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Stochastic Analysis of Levee Stability Subject to Variable Seepage Conditions

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Abstract

Levee stability is highly influenced by seepage. Specifically, hydraulic conductivity distribution within a levee influences pore pressure distribution and controls the drained strength of the soil. In this study the influence of hydraulic conductivity and blanket layer thickness on failure probability is evaluated within the context of reliability analyses that also include soil strength and unit weight as random variables. First-Order Reliability Method (FORM) is used to evaluate reliability, rank random variables by importance and to obtain sensitivity of the solution to each random variable and its distribution parameters. Stability is computed using Spencer's method of slices coupled to a finite element seepage code to directly evaluate pore pressure. In addition, response surface method solutions are compared to the direct reliability solution to assess response surface accuracy. The results show that blanket layer thickness is more important than most strength parameters and that the uncertainty in hydraulic conductivity is less important to the variance in safety factor for the case of a low-permeability blanket layer. Numerical challenges caused by implementing a finite element limit-state function are discussed. Response surface methods are found to give a reasonable approximation to the direct reliability solution when the design point is between response surface fitting points.

INTRODUCTION

The evaluation of the expected performance of levee systems is a challenging problem and given the many miles of levees that must be evaluated for potential failure it is essential to develop risk based assessment methodologies to guide efficient allocation of resources (NRC 2013). In Northern California, 100's of miles of levees along the Sacramento River present a particular challenge and have been the subject of extensive analyses on regional scale (URS and JBA 2008; DWR 2012). However, while the regional approach provides assessment of the global risk to the system, it does not consider specific local conditions or mechanisms. The purpose of the analyses presented herein is to develop a more site specific mechanistic model of levee performance. We build on prior work of the U.S. Army Corps of Engineers who used expert elicitation and the First-Order Second Moment reliability analysis to arrive at the risk of levee failure (USACE 2006; Perlea and Ketchum 2011). The advantage of the reliability analysis is that it incorporates uncertainty of input parameters and can provide important insights to guide future analysis and investment in these important engineering systems (Baecher and Christian 2003). In the approach presented herein we use the First-Order Reliability Method (FORM), which uses complete probability information to assess failure probability and is able to provide a suite of additional results that quantify the relationship of the solution to random variable input parameters (Der Kiureghian 2004). However, the solution requires iterative evaluation of the geotechnical response, making FORM a difficult method to implement in practice without a significant investment in time. A common solution to this obstacle is to use response surface methods, which use a variety of fitted functions to approximate geotechnical response (Li et al. 2016); however, their accuracy can be difficult to verify. By combining existing seepage, stability and reliability codes into a coupled code, we are able to incorporate hydraulic conductivity and blanket layer thickness into a reliability analysis. Response surface methods are also shown for comparison to provide insight to analysts without access to a coupled seepage, stability and reliability code.

MECHANISTIC LEVEE MODEL

An embankment cross-section was chosen to represent Sacramento River levees south of Sacramento in the Central Valley of California. A soil profile typically consists of discontinuous floodplain deposits where silt and clay surficial soils formed by overbank flows or low-relief marshland that are underlain by sandy meandering channel deposits. Active channels are confined by embankments, or levees, that have been constructed and maintained over 150 years using a variety of methods and materials and generally do not impound significant water levels outside of flood events. Levees are typically situated on top of fine-grained overbank deposits completely intersected by the confined channel, allowing a direct hydraulic connection between the river and relatively high-permeability channel deposits. This condition develops high pore pressures in the blanket layer on the land-side of the levee, reducing effective stress and stability of the slope as well as causing foundation erosion from high hydraulic gradients (under-seepage).

Seepage and stability are evaluated for a single levee cross-section (Figure 1) consisting of a symmetric 5.2 m embankment founded on a 5.5 m low-permeability blanket layer underlain by a 9.8 m aquifer, both of uniform thickness. A 6.1 m berm separates the water-side levee toe and channel, which is 10.7 m deep and intersects the blanket and aquifer. Channel and levee slopes are 2.5:1 (H:V) and the levee crest is 7.6 m wide. Embankment and aquifer soils were modeled as poorly graded sand with silt (SP-SM) and the blanket layer as silt (ML); engineering properties are summarized in Table 1.

