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LONG-TERM VARIATIONS IN SOLAR DIFFERENTIAL ROTATION AND SUNSPOT ACTIVITY

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Abstract. The solar equatorial rotation rate, determined from sunspot group data during the period 1879–2004, decreased over the last century, whereas the level of activity has increased considerably. The latitude gradient term of the solar rotation shows a significant modulation of about 79 year, which is consistent with what is expected for the existence of the Gleissberg cycle. Our analysis indicates that the level of activity will remain almost the same as the present cycle during the next few solar cycles (*i.e.*, during the current double Hale cycle), while the length of the next double Hale cycle in sunspot activity is predicted to be longer than the current one. We find evidence for the existence of a weak linear relationship between the equatorial rotation rate and the length of sunspot cycle. Finally, we find that the length of the current cycle will be as short as that of cycle 22, indicating that the present Hale cycle may be a combination of two shorter cycles.

1. Introduction

Both the amplitudes and the lengths of sunspot cycles vary. Studies of these variations are important for understanding the mechanism that drives the solar cycle (Bracewell, 1988; Dicke, 1988; Gokhale and Javaraiah, 1990, 1995; Juckett, 2003; Hathaway and Wilson, 2005). In addition, such studies are important to improve our knowledge of the connection between solar activity and space weather (*e.g.*, Eddy, 1976; Lassen and Friis-Christensen, 1995; Rozelot, 2001; Hiremath and Mandi, 2004; Georgieva *et al.*, 2005). It is well known that the lower amplitudes sunspot cycles have longer lengths and vice versa, but the exact physics behind this relationship is not yet known (see Solanki *et al.*, 2002; Hathaway *et al.*, 2003; Schüssler and Schmitt, 2004). The well known Gnevyshev–Ohl, or G–O, rule (Gnevyshev and Ohl, 1948) states that the sum of sunspot numbers over an odd number sunspot cycle exceeds that of its preceding even-numbered sunspot cycle. An important consequence of this empirical rule is that, beside providing information: a Hale cycle begins in an even cycle and ends in the following odd cycle, offering the possibility of predicting the level of activity of the latter from the former. Wilson (1988) showed that on the basis of the variation in the lengths of the cycles it is also possible to

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identify the pairs of sunspot cycles that constitute Hale cycles. The existence of a ∼80 year Gleissberg cycle in solar activity is also well known (Gleissberg, 1942; Hathaway, Wilson, and Reichmann, 1999). However, its statistical significance is not yet clear.

Many determinations of the solar cycle variation of the solar differential rotation, using different techniques and different data have been made (*e.g.*, Balthasar and Wöhl, 1980; LaBonte and Howard, 1982; Komm, Howard, and Harvey, 1993; Antia and Basu, 2000; Javaraiah, 2003a; Ulrich and Boyden, 2005). The results indicate the possibility of the differential rotation having a role in the mechanism of the solar cycle. Javaraiah (2003a) found ∼90 year variation in the differential rotation rate determined from sunspot data during the period 1879–2002. The relationship between the differences in the solar differential rotation rates during odd and even cycles, derived from the same sunspot data, strongly indicate that the solar cycles are not independent and are connected by the long-term variations in the solar equatorial rotation rate and the latitudinal gradient (Javaraiah, Bertello, and Ulrich, 2005). Therefore, the aforementioned ∼90 year variation in the differential rotation rate may be related to the Gleissberg cycle in solar activity. Hence, in this analysis we explore this correlation.

In the next section, we describe briefly the data and analysis. In Section 3, we compare the long-term variations in the solar differential rotation and those of the cycle length and strength over the last 110 years. In the same section, we show that the periodic modulation observed in the latitudinal gradient of rotation rate fits a sinusoidal function with a period of about 79.2 year. In Section 4, from the epochs and the strengths of the drops in the solar equatorial rotation rate (Javaraiah, 2003a,b), we derive some simple empirical rules on lengths of long-term variations of sunspot activity and predict the relative lengths of the double Hale cycle 7 and the current double Hale cycle 6. In Section 5, we show that the finding by Mendoza (1999) of the existence of an approximately linear relationship between the length of the sunspot cycle and the equatorial rotation rate, can only be marginally confirmed using our larger set of sunspot group data. In Section 6, we briefly discuss the implications of this result during the Maunder minimum.

2. Data and Analysis

Figure 1 shows the variation of the monthly Wolf number during the period 1749–2002. In Table I, we give the length, and strength, *i.e.*, the sums of the monthly averaged sunspots over the length of (1) sunspot cycle, (2) Hale cycle, (3) "double Hale cycle", and (4) Gleissberg cycle. We have taken the values of the strengths of the sunspot cycles and the 'Hale cycles (or double sunspot cycle)', and also the values of the lengths of the sunspot cycles, 1–21 from Wilson (1988). We determined the lengths and strengths of cycles 22 and 23 from the average monthly values taken from the website: ftp://ftp.science.mfsc.nasa.gov/ssl/pad/solar/greenwich.htm. We

