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PERTURBATIVE QCD

MASTER

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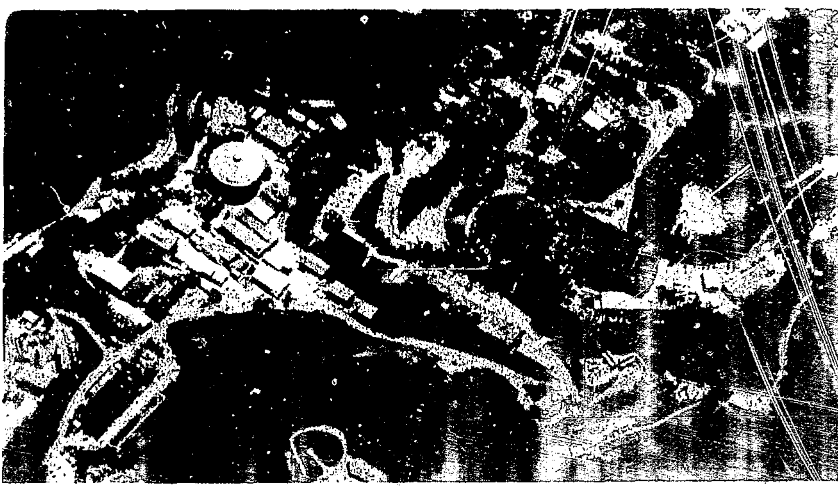

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PERTURBATIVE QCD

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INTRODUCTION

It is impossible in the time allocated for this talk to review the whole of perturbative QCD so I have decided to restrict discussion to a few specific subjects either because the are topical or because they illustrate some general problems in pe urbative QCD. This means that I shall have to omit discussion of ny interesting processes and I apologize to those authors whose wor is not discussed. I will not have time to discuss comparions of QCD with data.[1]

I would first like to review some general prope ies of the QCD perturbation expansion. Given some process P cal ılated through non-leading order, we are entitled to ask whether or not we have a reliable expansion.

$$P = A\ (1 + B\ \frac{\alpha}{\pi})$$

It is a matter of personal taste to decide how large $\frac{B\alpha}{\pi}$ an become before it is clear that we do not have any confidence t t the expansion is converging and hence that we have a reliable ediction. I shall take the attitude that $B \geq 6$ (corresponding to $\frac{B\alpha}{\pi} \sim 0.4$) indicates a problem. Unfortunately B is unambiguous if and only if the process is not partonic and A contains no α_s. $\left(\text{eg } R = \frac{\sigma(e\bar{e} \to \text{hadron})}{\sigma(ee \to \mu\bar{\mu})}\right)$

If A contains α_s there are two ambiguities affecting the size of B. They are the renormalization scheme used to define α_s and the scale at which it is evaluated.* Three popular renormalization schemes are used. They are well known but for completness are described in Table 1.

Table 1

Scheme	Prescription	Comment
MS[2)]	regulate by dimensional regularization. Remove poles ($\frac{1}{\varepsilon}$) appearing in 4-dimensions.	easy to use gauge invariant.
$\overline{\text{MS}}$[3)]	remove $\frac{1}{\varepsilon} + \log\ (4\pi) - \gamma_E$	as above
MOM[4)]	Subtract some 3 point vertex (momenta p_i) at a Euclidean point $p_i^2 = -\mu^2$	inconvenient gauge dependent, vertex dependent.

The relationships between these coupling constants is as follows.

$$\alpha_{\overline{MS}}(\mu^2) = \alpha_{MS}(\mu^2)[1 + .49\ \beta_0 \frac{\alpha}{\pi}]$$
$$\alpha_{MOM}(\mu^2) = \alpha_{MS}(\mu^2)[1 + 7.3\ \frac{\alpha}{\pi}] \quad (1)$$

* These two are really not distinct but it is convenient to think of them as being so.

where $\beta_o = 11 - 2n_f/3$ with n_f equal to the number of flavors. The above result is valid for four flavors in the MOM scheme defined in Landau gauge using the three gluon vertex. For five flavors 7.3 is replaced by 6.1. Other schemes are also possible, for example α could be defined through the value of R at some value of the center of mass energy $(\sqrt{s})$ $\frac{\alpha_R}{\pi} = (R/\Sigma\, e_i^2 - 1)$ where e_i is the charge of the i^{th} quark. There is no a piori reason to prefer any of these schemes, although the presence of log 4π and γ_E in the MS scheme makes it look rather odd. The MOM scheme is difficult to use, but this problem can be circumvented by working in the MS scheme and then using the relationship above. The gauge dependence of α_{MOM} is not a disadvantage.