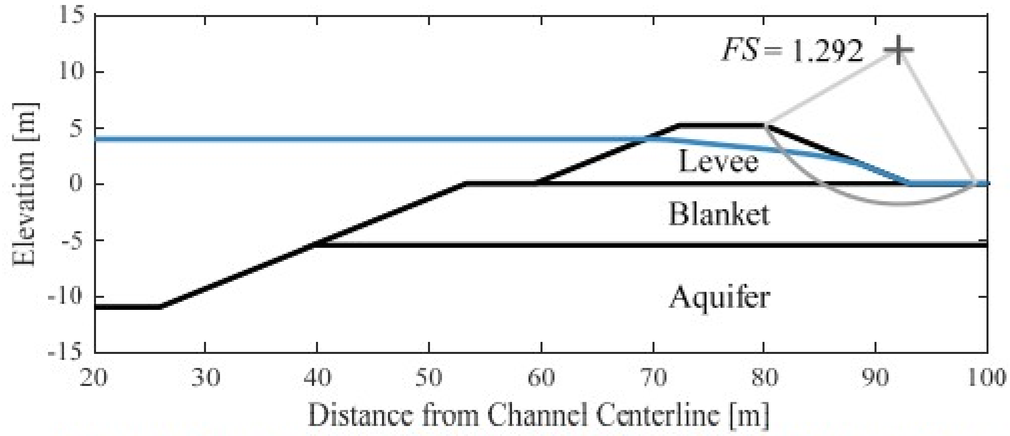

Figure 1. Cross-section with 200-yr WSE and critical failure surface.

Table 1. Soil properties

Layer	USCS	c' $[kN/m^2]$	ϕ' $[\degree]$	γ_{sat} $[kN/m^3]$	K_h $[m/s]$	K_v/K_h $[-]$	Thickness $[m]$
Embankment	SP-SM	0.00	38.0	18.9	8.00E−5	1.00	5.20
Blanket	ML	1.20	36.0	18.1	5.00E−8	0.25	5.49
Aquifer	SP-SM	0.00	35.0	18.9	8.00E−5	1.00	9.80

Pore pressures are computed with a saturated/unsaturated transient finite element code UNSAT1 (Neuman 1972). The saturated/unsaturated algorithm allows for a constant mesh to solve for the phreatic surface; time steps and unsaturated soil properties are chosen such that steady state conditions are reached in all analyses. A mesh organized in vertical columns contains approximately 9,000 rectangular elements varying in size from 1.5 m at the bottom and side model boundaries to 0.2 m in and near the embankment, with a maximum aspect ratio of 3.1. Three types of boundary conditions are applied: constant head on the vertical sides and water-side surface; impermeable boundaries on the aquifer base and levee crest; and, constant head seepage face conditions on the land-side slope and adjacent ground surface. The model was found to be insensitive to the presence of an aquitard below the aquifer; therefore, to improve computation time it is not included. Water surface elevations (WSE) are evaluated from the levee toe (datum) to crest; the 200-year WSE is at 4.0 m, about three-fourths of the levee height.

Stability is computed with Spencer's method of slices implemented in MATLAB (MathWorks 2015) by Tabarroki (2011) and uses an efficient genetic algorithm (Wang 2011) to search for circular and non-circular surfaces that minimize safety factor, *FS*. The stability code is modified to allow specification of pore pressures from the finite element seepage solution and computed safety factors are generally found to be within 0.01 of *FS* computed with commercial slope stability software.

Analyses described herein use the critical circular slip surface for the 200-year WSE (Figure 1) across all WSE to provide consistent geometry for comparing mechanistic and stochastic results. Safety factor becomes marginally stable as WSE approaches the crest, where FS = 0.98. Hydraulic gradient, i, which is hydraulic head, h, measured across the blanket vertically at the levee toe, $i = \Delta h/Z_B$, is shown on Figure 2a with FS.

