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Title
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Permalink
https://escholarship.org/uc/item/9qp0s911

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Publication Date
1996-11-01

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ECONOMIES OF SCALE AND SELF-FINANCING RULES WITH NONCOMPETITIVE FACTOR MARKETS

Kenneth A. Small

ABSTRACT

When a firm or public authority prices output at marginal cost, its profits are related to the degree of local economies of scale in its cost function. As is well known, this result extends to the case where some congestion-prone inputs are supplied by users. I show that contrary to common belief, the result holds even when scale economies are affected by a rising factor supply curve. In that case, constant returns to scale in production produces diseconomies of scale in the cost function, making marginal-cost pricing profitable. Examples are provided for a monopsonist both with and without price discrimination. In the latter case, second-best pricing is also considered: profits then are not governed in the usual way either by returns to scale in production or by scale economies in the cost function, but some useful bounds are provided.

ECONOMIES OF SCALE AND SELF-FINANCING RULES WITH NONCOMPETITIVE FACTOR MARKETS

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1. Introduction

As is well known, profits of a competitive firm have a sign determined by the degree of local returns to scale in its production. Profits are negative, zero, or positive if returns to scale are locally increasing, constant, or decreasing, respectively. This is sometimes called a <u>self-financing rule</u>.

But what of a firm that is a monopsonist in one or more of its factor markets? If the factor-supply functions are well-behaved, then there exists a well-defined cost function that takes them into account (Varian 1984, p. 105). The relationship between cost and output for this cost function defines the extent of <u>economies or diseconomies of scale</u>, which differs in general from the degree of returns to scale of production.[1] For example, suppose production takes place under constant returns to scale but one or more supply prices rise with factor usage (the other prices being constant); then the cost function will display diseconomies of scale because increasing output raises some input prices.

Do scale economies in such a situation tell us anything about profits? Clearly they do if profits are defined in the usual way as revenue minus costs. Thus, for example: "Where there are economies of scale, prices set at marginal cost will fail to cover total costs, thus requiring a subsidy." (Vickrey 1987, p. 315) As I show in the next section, this statement follows immediately from the definition of scale economies.

Self-financing rules have been extended to congestible facilities, most notably through the example of congested highways (Mohring and Harwitz, 1962, pp. 81-86; Strotz, 1965; Mohring, 1970, p. 696).[2] In this case marginal cost includes the imputed cost of inputs supplied by users

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[1]See Bannock et al. (1978, p. 388) for the terminological distinction between returns to scale and economies of scale. I thank Frank Gollop for bringing it to my attention. See also Eatwell (1987) and Silvestre (1987).

[2]The result is generalized to a variety of dynamic settings by Braid (1995) and Arnott and Kraus

(continued...)

rather than purchased in markets. More generally, congestible public goods may be provided efficiently at zero profits by governments, clubs, or firms when there are locally constant returns to scale — a condition that is guaranteed, for example, by perfect competition among firms with U-shaped average cost curves (Oakland, 1972; Berglas, 1976). Marginal-cost pricing of congestible facilities might occur due either to competition (Berglas, 1976; DeVany and Saving, 1980) or to prescription (Vickrey, 1987).

In stating and applying this result the distinction between returns to scale and economies of scale has been largely overlooked.[3] Yet empirical work on highway congestion exemplifies the importance of the distinction. This work typically estimates a production function or a cost function, using some combination of financial accounts and engineering assumptions. Those authors basing their case solely on the production function typically find increasing returns to scale, and thereby argue that marginal-cost pricing will produce a deficit.[4] Others argue that in the cost function, those increasing returns are offset by a rising supply price of urban land, possibly yielding no scale economies or even diseconomies so that marginal-cost pricing would produce a balanced budget or a surplus.[5]

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[2](...continued)
(1995).

[3]In the three cases cited, Mohring and Harwitz (1962) base their result on the "capital and congestion cost functions" (p. 85); Mohring (1970) on the production function (p. 696); and Strotz (1965) on the "returns to scale" of a government production function that converts "expenditure on roads, E," into congestion reduction (p. 135). Strotz's E is an amount of "homogeneous productive service" (p. 131) which is used to produce the aggregate consumption good, so in formal terms it really refers to a factor input rather than a cost, thereby justifying his use of the term "returns to scale."

[4]For example, Meyer et al. (1965), Mohring (1965, 1970); Kraus (1981b); Jansson (1984, pp. 220-222).

[5]For example, Fitch and Associates (1964), Vickrey (1965), Strotz (1965, p. 137), Keeler and Small (1977), Small et al. (1989), Newbery (1990). Strotz's discussion is especially intriguing. Strotz justifiably uses the term "returns to scale" (see earlier footnote); but when he speculates on the type of returns actually encountered, he argues for decreasing returns due to three factors: network effects (which Kraus 1981b shows to be an invalid argument); interchanges (which Kraus shows to be a valid argument though not sufficient empirically to reverse the increasing returns from highway width); and "more expensive

(continued...)

In this paper, I explicitly derive a self-financing rule based on cost functions when the firm (or public authority) is a monopsonist in one factor market, taken to be that for land. I consider cases both with and without price discrimination. I also consider both pure marginal-cost pricing and second-best pricing that corrects for the factor-market distortion, the latter case requiring a natural modification of the concept of "profit" for the result to hold. This formulation makes clear that profits are governed by the existence of scale economies or diseconomies of a cost function that incorporates whatever factor-supply elasticities actually face the firm. I show the result first in a general setting, then for the case of a congestible facility using the example of highway congestion.

My conclusion differs from that of Berechman and Pines (1991), who claim to resolve the conflict in favor of a rule based solely on the production function. They demonstrate that the degree of returns to scale of production determines the sign of "imputed profits;" these are defined as revenues minus a quantity I call "imputed costs." Imputed costs include a land component equal to the quantity of land used multiplied by its shadow price. A similar result is derived by Strotz (1965), who explicitly states: "the rent that equates the demand and supply of land is to be used in calculating the land cost of the road." (p. 164)

This formulation using imputed costs has the appeal that it more easily characterizes first-best pricing and investment rules. It also provides some nice results in models of clubs and local public goods. For example, under locally constant returns the imputed cost of providing a congestible public good is equal to the revenues from optimal user charges supplemented by a confiscatory tax on any urban land rents (Arnott, 1979; Berglas and Pines, 1981). But only for the factor-price taker do imputed profits correspond to real financial flows.[6] If one wishes to analyze the profits of a real fiscal entity, a rule involving the actual cost function is more

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[5](...continued)
construction ... *and land acquisition costs*" as the highway system is expanded (p. 137, emphasis added). This third argument appears to invoke a rising supply price.

