Lawrence Berkeley National Laboratory
LBL Publications

Title

Effective and N-Body Collisions on Subthreshold Kaon Production in High-Energy Nuclear Reactions

Permalink

https://escholarship.org/uc/item/9q56k438

Authors

Batko, G.
Randrup, Jorgen
Vetter, T.

Publication Date

1992-03-01

eScholarship.org Powered by the California Digital Library
University of California

LBL-31828
UC-413
Preprint

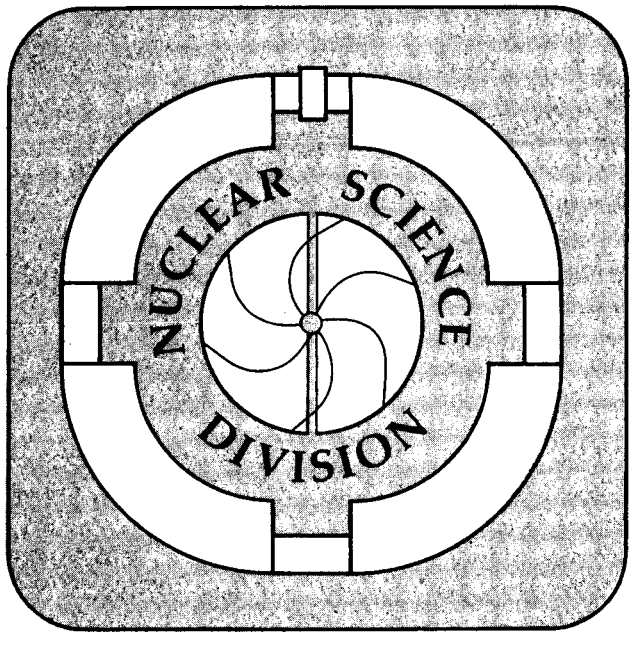
Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Submitted to Nuclear Physics A

Effect of *N*-Body Collisions on Subthreshold Kaon Production in High-Energy Nuclear Reactions

G. Batko, J. Randrup, and T. Vetter

March 1992



Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098

| LOAN COPY |
|Circulates |
|for 4 weeks| Bldg. 50 Library.
Copy 2
LBL-31828

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. Neither the United States Government nor any agency thereof, nor The Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or The Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or The Regents of the University of California and shall not be used for advertising or product endorsement purposes.

Lawrence Berkeley Laboratory is an equal opportunity employer.

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Nuclear Physics A LBL-31828

Effect of N-Body Collisions on Subthreshold Kaon Production in High-Energy Nuclear Reactions *

Gustavo Batko[1], *Jørgen Randrup*[1,2], *and Thomas Vetter*[3]

[1] Nuclear Science Division, Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720, USA

[2] Gesellschaft für Schwerioneneforschung mbH
D-6100 Darmstadt 11, Germany

[3] Institut für Theoretische Physik, Justus-Liebig-Universität
D-6300 Giessen, Germany

March 3, 1992

ABSTRACT:

We recently generalized the standard *BUU* transport model to incorporate N-body collisions. This model has now been further developed so as to allow the calculation of the energy spectra of kaons produced in nuclear reactions at relativistic energies. Applications to the systems Ca+Ca, Nb+Nb, and Au+Au at bombarding energies of 600 and 1000 MeV per nucleon suggest that the effect of N-body collisions on the kaon spectra is relatively minor.

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098; and by a NATO Science grant.

1 Introduction

With few exceptions, microscopic dynamical simulations of high-energy nuclear reactions have relied on the picture of sequential two-body scattering between the hadrons. About ten years ago, Kodama *et al.*[1] sought to generalize the intra-nuclear cascade description to take account of non-binary processes, by using the energy-dependent interaction range to determine the number of nucleons participating in a given scattering process. Somewhat later, Mrówczyński [2] discussed the application of transport theory to the description of nuclear matter and showed that elementary considerations lead to the expectation that three-particle collisions are significant in excited nuclear matter. Finally, quite recently, Bonasera and Gulminelli [3] studied the effect of three-body collisions on the basis of the *BUU* transport equation, employing a generalized mean free path in accordance with the analysis by Mrówczyński. Concurrently, we [4] have developed a generalized *BUU* transport equation that incorporates N-body collisions and made a survey study of the effect on a variety of commonly considered observables in high-energy nuclear reactions, such as collective flow and particle production.

In particular, we found that the total kaon yield is relatively unaffected by this incorporation of N-body collisions, and we understood that feature as a reflection of the fact that the effective temperature of the N-body clusters producing the kaons has little dependence of the cluster size N. In that first study we did not calculate the differential yields of the produced kaons. However, one might expect that the inclusion of N-body collisions will enhance the yield of high-energy kaons in reactions at subthreshold bombarding energies. We therefore find it worthwhile to examine quantitatively the effect of the N-body collisions on the spectra of kaons produced at subthreshold energies where the effect should be most noticeable.

Accordingly, we report here on an extension of our model to the calculation of differential production cross sections for particle production and the application to kaon production in symmetric nucleus-nucleus collisions at relativistic energies. In section 2 we briefly describe the model and its extension to calculation of kaon spectra. Then, in section 3, we present and discuss the calculated results. Finally, we conclude the study in section 4.

