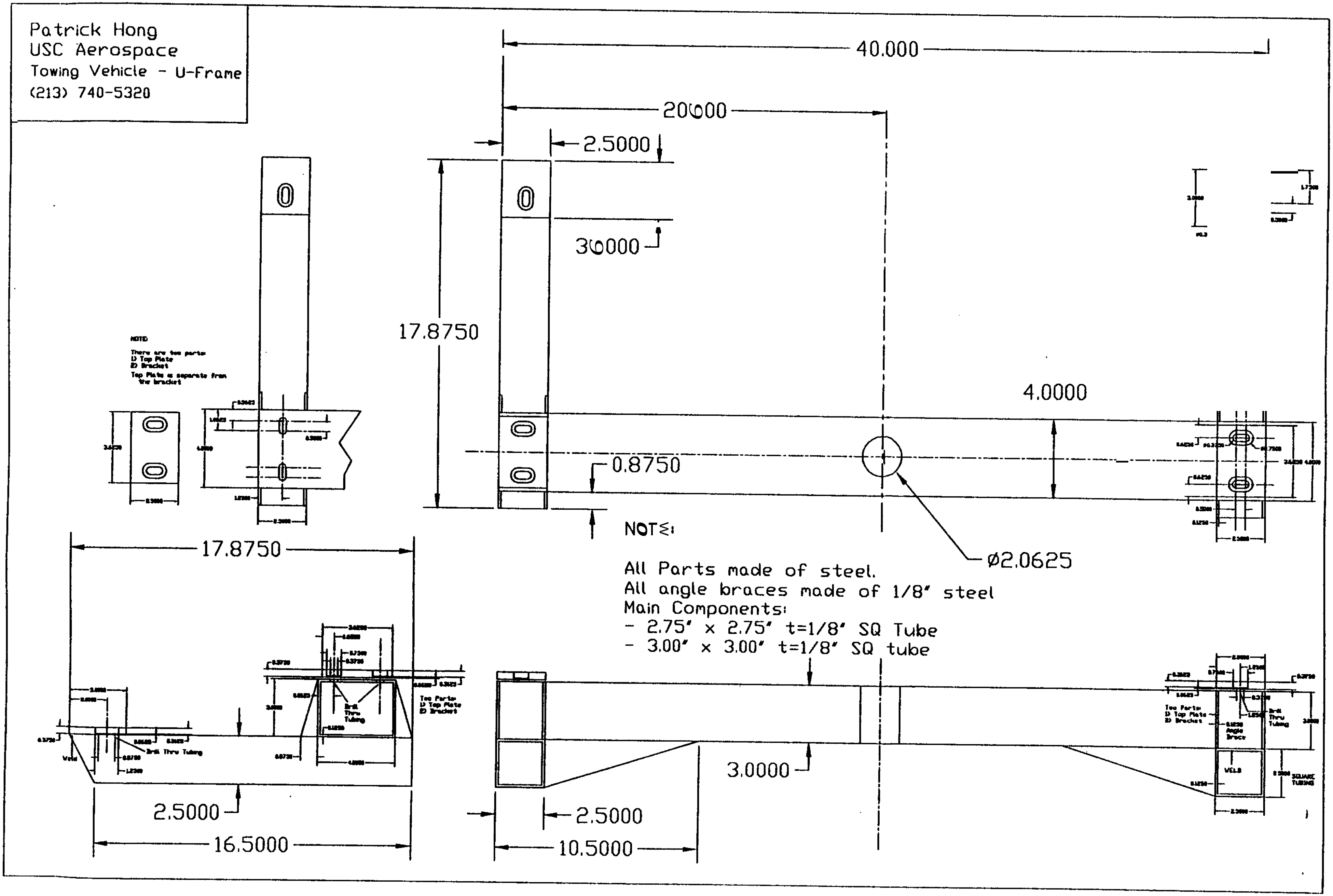
$$\Delta\ddot{X}' + 2\zeta\omega_n\Delta\dot{X}' + \omega_n^2\Delta X' = \frac{A}{M}\cos\omega t \tag{20}$$

ω_n is the undamped natural frequency, and ζ is the damping ratio.

APPENDIX B

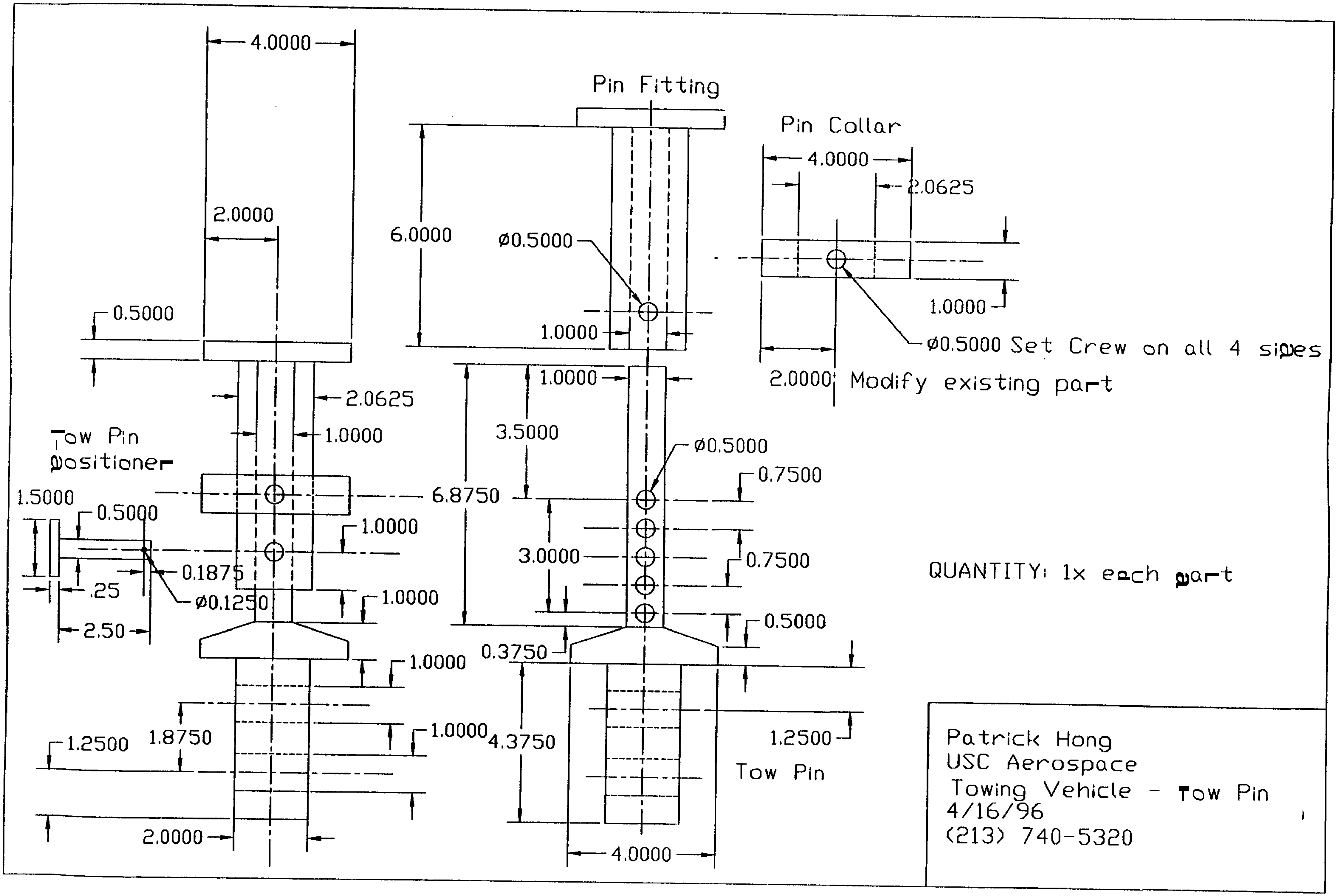
Assembly Drawing of the Tow Bar

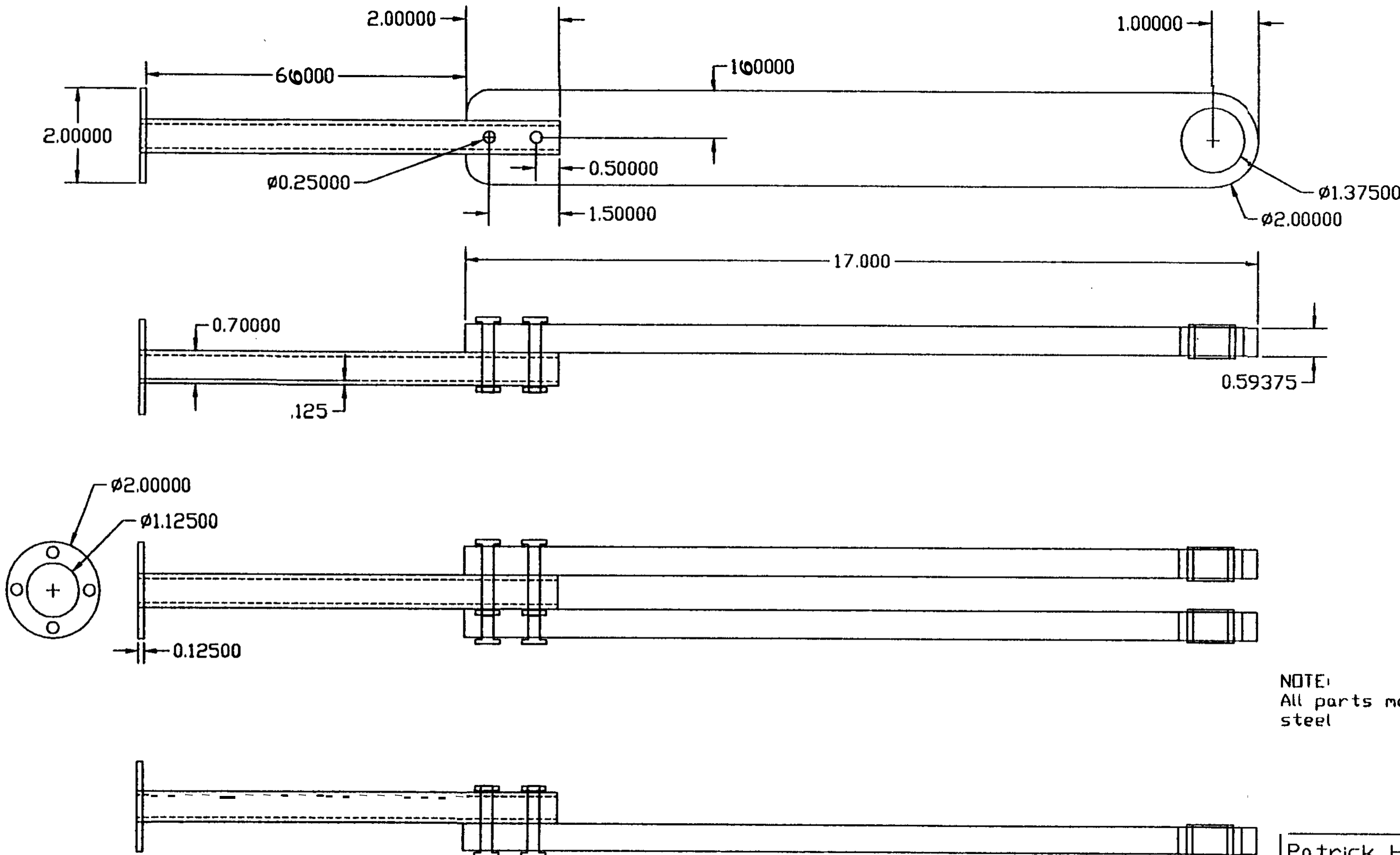
Patrick Hong
USC Aerospace
Towing Vehicle - U-Frame
(213) 740-5320



