Title
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The Minuet Method for Solving Sudoku Puzzles by Hand: A Summary

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This brief note shows how to solve Sudoku puzzles by hand. It is targeted to the Sudoku player who can solve intermediate and hard puzzles but sometimes fails on the extremely difficult. For a more extensive discussion see: https://arxiv.org/abs/1812.06778

The Minuet steps are:

1. **Basic Cleanup:** Solve the puzzle using all the tricks you know until either the puzzle is solved, or it locks up and your tricks don’t work anymore. Call this process a “basic cleanup.” After the cleanup you will have eliminated many numbers and be left with just a few “candidates” in each cell. [A cell’s candidates are numbers that can solve it without violating any of the rules of Sudoku that you can see at the moment.] The ultimate goal is to eliminate enough candidates so only one remains in each cell.

2. **Identify the Minuet dancers:** To unlock the puzzle identify a pair of candidates that form a logical dichotomy. In other words, one (and only one) of the two candidates is true and solves its cell. An example is the pair of candidates in any cell that has only two. Another example is a number that appears exactly twice as a candidate in any row (or column, or square). The candidates forming the dichotomy are the dancers. Enclose one dancer in a circle and the other in a square. The Minuet steps that follow will develop two tentative solutions assuming that each of the dancers is true, and exploit the consequences.

3. **Circle dance:** Assume that the circled candidate is true and do a basic cleanup. Eliminate as many additional candidates as possible. However, instead of erasing the eliminated candidates as we did in step 1, mark with over-dots those that are retained. If a cell contains only one over-dot, enclose the corresponding candidate in a circle. Scanning the puzzle for over-dots and circles will show you the tentative solution so you can further clean it up. If you get to a point where there is only one circled candidate in each unsolved cell, these candidates solve the puzzle and you are done. It is also possible that your cleanup may lead to a violation of the Sudoku rules. Then you know that the circle choice is false and should erase the encircled candidate of the original dichotomy. You should also erase all the (meaningless) over-dots, erase the square (but not its candidate, which you now know is true) and go back to step 1 to exploit this newly discovered truth. A third possibility is that neither of these two outcomes occurs. You should then retain the markings and proceed to step 4.

4. **Square dance:** Now repeat step 3 assuming that the square is true. Use under-dots to mark retained candidates and squares for singles. If when you are done each unsolved cell contains a single candidate (in a square) then these candidates solve the puzzle and you are done. It is also possible that the basic cleanup may lead to a violation of the Sudoku rules. Then the square choice is false and you should erase the candidate in the square, as well as all the meaningless under-dots. Since the circled dancer is true you can also erase all the eliminated candidates from the circle solution (i.e., those that remain unmarked in cells that contain either a circle or multiple over-dots) and go back to step 1 to exploit these simplifications. If neither of these two outcomes occurs, you should preserve all the markings and proceed to step 5.

5. **Joint Dance:** You arrive at this step only in the most difficult puzzles. You now have under-dots and over-dots all over the puzzle, and will consider the two sets together. Since we started with a dichotomy we know that unmarked candidates in cells that have been marked in steps 3 and 4 (i.e., cells that contain markings of both types) cannot be part of either solution. Therefore they can be erased. [This use of the dichotomy is the distinguishing feature of the Minuet method. The eliminations achieved in this way do not emerge by considering one tentative solution at a time as is customary.] After erasing all the unmarked candidates repeat Steps 3 and 4, and continue developing the tentative solutions; then go to 5 again etc. This Minuet-like alternation of dancing alone and together will often solve the puzzle. If it does not, and the puzzle locks up, erase all marks and go back to step 2; i.e., choose another two dancers and repeat the Minuet. The puzzle should eventually crack."
Choose dichotomies that promise significant simplifications during steps 3 and 4. Do this by scanning the puzzle.

To avoid too many marks, the author only uses them in cells where at least one candidate has been eliminated. Of course, other marking codes could be used. For example, you could mark the eliminated candidates instead of those that are retained. This, however, may not reveal the current solution as clearly. You should do what works for you—but without any erasures.

There is also a more dynamic way to perform the routine of steps 3-5, which the author prefers. He develops the tentative solutions of steps 3 and 4 concurrently (jumping from one dancer to the other frequently) and at those moments scans the joint markings in the puzzle for erasure opportunities using the joint-dance rules of step 5; he then returns to the concurrent development, etc. This variant of the method is faster than the original because the early scans often reveal allowable erasures that simplify the puzzle without having to wait until the individual dances have concluded.

After solving hundreds of the hardest puzzles, the Minuet method has never failed the author. For a single puzzle, the maximum number of Minuets has been 3.