2 Description of the model

In ref. [4] we augmented the *BUU* transport equation with a generalized collision integral, in order to take account of N-body collisions in the course of a nuclear reaction. This generalized transport theory can be briefly described as follows. Our starting point is the standard *BUU* model (see ref. [5]), specifically the numerical implementation made by Wolf *et al.*[6]. In that code a number (typically hundreds) of parallel manifestations of the nuclear A-body system are propagated in the same effective field obtained by averaging over these parallel systems after each time step. The code includes nucleons as well as the $\Delta(1232)$ and $N^*(1440)$ resonances, and it also treats the pions resulting from their decay. In addition to their motion in

the common field, the baryons in a given A-body system experience direct two-body collisions governed by the energy-dependent baryon-baryon scattering cross section $\sigma_{B_1B_2}(\epsilon_{12})$, where ϵ_{12} is the relative energy of the two baryons B_1 and B_2 (equal to their total kinetic energy in their *CM* frame). These collisions are subject to final-state Pauli blocking, where the blocking factors have also been obtained by averaging over all the parallel systems.

In our extension, this basic *BUU* simulation code is employed to provide a sequence of binary baryon-baryon collisions events. For each such event it is determined how many additional baryons are located in the proximity, on the basis of the appropriate interaction range associated with the energy-dependent two-body interaction cross section $\sigma_{B_1B_2}(\epsilon_{12})$. In this manner a "collision cluster" containing N baryons is identified, for that particular collision event. The differential distribution of these N baryons after this N-body collision is obtained by assuming, as a simple idealization, that there is a microcanonical redistribution of the available energy within each such cluster.[1]

This simple but general model was subsequently extended to particle production processes by allowing the two primary baryons B_1 and B_2 to produce particles in the usual fashion, after their interaction with the environment. The $N-2$ neighboring baryons thus act as an energy reservoir and a key quantity is the effective temperature τ_N of the cluster, since this quantity is a measure for the typical relative kinetic energy of the two baryons. In ref. [4] we found that the inclusion of N-body collisions has relatively little effect on the total yield of kaons in nuclear reactions at relativistic energies, and we ascribed this feature to the fact that τ_N depends little on the cluster size N, since the cluster members are all drawn from the same system.

The purpose of the present investigation is to extend the calculation to the *differential* distribution of the produced kaons. The method developed can readily be extended to the production of other particles as well.

The kaon is assumed to be produced in an elementary process of the form $B_1B_2 \rightarrow BYK^+$, where B denotes the final baryon and the hyperon Y is either a Λ or a Σ, and it is furthermore assumed that the corresponding differential cross section $d^3\sigma_{B_1B_2\rightarrow K^+}(\epsilon_{12})/d\boldsymbol{p}$ is known as a function of the relative energy ϵ_{12} of the two initial baryons B_1 and B_2. Since we shall apply the model to the subthreshold regime, it suffices to include only those processes in which the emerging baryon B is a nucleon and the hyperon Y is a Λ particle (the Δ lies about 300 MeV higher in mass and the Σ mass is about 75 MeV above m_Λ, and their inclusion would anyway not affect our conclusions).

In each such elementary process, the branching ratio $P_{B_1B_2\rightarrow K^+}(\boldsymbol{p})$ for producing a kaon with the specified momentum $\boldsymbol{p}$ is then given by the ratio between the associated differential production cross section and the elementary total interaction cross section,

$$P_{B_1B_2\rightarrow K^+}(\epsilon_{12};\boldsymbol{p}) = \frac{1}{\sigma_{B_1B_2}(\epsilon_{12})}\,\frac{d^3\sigma_{B_1B_2\rightarrow K^+}(\epsilon_{12})}{d\boldsymbol{p}}\,. \qquad (1)$$

[1] Several years ago, Shyam and Knoll [7] employed the same idealization in their statistical model for high-energy nuclear reactions based on Glauber-type dynamics in which all nucleons lying along the same line parallel to the beam form a similar collision cluster.

When N-body collisions are incorporated, the total energy available in the cluster, E^*, is first shared microcanonically between the N baryons in the cluster, before the two primary baryons produce the kaon. The resulting effective branching ratio is then obtained by averaging the elementary branching ratio (1) over all the microcanonical momentum configurations of the N-body cluster. In the present context it suffices to use non-relativistic kinematics, and so the effective production probability can be expressed in compact form as

$$P_{B_1B_2\cdots B_N\to K^+}(E^*;\boldsymbol{p}) =< P_{B_1B_2\to K^+}(\epsilon_{12}) >= \frac{\mathcal{I}^N_{E^*}[P_{B_1B_2\to K^+}]}{\mathcal{I}^N_{E^*}[1]} \ , \tag{2}$$

where we have used the N-particle phase-space functional

$$\mathcal{I}^N_E[f] = \prod_{i=1}^{N}[\int d\boldsymbol{p}_i]\ f(\boldsymbol{p}_1,\cdots,\boldsymbol{p}_N)\ \delta(\sum_{i=1}^{N}\boldsymbol{p}_i)\ \delta(E-\sum_{i=1}^{N}\frac{p_i^2}{2m_i}) \ . \tag{3}$$

The essential new ingredient in the model is the differential distribution of the produced kaon. Since this quantity is not well known, we assume, as a simple approximation, that the momentum distribution of the kaon is proportional to the phase space associated with the three particles in the outgoing channel. In the non-relativistic limit, the differential cross section is then given by

$$\frac{d^3\sigma_{B_1B_2\to K^+}(\epsilon_{12})}{dp} = \frac{4}{\pi^2}\left[1-\left(\frac{p}{p_{max}}\right)^2\right]^{\frac{1}{2}}\frac{\sigma_{B_1B_2\to K^+}(\epsilon_{12})}{p^3_{max}} \ , \tag{4}$$

where

$$p_{max} = \sqrt{2\mu(\epsilon_{12}-\epsilon_0)} \tag{5}$$

is the maximum momentum of the emitted kaon and μ is the corresponding reduced mass, $\mu^{-1} = m_K^{-1} + (m_B + m_\Lambda)^{-1}$. Furthermore, $\epsilon_0 = m_{K^+} + m_N + m_\Lambda - m_{B_1} - m_{B_2}$ is the threshold energy. [It should be noted that this quantity ϵ_0 may occasionally be negative, namely when one of the primary baryons is an $N^*(1440)$ and the other either an N^* as well or a $\Delta(1232)$; the energy tied up in these resonances is then available for the kaon production.] The resulting spectral shape is displayed by the solid curve in fig. 1 and may be compared with the data of Hogan *et al.*[8], taken at 2.54 GeV (corresponding to $E^* = 1.00$ GeV). It is seen that the adopted parametrization shows an acceptably good agreement with the data, and we expect that our approximation will be even better at the energies of present interest which are closer to the threshold.