[6]Hence Oakland (1972, p. 347) qualifies his generalized Mohring-Harwitz result as applying only in a "competitive economy" in which average and marginal production costs of the congestible public good are equal.

useful. One may wish to know, for example, whether a public agency will incur deficits, whether a regulated private firm will be financially viable, or whether the developer of a club-like community will make a profit. Such questions can be answered using a cost function that accounts for varying factor prices.

I demonstrate these results in a general context in section 2-4. I then derive (section 5) a Mohring-Harwitz type of model for congestible facilities as a special case of this more general one, in order to show that the results apply there as well. I conclude by arguing that authorities purchasing land for public projects do in fact face a rising supply curve, at least within the legal context prevailing in the United States.

2.   The Simple Mathematics of Self-Financing Rules

At the most general level, the self-financing rule is simply a consequence of basic definitions.[7] It applies to any enterprise that prices output at marginal cost, whether due to competition in output markets, to regulation, or to the policies set for a public agency. It can be stated concisely as:

> Proposition 1. Let $C(q)$ be any differentiable cost function, and let local scale economies be measured by the ratio of average to marginal cost,

$$s = \frac{C}{q \cdot dC/dq} \, . \tag{2.1}$$

> A firm or authority incurring this cost and setting output price equal to marginal cost earns profits, as a proportion of cost, equal to $(1-s)/s$.

> *Proof:* Revenues R are given by the denominator of (2.1), implying that profits relative to costs are:

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[7]See, for example, Baumol et al. (1988, p. 68), Kraus (1981a, p. 235), or Small (1992, p. 49). I regret that in my 1992 treatment I mistakenly used the term "returns to scale" to describe economies or diseconomies of scale.

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$$\frac{R}{C} = \frac{1}{s} \ . \tag{2.2a}$$

Equivalently,

$$\frac{R-C}{C} = \frac{1-s}{s} \ . \tag{2.2b}$$

Revenues exceed costs under diseconomies of scale ($s<1$), fall short under economies of scale ($s>1$), and are exactly in balance in the intermediate case ($s=1$).

This result easily generalizes to multiple outputs. Let $q=\{q_j\}$ and $p=\{p_j\}$ be vectors, and let s be the multiproduct scale economies associated with cost function $C(q)$, as defined by Bailey and Friedlaender (1982):

$$s = \frac{C}{\sum_j q_j \cdot \partial C/\partial q_j} \ . \tag{2.1'}$$

Then marginal-cost pricing implies that the denominator of this equation is equal to revenue, so that again (2.2) holds. For simplicity, I use the one-output case for the remainder of the paper.

My purpose is to explore the consequences of equations (2.2) when factor prices are not constant. This can be done most easily by focusing on the case where there is just one non-competitive factor market, which we may call "land". Let $x_a=(x_1, ..., x_{n-1})$ be the vector of input factors other than land, with w_a the corresponding vector of fixed factor prices. Let x_n be land input, and let $E_n(x_n)$ be the expenditure required to acquire x_n units of land. Output q is produced according to the production function

$$q = f(x_a, x_n) \ . \tag{2.3}$$

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The cost function is then defined as

$$C(q) - \underset{x_a, x_n}{\text{Min}} \left\{ w_a x_a + E_n(x_n) \quad : \quad q - f(x_a, x_n) \right\} .$$

(2.4)

We know from the envelope theorem that dC/dq is equal to the incremental cost of increasing output by increasing any one of the inputs:

$$\frac{dC}{dq} - \frac{\left(\partial[w_a x_a + E_n(x_n)]/\partial x_i\right)^*}{(\partial q/\partial x_i)^*} \qquad \text{for each } i$$

(2.5a)

$$- w_i / f_i^* \qquad \text{for each } i < n$$

(2.5b)

$$- (dE_n/dx_n)^* / f_n^* \qquad \text{for } i - n .$$

(2.5c)

where $f_i \equiv \partial f/\partial x_i$ and where the asterisk indicates that the quantity is evaluated at the solution $x^* \equiv (x_a^*, x_n^*)$ to (2.4). In particular, the usual equality holds between dC/dq and the short-run marginal cost defined by holding one or more factors constant in (2.4); that equality depends only on x^* being the solution to (2.4), not on its being optimal in any broader sense.

3.   Applications to Monopsony and Distorted Factor Prices

This section considers three specific cases of a non-competitive land market that might arise in public projects. The first two cases are monopsony, without and with price discrimination. The third case is a constant but artificially low price of land. In each case, I derive the cost function and note some associated optimization conditions. Where relevant, I also consider a second-best price that improves welfare compared to marginal-cost pricing.

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Monopsonist with No Price Discriminaion

In this case, land is supplied with a rising supply curve $w_n(x_n)$. (All the equations are identical if the supply curve is falling, with appropriate reversals of inequalities.) Thus $E_n(x_n) = w_n(x_n)x_n$, and (2.5c) becomes:

$$dC/dq = w_n^* \cdot (1+\omega_n^*) / f_n^* \tag{3.1}$$

where $\omega_n > 0$ is the inverse supply elasticity of x_n, defined by

$$\omega_n = \frac{x_n}{w_n} \frac{dw_n}{dx_n} . \tag{3.2}$$

It is useful to contrast this cost function with the imputed cost function analyzed by Berechman and Pines (1991),[8] which I denote by $\tilde{C}(q)$. Imputed cost is defined by first setting up a general equilibrium model with identical consumers, two produced goods, and aggregate resource constraints on factor inputs. One of the goods is produced according to (2.3). This model is then solved for all utility-maximizing quantities, leading to shadow factor prices $(\tilde{w}_a, \tilde{w}_n)$, and some output $\tilde{q}$ which we may consider a reference output. Finally, a minimization like that in (2.4) is performed except factor prices are held constant at these shadow prices; this minimization is performed at an arbitrary level of q, no just at $\tilde{q}$, so let us denote the resulting minimized cost by $\tilde{C}(q)$. This produces the theoretical results that the optimum can be decentralized by charging output price $\tilde{p} = d\tilde{C}/dq$; and that at this optimum, the sign of imputed profits

