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Continuous quantum measurement with particular output: pure wavefunction evolution instead of decoherence

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We consider a continuous measurement of a two-level system (double-dot) by weakly coupled detector (tunnel point contact nearby). While usual treatment leads to the gradual system decoherence due to the measurement, we show that the knowledge of the measurement result can restore the pure wavefunction at any time (this can be experimentally verified). The formalism allows to write a simple Langevin equation for the random evolution of the system density matrix which is reflected and caused by the stochastic detector output. Gradual wavefunction "collapse" and quantum Zeno effect are naturally described by the equation.

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The problem of quantum measurements has a long history, however, it still attracts considerable attention and causes discussions and even some controversy, mainly about the wavefunction "collapse" postulate (see, e.g., Ref. [1]). Among different modern approaches to this problem let us mention the idea to replace the collapse postulate by the gradual decoherence of the density matrix due to the interaction with the detector [2] and the approach of a stochastic evolution of the wavefunction (see, e.g., [3–5]). The renewed interest to the measurement problem is justified by the development of the experimental technique, which allows more and more experimental studies of the quantum measurement in optics and mesoscopic structures. The problem has also the close connection to the rapidly growing fields of quantum cryptography and quantum computing (see, e.g., [6]).

In the recent experiment [7] with "which-path" interferometer the suppression of Aharonov-Bohm interference due to the detection of which path an electron chooses, was observed. The weakly coupled quantum point contact was used as a detector. The interference suppression in this experiment can be quantitatively described by the decoherence due to the measurement process [8–11].

We will consider somewhat different setup: two quantum dots occupied by one electron and a weakly coupled detector (point contact nearby) measuring the position of the electron. The decoherence of the double-dot density matrix due to the measurement has been analyzed for this setup in Refs. [8,11]. In the present letter we answer the following questions: how the detector current looks like (as a function of time) and what is the proper double-dot density matrix for particular detector output. We show that the models of point contact considered in Refs. [8–10] describe an ideal detector. In this case the density matrix decoherence is just a consequence of ignoring the measurement result. The observer who follows the detector output knows the pure wavefunction at each moment of time. Moreover, a "mixed" density matrix can be gradually purified during the measurement.

Similar to Ref. [8] let us describe the double-dot system and the measuring point contact by the Hamiltonian $\mathcal{H} = \mathcal{H}_{DD} + \mathcal{H}_{PC} + \mathcal{H}_{int}$, where $\mathcal{H}_{DD} = (\epsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$ is the Hamiltonian of the double-dot, $\mathcal{H}_{PC} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r)$ describes the tunneling through the point contact (H and T are real), and $\mathcal{H}_{int} = \sum_{l,r} \Delta T c_2^\dagger c_2 (a_r^\dagger a_l + a_l^\dagger a_r)$, i.e. the tunneling matrix element for the point contact is T or $T + \Delta T$ depending on which dot is occupied. So, the average current $I_1 = 2\pi T^2 \rho_l \rho_r e^2 V/\hbar$ flows through the detector when the electron is in the first dot (V is voltage), while the current is $I_2 = I_1 + \Delta I = 2\pi(T + \Delta T)^2 \rho_l \rho_r e^2 V/\hbar$ when the second dot is occupied.

We make an important assumption of weak coupling between the double-dot and the detector (the better term would be "weakly responding" detector),

$$|\Delta I| \ll I_0 = (I_1 + I_2)/2, \tag{1}$$

so that many electrons, $N \gtrsim (I_0/\Delta I)^2$, should pass through the point contact before the observer is able to distinguish which dot is occupied. This assumption allows the classical description of the detector, namely the coherence between the quantum states with different number of electrons passed through the detector can be neglected [12].