NOTES:

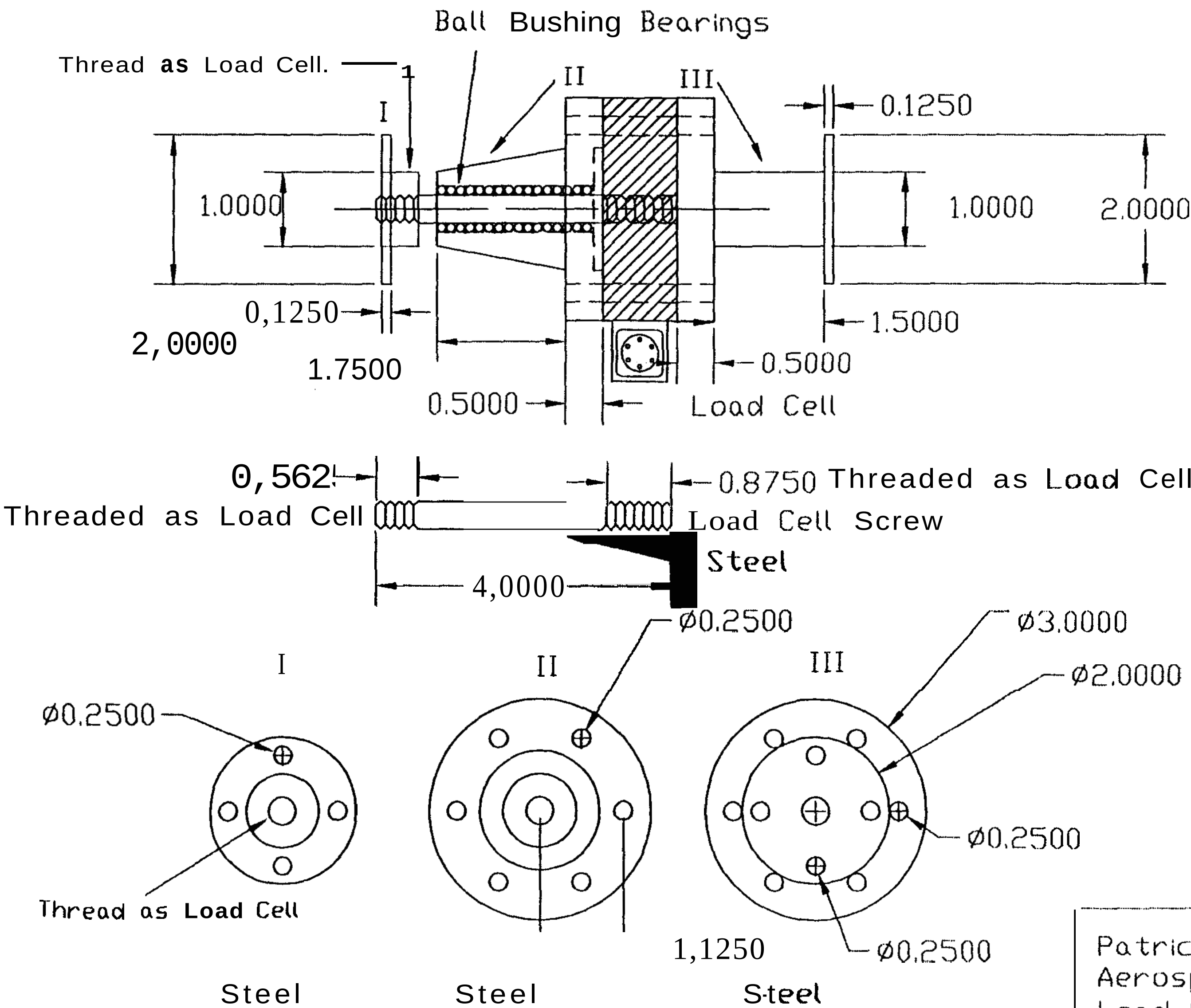
All Parts made of steel.
All angle braces made of 1/8" steel
Main Components:
- 2.75" x 2.75" t=1/8" SQ Tube
- 3.00" x 3.00" t=1/8" SQ tube


Pin Fitting
Pin Collar
ø0.5000 Set Crew on all 4 sides
Modify existing part
Tow Pin Positioner
Tow Pin
QUANTITY: 1x each part
Patrick Hong
USC Aerospace
Towing Vehicle - Tow Pin
4/16/96
(213) 740-5320



NOTE:
All parts made of
steel

Patrick Hong
USC Aerospace
Towing Vehicle-
Hitch Scissors
12/18/95



Load Cell Holder

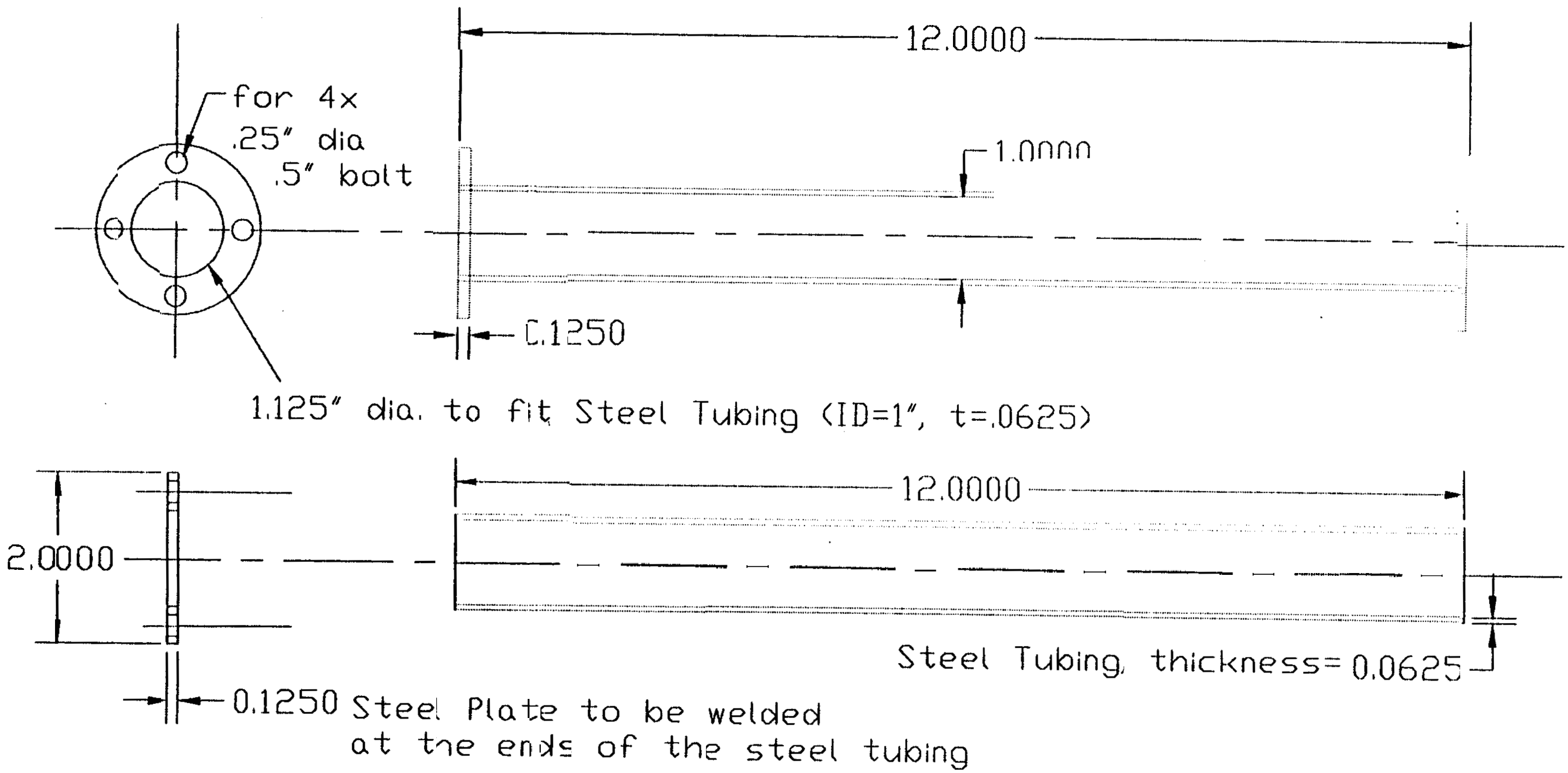
I = LC Connector transmits forces through the load cell screw to the load cell. Phis piece is threaded as the load cell and load cell screw. 4 holes are for bolts to connect 'to towbar sections.

II = LC Bushlng Holder contains ball bushing bearings to properly align the load cell screw with the load cell. the holder bolts to the load cell with matching load cell holes.

III = LC Hitch Holder connects the towbar and load cell to the hitch on the towing cur. The holder mounts to the load cell like the LC Bushing Holder, and mounts to the hitch like the LC Connector to the towbar sections.