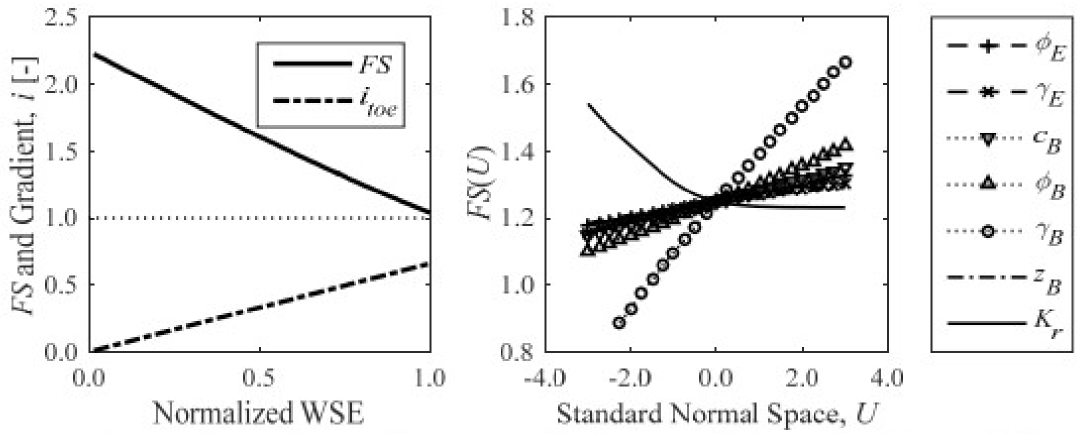

Figure 2. (a) FS, gradient at levee toe and breakout height and (b) FS sensitivity.

STOCHASTIC MODEL

Seven parameters are chosen to be modeled as random variables in the stochastic analysis: friction and unit weight for the embankment (ϕ_E, γ_E); cohesion, friction angle, unit weight and layer thickness for the blanket (ϕ_B, c_B, γ_B, Z_B); and the ratio of horizontal hydraulic conductivity (K_r). All variables are modeled with a normal distribution, excepting Z_B and K_r, which are modeled using truncated normal and lognormal, respectively (Table 2). The truncated normal distribution allows a lower bound to be set for blanket thickness, precluding negative values, and an upper bound to facilitate finite element seepage computations. In addition, a lower bound on Z_B prevents numerical issues from occurring when computing the FORM solution, as discussed later. Distribution parameters were selected such that each coefficient of variation ($c.o.v.=\sigma/\mu$) is consistent with data for typical Sacramento Valley soils and published values in general (e.g., Baecher and Christian 2003); all variables are considered uncorrelated in the analyses presented here.

Throughout the text U will refer to the standard normal space to facilitate understanding of random variable effects with respect to their distributions. All random variables, $\boldsymbol{X}$, are transformed to $\boldsymbol{U}$ such that U is normally distributed with zero mean and unit standard deviation (e.g., $\mu - \sigma$ is equivalent to $U = -1$), which is also a key theoretical underpinning of reliability analysis.

Table 2. Random variable distributions and parameters.

Random Variable	Distribution	c.o.v. [%]	Notes
ϕ'_E	Normal	5.0	For μ, see Table 1 (for all random variables)
γ_E	Normal	10.0	–
c'_B	Normal	30.0	–
ϕ'_B	Normal	5.0	–
γ_B	Normal	10.0	–
Z_B	Truncated Normal	5.6	$Z_{B,MIN}$ = 3.0 m; $Z_{B,MAX}$ = 10.0 m
$K_{a,h}$	Lognormal	200	Used to determine distribution of K_r; not included as random variable in FORM.
$K_{b,v}$	Lognormal	200	
K_r	Lognormal	2450	LN function of random variables: $K_r=K_{a,h}/K_{b,v}$, λ_r=7.38 and ζ_r=1.79

Since the hydraulic conductivity ratio controls pore pressure, K_r is included as a single lognormal random variable to simplify the stochastic model and reduce computation time. K_r is modeled as a function of two lognormal random variables such that $\lambda_r=\lambda_{a,h}/\lambda_{b,v}$ and $\zeta_r=\zeta_{a,h}/\zeta_{b,v}$. Despite K_r being a random variable, $K_{a,h}$ and $K_{b,v}$ are necessary inputs for the seepage code; however, infinite combinations of $K_{a,h}$ and $K_{b,v}$ can be chosen for a given K_r. Three cases are considered to evaluate this effect: Case 1, hold $K_{a,h}$ constant and vary $K_{b,v}$; Case 2, vary $K_{a,h}$ and hold $K_{b,v}$ constant; or, Case 3, vary $K_{a,h}$ and vary $K_{b,v}$. Constant values are set to the distribution mean. As shown in Figure 3, safety factor is insensitive to the absolute value of hydraulic conductivity when $U(Kr)\geq-2$. For all water surface elevations the FORM design point of K_r was in the range $0\leq U(Kr)\leq 0.2$; consequently, the reliability analyses do not depend on the absolute values of hydraulic conductivity in the seepage analysis and case 1 is used in all analyses.