For the total elementary cross section we adopt the functional form proposed by Schürmann and Zwermann [9],

$$\sigma_{NN\to K^+}(\epsilon_{12}) = 800\ \mu\mathrm{b}\ [\ p_{max}(\mathrm{GeV}/c)\ ]^4 \ , \tag{6}$$

and the contribution to the kaon production arising from the various other baryonic channels are related to the NN channel as in ref. [10].

In order to arrive at a tractable result, we shall make two simplifying approximations. The first exploits the fact that the energy dependence of the total cross section

$\sigma_{B_1B_2}(\epsilon_{12})$ is rather weak at those high energies for which kaon production is possible, and so we may to a good approximation replace this quantity by $\sigma_{B_1B_2}(\bar{\epsilon}_{12})$, where $\bar{\epsilon}_{12}$ is a suitable typical energy of the two baryons producing the kaon.[2] The effective branching ratio can then be expressed in terms of the effective kaon production cross section as

$$P_{B_1B_2\cdots B_N\to K^+}(E^*;\boldsymbol{p}) = \frac{1}{\sigma_{B_1B_2}(\bar{\epsilon}_{12})}\,\frac{\mathcal{I}^N_{E^*}[d^3\sigma_{B_1B_2\to K^+}(\epsilon_{12})/d\boldsymbol{p}]}{\mathcal{I}^N_{E^*}[1]}\,. \tag{7}$$

The second approximation consists in neglecting the fact that the differential production cross section $d^3\sigma_{B_1B_2\to K^+}(\epsilon_{12})/d\boldsymbol{p}$ refers the kaon momentum the *CM* frame of the two baryons B_1 and B_2, rather than to the *CM* frame of the cluster. This neglect of the Lorentz transformation of the kaon motion between these two frames introduces a certain local distortion of the resulting kaon spectrum. However, the quantitative effect is expected to be rather small for the following reason: Since the kaon production cross section is a steeply rising function of energy, there is a tendency, within a given cluster, to favor production from those momentum configurations for which the baryons move as only little, except for the relative motion of B_1 and B_2 (in order that the available energy E^* may be expended on producing a kaon); accordingly, the kaon is more likely to be produced when the two inertial frames have little relative motion.

The factorization brought out in eq. (7), invites the introduction of the effective differential cross section (in analogy with what was done in ref. [4] for the effective total kaon production cross section $\sigma_{B_1B_2\cdots B_N\to K^+}(E^*)$), and because of the two approximations discussed above, this quantity can be calculated in a similar manner, exploiting the fact that the $3N$-dimensional integral appearing in (2) can be reduced to one dimension [4],

$$\begin{aligned}\frac{d^3\sigma_{B_1B_2\cdots B_N\to K^+}(E^*;\boldsymbol{p})}{d\boldsymbol{p}} &= \frac{\mathcal{I}^N_{E^*}[d^3\sigma_{B_1B_2\to K^+}(\epsilon_{12})/d\boldsymbol{p}]}{\mathcal{I}^N_{E^*}[1]} \\ &= \frac{\Gamma(\frac{3}{2}N-\frac{3}{2})}{\Gamma(\frac{3}{2}N-3)\Gamma(\frac{3}{2})}\,(E^*)^{-\frac{3}{2}}\int_{\epsilon_0(p)}^{E^*} d\epsilon_{12}\,\frac{d^3\sigma_{B_1B_2\to K^+}(\epsilon_{12};\boldsymbol{p})}{d\boldsymbol{p}}\,\epsilon_{12}^{1/2}\left(1-\frac{\epsilon_{12}}{E^*}\right)^{\frac{3}{2}N-4}\,.\end{aligned} \tag{8}$$

In the integral over the relative energy ϵ_{12}, the quantity $\epsilon_0(p) = \epsilon_0 + p^2/2\mu$ denotes the production threshold which depends on the magnitude of the specified momentum $\boldsymbol{p}$ of the produced kaon. (The fact that $\epsilon_0(p)$ can be negative is no cause for concern, since the differential cross section $d^3\sigma_{B_1B_2\to K^+}(\epsilon_{12})$ is understood to vanish for $\epsilon_{12} \le 0$.) Thus, after changing to the dimensionless relative energy $t = \epsilon_{12}/E^*$ as the integration variable and using the adopted forms of the momentum profile (4) and the elementary

[2] In those cases where the two primary baryons are nucleons the total cross section $\sigma_{B_1B_2}(\epsilon_{12})$ is indeed rather energy independent at the relatively high kinetic energies for which kaon production is possible, but when one of the primary baryons is a resonance (Δ or N^*) the threshold is considerably lower and the reaction cross section may then exhibit a significant energy dependence. However, this feature plays no important role because the relative energy ϵ_{12} is usually not confined to such low values, and consequently our approximation is in fact fairly good, as has also been verified by numerical calculation.

production cross section (6), we arrive at the following expression for the effective branching ratio (7),

$$P_{B_1B_2\cdots B_N\to K^+}(E^*;\boldsymbol{p}) = \frac{800\ \mu b\ (\mathrm{GeV}/c)^{-4}}{\sigma_{B_1B_2}(\bar{\epsilon}_{12})}\frac{\Gamma(\frac{3}{2}N-\frac{3}{2})}{\Gamma(\frac{3}{2}N-3)\Gamma(\frac{3}{2})} \times\frac{4}{\pi^2}\sqrt{2\mu E^*}\int_{t_0(p)}^{1} dt\,[t-t_0(p)]^{1/2}\,t^{1/2}\,[1-t]^{\frac{3}{2}N-4}. \tag{9}$$

Again, the lower limit of the integral should never be negative.