Load Cell is purchased from a commerical manufacturer

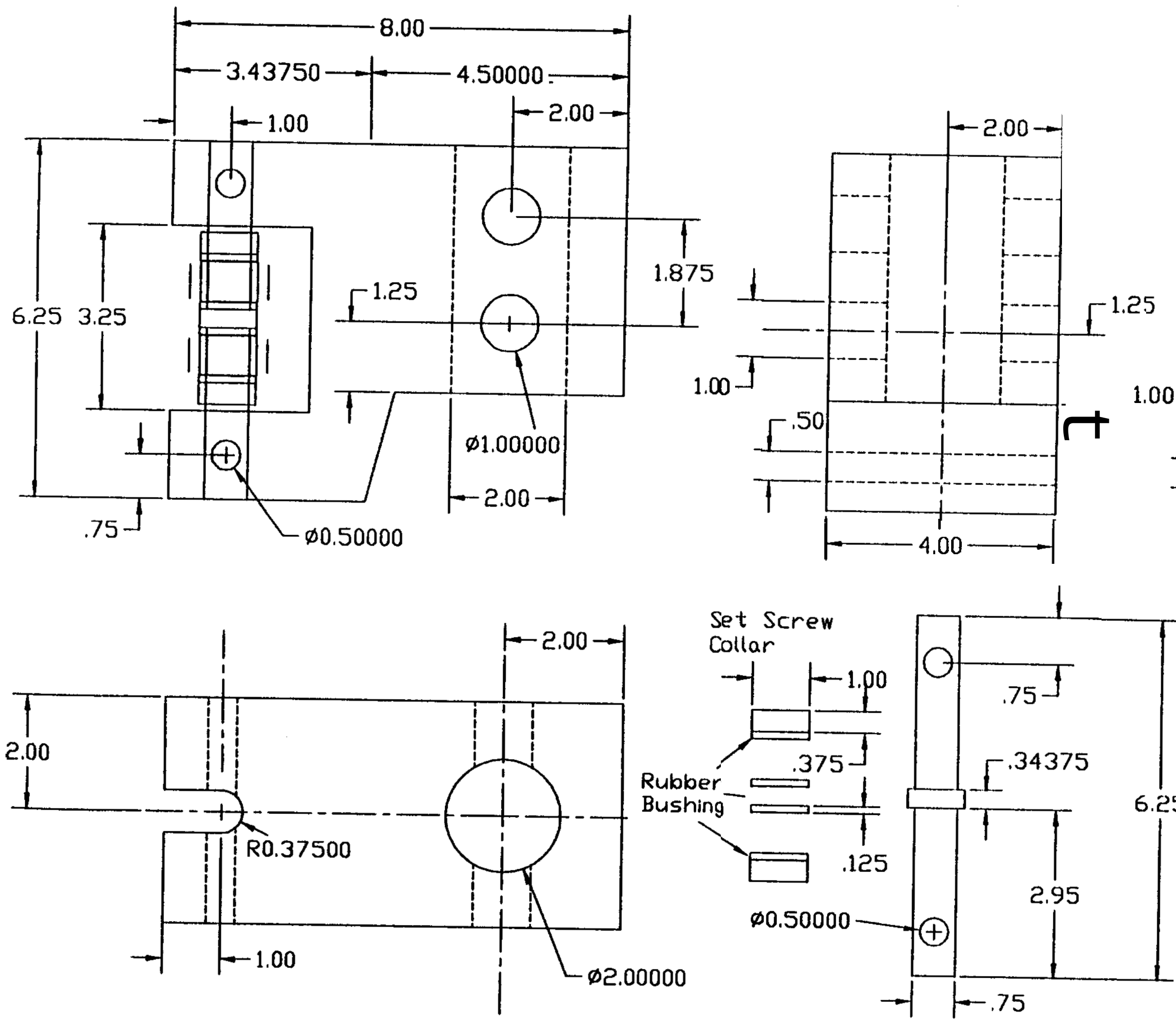
Patrick Hong
Aerospace
Load Cell Support
(213) 740-5320 10/16/95

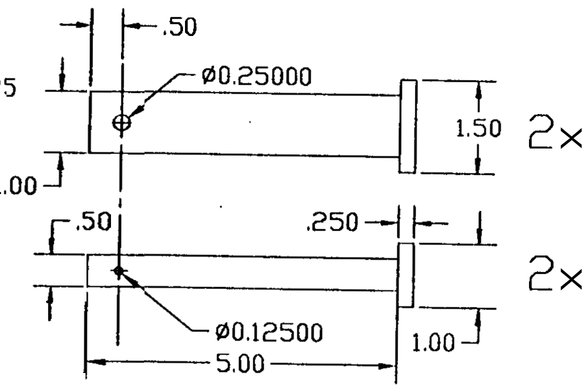


NOTES:

* Fabricate 24 units of above design

Patrick Hong
Tow Bar Extender Tubes
USC Aerospace
7/31/95
(213) 740-5320





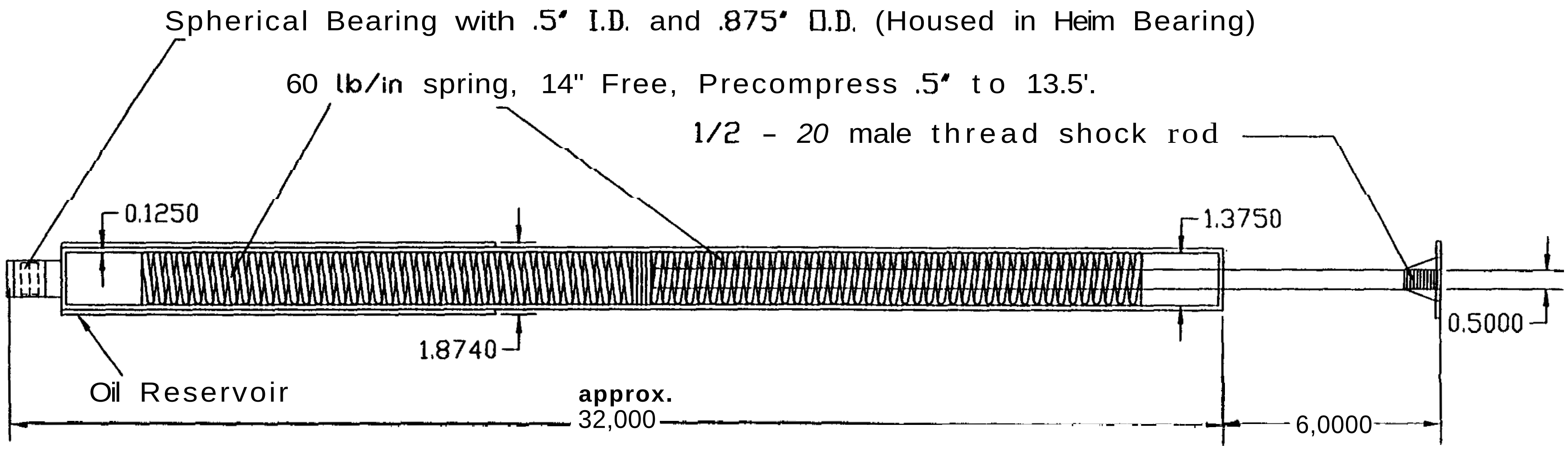
Shear Pin

Material:

Bracket = Steel
Hitch Pin = High Strength Steel
Shear Pin = High Strength Steel

*≡ Round all corners

Patrick Hong
USC Aerospace
Towing Vehicle - Hitch
(213) 740-5320
12/7/95

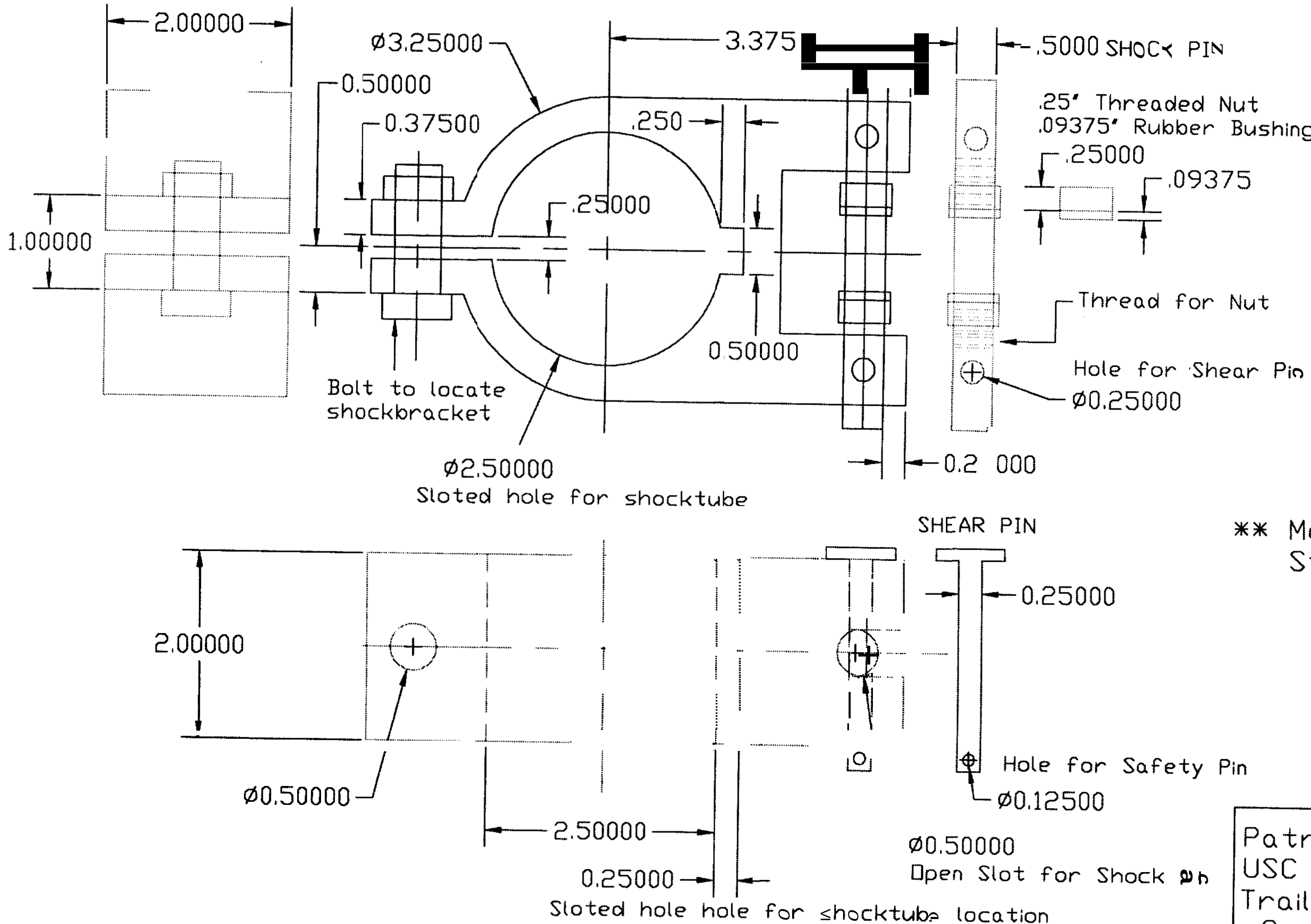


60 lb/in spring supplied by Works.
32 lb/in spring can be inserted:
1,359' O.D. / ,178" **wire**
10.4375' Free / 5.162" Solid
Precompress to 9.75'
Natural Freq: 0.57 Hz
Xeq at spacing 1, 70mph: 2.16'
Xeq at spacing 1, 40mph: 1.19'