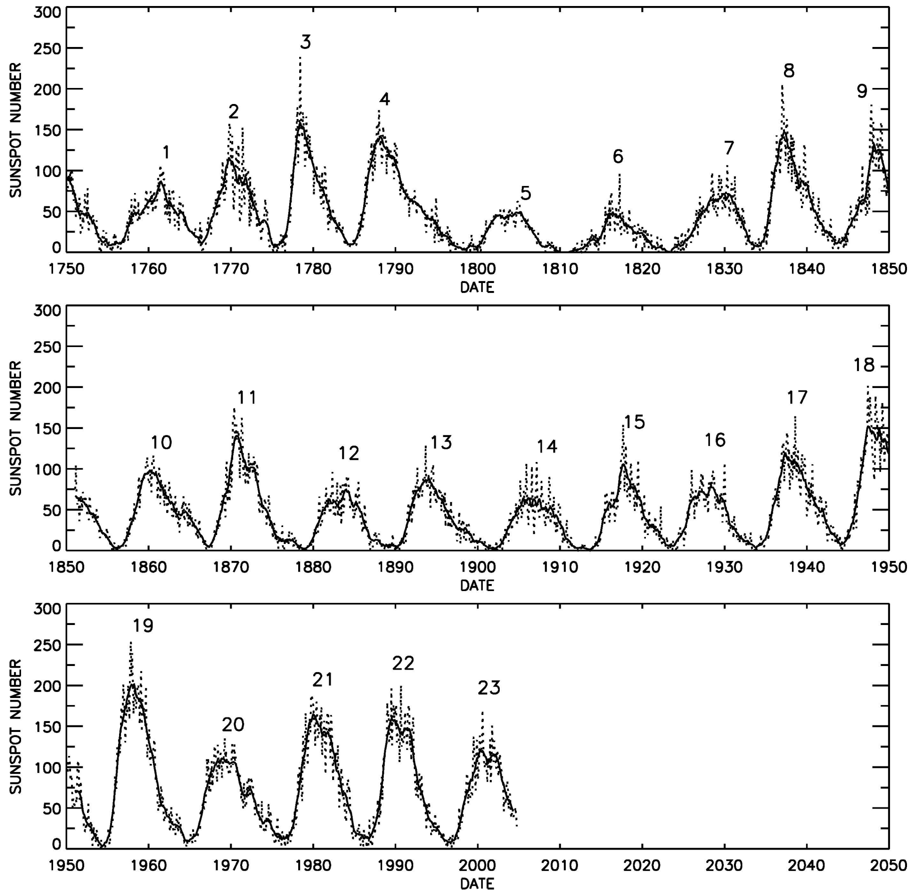

Figure 1. Monthly averaged (*dotted curve*) and smoothed (*continuous curve*) international sunspot numbers during the period 1749–2004 (ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS). The Waldmeir cycle number is marked near the each peak.

have also used the values of the lengths of sunspot cycles determined by Lassen and Friis-Christensen (1995) by means of sunspot minimum epochs with an applied five-coefficient smoothing filter.

We have used the values of the solar equatorial rotation rate (A) and the rotational latitude gradient (B) determined by Javaraiah (2003a) by fitting the Greenwich sunspot group data during the period 1879–1976, and the spot group data from the Solar Optical Observing Network of the US Air Force/US National Oceanic and Atmospheric Administration during 1977 January 1–2004 August 10, to the standard form of solar differential rotation: $\omega(\phi) = A + B \sin^2 \phi$, where $\omega(\phi)$ is the solar sidereal angular velocity at latitude ϕ. The details of the data reduction and the cycle-to-cycle modulations in A and B are discussed in Javaraiah (2003a,b) and Javaraiah, Bertello, and Ulrich (2005).

TABLE I

The strength and the length (in months) of sunspot cycle, Hale cycle, “double Hale cycle”, and Gleissberg cycle, respectively, during 1749–2004.

Sunspot cycle			Hale cycle			Double Hale cycle			Gleissberg cycle		
Number	Strength	Length	Number	Strength	Length	Number	Strength	Length	Number	Strength	Length
1	5647.2	135									
2	6438.7	109									
			1	13845.8	220						
3	7407.1	111									
						1	27359.1	530			
4	10100.6	163									
			2	13513.3	310						
5	3412.7	147									
6	2820.5	153									
			3	7588.6	280						
7	4768.1	127									
						2	23712.1	545			
8	7813.2	116									
			4	16123.5	265						
9	8310.3	149							1	47902.1	1098
10	6549.7	135									
			5	14056.1	276						
11	7506.4	141									
						3	24190.0	553			

(*Continued on next page*)

TABLE I
(*Continued*)

Sunspot cycle			Hale cycle			Double Hale cycle			Gleissberg cycle		
Number	Strength	Length	Number	Strength	Length	Number	Strength	Length	Number	Strength	Length
12	4598.2	134									
			6	10133.9	277						
13	5535.7	143									
14	4459.1	138									
			7	9778.9	258						
15	5319.8	120									
						4	21979.2	505			
16	4956.9	122									
			8	12200.3	247						
17	7243.4	125									
18	9087.4	122							2	60965.4	1016
			9	20556.5	248						
19	11469.1	126									
						5	38986.2	511			
20	8438.2	140									
			10	18429.7	263						
21	9991.5	123									
22	9424.7	124									
			11								
23	7930.0[a]	101[a]									

[a]Indicates the incompleteness of the current cycle 23.