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[8]This is the cost function referred to in their statement that "the homogeneity of the production function is exactly the reciprocal of the homogeneity of the corresponding cost function, *with fixed factor prices*..." (p. 178; italics in original). Their derivation is for the special case of highways, considered in section 3 below; but the same concepts apply to the more general case of this section.

$$R-C \;\sim\; \bar{p}q \;-\; \tilde{C}(q) \qquad\qquad\qquad (3.3)$$

is determined by what we may call *imputed economies of scale,*

$$r \;\sim\; \frac{\tilde{C}}{q \cdot \mathrm{d}\tilde{C}/\mathrm{d}q} \;, \qquad\qquad\qquad (3.4)$$

From standard duality theory, r is identical to the local returns to scale $r(\tilde{x})$ of the production function, evaluated at the corresponding input vector $\tilde{x} \equiv (\tilde{x}_a, \tilde{x}_n)$. That is, $r(x)$ is the elasticity of the production function with respect to a uniform increase in all inputs, starting at x.[9] We expect r normally to exceed s for the case considered here, reflecting the fact that the rising supply price of land creates an additional factor raising costs as output expands. More precisely, I prove the following result in the appendix:

> Proposition 2. For a given output, let x^* be the solution to the cost minimization problem (2.4) when the supply relationship is that of a non-discriminating monopsonist facing a rising supply curve $w_n(x_n)$, and let s be the economies of scale at the same output. Then $r(x^*) > s$.

It follows immediately that whenever returns to scale are constant, $r > s$ unambiguously. Hence using returns to scale r instead of economies of scale s in (2.2) would normally understate actual profits as measured by R-C, although doing so would correctly predict imputed profits $\tilde{R} - \tilde{C}$.

In general, it is difficult to say what the empirical counterpart of $\tilde{C}$ (q) might be, since the general-equilibrium shadow factor prices needed for its definition cannot be reliably estimated either from observed factor prices or from the solution to (2.4). This makes the sign of imputed profits of somewhat limited practical interest. Of course, if the authority actually

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[9]See, for example, Baumol et al. (1988), p. 21.

pays for land at its competitive rental price, the problem disappears and $\mathcal{C}$ becomes its actual cost function. I examine actual land-payment practices for the U.S. in Section 6.

In assessing the utility of results predicting actual profits under marginal-cost pricing, it is useful to ask: Why would a firm price at marginal cost? One reason might be that it has no market power in its output market. As shown by DeVany and Saving (1980), a profit-maximizing highway operator in such a situation will engage in marginal-cost pricing which takes precisely the form of congestion pricing as advocated, for example, by Walters (1961). For such a firm, choosing factor inputs according to (2.4) and setting output price equal to marginal cost as given by (2.5) may be viewed as two steps in a single profit-maximizing calculation (Varian 1984, pp. 104-105). Together they imply the well-known first-order condition

$$ pf_i - w_i(1+\omega_i) \tag{3.5} $$

where the inverse supply elasticities ω_i are all zero except for $i=n$. Equation (3.5) tells us that each factor is used to the point where the value of its marginal product is equated to its private marginal factor cost. In section 4, I use these conditions to derive the cost function explicitly for a simple production function in which both r and s are easy to see.

Anothers reason for charging marginal cost might be that a public authority is persuaded to do so by economists. (The technical term for this is *economist's fantasy*.) For example, short-run marginal-cost pricing is often recommended even when the amount of one factor is not optimal, on the assumption that that factor is fixed. As already noted, this is the same as charging dC/dq in this problem.

If land is used suboptimally but is variable, the problem is more subtle. Charging marginal cost is no longer second-best optimal given the distortion caused by the non-competitive land market. One could complain, therefore, that this authority just mentioned (or its economic advisor) is naive: it exploits its monopsony power in the land market, but refrains from exploiting any monopoly power it may have in the output market. Yet in many situations, such "naive" behavior is arguably more plausible than either first-best optimality or second-best

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output pricing. The public might well regard it as abhorrent to exploit power over highway users, while fiscally irresponsible <u>not</u> to obtain land at the lowest possible cost.

Still, we can consider the revenue implications of second-best pricing rule, formulated as follows: the firm minimizes its cost of producing any given output as in (2.4); but that output is determined by a pricing rule chosen to maximize the sum of profit, consumer surplus, and producer surplus. We expect this price to be lower than the profit-maximizing price, since expanding output provides some benefits to landowners, in the form of greater producer surplus (rents), that are not captured by the firm.

Let $p=P(q)$ be the inverse demand function for output, and let $x_a^*(q)$ and $x_n^*(q)$ be the factor demands that solve (2.4). The problem is then:

$$\text{Max}_q \left\{ \int_0^q P(\xi)d\xi - C(q) + S_n(q) \right\} \tag{3.6}$$

where

$$S_n(q) \equiv \int_0^{x_n^*(q)} \left[w_n^*(q) - w_n(x) \right] dx \tag{3.7}$$

is the producer surplus of suppliers of land, with $w_n^*(q) \equiv w_n[x_n^*(q)]$. The first-order condition[10] is

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[10]The second-order condition is met if $\tilde{C}$ is convex or at least not too concave compared to the slope of the output demand curve: see appendix.

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$$p^0 = dC/dq - dS_n/dq$$
$$= dC/dq - x_n^*(dw_n^*/dq) \qquad (3.8)$$
$$= dC/dq - \omega_n w_n^*(dx_n^*/dq) .$$

Assuming the output-elasticity of the derived demand for land is positive, this second-best price is less than the firm's marginal private cost.