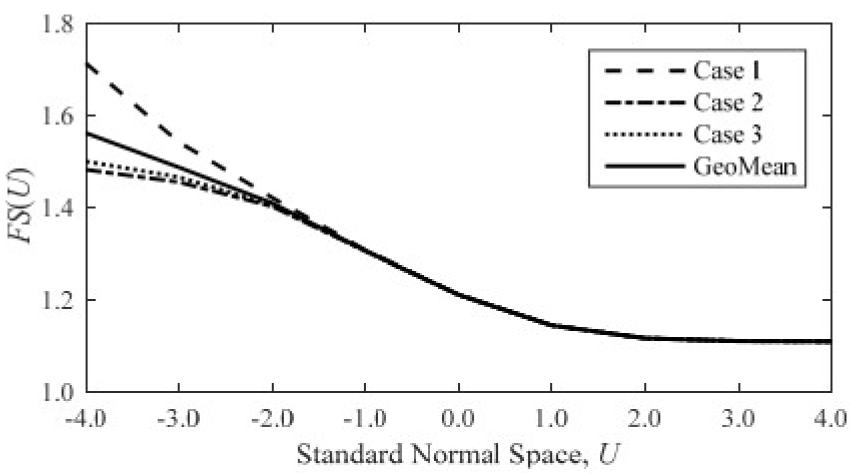

Figure 3. Sensitivity of FS to absolute value of $K_{a,h}$ and $K_{b,v}$

Reliability analysis is used to evaluate the probability of slope failure, taken as $P[FS(\boldsymbol{X}) \leq 1.0]$ using a limit-state function $g(\boldsymbol{X})=FS-1.0$. The FORM solution is used to solve for a design point, $\boldsymbol{u}^*$, such that $g(\boldsymbol{X}) \leq 0$ and probability density in the standard normal space is maximized (Der Kiureghian 2004); thus, the design point represents the set of random variables, $\boldsymbol{x}^*$, most likely to lead to failure. Failure probability is estimated using the reliability index, β, such that $p_f=\Phi(-\beta)$. FORM also produces quantities which illustrate the effect of each random variable and their distribution parameters on the solution. The importance vector, α, quantifies the effect of each variable on the variance of β, whereas sensitivity vectors quantify the impact of a parameter relative to mean and standard deviation $\partial\beta/\partial\mu$ and $\partial\beta/\partial\sigma$. Sensitivity vectors $\boldsymbol{\delta}$ and $\boldsymbol{\eta}$ are $\partial\beta/\partial\mu$ and $\partial\beta/\partial\sigma$ statistically scaled by the standard deviation of each random variable, respectively (e.g., $\delta_i=\partial\beta/\partial\sigma \cdot \sigma_i$). FORM is carried out with FERUM 4.1 (Bourinet 2010), which is implemented in MATLAB (MathWorks 2015) along with the seepage and stability codes discussed above.

RESPONSE SURFACE ANALYSIS

Although the reliability analyses described herein are directly integrated with a seepage and stability solution, a suite of response surface approximations was developed to evaluate accuracy when code integration is not possible. As described by Sudret and Der Kiureghian (2000), the method computes an approximate polynomial limit-state, $\hat{g}(\boldsymbol{x})=\boldsymbol{V}^T(\boldsymbol{x})\cdot\boldsymbol{a},$ where $\boldsymbol{V}^T(\boldsymbol{x})$ is a column vector of response surface variables x arranged such that individual terms are multiplied with coefficients, a, appropriately. Least-square error is minimized between a subset of fit points, evaluated at the response surface and approximated function, y, to solve for the coefficients per Faravelli (1989):

$$(1)\quad \boldsymbol{a}=(\boldsymbol{V}^{\mathrm{T}}(\boldsymbol{x}^{FP})\cdot\mathbf{V}(\boldsymbol{x}^{FP}))^{-1}\cdot\boldsymbol{V}^{\mathrm{T}}(\boldsymbol{x}^{FP})\cdot\hat{g}(\boldsymbol{x}^{FP})$$

Thus, the vectors $\boldsymbol{V}^T(\boldsymbol{x})$ has been expanded to matrix and $\hat{g}$ to a vector by inclusion of a particular set of fit points, $\boldsymbol{x}^{FP}$ while $\boldsymbol{a}$ contains a value for each coefficient selected by the analyst. Equation 1 incorporates the Moore-Penrose pseudo-inverse, allowing $\boldsymbol{V}^{\mathrm{T}}(\boldsymbol{x}^{FP})$ to be a non-square matrix.