3. Long-Term Evolution in the Rotation and Activity

Figure 2 shows the long-term evolution of *A* and *B* over a period of 11 solar cycles (12–22). Figure 3 shows the long-term evolution of the strength (R_{sum}) and length of the solar cycle over 17 cycles (6–22). Figure 2 shows a significant decrease in the

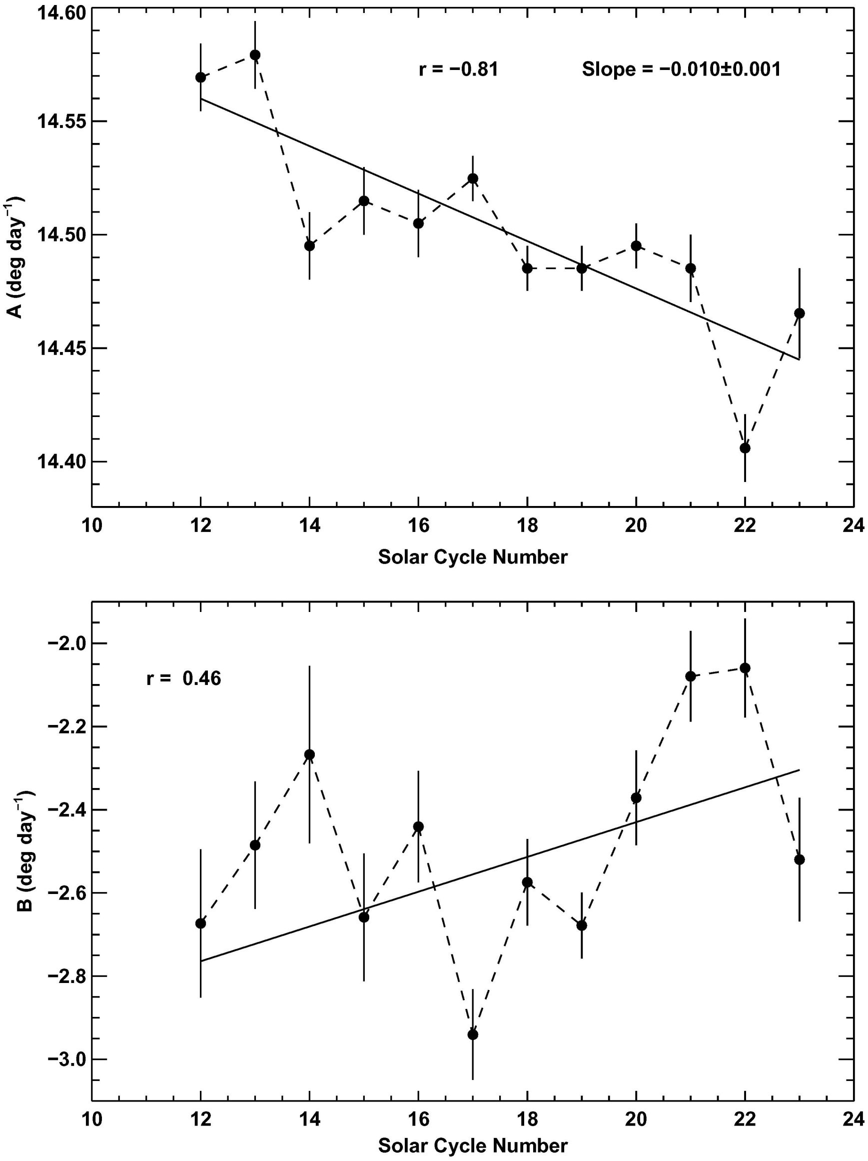

Figure 2. Long-term variations of *A* and *B* during cycles 12–22. For the correlations found to be significant, we also report the value of the slope obtained from the linear regression (note that cycle 23 is not yet complete).