Now suppose the central government instructs the highway authority to charge this second-best price. This of course will destroy the relationship between revenues and costs that occurs when the firm can maximize profits. But it is trivial to show that a simple per-unit subsidy, equal to the gap between price and marginal cost, restores the relationship:

$$R - C + q[(dC/dq)-p] = qp - C + q\,dC/dq - qp$$
$$= q\,dC/dq - C \qquad (3.9)$$
$$= C\cdot[(1-s)/s].$$

Note this result holds for any price, not just the second-best optimal price. It simply states that no matter what the output, if the firm receives a price for each unit equal to its marginal cost, then profits are a fraction $(1-s)/s$ of costs.

More interesting is the relationship between profits and returns to scale. One could speculate that the lower profits achieved under second-best pricing would be more accurately predicted by returns to scale r than by s. This turns out to be true when the production function is homothetic and ω_n is small. More precisely, I show in the appendix the following result:

Proposition 3. For a non-discriminating monopsonist selling output at the second-best price p^0 given in (3.8), the ratio of revenues to costs, $p^0 q/C$, differs from $1/r^*$ by:

$$\frac{p\,^{o}q}{C} - \frac{1}{r\,^{*}} - \frac{\omega_{n}w_{n}x_{n}}{rC}\left[1 - r\,^{*}\frac{q}{x_{n}^{*}}\frac{dx_{n}^{*}}{dq}\right] \tag{3.10}$$

where $r^{*} \equiv r(x^{*})$ is the degree of local returns to scale at the cost-minimizing input vector $x^{*} \equiv (x_{a}^{*}, x_{n}^{*})$. For a homothetic production function, the term in square brackets is positive; that is, $1/r^{*}$ underpredicts the the ratio of revenues to costs and equivalently $[(1-r^{*})/r^{*}]\cdot C$ underpredicts profits.

For the production function illustrated in Section 4, the term in square brackets is second-order in ω. So if the inverse supply elasticity of land is small, then r is a better approximation than s to use in predicting the sign of profits from second-best output pricing. However, neither will predict precisely.

Monopsonist With Perfect Price Discrimination

As is well known, perfect price discrimination in a factor market causes the firm to face the true social factor costs at the margin. As a result, the first-order conditions for maximizing profit in a competitive output market are the same as those for maximizing welfare; hence there is no distinction between first- and second-best output pricing. Those conditions, $pf_{i}=w_{i}$ for all factors i, could be written in terms of the imputed cost function $\tilde{C}$. But the corresponding self-financing rule would not be very useful, because the imputed cost function does not represent the actual outlays of the firm.

Instead, we can use the firm's actual cost function:

$$C(q) = \underset{x_{a}, x_{n}}{Min}\left\{ w_{a}x_{a} + \int_{0}^{x_{n}} w_{n}(\chi)d\chi \quad : \quad q = f(x_{a}, x_{n}) \right\}. \tag{3.11}$$

Differenting (3.11), we see that marginal cost is given by w_{i}/f_{i} for all i. Hence marginal-cost pricing produces the optimality conditions $pf_{i}=w_{i}$ for all factors i.

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Cost function (3.11) describes the true financial outflows for this firm, so the self-financing rule (2.2) is relevant. This cost function again accounts for the rising supply price of land, although in a different manner than (2.4).

Once again, the actual cost function C displays fewer scale economies, or more diseconomies, than does the imputed cost function $\tilde{C}$. This is because the two functions have the same marginal costs, as we have just seen, but C has a smaller average cost than $\tilde{C}$ due to smaller expenditures on inframarginal units of land. Therefore the ratio of average to marginal cost for C is less than for $\tilde{C}$. Using the imputed cost function instead of the actual one would impute a greater deficit, or a smaller surplus, than the firm will actually incur.

Authority Facing Constant but Artificially Low Land Price

In many situations the problem may not be so much monopsony as simply the use of market prices for land rental that are lower than the shadow prices. This may come about, for example, because public authorities are exempt from property and corporate income taxes (Vickrey, 1962) or because the price of central urban land is distorted downward due to unpriced congestion (Kanemoto, 1977; Arnott and MacKinnon, 1978).

Taking these distorted rents to be constant, the analysis of the cost function is identical to the conventional analysis; $r=s$ and equations (2.2) hold using either r or s on the right-hand side. The further question comes in considering second-best pricing. The objective is again to maximize (3.6), but now S_n is redefined as the (negative) social surplus caused by valuing land below its shadow cost:

$$S_n(q) \equiv \int_0^{x_n^*(q)} \left[\bar{w}_n - w_n(\chi_n) \right] d\chi_n \tag{3.12}$$

where $\bar{w}_n$ is the distorted rental price of land and $w_n(\bullet)$ is the shadow value (which could be constant). The first-order condition is

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$$p^{oo} = \frac{dC}{dq} + \left[w_n^* - \bar{w}_n\right]\frac{dx_n^*}{dq}$$ (3.13)

where p^{oo} is the second-best price. This price is higher than marginal cost, so the ratio of revenues to costs exceeds $1/r = 1/s$, by an amount

$$\frac{p^{oo}q}{C} - \frac{1}{r} = \frac{x_n^*(w_n^* - \bar{w}_n)}{C}\left[\frac{q}{x_n^*}\frac{dx_n^*}{dq}\right].$$ (3.14)

When the production function is homothetic, the output-elasticity in large parentheses is just $1/r$.

4. Example: Homogeneous Production, Constant-Elasticity Supply

The nature of the solutions discussed in the previous section are readily illustrated by the case of two inputs ($n=2$), with production according to the function:

$$q = f(x_1, x_n) = ax_1^\alpha x_n^\beta$$ (4.1)

and a supply curve for the second input given by:

$$w_n(x_n) = \omega_0 x_n^\omega .$$ (4.2)

The first input has constant price w_1. Returns to scale are therefore $r=\alpha+\beta$ and the inverse supply elasticity is ω.