The decoherence rate $\Gamma_d = (\sqrt{I_1/e} - \sqrt{I_2/e})^2/2$ of the double-dot density matrix $\sigma(t)$ due to the measurement by point contact has been obtained in Ref. [8]. In the weakly-coupled limit (1) it can be replaced by $\Gamma_d = (\Delta I)^2/8eI_0$ or by expression

$$\Gamma_d = (\Delta I)^2/4S_I, \tag{2}$$

where $S_I = 2eI_0$ is the usual Schottky formula for the shot noise spectral density S_I. Equation (2) has been also obtained in Refs. [9–11] for the quantum point contact as a detector, the difference in that case is $S_I = 2eI_0(1-\mathcal{T})$ where $\mathcal{T}$ is the transparency of the channel [13] (while above we implicitly assumed $\mathcal{T} \ll 1$ [14]). Notice that

the decoherence rate (2) was derived in Refs. [8–11] without any account of the information provided by the detector, implicitly assuming that the measurement result is just ignored. Now let us study how this additional information affects the double-dot density matrix.

We start with the completely classical case when there is no tunneling between dots ($H = 0$) and the initial density matrix of the system does not have nondiagonal elements, $\sigma_{12}(0) = \sigma_{12}(t) = 0$. We can assume that the electron is actually located in one of the dots, but we just do not know exactly in which one, and that is why we use probabilities $\sigma_{11}(0)$ and $\sigma_{22}(0) = 1-\sigma_{11}(0)$. The detector output is the fluctuating current $I(t)$. The fluctuations grow when we examine $I(t)$ at smaller time scales, so some averaging in time ("low-pass filtering") is necessary. Let us always work at sufficiently low frequencies, $f \sim \tau^{-1} \ll S_I/e^2$.

The probability P to have a particular value for the current averaged over time τ, $\langle I \rangle = \int_0^\tau I(t)dt$, is given by the distribution

$$P(\langle I \rangle, \tau) = \sigma_{11}(0)\, P_1(\langle I \rangle, \tau) + \sigma_{22}(0)\, P_2(\langle I \rangle, \tau), \quad (3)$$

$$P_i(\langle I \rangle, \tau) = (2\pi D)^{-1/2} \exp\left[-(\langle I \rangle - I_i)^2/2D\right], \quad (4)$$

where $D = S_I/2\tau$. After the measurement during time τ the observer acquires additional knowledge about the system and should change the probabilities σ_{ii} according to the standard Bayes formula for a posteriori probability. Hence,

$$\begin{aligned}
\sigma_{11}(\tau) &= \sigma_{11}(0) \exp[-(\langle I \rangle - I_1)^2/2D] \\
&\times \big\{ \sigma_{11}(0) \exp[-(\langle I \rangle - I_1)^2/2D] \\
&+ \sigma_{22}(0) \exp[-(\langle I \rangle - I_2)^2/2D] \big\}^{-1}, \\
\sigma_{22}(\tau) &= 1 - \sigma_{11}(\tau).
\end{aligned} \quad (5)$$

Now let us assume that the initial state is fully coherent, $\sigma_{12}(0) = \sqrt{\sigma_{11}(0)\sigma_{22}(0)}$ (while still $H = \epsilon = 0$). Since the detector is sensitive only to the position of electron, the detector current will behave *exactly* the same way. So, after the measurement during time τ we should assign the same values for $\sigma_{ii}(\tau)$ as in Eq. (5). Then the upper bound for the nondiagonal element is

$$\mathrm{Re}\,\sigma_{12}(\tau) \le |\sigma_{12}(\tau)| \le \sqrt{\sigma_{11}(\tau)\sigma_{22}(\tau)}. \quad (6)$$

If the actual measurement result is disregarded, then the upper bound for σ_{12} can be calculated using the probability distribution of different outcomes given by Eq. (3) and the upper bound (6) for each realization:

$$\begin{aligned}
\langle \mathrm{Re}\,\sigma_{12}(\tau) \rangle &\le \int \sqrt{\sigma_{11}(\tau)\sigma_{22}(\tau)}\, P(\langle I \rangle, \tau)\, d\langle I \rangle \\
&= \sqrt{\sigma_{11}(0)\sigma_{22}(0)}\, \exp[-(\Delta I)^2\tau/4S_I]. \quad (7)
\end{aligned}$$

This upper bound exactly coincides with the result given by decoherence approach (2). This fact forces us to accept somewhat surprising statement that Eq. (6) gives

not only the upper bound, but the true value of the nondiagonal matrix element, i.e. the pure state remains pure after the measurement (no decoherence occurs) if we know the measurement result.