As an illustration, we have also included in fig. 1 the spectral shapes associated with clusters with $N > 2$ (dashed curves). The available energy E^* for the various cluster sizes N has been obtained by assuming that the associated effective temperature $\tau_N = \frac{2}{3}E^*/(N-1)$ is the the same for all N, as our dynamical simulations suggest is roughly the case (see table 1). (As discussed in [4], the cluster temperature is related to the first moment of the beta distribution associated with the microcanonical constraint.) We note that the increase in the cluster size leads to a significant enhancement in the production of kaons with high momentum, while leaving the yield at lower momenta rather unaffected.

Since the branching ratio is so small, kaon production may be treated as a perturbative process. As a consequence, the influence of the kaon production on the collision dynamics is negligible and, furthermore, we may simply add up the contributions from each of the individual N-body collision obtained with our extended *BUU* model [4]. For a specific impact parameter, the total kaon differential multiplicity b is then given by

$$\frac{d^3\nu_{K^+}(b)}{d\boldsymbol{p}} = \frac{1}{\mathcal{N}}\sum_{collisions} P_{B_1B_2\cdots B_N\to K^+}(E^*;\boldsymbol{p})\ , \tag{10}$$

where $\mathcal{N}$ denotes the number of parallel systems utilized in the simulation. We note that for $N > 2$ the prescription used differs slightly from that employed in our first study [4], in which the typical energy $\bar{\epsilon}_{12}$ used in the denominator of the expression for the effective branching ratio (7) was evaluated by averaging the energy ϵ_{12} using the corresponding kaon production cross section as the weight function. There is very little sensitivity to the specific prescription adopted for $\bar{\epsilon}_{12}$ and in the present calculations we have simply used the incoming relative energy of the two primary baryons. It has been found that this difference affects the calculated kaon multiplicities by only a few per cent.

3 Results

We have applied the model to head-on collisions of ^{40}Ca+^{40}Ca, ^{93}Nb+^{93}Nb, and ^{197}Au+^{197}Au. In these calculations we have ignored the rescattering of the produced kaons by the other hadrons in the system. Although the kaon has a fairly long mean free path ($\lambda_{K^+} \approx 7$ fm in nuclear matter), the chance for scattering is far from

negligible and this effect may in fact modify the differential distribution significantly [11]. This complication need not be considered in the present investigation since we are only interested in assessing the relative effect of including the N-body collisions.

The contributions to the kaon multiplicities from the different cluster sizes N are summarized in fig. 2. The dashes show the result of the standard binary BUU calculation, while the result of incorporating the N-body collisions are indicated by the solid bars. It is seen that the total kaon yield remains almost unchanged when the N-body processes are included, for all the cases considered. The contributions to the kaon multiplicity from various cluster sizes $N = 2, \ldots, 6$ are shown by the solid histograms. As was already found previously [4], the kaon yield is determined, essentially, by the available energy per nucleon, due to the assumption of equipartition of the energy in the cluster. In order to understand the absolute sizes of the contributions from various values of N, it is useful to note that these quantities arise as the combination of two ingredients: one is the spectral profile associated with the particular value of N (as illustrated in fig. 1), and the other is the rate of occurence of such cluster sizes (which decreases steadily with N, as shown in fig. 3 of ref. [4]); the result is a roughly geometric decrease of the kaon multiplicity as a function of the cluster size N.

The differential kaon multiplicities corresponding to various cluster sizes N are displayed in fig. 3, as a function of the magnitude of the kaon momentum. In all cases, these spectral profiles are approximately of Maxwell-Boltzmann form, and the corresponding characteristic temperature is nearly independent of N, in each particular reaction. This fact makes it easy to understand why the kaon spectra are so relatively unaffected by the inclusion of N-body collisions: For each cluster size N the kaons have an approximately thermal shape with an effective temperature that depends little on N, and so the kaon spectra contain (nearly) no information about the size of the cluster involved in their production. To illustrate this key feature we list in table 1 the time average of the cluster temperatures $\tau_N = \frac{2}{3}E^*/(N-1)$ during the course of the reaction processes considered.

The resulting effect on the differential distribution is shown in fig. 4, where the invariant kaon multiplicity is shown as a function of the kaon kinetic energy. Qualitatively, all the shown reactions exhibit similar characteristics: The inclusion of N-body collisions slightly enhances the kaon emission, with the effect growing with the kaon momentum. However, the differences between the spectra calculated from a pure binary and the full N-body simulation are, in most cases, remarkably small. (The largest difference is observed at the highest kaon energies in Au+Au at 1 GeV per nucleon, but the numerical error is relatively large for those unlikely processes.) These results then suggest that, at least within the scope of the present model, the inclusion of N-body collision does not lead to a significant enhancement of the production of energetic kaons in the subthreshold energy regime. Of course, in the extreme tails of the spectra the relative enhancement will be large, but the associated absolute cross sections are then so small that the effect will probably be of little practical import.

