Title

Concepts and Principles for the Application of Nonlinear Structural Analysis in Bridge Design

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Publication Date

1997-05-01

REPORT NO.
UCB/SEMM-97/08

STRUCTURAL ENGINEERING
MECHANICS AND MATERIALS

CONCEPTS AND PRINCIPLES FOR THE APPLICATION OF NONLINEAR STRUCTURAL ANALYSIS IN BRIDGE DESIGN

BY

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MAY 1997

DEPARTMENT OF CIVIL ENGINEERING
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA

1. Report No. UCB/SEMM-97/08	2. Government Accession No.	3. Recipient Catalog No.
4. Title and Subtitle Concepts and Principles for the Application of Nonlinear Structural Analysis in Bridge Design		5. Report Date May 1996 6. Performing Organization Code UCB-ENG-8100
7. Authors Graham H. Powell Professor Emeritus of Civil Engineering University of California at Berkeley		8. Perf. Org. Report No. UCB/SEMM-97/08
9. Performing Organization Name and Address Department of Civil Engineering University of California Berkeley, CA 94720		10. Work Unit No. (TRAIS) 11. Contract or Grant No. 59T384
12. Sponsoring Agency Name and Address California Department of Transportation Office of Structures 1801 30th Street Sacramento, CA 94272		13. Type of Report Final 14. Sponsoring Agency Code E.A. 59-304-910154

15. Supplementary Notes

Prepared in cooperation with the State of California Department of Transportation.

16. Abstract

Engineers have decades of experience in the practical use of linear analysis. This type of analysis is relatively simple, and its role in structural design is well defined. However, engineers have much less experience in the practical use of nonlinear analysis. This type of analysis is much more complex, and its role in structural design is not well defined. This report is concerned with the broad problem of how to make effective use of nonlinear structural analysis in practical design, with emphasis on bridge structures. The report looks at several aspects of this broad problem. Some of these aspects are as follows.

(1) The reasons for using structural analysis, and the differences between linear and nonlinear analysis.
(2) Strength based versus damage based design.
(3) Demand-capacity concepts for design, and decision making based on demand-capacity comparisons.
(4) The importance of the behavior concept for the structure, and the value of capacity design.
(5) Modeling for linear and nonlinear analysis.
(6) Unanswered questions about how to use nonlinear analysis effectively in design, and the steps we must take to develop answers.

17. Key Words Bridges, Nonlinear, Structural Analysis, Structural Design		18. Distribution Statement Unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 100	22. Price

ABSTRACT

This report is based on two Caltrans research projects that have extended over about a five year period. The main focus of the research has been the development of tools for nonlinear structural analysis, in particular the computer programs DRAIN-2DX and DRAIN-3DX. These programs, and their libraries of nonlinear elements, are described in separate reports, and are not considered here.

This report is concerned with the broad problem of how to make effective use of nonlinear structural analysis in practical design. We have decades of experience in the practical use of linear analysis. This type of analysis is relatively simple, and its role in structural design is well defined. We have much less experience in the practical use of nonlinear analysis. This type of analysis is much more complex, and its role in structural design is not at all well defined.

This report looks at several aspects of the broad problem. Some of these aspects are as follows.

(1) The reasons for using structural analysis, and differences between linear and nonlinear analysis.
(2) Strength based versus damage based design.
(3) Demand-capacity concepts for design, and decision making based on demand-capacity comparisons.
(4) The importance of the behavior concept for the structure, and the value of capacity design.
(5) Modeling for linear and nonlinear analysis.
(6) Unanswered questions about how to use nonlinear analysis effectively in design, and the steps we must take to develop answers.

As this report shows, there are many important questions for which we do not yet have answers. The goal of the report is not to provide these answers, but to clarify the questions. When we understand the questions, we will be better able to devote resources to providing the answers.

ACKNOWLEDGEMENTS

A number of people have been involved in the Caltrans projects, and have had an influence of the thought expressed in this report. At the University of California they include Dr. Vipul Prakash, who did the coding on the DRAIN base programs, Dr. Scott Campbell, who developed DRAIN elements, and Professor Filip Filippou, who assisted with the project supervision. At Caltrans they particularly include (alphabetically) Omar Elkhayat, Brian Maroney, Richard Obisanya and Mark Yashinsky. The extensive discussions I have had with them has been invaluable in helping me to clarify my thoughts on the role of nonlinear analysis.

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1. INTRODUCTION.

1.1 OBJECTIVE AND SCOPE

Design for safety has traditionally been based on the perception that the safety of a structure depends on its strength. That is, traditional design is *strength based*. This type of design has been used for the design of countless structures, and has worked well for structures that are designed for gravity and wind loads. It has not worked so well, however, for structures that are designed to resist earthquake loads, because such structures are not really designed for strength. As a consequence, for design in earthquake prone regions, strength-based design is being replaced by *damage based* design. This is based on the perception that economy and safety are best achieved by allowing the structure to yield, but in such a way that it is not severely damaged. This type of design may also be referred to as *displacement based* or *ductility based* design.

Damage based design differs from strength based design in several important ways. In particular, for strength based design it is usually sufficiently accurate to use linear structural analysis, whereas rational damage based design requires nonlinear analysis.

This report is based on two Caltrans research projects that have extended over about a five year period. The main focus of the work has been the development of nonlinear analysis tools for use in the earthquake resistant design of bridges. Most of this work has been on the development of the computer programs DRAIN-2DX and DRAIN-3DX, and a number of nonlinear elements for these programs. The computer programs and nonlinear elements are described in detail in separate reports. Those reports focus on narrow issues of modeling and analysis, and are concerned with *how* to perform nonlinear analysis. The present report is more philosophical, and is concerned with the broad issues of *why* and *when* nonlinear analysis is needed. In the author's opinion these issues are critically important, and they have received much less attention than they deserve.

The overall problem is extremely broad, and it is impossible to cover its details in a single report. In any case, as this report emphasizes, there are many details that are not yet clearly understood and that require a great deal more study. This report is intended mainly a basis for discussion, and for planning future work.

This report leans towards applications in bridge design, but also considers building structures.

1.2. A NOTE ON REFERENCES

As noted, this report is a secondary product of the research that has been funded by Caltrans. It is not a how-to manual, nor a reference work, but merely a collection of thoughts that I hope will stimulate discussion and lead to more definitive reports in the future (not necessarily written by me). The report makes some observations that I believe are original, but obviously it draws on published books, papers and reports, by many distinguished researchers and practitioners. These works are not referenced, however, simply because there are so many of them and I have not had the time to prepare a reference list. I have mentioned the names of some researchers that I believe have been particularly influential, but this list is also far from complete.

1.3. REPORT CONTENTS

Chapter 2 discusses the role of structural analysis in design, demand-capacity concepts, a framework for decision making, and some differences between linear and nonlinear structural analysis. This lays the groundwork for later chapters.

Chapter 3 reviews traditional strength based design. This may seem elementary, since strength based design is well understood and is the basis of most design codes. It is worth reviewing in detail, however, because many of the key points are important for damage based design.

Chapter 4 uses a simplified example (a reinforced concrete bridge) to illustrate several aspects of damage based design, including the selection of demand-capacity measures, the importance of using capacity design principles, and the role of nonlinear structural analysis. This chapter shows that the structural analyses required for damage based design can be much more complex than those required for strength based design.

A conclusion from Chapter 4 is that some form of nonlinear analysis is required for rational damage based design. Chapter 5 discusses the broad problem of modeling for nonlinear analysis. Among other things, this chapter summarizes the causes of nonlinear behavior, briefly describes the properties of a number of nonlinear structural elements, and identifies several types of nonlinear analysis that might be performed. This chapter also emphasizes the importance of choosing a rational behavior concept for the structure, so that useful results can be obtained from a nonlinear analysis.

A conclusion from Chapter 5 is that nonlinear analysis is much more complex than linear analysis, and requires much more skill on the part of the analyst. Most engineers have limited experience, and will require guidelines for setting up nonlinear analysis models. Chapter 6 suggests a number of steps and rules that might be useful.

Finally, Chapter 7 presents a summary and conclusions. This chapter emphasizes that a great deal more research and development is needed before damage based design can be applied routinely.

2. STRUCTURAL ANALYSIS AND A FRAMEWORK FOR DECISION MAKING

2.1 PURPOSE

With today's computers and software it is a relatively simple task to create a linear computer model of a large structure, perform static and dynamic analyses, and use the results of these analyses to check the design of the structure. We may tend to think, therefore, that we can use essentially the same process for a nonlinear model. Granted, we will recognize that nonlinear analysis is more complex than linear analysis. Nevertheless, we might expect that we can create a model, get nonlinear analysis results, and check the design, in much the same way that we use linear analysis.

Unfortunately it is not that simple. When we use linear analysis there are many things that we take for granted and do not need to think about. When we use nonlinear analysis we must do a great deal of re-thinking. For example :

(1) We must have a clearer understanding of what we mean by structural performance, and how we use analysis to ensure adequate performance.

(2) We must define what we mean by "analysis results" and how we plan to use these results.

(3) We must recognize that nonlinear analysis is vastly different from linear analysis, is used for different reasons, and has a number of serious limitations.

The purpose of this chapter is to lay the groundwork for future chapters, by examining the role of structural analysis, with particular emphasis on decision making for design.

2.2 DEMAND AND CAPACITY

2.2.1 General

Structural analysis is not an end in itself -- it is a merely a tool to support decision making. Hence, if we are to make effective use of nonlinear analysis we must be clear on how it fits into the design process. The best way to achieve this is to define a rigorous framework for decision making. The advantages of such a framework are as follows.

(1) It forces us to break the complex process of decision making into its simplest possible parts.
(2) We can identify what decisions we must make, and exactly what we must do to make them.
(3) We can see whether we need nonlinear analysis.
(4) We can identify the weak links in the chain of reasoning, and the key problems that must be solved.

The framework described in this chapter is based on concepts of Demand and Capacity. These concepts are not new, of course. Over the years many engineers have espoused them, notably Paulay and Park in New Zealand, and Bertero, Krawinkler, Moehle, Priestley and many others in the United States. One goal of this report is to describe the framework in greater detail, especially as it relates to the role of nonlinear analysis.

2.2.2 Decision Making Based on Demand and Capacity

Essentially, the decision making process is as follows.

(1) Choose one or more demand-capacity measures that are related to structural performance.
(2) Compare numerical values of the demands against numerical values of the capacities. If demand does not exceed capacity, the performance is judged to be adequate.

Some key questions are as follows.

(1) What are the demand-capacity measures?
(2) How do we calculate the demand values?
(3) How do we calculate the capacity values?

2.2.3 Cost as the Demand-Capacity Measure

For damage based design, dollar value is the most rational choice for the demand-capacity measure, since if we can calculate the cost of damage we can make rational decisions on how to allocate resources. To use this measure, however, we must be able calculate all costs, accounting for structural damage, nonstructural damage, human injuries, disruption of operations, etc. Although this is a desirable goal, it is extremely difficult to achieve. Hence, although cost is the best measure for decision making, we do not yet have techniques for calculating the demand value with sufficient accuracy. The capacity value (the available resources) is relatively easy to calculate (politics aside), but since we can not calculate the demand value, cost is not a practical measure and we must use less direct measures. We will postpone the discussion of specific measures until later.

2.3 PERFORMANCE VS. PRESCRIPTIVE APPROACH

When we evaluate the performance of a structure or structural component based on demand-capacity comparisons, we are using the *performance approach*. However, it is often not feasible to make these comparisons. One reason, as in the preceding section, is that we do not have analysis tools to calculate demand and/or capacity values. A second reason is that the cost of using a performance approach in design can exceed any cost savings in construction.

Since the goal is to ensure that capacity exceeds demand, we may decide that we do not have to define explicit demand-capacity measures, or calculate demand and capacity values. Instead we can specify the materials, dimensions and/or details of the structure or component, so that we know the capacity exceeds the demand even though we have made no explicit calculations. That is, we can use indirect prescriptive rules rather than a direct performance analysis. This is the *prescriptive approach.*

There are many situations where the prescriptive approach is used. One example is the use of prescribed nailing patterns of plywood diaphragms in residential construction. In this case the purpose is to ensure adequate strength, and the prescriptive approach is used because it is not cost effective to perform explicit strength computations. A second example is the use of maximum flange width-to-thickness ratios to define a compact steel section. In this case the purpose is to ensure that the section can yield substantially in bending without local buckling, and it is difficult to formulate code requirements that are performance based. A third example is the use of specified shapes, sizes and spacings for ties in a reinforced concrete column. In this case one of the purposes is to ensure that the ductility of the column is not limited by buckling of the longitudinal reinforcement. It is not feasible to define demand-capacity measures related to buckling of the reinforcement, or to perform buckling analyses.

The prescriptive approach is perfectly valid, and it is necessary in these and many other cases. In general, however, the goal will be to seek direct performance measures.

2.4 FRAMEWORK FOR DECISION MAKING

2.4.1 Outline

The essential parts of the framework are as follows.

(1) Define the behavior concept for the structure and its components.
(2) Decide which modes of behavior are governed by prescriptive rules and which are subject to performance analysis.
(3) For those modes of behavior that are governed by prescriptive rules, specify the rules.
(4) For those modes of behavior that are subject to performance analysis:
   (a) Define the limit states.
   (b) Define the demand-capacity measures.
   (c) Specify how to determine the demand values.
   (d) Specify how to determine the capacity values.
   (e) Specify how to perform the demand-capacity comparisons.

Examples are presented later.

2.4.2 Some Important Questions

If we are to use this framework, there are many questions that we must be able to answer. Some of these questions are as follows.

(1) *What is the behavior concept?* Before we can even think of doing a performance analysis we must define the type of behavior that is required. This does not mean that we must decide all of the details before we can begin the design process. Structural design is iterative, where we progressively refine the details as the design progresses. We can begin with a broad concept for the structure, and use rough methods to assess its performance. Ultimately, however, we must choose clear behavior concepts for both the structure as a whole and its components, and we must specify clear methods for assessing their performance.

(2) *Which components and modes of behavior are to be governed by prescriptive rules, and which by performance analysis?* A single component can have several modes of structural behavior. We must identify all significant modes, and for each of them decide whether to use a prescriptive or performance approach.

(3) *For each component and mode of behavior that is governed by prescriptive rules, what are the rules?*

(4) *For each component and mode of behavior that is governed by performance analysis:*

(a) *What are the limit states?* Examples are (a) excessive deflection under gravity load plus a moderate earthquake (a serviceability limit state), (b) excessive damage under gravity load plus a strong earthquake (also a serviceability limit state), and (c) collapse under gravity load plus a strong earthquake (a strength or safety limit state).

(b) *For each limit state, what are the demand-capacity measures?* Before we perform any calculations we must decide what results we want to calculate. If the design is in a conceptual stage, we can use demand-capacity measures and calculation procedures that are loose and approximate. As the design gets progressively more detailed we must progressively refine the measures and the calculation procedures. However, until we define measures of some sort we can not perform any kind of performance analysis. Even when we use pure engineering judgment to evaluate a design, we must be using measures and calculation procedures of some kind.

(c) *Are these measures good enough for design purposes?* If demand is less than capacity we must be reasonably certain that the structure will behave as we intend. If this is not the case we must choose different measures. Note that the words "good enough" are extremely important. There will always be a great deal of uncertainty in structural design, and it will never be possible to be "perfect" or "exact". "Good enough" is more practical and much easier.

(d) *How do we calculate the demand values?* We must be able to calculate the demand values with good enough accuracy for design purposes. If we can not perform the calculations we must either choose different measures or develop better methods of calculation.

(e) *Is the method for calculating demand values repeatable?* If two engineers, working independently, apply the method to the same structure, will they get results that are close enough to each other for design purposes? If not, they have presumably made different assumptions, and the results are sensitive to these assumptions. The calculation procedure must be specified in greater detail.

(f) *How do we calculate the capacity values?* We must be able to calculate the capacity values with good enough accuracy for design purposes. Again, if we can not perform the calculations we must either choose different measures or develop better methods of calculation. Note that "calculation" does not necessarily mean that structural analysis is used. Capacity values are often obtained from experimental results, and we may need more experiments.

(g) *Is the method for calculating capacity values repeatable?*

(h) *How do we make the demand-capacity comparisons?* Having calculated the demand and capacity values, how will we compare them in order to make design decisions?

To these questions it is useful to add the following.

(5) *Are we using the simplest reasonable structural analyses?* Other things being equal, the simplest analysis is the best.

There are many areas of design where we know the answers to these questions. As shown in Chapter 3, we know the answers for traditional strength-based design. However, there are many areas in damage based design where the research and development has not yet been done, and we simply do not know the answers.

2.5 THE ROLE OF STRUCTURAL ANALYSIS

2.5.1 Reasons for Using Structural Analysis

Whenever we use structural analysis, especially nonlinear analysis, we must be clear on its purpose. The following are several reasons why we might use structural analysis.

(1) ***Demand Calculation for Design Evaluation***. A design for a structure exists, or there is an existing structure. For a specified design loading the structure must satisfy specified performance criteria. Structural analysis can be used to calculate demand values for a demand-capacity comparison. This is the most common use of structural analysis.

(2) ***Capacity Calculation for Design Evaluation***. For any demand-capacity comparison we need both demand and capacity values. In most cases we will get capacity values from code formulas, experiments, manufacturer's specifications, or engineering judgment. We may, however, use structural analysis to calculate capacities.

(3) ***Behavior Study***. The behavior of a structure is studied in a semi-quantitative way, in order to understand its behavior and help develop design concepts.

(4) ***Sensitivity Study***. If a design does not satisfy the criteria, a redesign is necessary. Analysis can provide sensitivity information for redesign.

(5) ***Vulnerability Assessment***. This involves behavior studies and design evaluations for an existing structure, in order to determine vulnerable components. An assessment is first made on the basis of strength. If the strength capacity is exceeded for a component, an attempt may be made to determine the damage consequences. This type of analysis is a combination of the preceding four, and is less specific than a formal design evaluation.

(6) ***Validation***. Analyses are often used to test theories or analysis models, and to develop procedures and guidelines for using these theories.

(7) ***Calibration***. Theories and analysis models are never fully rational, and always have empirical aspects. Analyses may be used to calibrate empirical models, and possibly develop guidelines for use. Validation confirms that a theory or model behaves as intended. Calibration ensures that the theory or model agrees with observed real-world behavior.

(8) ***Simulation***. The purpose of an analysis may be to predict what will happen to an actual structure under actual loads. This can be useful if the actual structure is a test specimen on, say, a shaking table, since the results can help validate or calibrate analysis models. It is probably a waste of time to attempt this for a real structure -- there are simply too many uncertainties. We may be tempted to think that structural analyses, including analyses of

complex structures subjected to earthquake loads, are accurate simulations. In most cases they simply are not.

2.5.2 Linear vs. Nonlinear Analysis

Structural analysis is performed not on an actual structure but on a model of the structure (an *analysis model*). In many cases a linear analysis model is good enough, and a linear analysis can be performed. In other cases a linear model is not good enough, and the analysis must be nonlinear. In the list of reasons in the preceding section, it is not stated whether the analysis is linear or nonlinear. Whether linear analysis is sufficient, or whether nonlinear analysis must be used, depends on the structure and the purpose of the analysis. About the only generalization that can be made is that if analysis is used to calculate strength capacity, it will almost always have to be nonlinear. The reason is that the strength of a structure or structural component almost always depends on nonlinear effects, and it will usually be impossible to account for these effects in a linear analysis.

Linear analysis is much simpler than nonlinear analysis for the following main reasons.

(1) For a linear model of a structural component, only the component stiffnesses are needed. Much more information is usually required for a nonlinear model, with the result that a nonlinear model tends to be much more complex than a linear model.

(2) For linear analysis superposition can be used. Hence, separate analyses can be performed for, say, gravity and lateral loads, and the results can be scaled and added to obtain results for the combined loads. Also, response spectrum methods, which rely on superposition of individual modal results, can be used for dynamic analysis. It is possible to superimpose the results of nonlinear analyses, but only in special cases.

(3) The computational algorithms required for linear analysis are simple and reliable. With a modern computer program we do not need to know the details of these algorithms, and we can treat the program as a "black box". This is not the case with nonlinear analysis. Nonlinear algorithms are, on the whole, neither simple nor reliable, and we must understand their features and limitations.

(4) Typically, the only results that we need from a linear analysis are the maximum structure displacements and the maximum forces for each member. We may need a much wider variety of results from a nonlinear analysis.

Also, the reason for performing a linear analysis will usually be clear (it will almost always be to calculate demands). The reason for performing a nonlinear analysis may not be so clear. For example, consider the following question.

> A type of nonlinear analysis that is increasingly being used in design is a static push-over analysis, where gravity load is applied to the analysis model and the lateral load is progressively increased, causing nonlinear behavior. *Is this a demand analysis, a capacity analysis or an analysis of some other kind?*

2.5.3 Is A Structural Analysis Needed?

There are many cases where a structural analysis, of any kind, is not needed. Often we will know, with good enough certainty and without performing any analyses, that the capacity exceeds the demand, and hence that the component or structure will have satisfactory performance. This may be all that we need to know to decide that the design can be accepted. Conversely, we may know that the capacity is less than the demand, and hence that the component or structure will not have satisfactory performance. This may be all that we need to know to decide that the design must be discarded.

The purpose of the exercise is to make design decisions, not to perform structural analyses.

2.5.4 The Role of Structural Analysis in This Report

This report is concerned primarily with the situation where we have created a design and must decide whether or not it has satisfactory performance. Hence, it is concerned mainly with the use of analysis to calculate demand and capacity values. This report does not address such issues as creative design, design synthesis or design optimization. Also it is not concerned with methods for estimating member sizes or with redesign strategies for cases where demands exceed capacities. Design is an iterative process, where a structure is conceived, evaluated, revised, re-evaluated, etc. This report is concerned mainly with the evaluation phase.

3. REVIEW OF CONVENTIONAL STRENGTH BASED DESIGN

3.1 PURPOSE

We are familiar with strength based design because it is the basis of most design codes and is taught in all basic design courses. We may not, however, have thought through the process in detail, and it is useful to review the assumptions and procedures. Many of the points covered in this review will be obvious to the experienced designer, and some may seem trivial. Nevertheless, these points are important for understanding the differences between strength based and damage based design.

3.2 BEHAVIOR CONCEPT AND DEMAND-CAPACITY MEASURES

The relationship between load and deflection for a hypothetical structure is illustrated in Figure 3.1. The concept for strength based design is that the applied load (the load demand) must not exceed the structure strength (the load capacity). The load demand is an estimate of the load that has a small probability of being exceeded in the life of the structure. This is usually obtained by multiplying the estimated service loads by load factors. In this report we are not concerned with the details of this process.

Ideally we would choose load as the demand-capacity measure, since this is directly related to strength. Unfortunately this is not a practical choice. One reason is that it is too difficult to calculate the capacity value (the collapse load) for a complex structure, especially for dynamic loads. An equally important reason is that it is more practical to design at the component (i.e., member and connection) level rather than the structure level.

For a frame structure the components are beams, columns, braces, connections, footings, piles, etc. For strength based design of these components the demand-capacity measures are usually axial force, shear force, bending moment and possibly torsional moment. In some cases, stress is used as a measure. We will use "P-V-M" to indicate these measures. They are well defined and well understood. It is unlikely, for example, that two engineers will have different definitions for the maximum bending moment in a column. This is significant because it is not necessarily the case when we consider measures for damage based design (for example, two engineers may well have different definitions for the maximum curvature).

When we use demand-capacity measures, we are using the performance approach to ensure that the structure behaves as intended. In practice we will also use the prescriptive approach. In this chapter we are concerned mainly with the performance approach.

3.3 ARE P-V-M VALUES GOOD ENOUGH MEASURES FOR DESIGN PURPOSES?

3.3.1 General

Clearly, P, V, M and stress are directly related to strength at the component level. Also, strength at the component level is clearly related to strength at the structure level. Hence, component P-V-M values should be valid demand-capacity measures. However, this is worth considering in more detail.

3.3.2 Simple Truss Example

For the following discussion we will consider only a truss, with simple bar components that can resist only axial force. Other components that must be considered for real structures include beams, columns, beam-to-column connections, base plates, footings, piles, etc. The details for such components are more complex than for a truss bar, but the principles are the same.

Consider the simple truss shown in Figure 3.2(a). Assume that all of the members have elastic-perfectly-plastic behavior, as shown in Figure 3.2(b). Also assume that the connections are ideally pinned for rotation, but are rigid for axial force. Since the truss is statically determinate, the load-displacement relationship is also elastic-perfectly-plastic, as shown in Figure 3.2(c). In this case the structure strength is reached when the weakest element yields, where "weakest" means the element with the largest axial force demand/capacity ratio.

The statically determinate truss in Figure 3.2 collapses as soon as its weakest member yields, because it has no ability to redistribute load. If the structure is statically indeterminate, but with only one redundant member as shown in Figure 3.3(a), it may be able to redistribute load after the weakest member yields. Hence the load-displacement relationship can be trilinear, as shown in Figure 3.3(c). Depending on the relative strengths and stiffnesses of the members, the strength reserve after first yield may be small or large.

As the number of redundant members increases, the load-displacement relationship becomes more complex, and the strength reserve between first yield and collapse tends to get larger. However, if the design of the structure is optimized to obtain minimum weight, the strength reserve may be small. This is shown in Figure 3.4.

It is unlikely that a real truss would have elastic-perfectly-plastic members. If it did, however, one way to design for strength would be as follows.

(1) Choose the design load combinations, and the load factors for each combination.
(2) Assume that the structure is linear (i.e. ignore yield, and assume the members have infinite strength).
(3) Estimate the member sizes.
(4) Perform a linear structural analysis for each load combination. Calculate the largest axial forces (tension and compression) for each member. These are the axial force demands.
(5) Size the members to have yield strengths (axial force capacities) at least equal to these demands.
(6) Repeat from Step (4), if necessary, to account for changes in the member sizes.

This method has the following features.

(1) The behavior of a member is fully defined by its stiffness and yield strength. For a simple truss bar these values can presumably be determined accurately.
(2) The yield strength is an accurate measure of the member strength capacity (for this problem).
(3) The structural analysis is "exact", since the members are designed to remain linear. Hence the calculated demand values are accurate (for the specified load combinations and load factors).
(4) We have ensured that no member yields under any load combination, and hence that the structure does not collapse under the factored loads.

(5) Hence we have satisfied the performance requirement for strength.

The method may not, however, give the most economical structure, since we have not taken advantage of force redistribution after first yield. In effect, we are using the strength at first yield as the load capacity of the structure, rather than the true strength. Unfortunately, unless we perform a nonlinear analysis we do not know much strength reserve is present in any specific structure.

3.3.3 More Realistic Members

In practice the members of an actual structure will rarely, if ever, have elastic-perfectly-plastic behavior, and hence the discussion in the preceding section is oversimplified. A more likely relationship between axial force and axial deformation for a truss member is shown in Figure 3.5. We can use essentially the same design method as before. The steps are as follows.

(1) Choose the design load combinations, and the load factors for each combination.
(2) Assume that the structure is linear (i.e. assume the members are linear and have infinite strength). Since the actual members are not linear we will have to linearize, for example as indicated in Figure 3.5. To ensure that different engineers get similar results, we must specify how the linearization is to be performed.
(3) Estimate the member sizes. Hence estimate the member stiffnesses, using the specified linearization method.
(4) Perform a linear structural analysis for each load combination. Calculate the axial force demands (tension and compression) for each member.
(5) Design the members to have tension and compression capacities at least equal to these demands. Because there are inevitable uncertainties in our knowledge of the material strengths and member behavior, we will usually calculate a nominal capacity then reduce it by a capacity reduction (ϕ) factor to get a "usable" capacity. This is shown in Figure 3.5.
(6) Repeat from Step (4), if necessary, to account for changes in the member sizes.

For the case with elastic-perfectly-plastic members we know the exact stiffnesses for Step (3), and hence the demands calculated in Step (4) are also exact. We are certain, therefore, that the method is sound (and almost certainly conservative). For the case with more realistic members, however, the stiffnesses are approximate, and hence the calculated axial force demands are also approximate. A key question is whether these demands are good enough for use in design.

One way of answering this question is shown in Figure 3.5. If the behavior of a linearized member, for forces up to the usable force capacity, is close to the behavior of the actual member, then the approximations in the structural analysis are small, and the calculated demands are good enough for design. Most members behave nearly linearly up to their force capacities, and hence in most cases it is good enough to use linear analysis (i.e., a linear approximation) to calculate strength demands.

3.3.4 Capacity Calculation

P-V-M demands can thus be estimated using linear structural analysis. The corresponding P-V-M capacities are usually estimated using formulas from design codes. These formulas are based largely on the results of experiments, with some support from analyses. They represent the results of decades of research and experience, and they are clearly good enough for strength based design.

3.3.5 More Complex members

The discussion in this section is oversimplified, because it is limited to truss members, for which only axial force needs to be considered. The reasoning is similar for more complex members, but the details are more difficult. For example, it is more difficult to estimate a linearized stiffness for a reinforced concrete column subjected to axial force, shear force and bending moment than for a steel truss bar subjected only to axial force. It is also more difficult to estimate the strength capacity, because reinforced concrete members subjected to combined loadings have complex behavior. In principle, however, the process is the same as for a simple bar.

3.3.6 Overstrength

Most structures will have significant overstrength (strength reserve), and the actual load capacity at the structure level will be larger than the factored design load. The main reasons are as follows.

(1) There will usually be redundancy, and hence redistribution of load.
(2) The structure will be designed for multiple load combinations, and the members will not all be fully stressed in all combinations.
(3) The members will usually be sized to be somewhat larger than the minimum required size.
(4) The code formulas for capacity typically underestimate the actual strength.

In general, we do not know the magnitude of the overstrength. As an approximation, if we know from experience, experiments and/or nonlinear analyses that a typical structure of a given type has, say, an overstrength of 50%, we may be justified in modifying the above procedure to make it less conservative. We can do this by (a) designing for smaller load factors, (b) increasing the member force capacities, or (c) both. This approach has risks, however, since it tends to be based on established design and construction methods. If new techniques are used to obtain more highly optimized designs, or if construction practices change to use different component details, the strength reserve as a proportion of first yield strength could be significantly smaller than that assumed.

3.3.7 An Apparent Inconsistency

Formulas for strength capacity account for nonlinear behavior such as yielding, crushing and buckling. The demand values, however, are calculated assuming linear elastic behavior. Is this consistent? That is, if we must account for nonlinear behavior on the capacity side, why can we ignore it on the demand side?

The answer is that there is no inconsistency. Once a demand-capacity measure has been chosen, the estimation of the demand value is a separate problem from the estimation of the capacity value. These calculations are performed separately, and they should not be confused with each other. For most strength-based designs, our judgments are (a) that it is good enough to calculate P-V-M demands using linear structural analysis, and (b) that it is good enough to calculate capacity values using code formulas. Both calculation methods are good enough, and hence it does not matter that we assume linear behavior on the demand side yet account for nonlinear behavior on the capacity side.

3.3.8 Key Points

This section has summarized the assumptions for conventional strength based design. The following are some key points.

(1) We assume that the strength capacity of the structure is the load at which the strength demand equals the capacity in the weakest member. This capacity will usually be a conservative estimate of the true capacity.

(2) With this assumption it is good enough to design at the component level, and to use P-V-M as demand-capacity measures.

(3) It is good enough, in most cases, to use linear structural analysis to estimate P-V-M demands, even though actual members are nonlinear.

(4) To ensure consistency in the estimation of demands, the method for obtaining member stiffnesses (i.e., for linearizing the member behavior) should be specified.

(5) The design load distributions, load combinations and load factors must be carefully chosen and clearly defined.

(6) It is good enough, in most cases, to use code equations to estimate P-V-M capacities.

(7) We can account for uncertainty, and/or change the level of safety, by adjusting the load factors and capacity factors. An advantage of Load and Resistance Factor Design is that it does this more rationally than Allowable Stress Design.

(8) The estimation of strength capacity is a separate problem from the estimation of strength demand. We must, however, use consistent demand-capacity measures (i.e., we must make "apples-to-apples" comparisons).

3.4 ISSUES FOR ESTIMATING DEMAND VALUES

3.4.1 General

As discussed in the preceding section, if the member behavior is elastic-perfectly-plastic there is no yielding up to the factored design load, and the P-V-M demands can be calculated accurately using linear structural analysis. Actual members are more complex, and if the factored loads were actually applied to the structure there would probably be some nonlinear behavior (yielding, cracking, crushing, etc.). As noted, however, we can reasonably expect that the amount of nonlinear behavior is small enough that we can use linear structural analysis to calculate the P-V-M demands.

We can not, however, accept this conclusion uncritically. A number of reasons for this are considered in the following sections. Note that we are considering only the demand side of the equation. Issues affecting the capacity side are considered later.

3.4.2 Stiffness Values and Repeatability of Results

If two engineers perform independent analyses of the same structure for the same loading, they should calculate similar P-V-M values (i.e., close enough for design purposes). If they do not, the process is seriously flawed.

When we perform a linear analysis, the P-V-M values that we calculate depend on the stiffnesses of the elements in the analysis model. Design codes are usually quite clear on how to calculate member capacities. However, they generally do not specify how to calculate linearized stiffnesses for structural analysis, leaving this largely to the judgment of the engineer. Different engineers could make different modeling assumptions, and hence could calculate significantly different member stiffnesses. For a frame structure, some of the modeling decisions that can affect stiffness values are as follows.

(1) The choice of Young's modulus (E), and of cross section axial and bending stiffnesses (EA and EI). This is easy for a bare steel section, but not so easy for a reinforced concrete section. For example, the bending stiffness of a reinforced concrete cross section depends on the axial force, since compressive axial force suppresses cracking and hence stiffens the section.
(2) For a member with a varying cross section, whether and how to account for the variation.
(3) Whether to account for shear deformations.
(4) Whether, and how, to account for the flexibility of member end connections.
(5) Whether to account for shear deformations in connection panel zones. Note that a connection could be assumed to be rigid in bending yet still have significant shear flexibility.
(6) Whether, and how, to account for P-Δ effects.
(7) Whether to assume that column footings are rigid, or to account for footing, soil and pile deformations.

