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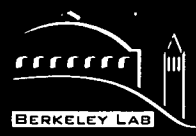
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Authors

Cheng, Wen-Hao
Sessler, Andrew M
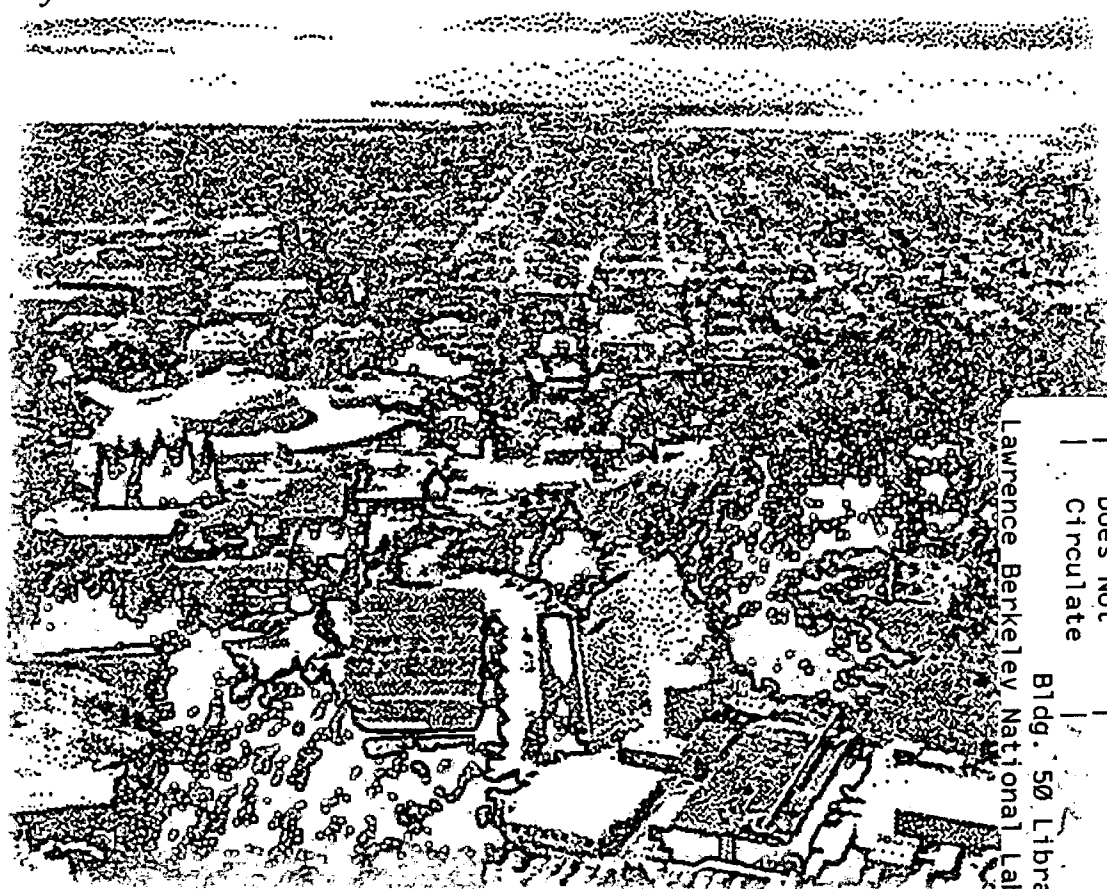Wurtele, Jonathan S

Publication Date

1996-11-01

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Wen-Hao Cheng, Andrew M. Sessler,
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Physics Division

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Damping of the Transverse Head-Tail Instability by Periodic Modulation of the Chromaticity[*]

Wen-Hao Cheng[1],
Andrew M. Sessler[2], and Jonathan S. Wurtele[1,2]
[1]Department of Physics, University of California
Berkeley, California 94720 USA
[2]Ernest Orlando Lawrence Berkeley National Laboratory
Berkeley, CA 94720 USA

November 1996

Damping of the Transverse Head-Tail Instability by Periodic Modulation of the Chromaticity

Wen-Hao Cheng[1], Andrew M. Sessler[2], and Jonathan S. Wurtele[1,2]

[1]Department of Physics, University of California, Berkeley
Berkeley, California 94720

[2]Lawrence Berkeley National Laboratory
Berkeley, California 94720

(Received)

An analytical and numerical study of the suppression of the transverse head-tail instability by modulating the chromaticity over a synchrotron period is presented. We find that a threshold can be developed, and it can be increased to a value larger than the strong head-tail instability threshold. The stability criterion derived agrees very well with the simulations. The underlying physical mechanisms of the damping scheme are the rotation of the head-tail phase such that the instability does not occur, and the Landau damping due to the incoherent betatron tune spread generated by the varying chromaticity.