Piston dim & design by Works,
Requiring damping: approx,
200 lb/ft/sec
in both directions

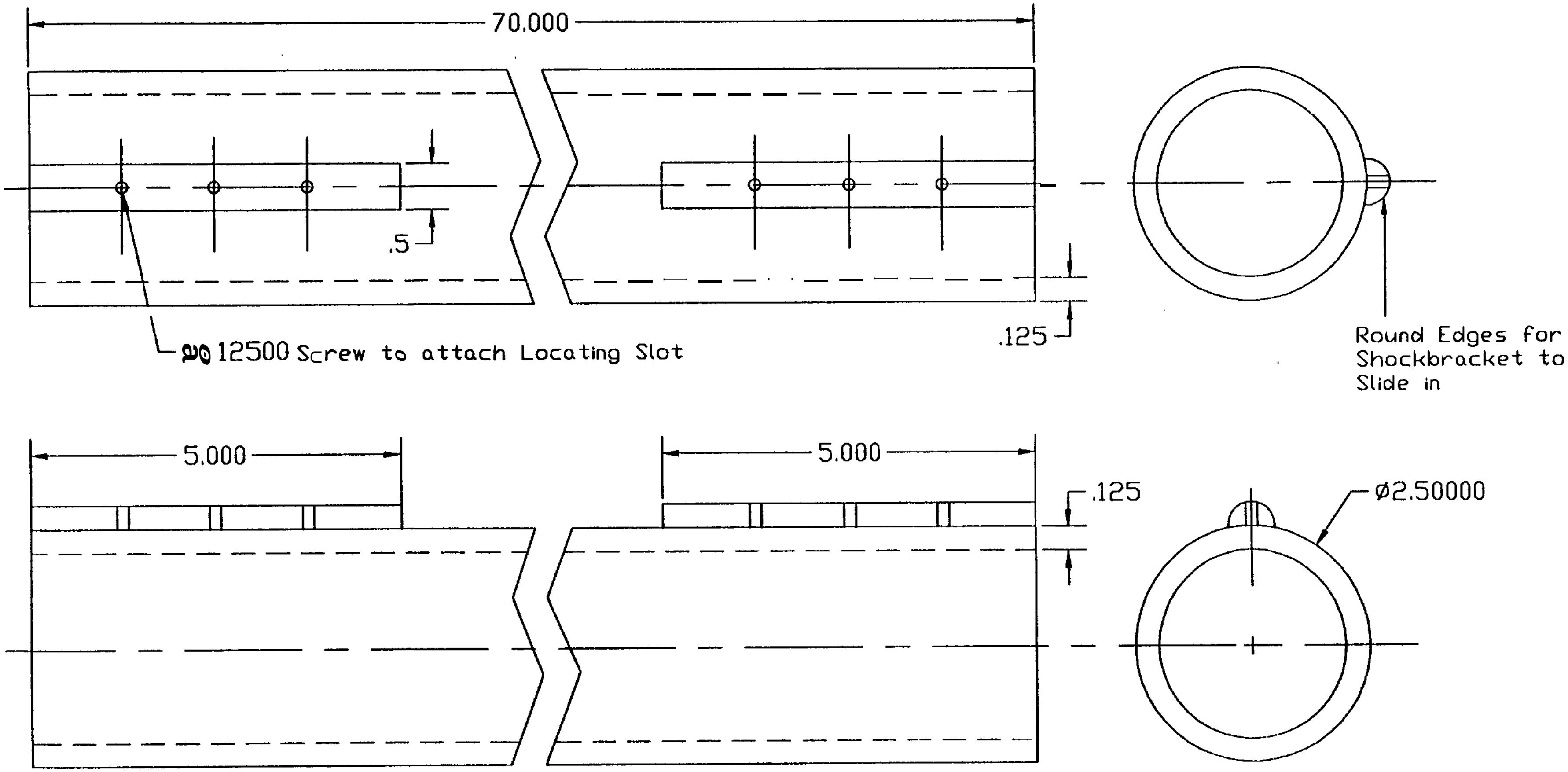
Shock Damper manufacutred by Works Performance Shocks:
Geilles Vaillancoort
8730 Shirley Ave
Northridge, CA 91324
TEL: (818) 701-1014
FAX: (818) 701-9043

Patrick Hong
USC Aerospace Engineering
Tow Bar Spring--Loaded Shock Absorber
7/14/95



** Materials -
Steel

Patrick Hong
USC Aerospace
Trailing Vehicle -
Spr/Shock Brckt
10/30/95



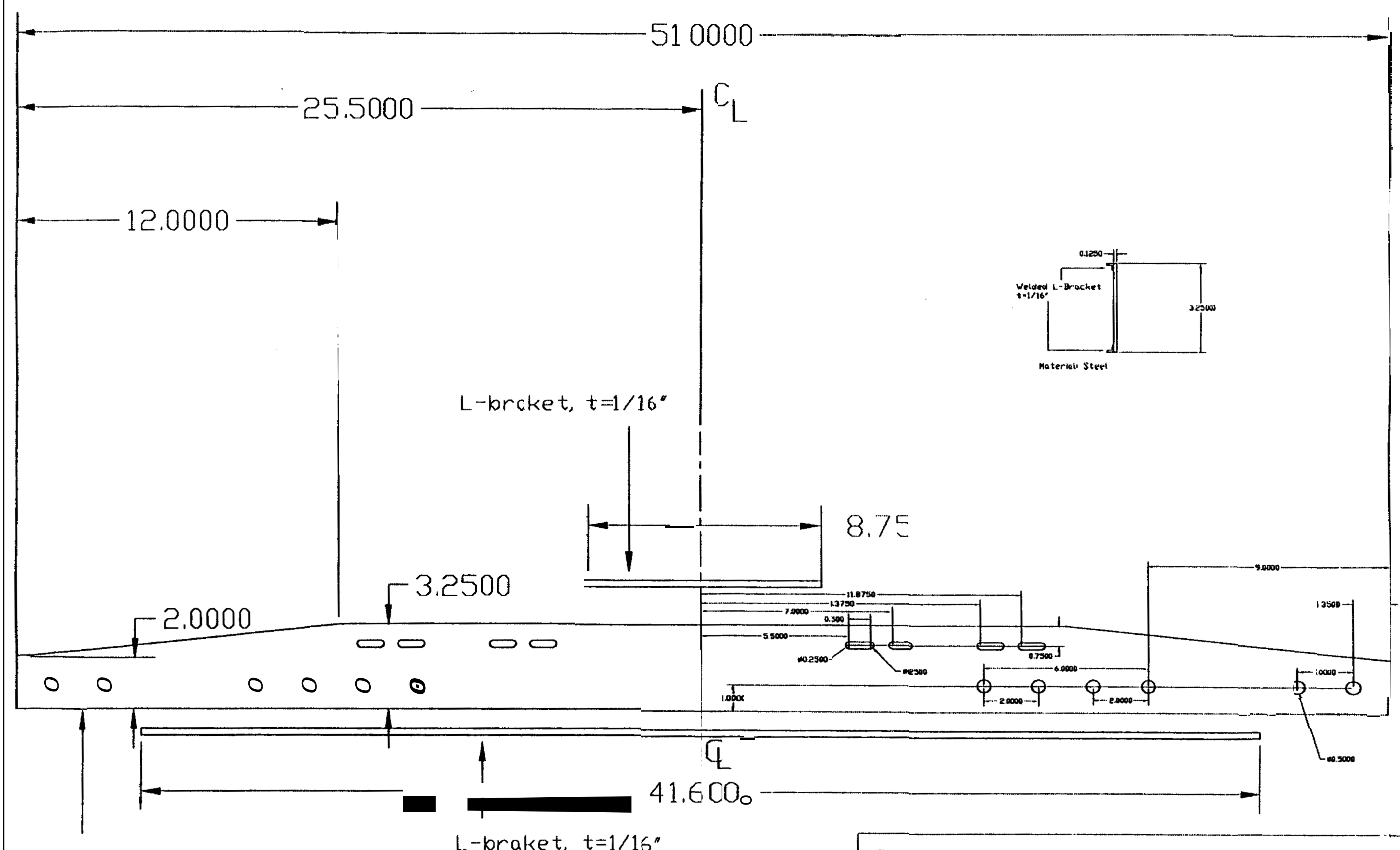
NOTES:

Total length of shocktube = 70".