It is conventional to express α in terms of a scale parameter Λ. The form usually used is

$$\alpha(\mu^2) = \frac{1}{4\pi\beta_o \log(\mu^2/\Lambda^2)} - \frac{\beta_1}{4\pi\beta_o^3} \frac{\log \log(\mu^2/\Lambda^2)}{\log^2(\mu^2/\Lambda^2)} \qquad (2)$$

This form is obtained by making an iterative solution of the two-loop renormalization group equation for α_s. It is not unique, a term of the form $E/\log^2(\mu^2/\Lambda^2)$ could be added to the right hand side and E is arbitrary. It is usually set to zero but other choices are possible.[5] Calculations are performed as power series in α and the introduction of Λ seems superfluous. Λ has the additional disadvantage that data quoted in terms of it appear to have very large errors. Quoting $\alpha_{\overline{MS}}$ (μ_o^2) for some value of μ_o^2 conveys all the necessary information without the disadvantages.

A second ambiguity is the scale μ^2 appearing in α_s. In a process charaterized by a single large momentum transfer Q^2 it is clear that $\mu^2 = Q^2$. But who is to say that $2Q^2$ or $Q^2/2$ is unreasonable? We have the following formula

$$\alpha_s(xQ^2) = \alpha_s(Q^2)[1 - \frac{\beta_o}{4} (\frac{\alpha}{\pi}) \log x] \qquad (3)$$

It is clear that such a shift is capable of capable of removing pieces proportional to β_o from B. The ambiguities in the choice of μ^2 are much worse if there is not one unique large scale. An example is large p_T hadron production where s,t, and u are all candidates for μ^2.[6] It is of course obvious that if A contains α_s to some high power, then the corresponding B will be particularly sensitive to scheme and scale ambiguities.

If the process P is partonic (e.g. Drell Yan, [7] or large p_T [6]) A will contain some parton distribution functions $(q(x, M^2))$; there is ambiguity in that the scale M^2 must be specified, and B depends on this choice. Usually $M = \mu$ is chosen but this is not required and indeed in some processes seems unreasonable.[6]

Based on the foregoing observations one can set up a set of satirical* rules for making a large correction small. If B is large and positive any or all of the following will help to reduce it.

1) Use α_{MOM}
2) Shift μ^2 from Q^2 to xQ^2 with $|x| < 1$ and find a physical reason to justify this choice of x.
3) As for (2) but change M^2 in the parton distributions.
4) Exponentiate something. (See later.)

If B is negative apply the converse of these rules. It is difficult to tell when an application of these rules is justified and when it is not. To really test ones understanding it is necessary to go to one more order; $P = A(1 + B\,(\frac{\alpha}{\pi}) + C(\frac{\alpha}{\pi})^2)$. Having made ones choice to fix B, C is unabiguous and one can ask whether it is large. Unfortunately we have no such calculation. (R is known to order α^2 but there A is independent of α_s and hence B is unambiguous.) Let us now see how these and other problems affict some actual calculations.

II. ONIA DECAYS

Consider the following ratio of widths for the $0^{-+}(\eta_0)$ ground state of an onium made of quarks of mass m_q and charge e_q.

$$\frac{\Gamma(\eta_0 \to \text{hadrons})}{\Gamma(\eta_0 \to \gamma\gamma)} = \frac{2}{9e_q^2}\;\frac{\alpha_s^2(\mu^2)}{\alpha^2}\;[1 + \frac{B\alpha}{\pi}] \tag{4}$$