A bunched beam traveling in a storage ring creates a deflecting force generated by the interactions of the particles and environment. The deflecting force, the so-called wake field, reacts and perturbs the beam, often causing transverse collective instabilities. These instabilities limit the peak current in the bunch. In this Letter, we analyze a new method for controlling such instabilities; namely, through a temporal variation of the ring parameters. We apply this method to a practical example, the head-tail (HT) instability.

In a storage ring, particles with different energy see different focusing strength in the quadrupoles, and thus have different betatron frequency. The ratio of the relative frequency difference to the relative momentum difference is called the chromaticity. The betatron angular frequency of an off-momentum particle is given by $\omega_\beta(\delta) = \omega_{\beta 0}(1 + \xi \delta)$, where ξ is the chromaticity, $\omega_{\beta 0}$ is the betatron angular frequency of the on-momentum particle, and $\delta = \Delta p/p$ is the relative momentum difference. Even if $\xi = 0$, there is an instability called the strong head-tail (SHT) instability. This instability has a threshold created by the particle's synchrotron oscillation, and when the threshold is exceeded, the bunch's motion grows exponentially. In practice, $\xi \neq 0$, there is still a SHT instability with a threshold; in addition, there is the head-tail instability due to chromatic effect, which has no stability threshold. The HT instability was observed in experiments [1], has been well analyzed [2], and has been confirmed by simulations [3].

The HT instability has been a concern for many circular accelerators in the world, for example, we may note the observations and simulations of single-bunch transverse excitation of the beam in the proton ring of the HERA collider at DESY [4], the observation of higher-order HT instability in the PS Booster of the LHC at CERN [5], and the investigation of the possible HT oscillation due to a transverse feedback kicker at KEK's B-Factory (KEKB) [6].

It is understood that, when $\xi/\eta > 0$, the in-phase mode which governs the bunch's transverse center of motion is damped, while the bunch's transverse size which is governed by the out-of-phase mode grows exponentially; when $\xi/\eta < 0$, the condition reverses [7], where $\eta = p\,dC/C\,dp - 1/\gamma^2$ is the slippage factor, $C = 2\pi R = cT_0$ is the circumference of the ring, $\gamma = (1 - \beta^2)^{-1/2}$, and $\beta = v/c \approx 1$ for a relativistic beam discussed in this Letter. Moreover, the growth rate of the out-of-phase mode when $\xi/\eta > 0$, is smaller than the growth rate of the in-phase mode when $\xi/\eta < 0$. Consequently, machine parameters are usually chosen such that ξ/η is positive and small, i.e. we need $\xi > 0\,(< 0)$ when the machine is operated above (below) transition. Damping mechanisms, such as radiation damping and Landau damping, may or may not stabilize the HT instability, depends on the damping time, the width of the incoherent tune spread, and so on.

As the sign of ξ/η is crucial to the stability of the two fundamental modes of head-tail oscillation, in analogous to the strong focusing principle, alternating the sign of ξ/η within a synchrotron period could stabilize both modes. Since varying η means transition crossing, which involves many unfavorable problems, such as vanishing Landau damping, large momentum spread, bunch-shape mismatch and nonlinear effects [8]; we propose, in this Letter, variation of the chromaticity in order to stabilize the HT instability.

While drafting this Letter, we were advised of the existence of the paper written by T. Nakamura of SPring-8 [9]. Nakamura suggested, as we have also (independently), the concept of chromaticity modulation, which contributes an incoherent tune spread that effectively Landau damps the transverse instabilities. In this Letter, going considerably beyond what Nakamura has done, we provide analysis, simulation results, and a stability criterion for the head-tail instability.