Monopsonist with No Price Discrimination

Solving the first-order conditions for the cost minimization of equation (2.4) yields cost-minimizing factor inputs:

$$x_1^* - (A\alpha/w_1)q^\mu$$
$$x_n^* - \left(A\gamma/\omega_0\right)^{1/(1+\omega)}q^\nu \tag{4.3}$$

where

$$A - a^{-\mu}(w_1/\alpha)^{\alpha\mu}(\omega_0/\gamma)^{\gamma\mu} \tag{4.4}$$

$$\mu - \frac{1}{\alpha+\gamma} - \frac{1}{r - \beta\omega/(1+\omega)} \tag{4.5}$$

$$\nu - \frac{\mu}{1+\omega} - \frac{1}{r+\alpha\omega} \tag{4.6}$$

$$\gamma - \frac{\beta}{1+\omega} \; . \tag{4.7}$$

If w_n were constant, use of each input would be proportional to $q^{1/r}$ and so would the cost function. But with w_n rising, the authority tilts its input mix increasingly toward x_1 as q rises, so that x_1 grows more rapidly, and x_n more slowly, than $q^{1/r}$. Expenditure on x_n, however, rises at the same rate as that on x_1, as can be seen by substituting (4.3) into the expression for cost. Doing so yields:

$$C(q) \sim (A/\mu)q^{\mu} \ .$$

(4.8)

Scale economies are therefore

$$s \sim 1/\mu \sim r - \omega\beta/(1+\omega) \ ,$$

(4.9)

which is less than r in accordance with Proposition 2.

Second-best pricing according to (3.8) yields

$$p^{\,o} = Aq^{\mu-1}(1-\omega\gamma\nu)$$

(4.10)

and

$$\frac{p^{\,o}q}{C} \sim \frac{1}{s}(1 - \omega\gamma\nu)$$

$$\sim \frac{1}{r}\left[1 + \frac{\alpha\omega^2\gamma}{s^2(1+\omega)}\right] \ .$$

(4.11)

This confirms that the ratio of second-best revenues to costs is overestimated by $1/s$ (to first order in inverse supply elasticity ω) while it is underestimated by $1/r$ (but only to second order in ω).

As a numerical example, suppose $\alpha=0.7$, $\beta=0.3$, and $\omega=0.3$. Then we have constant returns to scale in production $(r=1)$, but diseconomies of scale in costs since $s=0.93$ according to equation (4.9). With marginal-cost pricing, the ratio of revenues to total costs would be $1/s=1.074$. With second-best pricing, however, the ratio would be 1.013, which is closer to $1/r$ than to $1/s$.

<u>Monopsonist with Price Discrimination</u>

If we similarly calculate the optimal factor inputs for an authority solving the minimization problem in (3.11), we find that their output-elasticities are again μ and ν as in (4.3). $C(q)$ is again given by (4.8) except that A is redefined to a smaller value.[11] Hence scale economies are again $s = 1/\mu \equiv \alpha + [\beta/(1+\omega)]$.

5. User-Supplied Inputs With Congestion

The debate motivating these derivations arose in a congestion model that looks somewhat more complex than the one presented in Sections 2-4. However, it is actually a special case, as demonstrated in this section. This underlying equivalence should be no surprise, since it is why Mohring and Harwitz (1962) were able to derive the self-financing result for the case of highway congestion.

I use a standard model of highway pricing and investment, along the lines of Keeler and Small (1977) and many others. In order to capture the insight behind self-financing results in a congestion model, the congestion externality is quantified as a cost, thereby affecting both the marginal-cost price and the total costs. However, these user-borne costs are excluded both from revenues and costs in computing the authority's profits.

The equivalence between the two models is then demonstrated by reformulating the production relationships in the congestion model into a single production function like (2.3), in which one of the inputs is user-supplied and its "price" has the interpretation of an average user-perceived value. These production relationships are two: the congestion technology, and the production of highway capacity. Congestion technology relates the user-supplied input, which is taken to be x_1, to output q. Production of highway capacity involves all the other

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[11]The proportionality factors in (4.3) and (4.8) are altered as follows: (ω_0/γ) is replaced by (ω_0/β) in the factor A defined in (4.4), and A is multiplied by β instead of γ in the second of equations (4.3).

inputs. The two relationships are linked because capacity is a parameter in congestion technology. To complete the equivalence, we must relate scale economies for this general production function to those for the two underlying relationships, and we must relate "profits" under the general model to those of the highway authority.

The user-supplied input x_1 is here called "user time" and is written as the number of users times an average travel time, which is determined by the congestion technology:

$$x_1 = q \cdot t(q,X) . \tag{5.1}$$

where capacity X is a physical property of the highway. I assume $t(\bullet)$ is differentiable, monotonically increasing in q, and monotonically decreasing in X. Capacity is produced from the other inputs according to:

$$X = g(x_b, x_n) . \tag{5.2}$$

where $x_b \equiv (x_2,...,x_{n-1})$ is the vector of inputs other than time and land, with corresponding price vector w_b. By calling x_1 a user-supplied input, I mean that its cost is part of the perceived price p of travel:

$$p = \tau + w_1 t(q,X)$$

where τ is the money price charged for use of the road. In the transportation context, the factor price w_1 is conventionally called the value of time, and p is called the full price of travel.

To transform this formulation into that of sections 2-4, solve (5.1) for q as a function of x_1 and X, denoting the result as $q=H(x_1,X)$; this is possible because $t(\bullet)$ is monotonically increasing in q. Substituting (5.2), we can write q directly as a function of inputs:

$$q = H\big[x_1 g(x_b, x_n)\big] \equiv f(x_1, x_b, x_n) \ . \tag{5.3}$$

Equation (5.3) is in the form of the production function (2.3), with input vector x_a partitioned as (x_1, x_b). This production function gives the number of users who can use a system while maintaining a level of service yielding total travel time x_1, given the capacity that can be produced by inputs x_b, x_n. In other words, it gives the output made possible with inputs x_1, x_b, and x_n. Its degree of returns to scale, r, can be derived from the degree of returns to scale in producing capacity, r_g, and the degree of local homogeneity of the congestion function, h_t $\equiv \varepsilon_{tq} + \varepsilon_{tX}$, where the elasticies have signs $\varepsilon_{tq} > 0$ and $\varepsilon_{tX} < 0$. The result is shown in the appendix to be:

$$1 - r = \frac{-\varepsilon_{tX}(1 - r_g) + h_t}{1 - \varepsilon_{tX} + h_t} \ . \tag{5.4}$$

Capacity is usually defined in such a way that equal percentage increases in q and X have no effect on t: i.e., $h_t = 0$. (For a highway, this means congestion depends only on the volume-capacity ratio.) In that case, (5.4) shows that $(1-r)$ has the same sign as $(1-r_g)$, that is, $f(\bullet)$ has the same type of returns to scale as $g(\bullet)$.