Fortunately, for static loading the distribution of member forces in a structure depends only on the relative stiffness values, and not on the absolute values. Hence, if the stiffnesses of all members are overestimated or underestimated by the same factor, the member forces do not change. Also, the member force distribution often does not change much if the relative member stiffnesses are changed (for a statically determinate structure the member forces are independent of stiffness). Hence, the calculated member forces may be insensitive to differences in the modeling assumptions.

Note, however, that when we calculate structure deflections, rather than member forces, the calculated values depend on the *absolute* stiffnesses. If the stiffnesses of all members are increased by the same factor, the calculated deflections are decreased by this factor.

3.4.3 Earthquake Loads

For seismic resistant design it is necessary to account for dynamic effects when calculating the member force demands. For a given earthquake ground motion these demands depend on both the mode shapes and the modal periods of the structure. The mode shapes depend on the relative stiffnesses of the members. However, the modal periods depend on the absolute stiffnesses (the period of any mode is inversely proportional to the square root of the absolute stiffness). Hence, the modeling assumptions are more important for dynamic loading than for static loading.

The mode shapes and periods also depend on the mass of the structure. Hence, the modeling assumptions for mass are also important, including both the magnitude and the distribution of mass.

3.4.4 Earthquake Resistant Design with Yielding

In earthquake prone regions, most structures are designed to remain essentially linear in moderate earthquakes, but to yield substantially in strong earthquakes. In this chapter we are considering only strength based design. If significant yielding is allowed this is a different design concept, and damage based design should be used. The discussion in this chapter does not apply to design concepts that permit significant yielding.

3.4.5 Gap Opening and Closing

In most cases, linear analysis is good enough for calculating P-V-M demands. A possible exception is a structure with gaps, such as expansion joints.

There are three possible concepts for the behavior of a gap, as follows.

(1) It remains open (e.g., a fully functioning expansion joint).
(2) It remains closed (e.g., a bearing that has no uplift strength but always remains in compression and never uplifts).
(3) It opens and closes.

If the design concept is that the gap always remains open or always remains closed, we can use linear analysis. If the gap can open and close we will probably have to use some kind of nonlinear analysis. The reason is that it is generally not possible to identify a good enough linearized stiffness. This is illustrated in Figure 3.6.

If the design concept is that a gap must remain open, the demand-capacity measure is the deformation across the gap. The demand value can be calculated by linear structural analysis, assuming an open gap. Note that this value depends on absolute, not just relative, stiffnesses. The capacity value is the available gap width. If we calculate a demand that exceeds the capacity, then we must conclude that the gap will close. However, this does not mean that we must account for gap closure in the structural analysis. Instead, it means is that the design does not satisfy the performance requirements and must be revised. If we decide that the gap can be allowed to close we have chosen a different design concept, with different demand-capacity measures and different analysis needs. We may explore multiple concepts, but we must be careful to keep them separate.

If the design concept is that a gap must remain closed, the demand-capacity measure is the compression force across the gap. The demand value is calculated by linear structural analysis, assuming a closed gap, and the capacity value in tension is zero. Again, if we calculate a demand that exceeds the capacity, the design must be revised. We do not have to allow for gap opening in the analysis, and in the analysis model it is assumed that the gap can resist a tension force.

3.4.6 Elastic vs. Linear

Note that "elastic" is not the same as "linear". A component is linear if its stiffness is constant. It is elastic if any work done to deform the component is stored as recoverable strain energy.

Hence, a gap that opens and closes is nonlinear yet can be elastic. In an analysis model, a component that is linear must also be elastic, but a component that is elastic need not be linear. (For dynamic loads an "elastic" component could have viscous damping, which means that it absorbs energy and strictly speaking is not "elastic". The term will be used here to refer to the static behavior of a component.)

3.4.7 Are the Calculated Demands Accurate?

If we actually applied the factored design loads to a structure, and were able to measure P-V-M values, the actual values might not be very close to those calculated using linear analysis, especially for dynamic loads. Also, it is unlikely that the load magnitudes and distributions used for design will ever exist in the actual structure. Hence, it is unlikely that, for example, the bending moment diagram calculated for a frame structure will ever exist in the actual frame.

However, the purpose of the analysis is not to calculate the actual forces and moments, but to calculate demand values that are good enough for use in demand-capacity comparisons. It may be interesting, for academic reasons, to perform a detailed simulation of a structure, with the aim of determining its "true" behavior. However, this is not necessary for design, and in any case is probably impossible.

3.5 ISSUES FOR ESTIMATING CAPACITY VALUES

3.5.1 General

As already noted, the demand and capacity calculations are distinct from each other, linked only by the fact that consistent demand-capacity measures must be used.

The P-V-M capacities for frame components are usually obtained from formulas in design codes. These formulas are based largely on the results of experiments, with some support from analyses. They represent the results of decades of research and experience, and they are continually being updated. They are also simple enough that if two engineers evaluate the same formula they can be expected to get capacity values that are closely similar. In general, therefore, strength capacities calculated using conventional methods are clearly good enough for use in strength based design.

3.5.2 Capacity Calculation by Structural Analysis

Although strength capacities are usually obtained from design code formulas, analysis may be used if code formulas are unavailable or inadequate. For example, analysis is often used to calculate the bending moment capacity of complex reinforced concrete cross sections.

It is important to recognize, however, that capacity analysis is more difficult than demand analysis. For example we have seen that strength demand values can usually be calculated with good enough accuracy using linear structural analysis. In almost every case, however, the strength of a component depends on nonlinear modes of behavior, such as yielding and buckling. To calculate the strength capacity we must account for these modes of behavior in the analysis model, and in almost every case this will require a nonlinear analysis. Further, the nonlinear modes that affect strength are often complex, and can be extremely difficult to model. Two notable examples are shear failure in reinforced concrete columns and bond failure in girder-to-column connections.

Capacity analyses must thus be used cautiously. It is especially important that any analysis method used to calculate capacities be thoroughly calibrated against experiment.

3.5.3 Structure Capacity Calculation by "Advanced Analysis"

As already noted, design at the component level does not take advantage of load redistribution after yield, and hence does not ensure a consistent amount of overstrength. If we could calculate the strength at the structure level, using nonlinear analysis, we could account rationally for overstrength, and presumably design more economical structures. The demand-capacity measure would now be load factor, the demand values would simply be the design load factors, and the capacity values would follow from the calculated structure strength. The most important step would be the use of "advanced" structural analysis to calculate the strength capacity of the complete structure.

This "advanced analysis" approach is an attractive one, because it promises to be both more rational and more economical than the member-by-member approach. However, it is important to remember that any analysis calculates the strength not of the *real structure* but of the *analysis model*. The strength of a complex structure can be influenced by many different modes of behavior, and it is extremely difficult to account for all significant modes in the model of a complete structure. If a significant behavior mode is ignored (or not "captured") in the analysis model, the calculated strength could substantially overestimate the true strength.

There are many complex issues to be addressed before nonlinear analysis can be used reliably to calculate structure strength capacities. Most of these issues are also important for damage based design, as considered in later chapters.

3.6 ISSUES FOR DEMAND-CAPACITY COMPARISONS

3.6.1 General

Given demand and capacity values, decision making may be as simple as checking whether demand exceeds capacity. However, it is usually not quite so simple. The following sections identify some issues that may need to be addressed.

3.6.2 Form of Interaction Formula

If the strength of a component depends only on P, V or M, the strength capacity formulas are usually fairly simple. However, if the component strength is affected by interaction among P, V and/or M, the behavior and the formulas are more complex (and also generally less accurate). A decision must be made on what form of the formula to use.

For the purposes of discussion consider the case with simple linear interaction between P and M, with an interaction diagram as shown in Figure 3.7. For performing demand-capacity comparisons we can use the interaction formula in the following different ways.

(a) $$\frac{\text{Demand}}{\text{Capacity}} = \frac{P_D}{P_{C0}} + \frac{M_D}{M_{C0}}$$

where subscript D indicates demand and C indicates capacity. P_{C0} is the capacity under axial force alone, and M_{C0} is the capacity under moment alone.

(b) $$M_C = M_{C0}\left(1 - \frac{P_D}{P_{C0}}\right), \text{ then } \frac{\text{Demand}}{\text{Capacity}} = \frac{M_D}{M_C}.$$

(c) $$P_C = P_{C0}\left(1 - \frac{M_D}{M_{C0}}\right), \text{ then } \frac{\text{Demand}}{\text{Capacity}} = \frac{P_D}{P_C}.$$

If the component is fully loaded, these three forms all give a demand/capacity ratio of one. However, if the component is not fully loaded, or is overloaded, they give different values. This may or may not be important, but it should be noted.

Note that the formula for Case (b) shows that the moment capacity depends on the force demand. Similarly, for Case (c) the force capacity depends on the moment demand. Whenever there is interaction there will be such dependencies. In some cases capacity depends on demand, as in the above example. In other cases demand depends on capacity, as shown in later examples.

3.6.3 Interaction Formulas and Dynamic Loads

Suppose that a step-by-step dynamic analysis is performed to calculate demand values, and that the strength capacity is defined by the above interaction formula. If the maximum values of P_D and M_D are calculated separately and used in the interaction formula, the demand will tend to be overestimated, because the two maxima will generally not occur at the same time. A better approach is to calculate the value of the demand-capacity ratio at each time step. This is more difficult to do, however.

This aspect is also important if the maximum values are calculated using response spectra, since time of occurrence is not considered in the response spectrum approach.

3.6.4 Unity Factor

A decision must be made on whether to allow overstress in some components. For example, we might decide to allow demand-capacity ratios up to 1.02 for any component, and up to 1.05 for no more than 10% of the components.

3.7 SIMPLE vs. ELABORATE ANALYSES

If we can get as good a result using a quick hand calculation as using an elaborate computer model, the hand calculation is better (unless appearances are important, in which case the computer analysis may look better to a client). Usually, however, for linear analysis the more elaborate the analysis model the better the results. This is one reason why computer programs for structural analysis are so useful -- they allow us to get much better P-V-M demand values than we can get by hand. For a complex structure with complex loads, it is extremely useful if we can analyze a single elaborate model that includes all of the components with all of the loads, and calculate all the required P-V-M values.

It is even more useful if the computer program also calculates capacity values and demand-capacity ratios, especially if the program shows these ratios graphically in color-coded diagrams of the structure. The computer can then identify the "hot spots" in the structure, where the components are most heavily loaded. When we use linear analysis in this way we decide, before the analysis begins, that the design concept, the demand-capacity measures and the methods for calculating these measures are all good enough. That is, we make the important decisions, and we use the computer program only to perform the calculations and present the results. We do not expect the computer program to do the engineering for us.

We must be careful if we extrapolate from strength based design to damage based design. We may be tempted to conclude that if a computer program can identify the heavily loaded hot spots for strength based design, then it can also identify the heavily damaged hot spots for damage based design. In principle it is possible for a computer program to do this. However, we must first make the important decisions, and then use the computer program *only to perform the calculations*. As discussed in the following chapters, this is much more difficult for damage based design than for strength based design.

3.8 SUMMARY

This chapter is concerned only with conventional strength based design. The behavior concept is that the structure remains essentially elastic, with negligible inelastic deformation and hence negligible structural damage. If this is not the concept, the discussion in this chapter does not apply.

The key points made in this chapter are as follows.

(1) This report is concerned with the decision making process for evaluating the performance of existing designs. This process has four main steps, namely (a) choose demand-capacity measures, (b) estimate demand values, (c) estimate capacity values, and (d) make demand-capacity comparisons, and hence design decisions.

(2) In each step the key test is whether the method is good enough for practical design purposes. It need not be perfect.

(3) For strength based design of frame structures, axial forces, shear forces and moments acting on the members and connections (P-V-M) are good enough measures.

(4) Linear structural analysis is usually good enough for estimating demands.

(5) For linear analysis the only property that is needed for a component is its stiffness. Most components have essentially constant stiffness within the range of interest for strength based design. Nevertheless, the behavior is rarely truly linear, and for analysis it is *linearized*. Different engineers may make different linearizing (modeling) assumptions, and hence may get different results.

(6) For static load analysis, the P-V-M values depend only on the relative component stiffnesses. Hence, the P-V-M values may be relatively insensitive to the modeling assumptions. For dynamic load analysis, however, the P-V-M values depend on the absolute component stiffnesses, and are more sensitive to the modeling assumptions.

(7) For strength based design, the behavior concept is essentially elastic component behavior, which is not necessarily the same as linear. For example, a gap that opens and closes can be elastic yet strongly nonlinear. It can be difficult to linearize the behavior for such cases, and nonlinear elastic analysis may be needed.

(8) It is not necessary to model behavior that is not allowed by the behavior concept. For example, if a gap exists but is not allowed to close, it is not necessary to model gap closure. This is a general and important rule that applies also to damage based design.

(9) Other things being equal, the simplest analysis is the best. However, for linear structural analysis an elaborate analysis model is usually better than a simple one, and there are definite advantages to using sophisticated computer programs. It is important to note that the computer program is used only to perform calculations, not to do any engineering. The behavior concept and the demand-capacity measures have been chosen by the engineer, and the computer is merely doing calculations.

(10) Code equations are usually good enough for estimating P-V-M capacities. Sometimes, however, capacities can be estimated using structural analysis. This is a capacity analysis, as distinct from a demand analysis. Capacity analyses are almost always nonlinear.

(11) Code equations and capacity analysis must account for nonlinear behavior, whereas demand analysis can assume linear behavior. This is not inconsistent, since it satisfies the "good enough" requirement, and also because the demand and capacity calculations are largely independent. Indeed, these calculations have traditionally been performed by different people -- demand calculations by "analysts" and capacity calculations by "designers".

(12) Uncertainty is inevitable, on both the demand and capacity sides. It is accounted for using demand amplification (load) factors and capacity reduction factors.

(13) For many modes of behavior, prescriptive rules can be used to ensure satisfactory performance, and it is not necessary to choose demand-capacity measures.

(14) Design using P-V-M at the component level has the disadvantage that it does not ensure consistent overstrength. In the future it may be possible to use nonlinear analysis to calculate the strength capacity at the structure level. The demand-capacity measures would then be the loads on the structure. A great deal of research and development is needed before this becomes a practical option. The problems to be solved are essentially those considered in the rest of this report.

(15) We have decades of experience in strength based design. Damage based design is inherently more complex, and we do not yet have decades of experience.

4. SIMPLE EXAMPLE OF DAMAGE BASED DESIGN

4.1 PURPOSE

The preceding chapter reviewed the assumptions and procedures for strength based design. This chapter considers the assumptions and procedures for damage based design, using a simplified example. The purpose is to compare and contrast the two design methods, and to show that damage based design is a more complex process that places greater demands on the skills of the engineer. As before, this chapter emphasizes the decision making framework and the role of structural analysis. It does not present the computational details, and ignores a number of aspects that might be important in a practical problem.

4.2 SIMPLIFIED STRUCTURE

4.2.1 Structure and Behavior Concept

The structure is a two span, reinforced concrete, box girder bridge with two abutments and a single column bent. At the abutments the girder is supported on bearings that are flexible both laterally and longitudinally. There are gaps between the girder and the abutment back walls. Figure 4.1 shows a diagrammatic view. The dimensions and other details are not important.

Our primary concern for this example is earthquake resistance. The behavior concept is to allow flexural hinging near the base of the column. This is the only allowable mode of inelastic behavior, and in all other modes the structure is to remain essentially elastic. Also, the gaps at the abutments must remain open. This is not necessarily a practical concept.

4.2.2 Components and Modes of Behavior

For earthquake resistance the structural components of greatest interest are the columns, the column footing, the column-to-footing connection, and the end bearings. We will assume that the box girder, the bent cap and all other connections have ample strength and remain essentially elastic.

For a column the modes of behavior and the behavior concept are as follows.

(1) *Combined axial force and bending.* Inelastic hinging is allowed near the column base (the hinge region), while the rest of the column must remain essentially elastic. In the hinge region the confined concrete core can crush to some extent, the unconfined concrete shell can spall off, and the longitudinal reinforcement can yield. However, the amount of damage must be moderate, and the column must maintain its structural integrity. Outside the hinge region there must be no significant crushing of the concrete, yield of the reinforcement, or other inelastic modes of behavior.

(2) *Shear.* The column must remain essentially elastic in shear, in both the hinge region and the rest of the column.

(3) *Torsion.* Assume that torsion is negligible and can be ignored.

(4) *Buckling*. Assume that column buckling is not a concern.

For the column footing the modes of behavior are bending and shear. For the bent cap they are bending, shear and possibly torsion. The behavior must be essentially elastic in all of these modes.

For the column-to-footing and column-to-bent-cap connections, the modes of behavior are more complex, but are mainly of shearing type. There can be high bond stresses in the reinforcement, and large diagonal tension and compression forces in the connection panel zones. As noted, the connections must remain essentially elastic.

The end bearings are loaded in compression and deformed longitudinally and laterally. They must remain essentially elastic. The gaps between the girder and the abutments must remain open.

4.3 DEMAND-CAPACITY MEASURES

4.3.1 Elastic Modes of Behavior

For the elastic modes of behavior of the column the concern for design is strength, and the demand-capacity measures are P-V-M. The same is true for the footing, bent cap and connections. Bond stress may be used to ensure satisfactory bond performance.

4.3.2 Bearings and Gaps

For the end bearings the demand-capacity measures are vertical bearing force and lateral deformation. For the gaps at the abutments, the measure is the amount of deformation across the gap.

4.3.3 Column Hinging

Except for the hinge region, the column remains essentially elastic and the concern is strength. For the hinge region the concern is damage. There are a number of rational demand-capacity measures that we might use for damage, for example the following.

(1) Dollar cost of the damage.
(2) Loss of strength in the hinge region. If the loss of axial, bending and shear strength is not excessive, the structure will not collapse. If the strength loss is modest the structure can remain in service, and will be relatively inexpensive to repair.

The problem with such rational measures is that it can be difficult or impossible to estimate the demand value. For example, the permissible cost of the damage (the damage cost capacity) might be set at 25% of the replacement cost. However, there are no analysis tools available that we can use to estimate the cost of the damage caused by any given earthquake (the damage cost demand). Similarly, the loss of strength (the strength loss capacity) might be set at 25%. This is an easier problem, but there are few, if any, analysis tools that we can use to calculate strength loss for any given earthquake (the strength loss demand).

For this reason we must, with presently available technology, use less direct measures. Possible measures for members that yield in bending include strain, curvature, plastic hinge rotation, and deflection. For this example we will use deflection as the measure, specifically the lateral deflection at the top of the column. The reasons for this are explained later in this chapter, and we will see that strain, curvature and hinge rotation are also involved.

4.3.4 Prescriptive Rules

A number of prescriptive rules are likely to be needed to ensure satisfactory performance. Some of these are as follows.

(1) The minimum size and maximum spacing of column ties are likely to be specified, to ensure adequate confinement of the concrete and to prevent bar buckling.
(2) Lap lengths and bar anchorage details are likely to be specified to control bond slip.
(3) Shear reinforcement details may be specified for the hinge region, to ensure that shear integrity is maintained as the column yields in bending.

4.4 CAPACITY VALUES

4.4.1 General

Consider the calculation of capacity values first. The calculation of demand values is a separate problem, and is considered later.

4.4.2 Elastic Modes of Behavior

For column cross sections that are not in the hinge region, the behavior is essentially elastic in all modes of behavior, and conventional design code formulas can be used. The same is true for the footings and bent caps. Design codes tend to be less explicit about connection strength capacities, but formulas or analysis procedures are available.

If the column has an irregular shape, structural (stress) analysis can be used to calculate its bending strength capacity, for example using the BIAX computer program. Since the bridge can deflect both transversely and longitudinally, the strength is required for biaxial bending. Also, the bending moment capacity of a cross section depends on the axial force on the section.

4.4.3 Bearings and Gaps

The bearing force and lateral deformation capacities for the end bearings are likely to be obtained from the manufacturer's specifications. The lateral deformation capacity may depend on the bearing force, with a larger deformation capacity if the bearing is lightly loaded.

4.4.4 Column Hinging

Since the demand-capacity measure for column hinging is deflection, a value is required for the lateral deflection capacity of a column that hinges near its base. This defines the deflection that can be imposed at the top of the column before there is unacceptable damage in the hinge region. This deflection can be estimated by performing a nonlinear analysis of the column. The steps are as follows.

(1) Choose axial strain as the demand-capacity measure.

(2) Choose strain capacity values. At the concrete strain capacity in compression there must be no unacceptable damage to the concrete in a well confined core (the unconfined shell may spall off, but this is acceptable damage). The steel strain capacity in tension must be chosen to ensure that a bar that yields in tension in one earthquake cycle does not buckle in compression in a later cycle. This strain also limits the crack size, which helps ensure that shear strength is not lost in the hinge region under cyclic loading.
(3) Model the column, accounting for nonlinear behavior of the concrete and steel. This is discussed in more detail in the next section.
(4) Perform a static push-over analysis.
(5) Calculate the lateral deflection at the top of the column when the strain demand equals the capacity. This is the deflection capacity of the column.

In this case the purpose of the static push-over analysis is to calculate a capacity value. The analysis transforms from strain capacity to deflection capacity, accounting for the column aspect ratio and other effects. The key aspect to be clarified is how to model the column.

4.5 NONLINEAR MODELING OF COLUMN

4.5.1 Biaxial bending

The column can be deformed in both the longitudinal and transverse directions, and hence is subjected to biaxial bending. As a result, the deflection capacity depends on the direction of the deflection, and is not a single value. This is shown in Figure 4.2. The interaction surface can be found by performing a number of three dimensional analyses of the column. Alternatively, a pair of two dimensional analyses can be performed, one for each principal direction, and the shape of the interaction surface can be assumed, based on previous experience. These details are not important for the present discussion, and for simplicity only two dimensional behavior is considered.

4.5.2 A Possible Model

Figure 4.3 shows one possible nonlinear model of the column, for implementing Steps 3, 4 and 5 of Section 4.4.4. This model is essentially that recommended by Priestley. The analysis of this model requires several steps, as follows.

(1) Set up a model of the column cross section. Apply axial force, then calculate a relationship between curvature and maximum strain. This can be done using a number of computer programs, including BIAX and DRAIN. The process is shown diagrammatically in Figure 4.4. The value to be used for the axial force is considered in a later section.
(2) From the curvature-strain relationship get the curvature at which the strain capacity is reached, in either the concrete or steel. This is the curvature capacity.
(3) Estimate the yield curvature. One method is shown in Figure 4.4.
(4) Subtract the yield curvature from the curvature capacity. This is the plastic curvature capacity.
(5) Multiply the plastic curvature capacity by the length of the plastic zone to get the plastic hinge rotation capacity.
(6) Calculate the lateral deflection at the top of the column due to elastic deformations. This can be done by hand in most cases, but may require a linear computer analysis for a column with a variable cross section.

(7) Calculate the lateral deflection at the top of the column due to plastic hinge rotation. This is simply the hinge rotation multiplied by the lever arm, as shown in Figure 4.3.
(8) Add the elastic and plastic deflections. This is the deflection capacity of the column.

A key parameter in this analysis is the length of the plastic zone. If this length is short, the plastic hinge rotation capacity is small (Step 5), and hence the plastic lateral deflection is small (Step 7). If the length is larger, the plastic hinge rotation and plastic deflection are larger. It is important, therefore to use the "correct" length. Since there is no theoretical way to determine the length, it must be determined empirically, by calibrating against experimental results. This has been done by Priestley.

It may be noted that Steps 5, 6 and 7 can be combined, by using a simple nonlinear model such as that shown in Figure 4.5. This a model can easily be analyzed using, for example, the DRAIN-2DX computer program. Other analysis models can also be used. For example, Steps 1 through 8 can be considered in a DRAIN-2DX model such as that shown in Figure 4.6.

4.6 DEMAND VALUES

4.6.1 General

The deflection capacity calculation described in the preceding sections is not exact, but is good enough for making design decisions. A corresponding calculation must be made of the deflection demand for the column. That is, a value must be calculated for the deflection imposed on the column during a strong earthquake. This deflection need not be exact, but must be good enough for design purposes.

Strength (P-V-M) demands must also be calculated for the other components and modes of behavior. In the following sections, the deflection demand for damage based design of the column hinge region is considered first, followed by the P-V-M demands for the other components and modes of action.

4.6.2 Column Deflection Demand Using Linear Analysis

A common assumption for the seismic response of structures is the *equal displacements* assumption. This is illustrated in Figure 4.7(a). It is based mainly on observations from analyses of simple, single degree of freedom, linear and nonlinear models. Figure 4.7(b) illustrates the *equal energy* assumption. Analyses have shown that the equal displacements assumption is reasonable (i.e., good enough for design) if the elastic natural period of the structure is long, and the equal energy assumption is reasonable if the period is moderately long. The assumptions do not necessarily apply for complex, multi degree of freedom structures. For the present example assume that the structure period is long, and that the equal displacements assumption can be made.

It follows that linear structural analysis can be used to calculate the deformation demands, even though the behavior concept allows nonlinear behavior of the column. This means that a response spectrum analysis can be used, rather than the time history analysis that would be needed for a nonlinear analysis model. The analysis model must include the entire bridge, not just the column.

Note that linear analysis can be used because the behavior concept requires that the gaps at the abutments remain open. If this were not the case, nonlinear behavior of the structure would be caused by gap opening and closing, as well as by column hinging. In this case it does not necessarily follow that the equal displacements assumption is accurate, or that linear analysis can be used.

Note also that the calculated structure period for the analysis, and hence the calculated deflections, depend on the bending stiffness that is assumed for the column. To ensure that different engineers will get the same result, the method for calculating this stiffness must be specified, and not left to the engineer's judgment. If there is significant rotation of the footing at the column base, this effect should also be taken into account.

In a typical response spectrum analysis, the purpose is to calculate P-V-M demands for strength based design. If the structure stiffness is overestimated, the spectral acceleration values tend to increase. Hence, it tends to be conservative to ignore effects such as foundation flexibility. In the present case, however, the purpose is to calculate *deflection* demands. In effect, a displacement spectrum, not an acceleration spectrum, is being used. If the stiffness of the structure is overestimated, the spectral displacement values tend to decrease. Hence, it tends to be unconservative to ignore effects such as foundation flexibility. This is just one of the many differences that must be considered when damage based design is used.

4.6.3 Column Deflection Demand Using Nonlinear Analysis

If an equal displacements type of assumption is judged not to be reasonable, it is necessary to use nonlinear analysis. Since the only nonlinearity allowed by the behavior concept is hinging near the column base, this is the only nonlinearity that needs to be included in the analysis model. As already noted, it is pointless to model nonlinearities that are not allowed to occur.

4.6.4 Axial Force Demand on Column

In the capacity analysis outlined in Section 4.5, the column was loaded by a specified axial force. This is the axial force demand on the column, which for the purpose of the analysis is assumed to be constant. The axial force capacity is not explicitly calculated, but the capacity analysis ensures that the column cross section is able to sustain the axial force demand. The correct demand value must, however, be used in the analysis. The contributions to this demand will be as follows.

(1) Gravity dead load, multiplied by a load factor. The load factor must be decided.
(2) Gravity live load. The amount of live load assumed to act in conjunction with the earthquake load must be decided.
(3) Possible load due to vertical ground accelerations. The amount, if any, must be estimated.
(4) Possible load due to horizontal ground acceleration. This is likely to be small for a single column bent.

The axial force demand might be calculated by the analysis outlined in Section 4.6.2. However, this is not necessarily the case. A decision must be made on how to obtain the axial force demand.

4.6.5 Bending Moment and Shear Force Demands on Column

Damage associated with curvature in the hinge region is accounted for by considering deflection as the demand-capacity measure. Also, we know that the axial force capacity exceeds the demand. We must still consider the following

(1) Bending moment and shear force outside the hinge region. This part of the column must remain essentially linear.

(2) Shear force in the hinge region. This part of the column clearly does not remain linear. However, the amount of shear deformation must be small.

The analysis of the bridge structure, as outlined in Section 4.6.2, gives values for the column bending moments and shear forces. However, these are not the moment and shear demands. To calculate these demands we must return to the behavior concept.

This concept is that the column can crack, yield and crush in the hinge region, provided the damage is acceptable. This means that the bending moment in the hinge region can approach the full bending moment capacity of the column cross section. This capacity is calculated in the analysis of Section 4.5.2, Steps 1 and 2. Hence, since the bending moment in the hinge region (say at the center of the region) is known, the bending moment demand diagram for the complete column is also known. This is shown in Figure 4.8(a), which assumes that the inertia of the bridge superstructure dominates (i.e., there are negligible inertia forces within the column length) and that there is no significant bending moment at the top of the column (i.e. no significant torsion in the box girder). If the analysis of the bridge structure indicates significant bending moments at the top of the column, the bending moment demand diagram must be modified as indicated in Figure 4.8(b). In both cases the shear force demand is obtained as the slope of the bending moment diagram.

The bending moment and shear force demands in the elastic part of the column, and also the shear force demand in the hinge region, thus depend on the bending moment capacity in the hinge region. This capacity is not, however, the value calculated in Steps 1 and 2 of Section 4.5.2, because that calculation does not account for overstrength.

The actual strengths of the concrete and reinforcing steel are known only approximately. To obtain a conservative value of the column deflection capacity, the strengths used for the analysis in Section 4.5.2 are likely to be lower bound values. The actual strengths are almost certainly larger than these values. Hence, the actual bending moment capacity is almost certainly larger than the calculated value. To obtain conservative values of the bending moment and shear force demands it is necessary to recalculate the bending moment capacity using upper bound strength values.

This procedure for strength design of the column is a simple example of *capacity design*. A combination of capacity design and formal demand-capacity comparisons is essential for rational earthquake resistant design.

4.6.6 Strength Demands on Footings and Connections

To ensure that the footings and connections remain essentially elastic, the P-V-M demands must also be calculated using capacity design, based on upper bound estimates of the column bending strength.

4.6.7 Demands on Bearings and Abutment Gaps

The analysis of the bridge structure outlined in Section 4.6.2 is good enough to calculate the deformation demand for the column. Hence it is also good enough to calculate deformation demands for the bearings and gaps.

4.6.8 Summary

This section has outlined the steps that must be followed to calculate demand and capacity values for damage based design. Many details remain to be decided, even for this simple example. As the next section shows, the process is vastly simpler if strength based design is used.

4.7. COMPARISON WITH STRENGTH BASED DESIGN

If true strength based design is used, the behavior concept is that all components of the bridge remain essentially elastic. The process is simpler in the following ways.

(1) The demand-capacity measures are P-V-M for the all components, including all cross sections of the column.
(2) The capacity values are obtained by conventional methods.
(3) The demand values are obtained directly from a linear analysis of the bridge structure.

This is a much simpler process, where the engineer is required to have fewer skills and to exercise less judgment. True strength based design could be used, but conventional wisdom says that the resulting structure would be uneconomical.

4.8 CONCLUSION

Damage based design is more rational than strength based design, and more likely to ensure satisfactory seismic performance. However, it is also much more complex. As shown in the following chapters, many issues must be addressed before damage based design can become a routine process.

5. NONLINEAR MODELING AND ANALYSIS

5.1 PURPOSE

This chapter presents a broad overview of the problems associated with nonlinear modeling and analysis.

Most structural components have both complex behavior and uncertain material properties, and it is unlikely that we will be able to capture the nonlinear behavior exactly in an analysis model. When we set up a model we must decide which modes of behavior are important, and which can be ignored. We must also decide whether we must perform a dynamic analysis, or whether we can obtain good enough results from a simpler static analysis.

In this chapter, the essential features of an analysis model are first summarized. The causes of nonlinear behavior are then reviewed, and a number of nonlinear elements that might be used in an analysis model are briefly discussed. It is emphasized that for a practical structure it is not possible to create a model that captures all possible nonlinearities. Hence, it is important to select a clear behavior concept for the structure, in which the nonlinearity is deliberately limited to selected components and modes of behavior. A number of example concepts for earthquake resistant design are considered, and analysis models that can capture the nonlinearities are described briefly for each concept. For these examples it is assumed that a nonlinear dynamic analysis is to be performed. Such an analysis may not be feasible, however, and a number of simpler alternatives are noted. A critique is presented of the simplest of these alternative methods, namely the "Z Factor" (or R-Factor) method.

This chapter touches on many aspects of nonlinear modeling and analysis, but does so only briefly. To cover the topic in detail would require many more pages.

5.2 TYPES OF ANALYSIS MODEL

5.2.1 Discrete Model

For both linear and nonlinear analysis, computer programs will usually be used, and the analysis model will usually be a discrete (finite element) model consisting of nodes and elements. The main ingredients of a discrete model are illustrated in Figure 5.1. As shown this figure, a single component of the structure (e.g., a beam span) may be modeled using several elements. When an analysis model is set up, the most important part of the modeling process is the choice of the element types and their associated properties.

For a *linear* analysis of a frame structure, the most common element will be a beam-column element, for which the axial, flexural and (possibly) torsional and shear stiffnesses must be input. Only a few other element types will usually be needed (for example, spring elements to model flexible supports), and the results output for all elements are the P-V-M demand values. Solid finite elements may be used for more complex structures, or for components that can not be modeled using frame type elements (e.g., stubby shear walls). In such cases the results are usually stresses, which may be used directly as demand-capacity measures, or may be integrated to obtain P-V-M stress resultants.

For a *nonlinear* analysis, the most important part of the modeling process is again the choice of the element types and their associated properties. However, nonlinear elements are more complex than linear elements. The required element properties include more than just stiffness values, and the results output may be damage related demand-capacity measures, which are more complex to define and calculate than P-V-M values.

