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Publication Date

2024

DOI

10.1093/ptep/ptae057

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On the Magnitudes of some CKM Matrix Elements

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Received February 16, 2024; Revised April 10, 2024; Accepted April 12, 2024; Published April 13, 2024

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This write-up follows the presentation at the symposium, with emphasis on topics and ideas discussed there. It is purposefully informal, not a review of the field, and neither does it include a complete list of references. However, I hope that readers might find some comments useful or amusing, and may appreciate the challenges and reasons for excitement about recent progress and future opportunities in flavor physics.
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Subject Index B51, B52, B55, B57

1. Introduction

I was asked to talk at this symposium, celebrating the 50th anniversary of the Kobayashi–Maskawa theory [1], about the determinations of the magnitudes of elements of the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix [1,2]. This talk only covers (some of) those CKM elements that involve the third generation of quarks.

Given that much of the audience, as well as I, was not yet (or barely) alive when the paper that this symposium celebrates was written, I would like to emphasize how stunning the timeline was. After the discovery of CP violation [3] in 1964, Wolfenstein’s paper on the superweak model [4] was nearly instantaneous (the *Phys. Rev. Lett.* received dates are July 10 and August 31, 1964, respectively), whereas *Prog. Theor. Exp. Phys.* received the Kobayashi–Maskawa manuscript [1] on September 1, 1972. I am old enough to remember people talking about excluding the superweak theory as a goal for Belle and BaBar, and young enough to never have taken that seriously. Imagine, those 8 years between 1964 and 1972 were just as if ATLAS [5] and CMS [6] had discovered a new particle near 750 GeV in 2015, but all theory papers in the first 7 years had turned out to be wrong, and the correct model had only been proposed in 2023.

While this talk is about magnitudes of CKM elements, it is important to emphasize that testing the flavor sector of the standard model (SM) as a whole is a lot more interesting than any single measurement. It is the combination of many measurements that gives often the most interesting information, such as the constraints on beyond SM (BSM) contributions to flavor-changing neutral-current processes. (Wolfenstein used to say that, even though he invented the Wolfenstein parameters [7], he did not care what their values were, only whether their independent determinations gave consistent results.) For the success of this program, then, the interplay between experimental and theoretical developments, seeking to optimize theoretical cleanliness and experimental precision, has been crucial and also a lot of fun.

Since a substantial part of this talk concerns semileptonic decays, it may be amusing to note that semileptonic operators (to be precise, coefficients of operators composed of $l\bar{l}q\bar{q}$ fields)

account for 1053 (i.e. 42%) of the 2499 parameters of the dimension-6 baryon and lepton number conserving terms in the third-generation SMEFT [8] (558 CP -even and 495 CP -odd terms). Also, in the low energy effective theory below the weak scale, there are 1944 semileptonic parameters (i.e. 54%) of the 3631 terms [9] (1017 CP -even and 927 CP -odd).¹

2. The past

To proceed from the Kobayashi–Maskawa proposal to building asymmetric B factories, and pursuing their spectacularly rich physics program, required mixing angles and quark masses to have fortuitous values. (Technological developments were also critical, of course.) The combination of the value of m_b (about half of the mass of the Υ resonance, discovered in 1977 [10]) and the smallness of $|V_{cb}|$ (discovered via the long lifetime of b -flavored hadrons in 1983 [11,12]) resulted in b hadrons propagating long enough distances to be measured with detectors developed in the 1980s and 1990s. (If $|V_{cb}|$ was as large as the Cabibbo angle, $|V_{us}|$, then it would have been impossible to make time-dependent CP violation measurements at Belle and BaBar in the early 2000s.) Another critical ingredient was the much larger value of m_t than anticipated in the 1980s, enabling the ARGUS discovery of the B^0 – \bar{B}^0 oscillation [13]. The measurement of the ratio of B^0 decays after mixing vs. unmixed decays, $r = 0.21 \pm 0.08$, implied that m_t was much greater than the direct bound in 1987, $m_t > 23$ GeV. The comparable timescales of oscillation and decay, $\Delta m/\Gamma = 0.77$, and constraining $\Delta\Gamma \ll \Gamma$ for B^0 mixing were also important. Finally, CLEO [14] and ARGUS [15] established a nonzero value for $|V_{ub}|$ in 1989; if it were zero, the CKM matrix could not contain a physical CP violating phase, as the Jarlskog invariant is proportional to $|V_{ub}|$. These were all crucial to make (asymmetric) B factories exciting and plausible to pursue.

