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Matching theory and induction explain operant performance

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Matching theory is a general framework for understanding allocation of behavior among activities. It applies to choice in concurrent schedules and was extended to single schedules by assuming that other unrecorded behavior competes with operant behavior. Baum and Davison (2014) found that the competing activities apparently are induced by the "reinforcers" (phylogenetically important events, e.g., food) according to power functions. Combined with power-function induction, matching theory provides new equations with greater explanatory power. Four pigeons were exposed to conditions in which 7 different schedules of food delivery were presented within each experimental session. We replicated earlier results with variable-interval schedules: (a) a negatively accelerated increase of peck rate as food rate increased in the low range of food rates; (b) an upturn in pecking at higher rates; and (c) a downturn in pecking at extremely high food rates. When the contingency between pecking and food was removed, the food continued to induce pecking, even after 20 sessions with no contingency. A ratio schedule inserted in place of 1 variable-interval schedule maintained peck rates comparable to peck rates maintained by short interval schedules. We explained the results by fitting equations that combined matching theory, competition, and induction.

Key words: matching theory, induction, power function, variable-interval schedule, contingency

In its most general form, matching theory depends on three assumptions: (a) behavior takes time; (b) available time is limited; and (c) behavior takes up all the time available. That behavior takes time might seem obvious, because no action can be accomplished or measured at a moment (Baum, 1997, 2013). Even a rat's lever press or a pigeon's key peck takes time. That time is limited is simply a fact of life, because a day contains only 24 hours and a period of observation can only last so long. That behavior takes up all the time available stems from the recognition that a living organism behaves continuously; to be alive is to behave (Baum, 2013, 2018a, 2018b).

When these three assumptions are put together, they imply that the various activities of an organism must compete with one another for time. A limited time budget means that if one activity increases, others must decrease, and if one activity decreases, others must increase. Thus, activities take up time relative to one another, compete with one another for time, and depending on environmental factors such as rates of phylogenetically important events (PIEs, e.g., food and injury), the mix of activities will tend toward stability in a stable environment (Baum, 2012a, b, 2018a, b). This conception may be expressed in an equation that specifies time allocation among activities:

$$\frac{T_j}{\sum_{i=1}^{n} T_i} = \frac{V_j}{\sum_{i=1}^{n} V_i} \tag{1}$$

where T_i or T_j is time taken up by Activity i or Activity j, one of n activities, and V is a product of food rate, amount (e.g., Cording, McLean, & Grace, 2011; Davison & Baum, 2003), and immediacy (or delay; e.g., Chung & Herrnstein, 1967; Davison & Baum, 2007). Going beyond strict matching, incorporating Baum's (1974, 1979; Sutton, Grace, McLean, & Baum, 2008) modifications that introduced bias and sensitivity, we may substitute power functions of r, food rate, or a product of a number of power functions of other food-related variables, such as amount and immediacy:

$$V_i = \prod_{j=1}^{m} c_j x_{ij}^{s_j} \tag{2}$$

where x_{ij} is a variable such as rate, amount, or immediacy (up to m such). Baum (1974) initially

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called Equations 1 and 2 the "generalized matching law"; more recently, he called it the "law of allocation" (Baum, 2018a, 2018b). Baum and Rachlin (1969) used Equation 2 to propose a version for two alternatives that when generalized leads to Equation 1.

Equation 1 may be rewritten in various ways (Baum, 2012b). Equation 1 works well for explication of matching theory, but logarithmic forms work better for fitting to data (Baum, 1974, 1979; Baum & Rachlin, 1969).

Matching theory grew out of Herrnstein's (1961) original observation that relative responding approximately matched relative food rate in several pairs of concurrent variable-interval (VI) schedules. He expressed the relation as:

$$\frac{B_1}{B_1 + B_2} = \frac{r_1}{r_1 + r_2} \tag{3}$$

where B_1 and B_2 are response rates at Alternatives 1 and 2, and r_1 and r_2 are food rates obtained from Alternatives 1 and 2. Equation 3 is a specific form of Equation 1 for two activities, with T equal to B and V equal to r.

Almost immediately the question arose as to how Equation 3 could apply to single schedules. Herrnstein (1970, 1974) hypothesized that a second alternative does exist, even when only one schedule is programmed and one activity is recorded. He suggested that other activities occurred because an organism brought its own behavioral tendencies into the experiment. He designated the other activities other than the operant activity P_O (B_O here), and the matching equation for one schedule became:

$$\frac{B}{B + B_o} = \frac{r}{r + r_o} \tag{4}$$

with r_O representing the benefits obtained from B_O.