With $\mu = 2m_q$ and in the MS scheme $B \approx 22.$[8] In the momentum space scheme with $\mu = m_q$ $B = 2.$[9] The choice of μ can be justified by the fact that in lowest order the hadronic system consists of two gluons each carrying energy m_q. Let us now consider the lowest 1^{--} state.

$$\frac{\Gamma(1^{--} \to \text{hadrons})}{\Gamma(1^{--} \to e\bar{e})} = \frac{10(\pi^2-9)}{81\pi e_q^2}\;\frac{\alpha_s^3(\mu^2)}{\alpha}\;[1 + \frac{B\alpha}{\pi}] \tag{5}$$

Again we will use the MOM scheme and now since the hadronic system consists of three gluons in lowest order, take $\mu = 2m_q/3$ since this

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* All good satire is never far from the truth

is the average energy of each gluon. Unfortunately intuition fails for $B = -14$, a large correction. The calculation[10] was originally done in $\overline{MS}$ scheme where for $\mu = m_q$ $B = 9$. Of course we here have a rather extreme situation since the lowest order formula contains α_s^3. Nevertheless I think it is clear that we have a problem. There are ratios of widths one can form in onium decays which are independent of α_s in lowest order. For example consider the $\ell = 1, 2^{++}$ and 0^{++} state[11]

$$\frac{\Gamma(0^{++} \to \gamma\gamma)}{\Gamma(2^{++} \to \gamma\gamma)} = \frac{15}{4}\left(1 + 5.5\,\frac{\alpha_s}{\pi}\right) \tag{6}$$

Before leaving onium decays it is perhaps worth remarking that at some order in α_s all predictions of the type discussed above become sensitive to the onium binding force and predictions become unreliable unless one is in the region of quark masses and energy levels for which the potential is dominated by the Coulombic term.

III. DRELL-YAN

The usual parton model formula for the production of μ pairs of invarient mass Q in a hadronic process of total center of mass energy $\sqrt{s}$ is

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9Q^4} \sum_i e_i^2 \int \frac{dx_1 dx_2}{x_1 x_2}\, q_i(x_1, M^2)\, \bar{q}_i\, (x_2 M^2)\, \sigma \tag{7}$$

where $q_i(x, M^2)$ is the probability of extracting a parton of type i from a hadron with momentum fraction x. In lowest order

$$\sigma = \delta(1 - Z)$$

where $Z = \dfrac{Q^2}{s x_1 x_2}$. The corrections to this process are known[12] σ is replaced by

$$\sigma = \delta(1 - Z)\left[1 + \frac{2\alpha_s}{3\pi}\left[1 + \frac{4\pi^2}{3}\right]\right]$$
$$+ \frac{2\alpha_s}{3\pi}\left[\frac{3}{(1 - Z)_+} - 6 - 4Z + 2(1 + Z^2)\left(\frac{\log(1 - Z)}{1 - Z}\right)_+\right] \tag{8}$$

The corrections of course depend on M, M = Q yields like above result. The corrections are large. The lowest order formula does not depend on α_s so we are immune from scheme and scale dependence. It has been suggested that we take $M_i^2 = Q^2(1 - x_i)$,[13] this being the typical off-shellness of a parton in the process. However even after this modification the corrections are still large. Some

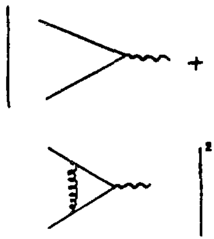

Figure 1(a)

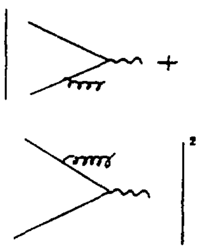

Figure 1(b)

progress can be made if one notices the piece proportional to π^2. This accounts for a large fraction of the correction, and is proportional to the lowest order $(\delta(1 - Z))$. This term has its origin in the failure to obtain an exact cancellation between real and virtual graphs. Suppose we evaluate the graph of Fig. 1(a) using a gluon mass λ to regulate the infra red divergence. We obtain a contribution to the Drell-Yan cross section of the form $\log^2 Q^2/\lambda^2$. A similar divergence is contained in the real graphs Fig.1(b). These generate a contribution proportional to $|\log^2(-Q^2/\lambda^2)|$. The sum of real and virtual graphs removes the λ dependence but leaves a π^2. Since this term is associated with the leading infrared divergence, and hence with the quark form factor one might expect that similar π^2 will appear in all orders. It is known that if one sums these leading divergences to all orders the result exponentiates into the so called Sudakov form factor.[14)] One might expect that the π^2 would also exponentiate.[15)] A more detailed argument on these lines leads to a form for σ as follows.[16)] The form is readily given in terms of the moments of