The mixing of neutral mesons plays a special role in determining CKM matrix elements and constraining BSM scenarios. This goes back to the successful prediction of m_c from K^0 – \bar{K}^0 mixing [16,17]. Since the 1970s, Δm_K and CP violation in the kaon sector have provided some of the strongest constraints on BSM scenarios, because flavor-changing neutral currents (FCNCs) are most strongly suppressed between the first and second generations of quarks. In the B^0 and B_s^0 systems the large top mass implies that the Glashow–Iliopoulos–Maiani (GIM) mechanism [18] does not yield strong suppressions, and FCNC decay rates and CP violating observables can be much larger than in K^0 or D^0 meson processes. After the initial measurement of B_s oscillation by CDF [19], by now $\Delta m_{B_s} = (17.7656 \pm 0.0057)$ ps⁻¹ [20] is one of the most precise CKM measurements, yielding a 1.6×10^{-4} relative uncertainty for $|V_{tb}V_{ts}| f_{B_s}^2 B_{B_s}$. The LHCb Collaboration’s textbook measurement of the time dependence of unmixed (“right-sign”) and mixed (“wrong-sign”) B_s decays is shown in Fig. 1. If the uncertainty of $f_{B_s}^2 B_{B_s}$ from lattice QCD did not dominate, the uncertainty of $|V_{tb}V_{ts}|$ would compete with that of $|V_{ud}|$ (the latter may actually be underestimated [21]).

The evaluation of FCNC B -decay amplitudes at the (multi)loop level also started around 1986–87, including two-loop calculations of $b \rightarrow s\gamma$ [22–24], important both for its sensitivity to BSM physics, and, in the SM, to $|V_{tb}V_{ts}|$. Remarkably, for $m_t \simeq 170$ GeV (unknown at that time), the prediction $\Gamma(B \rightarrow X_s\gamma)/\Gamma(B \rightarrow X_c e\bar{\nu}) \approx 0.003$ [22] is very close to the current central value [25]. In the presence of unavoidable experimental phase space cuts, the b -quark distribution function in the B meson [26,27] becomes important to describe the photon

¹I thank Aneesh Manohar for help with adding (the correct) integers.

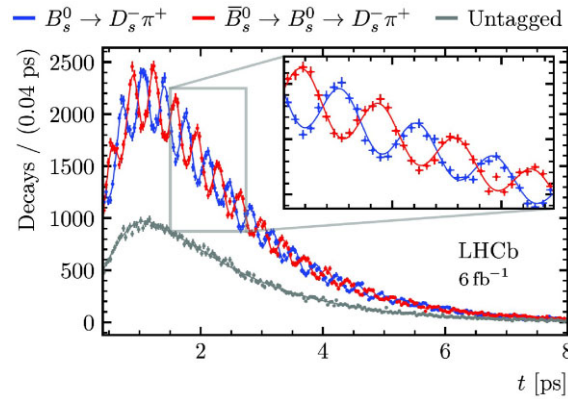


Fig. 1. Time dependence of “right-sign” (blue) and “wrong-sign” (red) B_s decays [20].

spectrum, which is largely BSM physics independent [28], can be extracted from the measured decay distributions [29], and plays a crucial role in the determination of $|V_{ub}|$ from inclusive decays.