Herrnstein (1970, 1974) further reasoned that the operant activity B and the other activity B_O must take up all the time available. Their sum must be a constant K, the total amount of behavior, which would be expressed in units of the operant activity B. Two implications followed. Firstly, the numerical value of K would depend on the topography of the operant activity. For example, K would differ for key pecking and lever pressing (deVilliers, 1977), and K would differ for interval and ratio schedules (Baum & Aparicio, 1999; Bell & Baum, 2017; Herrnstein & Heyman, 1979), because the functional units of the activities would differ. Thus, K equals both the total of behavior and approximates the response rate while the organism is responding, what Gilbert (1958) called the "tempo" of responding. Secondly, neither B_O nor r_O could be constant, because B_O would vary inversely with B, and r_O would vary directly with B_O (Herrnstein, 1979). Nevertheless, Herrnstein (1970) found that for curve fitting r_O could be treated (approximately but inaccurately) as constant.

Herrnstein (1979) recognized that r_O must vary, but suggested that the activity B_O was independent of food rate r, leaving open the question as to what sort of activity constitutes B_O. According to Herrnstein, B_O would consist of activity unrelated to food. Davison (2004) found experimental evidence supporting variation in B_O and indicating that B_O is likely composed of several different activities.

The concept of induction was introduced by Segal (1972) to characterize the effects of stimuli usually considered reinforcers or punishers on behavioral allocation independently of any contingencies. Segal showed that disparate phenomena such as classical conditioning, fixed action patterns, adjunctive activity, autoshaping, and aggression could all be viewed as examples of one process, induction. Baum (2012a) called the inducing events "phylogenetically important events," because their effects appear to have been selected during phylogeny. They include events traditionally viewed as reinforcers and punishers like food, water, mates, predators, and injury. Wetherington (1979) assembled results from her own research and that of others to show that food-induced drinking often follows Equation 4, where B is drinking, r is food rate and B_O is behavior other than drinking. One might guess that B_O consists, at least in part, of activities other than drinking also induced by food (Staddon, 1977).

Induction, as a process, resembles stimulus control. Like a discriminative stimulus, an inducing stimulus modulates the allocation of time among activities. Indeed, Baum (2012a) pointed out that stimulus control is an example of induction by a conditional inducer; the stimulus induces the change in behavioral allocation. Induction contrasts with elicitation,

one-to-one conjunction of discrete stimulus and discrete response, in that induction modulates the flow of behavior through time. Elicitation is induction conceived in a narrow time frame. Induction is inherently a molar process.

Baum (2012a) pointed out that rethinking reinforcement as induction eliminates the need to posit "strengthening" of operant activity by reinforcers and weakening by punishers. Instead, one may see operant activity as itself induced once covariance between the operant activity and the PIE has been established (Baum, 2018a). Various observations support this reasoning, such as reinstatement, in which an extinguished operant activity is brought back by noncontingent presentation of the "reinforcer" (Reid, 1958). Moreover, once operant activity is established, removing the contingency does not immediately result in elimination of the activity, which may persist indefinitely (Herrnstein, 1966). Thus, induction applies to both operant behavior and nonoperant activities sometimes known as "adjunctive" behavior (B_O).

Based on analysis of prior studies, Baum and Davison (2014) discovered that r_O depends on r, contrary to Herrnstein's (1970, 1974) reasoning. They also found that r_O is directly proportional to B_O and deduced that B_O consists largely or entirely of activities induced by the food besides the operant activity. Baum (2015) found that the induction of both operant behavior and the other induced behavior could be captured by power functions. On that basis, Baum and Davison offered an amended version of the matching relation (their Eqs. 8 and 12), which Baum (2015) expanded and which may be rewritten as:

$$\frac{B}{B + B_o + B_N} = \frac{c_1 r^{s_1}}{c_1 r^{s_1} + c_o r^{s_o} + r_N} \tag{5}$$

where B_N and r_N represent Herrnstein's original proposal that B_N would consist of activities unrelated to the PIE (e.g., food). All other expressions on the right side of Equation 5 depend on r, the food rate. The constants s_O and s_1 indicate sensitivity to r, and the constants c_O and c_1 are coefficients that account for differing units. The subscripts *1* and *O* correspond to the operant activity B and other food-induced activities, B_O.

For modeling purposes, B_N and r_N may be treated in at least two different ways. Baum and Davison (2014) assumed that B_N and r_N were substantial and had to be incorporated into the model of performance. In that approach, B and B_O both increase as r increases, and both replace B_N as r increases (Baum & Davison, 2014; Fig. 3). While agreeing that B, B_O, and B_N would take up all the time available and that B and B_O increase with r at the expense of B_N, we adopt a simpler approach and assume that r_N would usually be negligible in comparison with r, except possibly at extremely low food rates or when food is devalued by satiation. We reduce Equation 5 to:

$$\frac{B}{B + B_O + B_N} = \frac{c_1 r^{s_1}}{c_1 r^{s_1} + c_o r^{s_o}} \tag{6}$$