$$\sigma_n = \int Z^{n-1}\sigma(Z)dZ$$

$$c_n = \exp\left[\frac{\alpha_s(Q^2)}{2\pi}\left[\frac{4\pi^2}{3} + 4\log^2 n\right]\right]\left[1 + \frac{\alpha}{\pi} f(n)\right] \qquad (9)$$

with f(n) now small. Irrespective of whether one believes in this exponentiation it is interesting to note that most of the correction simply renormalizes the magnitude of the Drell-Yan cross-section but does not change its shape as a function Q^2/s. This has led to a parametization of the data by lowest order Drell-Yan multiplied by a so called K factor. The data seen to require a factor of order 2.3, but the K factor obtained above $\left(e^{\frac{\alpha}{2\pi}\{4\log^2 n - \frac{4\pi^2}{3}\}}\right)$ although of the right order of magnitude is not completely independent of Q^2/s.

Recently the QCD corrections to $\frac{1}{q_T}\frac{d}{dQdq_T}$ where q_T is the tranverse momentum of the μ pair have been calculated.[18] The authors organize their result in terms of a K factor defined by

$$K = \frac{\left(\frac{1}{q_T}\frac{d\sigma}{dQdQ_T}\right) \text{ leading order and correction}}{\left(\frac{1}{q_T}\frac{d\sigma}{dQdq_T}\right) \text{ leading order.}}$$

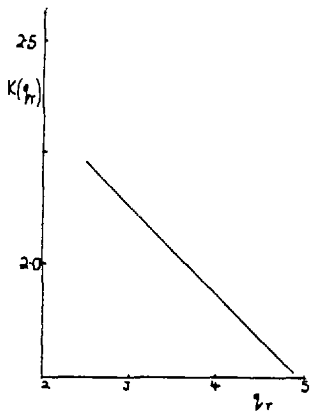

Figure 2

K is shown in figure 2 for Q = 6.5 and $\sqrt{s}$ = 19.4 GeV. Again we see the corrections are large. The perturbation expansion for large q_T muon pairs is valid provided $q_T \sim Q$. If $m_p \ll q_T \ll Q$ where m_p is the mass of the proton, then the perturbation expansion will be spoiled by coefficients of order $\log Q/q_T$. It is possible to resume the perturbation expansion picking out the leading term in $\log Q/q_T$ to all orders in α_s.[19] Unfortunately as yet we have no proof that such a resumation technique is well ordered, i.e. that terms coming from nonleading logs do not overwhelm those from the leading logs. It is far from clear that the expansion should well ordered since the leading terms sum to a factor of the type $e^{-\log^2 Q^2/q_T^2}$ and thus vanish as Q^2/q_T^2 goes to infinity despite the fact that every term in the expansion is large. An algorithm has been proposed for all orders in $\log Q/q_T$ but as yet a proof is lacking.[20] Possibly related to the absence of a proof are two other problems in Drell-Yan.

The whole Drell-Yan formalism for $d\sigma/dQ^2$ relies on the so called factorization theorem.[21] This theorem, proved with varying degrees of rigour, states basically that when corrections to a parton process are calculated divergences coming from the emission of soft gluons cancel and collinear singularites can be absorbed in a universal manner into definitions of the parton distributions. A two loop calculation of the Drell-Yan rate revealed a non-cancelling soft divergence.[22] This divergence is in higher twist so does not yet invalidate the factorization theorem. It raises two problems. The divergence can be removed by using a coherent state formulation,[23] which includes incoming gluons. This observation raises another question in that unlike the case of QED[24] we are unable to build a coherent state formalism valid to all orders. More worrying perhaps is the possibility that at some higher order the soft divergence problem will appear in leading twist.