We consider that the chromaticity is no longer a constant but a function of "time" s, where s measures the distance around the ring. The chromaticity can be expanded by a Fourier series in terms of the harmonics of the synchrotron phase advance ϕ, as

$$\xi(s) = \sum_{n=0} \xi_n \cos(n\phi + \theta_n), \tag{1}$$

where $\phi = \omega_s s/c$, ω_s is the synchrotron angular frequency, $n = 0$ corresponds to the case of constant chromaticity (DC) ξ_0, and θ_n is the phase difference between the chromaticity and energy variation.

The introduction of a time dependent part of the chromaticity generates an additional incoherent tune spread that contributes to the Landau damping, as was emphasized by Nakamura. Specifically, the constant part of chromaticity causes both the HT instability and Landau damping. However, Landau damping generated by the DC incoherent tune spread is not effective in stabilizing the weak instability. As will be shown in this Letter, the varying part of the chromaticity does not cause the HT instability, and consequently, Landau damping due to the AC (e.g., n=1) incoherent tune spread suppresses the instability due to the DC part of the chromaticity.

The incoherent chromatic tune spread due to the AC part of chromaticity can be estimated as $\sigma_\nu = \sqrt{3/8}\nu_{\beta 0}\xi_1\sigma_\delta$, for a Gaussian beam, where $\nu_{\beta 0} = \omega_{\beta 0}/\omega_0$, $\omega_0 = c/R$, $\sigma_\delta = (\omega_s/c\eta)\sigma_z$, σ_z is the rms bunch length. In obtaining the equation for the incoherent tune spread, we have adjusted θ_1 such that the chromaticity modulation is in-phase with the energy oscillation, i.e. $\xi = \xi_0 + \xi_1\sin\phi$, $\delta = (\omega_s/c\eta)r_z\sin\phi$, where (r_z, ϕ) are the action-angle variables in the longitudinal phase space. The AC part of the incoherent tune spread contributes to a Landau damping without driving the HT instability, and the damping rate per turn can be approximated as $\tau_{LD}^{-1}[1/\text{turn}] \approx 2\pi\sigma_\nu = 2\pi\sqrt{3/8}\nu_s\chi_1$, where $\nu_s = \omega_s/\omega_0$, and $\chi_1 = \omega_{\beta 0}\xi_1\sigma_z/c\eta$ is the AC part of head-tail phase. Note that the Landau damping time due to the AC part of chromaticity is independent of beam intensity and the impedance of a ring. Simulations of a bunched beam traversing an averaged impedance in a storage ring confirm this. The implication is that, within the tolerance of dynamic aperture reduction due to the chromaticity, one can increase the damping rate (by a large enough χ_1) to suppress the HT instability.

For an analysis of the effect of variable chromaticity, we assume the particle in a bunched beam experiences two forces: the external focusing force and the wake force generated from the interaction between the beam and cavities. We neglect any nonlinear synchrotron oscillation, the longitudinal wake force and the gradient of the transverse wake force. The synchrobetatron coupling effect on the longitudinal orbit is also ignored.

There are two parameters essential to the dynamics studied in this Letter:

$$\chi_n = \omega_{\beta 0}\xi_n\sigma_z/c\eta, \quad \Upsilon \equiv \pi N r_0|W_\perp|c^2/8\gamma C\omega_{\beta 0}\omega_s, \tag{2}$$

where χ_n is the phase shift between head and tail of a bunch for each harmonic n of the chromaticity, N is the number of particles in a bunch, $r_0 = e^2/m_0c^2$, and $W_\perp$ is the transverse wake function. The parameter Υ is approximately the ratio of betatron tune shift to the synchrotron tune. It can easily be shown, by a two particle model, that the onset of the SHT instability is where $\Upsilon \geq 1$ [7], when $\chi_1 = 0$. The well-known transverse Boussard criterion is also consistent with this condition [10]. In this work, we concentrate on the case of $n = 0 \& 1$, therefore we have three independent parameters under study: χ_0, χ_1, and Υ.

The effect of nonlinear chromaticity characterized by ξ_{01}, where ξ_{DC} is expanded as $\xi_{DC} = \xi_0 + \xi_{01}\delta$, plays a similar role to the AC component ξ_1. In fact, $\xi_1 \simeq \xi_{01}\sigma_\delta$. Since both ξ_{01} and σ_δ are usually small, the nonlinear part of the DC component ξ_{01} is not effective enough to suppress the HT instability.