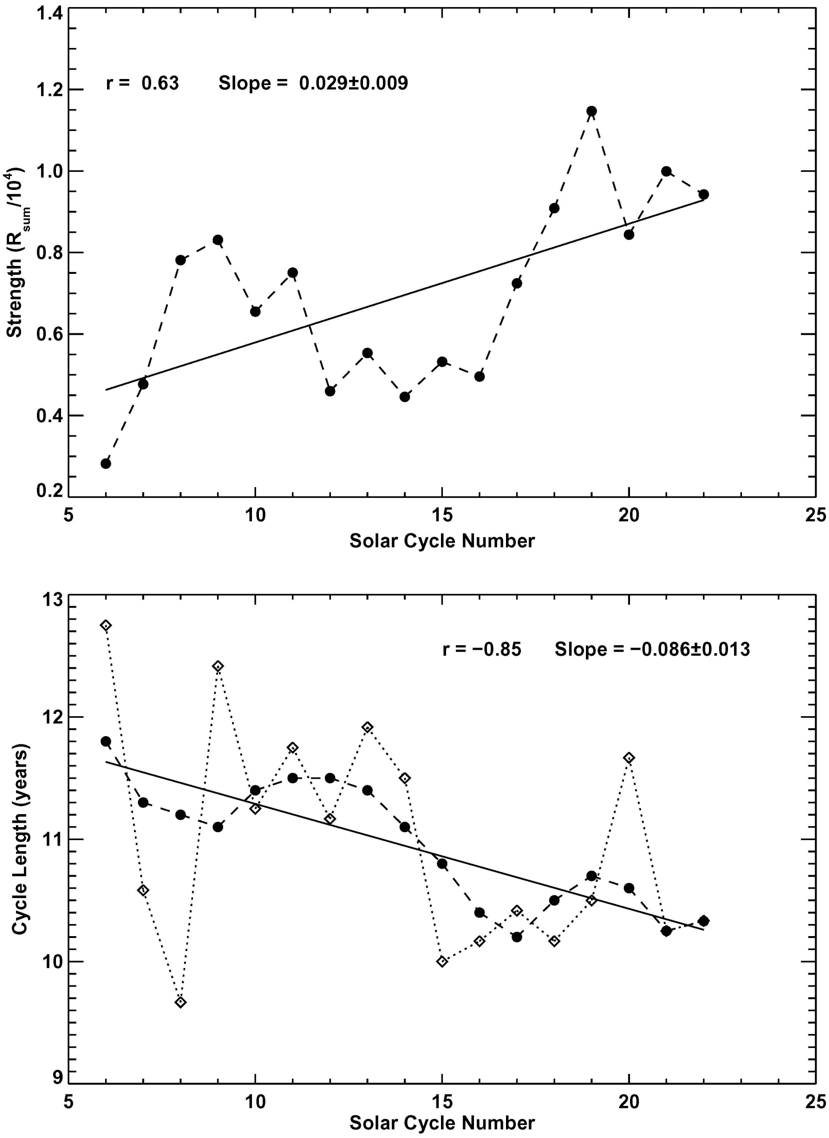

Figure 3. Same as Figure 2 but for strengths and lengths of the solar cycles 6–22. In case of length, the *dotted curve with squares* and the *dashed curve with filled circles* represent the values taken from Wilson (1988) and Lessen and Friis-Christensen (1995), respectively. For the latter, the values of r and *slope* are also given.

equatorial rotation rate over the last 11 solar cycles. We find no evidence for such a long-term decreasing trend in B. Figure 3 shows the strength of the cycle is more than doubled over the last 17 cycles. Solanki *et al.* (2004) found the level of solar activity during the last 70 years is exceptional and the previous period of such high activity occurred more than 8000 years ago. In case of length, the values determined by Lassen and Friis-Christensen (1995) show a significant decrease in the solar cycle length over 17 solar cycles, but no such trend in the values determined by Wilson (1988) from the relatively less smoothed sunspot data. Hence, it is difficult to draw a definite conclusion as to whether there exists a long-term variation in cycle length or not. The result indicates that the long-term decreasing trend in length is weak. In order to measure the strength of the observed correlations, we have performed a linear regression analysis on the pairs of variables, let say x and y, shown in Figures 2 and 3. Here we use the (weighted) Pearson's linear correlation coefficient r to measure the strength of the observed correlation, defined as:

$$r = \frac{\sum_i w_i(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i w_i(x_i - \bar{x})^2}\sqrt{\sum_i w_i(y_i - \bar{y})^2}} ,$$

where $\bar{x}$ and $\bar{y}$ are the means of the x_i and y_i, respectively. The weights w_i are given by $\sigma_i^{-2}/(\sum_i \sigma_i^{-2}/N)$, where σ_i is the standard measurement error associated with y_i and N is the total number of measurements. The calculated r for each pair of variables is indicated on the individual panels of Figures 2 and 3. If the null hypothesis is that x and y are uncorrelated ($r = 0$), it can be shown that under some generally acceptable assumptions, the statistic

$$t = r\sqrt{\frac{N-2}{1-r^2}}$$

is distributed like a Student's t-distribution with $\nu = N - 2$ degree of freedom (*e.g.*, Alder, 1977). In order to decide whether, for a sample of size N, a given correlation coefficient indicates a linear relationship, we test the hypothesis that the sample is chosen from a population for which $r = 0$ and, therefore, determine the probability that from such a population a sample of size N is taken for which the correlation coefficient equals or exceeds the absolute value of r calculated for the given sample. Here we use a two-tailed test of significance. If the probability p is less than 5%, we reject the hypothesis that the sample is taken from a population in which there is no linear relationship.

On the basis of the above criterion we found that A is well correlated with the solar cycle number ($r = -0.81$, $t = 4.361$, and $p = 0.1\%$). The variations of the strength ($r = 0.85$, $t = 4.942$, and $p = 0.1\%$) and length (the values determined by Lessen and Friis-Christensen, 1995) ($r = -0.81$, $t = 4.094$, and $p = 0.3\%$) are also significantly correlated with the solar cycle number, while the correlation between B and the solar cycle number is found to be statistically insignificant ($r = 0.46$, $t = 1.630$, and $p = 13.4\%$). For the correlations found to be significant, we have determined the coefficients of the linear regression, in particular the value of the slope which is indicated in Figures 2 and 3. The goodness-of-fit was calculated by comparing the value of the χ^2 merit function to the Chi-square probability distribution for $N - 2$ degrees of freedom. This test shows that all relationships are well described by a linear model, in agreement with the results of our analysis of r.