What about the cost function derived from $f(\bullet)$ allowing for a possible rising supply price of land? It can similarly be related to the cost function for producing capacity. To see this, let land be supplied according to any differentiable supply function such that expenditure on land is $E_n(x_n)$, as before. Define the total cost and capital cost functions:

$$C(q) = \underset{x_1, x_b, x_n}{Min} \left\{ w_1 x_1 + w_b x_b + E_n(x_n) \ : \ q = f(x_1, x_b, x_n) \right\} \tag{5.5}$$

and

$$K(X) \equiv \underset{x_b, x_n}{Min} \left\{ w_b \cdot x_b + W_n(x_n) \quad : \quad X = g(x_b, x_n) \right\} . \tag{5.6}$$

Their relationship is made apparent by noting that the constraints in (5.5) and (5.6) are just equations (5.3) and (5.2), respectively. Recall that these equations are equivalent under the transformation of variables from X to x_1 defined in (5.1). That is, (5.5) can be rewritten as a minimization over X instead of x_1, by substituting (5.1) for x_1 and (5.2) in place of the constraint:

$$
\begin{aligned}
C(q) &= \underset{X, x_b, x_n}{Min} \left\{ w_1 qt(q, X) + w_b \cdot x_b + W_n(x_n) \quad : \quad X = g(x_b, x_n) \right\} \\
&= \underset{X}{Min} \left\{ w_1 qt(q, X) + K(X) \right\}
\end{aligned} \tag{5.7}
$$

This equation gives the relationship between the capital cost function, $K(X)$, and the total cost function, $C(q)$.

We are now in a position to relate the scale economies of the two cost functions, $C(\cdot)$ and $K(\cdot)$. Denote these scale economies by s and s_K, respectively. The minimization in the second line of (5.7) implies the first-order condition

$$w_1 q(\partial t / \partial X) = -dK/dX . \tag{5.8}$$

Equation (5.7) also implies, using the envelope theorem, that

$$dC/dq = w_1 t + w_1 q(\partial t / \partial q) . \tag{5.9}$$

Manipulation (see appendix) yields:

$$\frac{1-s}{s} = \theta_1 h_t + \theta_K \left[\frac{1-s_K}{s_K} \right] \tag{5.10}$$

where $\theta_1 \equiv w_1 tq/C$ and $\theta_K \equiv K/C$ are the shares of user cost and capacity cost, respectively, in total cost. In the usual case when $h_t=0$, (5.10) shows that $(1-s)$ has the same sign as $(1-s_K)$. That is, the cost function (5.5) has the same type of scale economies as the capital cost function (5.6). Note that the condition $h_t=0$ can be regarded either as a normalization condition in defining "capacity" or as an assumption of constant returns in the congestion technology.[12]

Finally, how do "profits," as defined using cost function C, correspond to financial profits of the authority that finances capacity and practices congestion pricing? The answer is they are identical, because user costs are subtracted from both revenues and costs in going from one formulation to the other. Congestion pricing involves setting money price τ so that full price $p \equiv \tau + w_1 t$ is equal to marginal cost as given by (5.9). Financial profits are therefore

$$\begin{aligned}\tau q - K(X) &= (p-w_1 t)q - (C-w_1 tq) \\ &= pq - C\end{aligned} \tag{5.11}$$

which is identical to profits as defined in section 2. The ratio of the authority's financial profits to its cost, $(\tau q-K)/K$, is related to s_K as follows:

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[12] The latter interpretation is made, for example, by Mohring and Harwitz (1962) and Strotz (1965, p. 135).

$$\frac{\tau q - K}{K} = \frac{pq - C}{C} \cdot \frac{C}{K} \qquad \text{from (5.11)}$$

$$= \left[\frac{1-s}{s}\right] \cdot \frac{1}{\theta_K} \qquad \text{from (2.2b)}$$

$$= \left[\frac{1-s_K}{s_K}\right] + \frac{\theta_1}{\theta_K} \cdot h_t \qquad \text{from (5.10)} .$$

When $h_t=0$, this ratio is just $(1-s_K)/s_K$, exactly analagous to (2.2b); or equivalently, the ratio of congestion-pricing revenues to capital cost is $1/s_K$, as noted by Kraus (1982a, p. 236).

All the earlier results on the sign of profits therefore apply to the case of highway congestion. The sign of financial profit is governed by the degree of scale economies embedded in $C(\bullet)$, which is defined for general factor-market conditions and accounts for a possibly rising supply price for land. Scale economies in $C(\bullet)$ are related through (5.10) to scale economies in the capacity-cost function $K(\bullet)$. Under the usual assumption that $h_t=0$, $C(\bullet)$ has scale economies of the same type as $K(\bullet)$; hence the latter determines the sign of financial profits.

For completeness, note that (5.10) can also be written in the form

$$1-s = \frac{-\varepsilon_{tX}(1-s_K) + h_t}{1 - \varepsilon_{tX} + h_t} , \qquad (5.12)$$

exactly analogous to (5.4). (The derivation uses the first-order condition (5.8), which can be written as $\theta_1 \varepsilon_{tX} = -\theta_K/s_K$.) In the case of competitive factor markets, (5.12) would follow immediately from (5.4) using the usual duality results that $s=r$ and $s_K=r_g$ (Varian 1984, p. 68).

6.   The Context for Self-Financing Rules

To more fully appreciate the need for extending self-financing rules to monopsonistic input markets, consider the process of land assembly for a highway. The competitive price-

taking model of Berechman and Pines requires that we imagine the highway authority renting land from competitive owners, each of whom can at any time evict his highway tenant and lease his parcel to some other user at the prevailing market rent. This market rent is that which applies in equilibrium with the road fully built. But such a competitive equilibrium cannot exist because land must be irrevocably committed to the road prior to construction, typically through advance purchase. So the actual rental price paid for the land cannot be determined each period in a competitive market. Instead, we are in the realm of bilateral negotiations.

When land development is done privately, land assembly is often undertaken in great secrecy so that at least some of the land can be purchased at prices reflecting its value prior to any consideration of the new facility. Other parcels may have to be purchased at higher prices. This result resembles the discriminating monopolist.