A linearized Vlasov analysis of a many-particle system yields an eigenmode equation. The mode frequency, $\Omega^{(l)}$, can be approximated for the dominant radial mode ($j = 0$), as [11]

$$\Omega^{(l)} - \omega_{\beta 0} - l\omega_s \simeq -i8(\Upsilon\omega_s/T_0)N_g^{(l)}\tilde{Z}_{\text{eff}}^{(l)}, \tag{3}$$

where $N_g^{(l)} = \sum_q |g_{l0}(\chi_1, \chi_q - \chi_0)|^2$, g_{l0} is the frequency spectrum of the beam's perturbed density of the $(l,j) = (l,0)$ mode, l is the index of the azimuthal mode, $\chi_q = \omega_q\sigma_z/c$, $\omega_q = q\omega_0 + \omega_\beta + l\omega_s$, the transverse impedance is $Z_1^\perp(\omega_q) = -W_\perp\tilde{Z}(\omega_q)$, and the effective impedance is

$$\tilde{Z}_{\text{eff}}^{(l)} = \left[N_g^{(l)}\right]^{-1}\sum_q \tilde{Z}(\omega_q)|g_{l0}(\chi_1, \chi_q - \chi_0)|^2. \tag{4}$$

Note that, the number of azimuthal and radial nodes in the longitudinal phase space are, l and j, respectively. In the following study, we assume the beam distribution is Gaussian, and take a model-impedance function as: $\tilde{Z}(\omega) = 1/\omega - i\pi\delta(\omega)$. The coherent tune shift, given by the real part of the mode frequency, is

$$2\pi \Re(\Delta\nu) = -(4\Upsilon/l!\,2^l)\nu_s\chi_0^{2l}e^{-\chi_0^2}J_0^2(\chi_1/4),\tag{5}$$

where $\Delta\nu = (\Omega^{(l)} - \omega_{\beta 0})/\omega_0 - l\nu_s$, and $J_0(x)$ is the Bessel function. The growth rates per synchrotron period of the two fundamental modes, given in terms of the imaginary part of the mode frequency, $1/\tau_s^{(l)} = 2\pi\Im(\Delta\nu)/\nu_s$, are approximately

$$1/\tau_s^{(0)} \approx -4\Upsilon\,\mathrm{Erfi}\,(\chi_0)\,e^{-\chi_0^2}J_0^2(\chi_1/4),\tag{6}$$

$$1/\tau_s^{(1)} \approx \sqrt{\pi}\Upsilon\chi_0 L_{1/2}^{(-1/2)}(\chi_0^2)\,e^{-\chi_0^2}J_0^2(\chi_1/4),\tag{7}$$

where $\mathrm{Erfi}(x) = -i\,\mathrm{Erf}(ix)$, $\mathrm{Erf}(x)$ is the error function, and $L_k^{(l)}(x)$ is the Laguerre polynomial. One can see that, when $\chi_0 = 0$, the growth rate of HT instability is zero.

We can make a rough estimate of the stability criterion for the HT instability, which is that the incoherent tune spread is larger than the absolute value of the coherent tune shift; that is: $\sigma_\nu > |\Delta\nu|$. The approximate stability condition is therefore

$$\chi_1 > (8/\pi)\sqrt{2/3}N_l\Upsilon\left|\tilde{Z}_{\mathrm{eff}}^{(l)}(\chi_0)\right|,\tag{8}$$

where $N_l = \int d\omega_p|g_{l0}|^2$. Explicitly, expressed in terms of the accelerator parameters, we have

$$\xi_1 > c_l\frac{eI_0\left|Z_1^{\perp(l)}(\xi_0)\right|_{\mathrm{eff}}}{E}\left(\frac{R}{\sigma_z}\right)^2\left(\frac{\eta R}{\nu_s\nu_{\beta 0}^2}\right),\tag{9}$$

where $c_l = \sqrt{2/3}\Gamma(l+1/2)/\pi l!2^{l+1}$, $E = \gamma m_0 c^2$, and $I_0 = Nec/C$ which is the averaged current. When $0 < \chi_0 < 1$, the $l = 1$ mode is usually the dominant unstable mode, and $c_1 = 0.058$. In contrast, when $-1 < \chi_0 < 0$, the $l = 0$ mode is the dominant unstable mode and $c_0 = 0.23$.