Discrete models are almost always analyzed by the Direct Stiffness Method, using well established numerical techniques. Essentially the same numerical techniques are used for linear and nonlinear analysis, but nonlinear analysis requires additional steps.

5.2.2 Continuum Model

A discrete model has a finite number of elements and a finite number of displacement degrees of freedom. In contrast, a continuum model has an infinite number of elements, and may also have an infinite number of displacement degrees of freedom. Continuum models can be used for complete structures, but they are almost always used to model single elements. The element properties calculated for the continuum model are then used in a discrete model of the complete structure.

Figure 5.2 shows two examples, namely a beam element and a solid finite element. A true continuum has an infinite number of possible deformation modes, and in some cases an exact solution is used. For example, we can analyze a beam model exactly using the Stiffness Method, by forming and integrating the beam differential equation. We can also obtain an exact analysis for a beam model using the Flexibility Method. Usually, however, we will impose constraints to limit the number of allowable deformations of the element, and hence obtain an approximate solution. For example, we may assume that a beam element has a cubic displaced shape, which means that it has only four beam-type deformations (two rigid body modes and two bending modes). For the solid finite element we are not be able to, and do not want to, get an exact solution. Instead, we allow only eight deformations (three rigid body modes and five deformation modes). In all modes the element edges remain straight, which ensures that inter-element continuity is satisfied.

Discrete models are analyzed using standard numerical techniques, but a variety of techniques are used to analyze continuum models. In a computer program, computations associated with the discrete model are generic, and are performed at the structure level. Computations associated with continuum models for elements are specialized, and are performed at the element level.

5.2.3 Substructured Element

Often, the behavior of a structural component can not be captured using a single finite element. For example, a beam component as shown in Figure 5.3(a) might be modeled using a single beam element, as shown in Figure 5.3(b). If this element were based on the assumption that the displaced shape is a cubic function (i.e., the curvature variation is linear), the model would be accurate for linear analysis, because the displaced shape for a uniform beam is indeed cubic. It could be highly inaccurate after yield, however, because the displaced shape after yield is unlikely to be cubic.. The standard remedy for finite element analysis is to refine the mesh, by subdividing the component into several elements, as shown in Figure 5.3(c). This is inconvenient for frame analysis, however, since it requires that additional nodes be defined along the beam length. A more convenient solution is to use a single element, but to divide it into a number of subelements, as shown in Figure 5.3(d). In this case the computer program user defines nodes only at the beam-column joints, and uses a single element to model the beam component. The internal nodes and degrees of freedom are defined at the element level, and are not "seen" at the structure level.

Substructured elements combine discrete and continuum models. The computations for substructured elements combine standard numerical techniques with more specialized methods, all performed at the element level.

5.3 MATERIAL AND GEOMETRIC NONLINEARITY

Nonlinear behavior can be caused by nonlinear material behavior (material nonlinearity) and by change of shape of the structure and its components (geometric nonlinearity). Material nonlinearity can have a variety of causes, such as yield, cracking and frictional slip. Geometric nonlinearity can be caused by true large displacements, for example as in a cable structure, or by relatively small changes of shape, as in most frame structures. In the first case, the analysis must make use of true large displacements theory. In the second case, a simpler and more efficient method of analysis can be used, based on P-Δ theory.

In the following sections, the causes of material nonlinearity are discussed first, and a number of elements with material nonlinearity are briefly described. Methods of accounting for geometric nonlinearity are then considered.

5.4 CAUSES OF MATERIAL NONLINEARITY

5.4.1 Constitutive Relationships

An element accounts for material nonlinearity by means of a nonlinear action-deformation, or constitutive, relationship. Depending on the element type, this relationship can be between stress and strain, axial force and axial deformation, moment and curvature, hinge moment and hinge rotation, etc. Nonlinear relationships can take many different forms, depending on the underlying cause (or causes) of the nonlinearity, and on how accurately the nonlinearity is to be modeled.

In most cases there is so much uncertainty about the loads and material properties for an actual structure that there is little to be gained by using an elaborate nonlinear model. Nevertheless, the model should be reasonably realistic. One way to create a more realistic model is to identify the underlying causes of the nonlinear behavior, and to capture those causes in the constitutive relationship. In this section, a number of causes are identified, and the forms of the corresponding constitutive relationships are considered. The causes are as follows.

(1) Gap opening and closing.
(2) Brittle fracture.
(3) Plastic flow.
(4) Frictional slip.
(5) Inelastic volume change.

In addition, cases are considered where a constitutive relationship is used to model geometric nonlinearity.

Only static behavior is considered in this section. Dynamic behavior, especially viscous damping, is considered later.

5.4.2 Gap Opening and Closing

As a gap (or crack) opens and closes, the stiffness changes from zero and a large value. The simplest constitutive relationship is illustrated in Figure 5.4 In this case the gap closes suddenly, and the behavior in compression is linear and elastic. In practice it is unlikely that gap closure will be sudden. For example, an expansion joint in a bridge structure will probably accumulate compressible debris, and may have compressible filler material. Even if the gap is clean, its surface is likely to be rough, and as the gap closes the full compressive stiffness will not be developed immediately. It is also likely that a long expansion joint will close progressively over its length, not all at once.

Hence, although the action-deformation relationship shown in Figure 5.4 captures the essence of the behavior, it is an over-simplification.

5.4.3 Brittle Fracture

Figure 5.5(a) shows the simplest type of brittle fracture behavior. In this case the material is linear and elastic until the fracture strain is reached, then there is sudden failure. Figure 5.5(b) shows a more likely type of behavior. In this case the teeth are brittle, but each tooth fails at a different deformation. Part of the resistance to bond slip of a reinforcing bar comes from this type of mechanism.

A reinforced concrete member in tension exhibits similar behavior, as shown in Figure 5.5(c). If axial deformation is imposed on the reinforcing bar, as shown, some of the axial force is transferred to the concrete, by bond. As a result, the member is stiffer than the reinforcing bar alone. As the axial deformation is increased, the concrete tensile strength is exceeded, crack 1 forms, and the member suddenly becomes less stiff. If the axial deformation is increased further, additional cracks 2 and 3 form, with further loss of stiffness. When enough cracks form, the member stiffness is essentially the stiffness of the reinforcing bar alone. This type of behavior is usually referred to as "tension stiffening".

For an analysis model, the action-deformation relationship is usually smoothed out, for example as shown in Figure 5.5(d). Figure 5.5 shows only uniaxial force (or stress) and deformation (or strain). The behavior is more complex if there is multiaxial stress.

5.4.4 Plastic Flow

Metals yield essentially in shear, by deforming inelastically along slip bands. Cohesive soils (as distinct from frictional soils) exhibit similar behavior.

Figure 5.6(a) illustrates the special case of elastic-perfectly-plastic behavior, in simple tension and pure shear. Most materials will strain harden, for example as shown in Figure 5.6(b) through 5.6(e). For practical analysis it is usually good enough to assume simple hardening behavior, with a simple hysteresis loop based on kinematic hardening.

For the modeling of truss bars, beams and columns, the dominant stresses are longitudinal tension and compression, and a uniaxial model such as in Figure 5.6 is usually sufficient. In some cases, for example solid finite elements, multi-axial stresses must be considered, and the behavior is more complex. Figure 5.7 shows some aspects of the behavior under biaxial stresses, for the simple case of elastic-perfectly-plastic behavior. The material is typically assumed to be elastic until the von Mises yield surface is reached. It then becomes elastic-plastic (i.e., partially elastic and partially plastic). The behavior in the elastic-plastic range must be modeled using multi-axial plasticity theory, which is rather complex.

Multi-axial plasticity theory can be extended, largely by analogy, to other cases where there is interaction, for example P-M interaction in a column, as shown in Figure 5.8. In this case the axial force and bending moment are analogous to stress, and the axial strain and curvature are analogous to strain. This is a reasonable model for a steel member, since the nonlinear behavior in steel is caused by plastic flow. It is not such a good model for a reinforced concrete member, since much of the nonlinear behavior in concrete is caused by mechanisms other than plastic flow.

Figures 5.7 and 5.8 are both for elastic-perfectly-plastic behavior. The behavior, and the theory, are more complex when strain hardening is considered.

5.4.5 Frictional Slip

Figure 5.9 shows a bearing that can slide when the frictional resistance is overcome. The force, F, required to cause sliding is equal to μN, where N is the bearing force and μ is the coefficient of friction (assumed to be constant). There is thus interaction (coupling) between the bearing force and the sliding strength.

Materials such a granular soils, and to some extent concrete, exhibit frictional behavior. In this case the shear strength of the material depends on the amount of compressive stress. This is different from a metal or a cohesive soil, where the shear strength is independent of compressive stress.

In Figure 5.9, the sliding surface is smooth. Figure 5.10(a) shows a similar situation, but with normal and shear forces on a rough crack. As shown, if the crack slips in shear it must also open. This causes additional coupling between the normal (bearing) and shear (sliding) effects. A similar effect occurs in a dense granular material, but now the material must increase in volume (dilate) as it slips in shear. If the material is not free to expand (i.e., if it is confined, so that the dilation is restrained), slip in shear must be accompanied by an increase in compressive stress, which in turn causes an increase in shear strength. Hence, a granular material is stronger in shear if it is confined than if it is unconfined, and the strength increase depends on the amount of confinement. This is the main reason why confinement increases the strength of concrete.

Figure 5.10(b) shows another case where friction can have a substantial effect on behavior. The figure shows a reinforced concrete beam with a diagonal shear crack. Because the crack surface is rough, if there is slip along the crack there must also be extension normal to the crack. This extension has components in both the transverse and longitudinal directions. The transverse component causes extension of the transverse reinforcement, which thus resists shear force. The longitudinal component causes extension of the longitudinal reinforcement, which causes axial compression in the concrete, and hence increases shear strength. The interactions are complex, and are difficult to capture in an analysis model.

Under cyclic load, the simple friction bearing shown in Figure 5.9 has a hysteresis loop similar to that for an elastic-perfectly-plastic material. When the sliding surface is rough, as in Figure 5.10, reversal of the slip direction initially causes compaction, not dilation. Small particles may also break off from the crack surface (e.g., sand particles in concrete) preventing the crack from closing and also affecting the friction coefficient. As a result, cyclic shear behavior is complex and difficult to model.

5.4.6 Inelastic Volume Change (Crushing)

Under typical loading conditions, volume changes in structural materials such as steel, concrete and wood are small, although wood may crush significantly under excessive bearing stresses perpendicular to the grain. As already noted, dense granular soils, and also materials such as concrete, must dilate in order to fail in shear. The opposite may happen in loose granular soils, where shear deformation can cause compaction (and hence, in a saturated soil, can lead to liquefaction). However, these volume changes are a consequence of shearing actions, and there is little or no actual crushing of the material.

True crushing can occur, however, in lightweight cellular materials such as metal honeycomb and cellular plastics. In these cases, the cause of the inelastic volume change is buckling or fracture of the cell walls, not material yield under compressive stress. That is, the main cause is geometric nonlinearity that changes the internal structure.

5.4.7 Geometric Nonlinearity Modeled as Material Nonlinearity

Figure 5.11 shows the behavior of a strut that buckles in compression and yields in tension. In the loading cycle shown in the figure, the column first buckles elastically, at essentially constant force. The column becomes inelastic when a plastic hinge forms, and the axial strength then decreases. When the load is reversed the bar stiffens progressively as it straightens, then yields in tension. If the bar were simply to yield in compression, without buckling, the behavior would be simpler, as shown by the dashed line. The difference between the solid line and the dashed line is thus caused by geometric nonlinearity.

In most cases the constitutive relationship needs to account only for material nonlinearity. In this case, the relationship between axial force and axial deformation must account for both material and geometric nonlinearity. It may be possible to define a constitutive relationship that accounts for both types of nonlinearity (in effect, an equivalent material). As a general rule, however, it is better to model geometric nonlinearity directly.

5.4.8 Summary

As shown in this section, there are many causes of nonlinear behavior, and many possible constitutive relationships. For nonlinear analysis we must develop analysis models that capture the essential behavior of the structural components, and hence of the structure.

5.5 SOME ELEMENTS WITH MATERIAL NONLINEARITY

5.5.1 General

For linear analysis of frame structures, we can usually set up an analysis model using only a small number of different element types. These types include a beam column element, a truss bar element, a spring element for supports, a plane stress finite element for walls, and possibly a plate bending finite element for slabs. Nonlinear analysis is more complex, and we must consider a wider variety of element types, for the following reasons.

(1) For nonlinear analysis we must model a wider range of behavior, which requires a wider range of element types with more complex modes of behavior.

(2) Often, no single element in a computer program will behave exactly as we want, and we must combine several elements to obtain the required behavior. For this reason, nonlinear modeling requires that we be a lot more creative.

In the following sections, brief descriptions are presented of a number of elements that might be used for nonlinear modeling of frame structures such as buildings and bridges. The emphasis is on elements that are available in the DRAIN-2DX and -3DX programs. To simplify the descriptions, only two-dimensional elements are considered.

5.5.2 Simple Bar Elements

The simplest type of element is a bar that resists only tension and compression. The action-deformation (constitutive) relationship in this case is uniaxial, relating axial force to axial deformation. DRAIN-2DX and -3DX have two bar elements, namely Element Types 01 (a truss bar element) and 09 (a gap/cable element). The elements have simple force-extension relationships, as shown in Figures 5.12 and 5.13. Elements can be combined to obtain a variety of more complex relationships. A notable limitation of these elements is that they do not account for post-buckling strength loss of the type shown in Figure 5.11.

5.5.3 Zero Length Elements

In reality, all deformable components must have finite dimensions. Mathematically, however, it is possible for an element to have zero length yet be deformable. Zero length elements are useful for modeling components such as connections and bearings. The element stiffness and strength can be translational (e.g., a flexible bearing) and/or rotational (e.g., a semi-rigid beam-to-column connection). DRAIN-2DX and -3DX have three zero-length elements, namely Element Type 04 (a connection element), Type 05 (a bearing element with frictional slip) and Type 13 (a fracturing shear key element). The properties of element types 04 and 13 are shown in Figure 5.15. These elements both have uniaxial action-deformation relationships, and hence are similar to bar elements. As with the bar elements, more complex force-deformation and moment-rotation relationships can be obtained by combining elements. Element Type 05 accounts for frictional slip, as indicated in Figure 5.9. This element is more complex, because of the coupling between bearing force and friction strength.

5.5.4 Beam-Column Elements

Beam and column elements are inherently more complex than bar elements. Some reasons are as follows.

(1) A simple bar element has only one action, with one corresponding deformation (F and δ in Figures 5.12 through 5.14). Hence, the element has a simple 1-by-1 action-deformation relationship. In contrast, even a simple 2D beam element has three actions, namely axial force, bending moment at end i, and bending moment at end j. Hence, the action-

deformation relationship is multiaxial, and is characterized by at least a 3-by-3 constitutive matrix.

(2) Each cross section along the element length can have a different bending moment, and hence different nonlinear behavior. In general, inelastic behavior can spread continuously both along the element length and over the cross section depth.

(3) The nonlinear behavior at any cross section may be affected by P-M interaction. In a complex 3D element there could be interaction between axial force, bending moments about two axes, torsional moment, and shear forces along two axes (i.e., up to a 6-way interaction).

(4) Loads may be applied along the element length, as shown in Figure 5.3. These are element loads, as distinct from nodal loads.

In an actual beam, yield (or other types of nonlinear behavior such as cracking) will usually spread continuously along the element length. A beam element that accounts for this spreading can be termed a *distributed plasticity* model. Such models are usually complex, however, and most beam and column elements are based on *lumped plasticity* models, with zero-length plastic hinges. Figure 5.15(a) shows a simple *parallel* model, with two beam subelements in parallel. Element Type 02 in DRAIN-2DX is a parallel model of this type. Figure 5.15(b) shows a simple *series* model, with two plastic hinge subelements in series with a beam subelement.

More complex elements than these may be used, for example Element Type 08 in DRAIN-3DX, and Element Type 15 in DRAIN-2DX and -3DX. Element Type 08 is a true lumped plasticity model, whereas Element Type 15 has some distributed plasticity features. These elements can both be used for a simple beam model such as that in Figure 5.15(b), but they also have features that allow them to be used for models that are much more complex.

Some features of Element Type 15 are shown in Figure 5.16. A key feature of this element (and also of Element Type 08) is that cross sections are divided into fibers, each of which has a uniaxial force-extension or stress-strain relationship. With a fiber model, cross section properties such as the yield moment are not specified directly, but are calculated from the fiber properties. This has advantages if there is P-M interaction. If fibers are not used, the P-M interaction must be defined using an interaction surface, as shown in Figure 5.8, and the behavior after yield is defined by an extension of plasticity theory. This is reasonable for steel cross sections, but not so reasonable for reinforced concrete. The fiber model has the advantage that P-M interaction can be captured without resorting to plasticity theory. It also has the advantage that different properties can be specified for different fibers, for example, steel properties with yielding in tension and compression for some fibers, and concrete properties with cracking in tension and crushing in compression for other fibers.

There are many other possible models for beam and beam-column elements. One of these is a finite element model where, in effect, the curvature variation along the element length is defined by a finite element shape function. Two elements of this type are shown in simplified form in Figure 5.17. The element in Figure 5.17(a) has two nodes, with both translational and rotational displacements. This allows a cubic shape function to be defined for transverse displacement, corresponding to a linear curvature variation. The element in Figure 5.17(b) has four nodes, with only translational and rotational displacements. This also allows a cubic displacement shape function to be defined. If the actual curvature variation over the beam length is linear, such finite element with linear curvature is probably not accurate. To obtain accurate results it is usually necessary to use shorter elements (i.e., refine the finite element mesh).

5.5.5 Elements Constructed from Simpler Subelements

A wide variety of complex elements can be constructed by combining simpler elements. Three examples are as follows.

(1) A beam with both shear and flexural yielding can be modeled as shown in Figure 5.18(a). Note, however, that it is probably unwise to allow both types of yield. A component such as this should probably be designed to yield only in bending or only in shear.

(2) A beam with flexible end connections can be modeled as shown in Figure 5.18(b). The end connections can be modeled using a rotational zero length element (as shown at the left end), or alternatively using fibers and slaving constraints (as shown at the right end).

(3) A foundation that can bend and uplift can be modeled using beam and bar type elements, for example as shown in Figure 5.19.

Because nonlinear behavior can have many different causes and can take many different forms, it is often necessary to combine a number of elements to model the desired behavior. We must have a clear idea of the type of behavior that is to be modeled, and also be creative in combining the available elements to build the analysis model.

5.5.6 Solid Finite Elements

Most linear finite elements can be extended to account for material nonlinearity, by changing from a linear to a nonlinear constitutive relationship. Figure 5.20 shows a simple 2D solid element. Typically the element stiffness matrix will be determined using numerical integration, for example 2-by-2 point Gauss quadrature. This means that the material behavior is considered only at the integration points. Essentially, a linear element can be modified to consider material nonlinearity by replacing the linear constitutive relationship for the infinitesimal element at each integration point by a nonlinear relationship. This is not necessarily simple, however, because the state of stress is multiaxial (in the figure there are three interacting stress components, σ_{xx}, σ_{yy} and τ_{xy}). Hence, the theory for the constitutive relationship can be complex, especially for a complex material such as concrete.

DRAIN-2DX and -3DX do not include any nonlinear solid finite elements.

5.6 GEOMETRIC NONLINEARITY

5.6.1 True Large Displacements vs. P-Δ

In any structural analysis method there are three key ingredients, namely (1) continuity (geometric compatibility), (2) element action-deformation, and (3) equilibrium. Essentially, material nonlinearity is accounted for in the element action-deformation relationships, and geometric nonlinearity is accounted for in the continuity and equilibrium relationships. If large displacements are ignored, the continuity and equilibrium relationships are both linear. If large displacements are ignored, the continuity and equilibrium relationships are both linear. If large displacement effects can not be ignored, one or both of these relationships becomes nonlinear. There are two distinctly different types of analysis, as follows.

(1) *P-Δ analysis*. In this case the displacements are sufficiently small that linear continuity relationships can be used, but not linear equilibrium relationships. The continuity relationships are identical to those used in small displacements analysis. However, whereas in small displacements analysis the equilibrium relationships are formed in the undeformed position of the structure (and hence are linear), in P-Δ analysis they are formed in the deformed position, with some approximations.

(2) *True large displacements analysis*. In this case the displacements are so large that nonlinear continuity relationships must be used. Equilibrium must also be considered in the deformed position, without the approximations that are made for P-Δ analysis.

There are no general rules for deciding whether a small displacements, P-Δ or large displacements analysis is needed. For most civil engineering structures, however, it is unnecessary to consider true large displacements. If geometric nonlinearity must be considered, a P-Δ analysis will almost always be good enough. The reason is essentially as follows.

The displacements of a structure consist of translations and rotations, and it is the rotations that cause the geometric nonlinearity. Figure 5.21 shows a simple bar element in its undeformed and deformed positions. If the rotation of the bar is small, the relationship between the bar extension and the node displacements (i.e., the continuity relationship) is essentially linear. If the rotation is large, however, this relationship is nonlinear. Also, if the rotation is small, the bar forces in the deformed position have essentially the same orientations as in the undeformed position, and the relationship between the bar force and the nodal loads (i.e., the equilibrium relationship) is essentially linear. If the rotation is large, the equilibrium relationship in the deformed position is different from that in the undeformed position, and this relationship is nonlinear (because it depends on the rotation).

In most structures, rotations are quite small. For example, in a bridge or building, a value of 0.05 for the ratio of lateral drift to structure height would be very large. This corresponds, however, to a rotation, θ, of only 0.05 radians. For this value of θ, $\sin\theta = 0.04998$ and $\cos\theta = 0.99875$. For practical purposes, therefore, $\sin\theta = \theta$ and $\cos\theta = 1$. These are exactly the approximations made for P-Δ analysis. In most cases, therefore, a P-Δ analysis is accurate, and a true large displacements analysis is not needed. This is fortunate because a true large displacements analysis will typically require far more computational effort than a P-Δ analysis. Some computer programs have options for only small displacements and true large displacement analysis. Such programs can be grossly inefficient for the analysis of typical frame structures.

5.6.2 P-Δ and P-δ Effects

Figure 5.22(a) shows a cantilever that is modeled using a single beam-column element. Figure 5.22(b) shows the deflected shape, and Figure 5.22(c) shows the bending moment diagram. If equilibrium is considered in the undeformed position, the bending moment diagram is the linear diagram in Figure 5.22(b), with a maximum value of *Hh*. If equilibrium is considered in the deformed position, the bending moment diagram is the nonlinear diagram in Figure 5.22(c), with a maximum value of *Hh* + *P*Δ. As shown in Figure 5.22(c), there are two parts to the difference between the linear and nonlinear bending moment diagrams, one due to the "P-big-Δ" effect and the other due to the "P-small-δ" effect. If the element remained straight, only the P-Δ effect would be present. The P-Δ effect is thus associated with rigid body rotation of the element as the nodes displace, whereas the P-δ effect is associated with bending of the element within its length.

Note that the deflected shape in Figure 5.22(b) is for an elastic element. If a plastic hinge were to form at the base of the column, its deflected shape would be more nearly straight, as in Figure 5.22(d). The P-Δ effect is the same in both cases, but not the P-δ effect.

In the analysis of a discrete model, it is fairly easy to account for the P-Δ effect. It is more difficult, however, to account for the P-δ effect. The reason is that the P-Δ effect depends on only the element rigid body rotation, and not on its deformed shape. For example, in a beam-column element the rigid body rotation depends only on the translations of the nodes, and for any given nodal displacements this rotation is the same whether the element is elastic or yielded. Hence, it is not necessary to consider the yielded state of the element to account for the P-Δ effect. This is not the case for the P-δ effect. For any given nodal displacements, the deformed shapes of an elastic element and a yielded element are generally different. Hence, it is necessary to consider the yielded state, and the details of the deformed shape, to account for the P-δ effect.

For this reason, most elements account only for P-Δ effects. In the majority of cases this is reasonable, because P-δ effects tend to be small, and to have little effect on the behavior of the structure. If it is known that P-δ effects are significant (e.g., in a very long compression component), one solution is to subdivide the component into several elements, to account for P-Δ effects in each element, but to ignore P-δ effects. If sufficient elements are used, the P-δ effect is captured for the component as a whole. It is possible to account for the P-δ effect in a substructured beam-column element (see Figure 5.3), since it is already divided into shorter subelements.

5.7 DYNAMIC EFFECTS

5.7.1 General

Dynamic analysis is more complex than static analysis, because inertia and viscous forces must be considered, in addition to static forces. Usually the masses are simply lumped at the nodes of the analysis model, and as far as inertia forces are concerned there is little difference between linear and nonlinear analysis. If *consistent* element masses are used, however, there may be a difference. The reason is that the consistent mass matrix for an element depends on the deformed shape of the element, and as an element yields its deformed shape can change. Hence, the consistent mass matrix is not necessarily constant. If the internal inertia forces for a component are substantial (e.g., a long span beam), it is probably better to subdivide the component into several elements and lump the mass at the nodes, rather than use a consistent element mass matrix. It may also be possible to use a substructured element, with masses lumped at the internal nodes.

A more serious problem for nonlinear dynamic analysis is the modeling of viscous effects. This is considered in the following sections.

5.7.2 Viscous Damping in Linear Analysis

Experiments and observations show that real structures dissipate energy as they vibrate. There are many causes, including inelastic deformation of structural and nonstructural components, radiation of energy through the foundation, and deliberate damping of the structure using energy-absorbing devices. Some of the energy may be absorbed by true viscous damping, but it is likely that most of the energy is dissipated by other mechanisms. In a linear analysis model, however, the only way of absorbing energy is to add viscous damping. Hence, viscous damping is a modeling approximation. Only rarely will it capture the actual energy absorbing mechanisms.

In most linear analyses, a modal damping model is used, where each normal mode of vibration is damped independently, and the damping ratio can be chosen separately for each mode. This is convenient. However, it is not necessarily realistic, and it is not obvious what it means physically, in terms of viscous dashpots located throughout the structure.

An alternative to modal damping is Rayleigh damping. The modeling assumption for Rayleigh damping is that the damping matrix for the structure, $\underline{C}$, is given by:

$$\underline{C} = \alpha \underline{M} + \beta \underline{K}$$

where $\underline{M}$ = mass matrix, $\underline{K}$ = stiffness matrix, and α, β are scalar multipliers. With Rayleigh damping, each mode of vibration is damped independently, but the amount of damping can not be chosen separately for each mode. Instead, once the values of α and β have been chosen, the damping ratios for all modes are fixed. It is thus less flexible than modal damping, but has the minor advantage that it has a physical interpretation. This is shown in Figure 5.23. As shown, $\alpha\underline{M}$ damping corresponds to a series of viscous dashpots connecting the nodes to a fixed external point, and $\beta\underline{K}$ damping corresponds to viscous elements in parallel with the elastic elements of the model.

It is possible to extend Rayleigh damping to allow a different α value for each mass, and a different β value for each element. In this case, the modes of vibration will be coupled through the damping matrix, $\underline{C}$.

It is also possible to specify $\underline{C}$ directly, for example by including explicit viscous damping elements in the analysis model, and assembling the damping matrices for these elements.

The options for viscous damping in linear analysis are thus as follows.

(1) Modal damping.
(2) Basic Rayleigh damping (the same α and β values for all masses and elements).
(3) Extended Rayleigh damping (different α and β values for different masses and elements).
(4) Explicit viscous damping elements.

5.7.3 Viscous Damping in Nonlinear Analysis

Viscous damping is used in *linear* analysis because it is the only way to model energy absorption. In a *nonlinear* analysis the energy absorption will ideally be modeled directly, with no need for fictitious viscous damping. In practice, however, nonlinear analysis models will not capture all energy absorption mechanisms, and some viscous damping will usually be added to account for energy losses that are not modeled directly.

In some special cases, normal modes can be used for nonlinear analysis. In general, however, the normal modes change every time the structure yields, unloads, etc., and it is too expensive computationally to re-calculate the mode shapes and periods. Hence, we generally can not use modal damping, and we are limited to using Rayleigh type damping and/or to specifying explicit viscous elements.

For basic Rayleigh damping the damping matrix has the form:

$$\underline{C} = \alpha \underline{M} + \beta \underline{K}_0 \tag{5.1}$$

where $\underline{K}_0$ is the stiffness matrix in the initial unstressed state of the structure. In this case, $\underline{C}$ stays constant as the structure becomes nonlinear and its stiffness changes. This is a simple assumption, but in some cases it can lead to serious modeling errors.

Consider, for example, the case of a gap element that is initially closed. This element has a large initial stiffness, and hence a large viscous damper is present in the analysis model, in parallel with the gap element. This damper does not absorb much energy as long as the gap is closed, since the deformation of the stiff element is small. However, if the gap opens, the element deformation can be large, the viscous damper can absorb a lot of energy, and it can significantly stiffen the structure. The calculated response could thus be substantially in error.

This type of error can be overcome by two methods. The first method is to use extended Rayleigh damping, and to specify a small or zero value of β for the gap element. When the gap is closed the element is stiff and there is little energy absorption, so the lack of damping has only a small effect. When the gap is open the model is correct, since it does not predict a spuriously large energy absorption or add an artificially large stiffness. The second method is to use a damping matrix with the form:

$$\underline{C} = \alpha \underline{M} + \beta \underline{K}_T \tag{5.2}$$

where $\underline{K}_T$ is the current tangent stiffness matrix. In this case, when the gap opens the tangent stiffness of the gap element becomes zero, and hence the element has no viscous damping. Equation (5.2) has the disadvantage, however, that $\underline{C}$ changes each time $\underline{K}_T$ changes. This complicates the computation. Also, it is not obvious that it is a sound modeling assumption.

As with linear analysis, we can also add explicit viscous dampers to a nonlinear analysis model.

5.7.4 The Danger of Overestimating Energy Absorption

Whenever we specify viscous damping in a nonlinear analysis model, we must be careful not to "double dip". Suppose that we have set up an analysis model, and that we have performed linear analyses using modal damping. We can calculate α and β values for Rayleigh damping that give approximately the same ratios in the important modes, and hence give approximately the same linear analysis results. We might assume, therefore, that if we perform nonlinear analyses of the model, we can these same α and β values. If we do, however, we are almost certainly overestimating the total amount of energy absorption, because we will usually have elasto-plastic hysteresis as well as viscosity. In the linear model the viscous damping accounts for all energy absorption, but in the nonlinear model it accounts only for "miscellaneous" energy losses that are not modeled directly in the nonlinear elements. The values of α and β should thus be smaller for the nonlinear model.

Guidelines for the amount of viscous damping in nonlinear models are needed but have not yet been developed. In the meantime, when we perform a nonlinear dynamic analysis we should always check the energy balance for the analysis (if the computer program computes such a balance, which it should). This balance will show how much energy has been absorbed by viscous damping and how much by elasto-plastic hysteresis. Using engineering judgment, we must decide whether the viscous damping energy is a reasonable proportion of the total.

5.7.5 Key Points

Two key points made in this section are as follows.

(1) Viscous damping is a *modeling assumption*.
(2) For linear analysis viscous damping must account for all energy losses. For nonlinear analysis it must account only for those losses that are not considered directly in the analysis model.

For linear analysis we will typically use modal damping, and guidelines are available for choosing appropriate modal damping ratios. For nonlinear analysis we must use a different damping model, and guidelines are not available. It is important, therefore that we understand the modeling assumptions that we make when we specify, for example, basic Rayleigh damping. It is also important that we examine the energy balance for a dynamic analysis, and satisfy ourselves that the amount of energy absorbed by viscous damping is reasonable. Since there are currently no available guidelines, we must do this using our best engineering judgment.

5.8 NONLINEAR DYNAMIC ANALYSIS

For linear dynamic analysis, response spectrum methods are efficient, convenient, and good enough for most analyses. However, these methods rely on the superposition of modal responses, and hence are not applicable to nonlinear problems. *Inelastic* response spectra are useful conceptually, but they apply only to single degree-of-freedom structures.

Unless a linearized analysis model can be used (e.g., secant stiffness methods, as noted later), it is necessary to perform a time history analysis, integrating the equations of dynamic equilibrium step-by-step through time. This is computationally much more expensive than a response spectrum analysis, and has the following major problems.

(1) The calculated response tends to be sensitive to the ground motion. That is, even small changes in the assumed ground motion can lead to major changes in the calculated response. It is important, therefore, to perform analyses for a suite of possible ground motions. This is often not done.

(2) The ground motions records used in analysis are often generated artificially, to satisfy a chosen response spectrum. This spectrum is likely to be a composite of the spectra for several ground motions that are judged to be possible at the site. Hence, it tends to be more

severe than the spectrum for any single ground motion. Consequently, a ground motion that is generated to match the composite spectrum may be much stronger than any single motion that is reasonable for the site.

These potential errors tend to compensate for each other. However, this is not a sufficient reason for using an irrational procedure.

Note that it may still be possible to use mode shapes in nonlinear dynamic analysis -- just not modal superposition. A key feature of modal analysis is that it reduces the number of degrees of freedom in the analysis model. In a discrete node-element model, the number of degrees of freedom (and hence the number of equilibrium equations) is the number of nodal displacements. For a large analysis model, this can be a very large number. If we use mode shapes, the total number of mode shapes is also equal to the number of degrees of freedom -- when we use mode shapes we transform the degrees of freedom from the nodal displacements to the modal amplitudes. However, we do not have to consider all of the modes, for the following reasons.

(1) Many of the modes have zero associated mass. For a typical earthquake analysis, where all support points are assumed to move in-phase, these modes are not excited and they can be ignored.

(2) For earthquake loading, the higher (short period) modes tend not to be excited, and these modes can also be ignored.

Hence, if we use mode shapes we can reduce the number of degrees of freedom dramatically, and hence potentially reduce the computational effort. This approach is used effectively in computer programs such as 3D BASIS and the nonlinear version of SAP. Note that these programs still use step-by-step analysis.