Equation 6 states that operant activity B competes with other activities B_O that also are induced by the PIE (food). This competition between operant activity and nonoperant induced activity derives from Equations 1 and 2; it is central to understanding operant activity in the light of Equations 1 and 2. Which activity dominates depends on the relative sizes of s_O and s_1. In most experiments with VI schedules, s_1 exceeds s_O, and thus as r increases B increases relative to B_O. When we assume that $B + B_O + B_N$ equals a constant K and divide through by $c_1 r^{s_1}$, Equation 6 simplifies to:

$$B = \frac{K}{1 + c r^{-s}} \tag{7}$$

Where c equals c_O/c_1 and s equals s_1-s_O. The coefficient c accommodates differences in the units of the activities. The exponent s indicates the relative sensitivity of operant activity to PIE rate r compared with the nonoperant activity. If the exponent s is positive, it determines how rapidly response rate grows as r increases; the smaller is s, the slower is the increase in B. If s equals zero, B is invariant with r. If s is negative, s_O exceeds s_1, and B decreases as r increases, which would mean that B_O dominates B, as in "misbehavior" (Breland & Breland, 1961).

Figure 1 shows the results of fitting Equation 7 to the canonical data from Catania and Reynolds (1968), the same data against which

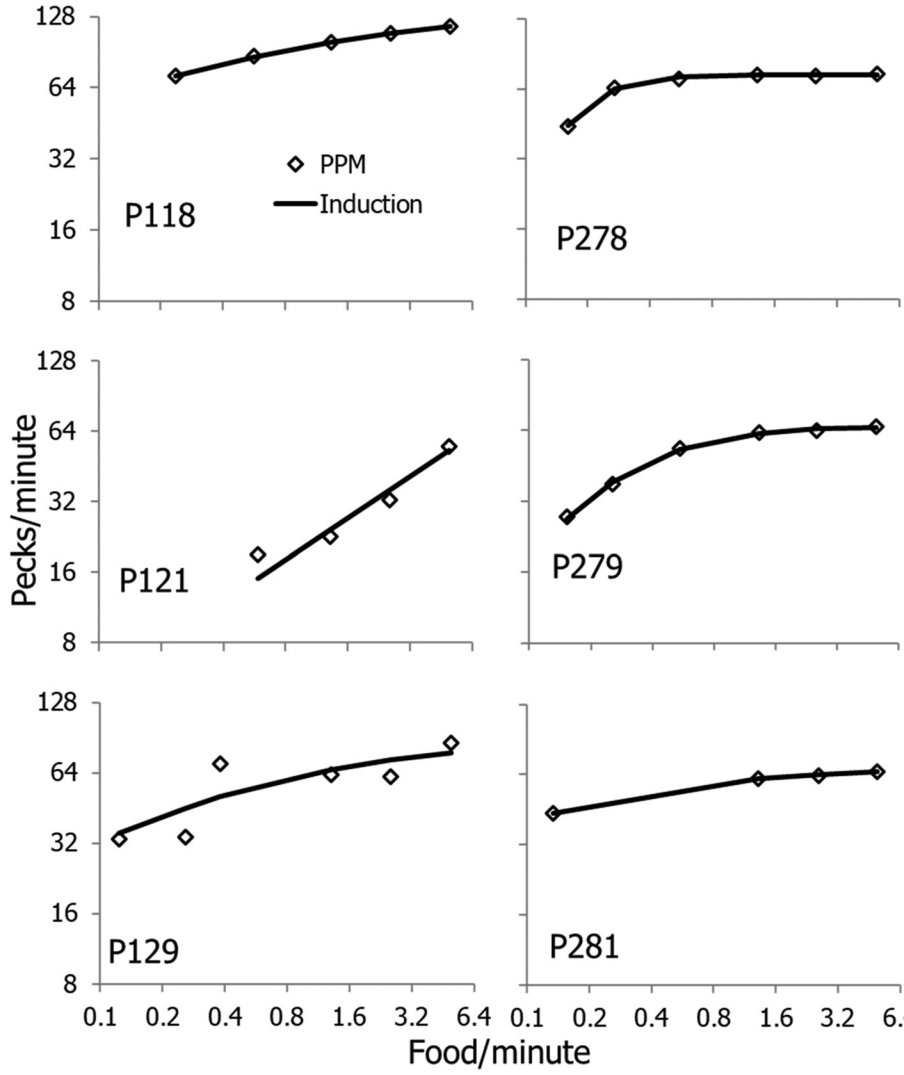

Fig. 1. Peck rates maintained by various VI schedules as a function of the food rates obtained. The solid curves show the fits of Eq. 7 to the performances. Note the logarithmic axes. Data are from Catania and Reynolds (1968).

Herrnstein (1970) tested Equation 4. The fits are excellent—better than Equation 4—but no one should be surprised, because Equation 4 is a special case of Equation 7 with s equal to 1.0 and c equal to r_O. The line of reasoning we are pursuing would suggest that Equation 7 captures competition between operant key pecking and other activities induced by the food rate r.