The development of heavy quark symmetry [30,31] and heavy quark effective theory (HQET) [32] also started around 1990, as well as putting the calculation of inclusive B -decay rates on rigorous foundations [33]. These have played crucial roles since then, for the determinations of $|V_{cb}|$ and $|V_{ub}|$, which are essential to constrain BSM physics by comparing measurements dominated by tree-level processes in the SM with loop-mediated FCNC measurements. In particular, the uncertainty of $|V_{cb}|$ is a large part of the uncertainty of the SM predictions for ϵ_K , $\mathcal{B}(K \rightarrow \pi \nu \bar{\nu})$, $\mathcal{B}(B_{d,s} \rightarrow \ell^+ \ell^-)$, etc. The measurement of $|V_{ub}|$ is especially important in constraining BSM physics, because it constitutes the dominant uncertainty in the side of the unitarity triangle (a graphical representation of the $V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$ relation; see Fig. 3 in Section 3 below) opposite to the currently best measured angle, $\beta(\equiv \phi_1)$. Moreover, the same theoretical tools can be used to analyze inclusive and exclusive FCNC decays, with complementary sensitivity to BSM physics, such as those mediated by $b \rightarrow s \gamma$, $b \rightarrow s \ell^+ \ell^-$, and $b \rightarrow s \nu \bar{\nu}$ transitions. (A recent illustration of how such combinations work is the semi-inclusive analysis of $B \rightarrow X_s \ell^+ \ell^-$ in the high- q^2 region, where the best constraints are derived [34] by comparing the data with a similar Belle measurement of $B \rightarrow X_u \ell \bar{\nu}$ [35], using the method of Ref. [36].)

Since the development of HQET, the determination of $|V_{cb}|$ from exclusive $B \rightarrow D^{(*)} \ell \bar{\nu}$ decays has relied on measuring the $q^2 = (p_\ell + p_\nu)^2$ distribution near the so-called zero recoil point, $w = 1$ [$w = (m_B^2 + m_{D^{(*)}}^2 - q^2)/(2m_B m_{D^{(*)}})$], where the $D^{(*)}$ is at rest in the restframe of the decaying B meson. The rates can be schematically written as

$$\frac{d\Gamma(B \rightarrow D^{(*)} \ell \bar{\nu})}{dw} = (\text{calculable terms}) |V_{cb}|^2 \begin{cases} (w^2 - 1)^{1/2} \mathcal{F}_*^2(w), & \text{for } B \rightarrow D^*, \\ (w^2 - 1)^{3/2} \mathcal{F}^2(w), & \text{for } B \rightarrow D. \end{cases} \quad (1)$$

Both $\mathcal{F}(w)$ and $\mathcal{F}_*(w)$ are equal to the Isgur–Wise function in the $m_{b,c} \gg \Lambda_{\text{QCD}}$ limit, and $\mathcal{F}_*(1) = 1$ is the basis for a model-independent determination of $|V_{cb}|$. There are calculable corrections in powers of $\alpha_s(m_{c,b})$, as well as terms suppressed by $\Lambda_{\text{QCD}}/m_{c,b}$, which can only

be parametrized, and that is where hadronic uncertainties enter. Schematically,

$$\begin{aligned}\mathcal{F}_*(1) &= 1_{\text{(Isgur-Wise)}} + c_A(\alpha_s) + \frac{0_{\text{(Luke)}}}{m_{c,b}} + \mathcal{O}(\Lambda_{\text{QCD}}^2/m_{c,b}^2), \\ \mathcal{F}(1) &= 1_{\text{(Isgur-Wise)}} + c_V(\alpha_s) + \mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}).\end{aligned}\quad (2)$$

The absence of the $\mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b})$ term for $B \rightarrow D^* \ell \bar{\nu}$ at zero recoil is a consequence of Luke's theorem [37]. Calculating corrections to the heavy quark limit in these decays is a vast subject. To achieve percent-level precision, lattice QCD appears to be the only tool [38]. FLAG [39] quotes $\mathcal{F}_*(1) = 0.904 \pm 0.012$ and $\mathcal{F}(1) = 1.054 \pm 0.009$, while the experimental data are $|V_{cb} \mathcal{F}_*(1)| = (34.77 \pm 0.36) \times 10^{-3}$ and $|V_{cb} \mathcal{F}(1)| = (41.26 \pm 0.97) \times 10^{-3}$ [40]. An important ingredient of the analysis that only received full attention in recent years, after the publication of unfolded measurements [41,42], is related to the functional form of fitting the data and extrapolating the rate to $w = 1$ (where phase space vanishes). In the experimental determinations of $|V_{cb}|$ before 2017, the model-independent Boyd–Grinstein–Lebed (BGL) parametrization [43–45] was supplemented by QCD sum rule results [46–48] for the subleading $\mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b})$ Isgur–Wise functions, to arrive at a fit prescription with fewer parameters [49] (with hard-to-quantify model dependence), which all prior BaBar and Belle analyses used. Recently two approaches were proposed to incorporate some $\mathcal{O}(\Lambda_{\text{QCD}}^2/m_c^2)$ corrections [50,51]. The question of how to fix the number of BGL expansion parameters that are fitted from a measurement of $B \rightarrow D^* \ell \bar{\nu}$ distributions with a given statistics is subject to debate [52–54], and different choices in recent experimental analyses [55,56] impact the extracted value of $|V_{cb}|$ significantly. Thus, in future measurements a detailed validation of the choices made with toy Monte Carlo studies seems essential. There are additional open questions related to (i) moderate tensions between HQET predictions, lattice QCD results, and form factor measurements; (ii) the role of the D^{**} states, etc. [57].