It can be added that we also employed an alternative prescription in which the kaon spectra was taken to be proportional to the phase-space distribution associated with all the $N+1$ particles in the final state of the cluster, as was done in ref. [7]. There is no significant difference in the results when compared to the above described

approach.

4 Concluding remarks

As a supplement to our recent incorporation of N-body collisions into *BUU* simulations of nuclear reactions [4], we have considered the differential production cross section of subthreshold kaons. The general assumption of microcanonical exchange of energy between the colliding baryons and their neighbors has been augmented by the assumption that the produced particle has a momentum distribution reflecting the corresponding final three-body phase space. The resulting model then provides a conceptually simple, yet very general, scheme for taking account of the proximity of additional baryons in the nuclear medium. We wish to emphasize that the model developed represents a very simple extreme in which there is full equilibration between the two primary baryons and their cluster partners. Therefore, at least within the limitations of incoherent sequential scattering processes, it may be expected that the model will provide a useful upper bound on the effect of N-body collisions in nuclear reactions.

We have specifically addressed the sensitivity of the differential kaon production cross sections to the inclusion of the N-body effects. Generally speaking, the kaon yields are affected only rather little, and we understand this in a general way as a result of the statistical sharing of the energy between the kaon and the other baryons involved in its production.

The relative enhancement grows with the kaon energy, as would be expected, but it does not achieve a significant magnitude until the absolute yields have become so small as to render the effect of little observational interest.

We wish to acknowledge stimulating discussions with Ulrich Mosel. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098; and by a NATO Science grant.

References

[1] T. Kodama, S.B. Duarte, K.C. Chung, and R.A.M.S. Nazareth, Phys. Rev. Lett. 49 (1982) 536.

[2] St. Mrówczyński, Phys. Rev. C32 (1985) 1784.

[3] A. Bonasera and F. Gulminelli, Phys. Lett. 259B (1991) 399; and preprint submitted to Nucl. Phys. A (1991).

[4] G. Batko, J. Randrup, and T. Vetter, Nucl. Phys. A (in press).

[5] G.F. Bertsch and S. Das Gupta, Phys. Rep. 160 (1988) 189.

[6] Gy. Wolf, G. Batko, W. Cassing, U. Mosel, K. Niita, and M. Schäfer, Nucl. Phys. A517 (1990) 615.

[7] R. Shyam and J. Knoll, Nucl. Phys. A426 (1984) 606.

[8] W.J. Hogan, P.A. Piroué, and J.S. Smith, Phys. Rev. 166 (1968) 1472.

[9] B. Schürmann and W. Zwermann, Phys. Lett B183 (1987) 31.

[10] J. Randrup and C.M. Ko, Nucl. Phys. A343 (1980) 519; *ibid.* A411 (1983) 537.

[11] J. Randrup, Phys. Lett. 99B (1981) 9.

	^{40}Ca+^{40}Ca		^{93}Nb+^{40}Nb		^{197}Au+^{197}Au	
	600	1000	600	1000	600	1000
Binary	157	233	151	227	147	221
N=2	152	223	145	215	141	209
N=3	133	198	131	193	128	190
N=4	127	186	125	185	124	181
N=5	123	180	121	178	121	177
N=6	121	179	121	178	120	177

Table 1: Effective temperatures.
The temporal average of the effective temperature $\tau_N = \frac{2}{3}E^*/(N-1)$ (in MeV) extracted from the dynamical simulations of head-on nuclear collisions at 600 and 1000 MeV per nucleon, for various cluster sizes N. The first line shows the effective temperatures for the standard BUU calculations in which only binary collisions occur.

Figure 1: Kaon differential yield.
The elementary kaon momentum distribution as a function of the magnitude of the kaon momentum, expressed in the *CM* system of the producing N-body cluster. The experimental data are taken by Hogan *et al.*[8] at 2.54 GeV, corresponding to an available energy of E^*=1.00 GeV. The solid curve is the assumed statistical shape for an elementary binary production process given by eq. (4). The dashed curves show the profiles resulting in various N-body clusters having the same effective temperature $\tau_N = \frac{2}{3}E^*/(N-1) = 0.67$ GeV, for $N = 3,4,5,6$.

Figure 2: Total kaon multiplicity.
The contributions to the K^+ multiplicity, ν_K, arising from N-body collisions of various cluster sizes N. The dashes indicate the results of the standard *BUU* simulation involving only binary collisions, while the solid histogram represents the contribution from various cluster sizes when N-body collisions are incorporated; the corresponding total multiplicity is indicated by the solid bar. The results were obtained from head-on collisions of Ca+Ca, Nb+Nb, and Au+Au at bombarding energies of 600 and 1000 MeV per nucleon.

Figure 3: Differential kaon multiplicity.
The contributions to the differential K^+ multiplicitiy arising from N-body collisions (clusters with $N = 2,3,4,5,6$ are shown by the solid lines in decreasing order, respectively), as functions of the momentum of the kaon emitted at $\theta = 90°$ in the overall *CM* system.

Figure 4: Invariant kaon spectrum.
The invariant K^+ energy spectrum as a function of the kinetic energy of the kaon at $\theta = 90°$ in the overall *CM* frame.