Figure 1 shows that Catania and Reynolds (1968) studied food rates only as high as five food deliveries per minute (fpm), equivalent to a variable-interval (VI) 12 s. A study by Baum (1993) included a wider range of high food rates, up to the limit of fixed-ratio (FR) 1. The top panel in Figure 2 shows results for VI schedules yoked to variable-ratio (VR) schedules from Baum's experiment. No food rates fell below about 0.33 fpm (VI 3 min) because the VR schedules would not sustain lower rates. Three phenomena are clear: (a) a negatively accelerated increase in response rate below about five fpm; (b) an upturn in response rate at medium-high food rates above five fpm; and (c) a downturn in response rate at the

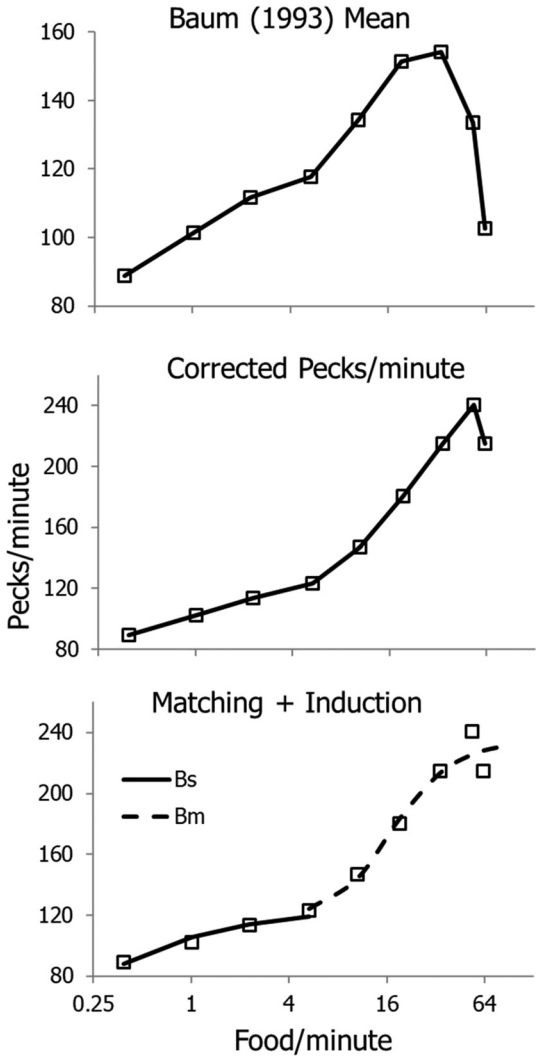

Fig. 2. Peck rates maintained by various VI schedules as a function of the food rates obtained. Top: mean performance of four pigeons. Middle: peck rates corrected according to Eqs. 8 and 9 by subtracting the obligatory timeout after food. Bottom: the solid curve shows Eq. 7 fitted to the low range of corrected food rates, and the broken curve shows Eq. 12 fitted to the upper range of corrected food rates. Note logarithmic x-axis. Data are from Baum (1993).

highest food rates. The negatively accelerated increase at low food rates corresponds to the data in Figure 1. The upturn and downturn are newer observations because they only appear when a wide range of food rates is studied. We turn first to explaining the downturn.

Downturn at Extremely High Food Rates

We treat the downturn separately because it may be understood independently of matching theory and induction. Baum (1993) showed that the downturn may be understood as an artifact of the physical arrangement of the experimental apparatus. It results from the distance separating the response key from the food hopper. Wetherington (1979), studying drinking, came to the same conclusion. She noted that the time required for a rat to move between a food hopper and a water tube constituted a delay that reduced the measured rate of drinking. Anyone who has watched a pigeon in a typical experimental setting has seen the pigeon, on finishing feeding, pull its head out of the hopper and reach up to peck the key. The motion requires only a fraction of a second, but at extreme food rates, even a pause of less than a second affects measured peck rate substantially. For example, if, with FR 1, food is occurring every second, and the delay equals half a second, the peck rate is halved. The downturn is explained by the obligatory timeout following food: the time required for the organism to move from the food hopper to the operandum. We correct for this delay (d) according to Equation 8:

$$B = \frac{B'}{1 - dr} \tag{8}$$

where B' is the measured response rate, B is the corrected response rate, and r is the food rate.

Estimating the delay d is tricky. The post-food pause (PFP) consists of three components: (a) the delay d already mentioned; (b) the time to peck the key after the delay; and (c) other behavior that intervenes. Baum (1993) corrected response rate by taking the biweight mean of the IRTs and calling the reciprocal the corrected rate. This method was largely successful, but not completely so.