5.9 MODELING EXAMPLES AND BEHAVIOR CONCEPTS

5.9.1 General

This section considers some aspects of setting up a nonlinear analysis model, using a number of example structures. Each example represents a different behavior concept that might be considered for earthquake resistant design. Most of the concepts are for structures where damage is permitted in a strong earthquake. They represent only a few of the concepts that could be used for earthquake resistant design. For each structure, a brief discussion is given of the type of analysis model that might be used, and the demand-capacity measures that might be chosen.

A modeling case study for a real structure could easily occupy a complete report, and hence the examples considered here are greatly simplified. Also, only 2D models are considered.

The discussions emphasize material nonlinearity. For each example it is assumed that a detailed nonlinear dynamic analysis is to be performed for combined gravity and earthquake loads, and that the purpose of the analysis is to calculate strength and damage demands. Alternatives to a detailed dynamic analysis are discussed later.

5.9.2 Unbraced Frame with Strong Columns and Weak Beams

In this concept the columns, connections and foundations remain essentially elastic, with essentially no damage. The beams are allowed to yield in bending, but remain essentially elastic in other modes (e.g., shear, local buckling, lateral-torsional buckling). Typically this means that only flexural yielding is permitted, and only at the locations shown in Figure 5.24.

In the analysis model the columns can be modeled as linear elements, except at the base and at the top. The beams must be modeled as nonlinear elements, allowing yield in bending. In most cases, especially for steel frames, a plastic hinge model will be good enough, for example using DRAIN Type 02 elements. A more elaborate model, using elements such as DRAIN Element Type 15, may be justified for reinforced concrete beams.

The demand-capacity measures for the columns are P-V-M, except where hinging is permitted at the base and top of the frame. The demand values are the P-V-M values from the analysis model, calculated using upper bound values for the beam strengths (i.e., using capacity design methods). The P-V-M capacities are obtained from standard design code formulas.

If plastic hinge models are used for the beams (and for the columns at the base and top), the demand-capacity measure for inelastic behavior at any hinge is likely to be one of the following.

(1) Maximum hinge rotation.
(2) Accumulated hinge rotation, as defined in Figure 5.25.
(3) Absorbed plastic work. This is essentially the accumulated hinge rotation multiplied by the plastic moment capacity.
(4) Some combination of maximum and accumulated rotation.

The demands are the values calculated in the analysis. The capacity must be obtained from design codes or experimental results.

If a more elaborate model (e.g., using DRAIN Element Type 15) is used, the demand-capacity measure will probably be strain. The use of this measure was discussed in Chapter 4. As noted in that chapter, it can be related to curvature and hinge rotation if desired. We must be careful if we calculate strain demands, since the calculated value can depend strongly on the properties we specify for the element. Guidelines for the proper use of complex elements such as Element Type 15 do not yet exist, and must be developed before these elements can be used routinely.

The columns and beams must both be designed to remain essentially elastic in shear. The shear force demands must be calculated using capacity design methods. For steel frames the cross sections at plastic hinges must be compact, to avoid local flange buckling, and the beams must be braced laterally to prevent lateral-torsional buckling, especially in the hinge regions. These aspects are likely to be covered by prescriptive rules. The connections and foundations must be designed to remain essentially elastic, again using capacity design methods.

5.9.3 Reinforced Concrete Bridge with Yielding Columns

In this concept, all components except the columns remain essentially elastic. The columns are allowed to yield in bending at chosen locations, but remain essentially elastic in other modes, especially shear. A structure of this type was discussed in some depth in Chapter 4. However, in that chapter a nonlinear dynamic analysis was not performed. Instead, the column deflection demand was calculated using linear analysis.

If a nonlinear dynamic analysis is performed, the demand-capacity measures could be the same as in Chapter 4 (deflection for inelastic bending of the column, P-V-M for other components and modes of action). The nonlinear analysis is then simply a replacement for the linear analysis, presumably giving more accurate deflection values. Alternatively, strain, curvature or hinge rotation could be used for the damage demand-capacity measure. The overall process is similar to that described in Chapter 4.

5.9.4 Steel Eccentrically Braced Frame

A model of an eccentrically braced frame is shown in Figure 5.26(a). All components except the shear links remain essentially elastic. The shear links are allowed to yield in shear, but remain essentially elastic in bending and other modes.

The columns and beams remain elastic, and hence are modeled as linear elements. The demand-capacity measures are P-V-M. The demand values are the P-V-M values from the analysis model, calculated using upper bound strengths for the shear links. The P-V-M capacities are obtained from standard design code formulas.

A shear link can be modeled as shown in Figure 5.26(b). The beam element accounts for bending of the link (which must be elastic), and the zero length shear element accounts for elastic and inelastic shear deformation. The actual shear deformation is distributed along the length of the link, whereas in the model it is lumped in the shear element. A detailed discussion of the properties to be assigned to the shear element is beyond the scope of this report, but they are approximately as shown.

The demand-capacity measure for shear deformation of the shear link is shear strain. The shear strain demand is the calculated deformation of the shear element divided by the link length. The shear strain capacity must be obtained from design codes or experimental results.

The demand-capacity measure for bending of the shear link is bending moment. The bending moment demand is the calculated maximum end moment. The bending moment capacity is less than the full plastic moment capacity of the cross section, because of M-V interaction (the web is yielded in shear, and is not available to carry moment). The M-V interaction diagram is essentially as shown in Figure 5.26(c), and the bending moment capacity is the plastic moment for the section minus the web.

Note that it is not necessary to account for M-V interaction in the analysis model. In the model it is assumed that the beam element is elastic (i.e., has infinite strength). If the calculated moment demand is less than the moment capacity, the shear link remains essentially elastic in bending, as intended, and the modeling assumption is correct. If the moment demand exceeds the moment capacity, this tells us that the design concept has not been realized, and the design must be revised. There is no point in allowing moment yield in the analysis model if it is not allowed by the behavior concept.

The link must be stiffened to ensure that there is no substantial local buckling of the flanges or web, and must be laterally braced to prevent lateral buckling. These aspects will usually be accounted for using prescriptive rules.

5.9.5 Structure With Seismic Isolation

In this concept, all components except the seismic isolators are typically required to remain essentially elastic. The isolators are allowed to yield or slide laterally, and must remain essentially elastic in bearing. The analysis model will typically consist mostly of linear elements, with nonlinear elements only for the seismic isolators, for example as indicated in Figure 5.27.

For the elastic components the demand-capacity measures are P-V-M. For yield or sliding of an isolator the demand-capacity measure is lateral deformation, or possibly accumulated deformation or energy absorption. The measure for bearing on an isolator is bearing force. The deformation and bearing capacities are provided by the manufacturer. There may be interaction between the bearing force and the lateral deformation.

5.9.6 Structure with Uplift at Supports

In this concept, all components except certain specified supports remain essentially elastic. The specified supports can uplift, causing gap opening and closing, for example as indicated in Figure 5.19. Energy absorbing devices might be placed at the supports.

5.9.7 Structure with Unspecified Yielding Components

In this concept the yielding components and modes are not explicitly identified as part of the design concept. It is presumed that they can be identified by analysis.

In the preceding concepts, nonlinear behavior is confined to relatively few structural components and behavior modes, all of which are well defined. The behavior is required to be essentially linear in all other modes and for all other components. This makes it possible to set up meaningful analysis models, and to apply capacity design methods. In this concept, the elastic and inelastic components and modes are not specified, and it is presumed that we can use analysis to tell us what they are. This is the *Simulation* reason noted in Section 2.4.1, and requires advanced analysis, as defined in Section 3.5.3. There are two flaws in this approach, as follows.

(1) As noted in Section 3.5.3, any analysis calculates the behavior not of the real structure but of the analysis model. The behavior of a complex structure can be influenced by many different modes of behavior, and it is extremely difficult to account for all significant modes in an analysis model. If a significant behavior mode is ignored, or if the value assigned to a property such as a material strength is inaccurate, the calculated behavior can be incorrect.

(2) In effect, we are asking a computer program to do our engineering for us.

For these reasons, this is not a sound behavior concept.

5.10 THE IMPORTANCE OF THE BEHAVIOR CONCEPT

The behavior concept that we choose for a structure has a major influence on the analysis model, and on the results that we must obtain from the analysis. If the behavior concept is that all the components of the structure must remain essentially elastic, we can use strength based demand-capacity measures and linear structural analysis. If we decide that we can accept significant damage, we have chosen a very different concept. In this case we will typically use a mixture of strength and damage based measures, and we may have to use nonlinear analysis.

If we are designing a new structure and are not sure what behavior concept to use, we may perform structural analyses to investigate different concepts (this is the *Behavior Study* reason noted in Section 2.4.1). If we are investigating an existing structure, we may perform analyses to assess its vulnerability (this is the *Vulnerability Study* reason). In these situations the behavior concept is an open question, and analysis is used to help provide answers. However, when we actually design a new structure, or when we design a retrofit for an existing structure, we will be using structural analysis to calculate demand and/or capacity values (these are the *Design Evaluation* reasons).

As noted in the preceding sections of this chapter, there are many possible causes of nonlinear behavior, and with present day technology we can capture only relatively simple nonlinearities in an analysis model. We are being unrealistic if we believe we can perform a nonlinear analysis that accounts for all possible nonlinear effects, calculates accurate load-deflection relationships, and identifies the locations and types of significant structural damage. What we must do, instead, is *choose* the structural components and modes of behavior for which significant damage (i.e., inelastic behavior) is to be permitted, and design for essentially elastic behavior in all other components and modes. We must thus develop a clear concept of how we want the structure to behave, and we must apply capacity design methods. Only by doing this can we be sure that our analysis models, and hence our analysis results, are good enough to use in design.

There is another reason for using capacity design, which may be even more important than the need to get good enough analysis results. This is that a structure designed using capacity methods is more likely to be damage tolerant, and more likely to be forgiving of inaccuracies in the analysis and design calculations. If our analysis models and/or design loads are inaccurate for some reason, we may underestimate the damage demands, or calculate inaccurate distributions of these demands. Such inaccuracies are less likely to be fatal if capacity design is used. Although the amount and distribution of damage in the actual structure may be different from that shown by the calculations, it is less likely that the damage will lead to unexpected or undesirable modes of behavior. This is because when we control the inelastic components and modes of behavior, we can usually design them with generous safety factors against damage. We can even use capacity design without calculating damage demands, by detailing the inelastic components to provide damage capacity that is larger than any likely demand (that is, we can use a prescriptive approach). The biggest danger in capacity design is not under-estimation of the damage demands, but under-estimation of the strength demands for components and modes of behavior that are supposed to remain elastic. If this mistake is made, there can be damage in components that have little damage capacity.

5.11 OTHER TYPES OF NONLINEAR ANALYSIS

5.11.1 General

For the examples in Section 5.9, we assumed that a nonlinear dynamic analysis was performed, to calculate damage demands for the inelastic modes of behavior and strength demands for the elastic modes. A nonlinear dynamic analysis is probably the best way to calculate demands, but such an analysis is complex and may not be feasible. We may be able to obtain good enough demand values using nonlinear static analysis, or using simplified dynamic analysis. We may also use nonlinear analysis to calculate capacity values. Some types of analysis that might be used are as follows.

5.11.2 Static Pushover Analysis to Calculate Demands

As an alternative to a dynamic analysis, it may be possible to calculate good enough damage (deformation) and strength (P-V-M) demands by performing a nonlinear static push-over analysis. For the case of a building frame, an estimate is made of the lateral displacement at the roof under the design earthquake, usually by performing a linear dynamic analysis. An estimate is also made of the lateral load distribution over the height of the structure, to approximate the effect of the earthquake. Gravity load is then applied to the analysis model, and the lateral load is increased until the estimated roof displacement is reached. The damage and strength demands on the components are the calculated values at this displacement. The analysis model can be simpler than that required for a dynamic analysis, because cyclic behavior of the components does not have to be modeled.

This method is based on two main assumptions, as follows.

(1) Reasonably accurate estimate of the roof displacement can be made using a linear analysis.

(2) The static deformations of the structure for the chosen lateral load distribution are reasonably close to the dynamic deformations in an earthquake. To account for uncertainty it may be necessary to consider more than one lateral load distribution.

5.11.3 Static Pushover Analysis to Calculate Displacement Capacity

When static pushover analysis is used as in the preceding section, the demand-capacity measures for damage are deformations at the component level (e.g., plastic hinge rotations). As an alternative, lateral displacement at the roof could be used as the demand-capacity measure, and the static pushover analysis is then used to transform from component deformation capacity to structure displacement capacity. Given plastic hinge rotation capacities for the beams in a frame structure, lateral load is applied until the capacity is reached at the most critical beam. The roof displacement at this load is the displacement capacity of the frame. In Chapter 4, this method was used to calculate the deflection capacity of the bridge column.

5.11.4 Static Pushover Analysis to Calculate Strength Capacity

Consider a structure in which the components all have unlimited ductility (i.e., they can yield an unlimited amount without losing significant strength). Also assume that deflections are not important, and that only static loads are applied. In this case the structure can be designed considering strength only, using load as the demand-capacity measure. For a specified gravity load, the lateral load demand can be the design load. The lateral load capacity is then the lateral load required to cause collapse, which can be calculated using a static pushover analysis.

If P-Δ effects are ignored, the relationship between lateral load and lateral deflection will have the form shown in Figure 5.28. Collapse occurs in the analysis model when enough elements have yielded to form a collapse mechanism. An interesting feature of this collapse load is that it depends only on the element strengths, and is independent of the element stiffnesses. In particular, the elements can be assumed to be rigid-plastic if desired. In this case the deflections of the model are zero until a collapse mechanism forms, at the rigid-plastic collapse load. As shown in Figure 5.28, this collapse load is exactly the same as the elastic-plastic collapse load. If the element stiffnesses are changed, the sequence of element yield can change, and the inelastic deformations (the ductility demands) also change. However, the final mechanism and collapse load are the same. It is interesting to note that this is the opposite of linear analysis. For linear analysis, with unlimited element strength, only the element *stiffnesses* are important, whereas for elastic-plastic analysis, with unlimited element ductility, only the element *strengths* are important.

It is unlikely, however, that a real structure will have this type of behavior. There are two main reasons, as follows.

(1) Real structural components do not have unlimited ductility. If a significant number of components reach their ductile capacities before the collapse mechanism forms, the actual collapse load is likely to be substantially less than the rigid-plastic collapse load. This is shown in Figure 5.29(a).

(2) P-Δ effects may be significant. As shown in Figure 5.29(b), P-Δ effects change the load-deflection relationship, and can substantially reduce the collapse load.

Because of this, both the stiffnesses and the strengths of the elements are important in the analysis model. Changes in stiffness affect both the magnitude and the distribution of the element deformations, and hence the ductility demands. Changes in stiffness also affect the structure deflections, and hence the magnitude of the P-Δ effect. Hence, rigid-plastic analysis is not of much practical value. The same is true of simple elastic-plastic analysis with unlimited ductility and small displacements.

5.11.5 Load Reversal with P-Δ Effect

If the P-Δ effect is small, and if there is no strength degradation due to limited ductility, the behavior under reversed loading will be as indicated by the dashed line in Figure 5.30. If the P-Δ effect is large, however, the behavior will be different, as shown by the solid line. If the structure is deflected substantially in one direction, it becomes, in effect, stronger in one direction than in the other. In an earthquake, therefore, the structure will tend to drift in the weaker direction. In this case, dynamic analysis for earthquake loads will be sensitive to the ground motion. An analysis with one motion may indicate modest drifts, while an analysis with a similar motion may indicate complete collapse.

This effect is probably real. If there are substantial P-Δ effects in a real structure, and it is deflected past the point of maximum strength, it will not necessarily collapse, because the loading is dynamic, not static. However, the possibility of a progressive collapse is greatly increased.

5.11.6 Static Pushover Analysis to Develop Simplified Models

A static pushover analysis can also be used to calculate the lateral load-deflection relationship for a structure. The load-deflection relationship can be used to develop a model of the structure that uses fewer and simpler elements, but has essentially the same load-deflection relationship. The simpler model can then be used in a nonlinear dynamic analysis. The simplest model is a nonlinear spring, which gives a single degree-of-freedom model for dynamic analysis. The analysis also gives the deformation capacity of this spring, as described in the Section 5.11.3.

5.11.7 Linearized Analysis Using Secant Stiffness

Figure 5.31 shows the type of load-deflection relationship that might be calculated from a static pushover analysis. If the displacement demand were known, this relationship could be replaced by a linear relationship, defined by the secant stiffness as shown. The deflection demand is not known, but it may be possible to calculate it by an iterative procedure. Given the load-deflection relationship (from a static pushover analysis), the steps are as follows.

(1) Estimate the deflection demand, and hence calculate the secant stiffness.
(2) Perform a linear dynamic analysis using this stiffness. Calculate a deflection demand.
(3) Compare the calculated demand with the estimate. If they are similar the procedure has converged. If not, calculate a new secant stiffness and repeat.

The big advantage of this method is that it requires only linear dynamic analysis, which can be a response spectrum analysis. It is not quite this simple in practice, however, for the following reasons.

(1) Although a response spectrum analysis gives a single maximum displacement, in the actual dynamic response the displacement amplitude varies from cycle to cycle. Hence, a secant stiffness based on the maximum displacement may not be a representative value. A secant stiffness based on a smaller deflection (e.g., 80% of the maximum) is likely to be better.

(2) For the linear dynamic analysis, a viscous damping coefficient is needed as well as a stiffness. The amount of viscous damping must be calculated so that the viscous energy absorption in the linear analysis approximates the actual hysteretic energy absorption due to inelastic behavior. This energy is the area under the hysteresis loop, which might be as shown in Figure 5.31. The hysteresis loop can be estimated based on the load-displacement relationship from the static pushover analysis, or can be calculated using a cyclic pushover analysis.

The secant stiffness method has some serious weaknesses, but it has the major advantage that it requires only linear dynamic analysis. For some applications it can give good enough results for design purposes.

5.11.8 Linear Analysis Using Z Factors

As noted in earlier chapters, it is common practice to use the equal displacements assumption for calculating the displacements due to earthquake ground motions. With this assumption, the displacement demands for a nonlinear structure can be calculated using linear analysis. Studies have shown that the assumption is reasonable for single-degree-of-freedom structures with a variety of different types of nonlinear behavior.

The well known Z Factor (or R Factor, or μ Factor) method uses the same assumption, but applies it in a different way. The steps are essentially as follows.

(1) The ductile capacity ratio of the structure is known. This is the deflection at which the damage becomes unacceptable, expressed as a multiple, μ, of the yield deflection.

(2) The strength demand for the structure is calculated using linear analysis (usually a linear dynamic analysis).

(3) The structure is designed to have a strength capacity at least equal to $1/\mu$ times this linear strength demand.

(4) If the equal displacements assumption is correct, the ductility demand on the structure will not exceed μ, and hence the structure will not be excessively damaged.

This is an attractive method, for two reasons. First, conventional strength-based design can be used. Second, linear structural analysis can be used. It can be a valid method, but as shown in the following section, it must be used with caution.

5.12 THE Z FACTOR METHOD AND DAMAGE CONCENTRATION

5.12.1 General

The problem with the basic Z Factor method, using the steps listed in the preceding section, is that it not only makes the equal displacements assumption, which is often reasonable, but also assumes that the distribution of inelastic deformations throughout the structure is the same as the distribution of elastic deformations, which is usually not reasonable. In most cases the inelastic deformations, and hence the damage, are concentrated in relatively few components of the structure. This is *damage concentration*. It must be taken into account if the Z Factor method is to be good enough for design purposes.

5.12.2 Simple Column Example

In Chapter 4, we considered the design of a bridge column using displacement as the demand-capacity measure. The demand value was calculated using linear analysis, and the capacity value was calculated using nonlinear analysis. For design of the same column using the Z Factor method, the demand-capacity measure is a strength quantity. For this discussion, use bending moment at the base of the column. The steps for Z Factor design are essentially as follows.

(1) Estimate the ductile capacity ratio, μ. For example, say $\mu = 6$. Since the strength measure is bending moment, it is natural to express this ratio in terms of curvature. The ductile capacity ratio is the curvature capacity divided by the yield curvature. It can be determined from experience, by experiment, or by analysis using a computer program such as BIAX.

(2) For the design earthquake, and using linear analysis, calculate the maximum deflection at the top of the column, Δ_{linear}, and the maximum bending moment at the base of the column, M_{linear}. Divide M_{linear} by 6 to get the bending moment demand.

(3) Design the column to have a bending moment capacity at least equal to this demand.

For the Z Factor method to be sound, the maximum curvature in the column must be 6 times the yield curvature. As the following analysis shows, this is probably not the case.

Figure 5.32(a) shows a simplified bridge column. If P-Δ effects are ignored, the bending moment diagram is as shown in Figure 5.32(b). For the purposes of discussion, let the moment-curvature relationship for the column be as shown for Case A in Figure 5.32(c). For this case, the column has uniform strength over its height, with the same moment-curvature relationship at all cross sections. The yield moment is M_y, as shown. Since $\mu = 6$, this is chosen to be 1/6 times the maximum linear moment demand, or $M_{linear}/6$.

Figure 5.33 shows two states for the column. In the first state, shown in Figure 5.33(a), the deflection at the top of the column is the first yield deflection. The bending moment in this state is $M_{linear}/6$, and the deflection is $\Delta_{linear}/6$. Since the column is linear up to this state, the curvature is proportional to the bending moment. Hence, the curvature variation is a straight line, as shown. In the second state, shown in Figure 5.33(b), the deflection at the top of the column is Δ_{linear}. The moment at the base is not M_{linear}, however, because the column has yielded. Instead, the bending moment and curvature diagrams can be calculated to be as shown (the calculations were done using the Moment -Area method). As shown, the maximum curvature is 11.35 times the yield curvature, not 6 times. Hence, the Z Factor method does not give the required ductility demand.

Alternatively, consider Case B, where the column does not have uniform strength. Instead, for this case the moment capacity varies linearly, as indicated in Figure 5.32(c), so that the yield moment at every section is 1/6 times the maximum linear moment at that section. Also, the shape of the moment-curvature diagram is the same at all cross sections, as shown.

For this case, Figure 5.34 shows the same two states as Figure 5.33. The first state, at first yield, is the same as before. However, the second state, with a deflection Δ_{linear}, is very different. Now, the curvature diagram remains a straight line, even after yield, and the maximum curvature is exactly 6 times the yield curvature. In this case, therefore, the Z factor method does give the expected ductility demand.

The Case B design gives the desired behavior because the variation of plastic curvature over the column height is the same as the variation of elastic curvature. The ductility demand ratio is thus equal to 6 at all sections. The Case A design does not give the desired behavior, because the plastic curvature is concentrated near the base of the column. The ductility demand ratios in this case are less than 6 at some cross sections and larger than 6 at others.

In order to obtain the desired ductility demand ratio for the Case A design, it is necessary to make the column stronger. For this example, make the yield moment $2.33M_{linear}/6$, keeping its stiffness the same. The state of the column when the deflection is Δ_{linear} is now as shown in Figure 5.35. The yield curvature for the column is larger by a factor of 2.33, and hence the ductility demand ratio at the column base is 13.98/2.33 = 6, as required.

The factor of 2.33 can be interpreted as a *damage concentration factor*. If there is no damage concentration, as in Case B, the required moment capacity at the base of the column is $M_{linear}/6$, or in the general case M_{linear}/μ. If damage concentration is present, as in Case A, the required moment capacity is $2.33M_{linear}/6$, or in the general case $\delta M_{linear}/\mu$, where δ is the damage concentration factor. If the value of δ is known, or if a reasonable value can be chosen (based on experience, analyses of similar structures, design code requirements, etc.), the Z Factor method becomes more rational.

The problem, of course, is to estimate δ. In the example used here, a hardening ratio of 0.05 was assumed for the moment-curvature relationship. If a hardening ratio of 0.10 is used, with $\mu = 6$ as before, there is less damage concentration, and the value of δ is 1.72, rather than 2.33. That is, δ is sensitive to the modeling assumptions.

The observations for this example can be extended to more general structures. If it were possible to design a structure such that the plastic deformations are proportional to the elastic deformations, in all components and in all modes of behavior, the Z Factor method would be exact. This is almost certainly unrealistic. Nevertheless, the Z Factor method will tend to be more accurate if the inelastic deformations are distributed more or less uniformly through the structure. Conversely, the method will tend to underestimate the ductility demands more severely as the inelastic deformations become more concentrated.

The obvious approach is to extend the Z-factor method to calculate damage concentration factors. To retain the advantages of the Z factor method, this must be done using linear analysis. A possible method is to use an iterative secant stiffness method, where the secant stiffnesses at iteration i depend on the strength demand-capacity ratios at iteration i-1. With this method, heavily stressed components are relatively less stiff, and hence have larger deformations, corresponding to larger amounts of damage concentration.

5.12.3 A Nasty Paradox

In the example of the preceding section, in order to get a curvature ductility ratio of 6 for the Case A design (uniform strength over column height) we had to increase the strength of the column. When we use a strength equal to $M_{linear}/6$, the maximum curvature is 11.35 times the yield curvature. When we increase the strength to $2.33M_{linear}/6$, assuming the same stiffness and deflections, the maximum curvature is 6.0 times the (increased) yield curvature. Note, however, that although we decreased the ductility ratio when we increased the column strength, we increased the maximum curvature, by the ratio (2.33)(6.0)/(11.35) = 13.98/11.35 = 1.23. The stronger column must thus be able to sustain a larger curvature than the weaker one. In this example we kept the stiffness of the column constant, which suggests essentially that we kept the same size and increased the amount of reinforcement and/or the concrete strength. This could, unfortunately, decrease the curvature capacity of the column, making the stronger column worse than the weaker column. This is a nasty paradox. It is implicit in the Z-Factor method that strengthening a component will reduce the ductility demand, and hence presumably reduce the damage. The example of the preceding section suggests that this may not be the case.

Note, also, that for this example, maximum curvature was used as the demand-capacity measure, a bilinear moment-curvature relationship was assumed, and an "exact" method of analysis was used. A similar column was considered in Chapter 4, but a completely different method of analysis was used, and it does not necessarily follow that the same conclusions would be reached using that method.

5.13 CONCLUSION

This chapter has touched on many aspects of nonlinear modeling and analysis. One of the purposes of the chapter is to emphasize that nonlinear analysis is much more complex than linear analysis, and that many philosophical, theoretical, computational and practical issues must be resolved before it can be used routinely in design. In particular, there is a need for detailed guidelines on constructing nonlinear models, selecting demand-capacity measures, performing nonlinear analyses, and interpreting the analysis results. These are complex tasks that require a great deal more work, covering both theory and practice, and requiring close collaboration between analysts and designers.

6. SOME STEPS AND RULES FOR NONLINEAR MODELING

6.1 PURPOSE

In most cases we use structural analysis to calculate demand values. Given a structure, the components that make up the structure, and a set of demand-capacity measures for each component, the main task is to set up an analysis model that will give good enough demand values for design purposes. If nonlinear analysis is required, most engineers will need assistance with this task, in the form of rules and guidelines for setting up meaningful and consistent models. At the present time such rules and guidelines do not exist, and most modeling decisions are left to the engineer's judgment.

The purpose of this chapter is to suggest a method for formulating modeling rules, with emphasis on models for demand analysis. It is important to note that the suggested method is in preliminary form, and a lot more effort is needed to develop practical rules.

6.2 OVERALL STEPS FOR SETTING UP A MODEL

The overall steps for setting up an analysis model are as follows.

(1) Identify the structural components (members and connections).
(2) For each component identify the significant modes of behavior.
(3) For each mode of behavior decide whether it is to be governed by (a) performance analysis or (b) prescriptive rules.
(4) For each mode of behavior that is governed by performance analysis, proceed as described in Section 6.3.
(5) For each mode of behavior that is governed by prescriptive rules, proceed as described in Section 6.4.

Ultimately we must consider all modes of behavior for all components. However, in the early stages of a design we may consider only the most important components and behavior modes.

6.3 STEPS IF BEHAVIOR MODE IS GOVERNED BY PERFORMANCE ANALYSIS

6.3.1 General

For each behavior mode that is governed by performance analysis, we must choose one or more demand-capacity measures, and we must have methods for calculating the demand and capacity values. In this chapter we are concerned only with demand calculation. We must set up an analysis model, perform the analysis, and obtain the demand values from the analysis results.

For each behavior mode, use the following procedure to decide whether, and how, to include the mode in an analysis model.

6.3.2 Categorize According to Type of Behavior

Assign the behavior mode to one of the following categories.

Performance, Zero Deformation (PZ). For this type of mode there is no significant deformation, and no significant damage. An example is moment transfer through a beam-to-column connection that is assumed to be rigid.

Performance, Linear Elastic Deformation (PL). For this type of mode there is significant deformation, but the behavior is essentially linear and elastic, and there is no significant damage. An example is column bending in a frame with strong columns and weak beams.

Performance, Nonlinear Elastic Deformation (PNE). For this type of mode there is significant deformation, but again the behavior is essentially elastic and there is no significant damage. However, the behavior is nonlinear. An example is expansion and contraction across a joint that can open and close without damage.

Performance, Inelastic Deformation (PNI). For this type of mode there is significant inelastic deformation, nonlinear behavior, and significant damage. An example is beam bending in a frame with strong columns and weak beams.

Note that these categories are for individual behavior modes, not complete components. A single component can have several behavior modes, belonging to different categories.

6.3.3 Modeling and Demand Calculation for PZ Modes

PZ modes have essentially zero deformations. In a discrete analysis model, all elements are deformable[1], and for each deformation there is a corresponding action (axial force, bending moment, etc.). For analysis, the deformations for a **PZ** mode are assumed to be zero. Hence, there is no need for an element deformation, and there may not be a need for an element. Some examples are as follows

(1) *Rigid beam-to-column connection.* Beam-to-column connections are typically assumed to be rigid, and are modeled as shown in Figure 6.1(a). In this model there is no element at all for the connection, even though it is a component of the structure. The main demand-capacity measure is the bending moment transmitted through the connection.

Note that if the connection were deformable, it would have to be modeled using an element. This could be a special connection element, as indicated in Figure 6.1(b). Alternatively the connection could be part of the beam element, as indicated in Figure 6.1(c).

(2) *Rigid support.* A stiff support is often assumed to be rigid, and modeled using nodal restraints as shown in Figure 6.1(a). As in the preceding example there is no element. The demand-capacity measures are the reaction forces and moments.

If the support were deformable, it would have to be modeled using one or more elements, for example a set of support springs.

[1]An analysis model can have regions that are literally rigid. These regions can not be modeled using elements, because the element stiffness would have to be literally infinite. Rigid regions must thus be modeled using "slaving constraints". Note that an element that has a very large, but not infinite, stiffness is not literally rigid, just very stiff.

(3) *Lateral-torsional buckling of a beam.* The main mode of action for a beam is bending. However, if the beam is slender and/or has little lateral support, it may buckle in a lateral-torsional mode. If the behavior concept allows significant lateral-torsional deformation, this deformation must be accounted for in the analysis model. In almost every case, however, the behavior concept will require that lateral-torsional deformations be negligible, and hence in the analysis model they can be zero. This is fortunate, because beam-type elements generally do not account for lateral-torsional deformations. The demand-capacity measure is the beam bending moment.

In the first two examples there are no elements, and hence the demand values are not calculated directly during the analysis, as element actions. Instead, they must be calculated indirectly, using equilibrium. For example, the bending moment transmitted through the connection in Figure 6.1(a) is equal to the end moment in the girder. Similarly, the moment reaction in Figure 6.2(a) is the moment required to satisfy rotational equilibrium at the restrained node. Some computer programs calculate reactions at restrained nodes. Often, however, additional calculations will be needed to obtain the demand values from the computer output.

To simplify the calculation of demand values for certain **PZ** modes, we may re-classify them as **PL** modes, and model them using deformable elements, for example, as shown in Figures 6.1(b) and 6.2(b). One disadvantage of this approach is that it increases the number of nodes and elements in the analysis model. A second disadvantage is that to model rigid components we may be tempted to specify very large stiffnesses for the elements, for example a stiffness of 10^{10} units for the rotational stiffness of the connection element in Figure 6.1(b). Such large stiffnesses can, in some cases, lead to numerical sensitivity problems in the analysis. It is important to note that there is no such thing as a truly rigid connection or support, and when we assume a component is rigid we are making an approximation. If we decide to model such a component using a deformable element, we should estimate its actual stiffness, and use this stiffness in the analysis. Actual stiffnesses will rarely have astronomical values.

We must remember, also, that when we use capacity design methods we must base the strength demands for elastic components on upper bound strengths for the inelastic components, and in some cases we will calculate these strength demands separately from the computer analysis. For example, the moment demand on a beam-to-column connection such as that in Figure 6.1 might be the upper bound moment capacity of the adjacent beam, which is not necessarily the same as the moment calculated for the analysis model. If we get strength demands directly from the analysis model, we must be careful to use upper bound strengths for the inelastic components, otherwise the P-V-M results from the analysis may be too small.

6.3.4 Modeling and Demand Calculation for PL Modes

There are significant deformations associated with **PL** modes. Hence, for each **PL** mode there must usually be one or more elements in the analysis model, and the behavior mode must be modeled by deformations of these elements. Some examples are as follows.

(1) *Column in a strong column, weak beam frame.* The column is designed to remain essentially elastic in all modes, and hence must be modeled using elastic elements. Each column will usually be modeled using a single beam-column element, with axial, flexural and (for the 3D case) torsional deformations. In some cases a column may be divided into several elements, for example to account for cross section variations. For bending behavior, the demand-capacity measure can be bending moment alone if interaction is

ignored, or combined axial force, bending moment and possibly torsional moment if interaction must be considered. The bending moment demand may be amplified to account for P-Δ and P-δ effects.

The computer program output may be demand values only, or demand values, capacity values and demand-capacity ratios. Note that if the demand-capacity ratio is greater than one, this indicates that the column is not strong enough. This means that the design must be revised, not that the column must be modeled using an inelastic element.