A code has been developed to simulate a bunched beam traversing a ring with a transverse impedance. A bunch beam is loaded with a bi-Gaussian distribution in both longitudinal and transverse phase spaces. All results are numerically converged when the number of macro-particles simulated is larger than 400. Since χ_0 is usually chosen as a positive parameter in accelerators, we only show the figures of numerical work for $\chi_0 > 0$. Simulations, nevertheless, confirm the growth rates and stability criterion for both sign of χ_0.

In Fig. 1, we show the results of multi-particle simulations. The curve of $\langle y \rangle$ presented in this Letter has been averaged over a synchrotron period. For a beam with initial centroid offset, the bunch centroid motion is initially dominated by the $l = 0$ mode, which is a damping mode when $\chi_0 > 0$; the higher order unstable modes then cause the growth of averaged bunch-center after the initial damping. The varying chromaticity, nonetheless, Landau damps all the higher order unstable modes when χ_1 is larger than the HT stability theshold estimated in Eq. (8).

The estimate for stability in Eq. (8) is usually sufficient for the bunch centroid motions. A rigorous stability criterion can be derived by incorporating the incoherent tune spread in the Vlasov analysis. We first write down the betatron phase advance,

$$\Phi_\beta = \frac{\omega_{\beta 0}}{c}s + \frac{\omega_{\xi 1}}{2c}\phi r_z - \frac{\omega_{\xi 0}}{c}r_z\cos\phi - \frac{\omega_{\xi 1}}{4c}r_z\sin(2\phi),\tag{10}$$

where $\omega_{\xi(0,1)} = \omega_{\beta 0}\xi_{(0,1)}/\eta$, and the tune generated by the in-phase oscillation between the chromaticity modulation and the energy oscillation is included. Following the well-known technique [12], one can find the dispersion relation of the most dominat radial mode, which is

$$V + iU = \frac{8\Upsilon}{2\pi}N_l\left\{\Re\left[\tilde{Z}_{\mathrm{eff}}^{(l)}\right] + i\Im\left[\tilde{Z}_{\mathrm{eff}}^{(l)}\right]\right\},\tag{11}$$

where $V + iU$ is the so called "beam transfer function" [7]. For a Gaussian beam with the model-impedance, we have $V + iU =$

$$\frac{-i\chi_1^2/2}{\sqrt{2\pi}\chi_1 - 2\pi\nu e^{-2\nu^2/\chi_1^2}\left[\mathrm{Erfi}\left(\frac{\sqrt{2}\nu}{\chi_1}\right) - i\right]},\tag{12}$$

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$$\frac{-i\chi_1^4}{\sqrt{2\pi}\,(\chi_1^3+4\nu^2\chi_1)-8\pi\nu^3 e^{-2\nu^2/\chi_1^2}\left[\mathrm{Erfi}\left(\frac{\sqrt{2}\nu}{\chi_1}\right)-i\right]},\tag{13}$$

for the $l = 0$ and $l = 1$ modes, respectively, where $\nu = \Delta\nu/\nu_s$. Examination of the dispersion relation shows that the SHT threshold can be enlarged by increasing χ_1.

Multiparticle simulations show that the rms-emittance of a Gaussian beam is stabilized, when the the value of χ_1 approaches the stability threshold of Eq. (11) [cf. Fig. 2], where $\varepsilon_{\mathrm{rms}} = (\langle y^2\rangle\langle P_y^2\rangle - \langle yP_y\rangle^2)^{1/2}$, $P_y = (c/\omega_{\beta0})\,dy/ds$, and the bracket $\langle\rangle$ means a phase-space ensemble average. We find that, the results of simulation of the bunch centroid motion agree very well with the approximate stability limits, and the results of emittance growth agree with the exact stability criterion [cf. Fig. 3]. Figs. 4 show the simulation results of stabilization of the SHT effect by a large enough χ_1, when $\chi_0 = 0$. This implies that the limitation of peak current in a storage ring can be increased by the varying chromaticity scheme.