It may be interesting to note here that A is slightly increased from cycle 14 to cycle 17 (see Figure 2), *i.e.*, during the double Hale cycle 4 which followed the big drop from cycle 13 to cycle 14. It is difficult to draw such a conclusion about the variation of A during the double Hale cycle 5 which was preceded by the moderate drop from cycle 17 to cycle 18. However, the pattern supports the idea that A tends to slightly increase monotonically during each double Hale cycle. Therefore, A is expected to be slightly increasing from cycle 22 to cycle 25, *i.e*, during the current Hale cycle 6 which is preceded by the big drop from cycle 21 to cycle 22. The current value of A for cycle 23 is indeed significantly larger than that of cycle 22. Javaraiah (2003b) predicted that the current double Hale cycle (which comprises cycles 22, 23, 24, and 25) is expected to be considerably weaker than the previous one. It is interesting to note here that recent studies of sunspot activity indicated that the present epoch is at the onset of an upcoming minimum in the long-term solar variability (*e.g.*, Bonev, Penev, and Sello, 2004, and references therein).

A general tendency of an anti-correlation between level of activity and rate of rotation has been reported in several studies (*e.g.*, Hathaway and Wilson, 1990; Obridko and Shelting, 2001; Javaraiah, Bertello, and Ulrich, 2005). Figures 2 and 3 also show that while A is systematically decreasing the strength is increasing, indicating an anti-correlation between A and amount of activity on a time scale of about 100 years. According to an empirical rule derived by Javaraiah (2003b) the size of a drop in A is a predictor of the strength of the double Hale cycle which follows the drop. That is, a weaker double Hale cycle is preceded by a larger drop in A, suggesting that the drop in A suppressed the activity during the double Hale cycle. This contradicts the aforementioned anti-correlation between the amount of activity and rotation rate. However, the decreasing trend in A is mainly because of the big and moderate drops that occurred in the intervals of about four cycles. (In fact, as mentioned above there is a small monotonic increase in A after each big drop.) As the drop of A is bigger, its effect on activity seems to be greater and persisting for relatively a longer time (up to about 44 years and even more).

In Figure 2, as pointed out by Javaraiah (2003a,b), there is a suggestion of about a 90 year cycle in B. In order to investigate the existence of such a periodicity not only in the B time series but also in the other parameters that are plotted in Figures 2 and 3, we have removed the estimated linear trend from the data and fit the residuals to a model which consists of a simple cosine function above a constant background. The results of the least-square fits are shown in Figures 4 and 5 for the linear case, the goodness-of-fit was calculated by comparing the value of the χ^2 merit function to the Chi-square probability distribution for $N - 4$ degrees of freedom. With the exception of the A coefficient time series, this test shows that the relationships between the residuals and the solar cycle number are well described by such a simple model, and the deviations are not significant on the basis of the 5% level of significance. The calculated periodicity for B is 7.4 ± 0.5 solar cycles. If we use as a mean value for the solar cycle the average of the lengths given in Table I, this periodicity corresponds to about 79.2 year, a value that is consistent