(2) *Expansion joint that is designed to remain open.* An example is shown in Figure 6.2. The demand-capacity measure in this case is deformation across the joint. The joint can be modeled without using an element, for example as shown in Figure 6.2(a). However, to obtain the deformation of the joint in this case we must calculate the displacement difference between Node A and Node B. If the computer program has an option to calculate relative displacements, this can easily be done. If not, it is probably easier to add a simple element connecting the two nodes, as shown in Figure 6.2(b). The deformation of this element is the required joint deformation (in effect, the element is a deformation meter).

(3) *Bending in the shear link of an eccentrically braced frame.* The behavior concept for the shear link in an eccentrically braced frame is that it can yield in shear but must remaining essentially elastic in bending. Hence, elastic bending exists in parallel with inelastic shear in a single component. The demand-capacity measure for bending is bending moment, and for shear is shear strain. Bending is a **PL** mode, whereas shear is a **PNI** mode.

The modeling of shear links was considered in Section 5.9.4. Section 6.4 presents additional discussion on cases where elastic and inelastic modes of behavior are present in a single component.

6.3.5 Modeling and Demand Calculation for PNE Modes

As with **PL** modes, there are significant deformations associated with **PNE** modes. For each **PNE** mode there must be one or more elements in the analysis model, and the behavior mode must be modeled by deformations of these elements. Since there is no significant damage, the elements will be elastic. However, since the behavior is nonlinear we may have to use nonlinear elements. Some examples are as follows.

(1) *Expansion joint that can close and re-open.* An expansion joint that must remain essentially elastic can be modeled using one or more gap elements. The demand-capacity measure is gap opening in tension, and bearing force in compression.

(2) *Column baseplate that allows uplift in tension.* This is similar to an expansion joint, except that the gap is initially closed.

(3) *Prestressed concrete beam that can crack in bending.* A prestressed concrete beam might be designed to allow substantial cracking when the concrete tension stress due to bending exceeds the prestress, with the requirements that (a) the cracks close fully on load reversal, (b) there is no loss of prestress, and (c) there is no concrete crushing. The bending of such

a beam is thus significantly nonlinear, but is elastic. This might be a difficult concept to realize in practice.

If the behavior of a component is nonlinear, it does not necessarily follow that the corresponding element or elements must also be nonlinear. As noted in Chapter 5, in some cases it is possible to calculate deformations and/or actions with good enough accuracy using linear (more correctly "linearized") element properties. This is inherently risky, however. When a linear model is used to analyze nonlinear behavior, the model tends to be empirical rather than rational. This means that the linear analysis model must be calibrated against either experiments or analysis models that are more accurate. As with any empirical model or theory, the calibrated model may be accurate for only the range of parameters considered in the calibration.

6.3.6 Modeling and Demand Calculation for PNI Modes

As with **PL** and **PNE** modes, there are significant deformations associated with **PNI** modes. For each **PNI** mode there must be one or more elements in the analysis model, and the behavior mode must be modeled by deformations of these elements. Since significant damage is allowed for **PNI** modes, the elements will be both nonlinear and inelastic. Some examples are as follows.

(1) *Beam that yields in bending.* The demand-capacity measure for the yielding region is damage based, for example plastic hinge rotation.

(2) *Bridge column that yields, cracks and crushes under combined bending and axial force.* Possible analysis models and demand-capacity measures for this case were considered in Chapter 4.

(3) *Shear link in an eccentrically braced frame.* The analysis model and demand-capacity measures for this case were considered in Section 5.9.4.

(4) *Seismic isolator that yields or slips.* A seismic isolator can usually be modeled using relatively simple nonlinear elements. In the lateral direction the demand-capacity measure is probably deformation, and in the vertical direction it is probably bearing force. The bearing mode of behavior could be a **PZ** mode, a **PL** mode, or (if uplift is permitted) a **PNE** mode.

6.4 ELASTIC MODES IN PARALLEL WITH INELASTIC MODES

The examples of the preceding section included a bridge column yielding in combined axial force and bending, and a shear link yielding only in shear. In both of these cases, inelastic and elastic modes of behavior occur in parallel. In the case of the bridge column, the behavior in shear must be essentially elastic. In the case of the shear link, the behavior in bending must be essentially elastic. This has important implications on the capacity side, since it means that the column must be designed to ensure negligible damage in shear, and the shear link must be designed to ensure negligible damage in bending, otherwise the behavior concept has not been realized. It also has implications on the demand side, which is our concern in this chapter.

The element or elements that are used to model the bridge column must include inelastic behavior under combined axial force and bending. However, these elements need not, and should not, include inelastic behavior in shear. This is a general rule -- model only those aspects of behavior that are permitted by the behavior concept.

Shear deformations that are *elastic* may, however, be included in the beam-column element. If elastic shear deformation is modeled, shear is a **PL** mode, otherwise it is a **PZ** mode. The decision on whether to include or ignore shear deformations must be based on whether shear deformations have significant or insignificant effects on the behavior of the analysis model. If the column is short, and we believe shear deformations can be significant, we should include them. If we believe they are insignificant, we can ignore them. Note that including or ignoring shear deformations has little or nothing to do with calculating shear force demands. This is a situation that requires capacity design, and shear force demands must be obtained from bending moment capacities.

The element or elements that are used to model a shear link are the opposite of those for the column -- they must be inelastic for shear but elastic for bending. Hence, it is not necessary to model flexural hinging. This is the same general rule as in the bridge column example -- model only those aspects of behavior that are permitted by the behavior concept.

6.5 STEPS IF BEHAVIOR MODE IS GOVERNED BY PRESCRIPTIVE RULES

6.5.1 General

The modeling rules for modes of behavior that are governed by prescriptive rules are similar to those for modes that are governed by performance analysis. However, demand-capacity measures are not needed, and demand values do not have to be calculated. Instead, rules must be specified to ensure that the required behavior is obtained.

For each mode, use the procedure described in the following sections to decide whether, and how, to include the mode in the analysis model.

6.5.2 Categorize According to Type of Behavior

Assign the mode of behavior to one of the following categories.

Rules, Zero Deformation (RZ). For this type of mode there is no significant deformation, and no significant damage. Two examples are (a) reinforcing bar buckling in a reinforced concrete column, and (b) local flange buckling in a steel girder.

Rules, Linear Elastic Deformation (RL). For this type of mode there is significant deformation, no significant damage, and the behavior is essentially linear. An example is the seismic gap between buildings, which is usually specified by rules, and is intended to be large enough to accommodate any foreseeable relative displacement.

Rules, Nonlinear Elastic Deformation (RNE). For this type of mode there is significant deformation, no significant damage, and the behavior is substantially nonlinear. An example is bending and buckling of a tension wind brace in a light steel building. The bar is designed for strength in tension, but it is allowed to buckle in compression, and it must support its own weight as a beam. Its behavior in bending and compression is assumed to be elastic if it satisfies certain slenderness requirements. It may be necessary to model it as an element that can resist only tension.

Rules, Inelastic Deformation (RNI). For this type of mode there is significant inelastic deformation, and significant damage. Inevitably, the behavior is substantially nonlinear. An example is yield of a reinforced concrete member in bending, where the hinge region is detailed to ensure that it can sustain any foreseeable amount of hinge rotation without suffering unacceptable damage.

As before, these categories are for individual behavior modes, not complete components. A single component can have several behavior modes, belonging to different categories. Also, in any component, some modes of behavior might be governed by prescriptive rules, and others by performance analysis.

6.5.3 Modeling of RZ Modes

RZ modes have essentially zero deformations, and it is not necessary to calculate any demand values. In the analysis model they are simply ignored.

6.5.4 Modeling of RL, RNE and RNI Modes

There are significant deformations associated with **RL**, **RNE** and **RNI** modes. For each mode there must be one or more elements in the analysis model, and the behavior mode must be modeled by deformations of these elements. The modeling decisions are similar to those for **PL**, **PNE** and **PNI** modes. However, the actions and deformations need not be calculated, since there is no need to calculate demand values. This simplifies the analysis.

6.6 SOME GENERAL MODELING RULES

6.6.1 General

The preceding sections have outlined rules for modeling specific components and behavior modes. There are also a number of general rules that can be formulated, as follows.

6.6.2 Rule 1 : First define the behavior concept, then set up the analysis model.

Before starting the journey, plan the route. As already noted, the goal is to make design decisions, not to perform structural analyses.

6.6.3 Rule 2 : For each different behavior concept, set up a different analysis model.

There will be substantial differences between behavior concepts, and corresponding differences between models. We may be able to set up a single model, and adapt it for specific concepts by turning different nonlinear features on or off, but each specific model will be different.

6.6.4 Rule 3 : Model only the behavior that is permitted by the behavior concept.

If a mode of behavior is required to be elastic, there is no need to model it as inelastic. If a building frame is required to have strong columns, model the columns using elastic elements. If the behavior concept does not allow a gap to close, model it as always open. If column buckling is not allowed, do not try to include it in the model.

If a mode of behavior is not permitted by the behavior concept, it is a waste of time and effort to model it.

6.6.5 Rule 4 : Most modes of behavior should be elastic.

This is not a hard-and-fast rule. However, if a single component is allowed to be inelastic in several different modes, it is likely that the behavior concept is either unsound or has not been defined in sufficient detail. It will also be difficult to model the component. To ensure reliable earthquake resistant behavior, the behavior concept must be simple. This means that inelastic behavior must usually be permitted in only one or two modes for any component.

Conversely, if there are so few inelastic components that the structure lacks redundancy, in the sense that unexpected poor performance of a few components can lead to major damage, the behavior concept may also be unsound.

6.6.6 Rule 5 : Use P-Δ analysis, not true large displacements.

A P-Δ analysis is good enough for the majority of structures, and is usually simpler and more efficient computationally than a true large displacements analysis.

6.6.7 Rule 6 : The model does not have to be exact, just good enough.

The goal of an analysis is to obtain results (usually demand values) that are good enough for making design decisions. The goal is not to simulate the exact behavior of the structure.

6.6.8 Rule 7 : Don't analyze. Design.

The purpose of the exercise is to make design decisions, not to perform structural analyses.

7. SUMMARY AND RECOMMENDATIONS

7.1 PURPOSE

As stated in the introduction, this report is not a how-to manual or a reference work, but merely a collection of thoughts on the broad topics of damage based design and the use of nonlinear structural analysis. In this chapter, the main points that have been made in the report are summarized, and some recommendations are made for future work.

7.2 SUMMARY

7.2.1 The Need for a Decision-Making Framework

Structural analysis is not an end in itself -- it is a merely a tool to support decision making. Hence, if we are to make effective use of nonlinear analysis we must be clear on how it fits into the design process. The best way to achieve this is to define a rigorous framework for decision making. The advantages of such a framework are as follows.

(1) It forces us to break the complex process of decision making into its simplest possible parts.
(2) We can identify what decisions we must make, and exactly what we must do to make them.
(3) We can see whether we need nonlinear analysis.
(4) We can identify the weak links in the chain of reasoning, and the key problems that must be solved.

7.2.2 Outline of Framework

The essential parts of the framework are as follows.

(1) Define the behavior concept for the structure and its components.
(2) Decide which modes of behavior are governed by prescriptive rules and which are subject to performance analysis.
(3) For those modes of behavior that are governed by prescriptive rules, specify the rules.
(4) For those modes of behavior that are subject to performance analysis:
    (a) Define the limit states.
    (b) Define the demand-capacity measures.
    (c) Specify how to calculate the demand values.
    (d) Specify how to calculate the capacity values.
    (e) Specify how to perform the demand-capacity comparisons.

7.2.3 The "Good Enough" Rule

There will always be a great deal of uncertainty in structural design, and it will never be possible to be "perfect" or "exact". "Good enough" is more practical and much easier. This applies to the selection of demand-capacity measures, and the calculation of demand and capacity values.

7.2.4 Reasons for Using Structural Analysis

The following are several reasons why we might use structural analysis.

(1) Demand Calculation for Design Evaluation.
(2) Capacity Calculation for Design Evaluation.
(3) Behavior Study.
(4) Sensitivity Study.
(5) Vulnerability Assessment.
(6) Validation.
(7) Calibration.
(8) Simulation.

Whenever we use structural analysis, especially nonlinear analysis, we must be clear on the purpose. The reason for most analyses is Demand Calculation.

7.2.5 Linear vs. Nonlinear Analysis

Linear analysis is much simpler than nonlinear analysis, for the following main reasons.

(1) For a linear model of a structural component, only the component stiffnesses are needed. Much more information is usually required for a nonlinear model.
(2) For linear analysis superposition can be used. Separate analyses can be performed for, say, gravity and lateral loads, and the results can be scaled and added to obtain results for the combined loads. Also, response spectrum methods can be used. Superposition can only rarely be used for nonlinear analysis.
(3) The computational algorithms required for linear analysis are simple and reliable. Nonlinear algorithms are neither simple nor reliable, and we must understand their features and limitations.
(4) Typically, the only results that we need from a linear analysis are the maximum structure displacements and the maximum forces for each member. We need a much wider variety of results from a nonlinear analysis.

Also, the reason for performing a linear analysis is usually clear -- it is almost always to calculate strength (P-V-M) demands. The reason for performing a nonlinear analysis may not be so clear.

7.2.6 Is A Structural Analysis Needed?

There are many cases where a structural analysis, of any kind, is not needed. Often we will know, with good enough certainty and without performing any analyses, that the capacity exceeds the demand, and hence that the component or structure will have satisfactory performance. This may be all that we need to know to decide that the design can be accepted. Conversely, we may know that the capacity is less than the demand, and hence that the component or structure will not have satisfactory performance. This may be all that we need to know to decide that the design must be revised.

7.2.7 Conventional Strength-Based Design

This key points for conventional strength based design are as follows.

(1) We assume that the strength capacity of the structure is the load at which the strength demand equals the capacity in the weakest component. This capacity will usually be a conservative estimate of the true capacity.

(2) With this assumption it is good enough to design at the component level, and to use P-V-M as demand-capacity measures.

(3) It is good enough, in most cases, to use linear structural analysis to estimate P-V-M demands, even though actual components are often nonlinear.

(4) To ensure consistency in the estimation of demands, the method for obtaining component stiffnesses (i.e., for linearizing the behavior of the structure) must be specified.

(5) The design load distributions, load combinations and load factors must be carefully chosen and clearly defined.

(6) It is good enough, in most cases, to use code equations to estimate P-V-M capacities.

(7) We can account for uncertainty, and/or change the level of safety, by adjusting the load factors and capacity factors. An advantage of Load and Resistance Factor Design is that it does this more rationally than Allowable Stress Design.

(8) The estimation of strength capacity is a separate problem from the estimation of strength demand. We must, however, use consistent demand-capacity measures (i.e., we must make "apples-to-apples" comparisons).

7.2.8 Elastic vs. Linear

"Elastic" is not the same as "linear". A component is linear if its stiffness is constant. It is elastic if any work done to deform the component is stored as recoverable strain energy. A component can be nonlinear and elastic.

7.2.9 Simple vs. Elaborate Analyses

For linear structural analysis an elaborate analysis model is usually better than a simple one, and there are definite advantages to using sophisticated computer programs. It is important to note, however, that the computer program is used only to perform calculations, not to do any engineering. When linear analysis is used to calculate P-V-M demands, the behavior concept and the demand-capacity measures have been chosen by the engineer, and the computer is merely doing calculations.

We must be careful if we extrapolate from strength based design to damage based design. We may be tempted to conclude that if a computer program can identify the heavily loaded "hot spots" for strength based design, then it can also identify the heavily damaged hot spots for damage based design. In principle it is possible for a computer program to do this. However, we must first make the important decisions, and then use the computer program *only to perform the calculations*. This is much more difficult for damage based design than for strength based design.

7.2.10 Advanced Analysis to Calculate Load Capacity

Strength based design using P-V-M at the component level has the disadvantage that it does not ensure consistent overstrength. In the future it may be possible to use "advanced" nonlinear analysis to calculate the strength capacity at the structure level. The demand-capacity measures would then be the loads on the structure. However, a great deal of research and development is needed before this becomes a practical option.

7.2.11 Causes of Nonlinearity

There are two broad types of nonlinearity, namely *material* and *geometric*. The main causes of material nonlinearity are (1) gap opening and closing, (2) brittle fracture, (3) plastic flow, (4) frictional slip, and (5) inelastic volume change. Geometric nonlinearity is caused by change of shape of the structure (specifically, large rotations).

7.2.12 P-Δ vs. Large Displacements Analysis

If the change of shape is modest (i.e., if the rotations are only moderately largel), P-Δ analysis can be used. This is the case for most structures. If the change of shape is large, true large displacements analysis must be used. P-Δ analysis can be much more efficient computationally than true large displacements analysis.

7.2.13 The Need for Creative Modeling

Because nonlinear behavior can have many different causes and can take many different forms, it is often necessary to combine several elements to model the desired behavior. We must have a clear idea of the type of behavior that is to be modeled, and also be creative in combining the available elements to build the analysis model.

7.2.14 Viscous Damping in Nonlinear Analysis

Viscous damping is a *modeling assumption*. It is used in linear analysis because it is the only way to model energy absorption. In a nonlinear analysis the energy absorption would ideally be modeled directly, with no need for viscous damping. In practice, however, nonlinear analysis models will not capture all energy absorption mechanisms, and some viscous damping will usually be needed to account for energy losses that are not modeled directly.

When we specify viscous damping in a nonlinear analysis model, we must be careful not to "double dip". In a nonlinear analysis model, viscous damping must be used to account only for energy losses that are not modeled directly in the nonlinear elements.

7.2.15 Viscous Damping Options

The options for viscous damping in linear analysis are as follows.

(1) Modal damping. This is the most commonly used option.
(2) Basic Rayleigh damping (the same α and β values for all masses and elements).
(3) Extended Rayleigh damping (different α and β values for different masses and elements).
(4) Explicit viscous damping elements.

For nonlinear analysis it is usually impractical to uncouple into normal modes, and hence modal damping is usually not an option. Basic Rayleigh damping is often used, but it has serious limitations. Extended Rayleigh damping is more flexible, but it is still just a modeling assumption.

7.2.16 Importance of the Behavior Concept

The behavior concept that we choose for a structure has a major influence on the analysis model, and on the results that we must obtain from the analysis. If the behavior concept is that all the components of the structure must remain essentially elastic, we can use strength based demand-capacity measures and linear structural analysis. If we decide that we can accept significant damage, we have chosen a very different concept. In this case we will typically use a mixture of strength based and damage based measures, and we may have to use nonlinear analysis.

7.2.17 The Importance of Using Capacity Design

There are many possible causes of nonlinear behavior, and with present day technology we can capture only relatively simple nonlinearities in an analysis model. We are being unrealistic if we believe we can perform a nonlinear analysis that accounts for all possible nonlinear effects, calculates accurate load-deflection relationships, and identifies the locations and types of significant structural damage. What we must do, instead, is *choose* the structural components and modes of behavior for which significant damage (i.e., inelastic behavior) is to be permitted, and design for essentially elastic behavior in all other components and modes. We must thus develop a clear concept of how we want the structure to behave, and we must apply capacity design methods. Only by doing this can we be sure that our analysis models, and hence our analysis results, are good enough to use in design.

There is another reason for using capacity design, which may be even more important than the need to get good enough analysis results. This is that a structure designed using capacity methods is more likely to be damage tolerant, and more likely to be forgiving of inaccuracies in the analysis and design calculations. If our analysis models and/or design loads are inaccurate for some reason, we may underestimate the damage demands, or calculate inaccurate distributions of these demands. Such inaccuracies are less likely to be fatal if capacity design is used. Although the amount and distribution of damage in the actual structure may be different from that shown by the calculations, it is less likely that the damage will lead to unexpected or undesirable modes of behavior. This is because when we control the inelastic components and modes of behavior, we can usually design them with generous safety factors against damage. We can even use capacity design without calculating damage demands, by detailing the inelastic components to provide damage capacity that is larger than any likely demand (that is, we can use a prescriptive approach). The biggest danger in capacity design is not under-estimation of the damage demands, but under-estimation of the strength demands for components and modes of behavior that are supposed to remain elastic. If this mistake is made there can be damage in components that have little damage capacity.

7.2.18 Alternatives to Nonlinear Dynamic Analysis

Nonlinear dynamic analysis may be necessary, but alternatives may be good enough for design purposes. Alternatives include static pushover analysis, dynamic secant stiffness analysis, and the Z-Factor (or R-Factor) method.

7.2.19 Weakness of the Z-Factor Method

The Z-Factor method of design is convenient because it uses conventional strength based design and linear analysis. However, because of damage concentration the ductility demands will generally be larger than the intended values.

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7.2.20 Steps for Setting up Nonlinear Analysis Models

Tentatively, the steps for setting up a nonlinear analysis model can be organized as follows.

(1) Identify the structural components (members and connections).
(2) For each component identify the significant modes of behavior.
(3) For each mode of behavior decide whether it is to be governed by (a) performance analysis or (b) prescriptive rules.
(4) For each mode of behavior that is governed by performance analysis, assign it to one of the following categories.
    (a) Zero Deformation.
    (b) Linear Elastic Deformation.
    (c) Nonlinear Elastic Deformation.
    (d) Inelastic Deformation.
(5) For each mode of behavior that is governed by prescriptive rules, assign it to one of the same four categories.
(6) Set up the model. Based on the assigned category, guidelines can be set up to assist with the modeling process.

7.2.21 Some General Rules

The following are some general rules.

Rule 1 : First define the behavior concept, then set up the analysis model. Before starting the journey, plan the route.

Rule 2 : For each different behavior concept, set up a different analysis model. It may be possible to set up a single model, and adapt it for specific concepts by turning different nonlinear features on or off. However, each specific model will be different.

Rule 3 : Model only the behavior that is permitted by the behavior concept. If a mode of behavior is not permitted by the behavior concept, it is a waste of time and effort to model it.

Rule 4 : Most modes of behavior should be elastic. To ensure reliable earthquake resistant behavior, the behavior concept must be simple. This means that inelastic behavior must usually be permitted in only one or two modes of behavior for any component.

Rule 5 : Use P-Δ analysis, not true large displacements. A P-Δ analysis is good enough for the majority of structures, and is usually simpler and more efficient computationally.

Rule 6 : The model does not have to be exact, just good enough. The goal is to obtain results that are good enough for making design decisions.

***Rule** 7* : Don't analyze, design. The purpose of the exercise is to make design decisions, not to perform structural analyses.

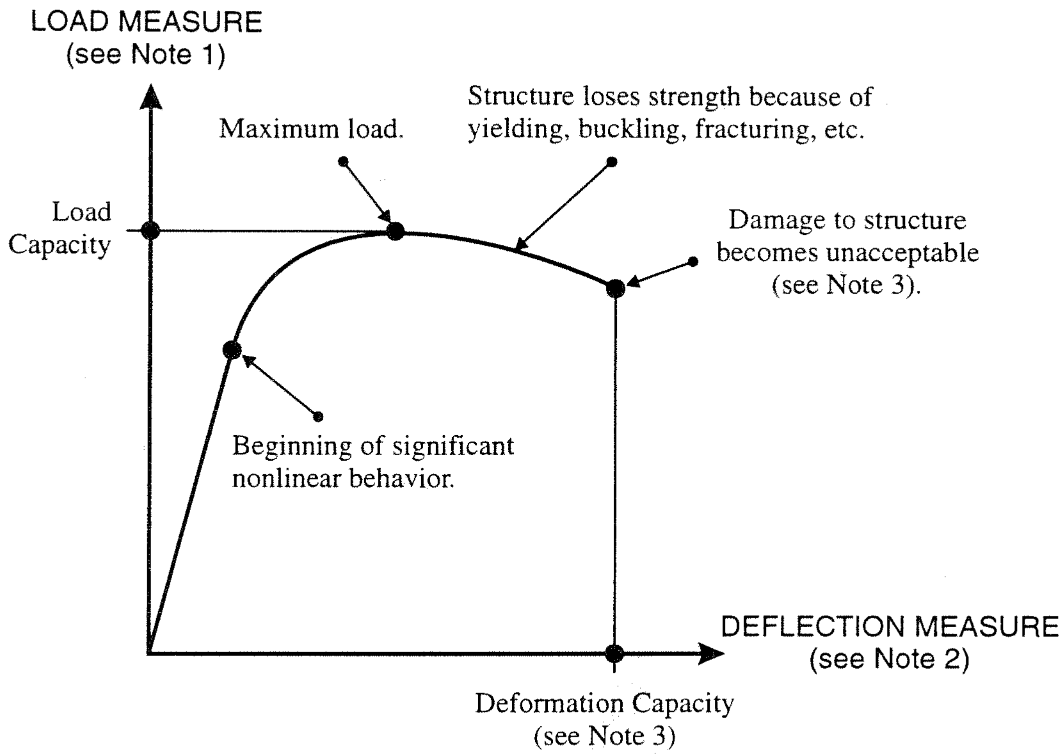
7.3 RECOMMENDATIONS

Nonlinear analysis is much more complex than linear analysis. One goal of this report is to identify the many philosophical, theoretical, computational and practical issues must be resolved before nonlinear analysis can be used routinely in design. When we use nonlinear analysis at the present time, we tend to place far too much responsibility on the individual engineer. Often they are younger engineers, who have little experience yet are expected to make complex modeling and analysis decisions. We have decades of experience in strength based design, and over those decades we have developed fairly well defined analysis methods and guidelines for their use. Damage based design is inherently more complex, and we do not yet have decades of experience. Before we can apply damage based design in a rational way, we must develop the methods and guidelines. This is, of course, being done, but in the author's opinion to only a limited extent, and with an insufficient commitment of resources.

It is recommended that detailed guidelines be developed on topics such as the following.

(1) Selection of behavior concepts for damage based design.
(2) Selection of damage-based limit states and demand-capacity measures for different structures and structural components.
(3) Definition of analysis models for demand calculation.
(4) Viscous damping for nonlinear analysis.
(5) Selection of design ground motions for use in nonlinear analysis.
(6) Use of nonlinear dynamic analysis.
(7) Use of alternatives to dynamic analysis, such as static push-over, secant stiffness, and Z-Factor methods.
(8) Processing of analysis results to obtain demand values.
(9) Calculation of capacity values.
(10) Decision making based on demand-capacity comparisons.
(11) Application of capacity design methods.

These are complex tasks that require a great of work, covering both theory and practice, and requiring close collaboration between analysts and designers.



Note 1. Examples of load measures are (a) total load and (b) load factor (multiple of design load). Load factor is dimensionless, and is the best type of measure. For earthquake resistant design we will usually be concerned with the lateral load factor, when the gravity load factor is held constant.

Note 2. Examples of deflection measures are (a) lateral deflection and (b) drift ratio (lateral deflection divided by structure height). Drift ratio is dimensionless, and is the best type of measure.

Note 3. The deformation capacity is not used in simple strength based design, since the behavior concept is that the load never exceeds the load capacity. The deformation capacity is, however, of fundamental importance in damage based design. Although load capacity is well defined (at least in this simple example), it may not be easy to define what is meant by "unacceptable damage".

FIGURE 3.1 LOAD-DEFLECTION RELATIONSHIP FOR A STRUCTURE

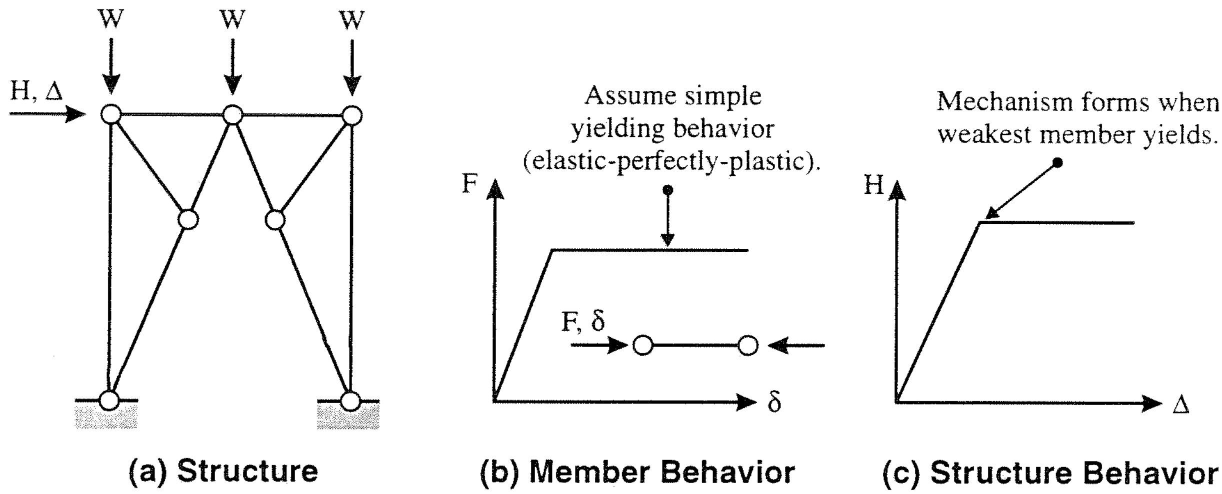


FIGURE 3.2 SIMPLE TRUSS EXAMPLE (STATICALLY DETERMINATE)

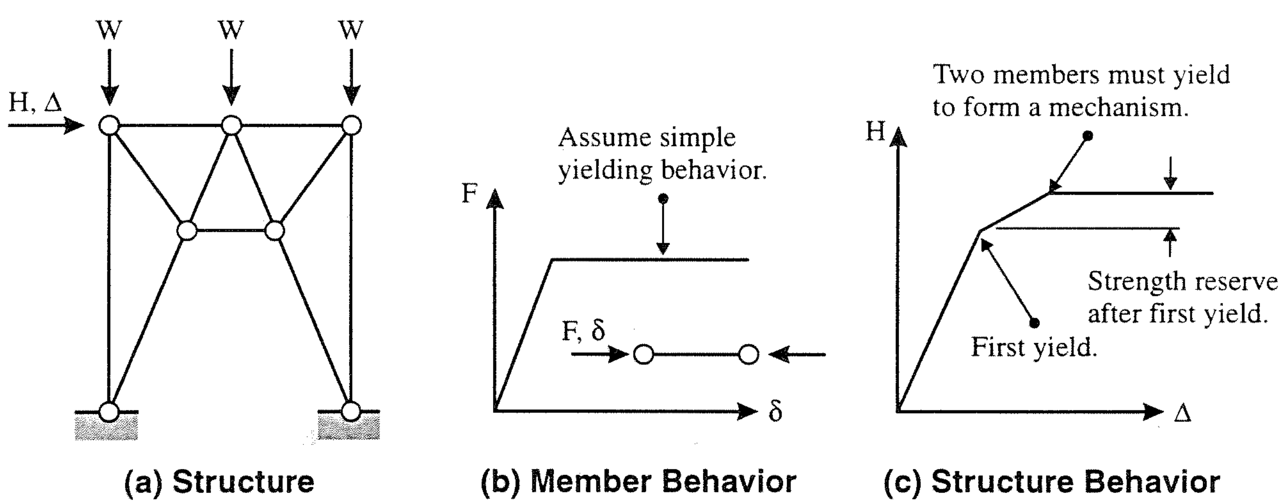


FIGURE 3.3 SIMPLE TRUSS EXAMPLE (STATICALLY INDETERMINATE)

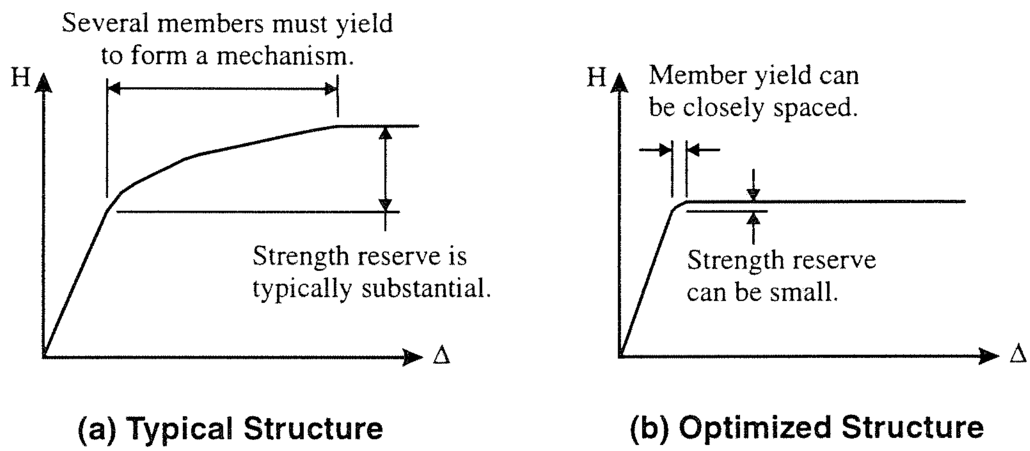


FIGURE 3.4 BEHAVIOR IF STRUCTURE IS MORE HIGHLY INDETERMINATE

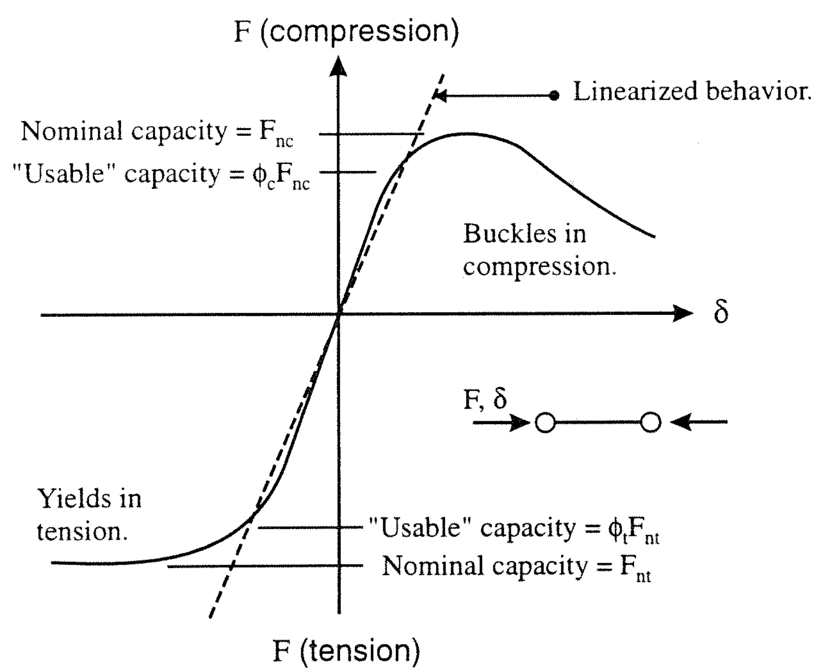


FIGURE 3.5 BEHAVIOR OF AN ACTUAL TRUSS BAR

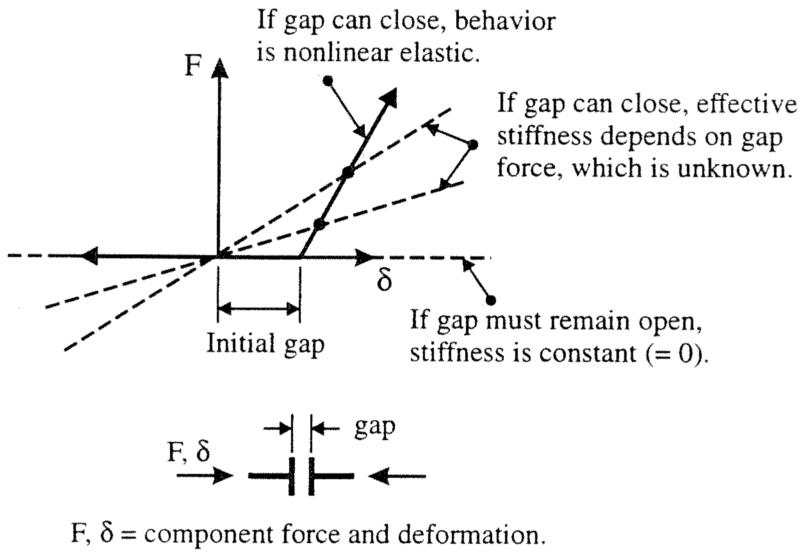


FIGURE 3.6 GAP COMPONENT (E.G., EXPANSION JOINT)