Simultaneously, we proved that the point contact detector considered in Refs. [8–10] causes the slowest possible decoherence of the measured system, and hence represents an ideal detector in this sense. In contrast, the result of Ref. [15] shows that a single-electron transistor with large tunnel resistances and biased by relatively large voltage is not an ideal detector (the non-ideal detector has been also considered in Ref. [11]).

If the initial state of the double-dot is not purely coherent, $|\sigma_{12}(0)| < \sqrt{\sigma_{11}(0)\sigma_{22}(0)}$, it can be treated as the statistical combination of purely coherent and purely incoherent states with the same $\sigma_{11}(0)$ and $\sigma_{22}(0)$, then

$$\sigma_{12}(\tau) = \sigma_{12}(0)\left[\sigma_{11}(\tau)\sigma_{22}(\tau)/\sigma_{11}(0)\sigma_{22}(0)\right]^{1/2}. \quad (8)$$

Eq. (8) together with Eq. (5) is the central result of the present letter.

Equations (3)–(5) and (8) can be used to simulate the detector output $I(t)$ and the corresponding evolution of the density matrix. For example, in the Monte-Carlo method we should first choose the timestep τ satisfying inequalities $e^2/S_I \ll \tau \ll S_I/(\Delta I)^2$ and draw a random number for $\langle I \rangle$ according to the distribution (3). Then we update $\sigma_{11}(t)$ and $\sigma_{22}(t)$ using this value of $\langle I \rangle$ and repeat the procedure many times (the distribution for the current averaged over the interval $\Delta t = \tau$ is new every timestep because of changing $\sigma_{ii}(t)$ which are used in Eq. (3)). The nondiagonal matrix element can be calculated at any time using Eq. (8).

This Monte-Carlo procedure is equivalent to the following nonlinear Langevin-type equation for the density matrix evolution (equation for σ_{11} is sufficient):

$$\dot{\sigma}_{11} = \mathcal{R}, \quad \mathcal{R} = -\sigma_{11}\sigma_{22}\,\frac{2\Delta I}{S_I}\,[I(t) - I_0] \quad (9)$$

$$= -\sigma_{11}\sigma_{22}\,\frac{2\Delta I}{S_I}\left[\frac{\sigma_{22} - \sigma_{11}}{2}\,\Delta I + \xi(t)\right], \quad (10)$$

where the random process $\xi(t)$ has zero average and the low frequency spectral density $S_\xi = S_I$. The second expression for $\mathcal{R}$ allows to simulate the measurement while the first one can be used to calculate the density matrix for given $I(t)$ (that can be done easier using Eq. (5)). Notice that Eq. (10) is closely connected with the Quantum State Diffusion approach of Refs. [3–5].

Figure 1 shows a particular result of the Monte-Carlo simulation for the symmetric initial state, $\sigma_{11}(0) = \sigma_{22}(0) = 1/2$. Thick line shows the random evolution of $\sigma_{11}(t)$. Equation (10) describes the gradual localization in one of the dots (first dot in case of Fig. 1). Let us define the typical localization time as $\tau_{loc} = 2S_I/(\Delta I)^2$ (we choose the exponential factor at $\sigma_{11} = \sigma_{22} = 1/2$). Then

it is exactly equal to the time $\tau_{dis} = 2S_I/(\Delta I)^2$ necessary for the observer to distinguish between two states (defined as the relative shift of two Gaussians (4) by two standard deviations), and $\tau_{loc} = \tau_d/2$ where $\tau_d = \Gamma_d^{-1}$. It is easy to prove that the probability of final localization in the first dot is equal to $\sigma_{11}(0)$, because $\sigma_{ii}(\tau)$ averaged over realizations is conserved.

The detector current $I(t)$ basically follows the evolution of $\sigma_{ii}(t)$ but also contains the noise which depends on the bandwidth. The dashed line in Fig. 1 shows the current averaged over the "running window" with the duration $\Delta t = S_I/(\Delta I)^2$ while the thin solid line is current $\langle I \rangle$ averaged starting from $t = 0$. Notice that our result for $I(t)$ directly contradicts the point of view presented in Ref. [11].