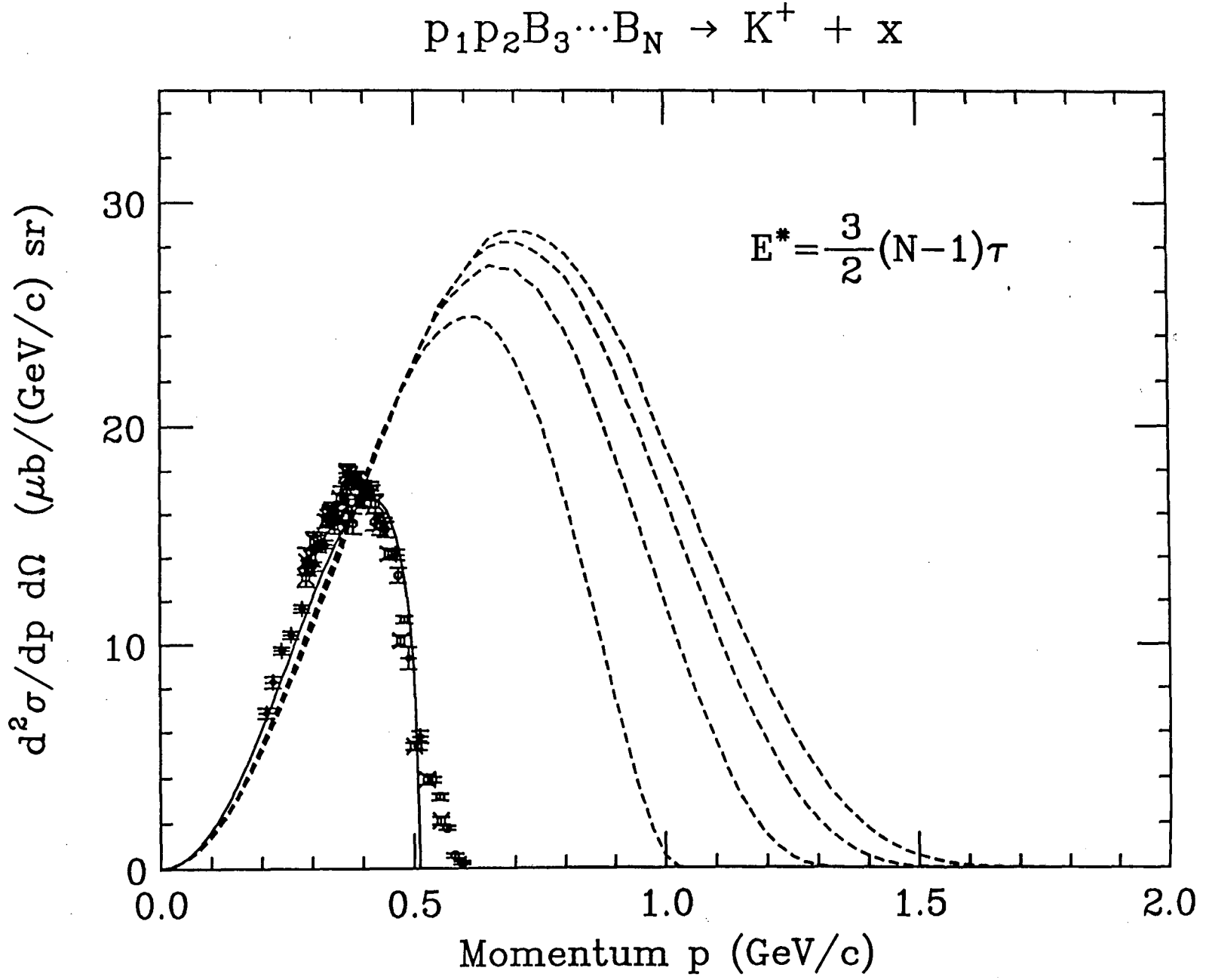

$p_1p_2B_3\cdots B_N \rightarrow K^+ + x$
$E^* = \frac{3}{2}(N-1)\tau$
$d^2\sigma/dp\ d\Omega\ (\mu b/(GeV/c)\ sr)$
Momentum p (GeV/c)
0
10
20
30
0.0
0.5
1.0
1.5
2.0