For the determination of $|V_{cb}|$ from inclusive $B \rightarrow X_c \ell \bar{\nu}$ decay, instead of identifying all particles, final-state hadrons that can be produced by the strong interaction are summed over, subject to constraints determined by short-distance physics, e.g. the energy of the charged lepton. Although hadronization is nonperturbative, it occurs on much longer distance (and time) scales than the underlying weak decay, and therefore one can use an operator product expansion (OPE) to calculate the decay rate in a systematic expansion. The leading order corresponds to the free b -quark decay rate, and perturbative and nonperturbative corrections can systematically be accounted for. The first few dominant nonperturbative parameters can be extracted by fitting differential decay distributions in the same $B \rightarrow X_c \ell \bar{\nu}$ decays, an approach that has been pursued for over 20 years now (mainly using fits in the so-called $1S$ [58,59] and kinetic [60,61] mass schemes). Recently three-loop α_s^3 corrections were calculated [62–64], α_s corrections are known up to $\mathcal{O}(\Lambda_{\text{QCD}}^3/m^3)$ [65], and lattice QCD calculations of inclusive rates are being developed (see Ref. [66] and references therein).

Given that determinations of $|V_{cb}|$ from exclusive and inclusive decays have been in persistent tension (at an inconclusive, couple of sigma level), it is possible that a detailed understanding of the composition of the inclusive rate in terms of the sum over exclusive modes will be needed, in order to understand the resolution of this tension (whether on the theoretical or experimental side, or a combination). The upcoming much larger datasets should allow this to be explored and hopefully resolved.

Determinations of $|V_{cb}|$, which are currently less precise, can also be obtained from the measurement of $B_s \rightarrow D_s^{(*)} \mu \bar{\nu}$ decay [67]. In addition, semileptonic decays to τ leptons measured in $B \rightarrow D^{(*)} \tau \bar{\nu}$ and related modes are also sensitive to $|V_{cb}|$. The most precise data involving τ leptons are the $|V_{cb}|$ -independent ratios, $\mathcal{B}(B \rightarrow D^{(*)} \tau \bar{\nu})/\mathcal{B}(B \rightarrow D^{(*)} \ell \bar{\nu})$, measured by BaBar, Belle, and LHCb for the last 10 years. If the current, approximately 3σ [40], hint of lepton nonuniversality prevails, the determination of $|V_{cb}|$ becomes more complicated, since one would need to know the size of the BSM contributions in the e and μ modes. It is due to heavy quark symmetry that the SM predictions for these ratios of rates is under good theoretical control [51,68], even independent of lattice QCD.

3. The present

There was little to say about the present at the time of this symposium, in February 2023, since neither LHC nor Belle was taking data, during LHC's LS2 and the Belle II upgrade. However, ChatGPT was fairly new, and I was amused by the poem it composed about Kobayashi, Maskawa, and Belle II, shown in Fig. 2. (I said that this was the first and last time I'd use ChatGPT in a talk, which I hope to continue to obey.)

Possibly the most concise illustration of the achievements of the last two decades is what used to be the “money plot” for BaBar and Belle, summarizing the main constraints on the unitarity triangle, shown in Fig. 3 (left plot). These measurements of primarily $B_{u,d}$ decays (some of them further refined by LHCb) established the Kobayashi–Maskawa mechanism as the dominant source of CP violation in flavor-changing processes in the quark sector. Although this SM CKM fit shows impressive and nontrivial consistency, the implications of the agreement among different measurements are often overstated. The determination of CKM parameters only using dominantly tree-level decays yields the plot on the right in Fig. 3. The constraints shown in the left plot in Fig. 3 but not on the right all involve neutral meson mixing. Allowing BSM contributions, FCNC processes may be altered significantly and additional parameters related to CP and flavor violation arise, so the fits become less constraining. The conclusion

Kobayashi and Maskawa, their theories we celebrate,
 Their work on broken symmetry, opened a new physics debate.