Now let us consider the general case of the double-dot with non-zero tunneling H. If the frequency Ω of "internal" oscillations is sufficiently low, $\Omega = (4H^2 + \epsilon^2)^{1/2}/\hbar \ll S_I/e^2$, we can use the same formalism just adding the slow evolution due to finite H (the product $\Omega\tau_{loc}$ is *arbitrary*). The particular realization can be either simulated by Monte-Carlo procedure similar to that outlined above [now update of $\sigma_{12}(t)$ using Eq. (8) should be necessarily done at each timestep] or equivalently described by the coupled Langevin equations

$$\dot\sigma_{11} = -\dot\sigma_{22} = (-2H/\hbar)\,\text{Im}(\sigma_{12}) + \mathcal{R}, \qquad (11)$$

$$\dot\sigma_{12} = \frac{i\epsilon}{\hbar}\sigma_{12} + \frac{iH}{\hbar}(\sigma_{11} - \sigma_{22}) + \frac{\sigma_{22} - \sigma_{11}}{2\sigma_{11}\sigma_{22}}\mathcal{R}\sigma_{12}$$
$$-\gamma_d\sigma_{12}, \qquad (12)$$

where $\gamma_d = 0$ for an ideal detector (see below).

Figure 2 shows particular results of the Monte-Carlo simulations for the double-dot with $\epsilon = H$ and different strength of the interaction with an ideal detector. The electron is initially located in the first dot, $\sigma_{11}(0) = 1$. The dashed line shows the evolution of σ_{11} with no detector. Notice that because of $\epsilon \neq 0$, the initial asymmetry of the electron location remains in this case for infinite time. When the interaction with detector, $\mathcal{C} = \hbar(\Delta I)^2/S_I H$, is relatively small (top solid line), the evolution of σ_{11} is close to that without the detector. However, the electron gradually "forgets" the initial asymmetry and the evolution can be described as the slow variation of the phase and amplitude of oscillations (recall that the wavefunction remains pure).

When the coupling with the detector increases, the evolution significantly changes (middle and bottom curves in Fig. 2). First, the transition between dots slows down (Quantum Zeno effect). Second, while the frequency of transitions decreases with increasing interaction with detector (at sufficiently strong coupling), the time of a transition decreases, so eventually we can say about uncorrelated "quantum jumps" between states.

In a regime of small coupling with detector, $\mathcal{C} \ll 1$, the detector output is too noisy to follow the evolution of σ_{ii} and, correspondingly, only slightly affects the oscillations. On contrary, when $\mathcal{C} \gg 1$ the detector accurately informs about the position of electron and simultaneously destroys the oscillations.

Equations (11)–(12) can be generalized for a non-ideal detector, $\Gamma_d > (\Delta I)^2/4S_I$ (as in Refs. [11,15]), which gives an observer less information than possible in principle. Let us model it as two ideal detectors "in parallel" with unaccessible output of the second detector. Then the information loss can be represented by the extra decoherence term $-\gamma_d\sigma_{12}$ in Eq. (12) where $\gamma_d = \Gamma_d - (\Delta I)^2/4S_I$. The limiting case of a nonideal detector is the detector with no output (just an environment, $\Delta I = 0$) or with disregarded output. Then the evolution equations (11)–(12) reduce to the standard decoherence approach.

For nonideal detector it is meaningful to keep our old definition of the localization time, $\tau_{loc} = \tau_{dis} = 2S_I/(\Delta I)^2$ while $\tau_d < 2\tau_{loc}$. So, we consider localization time not as a real physical quantity (that is meaningless because observer cannot check it) but as a quantity related to observer's information. Similarly, the effective decoherence time is defined as $\tau'_d = \gamma_d^{-1}$.

Equations (11)–(12) with the term $\mathcal{R}$ given by Eq. (9) can be used to obtain the evolution of the density matrix in an experiment provided the known detector output $I(t)$ and initial condition $\sigma_{ij}(0)$. Notice that even if the initial state is completely random, $\sigma_{11} = \sigma_{22} = 1/2$, $\sigma_{12} = 0$, the nondiagonal matrix element appears during the measurement due to acquired information, so that sufficiently long observation with an ideal detector leads to almost pure wavefunction. Such a purification of the density matrix described by Eqs. (11)–(12) is analogous to the localization at $H = 0$.