Figure 1

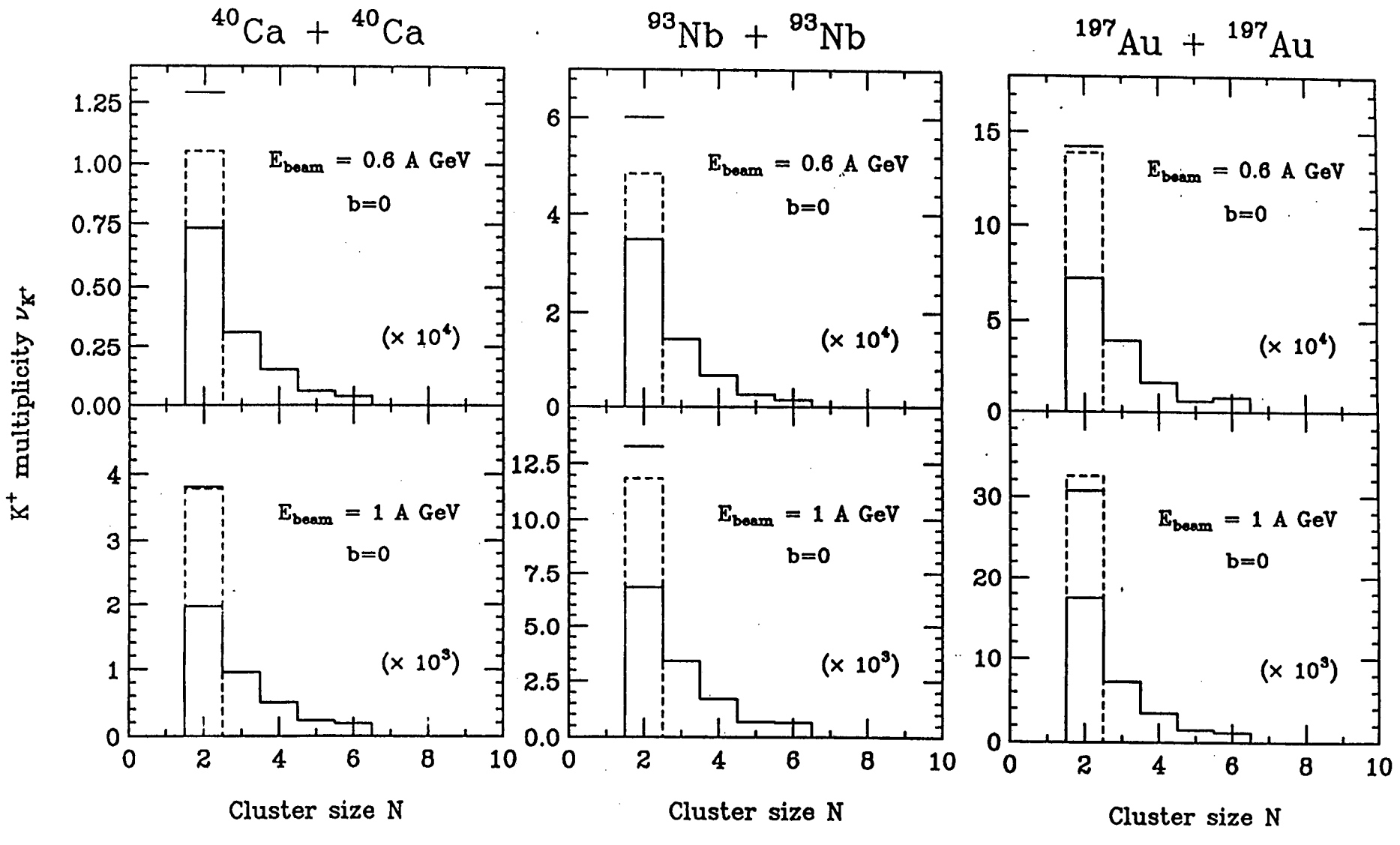

$^{40}Ca + ^{40}Ca$
$^{93}Nb + ^{93}Nb$
$^{197}Au + ^{197}Au$
K^+ multiplicity ν_{K^+}
E_{beam} = 0.6 A GeV
b=0
(× 10^4)
E_{beam} = 1 A GeV
b=0
(× 10^3)
Cluster size N