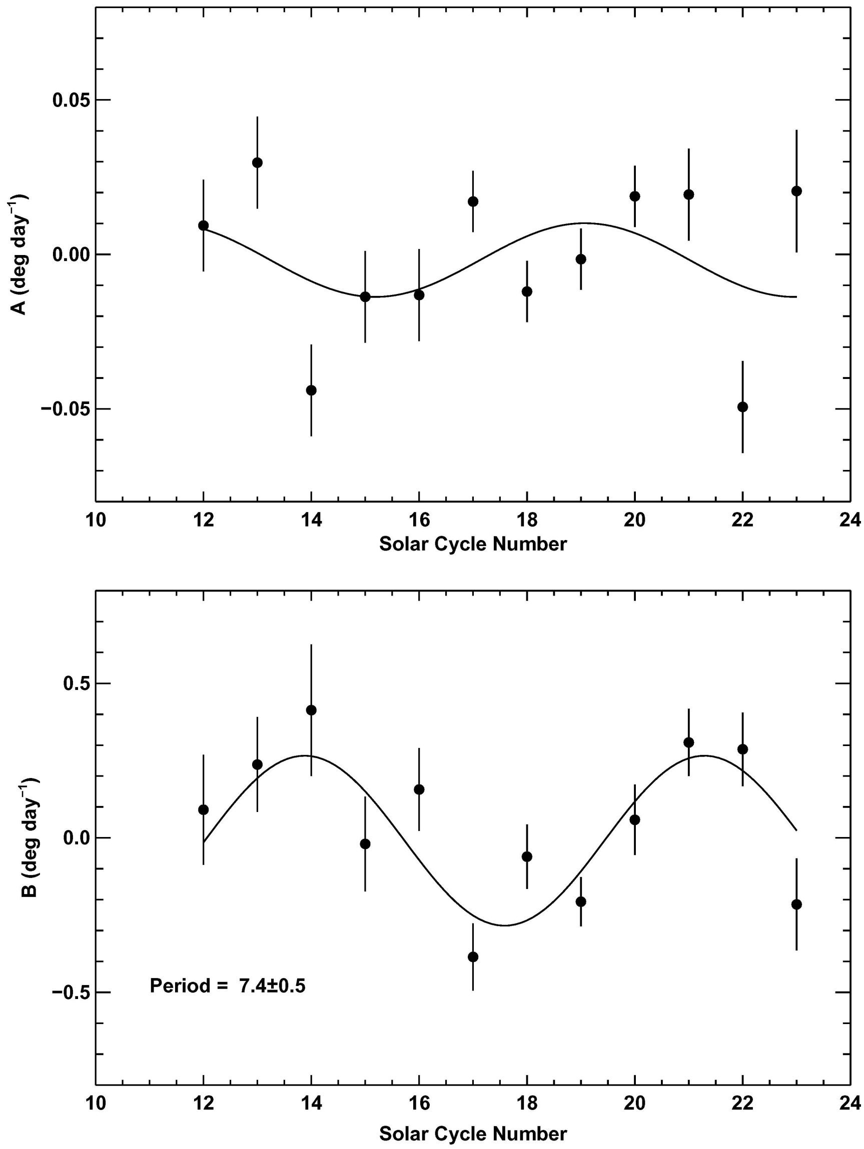

Figure 4. Residuals of the quantities plotted in Figure 2, after removing the estimated linear trend. The *solid line* in each plot is the *cosine function* fit to the residuals. The relationship between the residual of B and the solar cycle number is well described by this simple model. The resulting period, in units of solar cycle number, is indicated for this case.

with what is expected for the existence of the Gleissberg cycle. As expected, both cycle strength (see also Hathaway, Wilson, and Reichmann, 1999) and length (more smoothed time series) show similar periodicity. The level of sunspot activity is now in the declining phase of the current Gliessberg cycle, whose minimum is expected near cycle 25.

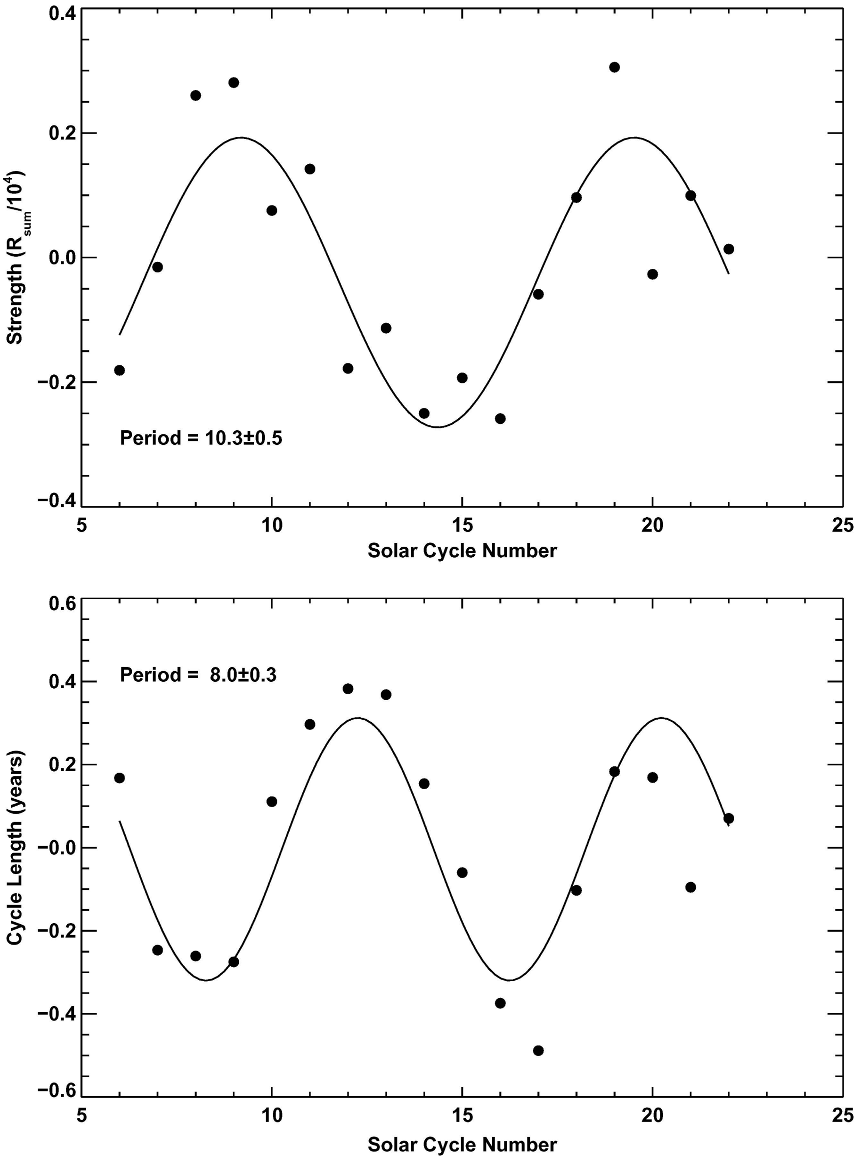

Figure 5. Same as Figure 4 but for the quantities plotted in Figure 3 (in case of lengths, for the values taken from Friis-Christens (1995)).