Let us briefly discuss the philosophical aspect of the developed formalism. The statement that the pure wavefunction remains pure during the continuous measurement by an ideal detector (with known result) may seem surprising at first, however, we easily recognize the direct analogy to the "orthodox" situation of a "sharp" measurement (the wavefunction is pure after the "collapse"). Another important point is that the density matrix is in some sense observer-dependent. For example, if two observers have different level of access to the detector information (as, for example, in the model of nonideal detector considered above), then the density matrix for them will be different. Nevertheless, the observer with less information still can use his density matrix for all purposes, just his predictions will be less accurate. So, instead of the "actual" density matrix, it is better to discuss only the "accessible" density matrix, that is fully consistent with the "orthodox" (Copenhagen) point of view.

If the knowledge of the detector output is not used in the experiment, the decoherence approach is suitable. On contrary, one can devise an experiment in which the

subsequent system evolution depends on the preceding measurement result; then the proper description is given by Eqs. (11)–(12).

For example, let us consider the double-dot with $H = 0$ and fully coherent symmetric initial state. According to our formalism, after the measurement by an ideal detector during time τ (most interesting case is $\tau \lesssim \tau_{loc}$) the wavefunction remains pure but becomes asymmetric (Eqs. (5) and (8)). To prove this, for example, an experimentalist should switch off the detector at $t = \tau$, reduce the barrier between the dots (create finite H), and create the energy difference $\epsilon = [(1 - 4|\sigma_{12}|^2)^{1/2} - 1]H\mathrm{Re}\sigma_{12}/|\sigma_{12}|^2$; then after the time period $\Delta t = [\pi - \arcsin(\mathrm{Im}\sigma_{12}\,\hbar\Omega/H)]/\Omega$ the electron will be moved to the first dot with the probability equal to unity, that can be checked by the detector switched on again. Alternatively, using the knowledge of $\sigma_{ij}(\tau)$ an experimentalist can exactly prepare the ground state of the coupled double-dot system and check it, for example, by the photon absorption. Another experimental idea is to start with completely random state of the double-dot with finite H and then gradually (most interesting case is $\Omega\tau_{loc} \lesssim 1$) obtain almost pure wavefunction using the detector output $I(t)$ and Eqs. (11)–(12). The final test of the wavefunction is similar to that considered above.

An experiment of this kind can verify the formalism developed in the present letter. While such an experiment is still a challenge for the present-day technology, we hope that it can be realized in the near future.

In conclusion, we developed a simple formalism for the evolution of double-dot density matrix with account of the result of the continuous measurement by weakly coupled (weakly responding) point contact. The formalism is suitable for any two-level system measured by weakly coupled detector.

FIG. 1. Thick line: particular Monte-Carlo realization of σ_{11} evolution in time during the measurement of uncoupled dots, $H = 0$. Initial state is symmetric, $\sigma_{11}(0) = \sigma_{22}(0) = 1/2$, while the measurement leads to gradual localization. Initially pure wavefunction remains pure at any time t. Thin line shows the corresponding detector current $\langle I \rangle$ averaged over the whole time interval starting from $t = 0$ while the dashed line is the current averaged over the running window with duration $S_I/(\Delta I)^2$.

FIG. 2. Random evolution of σ_{11} (particular Monte-Carlo realizations) for asymmetric double-dot, $\epsilon = H$, with the electron initially in the first dot, $\sigma_{11}(0) = 1$, for different strength of coupling with detector: $\mathcal{C} = \hbar(\Delta I)^2/S_I H = 0.3$, 3, and 30 from top to bottom. Dashed line represents $\mathcal{C} = 0$ (unmeasured double-dot). Increasing coupling with detector destroys the quantum oscillations (while wavefunction remains pure at any t), slows down the transitions between states (Quantum Zeno effect), and for $\mathcal{C} \gg 1$ leads to uncorrelated jumps between well localized states.

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