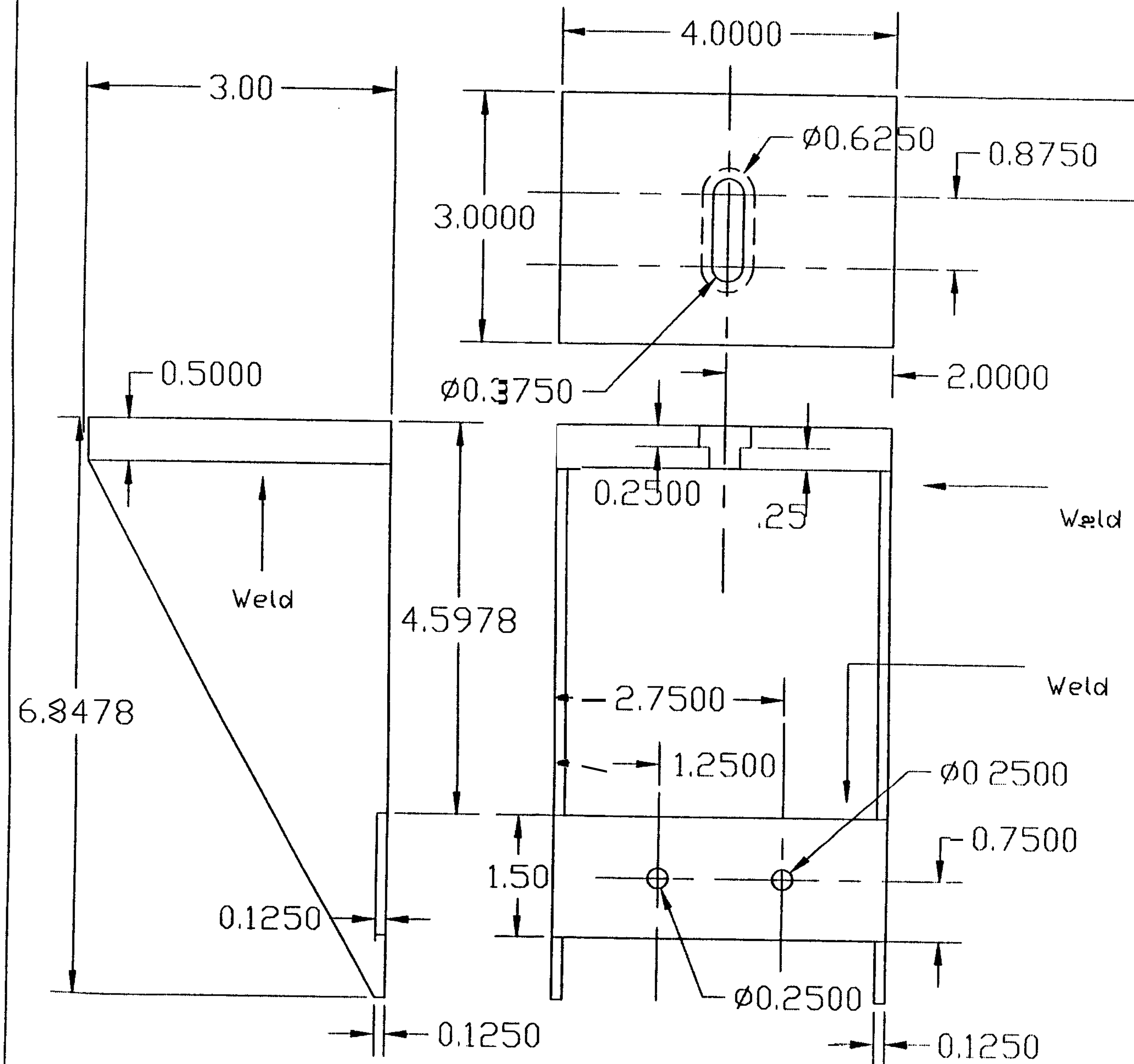
There are 10" of locating slot for shockbracket off each end of the shocktube.

The shocktube is made of t=.125" steel tubing

Patrick Hong   USC Aerospace
Trailing Vehicle - Support Tube
10/30/95

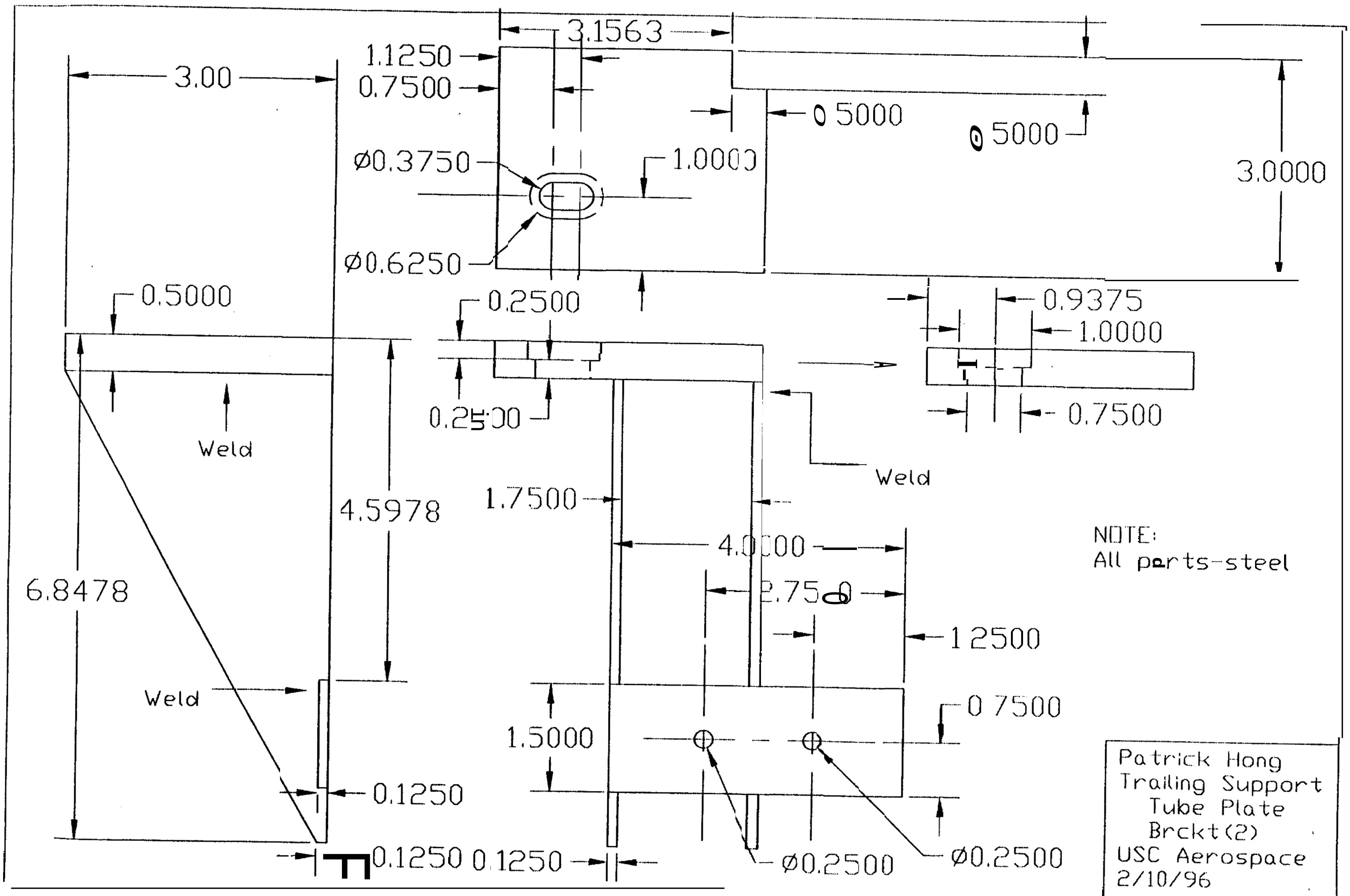


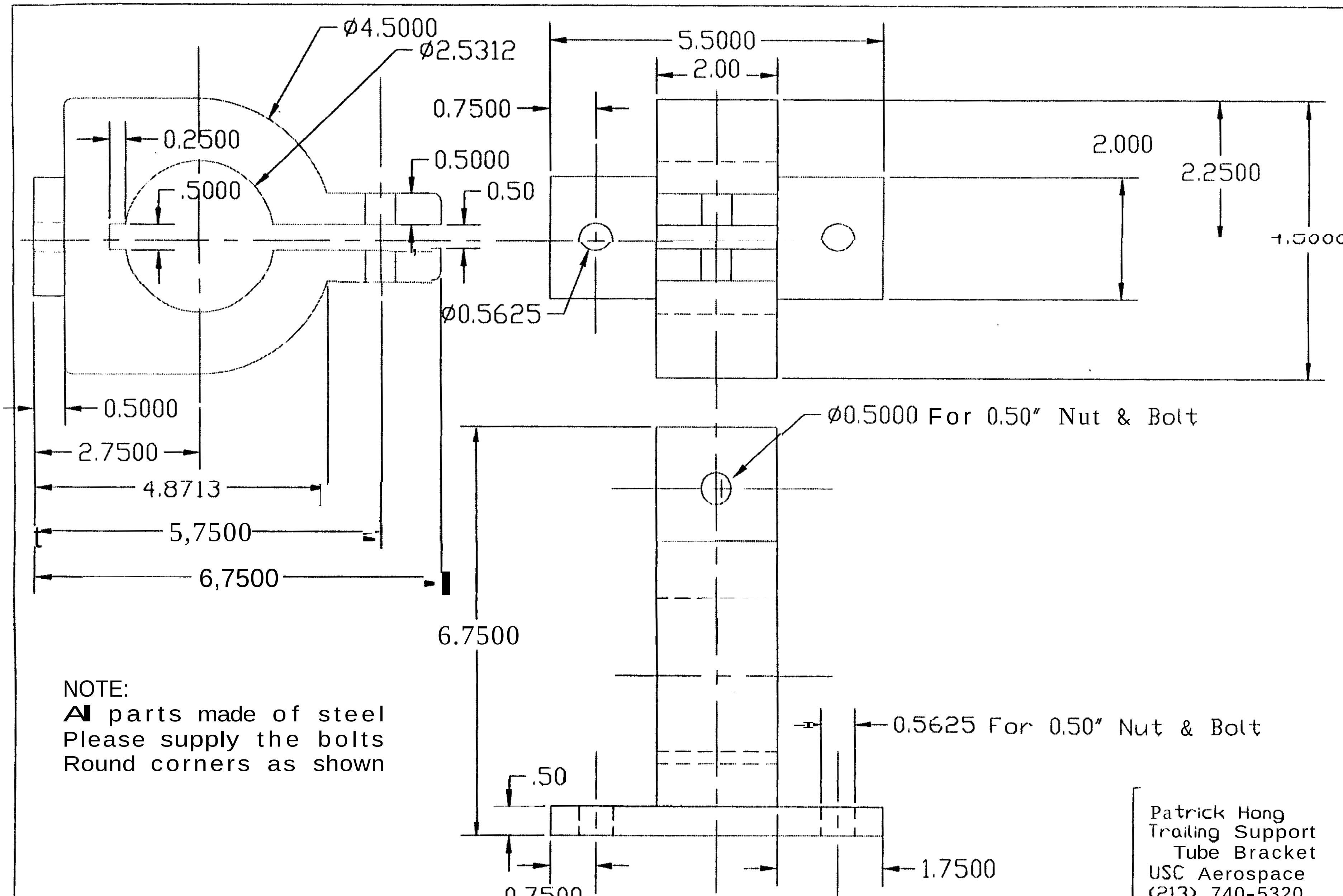
Patrick Hong, USC Aerospace
Trailing Support Tube Plate
2/10/96 (213) 740-5320