A better approach derives from the understanding that response rate has an upper limit. An organism can only operate the microswitch attached to a key or lever just so fast; a finite amount of time is required. Killeen and Nevin (2018) called that minimal time δ. We may assume that the maximal rate in Figure 2 (top) is at or near the upper limit. The lower rates to the right of this maximum, when corrected, should equal that maximum. If B_2 is the

maximum and B_1 is the peck rate at the highest food rate, we can equate these two corrected rates (Eq. 8), solve for d and derive an estimate of d:

$$d = \frac{\frac{1}{B_2} - \frac{1}{B_1}}{\frac{r_2}{B_2} - \frac{r_1}{B_1}} \quad (9)$$

Estimated this way from Baum's (1993) data, d equals exactly half a second, a plausibly short time for a pigeon to remove its head from the food hopper and raise it toward the key. Applying Equation 8 to all the peck rates in the top graph of Figure 2 results in the corrected rates in the middle graph of Figure 2. All peck rates are increased above their uncorrected levels, but the lower the rate, the less the correction.

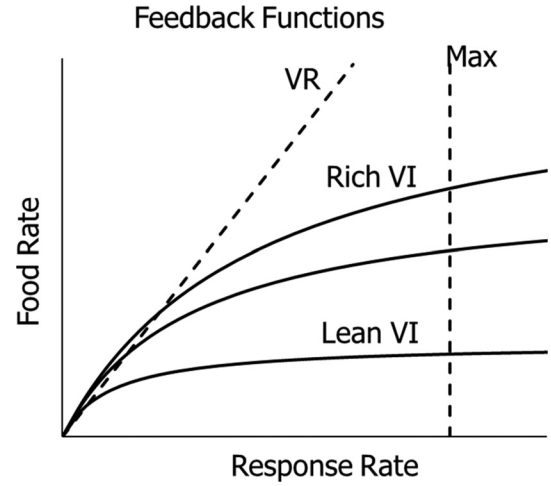

Fig. 3. Feedback functions for 3 VI schedules (solid curves) and a VR schedule (broken diagonal line). The vertical broken line shows the maximum response rate attainable. The VI curves depart from the VR line in order, from leanest to richest. The richer the VI schedule, the more it functions like a ratio schedule.

Upturn at High Food Rates

Baum (2015) found that matching theory readily accounted for performance on VR schedules, but VI schedules presented more of a challenge. The upturn in peck rate on VI schedules as food rate increases beyond the levels usually studied (e.g., Fig. 1) is absent for VR schedules (e.g., Baum, 1981a, 1993) and requires additional assumptions for its explanation. Baum (2015) showed that the unadorned matching relation (Eq. 4 here) could not account for the upturn at all, but that the power-function version (Eq. 6 here) provided some upward curvature. Although the power-function version roughly approximated the upturn, it failed to capture the shift from responding at relatively low food rates to higher food rates that was represented in the inflection point seen in the top graph of Figure 2 and in all four pigeons' data in the original study (see Baum, 2015, Fig. 3). Thus, an additional factor was needed to explain the upturn.

The upturn may be understood as an effect of a change in the contingency between pecking and feeding—as the VI gets smaller and smaller it becomes more and more ratio-like. Figure 3 illustrates this shift. The figure shows feedback functions for three VI schedules and a ratio schedule. Each function shows how food rate varies with peck rate on one schedule. VI feedback functions are the solid curves. The lowest VI feedback function is relatively lean. The highest VI feedback function is relatively rich. All three VI functions are steepest when response rate is low and tend to level off as response rate increases. The vertical broken line shows the maximum response rate attainable. The diagonal broken line shows a feedback function for a ratio (VR) schedule, in which food rate is directly proportional to response rate. At relatively low response rates, the four functions are indistinguishable, but as response rate increases, the VI functions fall away from the VR function. They fall away in order—the lowest first and the highest last. Thus, the richer the VI schedule the more it becomes ratio-like as PIE rate increases with response rate. The richest VI function retains some slope even at the maximum response rate. With this observation in hand, we now need to ask why response rates on ratio schedules exceed those on interval schedules. Baum (1993) showed that the rate difference is too large to be explained by a difference in r_O in Equation 4; another explanation is needed.

In contrast with interval schedules, in which selection for high response rates decreases as response rate increases, as the curves in Figure 3 indicate, ratio schedules always select for higher response rate, no matter how high the response rate, as the line in Figure 3 indicates. This selection occurs in ratio schedules even when food rate is equated to that in a

comparable VI schedule (Baum, 1993). Any variant in topography that results in a higher response rate is selected. Palya (1992) and Baum (1993) noted that pigeons' interactions with a response key differ from the pecking characteristic of VI schedules when the payoff schedule is a ratio schedule. Instead, pigeons tend to "flick" (Palya), "nibble," or "swipe" at the key, causing multiple operations of the key's switch for a single back-and-forth head motion. The tight feedback relation of the ratio schedule favors and selects this different activity with respect to the response key. Baum (2015) suggested that the upturn represents a change in the topography of the pigeons' interactions with the response key as food rate increased beyond the inflection point. He wrote, "Thus, the up-turn in peck rates might represent a mix of two topographies—say, multi-operation (flicking or swiping) and single-operation (pecking), with multi-operation increasingly replacing single-operation as food rate increases from moderate to high" (p. 31). The ratio-like activity replaces the interval-like activity ("pecking") as food rate rises and the VI schedule becomes more ratio-like. Baum (2015) suggested that the multi-operation topography could be incorporated into matching theory by characterizing it as generating a higher tempo of key operation, K_m.