Occasionally, public land assembly occurs in the same way, as in the famous case of the Los Angeles water district's purchase of riparian land in Owens Valley, California. More often, public land assembly occurs through negotiations in a context of eminent domain, the doctrine by which the public can force a sale at a price determined by a court. Fischel (1995) offers a fascinating overview of the relevant case law in the United States, which provides insights into the actual nature of the supply function faced by an authority building any large project involving land assembly.[13]

In one instance, court rulings provide an outcome resembling that of a non-discriminating monopsonist. In Florida, alone among the fifty states in the U.S., land taken for a highway through eminent domain is priced at an amount that includes the increase in value induced by the highway (Bingham, 1985, pp. 11-7 through 11-11).

The overwhelming tendency, however, is to value land taken for a public project well below its market value with the project in place. According to Fischel, court cases have followed cycles. Early in the development of a particular technology such as railroads, when the benefits from facilitating projects were perceived to be very high, rules for compensation were relatively favorable to the agency acquiring land. Later, as the magnitude of benefits

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[13]See especially chapter 2. I am indebted to Fischel also for pointing me, in personal discussion, to the articles by Francis and Bingham cited below.

became less compelling and issues of horizontal equity were given more scope, the rules changed toward requiring higher compensation.

For example, in the early days of both railroads and interstate highways, the courts frequently reduced the price paid for land by a "benefit offset." This reflected the fact that for many landowners, only part of the owner's parcel was taken, while the remainder of the parcel rose in market value because of the project. In the case of urban elevated railways, this same theory was at first applied to property in the form of easements for light and air, which were deemed to be implicitly taken by the builder of an elevated structure in the middle of a street. Later, such offsets were prohibited, thereby raising the cost of acquiring those easements. In the case of interstate highways, similarly, compensation practices allowed rather low compensation during the 1950s and 1960s; but starting in 1970, both federal law and state court decisions added many new compensation requirements such as for relocation costs, blight caused by prior announcement of the project, and loss of business goodwill. Cordes and Weisbrod (1979) provide empirical evidence that such compensation practices did affect the amount of highway construction undertaken. Hence, their impact on the price of land acts in the manner postulated in the investment models of in this paper.

The benefit offset was haphazard, leading to differing payments for similar land purchases depending on how much other land was part of a given parcel. Nevertheless, it produced a very rough kind of rising supply price resembling that facing a perfectly discriminating monopsonist. Given the route and the size of the landholdings within which the road passed, the acquisition of a very small strip of land might be essentially free because there would be enough other land in the parcels for the benefit offset to be virtually complete. A larger purchase would more often encounter the need to purchase parcels for which the benefit offset would be only a fraction of the cost of the land actually purchased. For such parcels, marginal increases in the project's scope would reduce the benefit offset by an amount reflecting the full access value of land taken, which means land at the margin would effectively be supplied at the full market price with the road in place.

Two California practices illustrate other mechanisms by which highway authorities may effectively be faced with a rising supply curve for land (Francis, 1984, p. 449). California, like

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all states except Florida, normally allows a public authority to exclude from its land payments the land value induced by the highway itself. However, if land not originally planned to be used is subsequently taken due to a change in plans, induced value would be compensated. Similarly, a 1971 court decision required that land taken for a freeway be valued at an amount that includes any induced value caused by an intersecting freeway. Both of these practices mean that as the highway system is expanded, the authority must pay a land price that reflects at least in a crude way the increasing scarcity value of the land that results from the highway system itself.

Of course, landowners may impose legal costs or delays on highway authorities in order to increase their payments. However, there seems no evidence that this is either systematic or widespread. Probably it means that most land is purchased for slightly more than the courts would require, but it does not affect the relationship between price and scarcity.

Thus, it seems clear that highway authorities face a rising supply price for land. Furthermore, in many cases the nature of compensation rules leads to the landowners being left with little producer surplus, corresponding to the case of the perfectly discriminating monopsonist. This is the least problematic case for our purposes because marginal-cost pricing is first-best. In such a case, the condition for a fully enlightened highway authority to be self-financing is there are no positive economies of scale in its cost function, taking account of the rising price of land.

The question arises whether we should include local land value increases induced by the highway itself, if they are in part offset by land value decreases elsewhere. Strictly speaking, the arithmetic of equations (2.1)-(2.4) holds regardless of the cause of the rising supply price faced by the highway authority. However, it is probably better for two reasons to limit the use of these equations to sketch planning, that is the analysis of alternative levels of highway provision throughout an urban area, as for example in Fitch and Associates (1964). One reason is that normative implications of marginal-cost pricing become more cloudy if a highway competes for traffic with other highways that are not optimally priced. The other reason is ambiguity about what the price of land is a function of. When land price is increased because of incidental business from traffic carried by the road, it is really output q rather than land input x_n that is affecting land price, which would alter the choice of output price. When land price is

increased by improved accessibility, either volume-capacity ratios q/X must have decreased or there must have been some improvement in the highway's characteristics other than increased capacity, such as faster off-peak times or improved safety; Larsen (1993) notes that these are often correlated with capacity and that this alters the Mohring-Harwitz result. Both of these possibilities suggest useful extensions of the model, but are accounted for neither here nor in any of the self-financing literature discussed above.

7. Conclusion

There are practical reasons, then, to be interested in the finances of public authorities that face noncompetitive factor markets for land. This paper has shown how the self-financing rule, usually applied only to first-best optimal investment and pricing, can be extended to such situations. The extended rule relates revenues from marginal-cost pricing, whether or not that pricing is first- or even second-best optimal, to the actual costs incurred by the authority. The relationship hinges on the degree of scale economies of the actual cost function, which are diminished to the extent that the authority faces a rising supply price of land.