NOTE:
All parts - steel

Patrick Hong
Trailing Support
Tube Plate Brckts (1)
USC Aerospace
2/10/96
(213) 740-5320


3.1563
1.1250
0.7500
3.00
0.5000
0.5000
Ø0.3750
1.0000
3.0000
Ø0.6250
0.5000
0.2500
0.9375
1.0000
0.2500
0.7500
Weld
Weld
4.5978
1.7500
6.8478
4.0000
NOTE:
All parts-steel
2.7500
1.2500
Weld
0.7500
1.5000
0.1250
0.1250
0.1250
Ø0.2500
Ø0.2500
Patrick Hong
Trailing Support
Tube Plate
Brckt(2)
USC Aerospace
2/10/96



NOTE:
All parts made of steel
Please supply the bolts
Round corners as shown

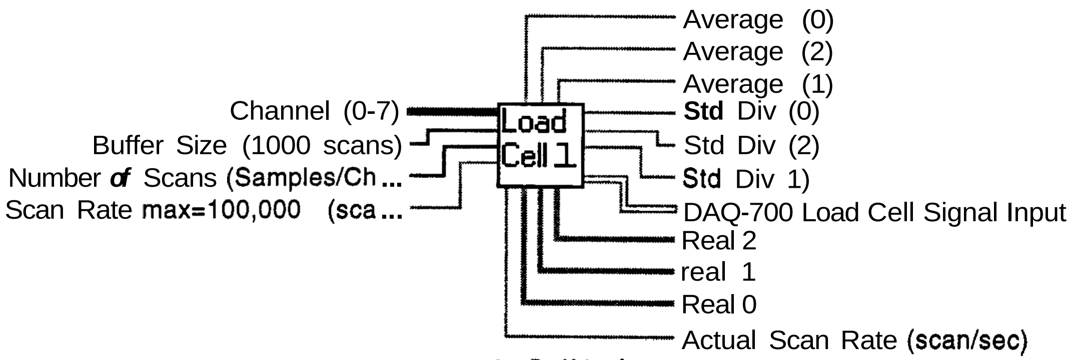
Patrick Hong
Trailing Support
Tube Bracket
USC Aerospace
(213) 740-5320