In a recent paper interactions in the initial state have been considered.[25] The authors consider interactions between spectators and active partons (Figure 3) by soft gluon exchange. The exchange of these soft gluons does not affect the shape of $\frac{d\sigma}{dQ^2}$ but does affect the normalization. In the usual formula there is a factor of 1/3 coming form the fact that the two annihilating quarks must be the same color. The gluon exchanges enable the color to fluctuate changing this factor. Thus the usual formula should be multiplied by a factor A with $1 \leqslant A \leqslant 3$.

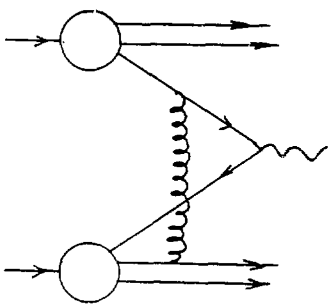

Figure 3

It should be clear from the foregoing that we are far from being able to make believable quantitative calculations in Drell-Yan. Indeed all recent progress has been to confuse and complicate the situation, one can only hope that the next year will bring more positive developments.

IV. EVENT SHAPES IN $e\bar{e}$

Consider $e\bar{e} \to A + B +$ anything and let the angle between A and B be $(\pi - \theta)$. These are three regions of phase space to consider.

(a) $\theta \sim 0$
(b) $\theta \sim \pi$
(c) the rest.

In the final region orthodox perturbation theory is applicable, but in the other regions the expansion is ruined by coefficients proportional to log θ or log(π - θ). The situation here is similar to the intermediate q_T range in Drell-Yan. Leading logs can be re-sumed and lead to Sudakov like factors.[26] However unlike the Drell-Yan case a re-summation scheme valid to all orders can be established.[27] This represents considerable progress but pressure of space prevents detailed discussion the interested reader should consult Ref. 27.

There is some controversy over the higher order corrections in region (c). Three groups have completed a calculation of the structure through order α_s^2.[28-30] Two of these groups[28,29] cast their result in terms of thrust (or the shape parameter C).

$$\frac{1}{\sigma}\frac{d\sigma}{dT} = A_o(T)\,\frac{\alpha_{\overline{MS}}(S)}{\pi} + A_1(T)\left(\frac{\alpha}{\pi}\right)^2 \qquad (10)$$

where S is the center of mass energy squared and $T = \mathrm{Max}\,\frac{\Sigma|p_{i||}|}{\Sigma p_i}$ where p_i is the momentum of the i^{th} particle and $p_{i||}$ is its component along some axis. A_o receives contribution from three body final states ($q\bar{q}g$) and A_1 from three and four body find states. The two groups agree on the values of A_o and A_1 (Figure 4). Notice that the shape of A_1 is similar to A_o and that the correction is large

$$\int_{0.5}^{0.85} \frac{1}{\sigma}\frac{d\sigma}{dT}\,dT = B_o\,\frac{\alpha_s(s)}{\pi}\left[1 + 17\frac{\alpha}{\pi}\right]$$

where B_o is a constant. Near $T = 1$ there are very large corrections due to the presence of $\log^2(1 - T)$ (c.f. $\log^2\theta$ discussed above.) It has been claimed that the origin of the large corrections can be recognized and that they can then be summed to all orders.[31] There are π^2's similar to those in Drell-Yan, these along with $\log^2(1 - T)$ will exponentiate. In addition it is recognized that the scale μ^2

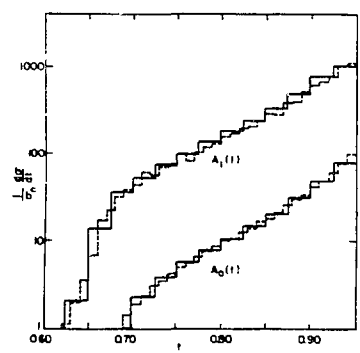

Figure 4

should perhaps not be S but something smaller and T dependent. These operations will reduce the size of the correction but unfortunately the arguments are not rigorous, and in addition unlike the Drell-Yan case a complete analytic formula is not available.