In summary, the chromaticity of a storage ring, which causes the head-tail instability, usually needs to be controlled by sextupoles. We have shown that, by the varying chromaticity scheme, the head-tail instability is suppressed, and, futhermore, a stability threshold is developed. With a large enough allowable AC part of the chromaticity, one could even make larger the threshold of the strong head-tail instability. The physics of the underlying mechanism is simple: strong focusing principle and Landau damping. Studies of practical operation issue, such as rapid modulated sextupole magnets, and theoretical issues, such as the reduction of dynamic apertures, and exact calculations of the azimuthal mode-coupling, are required. Also, of course, the practical aspects of varying chromaticity must be compared with the other schemes that also introduce an incoherent tune spread, e.g., space-charge, ion-trapping, rf-nonlinearity, and octupole magnets. Temporal variation of accelerator parameters might be used in the control of other instabilities.

[1] The SPEAR Group, *Proc. 9th Int. Conf. High Energy Accel.*, SLAC, 1974, p.338; J. Gareyte & F. Sacherer, *Proc. 9th Int. Conf. High Energy Accel.*, SLAC, 1974, p.341.

[2] C. Pellegrini, Nuovo Cimento **64A**, 447 (1969); M. Sands, SLAC Reports TN-69-8 and TN-69-10 (1969); F. Sacherer, *Proc. 9th Int. Conf. High Energy Accel.*, SLAC, 1974, p.347.

[3] R. H. Siemann, Nucl. Instr. Meth. **221**, 293 (1984).

[4] E. Gianfelice, DESY-HERA 94-3, p.25, (1994); F. Galluccio, Part. Accel. **50**, 141 (1995).

[5] R. Cappi, Part. Accel. **28**, 117 (1995).

[6] E. Kikutani, Part. Accel. **52**, 251 (1996).

[7] A. W. Chao, *Physics of Collective Instabilities in High Energy Accelerators*, (John Wiley & Sons, New York, 1993).

[8] K. Takayama, Part. Accel. **14**, 201 (1984); J. Wei & S. Y. Lee, Part. Accel. **28**, 77 (1990).

[9] T. Nakamura, *Proc. 1995 IEEE Part. Accel. Conf.*, Vol. 5, 1995, p.3100.

[10] D. Boussard, CERN Lab II/RF/Int 75-2 (1975).

[11] The derivations of the eigenmode analysis and dispersion relation are outlined in the manuscript by W.-H. Cheng, A. M. Sessler, and J. S. Wurtele, (to be submitted to Physical Review E).

[12] L. D. Landau, J. Phys. U.S.S.R., **10**, 25 (1946); N. G. van Kampen, Physica, **21**, 949, (1955); F. J. Sacherer, CERN/SI-BR/72-5, 1972.
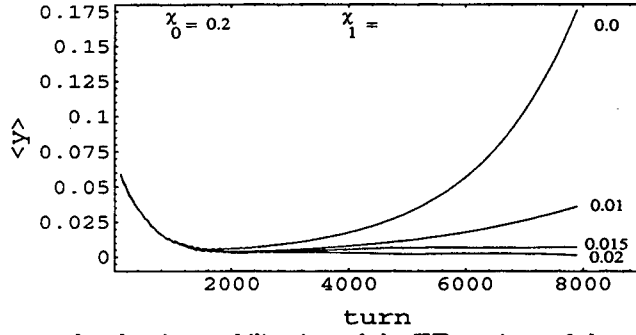
FIG. 1. Multi-particle simulation results showing stabilization of the HT motions of the centroid of a Gaussian beam by χ_1, where $\chi_0 = 0.2$, $\Upsilon = 0.22$. The estimated stability threshold for the $l = 1$ mode, according to Eq. (8), is where $\chi_1 \geq 0.0127$.
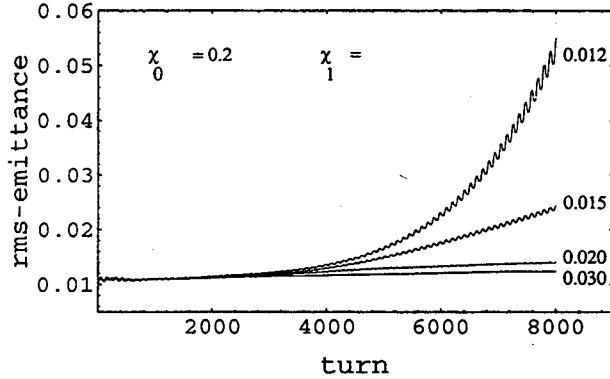