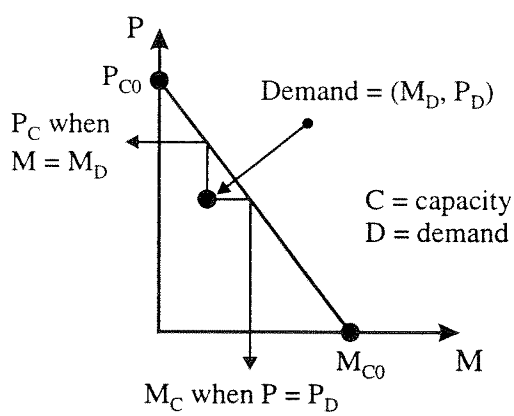


FIGURE 3.7 SIMPLE INTERACTION SURFACE

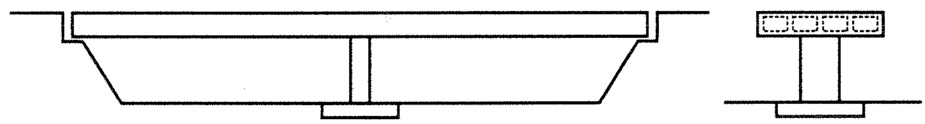


FIGURE 4.1 SIMPLIFIED BRIDGE EXAMPLE

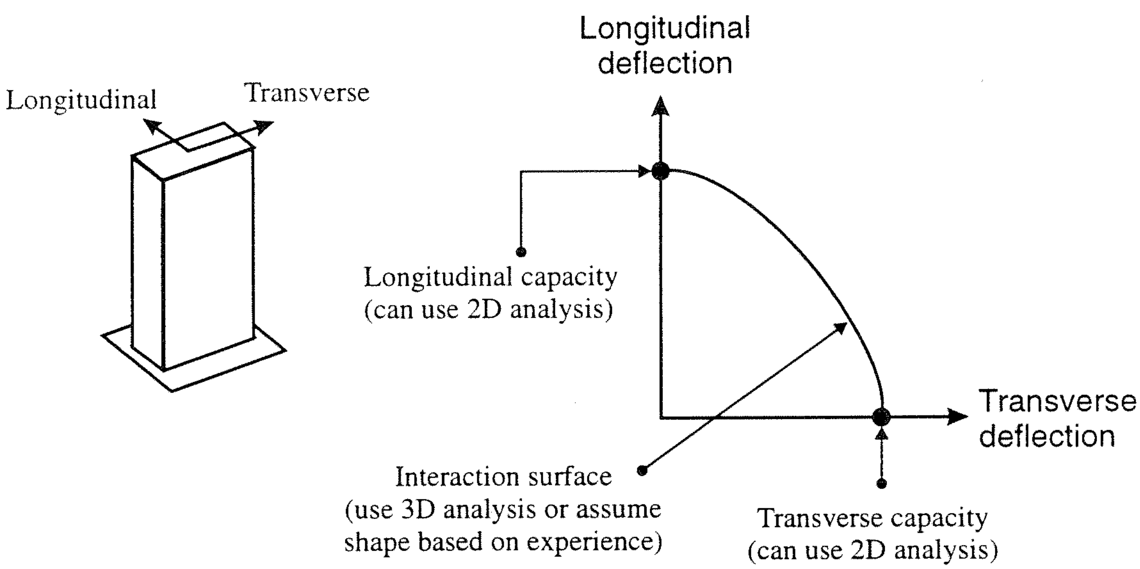


FIGURE 4.2 COLUMN DEFLECTION CAPACITY

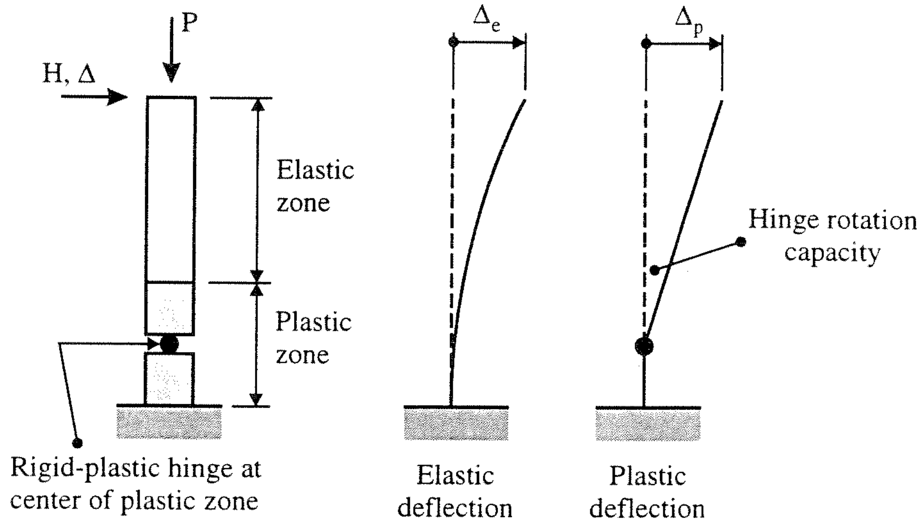


FIGURE 4.3 COLUMN MODEL FOR DEFLECTION CAPACITY

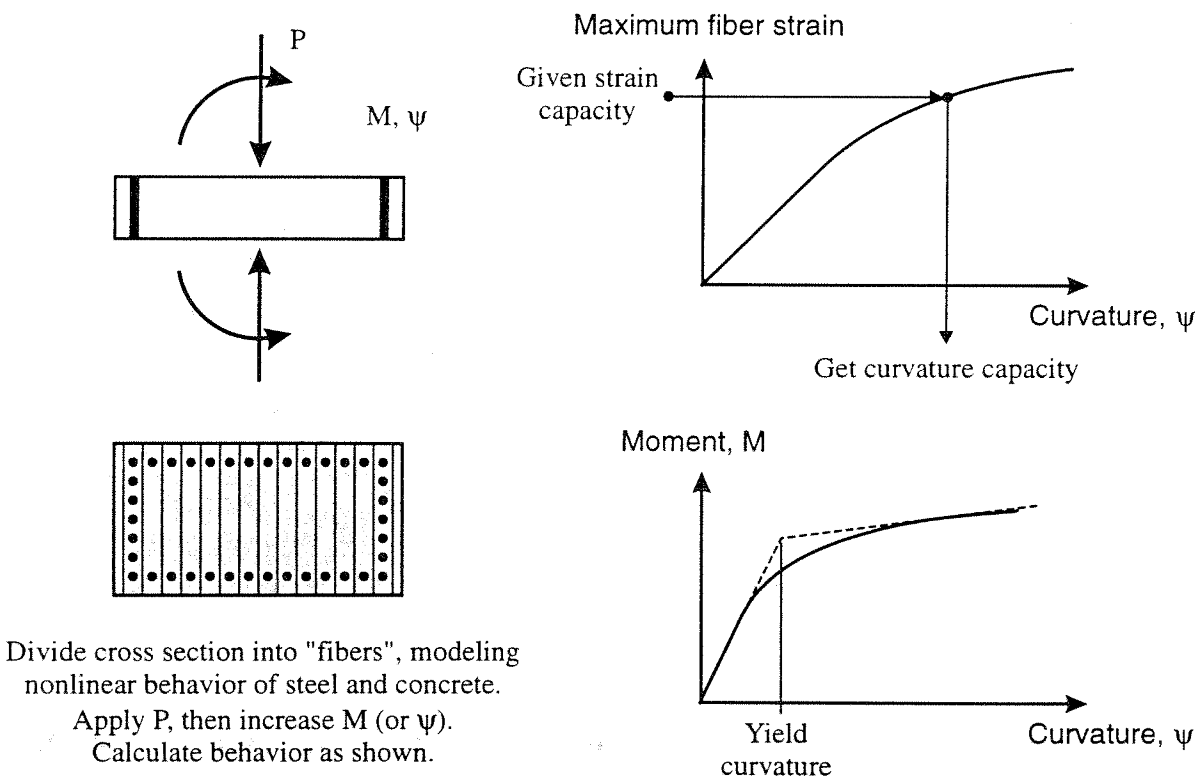


FIGURE 4.4 CROSS SECTION MODEL FOR CURVATURE CAPACITY

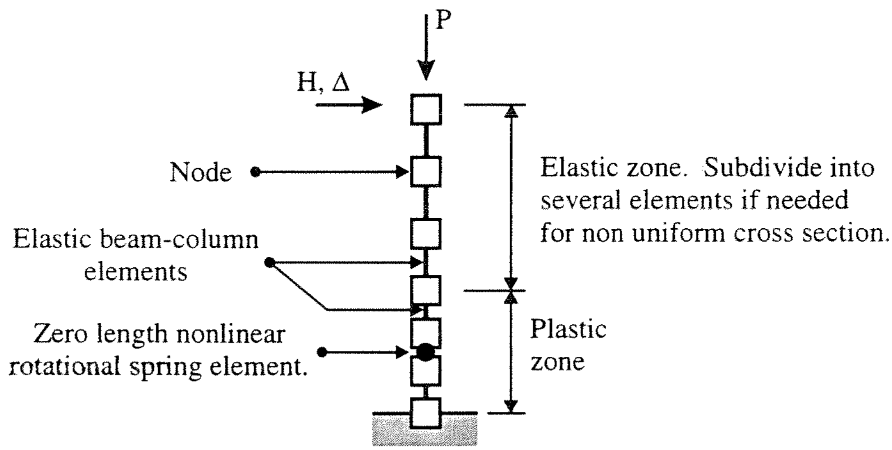
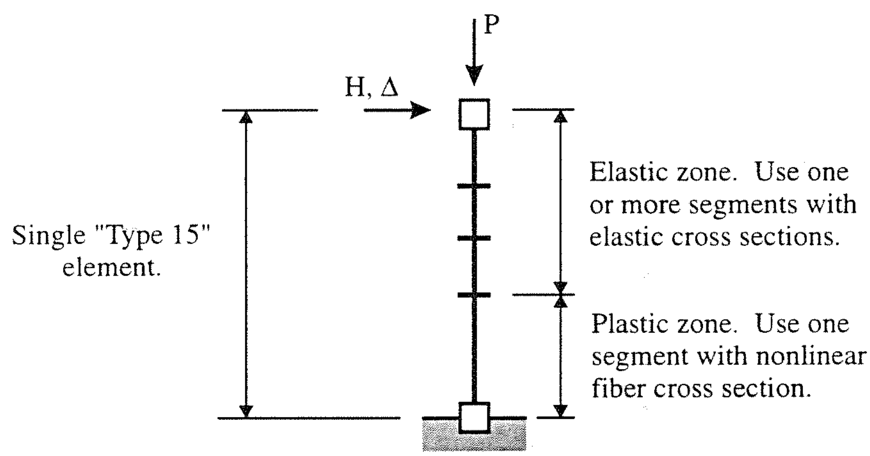
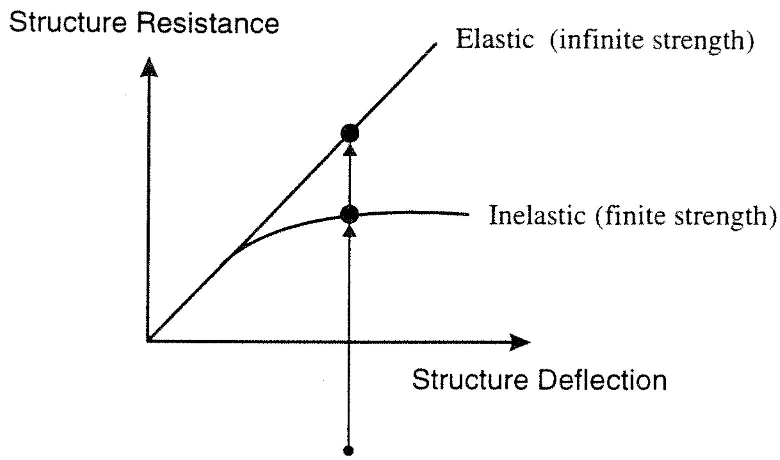


FIGURE 4.5 MODEL FOR NONLINEAR COMPUTER ANALYSIS



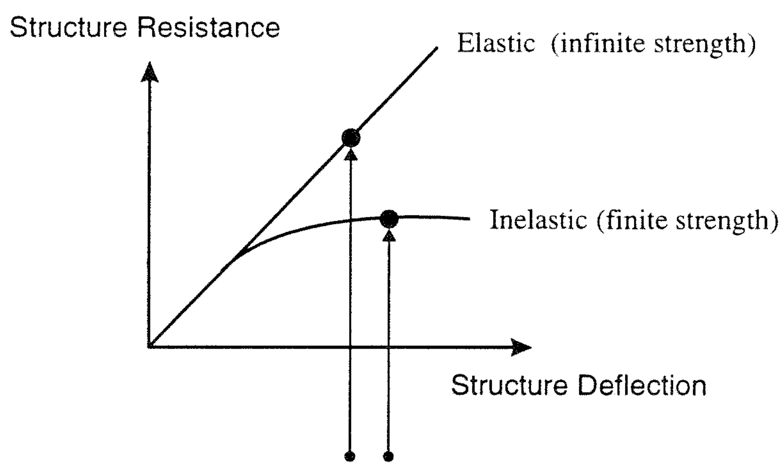
Deflection capacity is reached when maximum steel or concrete fiber strain in nonlinear segment reaches strain capacity.

FIGURE 4.6 DRAIN-2DX MODEL FOR COMPLETE CAPACITY ANALYSIS



For any given earthquake, maximum deflections are same for elastic and inelastic structures.

(a) Equal Displacements



For any given earthquake, areas under curves are same for elastic and inelastic structures.

(b) Equal Energy

FIGURE 4.7 EQUAL DISPLACEMENTS AND EQUAL ENERGY ASSUMPTIONS

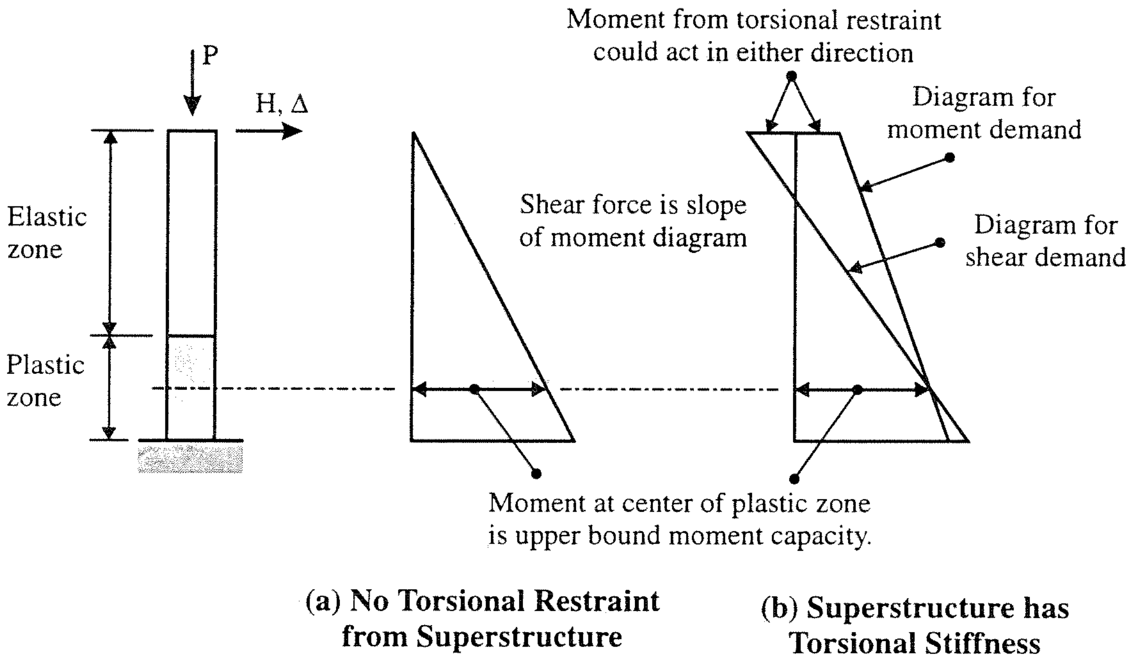


FIGURE 4.8 BENDING MOMENT DIAGRAMS FOR CALCULATING COLUMN SHEAR DEMAND AND ELASTIC ZONE MOMENT DEMAND

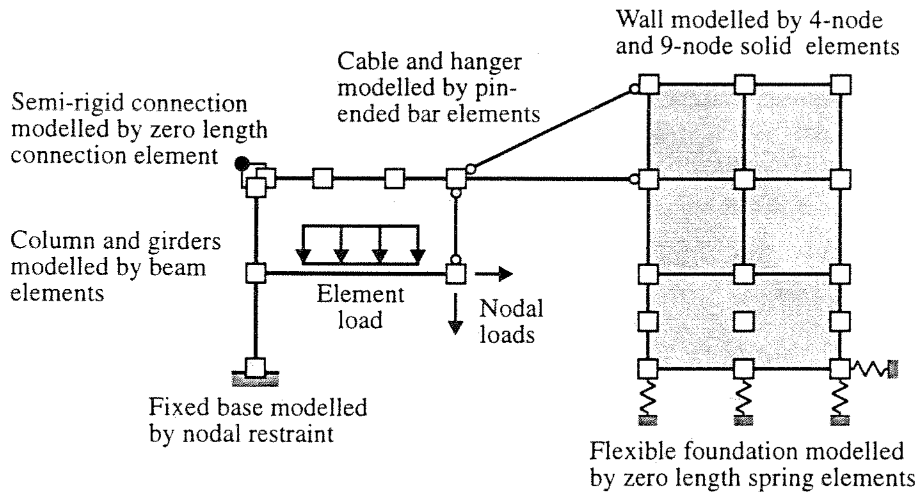
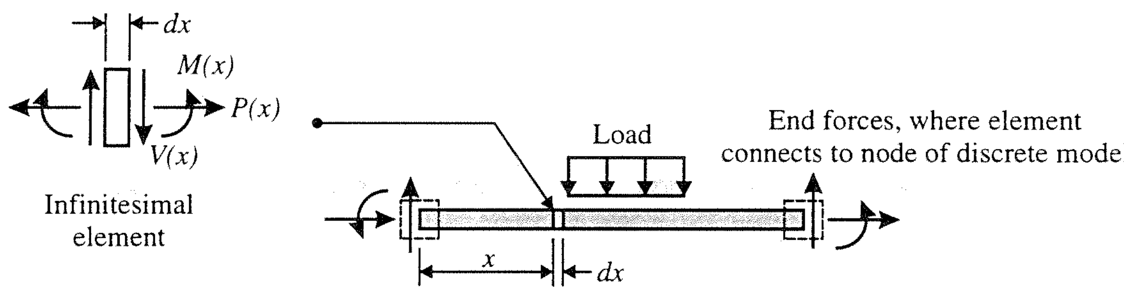
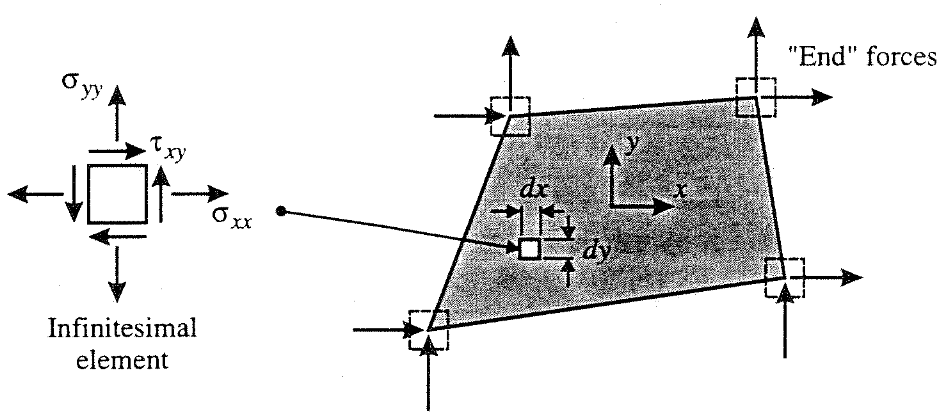


FIGURE 5.1 INGREDIENTS OF A DISCRETE (FINITE ELEMENT) MODEL

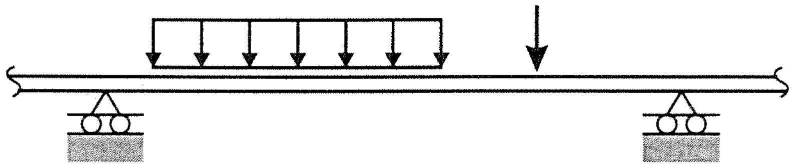


(a) Beam Element

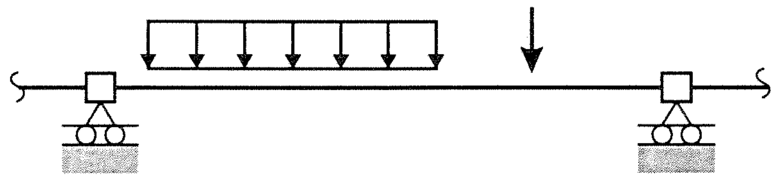


(b) 2D Solid Finite Element

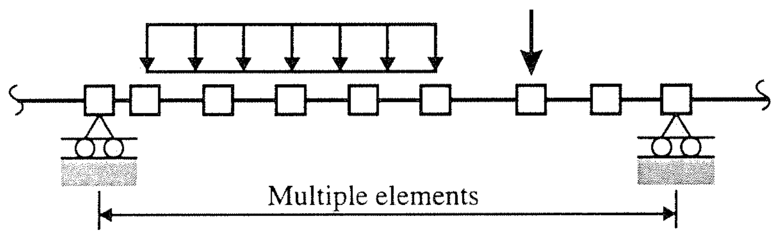
FIGURE 5.2 INGREDIENTS OF A CONTINUUM MODEL



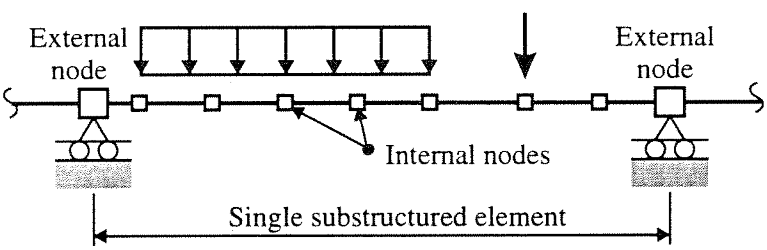
(a) Span of a continuous beam



(b) It may be inaccurate to model complete span with a single element.

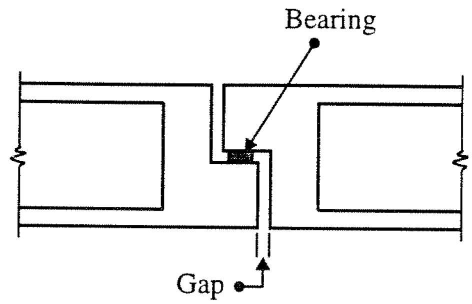


(c) We can add nodes (refine mesh), but this can be inconvenient.

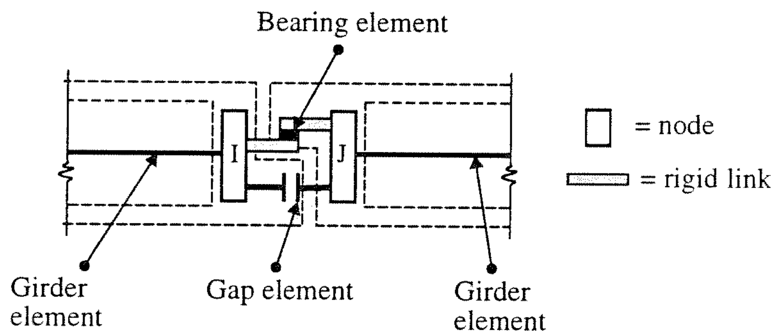


(d) A single substructured element can be more convenient.