At least one fit point is required for every coefficient specified by a particular response surface equation. For example, a quadratic equation for two variables would contain 5 coefficients if one constant term is incorporated: $\hat{g}(\boldsymbol{x})=a_1+a_2x_1+a_3x_2+a_4x_1^2+a_5x_2^2.$ Various response surface models were computed and compared to the seepage-stability and reliability solutions to evaluate their accuracy with respect to the seepage parameters Z_B and K_r, specifically: polynomials of order one through four; several subsets of random variables; and, fit points selected at $\pm1\sigma$, $\pm2\sigma$ and $\pm3\sigma$ for each random variable.

RESULTS

Failure probability increases monotonically from a negligible value at low WSE to 0.098 at the 200-yr WSE and a maximum value of 0.531 with WSE at the levee crest, corresponding to reliability indices of 1.291 and −0.078, respectively (Figure 4a). Solutions were not possible for WSE in the bottom third of the levee due to the inability of the stability code to compute FS for low values of U. As illustrated in Figure 4b, the design point in U-space for each random variable moves toward U=0 for all random variables, with K_r the only exception.

Importance and sensitivity vector trends were found to be consistent for WSE in the upper two-thirds of the levee profile; therefore, results and relationships are described only for the 200-yr WSE (4.0 m), but may generally be applied to all other locations (Table 3). As illustrated by α, all random variables except K_r are capacity variables (i.e., inversely related to failure probability), and the importance of each variable is ranked as follows:

$$\mathrm{Imp}(\gamma_B) > \mathrm{Imp}(Z_B) > \mathrm{Imp}(\gamma_E) > \mathrm{Imp}(\phi'_B) > \mathrm{Imp}(\phi'_E) > \mathrm{Imp}(\gamma_E) > \mathrm{Imp}(K_r)$$

While α describes the impact of a random variable on β variance, the sensitivity vector δ show that the absolute value of β is most sensitive to a change in Z_B, the toe blanket thickness, followed closely by γ_B and K_r. Thus, while K_r is the least important variable with regard to variance, a unit-change in the absolute value of K_r would have the third-largest impact on β relative to other random variables. Finally, the sensitivity vector $\boldsymbol{\eta}$ shows the dominating effect that standard deviation for Z_B has on failure probability: a 10% reduction in standard deviation for Z_B would result in $\Delta\beta$=0.3, changing computed failure probability from 0.098 to 0.117. Sensitivity is illustrated on Figure 4a, where the dashed curves represent computed failure probability and reliability index if the distribution means of γ_B, Z_B and K_r were increased by ±σ (i.e., the sensitivity δ_i), providing a graphical understanding of the solution variance.

Table 3. Design point, importance and sensitivity vectors for 200-yr WSE.

RV	x^*	u^*	α	δ	η
ϕ'_E	37.8	−1.31E−01	−1.01E−01	1.01E−01	−1.32E−02
γ_E	18.3	−2.80E−01	−2.17E−01	2.17E−01	−6.05E−02
c'_B	1.11	−2.32E−01	−1.80E−01	1.80E−01	−4.18E−02
ϕ'_B	35.6	−2.21E−01	−1.71E−01	1.71E−01	−3.76E−02
γ_B	16.3	−9.91E−01	−7.67E−01	7.67E−01	−7.60E−01
Z_B	4.56	−6.93E−01	−5.38E−01	7.95E−01	3.09E+00
K_r	1830	7.42E−02	5.76E−02	−3.02E−01	2.95E−02