Their findings changed our understanding, of the universe's design,
 And paved the way for Belle II, a new experiment divine.

At the SuperKEKB accelerator, the search for answers began,
 To study the B meson, and understand the weak force's plan.

With precision and accuracy, the data was analyzed and revealed,
 Uncovering secrets of the universe, and how it has been sealed.

The legacy of Kobayashi and Maskawa, lives on in Belle II's quest,
 To push the boundaries of knowledge, and put their theories to the test.

Their names will always be remembered, in the annals of time,
 For their contribution to physics, and their search for truth sublime.

Fig. 2. ChatGPT: write a poem about Kobayashi, Maskawa, and Belle II (January 29, 2023).

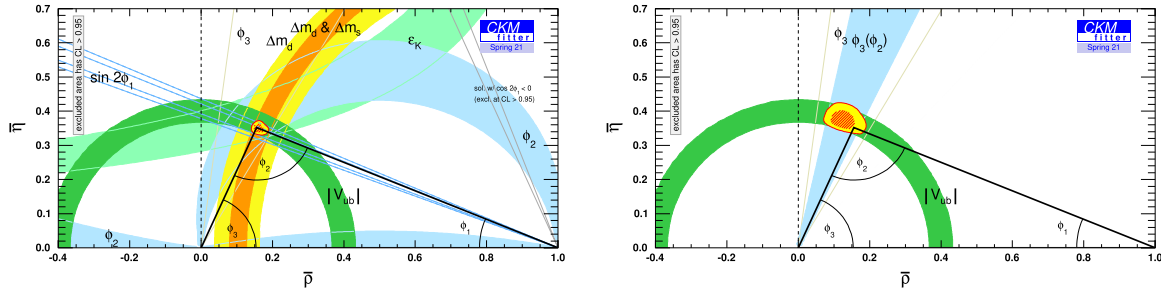


Fig. 3. Main constraints on the unitarity triangle (left), and their subset dominated by tree-level SM processes (right). The colored regions show 95% CL. (From Ref. [69]²).

of such analyses, some of which will be discussed below, is that $\mathcal{O}(20\%)$ BSM contributions to most FCNC processes are still allowed. As a result of LHCb measurements in Runs 1 and 2, the constraints in the B_s sector have become comparably strong to constraints in the B_d sector.

4. The future

In the next one to two decades, the LHC experiments [70,71], Belle II [72], BES III [73], and their planned and possible upgrades will increase b -, c -, and τ -decay datasets by almost two orders of magnitude. If the FCC (future circular collider) is built, it will have an extremely rich flavor physics program [74,75], studying 10^{12} b -flavored hadrons from the decay of Z bosons, in a clean experimental environment. An intriguing aspect of the FCC- ee flavor physics program is that, in its $W^+ W^-$ phase, the decay of 10^8 W bosons will allow a determination of $|V_{cb}|$ with about 0.3% precision [76], independent of $|V_{cb}|$ measurements in B decays.

The measurement of $|V_{ub}|$ from inclusive semileptonic decays belongs in my mind largely to the future, because the existing determinations all include some unaccounted model dependence, which we know how to eliminate. (The determination of $|V_{ub}|$ from exclusive decays seems fully in the domain of lattice QCD, at the relevant precision [38].) Similar to the case of $B \rightarrow X_s \gamma$ discussed above, also in the analysis of inclusive $B \rightarrow X_u \ell \bar{\nu}$ decays, significant experimental cuts on the phase space are required to suppress the much larger $B \rightarrow X_c \ell \bar{\nu}$ backgrounds. As a result, instead of local HQET matrix elements, the nonperturbative quantities that determine the rate are the b -quark distribution functions in the B meson [26,27] (except for the $q^2 > (m_B - m_D)^2$ cut [77]). At leading order in Λ_{QCD}/m_b there is only one such function, which is identical for $B \rightarrow X_s \gamma$ and $B \rightarrow X_u \ell \bar{\nu}$, whereas, at subleading order, several different distribution functions occur. For $B \rightarrow X_u \ell \bar{\nu}$, the consistent treatment of the shape function region (where the hadronic final state is “jetty”) and the local OPE region is more complicated than in the case of $B \rightarrow X_s \gamma$.