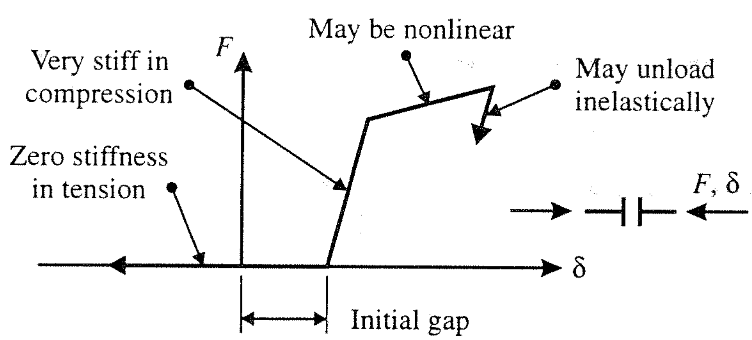
FIGURE 5.3 SUBSTRUCTURED BEAM ELEMENT



(a) Simple Expansion Joint

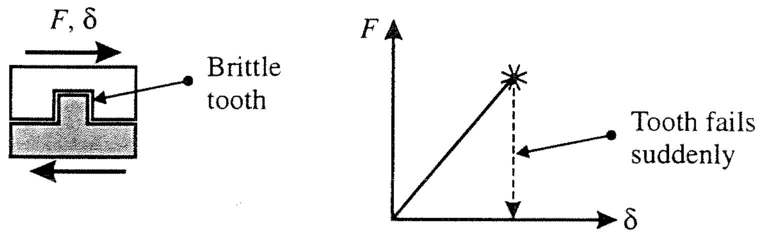


(b) Joint Model

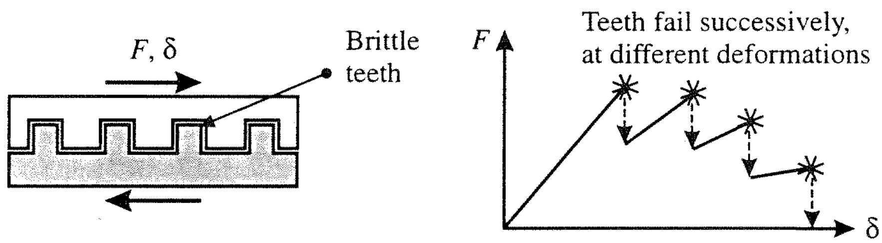


(c) Properties of Gap Element

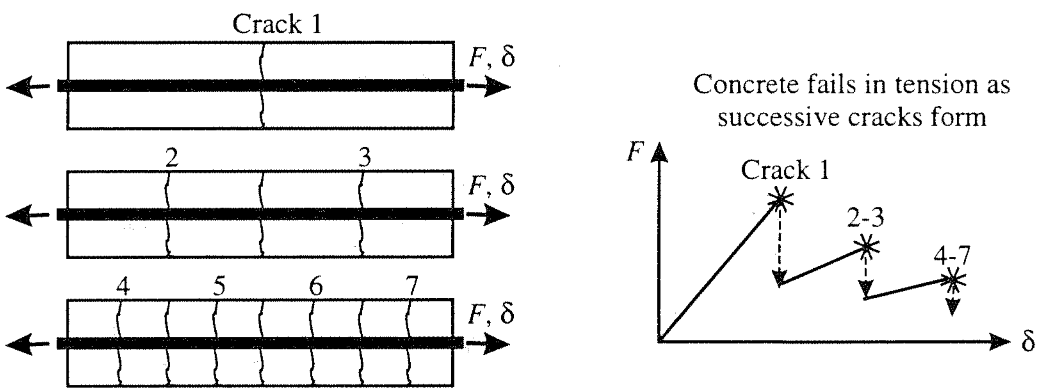
FIGURE 5.4 GAP OPENING AND CLOSING



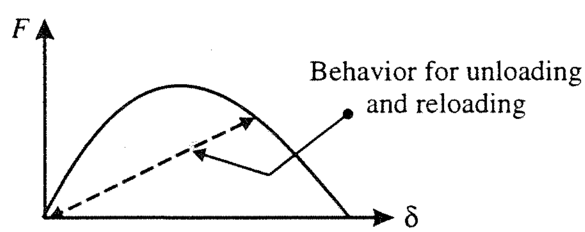
(a) Single Brittle Tooth



(b) Series of Brittle Teeth

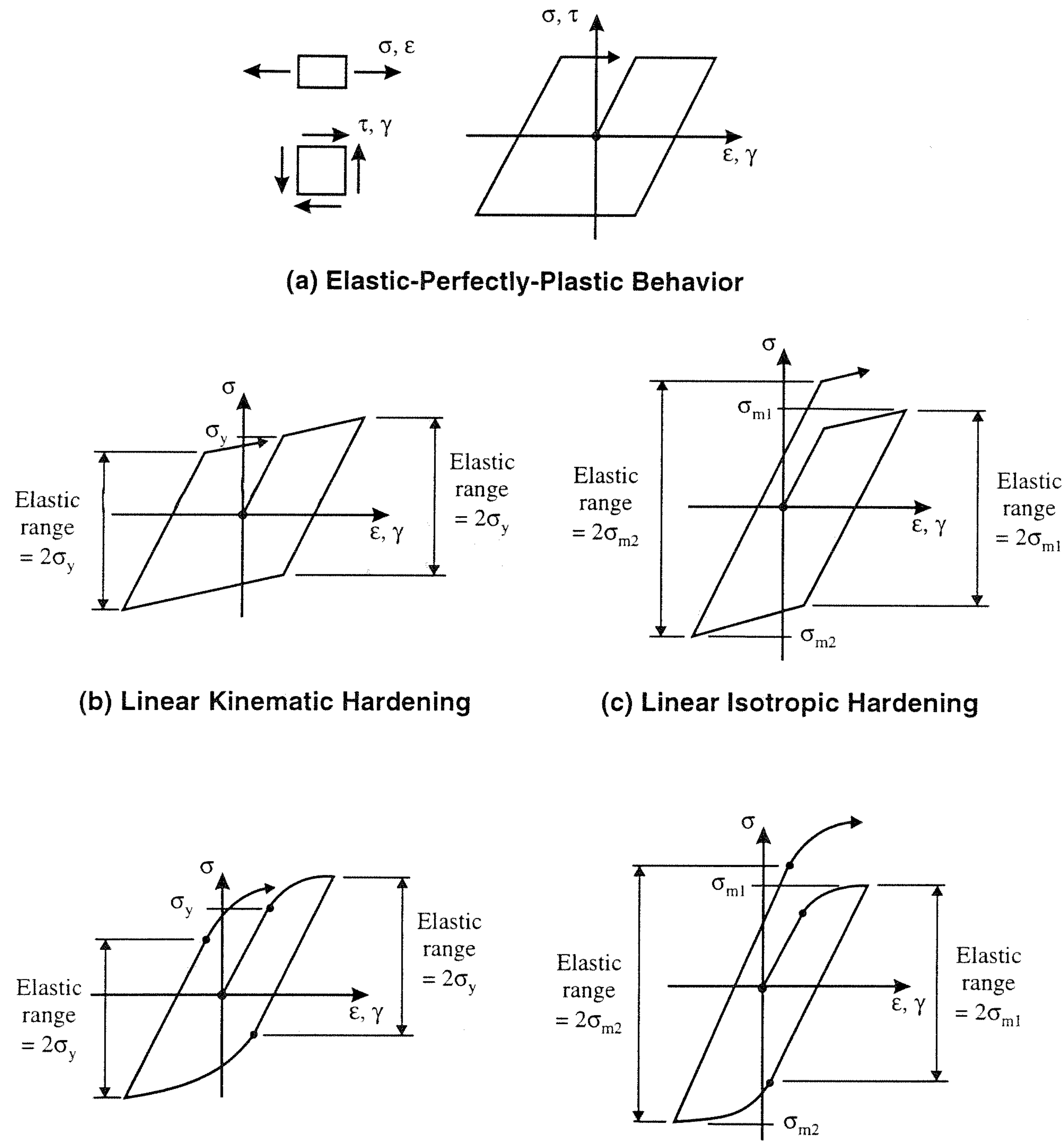


(c) Reinforced Concrete in Tension



(d) Smoothed Action-Deformation Relationship

FIGURE 5.5 BRITTLE FRACTURE



(a) Elastic-Perfectly-Plastic Behavior

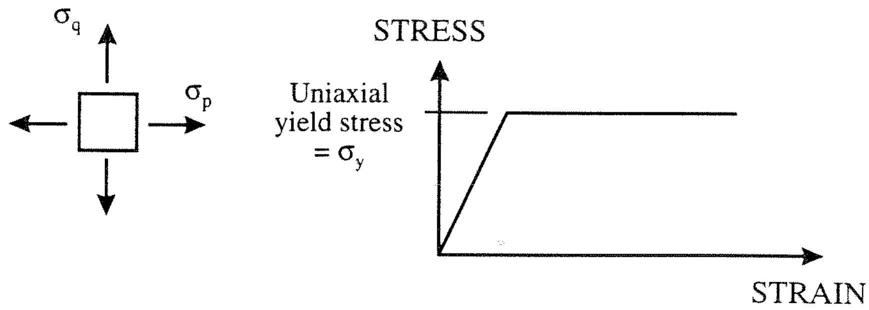
(b) Linear Kinematic Hardening

(c) Linear Isotropic Hardening

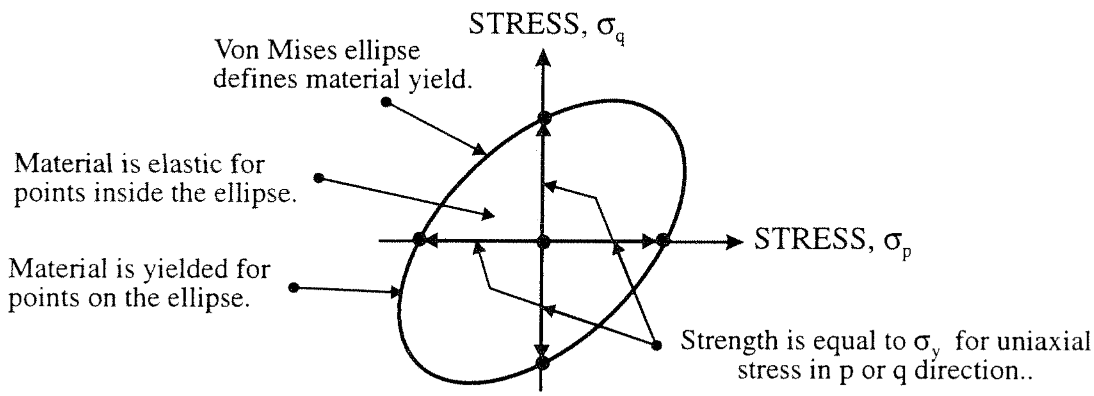
(d) Nonlinear Kinematic Hardening

(e) Nonlinear Isotropic Hardening

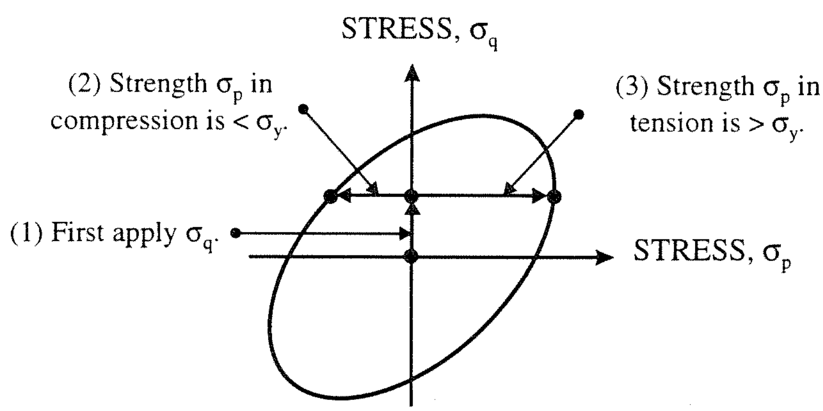
FIGURE 5.6 PLASTIC BEHAVIOR



(a) Biaxial Stress with Elastic-Perfectly-Plastic Behavior



(b) Interaction by Von Mises Theory



(c) Effect of Stress σ_q on Strength σ_p

FIGURE 5.7 EFFECT OF STRESS INTERACTION ON STRENGTH OF STEEL

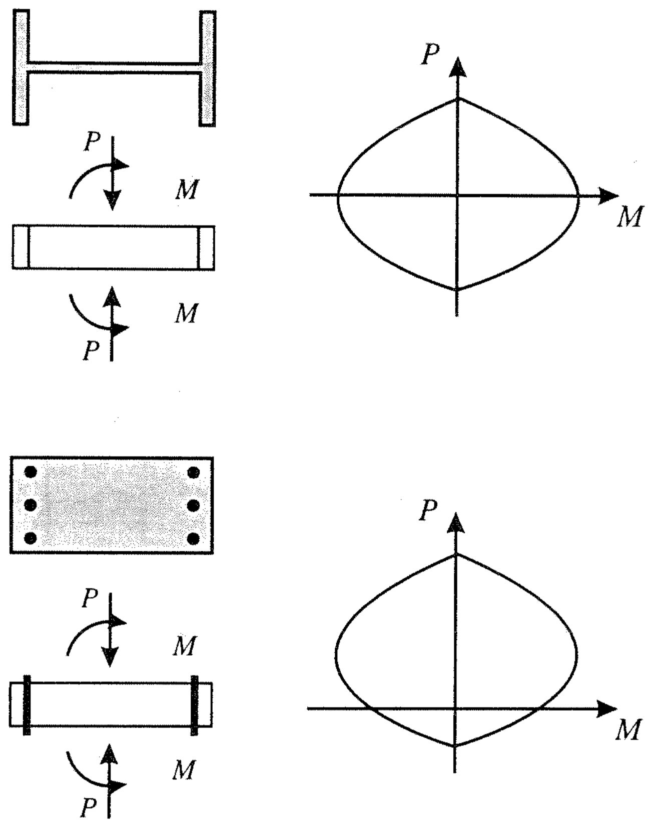


FIGURE 5.8 P-M INTERACTION FOR STEEL AND CONCRETE COLUMNS

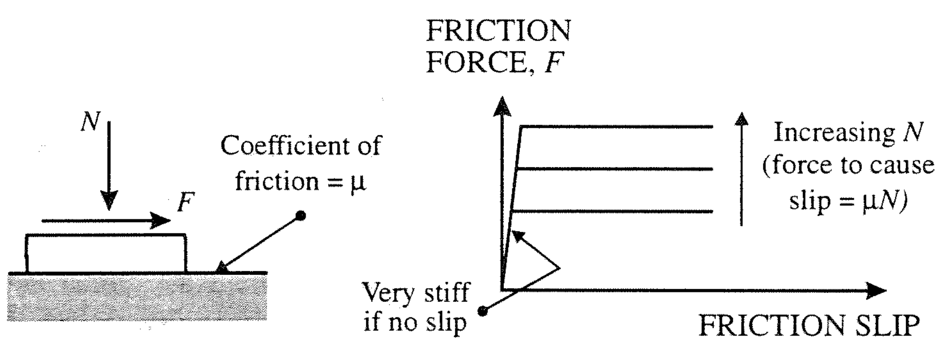
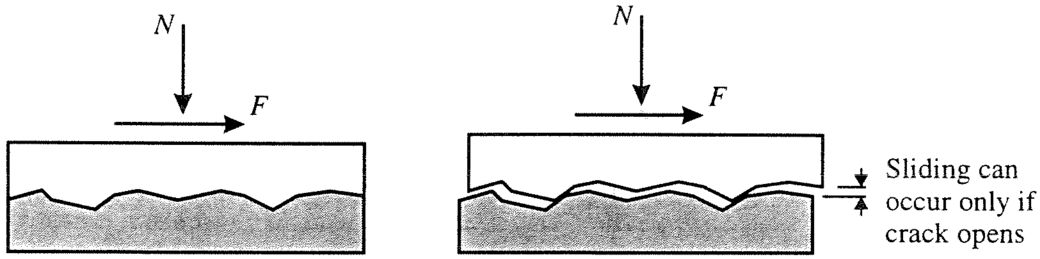
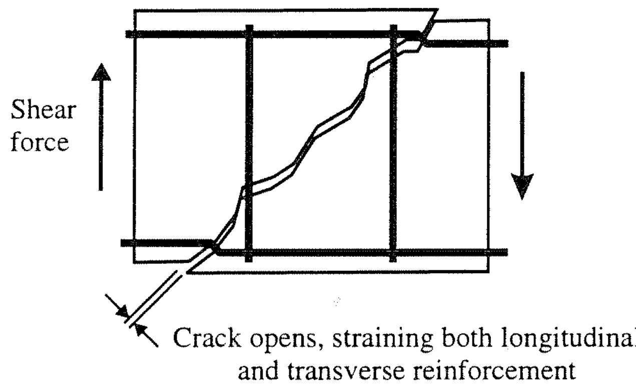


FIGURE 5.9 FRICTIONAL SLIP



(a) Frictional Slip on a Rough Surface



(b) Deformation on a Diagonal Shear Crack

FIGURE 5.10 FRICTIONAL SLIP ON ROUGH SURFACES

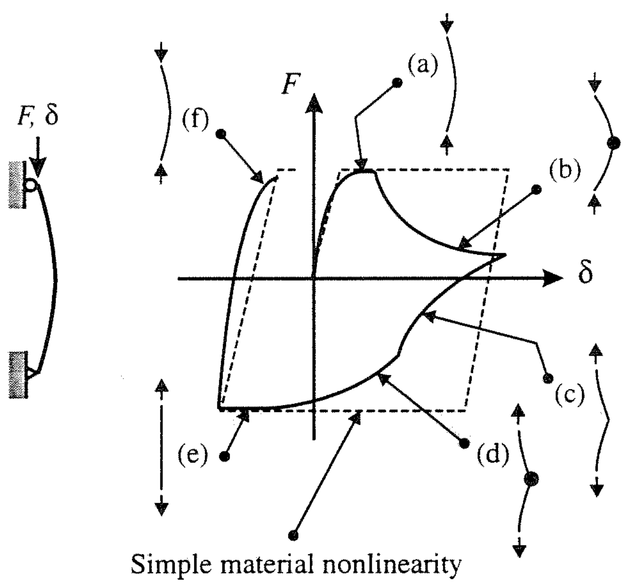
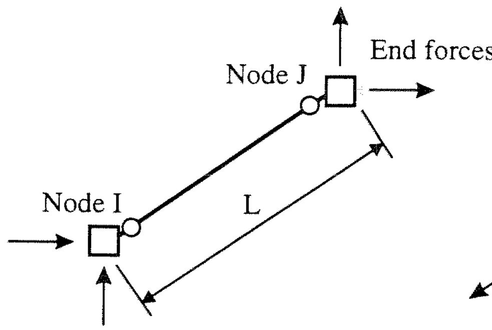
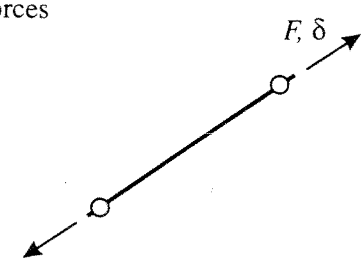


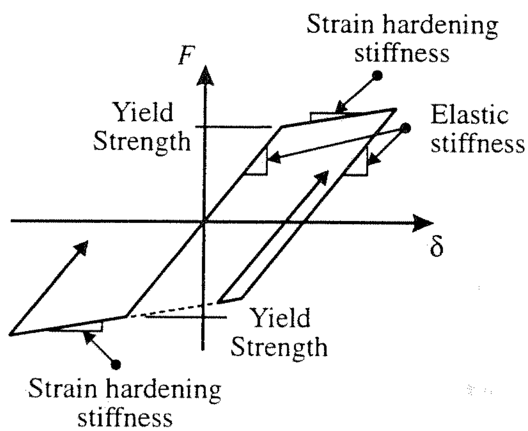
FIGURE 5.11 BUCKLING STRUT BEHAVIOR (COMBINED GEOMETRIC AND MATERIAL NONLINEARITY)



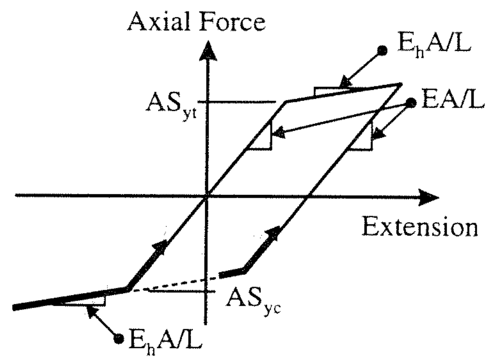
(a) Element and Nodal Forces



(b) Action and Deformation

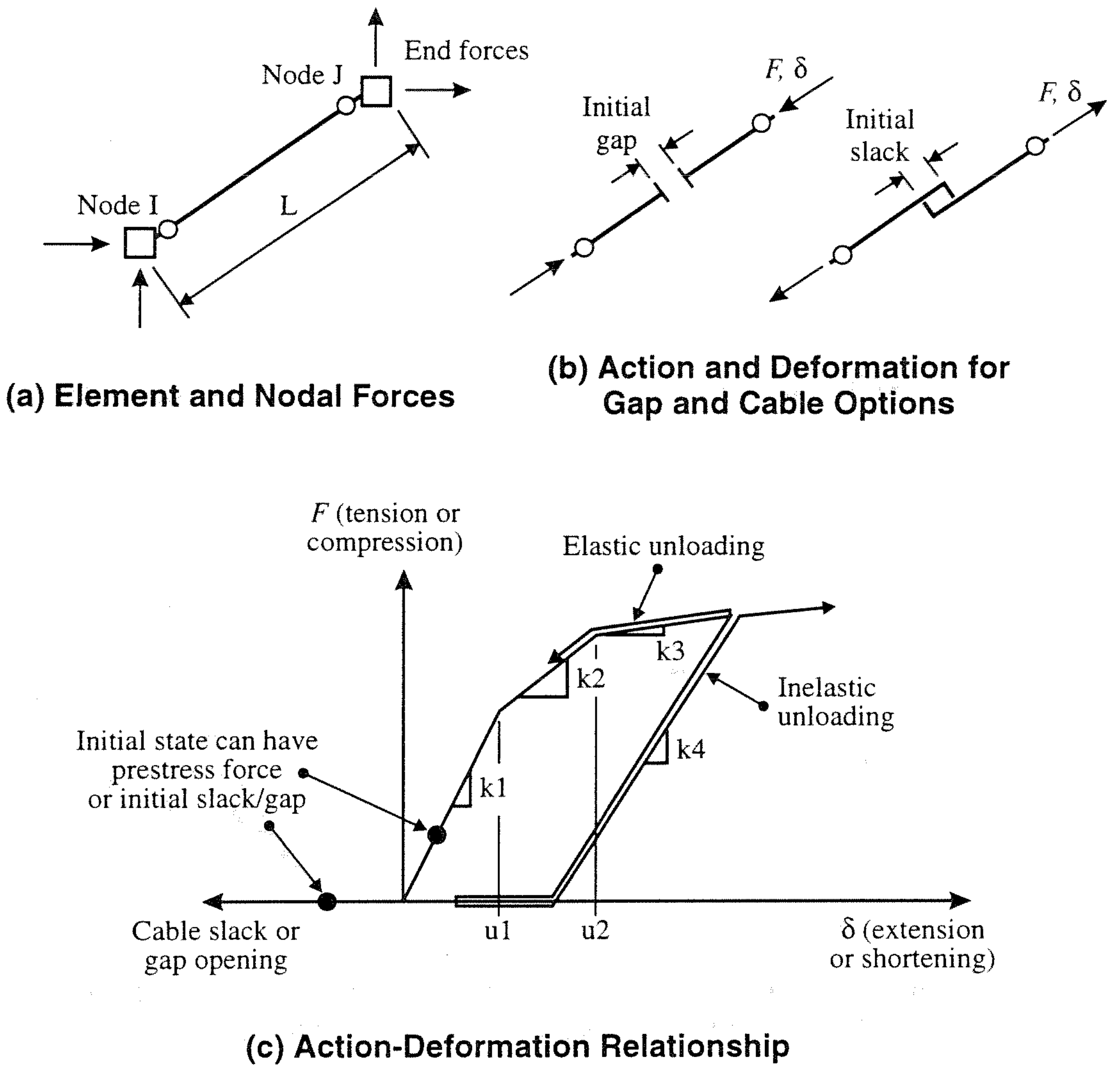


(c) Action-Deformation Relationship for Yielding Option in Compression



(c) Action-Deformation Relationship for Buckling Option in Compression

FIGURE 5.12 SIMPLE TRUSS BAR ELEMENT (DRAIN ELEMENT TYPE 01)



(a) Element and Nodal Forces

(b) Action and Deformation for Gap and Cable Options

(c) Action-Deformation Relationship

FIGURE 5.13 GAP/CABLE ELEMENT (DRAIN ELEMENT TYPE 09)

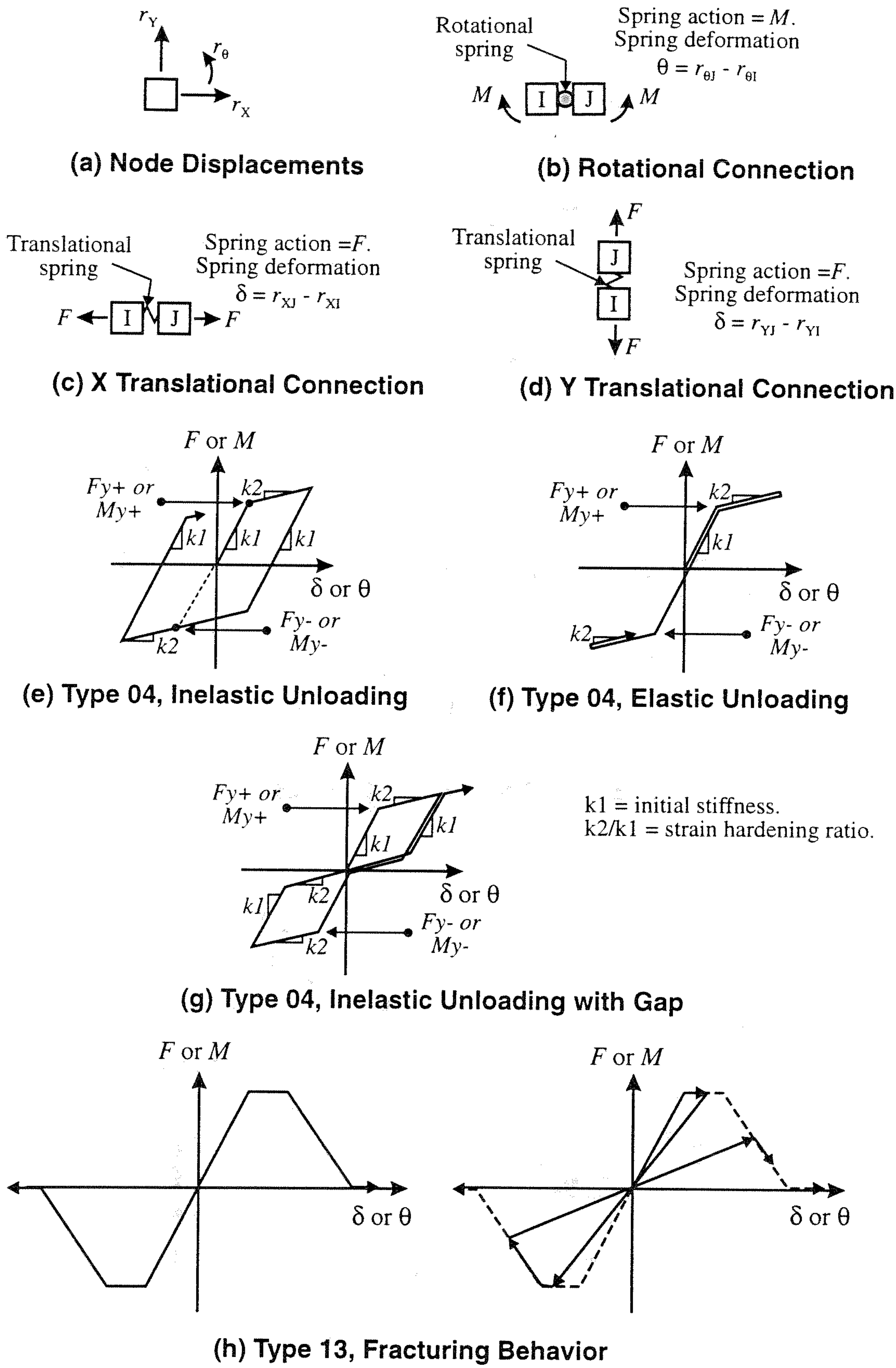
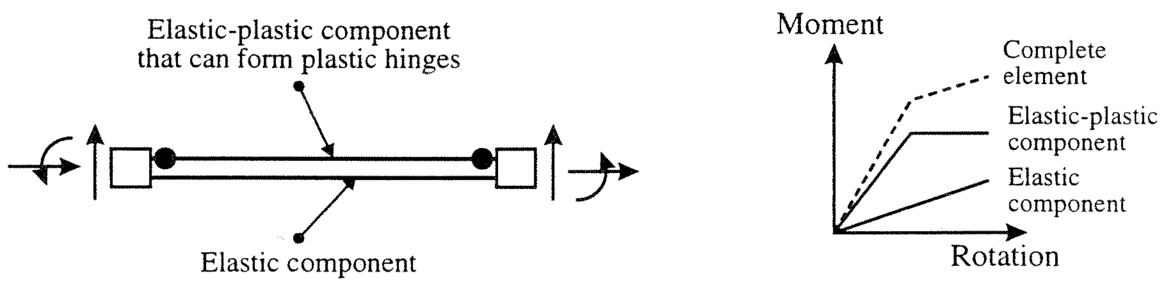
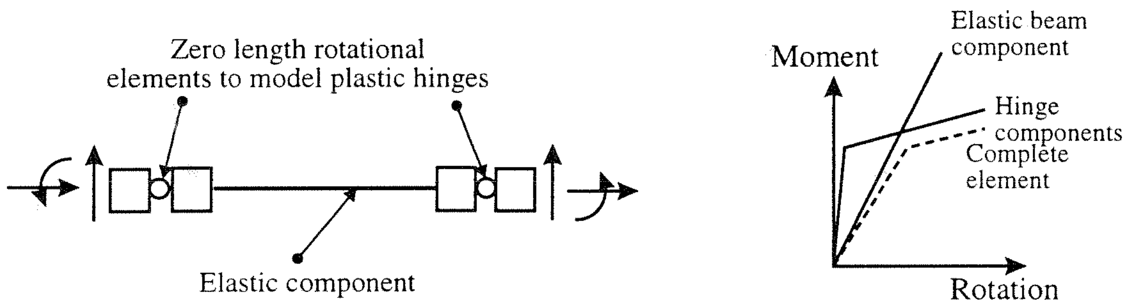


FIGURE 5.14 ZERO LENGTH CONNECTION ELEMENTS
(DRAIN ELEMENT TYPES 04 AND 14)



(a) Parallel Model (DRAIN-2DX Element Type 02)



(b) Series Model

FIGURE 5.15 SIMPLE MODELS FOR BEAM ELEMENTS

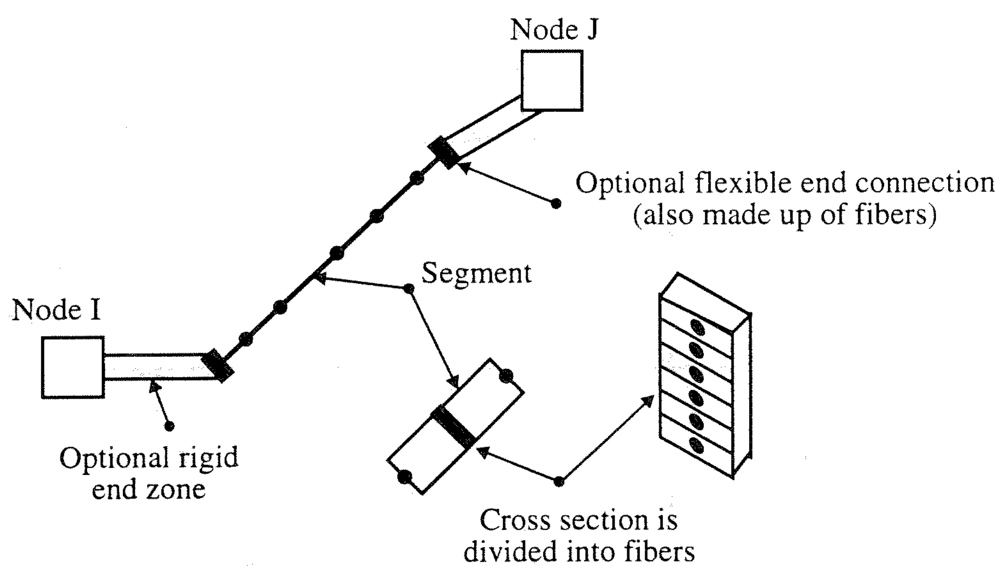
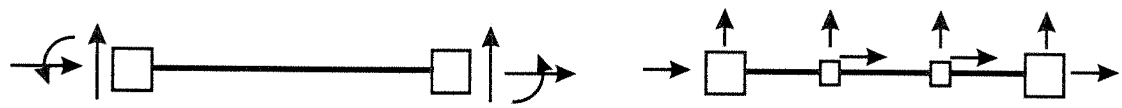


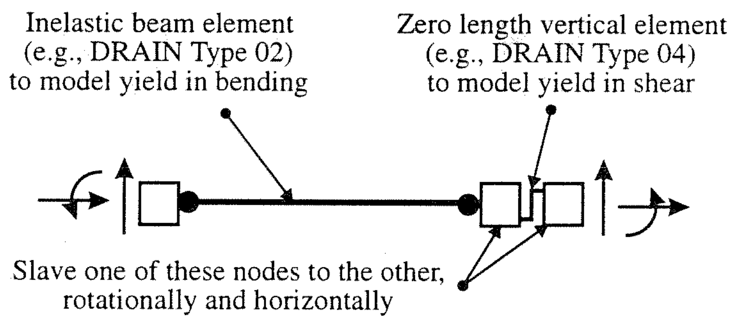
FIGURE 5.16 SOME FEATURES OF DRAIN ELEMENT TYPE 15



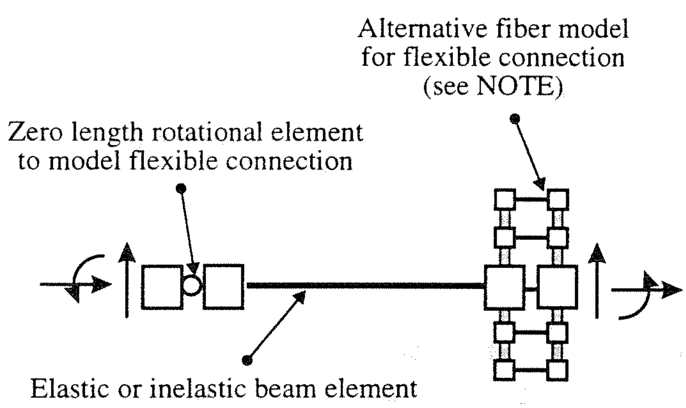
(a) Two-Node Model

(b) Four-Node Model

FIGURE 5.17 FINITE ELEMENT BEAM MODELS



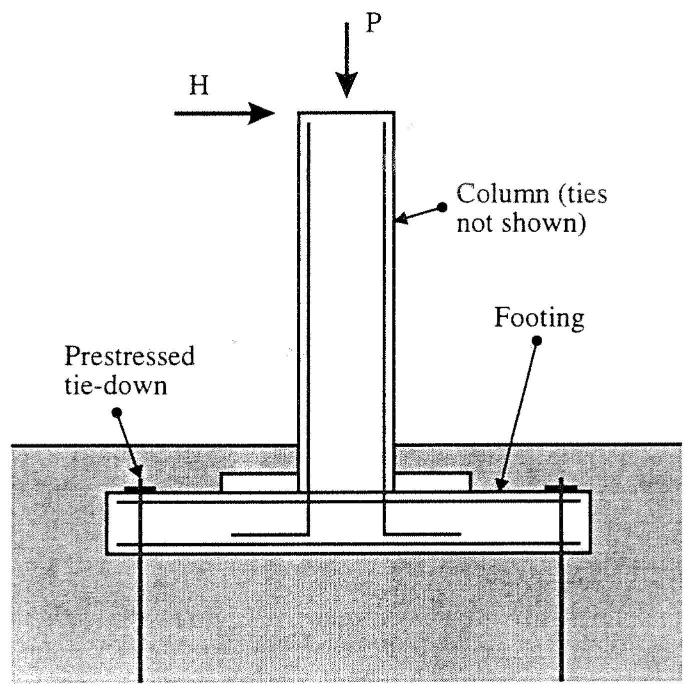
(a) Beam with Both Flexural and Shear Yield



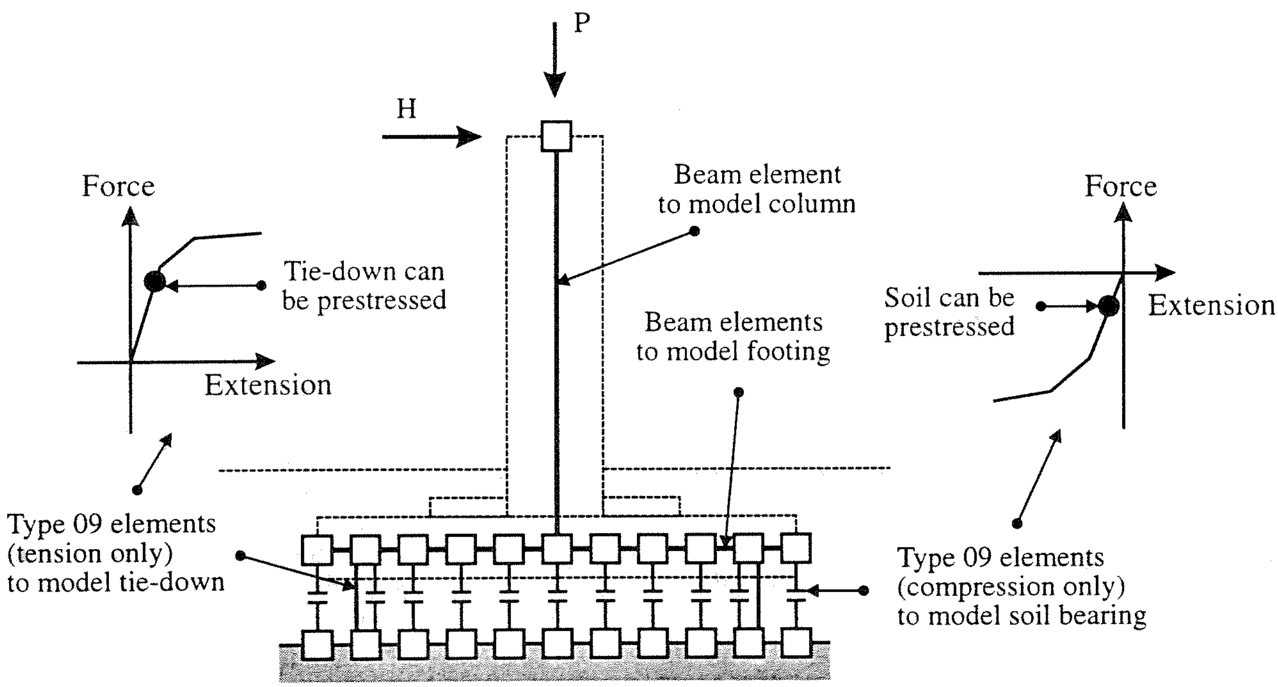
NOTE:
(1) Nodes on beam axis are main nodes.
(2) Position secondary nodes at fiber heights.
(3) Use rigid link slaving to connect secondary nodes to main nodes. This creates two plane sections.
(4) Connect secondary nodes (and main nodes if desired) with elements that model fibers. These can be truss bar elements (Type 01), zero length connection elements (Type 04) and/or gap/cable elements (Type 09).
(5) Connect the two main nodes with a stiff vertical element to provide shear resistance (otherwise model is unstable).