FIG. 2. Multi-particle simulation result showing stabilization of the HT motions of the rms-emittance of a Gaussian beam when $\chi_1 \geq 0.026$ – the theoretical stability threshold of the $l = 1$ mode [cf. Eq. (11)]. Here $\chi_0 = 0.2$, $\Upsilon = 0.22$.
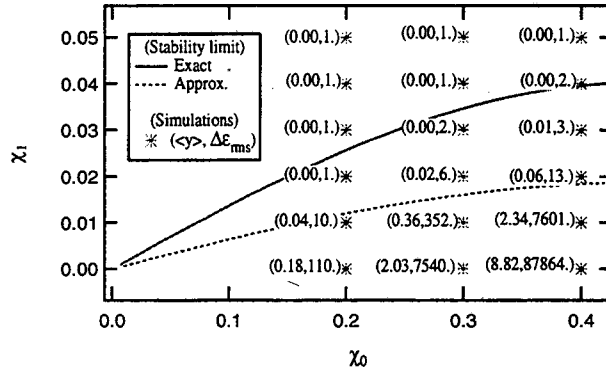

FIG. 3. Stability limits of a Gaussian beam with the model-impedance function for the $l = 1$ mode, in the AC (χ_1) vs. DC (χ_0) space. Here $\Upsilon = 0.22$, $\langle y \rangle$ is the averaged centroid motion at 8000 turns, $\Delta\varepsilon_{rms} = \varepsilon_{rms}(8000)/\varepsilon_{rms}(0)$, and the approximate and exact stable limits are plotted according to the criteria shown in Eqs. (8) and (11), respectively. The region below the solid (dashed) line is stable for the bunch's rms-emittance (centroid) motion. Note that, $\langle y \rangle(0) = 0.1[cm]$, $\varepsilon_{rms}(0) = 0.01[cm]$, and $\Delta\varepsilon_{rms}$ is rounded to the closest integer.
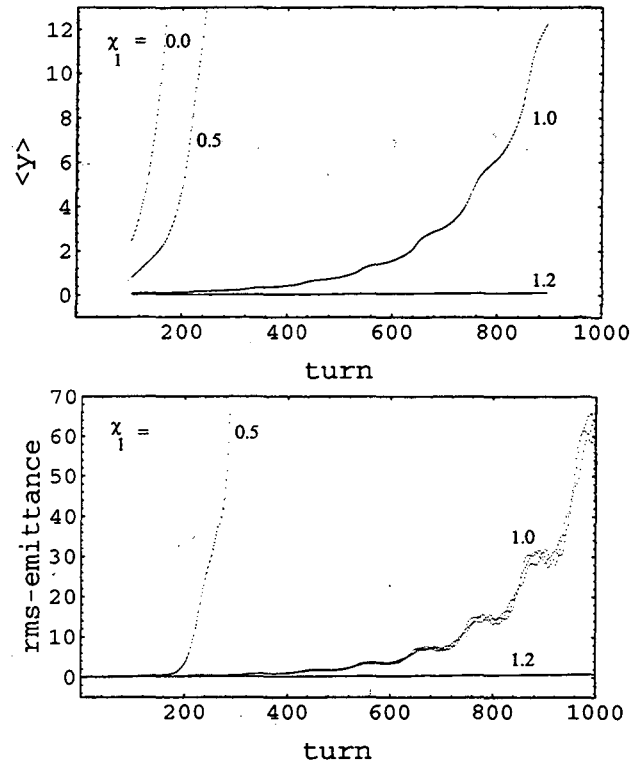
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FIG. 4. Multi-particle simulation results showing stabilization of the SHT motions of the centroid and rms-emittance of a Gaussian beam by χ_1. The SHT stability limit is $\Upsilon < 1$, when $\chi_1 = 0$. In these figures, $\chi_0 = 0$, $\Upsilon = 1.65$, $\langle y \rangle(0) = 0.1$[cm], and $\varepsilon_{\mathrm{rms}}(0) = 0.01$[cm].

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