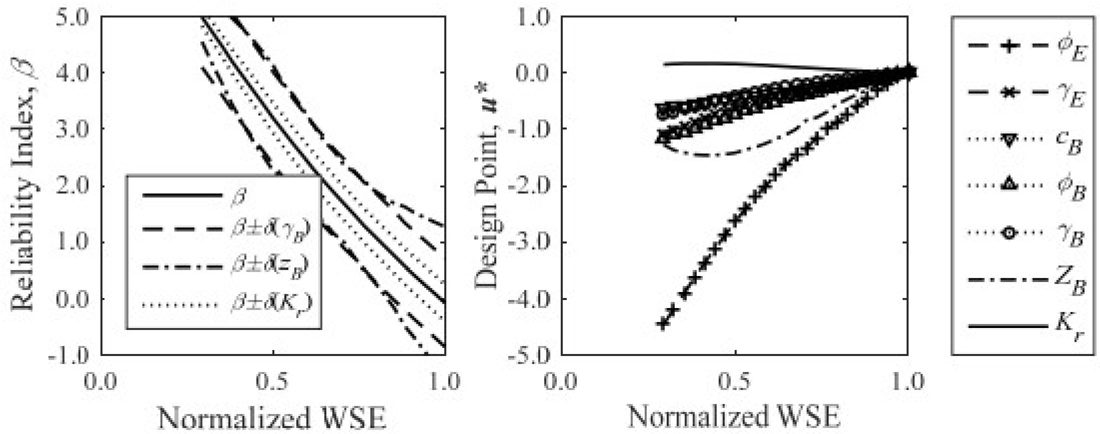

Figure 4. (a) Reliability index, *β*, and (b) design point, *u, for all WSE.**

A quadratic response surface with fit points at U={−1, 0, +1} generally provides sufficient accuracy for all random variables considered in this study, with the exception of K_r. Relative error across a range of values produced residuals within a few percent of the true solution when evaluation points were close to the fit points used to generate a response surface. As shown in Figure 2b, the shape of $FS(K_r)$ is characterized by a sharply decreasing linear segment transitioning abruptly near to a relatively horizontal linear segment, which is poorly represented by a quadratic function. Response surface error is presented in Figure 5a for quadratic, cubic and quartic models where all variables were set simultaneously to values of $-3 \leq U \leq +3$. Cubic models required a third fit point, which was either U=2 or 3; the quartic surface used both points, as it requires 4 total. Clearly a quartic function best fits the levee response, although cubic response surfaces approximate the general shape, regardless of the fit point used. Regardless, all four models quickly accumulate error once applied beyond ΔU=0.25 outside any fit point used.

Response surface accuracy is significantly improved and numerical errors avoided when fit points of each random variable are transformed to the standard normal space prior to solving Equation 1. To compare accuracy of reliability analyses using a response surface, *FS* was computed at the true design point, the results of which are presented on Figure 5b using the same four models in Figure 5a. If the response surface exactly matches the true solution it should compute *FS*=0 at the true design point (subject to the reliability algorithm tolerance). Despite differences in polynomial order, all four models produce consistent errors in computed *FS*, which are reduced from 1.15 to 0.01 as WSE approaches the crest. The decreasing trend and remarkable accuracy at high WSE are explained by Figure 4b: as WSE increases, the design point converge to u_i*=0 for all random variables, which is where the response surfaces are most accurate.

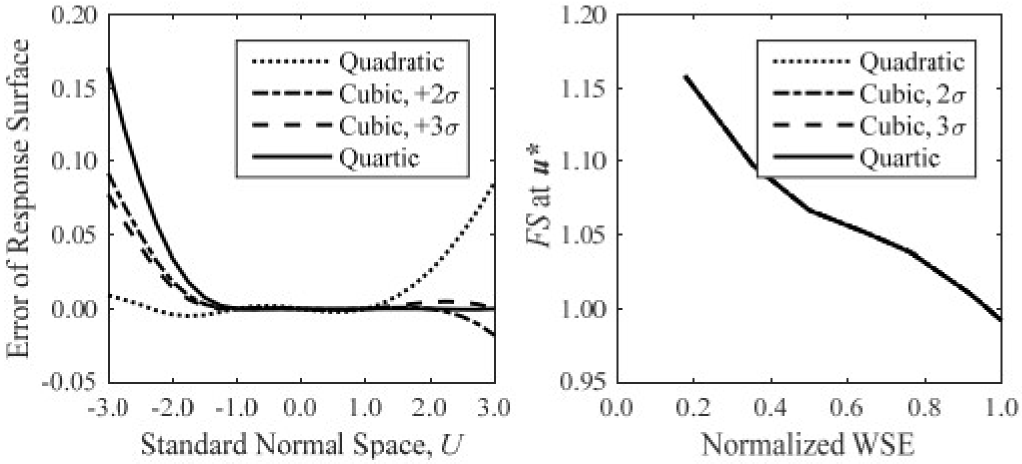

Figure 5. Accuracy of response surface (a) in U-space and (b) at the true design point.