A third group of authors[30] prefers to discuss events in terms of jets. To do this they require that all but a fraction of the total energy $\varepsilon/2$ be deposited into three cones of opening angle δ. They are therefore excluding some fraction of the four body final states included by the groups calculating $\frac{d\sigma}{dT}$. The cross-section is presented in terms of $\frac{d\sigma}{dx_{max}}$ where x_{max} = largest $\left(2E_i/\sqrt{s}\right)$ and E_i is the energy of the i^{th} jet. Of course $\frac{d\sigma}{dx_{max}}$ depends on ε and δ. Figure 5 shows the result. The variation with ε and δ is quite strong. The authors claim that for a range of ε, δ the corrections are small. A calculation of the same quantity using the matrix elements of Ref. 28 obtains the same result[32] showing that there is no disagreement between the various groups. The range of ε, δ for which the correction is less than 30% is reported to be $\varepsilon \geqslant 0.05$, $60^\circ \leqslant \delta \leqslant 36^\circ$. It is clear then that if one wants to compare with data and extract a meaningful value of α_s, one must impose these ε, δ cuts on the data. Of course hadronization will smear ε and δ restricting the range of validity still further.

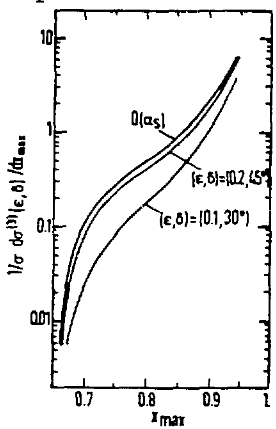

Figure 5

So what is the status of the perturbation expansion in $e\bar{e}$? It is clear that predictions for $\frac{d\sigma}{dT}$ are not reliable. The introduction of ε, δ provides an additional ambiguity to be added to the list in the introduction. It should not be surprising that for some ε, δ the corrections can be made small. I do not know of any a priori argument to arrive at the "correct" values of ε, δ, and it is possible that we have merely defined the problem away to this order and that it will return with a vengence when next order is calculated. As with most processes the acid test awaits the calculation of yet one more order.

V. HIGHER TWIST

Corrections due to higher twist afflict all processes but it is usual to worry about it only if the QCD pertubation in leading twist is well behaved. It is most relevant for attempts to extract α_s from deep inelastic scattering data. It is clear from most discussion of higher twist that it is most important near the kinematic boundary $x = 1$.[33] Most models of higher twist parameterize it in terms of some x function which is usually not normalizable. Some of these models are based on the concept of diquarks. None of these models can be derived from QCD and their failure would not cause me to declare that QCD were wrong. Phenomenological models and their relationship to the data are considered by the next speaker[34] so I will not discuss them here. I would like to refer briefy to an attempt to calculate higher twist using Bag model wave functions.[35] Consider the moments of a non singlet structure function

$$\int x^{n-2} F(x, Q^2)dx = \sum_{p=0}^{\infty} \frac{A_{p,n}(\log Q^2)^{d_{p,n}}}{(Q^2)^p} \tag{11}$$

The usual QCD term corresponds to $p = 0$ in this series. The $d_{p,n}$'s are calculable in QCD perturbation theory but the $A_{p,n}$'s are not. The A's are related to matrix element of operators evaluated in the proton state.

$$A_{np} \approx \langle P|\theta_{n,p}|P\rangle$$

where θ is some local operator. Given a model for the proton wave function the A's can be calculated. The leading twist matrix elements ($p = 0$) have already been evaluated.[36] The calculation is not straightforward as some modification of the bag wave functions is necessary.[36] However it was concluded that the $A_{\alpha,n}$'s were about the correct order of magnitude when compared to data. For $n = 2$ A_2 has been calculated recently.[35] We have

$$M_2(Q^2) = M_2^{T=2}(Q^2) - \frac{\Delta^2}{Q^2} + 0\left(\frac{1}{Q^4}\right) \tag{12}$$

with $\Delta \approx 100$ MeV. Using the earlier estimate for the twist two term $\left(M_2^{T=2}(Q^2)\right)$,[36] the twist four term is of order 1% at $Q^2 = 5\ \text{GeV}^2$. This is remarkably small. However as I remarked above higher twist is expected to be most important for large n (x near 1); it will be interesting to see such calculations.