Figure 2

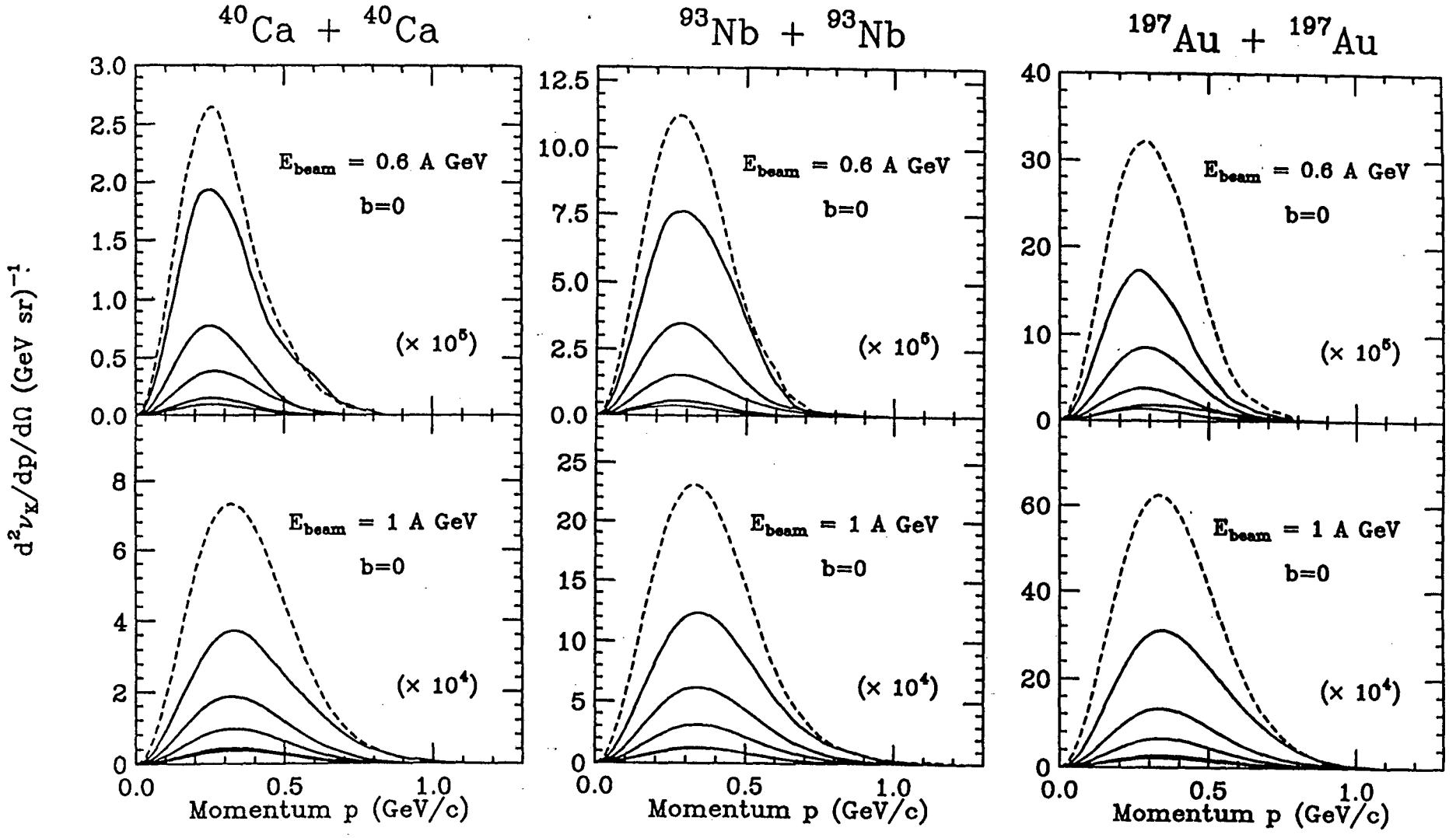

$^{40}Ca + ^{40}Ca$
$^{93}Nb + ^{93}Nb$
$^{197}Au + ^{197}Au$
$d^2\nu_K/dp/d\Omega$ (GeV sr)$^{-1}$
E_{beam} = 0.6 A GeV
b=0
(× 10^5)
E_{beam} = 1 A GeV
b=0
(× 10^4)
Momentum p (GeV/c)

Figure 3

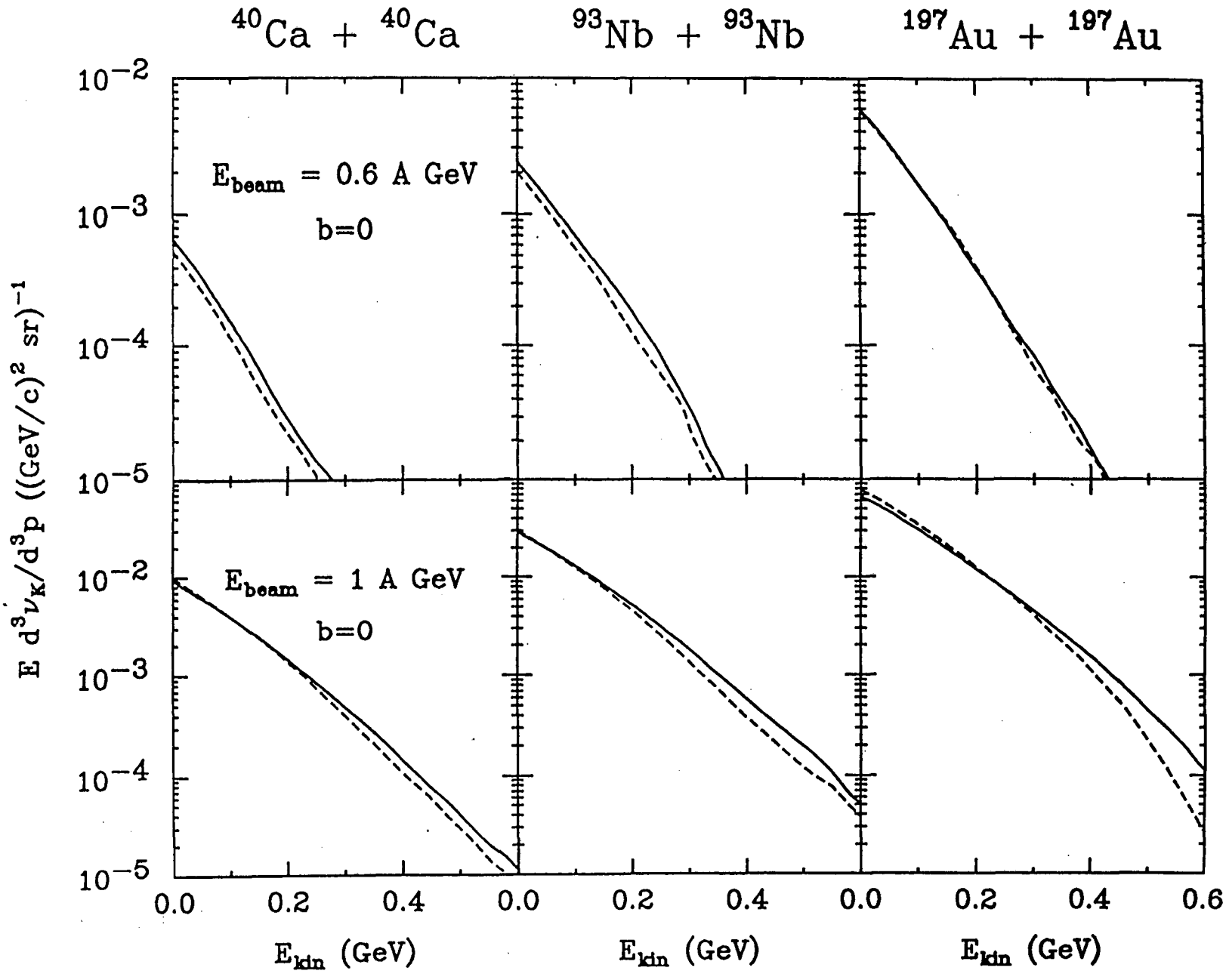

^{40}Ca + ^{40}Ca
^{93}Nb + ^{93}Nb
^{197}Au + ^{197}Au
E_{beam} = 0.6 A GeV
b=0
E_{beam} = 1 A GeV
b=0
E d^3N_K/d^3p ((GeV/c)^2 sr)$^{-1}$
E_{kin} (GeV)
E_{kin} (GeV)
E_{kin} (GeV)

Figure 4

LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
TECHNICAL INFORMATION DEPARTMENT
BERKELEY, CALIFORNIA 94720