4. Prediction of Relative Lengths of Double Hale Cycles 6 and 7

Using the pattern of long-term variations in *A*, Javaraiah (2003b) derived some simple empirical rules that allow the prediction of strengths of the future double Hale cycles. From Figure 2 and Table I it seems that such empirical rules can also be derived for the length of long-term variations in sunspot activity. In Table I, it can be seen that the known approximate inverse relationship between the amplitude and the length of a sunspot cycle seems to be applicable also to the ‘double sunspot cycles’ or Hale cycles, the double Hale cycles, and the Gleissberg cycles in sunspot activity (*i.e.*, there is an evidence for the existence of a negative correlation between strength and length of the variation whose time scale is longer than 11 years also). However, we notice that within some Hale cycles the behavior of the even–odd cycles pairs differ from the inverse relationship between the cycle length and strength. Such an anomaly seems to be more evident in the constituents (double Hale cycles) of the Gleissberg cycles. For example, it can be seen that the double Hale cycle 2 is considerably weaker and also shorter than the double Hale cycle 3. The double Hale cycle 4 is also considerably weaker and shorter than the double Hale cycle 5. That is, there is a suggestion that within a Gleissberg cycle, the preceding double Hale cycle which followed the big drop in A is slightly shorter than the double Hale cycle which followed the moderate drop (this can be seen with the help of Figure 2 and Table I). Therefore, the length of the current double Hale cycle 6 which follows the big drop in A from cycle 21 to cycle 22 is expected to be shorter than that of the double Hale cycle 7, which is supposed to be preceded by a moderate drop in A from cycle 25 to cycle 26.

5. Relationship Between Equatorial Rotation Rate and Cycle Length

Mendoza (1999) obtained the following linear relation between A (in deg day^{-1}) and cycle length (L, in years): $A = 0.08L + 13.66\,\text{deg day}^{-1}$ (with correlation coefficient, $r = 0.69$ and ± 0.03 error in the *slope*), using the values of A determined by Balthasar, Vázquez, and Wöhl (1986) for cycles 12–20 and the lengths of cycles determined by Lassen and Friis-Christensen (1995). In Figure 6, we show the correlation between A and L as determined in this work. We also used the values of the lengths of the cycles 12–22 determined by Lassen and Friis-Christensen (1995), but we have used the values of A shown in Figure 2. The uncertainties in these values of A are relatively less than those determined by Balthasar, Vázquez, and Wöhl (1986) because some extra precautions, including those suggested by Ward (1966), were taken into account in the data reduction (see Javaraiah and Gokhale, 1995) and we have also used the updated Greenwich data (see Javaraiah, 2003a). We obtained the following linear regression fit between A and L:

$$A = (0.06 \pm 0.01)L + 13.90 \pm 0.10\ \text{deg day}^{-1}. \qquad (1)$$

The correlation coefficient, however, is just marginal, $r = 0.57$ and it indicates a significant probability that the sample is taken from a population in which there is no linear relationship between the two variables ($t = 2.088$, $p = 6.6\%$). In conclusion, we find only a very weak evidence for the existence of a linear relationship between A and L.

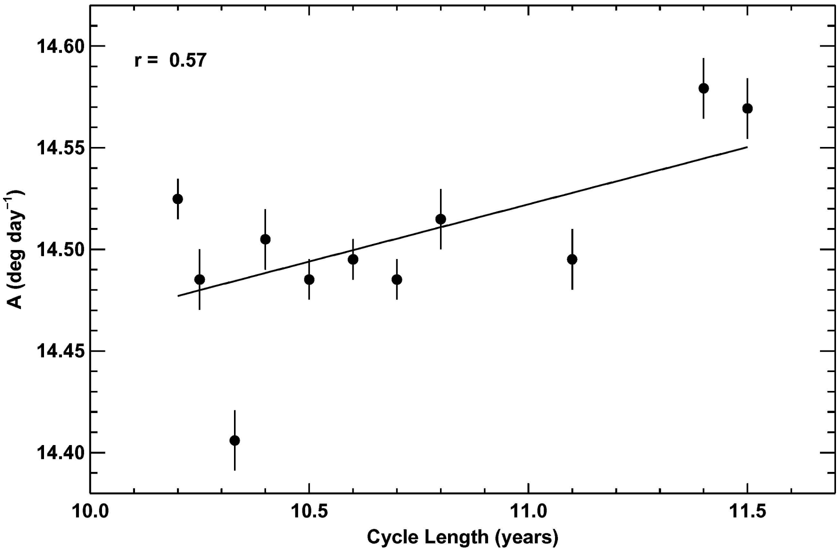

Figure 6. Plot of the cycle length (*L*) vs. equatorial rotation rate, *A* and the corresponding linear regression given by Equation (1). The value of the correlation coefficient *r* is also shown.

While only about 75% of the current cycle 23 is completed, we can use the current mean value of 14.46 deg day^{-1} for *A* during this cycle. We have already used this value to derive a linear relationship between the equatorial rotation rates during odd and even cycle (Javaraiah, Bertello, and Ulrich, 2005). If we use this value in Equation (1), we find that the length of cycle 23 would be 9.3 ± 2 years. Therefore, since cycle 22 is a short cycle (length is 10.3 year), the present Hale cycle may be a combination of two short cycles (22 and 23) and may be satisfying the 'bimodelity' of sunspot cycles, a characteristic of the sunspot cycles which was found by Wilson (1988). The aforesaid predicted length of the current cycle 23 suggests that this cycle, which began in May of 1996, is expected to end in late 2005. A similar conclusion is drawn by Hathaway and Wilson (2005) from the anti-correlation between the period of a cycle and the amplitude of following cycle.