Before leaving higher twist I would like to discuss the angular distribution of Drell-Yan pairs. In $\pi N \to \mu^-\mu^+ x$ the μ^+ angular distribution can be parameterized as $1 + \alpha(x_1)\cos\theta$ where x_1 is the momentum fraction of the anti-quark. QCD predicts $\alpha = 1$. There is a prediction[37] of the dependence of α with x_1 (Figure 6), due to higher twist terms.

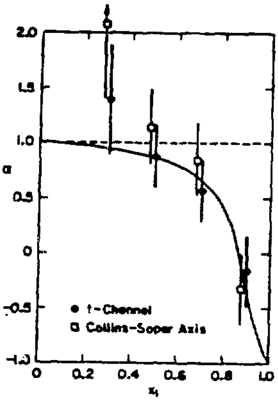

Figure 6

Some data are shown.[38] The predictions are not entirely free of parameters but the agreement is startling. Recent data[39] conflicts strongly with the model, but it appears that there may be a problem in the data analysis. The model contains an arbitrary parameter, called p_T, which sets the scale of the higher twist term. It appears from Ref. 39 that this parameter was taken to be the p_T of the μ pair which it is not.[40] The model may not yet be ruled out.

VI. DOES α_s RUN?

I would like to make some miscellaneous comments about α_s before drawing conclusions. It has been pointed out that $\alpha_s(\mu^2)$ is strictly defined only in the Euclidean region for $\mu^2 < 0$.[41] For $\mu^2 > 0$ it is normal to use $\alpha_s(-\mu^2)$, but maybe one should use $|\alpha_s(+\mu^2)|^2$. These differ only at small μ^2, in the latter case the coupling constant tends to freeze as μ^2 reduces rather than increasing. A similar effect can be produced with a gluon mass.[42] Of course this region of small μ^2 is the region where α_s changes most rapidly so a modification of the type indicated would be helpful for comparison with certain data. There is a QCD prediction for asymptotic behaviour of the baryon form factor $g_m(Q^2)$[43]

$$g_m(Q^2) = \frac{A}{Q^4}\,[\alpha_s^2(Q^2)]^{2+4/3\ \beta_o}$$

where the normalization A is not calculable. We can therefore predict the Q^2 evolution but not the normalization. Experimentally[44] $Q^4 g_m(Q^2)$ is roughly Q^2 independent and the data are in a Q^2 range where α_s is expected to vary rapidly. Note here that $Q^2 < 0$ so discussion of Ref.41) is not applicable. A similar experimental conclusion as to the non-running of α_s can be draw from wide angle exclusive data.[45]

The only positive evidence of a running α_s is from deep inelastic scattering but extracting precise values seems to be difficult.[34] To sum up it appears that there is no process where we have a quatitative disagreement between QCD and experiment. Unfortunately it seems we are far from a definitive quantitative sucess.