DISCUSSION

Inclusion of seepage parameters in reliability analyses allows comparison of the influence of Z_B and K_r relative to that of strength parameters. While the results presented here do not produce unexpected results, they do illustrate a key deficiency in standard practice. Geotechnical investigations typically focus a disproportionate effort to selecting strength parameters for use in mechanistic models; however, reliability analyses indicate blanket thickness is more important than almost every strength parameter, with γ_B the only exception. Therefore, it may be warranted for similar problems to invest more investigation and testing effort to reducing uncertainty of γ_B and Z_B.

Incorporating a finite element seepage analysis into the reliability analysis required a consistent finite element mesh and sufficient hydraulic conductivity floating point precision to converge to a design point. Inclusion of blanket layer thickness, Z_B, as a random variable allowed for pore pressures and *FS* to be computed consistently across a range of values of Z_B. Pore pressures applied at the base of each slice in the slope stability algorithm are interpolated between finite element nodes using shape functions is therefore dependent on the finite element mesh. Early attempts at solving the seepage-stability algorithm with FORM could not find a solution due to small-scale variations in the computed *FS*, caused by a mesh that was changing at each iteration. For example, ΔZ_B =1 m generally resulted in ΔFS~0.05 but local variations of ΔFS~0.005 occurred on a much smaller scale of Z_B (Figure 6). FORM requires a continuously differentiable limit-state function and local variations in the derivative prevent convergence to a design point. The solution was to allow nodal points nearest the blanket layer base to be adjusted vertically to match the value of Z_B required by each FORM iteration while ensuring that failure surface elements remain constant. The minimum value of the truncated normal distribution for Z_B was therefore set to prevent the FORM algorithm from modifying the mesh within the sliding mass during an iteration. Similar localized variations of safety factor occurred due to perturbations in K_r: ΔK_r=0.1 resulted in ΔFS~0.001 with localized variations of *ΔFS*~0.0001. In this case, however, the problem was caused by insufficient specification of floating point precision in the seepage finite element code.

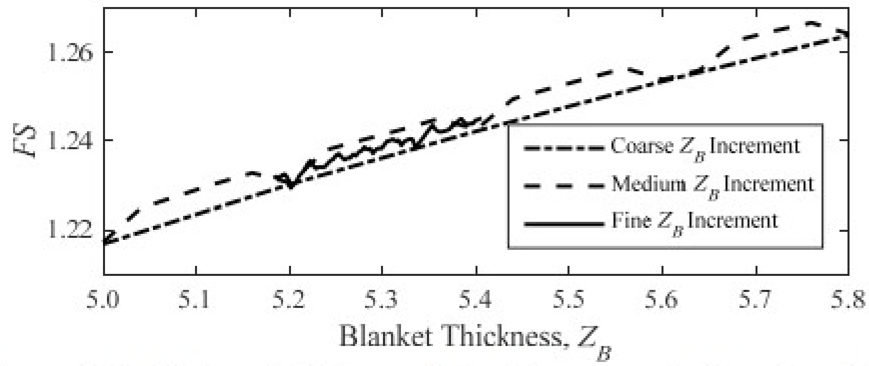

Figure 6. Variation of FS due to finite element mesh changing with Z_B.

Computation time was dramatically affected by the number of iterations required for each FORM analysis, which typically required 3 to 10 iterations to reach a solution. Approximately 5 seconds are required to complete a coupled seepage-stability evaluation, of which 10 evaluations are generally required per FORM iteration on a computer with 16 GB memory and a 4-core 3.4 GHz processor. Therefore, a single reliability analysis can be completed within 3 minutes, but that time could also increase by an order of magnitude if a large number of iterations are required. Improving selection of computation settings and gradient response of the limit-state function could significantly reduce the variability in computation time.

CONCLUSION

We present a FORM analysis of seepage-induced slope stability using a code that couples saturated-unsaturated seepage and slope stability computations within the reliability analysis. Blanket thickness importance was 2 to 3 times that of other strength parameters for the levee example considered, but 0.7 times that of blanket unit weight. Importance of hydraulic conductivity is an order of magnitude less. Model error on the order of 1% caused problems in the FORM algorithm; care must be taken to ensure smooth levee response computations when incorporating finite element solutions in reliability analyses. Response surface methods can produce accuracies on the order of 1% for safety factor and reliability when evaluated at points within the region of fit points used to develop the model. If a response surface is used for reliability analysis the computed design point should be confirmed to lie within the region of fit points.

ACKNOWLEDGEMENTS

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