Just like the determination of $|V_{cb}|$ from inclusive decays relies on extracting HQET matrix elements and $|V_{cb}|$ itself from the measurements of $B \rightarrow X_c \ell \bar{\nu}$ absolute rates and distributions, the determination of $|V_{ub}|$ from inclusive decays should utilize a similar strategy, fitting $B \rightarrow X_u \ell \bar{\nu}$ rates and distributions (as well as the $B \rightarrow X_s \gamma$ spectrum), to b -quark distribution functions and $|V_{ub}|$. To develop such a self-consistent theoretical approach and fitting tool has been the goal of SIMBA [29]. Figure 4 shows a toy Monte Carlo study (as an illustration, not using the state-of-the-art theory yet), fitting the $B \rightarrow X_u \ell \bar{\nu}$ charged lepton (left) and hadronic invariant mass (right) spectra, with statistics corresponding to 75/ab [78]. The sensitivity is

²See also <http://ckmfitter.in2p3.fr/> for updates.

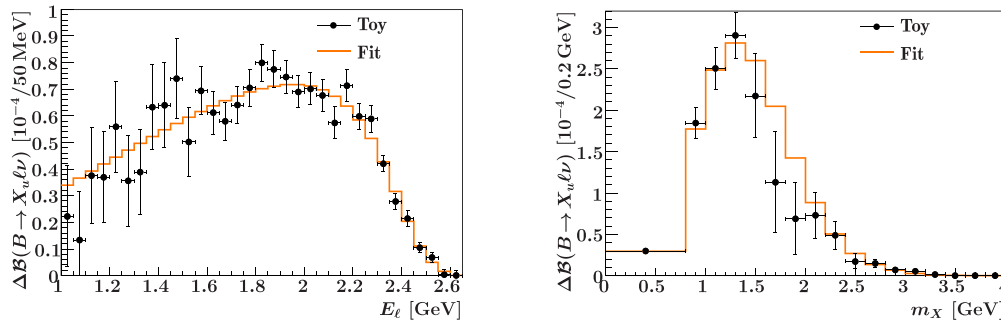


Fig. 4. A (not state-of-the-art) Monte Carlo study with SIMBA, fitting the charged lepton (left) and hadronic invariant mass (right) spectra, with statistics corresponding to 75/ab [78].

actually underestimated, as it assumes uncertainties and correlations similar to a BaBar full hadronic reconstruction analysis, compared to which the Belle II hadronic tagging efficiency is already better. This suggests that, depending on the ability to constrain (linear combinations of) subleading shape functions, an uncertainty on $|V_{ub}|$ at the few percent level may be achievable at Belle II.

As a slight aside, the measurements of $B_{d,s} \rightarrow \mu^+\mu^-$ are usually discussed for their BSM sensitivity, for good reasons. While $\mathcal{B}(B_s \rightarrow \mu^+\mu^-) = (2.95 \pm 0.41) \times 10^{-9}$ [40] has been measured by three LHC collaborations, the decay $B_d \rightarrow \mu^+\mu^-$, which has a branching ratio around 10^{-10} in the SM, has not been observed. Its uncertainty is not expected to reach 10% even by the end of HL-LHC data taking. It is an example of a channel whose interpretation will not become theory limited in any planned experiment, and greater statistics would always teach us about short-distance physics. In terms of an operator analysis, its mass-scale sensitivity is comparable to $K \rightarrow \pi\nu\bar{\nu}$. Amusingly, the theoretically cleanest determination of $|V_{ub}|$ that I know of, in principle, setting experimental realities aside, could come from the ratio $\mathcal{B}(B^- \rightarrow \ell\nu)/\mathcal{B}(B^0 \rightarrow \mu^+\mu^-)$ relying only on perturbative QCD calculations and isospin.