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REFERENCES

1. The reader may like to consult two other recent reviews.
   P. V. Landshoff, Talk given at EPS meeting Lisbon, July (1981).
   A. Buras, Talk given at the Lepton Photon meeting Bonn, Aug. (1981).
2. G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972).
3. W. Bardeen, A. Buras, D. Duke, T. Muta, Phys. Rev., D18, 3998 (1978).
4. W. Celmaster and R. J. Gonslaves, Phys. Rev., D20, 1420 (1979).
5. E. Monsay and C. Rosenzweig, Phys. Rev., D23, 1217 (1981).
6. R. K. Ellis, M. Furman, H. Haber, I. Hinchlifre, Nucl. Phys., B173, 397, (1980).
7. S. Drell and T. M. Yan, Phys. Rev. Lett. 25 316 (1970).
8. R. Barbieri, et al., Nucl. Phys., B154 535 (1979).
9. W. Celmaster, D. Sivers, Phys. Rev., D23, 227 (1981).
10. G. P. Lepage and P. B. Mackenzie, CLNS/81-498.
11. A. Barbieri, M. Caffo, R. Gatto, E. Remiddi, Phys. Lett. 95B 93, (1980).
12. G. Altarelli, R. K. Ellis, G. Martinelli, Nucl. Phys., B157, 461 (1979).
    J. Kubar-Andre and F. E. Paige, Phys. Rev., D19, 221 (1979).
13. S. Brodsky, SLAC, PUB 2447.
14. V. Sudakov, Zh eksp theor Fiz, 30, 87 (1956).
15. The argument is not rigorous and the existance of a non-Abelian Sudukov form factor has not been proved. See A. Mueller, Phys. Rev. D20, 2037 (1979).
16. A. Parisi, Phys. Lett., 90B, 295 (1980).
17. See for example J. Bergan, talk given at Cracow School of Physics, June (1981). Saclay-OPLPE-81-03.
18. R. K. Ellis, A. Martinelli, R. Petronzio, CERN TH 3079 (1981).
19. Yu. L. Dokshitzer, et al., Phys. Rep. 58C, 269 (1979
    G. Altarelli, A. Parisi, R. Petronzion, Phys. Lett. 76B, 356 (1978).
20. J. Collins, D. Soper, Talk presented at Moriond, Jan. 81 and Refs. therein.

21. R. K. Ellis, et al., Nucl. Phys., B146, 29 (1978).
S. B. Libby and G. Sterman, Phys. Rev., D18, 3252 (1978).
A. Mueller, Phys. Rev., D18, 3708 (1978).
22. A. Andrassi, et al., Nucl. Phys., B182, 104 (1981).
C. Di' Lieto et al., Nucl. Phys., B183, 223 (1981).
23. C. A. Nelson, Nucl. Phys., B181, 141 (1981).
M. Greco, et al., Phys. Lett., 77B, 282 (1978).
24. N. Papanicolaou, Phys. Rep., 24C, 229 (1976).
25. G. T. Bodwin, S. Brodsky and P. Lepage, SLAC Pub. 2787 (1981).
26. For a review see Dokshitzer, et al., Ref. 19.
27. J. Collins and D. Soper, Talk given at QCD conference Tallahassee, Florida, March (1981),
28. R. K. Ellis, D. A. Ross, T. Terrano, Nucl. Phys. B178, 421 (1981).
R. K. Ellis, D. A. Ross, CERN TH 3131, July (1981).
29. J. A. M. Vermassen, K. J. F. Gaemers, S. J. Oldham, CERN TH 3002, Dec. (1980).
30. K. Fabricus, I. Schmitt, G. Kramer, A. Schierholz, DESY 811035.
31. D. A. Ross, University of Southampton preprint SHEP 80/81-4 (1980).
32. S. Sharpe, LBL-13018, July (1981).
33. A complete list of referances is impossible, see for example Ref. 13 and A. Donnachie and P. V. Lanshoff, Phys. Lett. 95B, 437 (1980).
H. D. Politzer, Nucl. Phys., B172, 349 (1980).
34. Clara Metuzzi, these proceedings.
35. R. L. Jaffe and M. Soldgate. Talk given at the QCD meeting in Talahasse Florida, Feb. (1981). Also CTP 937 to appear in Phys. Lett.
36. R. L. Jaffe and G. G. Ross, Phys. Lett., 93B, 313 (1980).
37. E. L. Berger and S. Brodsky, Phys. Rev. Lett., 42, 940 (1979).
38. K. J. Anderson et al., Phys. Rev. Lett., 43 1219 (1970).
39. J. Badier, et al., CERN-EP/81-71.
40. I am grateful to S. Brodsky for a discussion of this point.
41. M. R. Pennington and G. G. Ross, Phys. Lett. 102B, 167 (1981).
42. G. Parisi and R. Petronzio, Phys. Lett. 96B, 51 (1980).
43. S. Brodsky, P. Lepage, S. A. A. Zaida, Phys. Rev., D23, 1152 (1981). The validity of this prediction has been questioned for a discussion see A. Duncan and A. Mueller, Phys. Rev., D21, 1636 (1980).
44. M. D. Mestayer, SLAC report No. 216 (1978).
45. For a detailed discussion see P. V. Landshoff, Ref. 1.