6. Discussion

From the analysis of the cycle-to-cycle variation in the solar equatorial rotation rate, determined from the data on sunspot groups during the period 1879–2002, Javaraiah (2003b) derived some simple empirical rules which predict the strengths of the future double Hale cycles of sunspot activity. We show here that such rules can also be drawn for the length of the long-term variations of sunspot activity, by combining information about sunspot numbers and the differential rotation. We strongly feel that this is reasonable since the variations of solar rotation are most likely related to variations in solar activity. This study and the results discussed in Javaraiah (2003b) suggest that the solar activity behaves systematically on time scales of 11 years to at least 100 years.

Figure 1 shows that the pair of two consecutive cycles, cycle 4 and cycle 5, violate the G–O rule. Wilson (1988) noticed that due to the exceptionally long length of cycle 4, this pair of cycles does not satisfy the 'bimodelity' rule of sunspot cycles he derived from his analysis. He attributed this result to the lack of reliable observations during these two cycles. On the other hand, the cycle pair 22–23 seems to satisfy the 'bimodelity' rule of sunspot cycles, although they violate the G–O rule. However, in the case of the cycle pair 4–5, it is still not clear whether the G–O rule was actually violated or if there was an additional weak cycle in the 1790s as reported by Usoskin, Mursula, and Kovaltsov (2003).

There are indications from the variations in cosmogenic isotopes, such as ^{10}Be, that the cyclic behavior of solar activity might have existed during the Maunder minimum, 1645–1715 (*e.g.*, Beer, Tobias, and Weiss, 1998). Using the Hevelius' solar drawings during the period from the autumn of 1642 through the autumn 1644, Eddy, Gilman, and Totter (1976) found that the mean value of A over that period is $\sim$4% higher than the mean value of A during modern times. In view of this result, many scientists extrapolated the results found from the data on sunspot groups during the modern time to the Maunder minimum period. Recently, Mendoza (1999) used the linear relationship between A and cycle length found for cycles 12–20 (see Section 5) and the mean value of A in the period 1642–1644 (which is 4% higher than the mean A during the modern time) to find that the length of a solar cycle during the Maunder minimum should have been $\sim$17 year. If, as mean value of A, we use 14.51 deg day^{-1} determined by Javaraiah (2003a) increased by 4% from Equation (1) we get that the length of a cycle during the Maunder minimum should have been $\sim$20 year. This is somewhat closer to the value of $\sim$22 year found by Usoskin, Mursula, and Kovaltsov (2000) using delayed component techniques. However, using the observations made by La Hire a half century later than Hevelius, Nesme-Ribes *et al.* (1993) found that the rotation should have been slower during the Maunder minimum than during the modern time. Recently, Vaquero, Sánchez-Bajo, and Gallego (2002) analyzed Flamstead's observations of sunspots during the period from 1684 April 25 to 1684 May 8 and obtained a synodic rotation rate of 12.8 ± 0.2 deg day^{-1}, which is about 5% lower than the modern value. From the long-term temporal variations of A and the level of sunspot activity during the current era, Javaraiah (2003b) found that a 10 times bigger drop in A might have occurred near the beginning of the Maunder minimum, as compared to the value observed during modern times. This conjecture is consistent with both results, *viz.*, about 5% low value, $\sim$13.8 deg day^{-1} (sidereal), in the deep Maunder minimum found by Vaquero, Sánchez–Bajo, and Gallego (2002) and about 4% higher value was found by Eddy, Gilman, and Totter (1976), for the period 1642–1644, *i.e.*, just before the beginning of the Maunder minimum. If we use the aforesaid low value, from Equation (1) we get a negative value for a cycle length. This is meaningless. Therefore, Equation (1), which corresponds to a positive correlation between *A* and *L* during modern times, cannot be applied for determining the lengths of the cycles during the Maunder minimum. On the other hand, during the deep Maunder minimum, the activity was nearly completely absent (maybe only about 10% of the amount of activity during modern times, Javaraiah, 2003b), which makes it difficult to identify clearly the beginnings and the endings of the cycles.

7. Conclusions

In this paper, we have investigated the long-term variations of solar differential rotation and sunspot activity using the sunspot group data for the period 1879–2004. This analysis allowed us also to make some predictions on the future behavior of these quantities. The following conclusions are drawn from this work:

(1) The solar equatorial rotation rate significantly decreased over the last century, whereas the level of activity has increased considerably.
(2) The latitudinal gradient component of the solar rotation rate shows a strong periodicity of about 79.2 year, confirming the existence of the Gleissberg cycle in this component found in an earlier study (Javaraiah, 2003a).
(3) The equatorial rotation is expected to slightly increase from cycle 22 to cycle 25, *i.e.*, during the current double Hale cycle 6, while the level of activity is expected to be weaker than that of the previous double Hale cycle.
(4) The length of the next 'double Hale cycle' is expected to be longer than the current one.
(5) The existence of an approximate linear relationship between the equatorial rotation rate and the length of sunspot cycle, as found by Mendoza (1999), could not be proven using our relatively longer time series of sunspot group data. We find only a very weak evidence for this linear relationship.
(6) The current Hale cycle 11 seems to be a combination of two short sunspot cycles, viz., the cycles 22 and 23, suggesting the current cycle 23 may be ending at early 2006.

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