Appendix

Proof of Proposition 2

Let $x=(x_a, x_n)$ be any input vector, and define returns to scale as the elasticity of the production function with respect to an increase in inputs along the ray defined by x:

$$r(x) = \frac{t}{q}\frac{df(tx)}{dt}\Big|_{t=1} . \tag{A.1}$$

Writing out the derivative in this equation and applying the first order conditions (2.5) at x^* $\equiv (x_a^*, x_n^*)$, with $\lambda \equiv dC/dq$, yields

$$
\begin{aligned}
r(x^*) &= (t/q)\sum_{i=1}^{n} f_i(tx^*)x_i^* \Big|_{t=1} \\
&= (1/q)\left[\sum_{i=1}^{n-1}(w_i/\lambda)x_i^* + (w_n/\lambda)(1+\omega_n^*)x_n^*\right] \\
&= \frac{1}{q(\partial C/\partial q)}\left(C + \omega_n^* w_n^* x_n^*\right) \\
&= s\cdot\left[1 + (\omega_n^* w_n^* x_n^*/C)\right] .
\end{aligned}
\tag{A.2}
$$

which is greater than s for positive ω_n^*.

The intuition is that even though the non-discriminating monopsonist must pay the same price for all units of land (so that $\tilde{C}=C$ at reference output $\tilde{q}$), the marginal costs differ. This is because in calculating C, but not $\tilde{C}$, the price of land will rise with increasing q. Hence the ratio of average to marginal cost is larger for $\tilde{C}$ than for C, i.e. $\tilde{C}$ shows greater economies of scale.

Second-Order Condition for Equation (3.8)

The maximand $\mathcal{L}$ in equation (3.8) contains the quantity $C(q)-S_n(q)$, which can be written as

$$C(q)-S_n(q) = [w_a x_a^* + w_n^* x_n^*] - [w_n^* x_n^* - \int_0^{x_n^*} w_n(\chi)d\chi]$$

(A.3)

in which the middle two terms cancel. Therefore

$$\frac{d\mathcal{L}}{dq} = P(q) - w_a\frac{dx_a^*}{dq} - w_n^*\frac{dx_n^*}{dq}$$

(A.4)

$$\frac{d^2\mathcal{L}}{dq^2} = P'(q) - w_a\frac{d^2x_a^*}{dq^2} - w_n^*\frac{d^2x_n^*}{dq^2} - \frac{dw_n^*}{dq}\frac{dx_n^*}{dq}$$

$$= P'(q) - \tilde{C}''(q) - \frac{dw_n^*}{dq}\frac{dx_n^*}{dq}$$

(A.5)

since $\tilde{C}$ is defined like C except holding w_n constant at w_n^*. The last term is negative, so we are assured of a maximum if $P'-\tilde{C}''<0$ or if $P'-\tilde{C}''$ is positive but not so large as to overcome the last term. Demand is downward sloping ($P'<0$), so if $\tilde{C}$ is convex or not too concave compared to P', the second-order condition holds. Note that $P'-\tilde{C}''<0$ is the second-order condition for optimality of marginal-cost pricing in the normal case of competitive factor markets, so the requirement here is weaker than the one usually imposed.

Proof of Proposition 3

Equation (A.2) implies that

$$\frac{1}{s} - \frac{1}{r^*} - \frac{\omega_n^* w_n^* x_n^*}{r^* C} \tag{A.6}$$

Applying definition (3.8) of p^o yields (3.10) directly.

Now suppose the production function is homothetic, and consider the expansion paths for inputs as q increases. If ω_n^* were zero, these expansion paths would be rays from the origin, and each input would grow proportionally to q^{1/r^*}. Hence the output-elasticity in the last term in (3.10) would be $1/r^*$, so that the term in square brackets would vanish. With ω_n^* positive, use of land grows more slowly than q^{1/r^*} because the price of land rises; so the elasticity in (3.10) is less than $1/r^*$, causing the term in square brackets to be positive.

Derivation of Equation (5.4)

Let r, h_t, and r_g be the degrees of local homogeneity of $f(\bullet)$, $t(\bullet)$, and $g(\bullet)$, respectively. That is, they are defined by

$$f_1 x_1 + f_b x_b + f_n x_n - rq \tag{A.7}$$

$$t_q q + t_X X - h_t t \tag{A.8}$$

$$g_b x_b + g_n x_n - r_g X . \tag{A.9}$$

Substituting the definition (5.3) of f into the definition (5.1) of x_1,

$$x_1 - f(x_1, x_b, x_n) \cdot t\left[f(x_1, x_b, x_n),\ g(x_b, x_n)\right] . \tag{A.10}$$

Differentiating with respect to x_1, x_b, and x_n yields:

$$1 - f_1 t + f \cdot t_q f_1 \tag{A.11}$$

$$0 - f_b t + f \cdot t_q f_b + f \cdot t_X g_b \tag{A.12}$$

$$0 - f_n t + f \cdot t_q f_n + f \cdot t_X g_n . \tag{A.13}$$

Multiply these equations by x_1, x_b, and x_n, respectively, and add them (remembering that $f=q$). Using (A.7) and (A.9) to simplify, this yields:

$$x_1 - rq \cdot (t + q t_q) + q t_X r_g X \tag{A.14}$$

or, recalling that $x_1 = qt$,

$$qt(1-r) - q \cdot (rq t_q + r_g X t_X) . \tag{A.15}$$

Applying (A.7) and dividing by qt,

$$1 - r - r \cdot h_t + (r_g - r) \cdot \varepsilon_{tX} . \tag{A.16}$$

Solving for r yields (5.4).

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Derivation of Equation (5.10)

Returns to scale are $s \equiv C/(q\mathrm{d}C/\mathrm{d}q)$, $s_K \equiv K/(X\mathrm{d}K/\mathrm{d}X)$, and h_t as defined by (A.8). Applying these definitions to (5.9) and multiplying by q yields:

$$\frac{C}{s} = w_1 qt + w_1 q^2 t_q . \tag{A.17}$$

Subtracting $C \equiv w_1 qt + K$ from each side,

$$\begin{aligned}
C \cdot \left[\frac{1}{s} - 1 \right] &= w_1 q^2 t_q - K \\
&= w_1 q \cdot (h_t t - t_X X) - K \qquad \text{(from } A.8) \\
&= w_1 qth_t + K_X X - K \qquad \text{(from 5.8)} \\
&= w_1 qth_t + K \cdot \left[\frac{1}{s_K} - 1 \right] \qquad \text{(from definition } of\ s_K) .
\end{aligned} \tag{A.18}$$

Dividing by C yields (5.10).

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