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Emittance growth due to random force error

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Abstract

In references [1] and [2], the authors showed that the emittance growth rate due to the force error of macroparticle sampling will be proportional to the step size of simulation. In this note, we show that for the random force error, the emittance growth rate will be independent of the step size, which is consistent with the simulation observation.

1 In the Eq. 17 of reference [1] and the Eq. 27 of reference [2], the authors
2 showed that the emittance growth rate due to the error from sampling a smooth
3 particle distribution function with finite number of macroparticles in the particle-
4 in-cell (PIC) simulation is proportional to the step size used in the simulation.
5 This is puzzling since if the step size becomes smaller and tends to zero, the arti-
6 ficial emittance growth rate would become smaller and vanish eventually. Using
7 the same example of Fig. 2 in reference [1], for a linear focusing-drift-defocusing-
8 drift (FODO) lattice, we ran the simulation using 25,000 macroparticles with
9 the nominal integration step size and half of the nominal step size. The four di-
10 mensional emittance growth evolution of the 1 GeV, 30 A current proton beam
11 using the two step sizes is shown in Fig. 1 It is seen that the emittance growths
12 from both step sizes closely follow each other. The emittance growth rate is
13 independent of the step size. In the following, we will show that for the random
14 force error, the artificial emittance growth rate is independent of step size.

15 In the PIC simulation of high intensity beams, the motion of macroparticle

Figure 1: The 4D emittance growth evolution in a FODO lattice using 25 thousand macroparticles with nominal and half nominal step sizes in the simulation.

will be subject to both the force from external focusing/acceleration and the space-charge force from charged particle Coulomb interactions. Due to the use of finite number of macroparticles to sample the particle distribution function and the use of finite number of grid points, the space-charge force calculated from the numerical solution of the Poisson equation in PIC simulation will not be exact and contains a numerical error. Following the concept of splitting operator, here, we focus only on the effect of numerical space-charge force error in the simulation. If the error of space-charge force is a random error, the equations of motion governing macroparticles will be a group of stochastic differential equations. Using the Euler method for stochastic differential equations, after one step τ , the new coordinates (in one-dimension) can be approximated as [3]:

$$x_2 = x_1 \quad (1)$$

$$x'_2 = x'_1 + \delta F \sqrt{\tau} \quad (2)$$

where δF denotes the random Gaussian force error with zero mean and standard deviation of $\sqrt{\langle (\delta F)^2 \rangle}$. The new emittance under the effect of this force will

29 be:

$$\begin{aligned}
\epsilon_2^2 &= \langle x_2^2 \rangle \langle x_2'^2 \rangle - \langle x_2 x_2' \rangle^2 \\
&= \langle x_1^2 \rangle \langle x_1'^2 \rangle - \langle x_1 x_1' \rangle^2 + 2(\langle x_1^2 \rangle \langle x_1' \delta F \rangle - \langle x_1 x_1' \rangle \langle x_1 \delta F \rangle) \sqrt{\tau} \\
&\quad + (\langle x_1^2 \rangle \langle \delta F^2 \rangle - \langle x_1 \delta F \rangle^2) \tau
\end{aligned} \tag{3}$$

30 where $\langle \rangle$ denotes the average with respect to the particle distribution. The
31 above equation can be rewritten as:

$$\begin{aligned}
\epsilon_2^2 &= \epsilon_1^2 + 2(\langle x_1^2 \rangle \langle x_1' \delta F \rangle - \langle x_1 x_1' \rangle \langle x_1 \delta F \rangle) \sqrt{\tau} \\
&\quad + (\langle x_1^2 \rangle \langle \delta F^2 \rangle - \langle x_1 \delta F \rangle^2) \tau
\end{aligned} \tag{4}$$

32 Using the assumption that δF is a random Gaussian error and independent of
33 x and x' , i.e. $\langle x \delta F \rangle = 0$, and $\langle x' \delta F \rangle = 0$, we obtain:

$$\epsilon_2^2 = \epsilon_1^2 + \langle x_1^2 \rangle \langle \delta F^2 \rangle \tau \tag{5}$$

34 The emittance growth will be:

$$\Delta \epsilon \approx \frac{1}{2} \langle x^2 \rangle \langle (\delta F)^2 \rangle \tau / \epsilon, \tag{6}$$

35 and the emittance growth rate will be:

$$\frac{\Delta \epsilon}{\tau} \approx \frac{1}{2} \frac{\langle x^2 \rangle \langle (\delta F)^2 \rangle}{\epsilon} \tag{7}$$

36 This emittance growth rate due to the random force error is independent of the
37 time step size and is proportional to the variance of the random force error that
38 is inversely proportional to the number of macroparticles used in the simulation.

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