4.1. Beyond SM sensitivity in $B-\bar{B}$ mixing

It has long been known that the mixing of neutral mesons is particularly sensitive to BSM physics and probes some of the highest scales (the smallness of $\Delta m_K/m_K \approx 7 \times 10^{-15}$ has been known since the 1960s). In a large class of models, the dominant effect of BSM physics is to modify the mixing amplitudes of neutral mesons, and tree-level decays are not affected. In this case, the 3×3 CKM matrix remains unitary, and the BSM effects can be parametrized by two real parameters for each neutral meson system. The mixing of B_q^0 mesons (where $q = d, s$) are simplest to analyze, as they are dominated by short-distance physics. Writing the mixing amplitude as $M_{12}^q = M_{12}^q(\text{SM}) (1 + h_q e^{2i\sigma_q})$, one has to redo the CKM fit including the effects of h and σ on the observables. The resulting constraints on h_d and h_s , the magnitudes of the BSM contributions relative to the SM in B_d and B_s mixing, respectively, are shown in the left plot in Fig. 5, assuming independent BSM contributions. This shows that order (10–20)% corrections to M_{12} are still allowed. (Evidence for $h_q \neq 0$ would rule out the SM, and the black dot indicating the best-fitting point in the left plot shows a slight pull.) Similar conclusions apply to other neutral meson mixings, as well as many other $\Delta F = 1$ FCNC decays, such as $B \rightarrow X\gamma$, $B \rightarrow X\ell^+\ell^-$, $B_{d,s} \rightarrow \ell^+\ell^-$, $K \rightarrow \pi\nu\bar{\nu}$, etc. The same analysis, for a future time, using sensitivity projections for 50/ab LHCb data and 50/ab Belle II data [79] (referred to there as “Phase I”) are shown in the

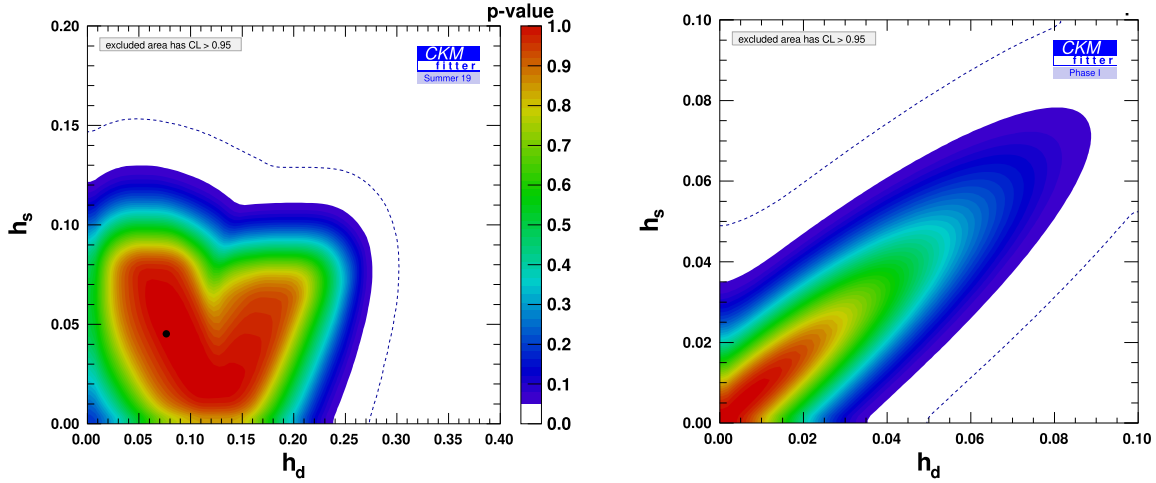


Fig. 5. Constraints on h_d – h_s at present (left plot) and expected future sensitivity assuming 50/ab LHCb data, and 50/ab Belle II data (right plot). The colors indicate the confidence levels as shown, while the dashed lines show 3σ limits. (From Ref. [79].).

Table 1. The scale of the $(C_q^2/\Lambda^2)(\bar{b}_L\gamma^\mu q_L)^2$ operator probed (in TeV, at 95% CL) by B_d and B_s mixings at present and at “Phase I” (described in the text) [79]. The impact of the SM-like hierarchy of couplings and/or loop suppression is shown.