A key question that Baum (2015) left unanswered was how to integrate the concave downward curve in the low-moderate range with the upturn at the high end of that range. He showed only that the upturn could be fitted by power-function induction. The shift marked by the inflection point (at about five fpm in Fig. 2, top) must be incorporated to explain the ascent from moderate peck rates to the increasing peck rates. At the inflection point, where the VI feedback function begins to have some resemblance to a VR feedback function—slight at first but increasing—multi-operation topography of interaction with the key begins to mix with the single-operation topography, slightly at first, but increasingly. One possibility is that a threshold exists at which the VI feedback function introduces enough selection for high peck rates to begin selecting the multi-operation topography within the pigeon's interactions with the key (Fig. 3). Below the threshold, selection for multi-operation is negligible, but above the threshold, multi-operation becomes significant. Above the threshold, the multi-operation topography would be induced and would compete with the single-operation topography.

We propose for VI schedules that above a certain threshold food rate, h, this new activity, with its multi-operation topography, begins to be induced by the food and increases as the food rate increases, displacing pecking. Each of the two activities determines a different *tempo*—that is, the rate of switch operating while the organism is engaged with the response key and the maximum attainable by the activity (Gilbert, 1958). The mix of activities during the upturn may be represented by p, the proportion of the ratio-characteristic activity:

$$B = (1 - p)K + pK_m \tag{10}$$

where K is the tempo of pecking and K_m is the tempo of switch operating for "nibbling" or "swiping" (m for multiple); because the units of multi-operation take less time than the units of single-operation, and the tempo equals the reciprocal of the time per key operation, K_m always exceeds K. At the threshold h, p equals zero, but beyond h, p grows with r until p equals 1.0 and K_m dominates the responding. For power-function induction (Eq. 6), p increases according to:

$$p = \frac{c_m(r-h)^{a_m}}{c_m(r-h)^{a_m} + c_s(r-h)^{a_s}} \tag{11}$$

Where c_m and a_m are the coefficient and exponent for the ratio topography, and c_s and a_s are the coefficient and exponent for the interval topography. Equation 11 combined with Equation 10 simplifies to:

$$B = K + \frac{K_m - K}{1 + q(r-h)^{-a}} \tag{12}$$

where q equals c_s/c_m and a equals a_m-a_s. As in Equation 7, the coefficient (q here) is required to accommodate differences in units of the two activities. The exponent a must be positive, because the proportion of ratio-like activity increases as food rate increases; ratio-like responding dominates over interval-like responding.

The result of fitting Equation 12 to the upturn in Figure 2 is shown by the broken curve in the bottom graph in Figure 2. The threshold h was set at five food deliveries per minute (fpm), and the fitted parameters were:

K_m = 236; K = 124; q = 91; a = 1.75. The estimate of a tells us that increasing food rate induces the ratio-like activity faster above threshold, because a_m exceeds a_s. The solid curve in the graph shows the result of fitting Equation 7 to the peck rates for r below threshold. The fitted parameters were: c = 0.182; s = 0.856. When we tried to fit the corrected peck rates in Figure 2 (middle) without assuming a threshold, the estimates of q and a came out to 1269 and 2.49, which seem implausibly high; these values result from simulating a threshold by making multi-operation responding extremely low in the low-moderate range of r and by making it ramp up quickly in the moderate-high range of r.

Although the present line of theory indicates that a key, lever, or button might be operated in a variety of ways, in this paper, we will use the traditional terms "peck" and "response" to maintain contact with previous research. A pigeon does not always peck a key to operate the switch, but "switch operation" seems awkward, when "peck" or "response" is usual, and for want of a better term, we will stay with the usual ones, except when trying to be precise.

The present experiment aimed to provide data on pecking on a wide range of VI schedules, including high food rates where Baum (1993) observed downturns in responding. In doing so, we tested a different method for gathering the data on various VI schedules. Instead of presenting each schedule by itself for many sessions, we presented seven different schedules within each session. Each component schedule was preceded by a blackout, but otherwise they were to be discriminated only according to the rate of food delivery; no signal indicated which schedule was in force, except when a ratio schedule was inserted in place of a VI schedule. We were able to use the data so generated to test further this combination of matching theory with induction described above.

Method

Subjects

Four White Carneau pigeons, numbered 022, 031, 034 and 119 participated as subjects. Pigeons were maintained at 85% of their free-feeding weight, +/- 15 g, through appropriate postsession feedings. All had previous experience with a variety of experimental procedures. Subjects were housed individually in a vivarium with a 12:12 light–dark cycle (lights on at 0700 hr) and had free access to water and grit.