(b) Beam with Flexible End Connections

FIGURE 5.18 MODELS FOR BEAM ELEMENTS



(a) Diagrammatic View of Structure



(b) Possible DRAIN-2DX Model

FIGURE 5.19 MODEL OF COLUMN WITH UPLIFTING FOUNDATION

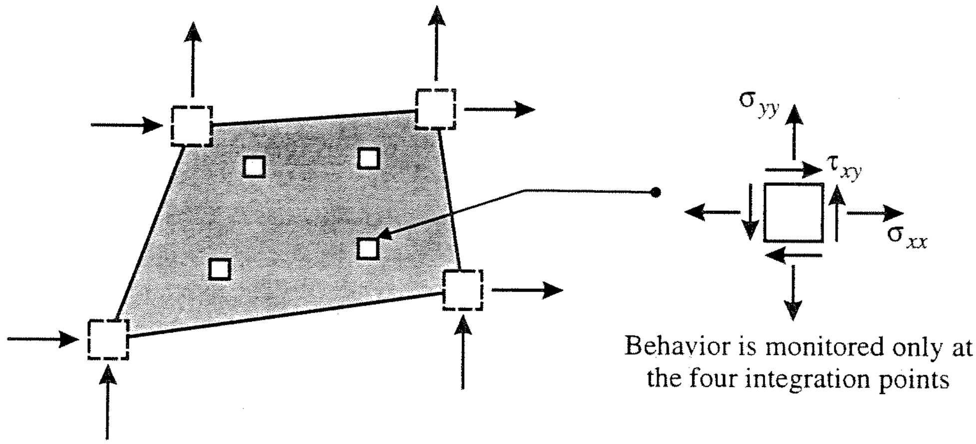


FIGURE 5.20 SOLID FINITE ELEMENT

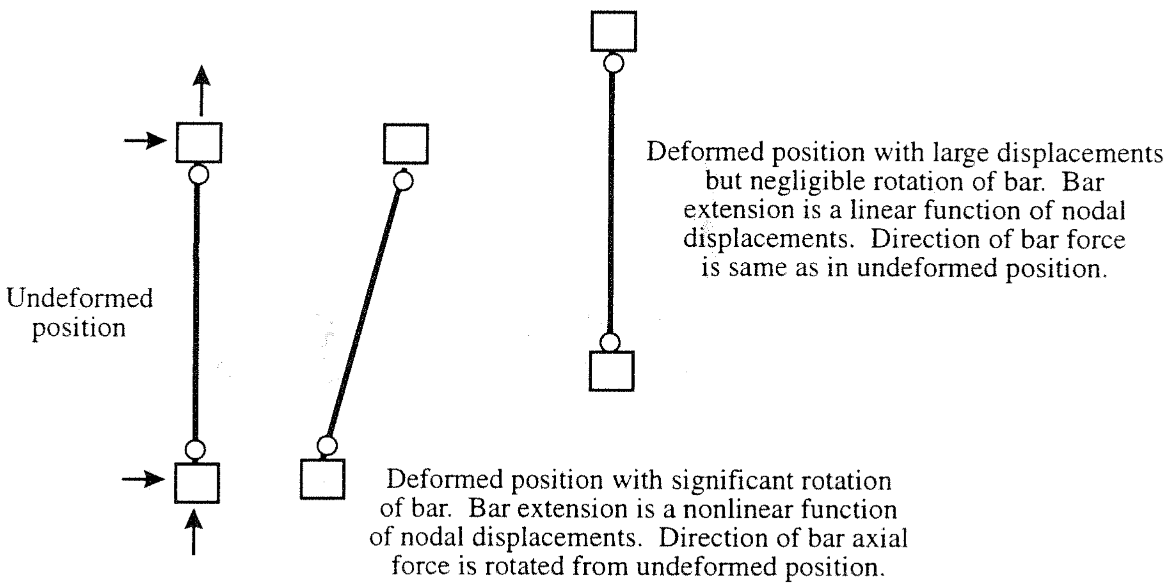


FIGURE 5.21 LARGE DISPLACEMENTS OF A TRUSS BAR ELEMENT

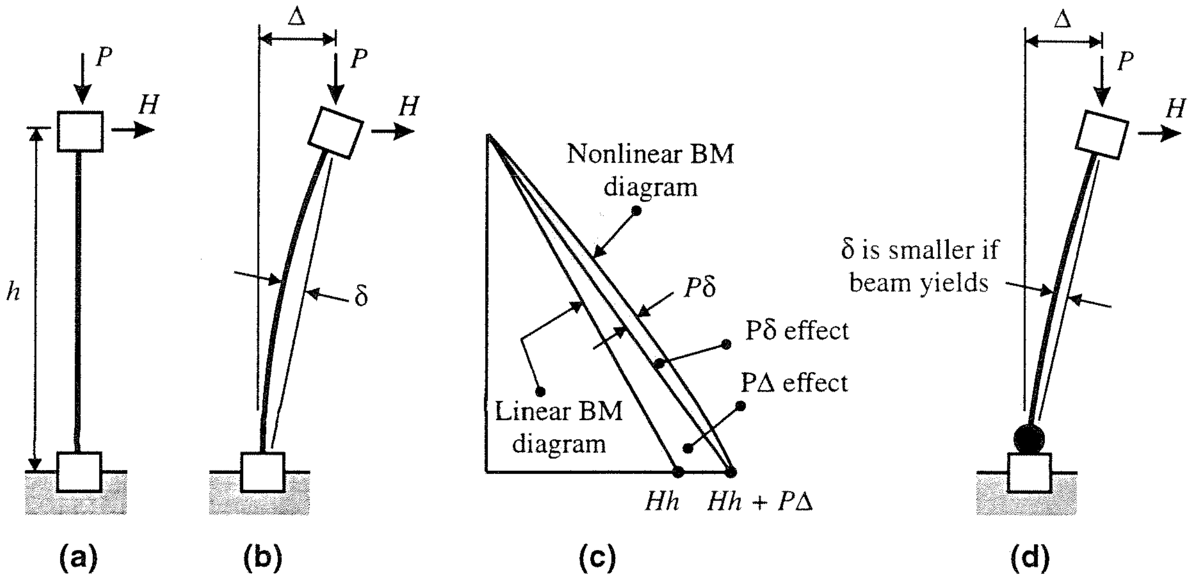


FIGURE 5.22 P-Δ AND P-δ EFFECTS

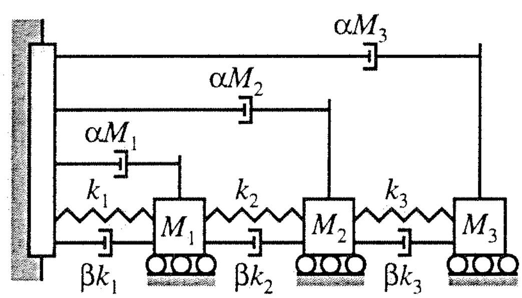


FIGURE 5.23 PHYSICAL MEANING OF RAYLEIGH DAMPING

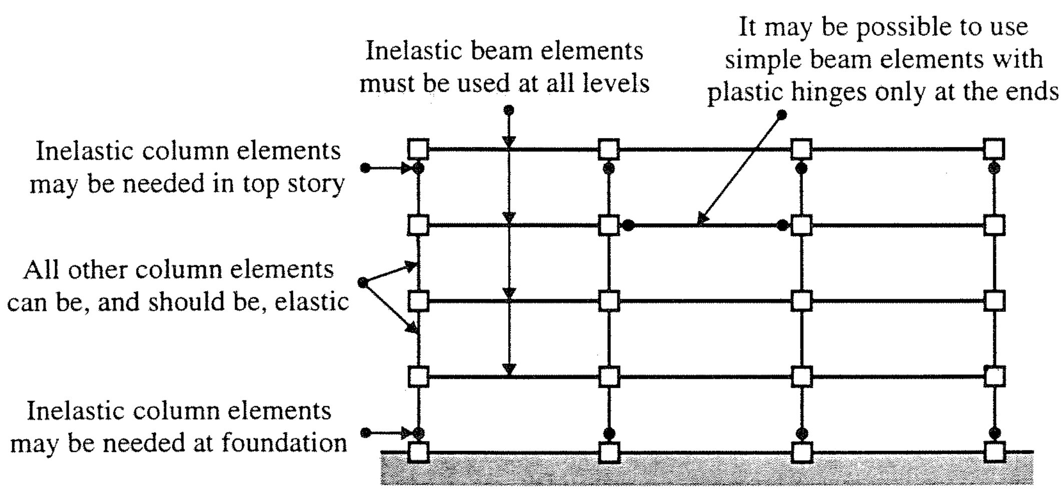


FIGURE 5.24 STRONG COLUMN, WEAK BEAM FRAME MODEL

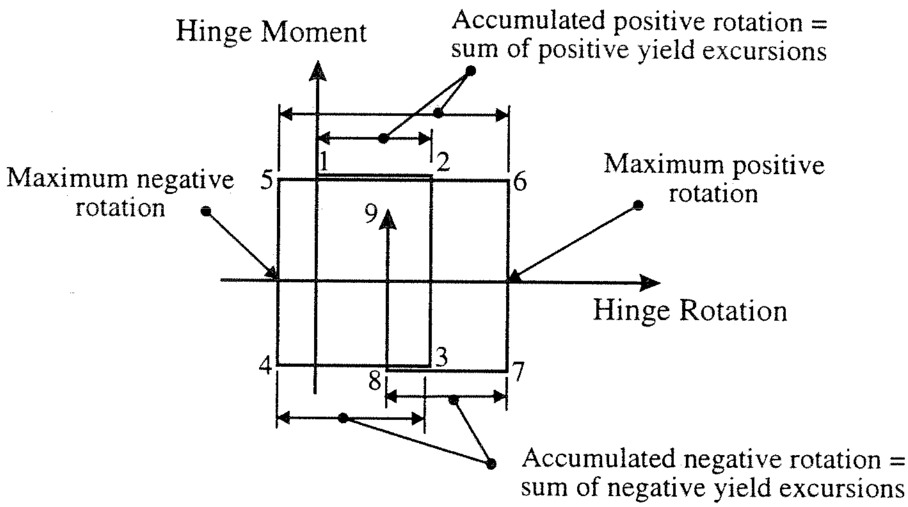


FIGURE 5.25 POSSIBLE DEMAND-CAPACITY MEASURES FOR PLASTIC HINGE

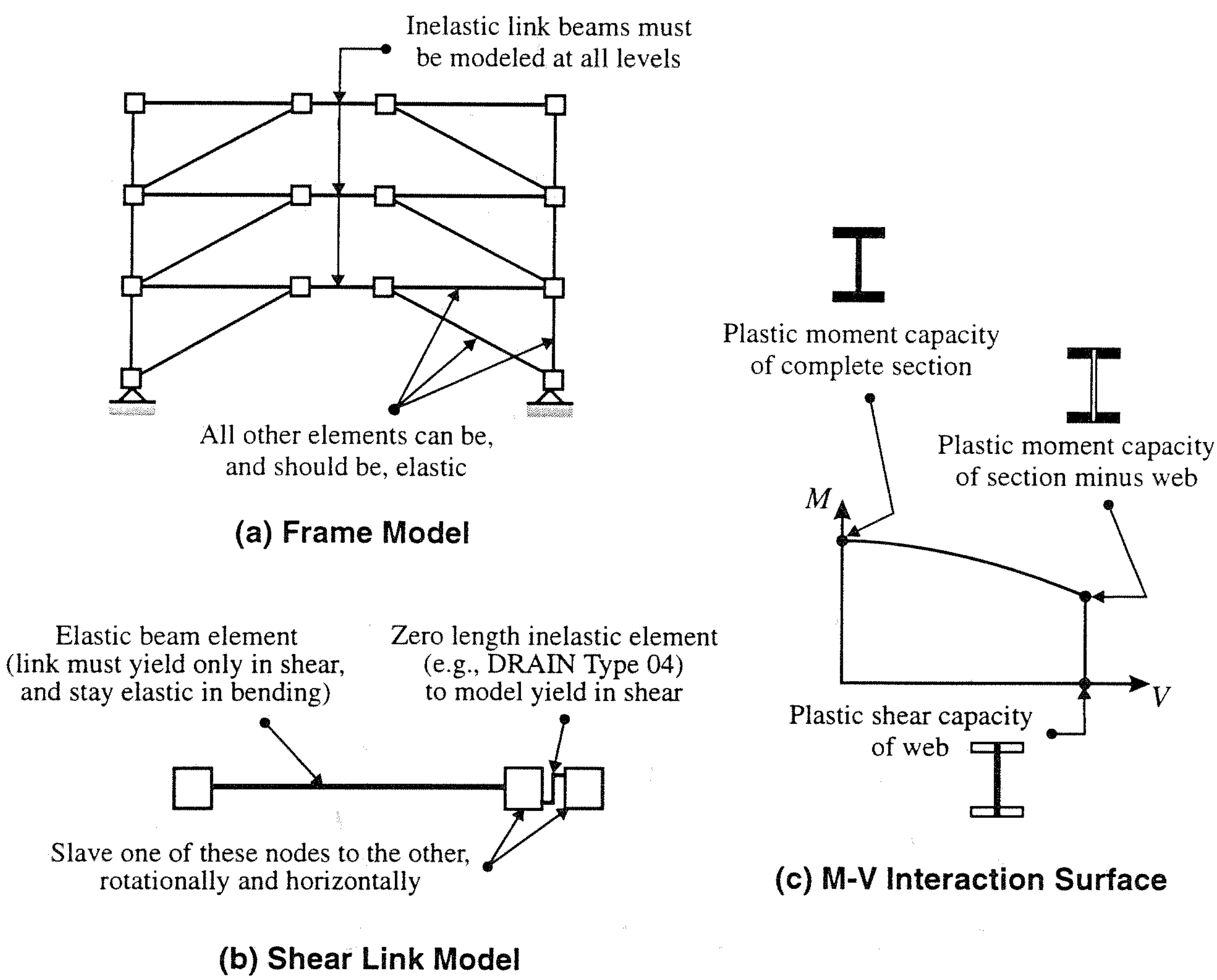


FIGURE 5.26 ECCENTRICALLY BRACED FRAME

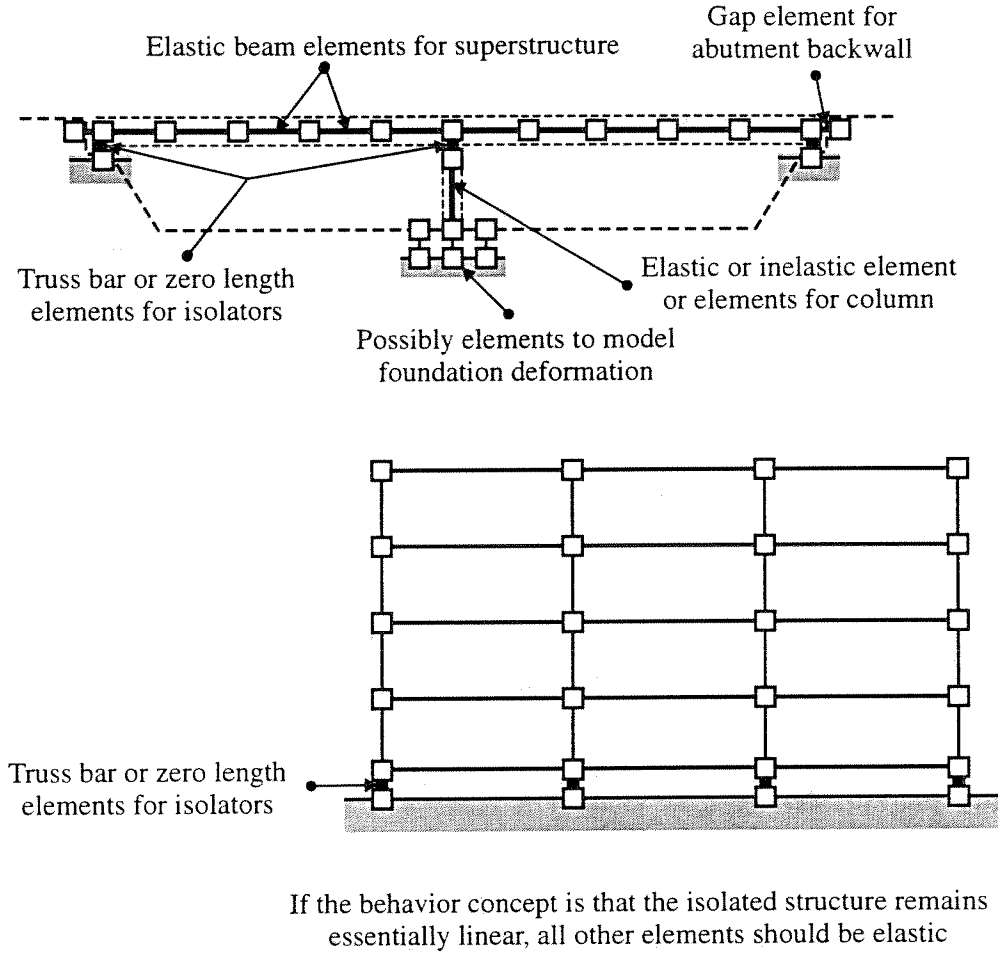


FIGURE 5.27 STRUCTURES WITH SEISMIC ISOLATION

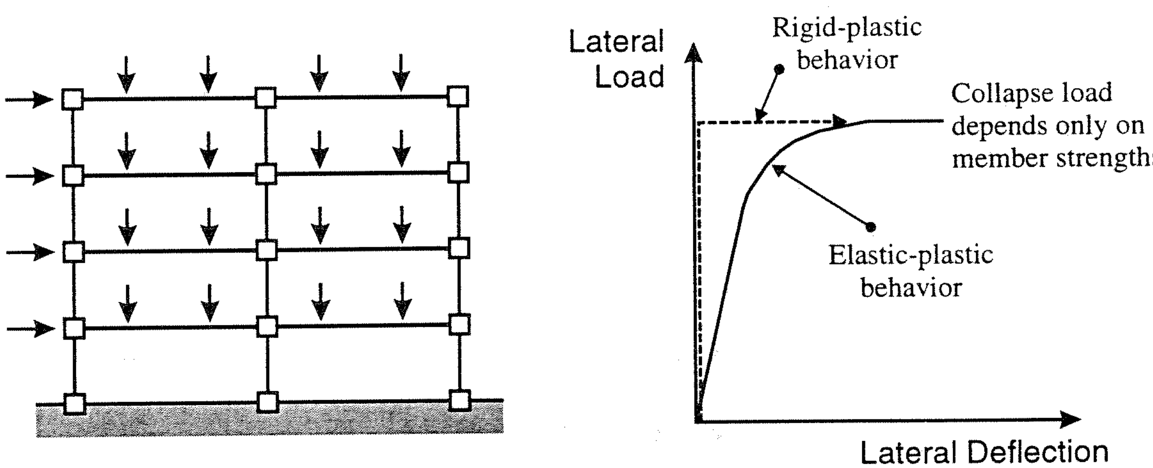


FIGURE 5.28 STATIC PUSHOVER BEHAVIOR WITH UNLIMITED DUCTILITY AND NO P-Δ

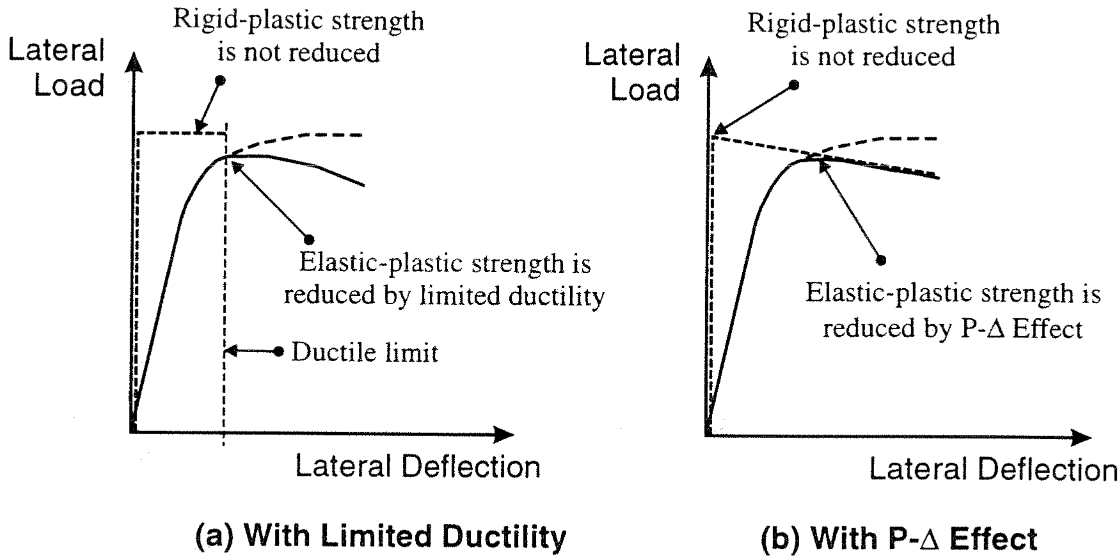


FIGURE 5.29 STATIC PUSHOVER BEHAVIOR WITH LIMITED DUCTILITY AND/OR P-Δ EFFECTS

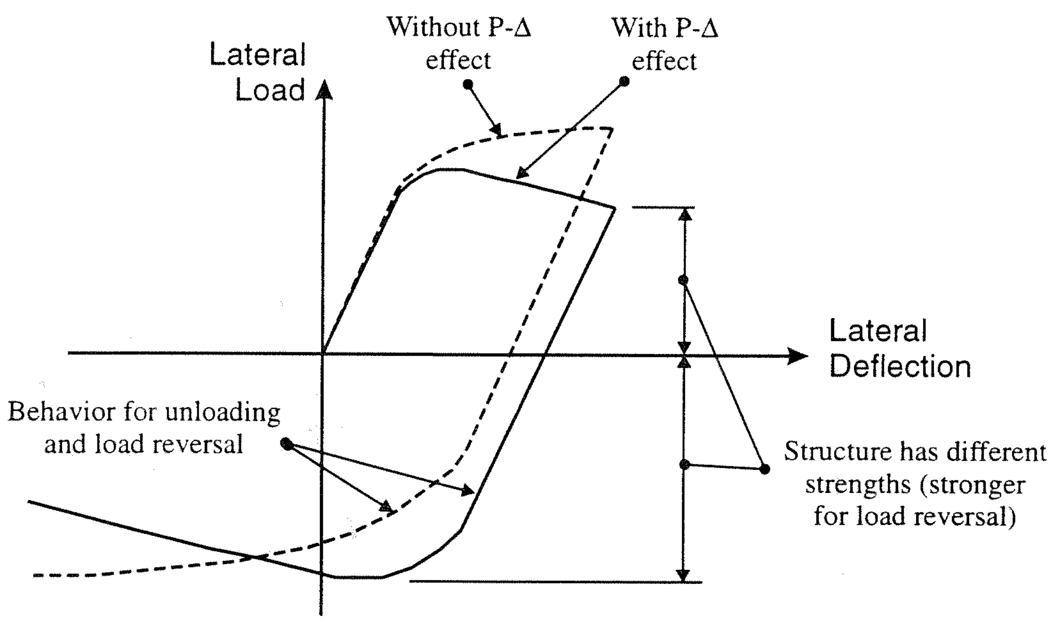


FIGURE 5.30 P-Δ EFFFECT WHEN LOAD REVERSES

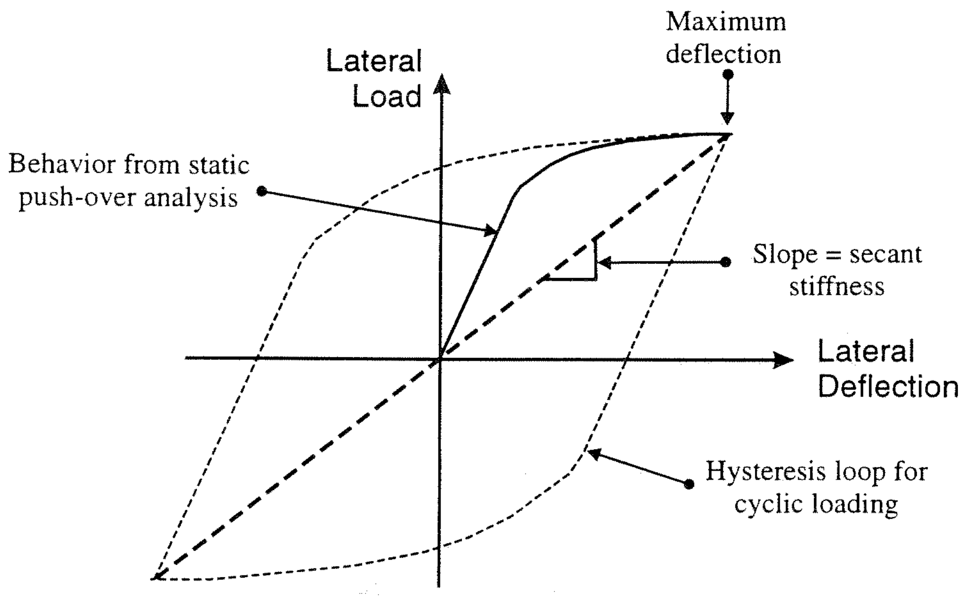


FIGURE 5.31 SECANT STIFFNESS

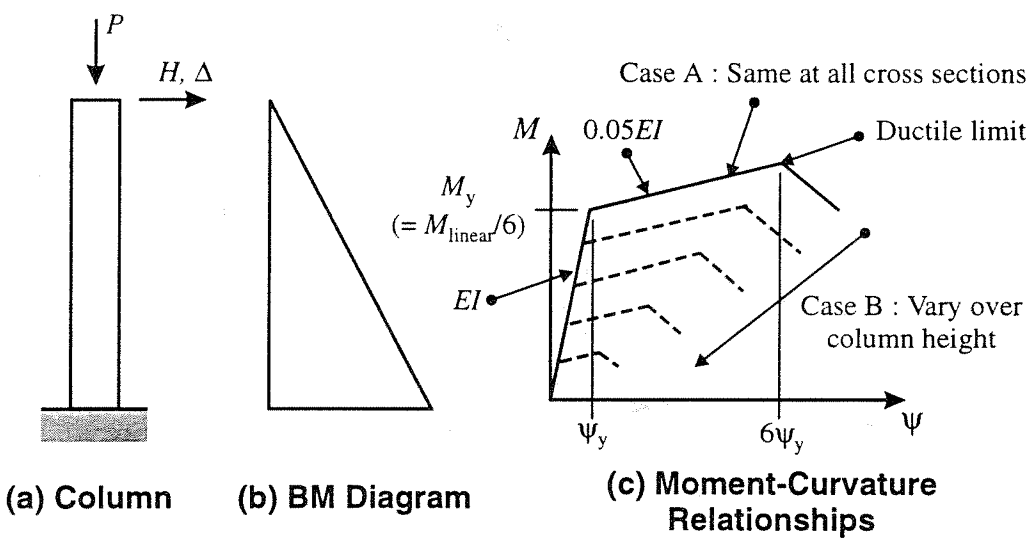
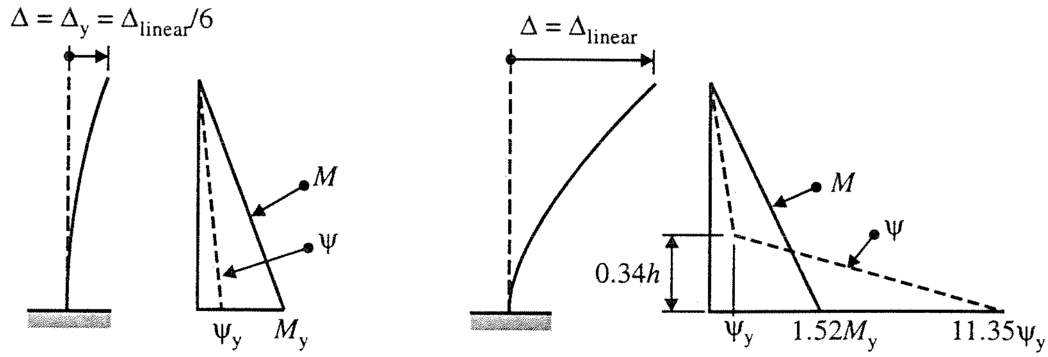
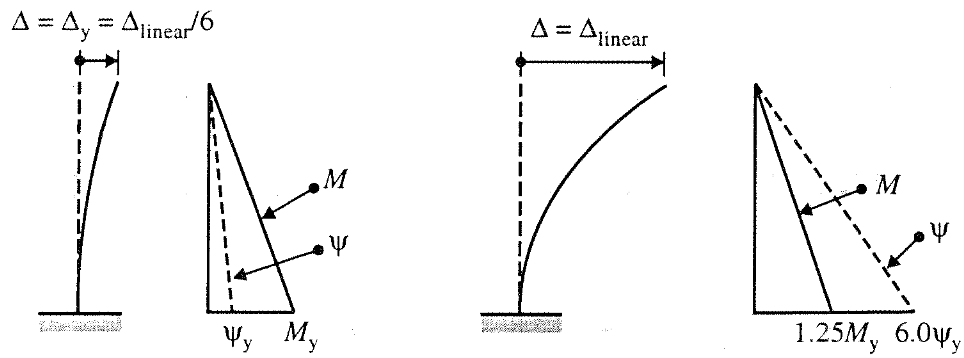


FIGURE 5.32 COLUMN EXAMPLE FOR Z-FACTOR STUDY



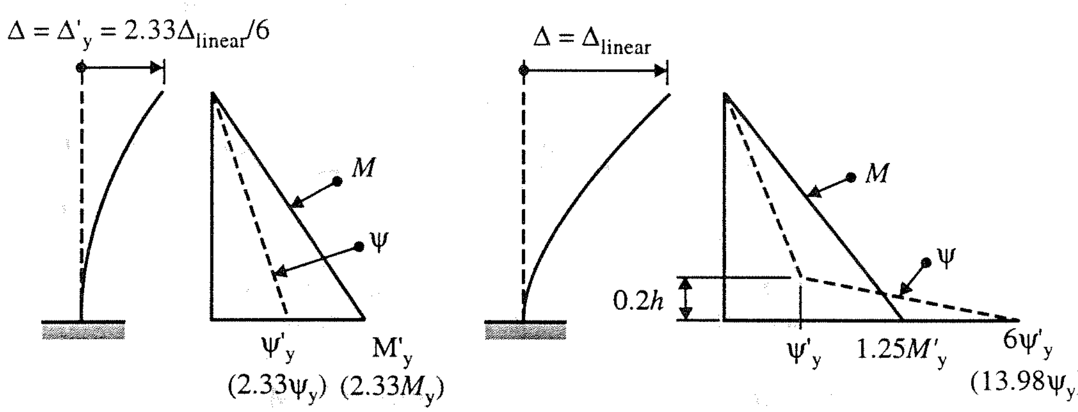
(a) First Yield (b) Maximum Deflection

FIGURE 5.33 BEHAVIOR FOR CASE A



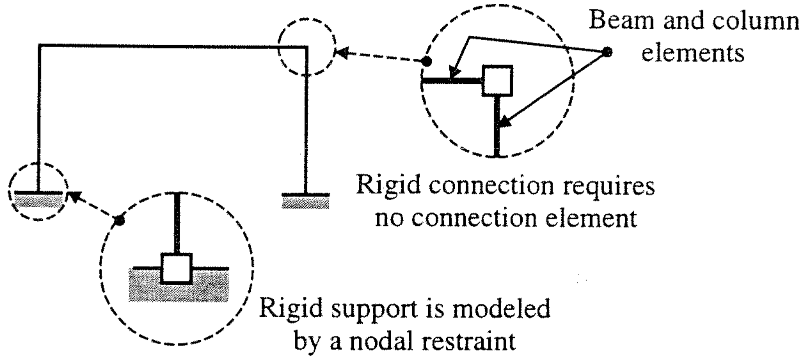
(a) First Yield (b) Maximum Deflection

FIGURE 5.34 BEHAVIOR FOR CASE B

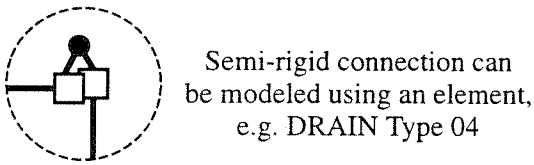


(a) First Yield (b) Maximum Deflection

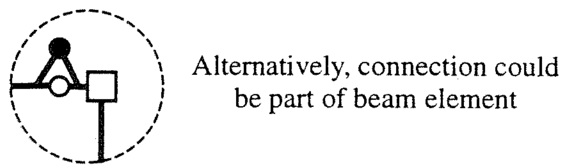
FIGURE 5.35 BEHAVIOR WITH STRONGER COLUMN



(a) Rigid Connection and Support

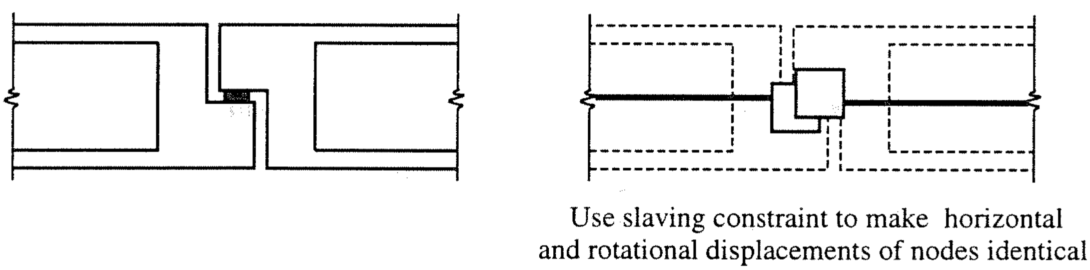


(b) Semi-Rigid Connection

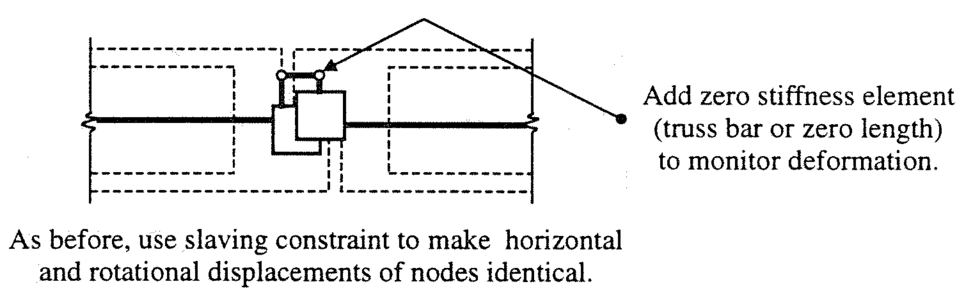


(c) Alternative for Semi-Rigid Connection

FIGURE 6.1 MODEL OF BEAM-TO-COLUMN CONNECTION



(a) Joint Modeled with no Element



(a) Joint Modeled with an Element to Monitor Joint Deformation

FIGURE 6.2 MODEL OF EXPANSION JOINT