APPENDIX C

LabView Codes for the Tow Bar Data Acquisition

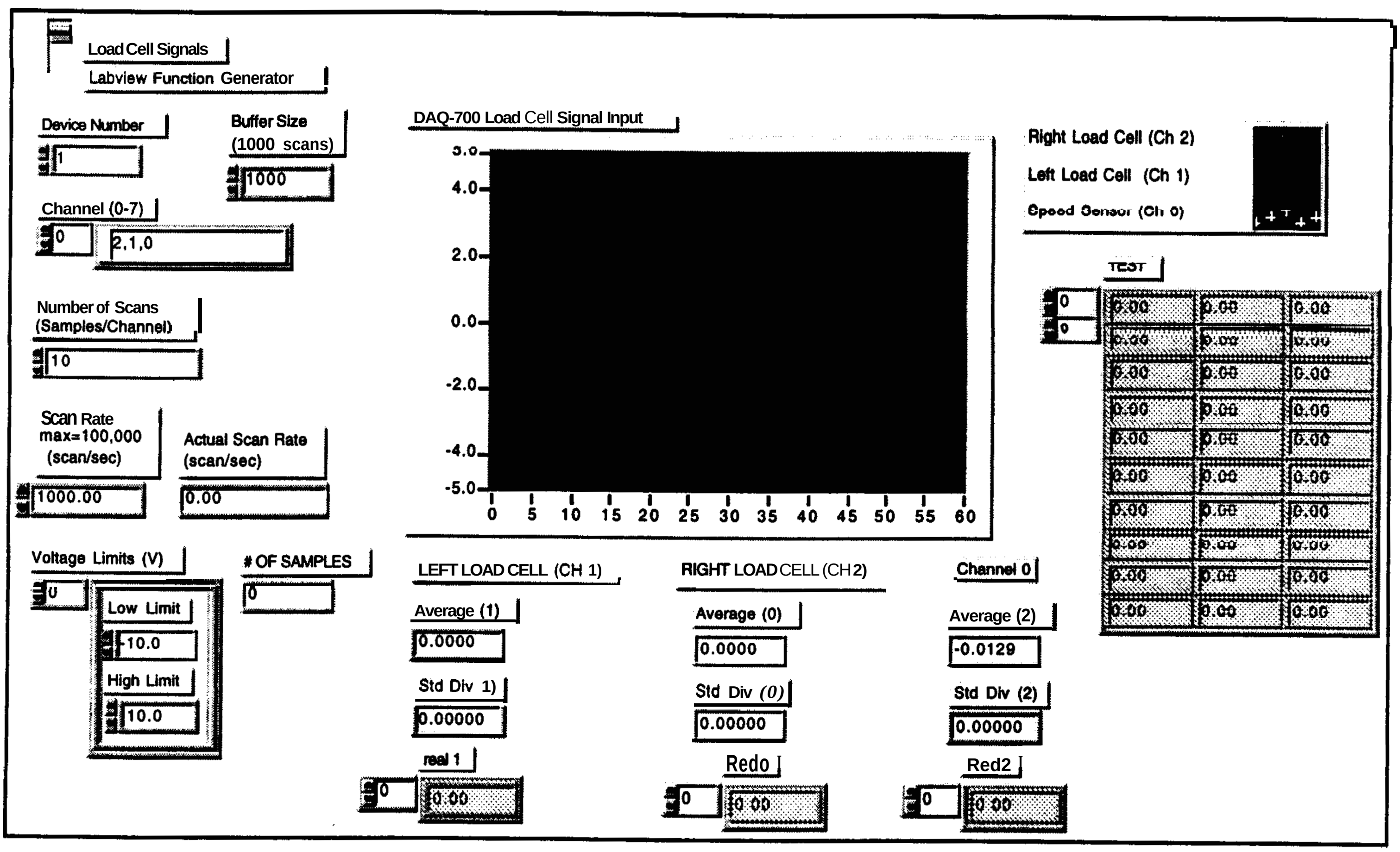
Load Cell1.vi
1/22/97 2:18 PM

Connector Pane

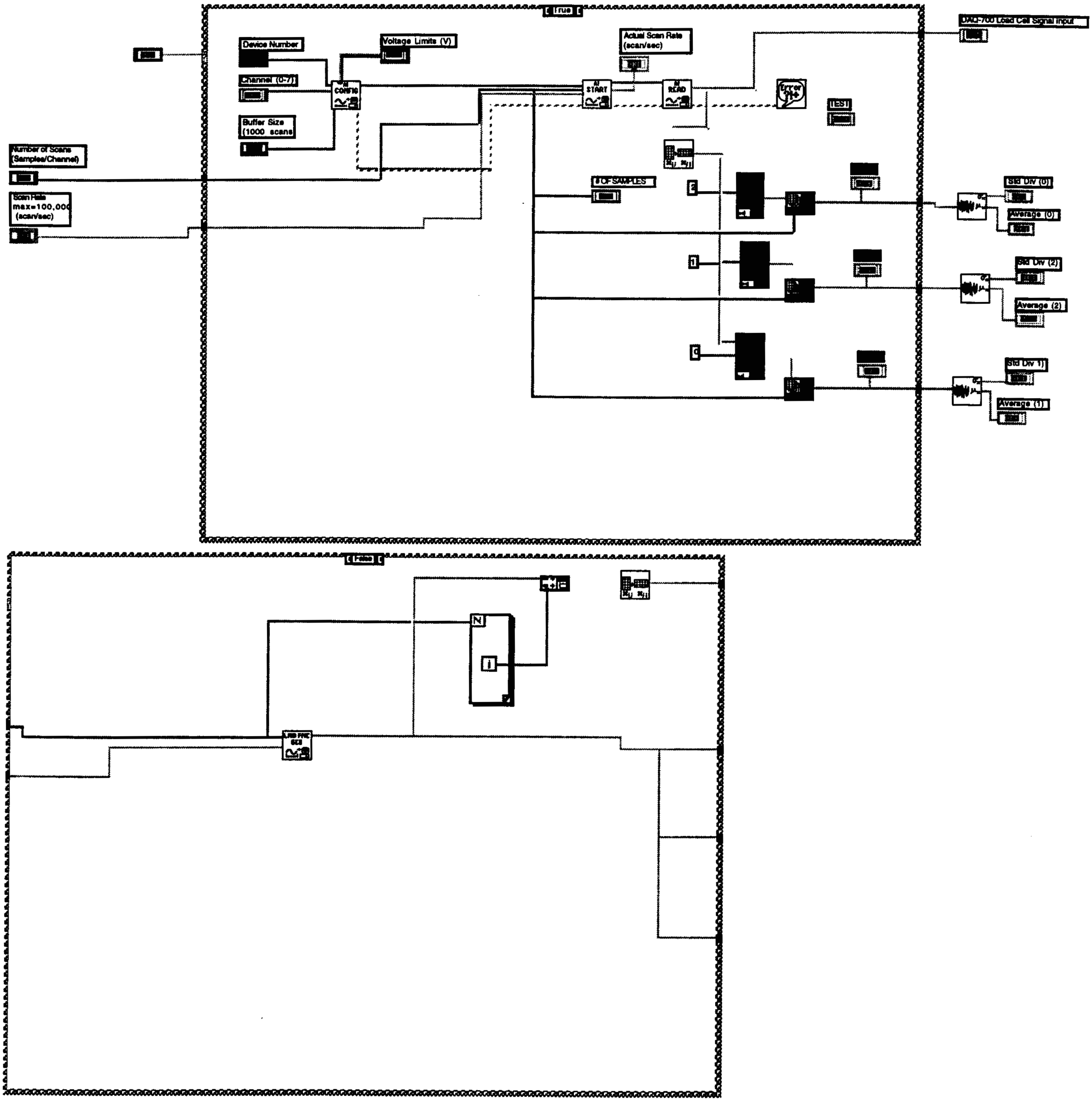


Load Cell1.vi

Front Panel



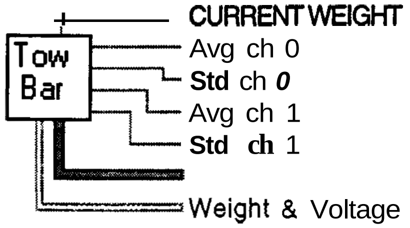
Block Diagram



APPENDIX C

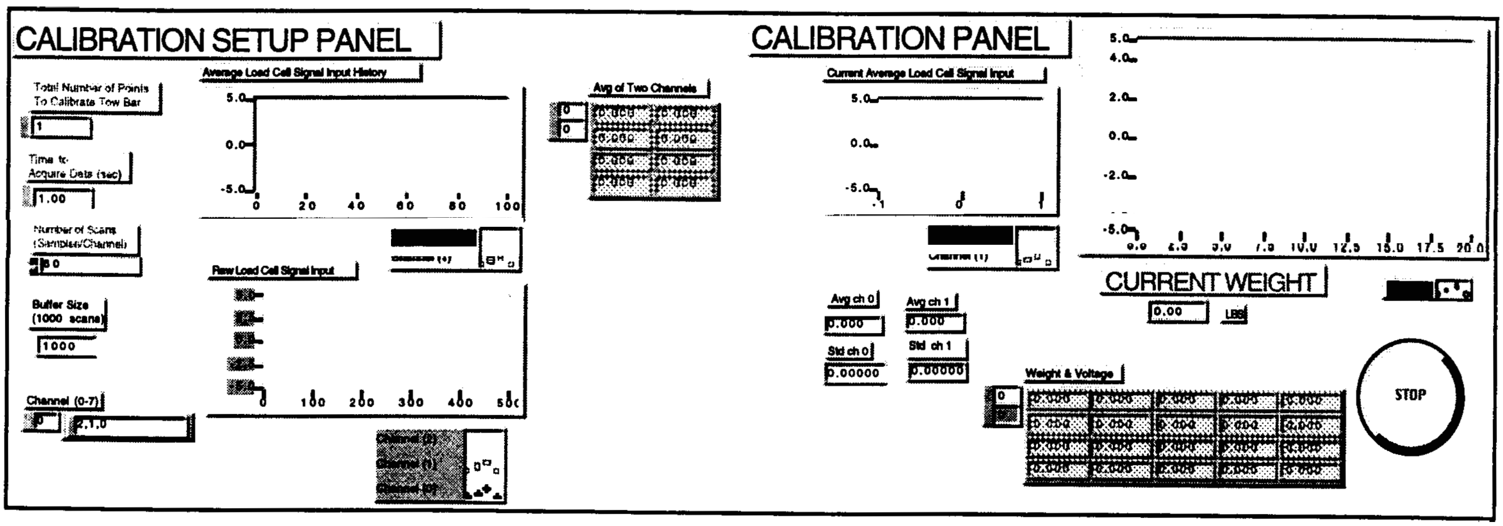
LabView Codes for the Tow Bar Data Aquisition