Apparatus

Four standard three-key operant chambers were used. The dimensions were 35 cm deep x 35 cm wide x 35 cm high. Response keys were located 26 cm above the floor and could be illuminated red, green, or white, but only the center key was used in the experiment. A houselight was mounted 7 cm above the center key in each chamber for ambient illumination, and a grain magazine with a 6 cm x 5 cm aperture was located 13 cm below the center key. A magazine light was turned on when wheat was made available. A force of approximately 0.10 N was required to operate each key, and each response resulted in an audible feedback click. Chambers were enclosed in sound-attenuating boxes and fitted with ventilation fans for masking extraneous noises. The experiment was controlled with a MED-PC system that was interfaced to an IBM-compatible microcomputer located in an adjacent room.

Procedure

Experimental sessions were conducted daily with rare exceptions. Each session consisted of seven components, each presenting a different schedule of food for pecks at the center key, which was illuminated white or red, except during the 2.5-s food deliveries and during blackouts. The first component of a session was preceded by a 15-s blackout; after that, a 30-s blackout preceded each component. The seven components were presented randomly without replacement. Components were unsignaled, except in Conditions 6 and 7, in which the random-ratio (RR) component or its noncontingent equivalent was signaled by a red key light, as explained below. Sessions usually ended after all seven components had occurred, but occasionally a session terminated early if the session exceeded 60 min. Every operation of the response key that did not produce food produced a flicker (0.05 s) of the key light. All inter-event times were recorded within components with a resolution of 0.01 s.

Each condition presented a set of seven components, each with a different schedule of

Table 1

Order of conditions, listing schedules and number of sessions each condition lasted

Condition	Pigeons	Schedules	Sessions
1 (VIs)	022, 043	FR 1, VI 1, VI 2, VI 8, VI 32, VI 128, VI 512	63, 70
	031, 119	FR 1, VI 1, VI 4, VI 16, VI 64, VI 256, VI 512	70
2 (VIs)	022, 043	FR 1, VI 1, VI 4, VI 16, VI 64, VI 256, VI 1024	83
	031, 119	FR 1, VI 1, VI 2, VI 8, VI 32, VI 128, VI 1024	83
3 (VT)	022, 043	FR 1, VT 1, VT 4, VT 16, VT 64, VT 256, VT 1024	10
	031, 119	FR 1, VT 1, VT 2, VT 8, VT 32, VT 128, VT 1024	10
4 (VI + EXT)	022, 043	FR 1, VI 1, VI 4, VI 16, VI 64, VI 256, EXT	35
	031, 119	FR 1, VI 1, VI 2, VI 8, VI 32, VI 128, EXT	35
5 (VI + EXT)	022, 043	FR 1, VI 1, VI 2, VI 8, VI 32, VI 128, EXT	41
	031, 119	FR 1, VI 1, VI 4, VI 16, VI 64, VI 256, EXT	41, 40
6 (VI + RR)	022, 043	RR[a], FR 1, VI 4, VI 16, VI 128, VI 256, VI 512	107, 105
	031, 119	RR[b], FR 1, VI 4, VI 16, VI 128, VI 256, VI 512	103, 107
7 (VT)	022, 043	VT (=RR), FR 1, VT 4, VT 16, VT 128, VT 256, VT 512	20
	031, 119	VT (=RR), FR 1, VT 4, VT 16, VT 128, VT 256, VT 512	20

a: RR 8.62, RR 4.39; b: RR 4.93, RR 7.14.

Note. All times are in seconds. EXT stands for extinction, which was programmed as VT 1024, but without delivering the food. In Condition 7, the RR schedule was replaced with a VT ("=RR") that matched the food rate produced by the RR in Condition 6. "NC" indicates noncontingent food.

food delivery. Table 1 shows the schedules in each condition and the number of sessions each condition lasted. Conditions 1 and 2, which included only VI schedules and FR 1, aimed at replicating previous research (e.g., Baum, 1993). Conditions 4 and 5, which contained an extinction component, tested for the pigeons' ability to discriminate a long absence of food. Conditions 3 and 7 tested the effects of removing contingencies to see how long the food might continue to induce the operant activity. Except for Conditions 3 and 7, in which food was delivered noncontingently according to variable-time (VT) schedules, conditions changed when visual inspection indicated that all seven response rates appeared stable from session to session. In Condition 6 (VI + RR), a RR schedule replaced one VI schedule, with the aim of comparing performance on an actual ratio schedule with performance on short VI schedules that might be ratio-like, as in Figure 3. The RR parameter was set to a probability that, given the pigeon's peck rate, would produce a food rate in the middle-high range. The response key was illuminated red during the RR component. In Conditions 3 and 7 (VT), food was delivered noncontingently, except for FR 1 (fixed ratio 1 or "continuous reinforcement"). In Condition 7, in the component that had been the RR component, food was delivered at the same rate as in the previous RR component (Condition 6), and the key was illuminated red.