Couplings	BSM loop order	Sensitivity for summer 2019 [TeV]		Phase I sensitivity [TeV]	
		B_d mixing	B_s mixing	B_d mixing	B_s mixing
$ C_{ij} = V_{ii}V_{ij}^* $ (CKM-like)	Tree level	9	13	17	18
	One loop	0.7	1.0	1.3	1.4
$ C_{ij} = 1$ (No hierarchy)	Tree level	1×10^3	3×10^2	2×10^3	4×10^2
	One loop	80	20	2×10^2	30

right plot in Fig. 5. With even higher statistics expected with the LHCb Upgrade II [80] and a possible upgrade of Belle II [81] (and especially FCC- ee), improving the determination of $|V_{cb}|$ beyond current projections becomes crucial for this type of analysis to continue to improve its sensitivity [79]. If new developments result in theory uncertainties not becoming a limiting factor for this type of analysis, then the statistical sensitivity could reach [79] $h_d \lesssim 0.02$ and $h_s \lesssim 0.01$ at 95% CL, with the full LHCb Upgrade II and the possible Belle II upgrade datasets.

The correspondence between the constraints on $h_{d,s}$ and the scale of BSM physics depends on the operator(s) that contribute. As the simplest example, if new contributions add a term $(C_q^2/\Lambda^2)(\bar{b}_L\gamma^\mu q_L)^2$ to the same operator that describes B mixing in the SM, then one finds

$$h_q \simeq \frac{|C_q|^2}{|V_{ib}^* V_{iq}|^2} \left(\frac{4.5 \text{ TeV}}{\Lambda} \right)^2. \tag{3}$$

It is then straightforward to translate the constraints in Fig. 5 to the scale of BSM physics probed; a summary of expected sensitivities is shown in Table 1 for different structures. These sensitivities, even with SM-like loop- and CKM-suppressed coefficients, are comparable to the scales probed by the LHC. The factor of 1.5–2 increase in mass-scale sensitivity at Phase I compared to the present is greater than the improvement for many other searches during the HL-LHC.

5. Final remarks

Pursuing ever more precise tests of the flavor sector remains exciting and well motivated. If there is BSM physics below the few tens of TeV scale, which we hope to probe directly in current and planned experiments, it cannot have a generic flavor structure, as that is already ruled out by orders of magnitudes. There is thus an unavoidable complementarity between direct searches of BSM physics, and learning about its signature (or lack of signature) in precision experiments. In a large class of scenarios one may expect deviations from the SM close to the current experimental precision, so there is significant discovery potential any time experimental sensitivities improve substantially.

As in the past, theory will be important for the interpretation of the measurements and for maximizing their sensitivity to BSM physics, and experimental results will be essential inputs of theory considerations, and triggers for new developments. In particular, the reduction of the uncertainties of $|V_{cb}|$ and $|V_{ub}|$ are crucial. In each case, with new measurements, the tension between inclusive and exclusive measurements will hopefully shrink, possibly yet-unknown subtleties understood, and the source of current tensions identified. They may lie either on the theory side or on the experimental side, or a combination. In the case of the determination of $|V_{ub}|$ from inclusive decays, we know how qualitatively better measurements can be done than those implemented so far. In addition, the determinations from exclusive semileptonic decays will improve with statistics and better lattice QCD calculations. Both for $|V_{cb}|$ and $|V_{ub}|$, measurements from two-body leptonic decays will also become available, and will become competitive in precision as statistics grows.

These experimental improvements need much larger datasets than available so far, and there is a suite of measurements (which have been identified, and will likely become further enlarged by future theory developments), in which theoretical uncertainties will not limit probing short-distance physics. However, I do not think that anyone has seriously explored what the largest B -decay datasets may be, which would improve sensitivity to short-distance physics.³ My understanding is that the sensitivity to possible signatures of BSM physics would continue to improve with datasets well beyond all upgrades contemplated at LHCb and Belle II, and even at the Tera- Z run of the FCC.

Conflict of interest statement. None declared.

Acknowledgments

I thank the organizers for the invitation to this very enjoyable symposium, and for their patience regarding the completion of this write-up. I thank Shoji Hashimoto and Aneesh Manohar for helpful conversations. This work was supported in part by the Office of High Energy Physics of the US Department of Energy under contract DE-AC02-05CH11231.

Funding

Open Access funding: SCOAP³.

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³I vividly remember Sanda-san at the Workshops on Higher Luminosity B Factory at Izu in 2003 [82], saying (as a joke? or not as a joke?) that the question was not whether the design luminosity of the next-generation B factory should be 10^{35} or 10^{36} , but rather if it should be 10^{53} or 10^{63} .

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