Connector Pane



Towbar Calib1.vi

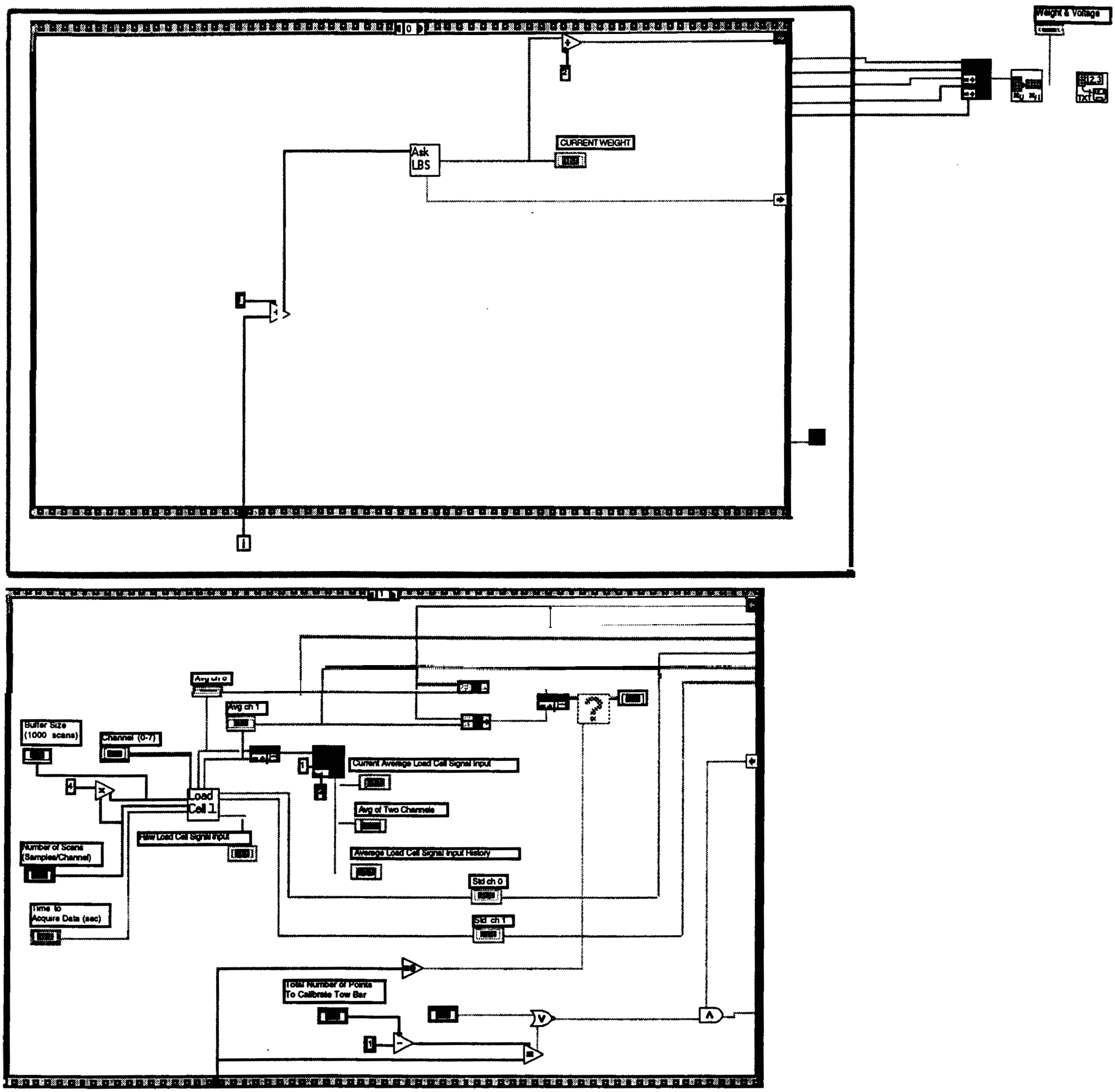
Front Panel



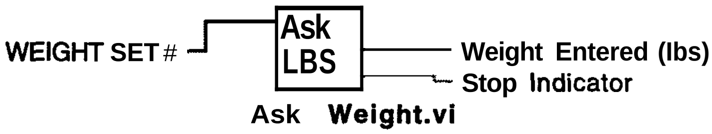
Tow Bar

Towbar Calibl.vi
1/22/97 2:23 PM

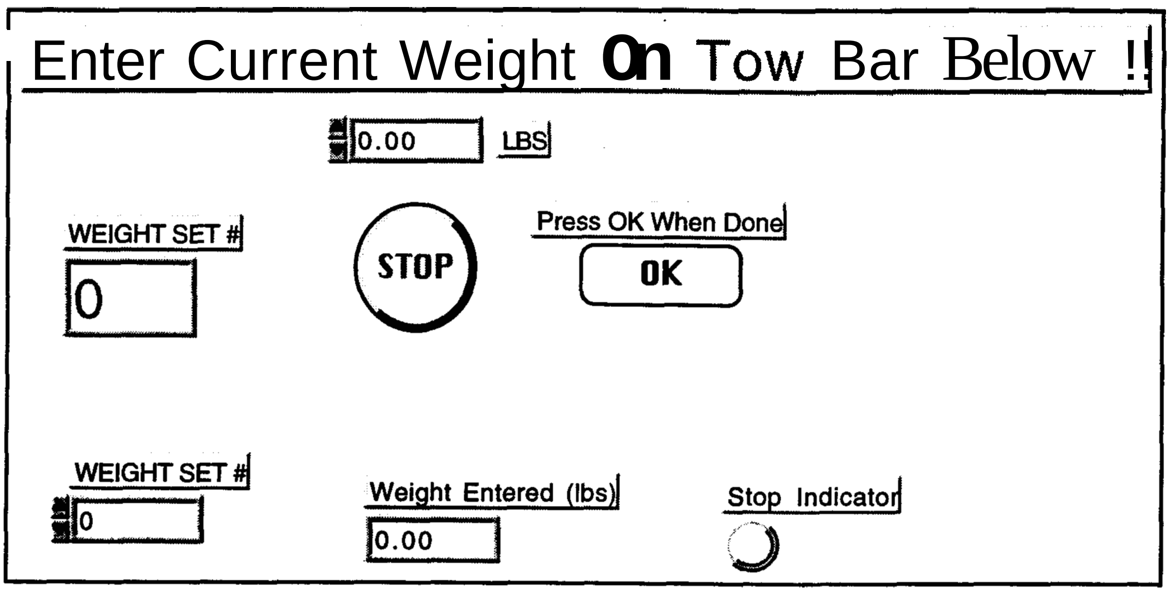
Block Diagram



Connector Pane



Front Panel



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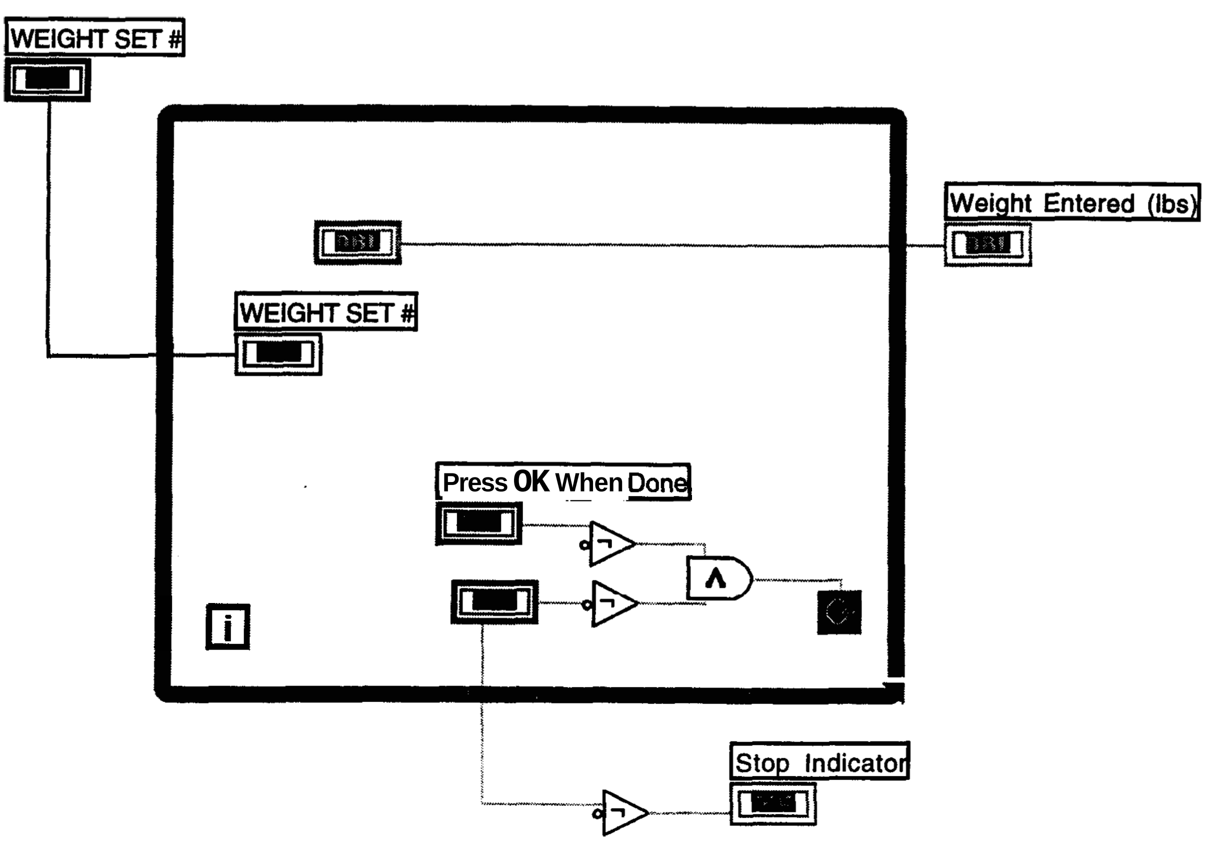

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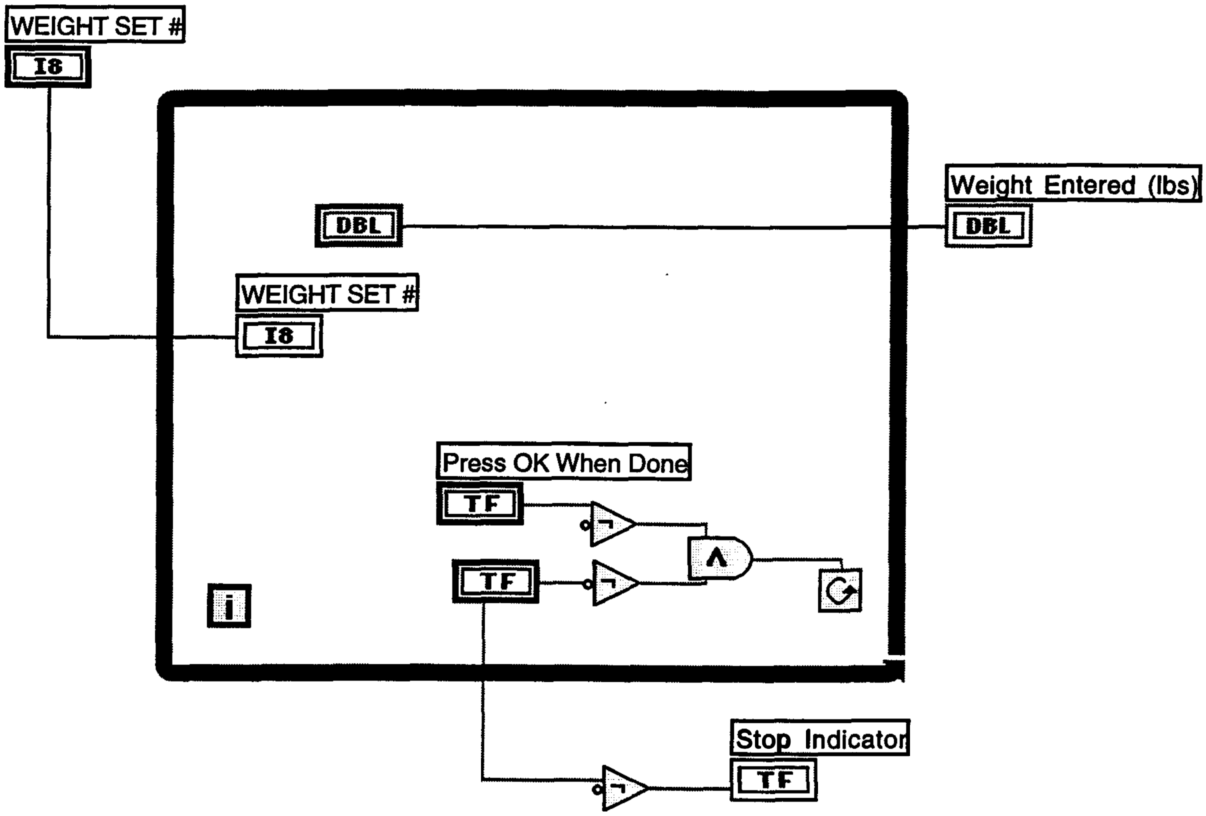
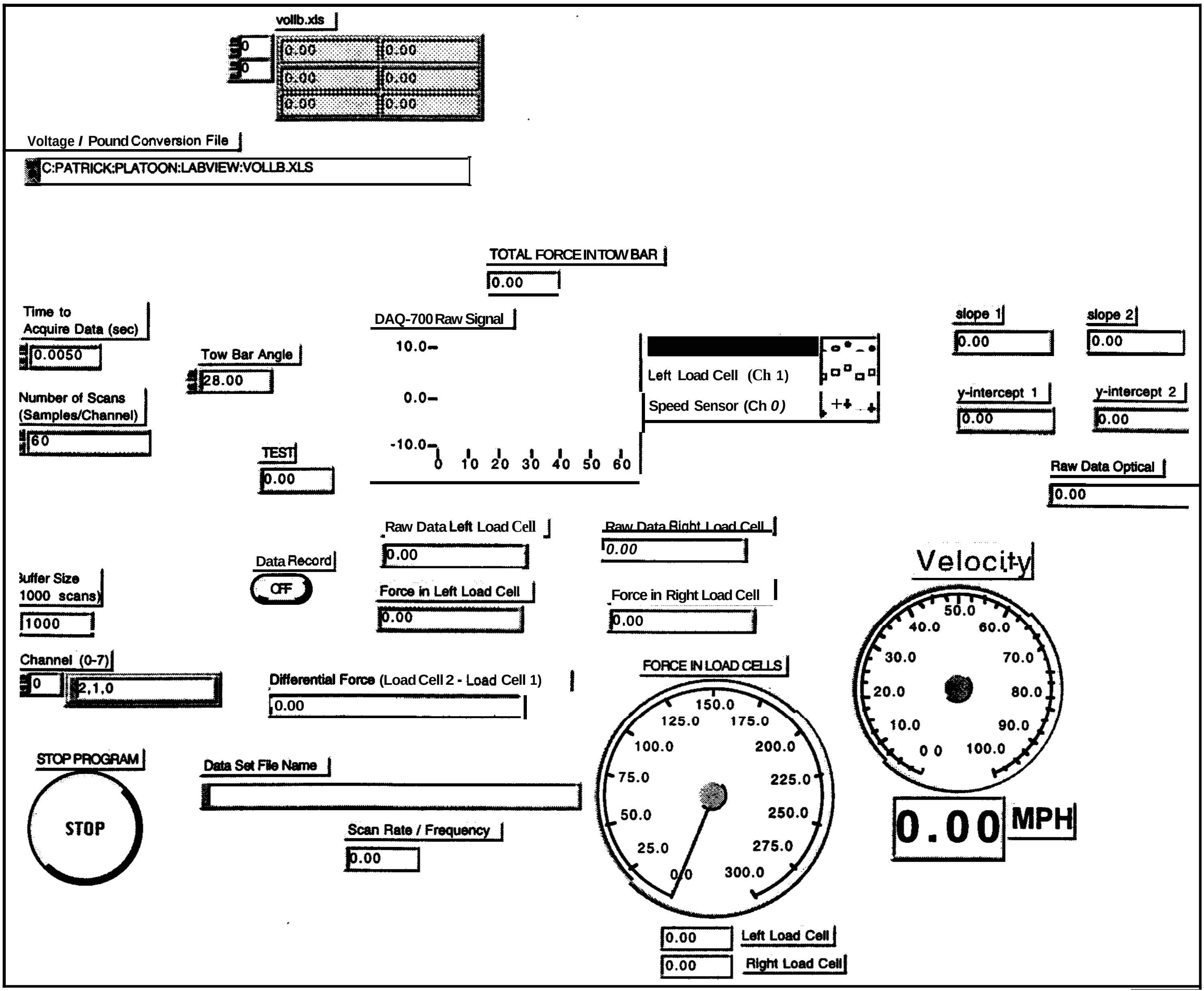


road test data acq.vi
1/22/97 2:27 PM

Connector Pane

road test data acq.vi

Front Panel



road test data acq.vi
7/17/96 1:28 PM

Block Diagram