All VI and VT schedules were programmed by selecting without replacement from a list of 12 intervals generated by the method of Fleshler and Hoffman (1962). These intervals ranged from 0.05 s to 3.48 s, with a mean of 1.0. Within a component, each of these intervals was multiplied by the appropriate schedule factor to create the interval to be timed. Each component ended after a certain number of

Table 2

Number of food deliveries in a component for each schedule and bin width used in analysis

Schedule	Food Deliveries	Bin Width (s)
FR 1, VI 1, VI 2, VI 4	15	0.5
VI 8	9	1.0
VI 16	9	2.0
VI 32	9	3.0
VI 64	9	6.0
VI 128	6	8.0
VI 256	6	15.0
VI 512	6	30.0
VI 1024, EXT	3	40.0
RR	15	6.0

Note. All times are in seconds. The number of food deliveries was the same for VI and VT schedules.

food deliveries. Table 2 shows the number of food deliveries after which each component schedule ended. Table 2 also shows the bin width, in seconds, used for each schedule in analysis (explained below). Because the pigeons were already experienced, they were given seven (P022) or 11 (P031, P043, P119) sessions of pretraining on a similar procedure, in which the leanest component schedule was increased from VI 64 s to VI 128 s to VI 256 s, and then started directly on Condition 1 (VIs).

Results

Data from the final sessions of each condition were analyzed: for Conditions 1 (VIs), 2 (VIs), and 6 (VI + RR), the final 50 sessions; for Conditions 4 (VI + EXT) and 5 (VI + EXT), the final 30 sessions; and for Conditions 3 (NC) and 7 (NC), the final five sessions. To pool the data across all the presentations of a component, we divided each component presentation into equal consecutive time intervals ("bins" hereafter), starting with the beginning of the component presentation. Table 2 shows the bin widths. We calculated peck rate and food rate in each bin by pooling all the pecks and food deliveries in each bin across component presentations. The time base equaled the number of occurrences of the bin times the width. Because the component presentations were of variable duration, the number of times a bin occurred decreased as the bin was later in the component. The bin widths were arrived at by trial and error. Too small a bin results in too much variability in peck rate, making estimation of stable peck rate within the component unreliable. Too large a bin results in too little variability, blurring the within-component change in rate and making estimation of stable peck rate impossible. Between these two extremes, we were able to find a bin duration that worked well—that is, for each component schedule, a bin width was found that revealed the trend in peck rate and the stable rate arrived at in the component (see appendix for examples). In the richer components, peck rate increased and leveled off, and in the leaner components, peck rate decreased and leveled off.

Except for Conditions 3 (VT) and 7 (VT), in which the final 5 days were averaged, stable within-component peck rates were estimated as follows. In Conditions 1 (VIs), 2 (VIs), and 6 (VI + RR), all bins that occurred at least 40 times (out of a maximum of 50) were included, and in Conditions 4 (VI + EXT) and 5 (VI + EXT), all bins that occurred at least 24 times (out of a maximum of 30) were included. A regression line of peck rate versus bin order was calculated for the first bin to the last bin, then from the second bin to the last bin, then for the third bin to the last bin, and so on. The slopes of these regression lines were examined to find the bin for which the slope was closest to zero. Peck rate was averaged from the bin with the lowest slope to the final bin to estimate the stable peck rate (see Bell & Baum, 2017, for an application of this method across sessions). Figure A1 in the appendix shows two examples to illustrate how the estimation of stable rate was done. Food rate in a component was estimated by adding up the total number of food deliveries in all the presentations of the component and dividing by the total number of bins that occurred at least once multiplied by the bin width.

Figure 4 shows peck rate plotted against food rate from Conditions 1 (VIs), 2 (VIs), 4 (VI + EXT), 5 (VI + EXT), and 6 (VI + RR) for each of the four pigeons. Except for P031 in Condition 4, P043 in Condition 5, and P119 in Condition 4, the extinction components were generally not discriminated. The filled diamonds show the response rates for the RR schedules in Condition 6. The rate is noticeably higher than the VI rates only for P119. The solid lines show the means.

All four pigeons' peck rates, to one degree or another, conformed to the pattern seen in Figure 2 (top), except those for P031 showed little discrimination among the schedules, and those for P022 did not flatten out at the low end of food rates. All four show at least some increase in peck rate in the middle range, and all show the drop in peck rate at the extreme of food rates, except for P031's rates, which leveled off at the highest food rates. Thus, the results of this within-session procedure replicated Baum's (1993) results in three of our four pigeons.

Figure 5 shows the results of omitting the contingencies between pecking and food deliveries in Condition 3 for the last five of 10 sessions (squares). Peck rates and food rates were calculated for the latter two-thirds of component food deliveries averaged across the five

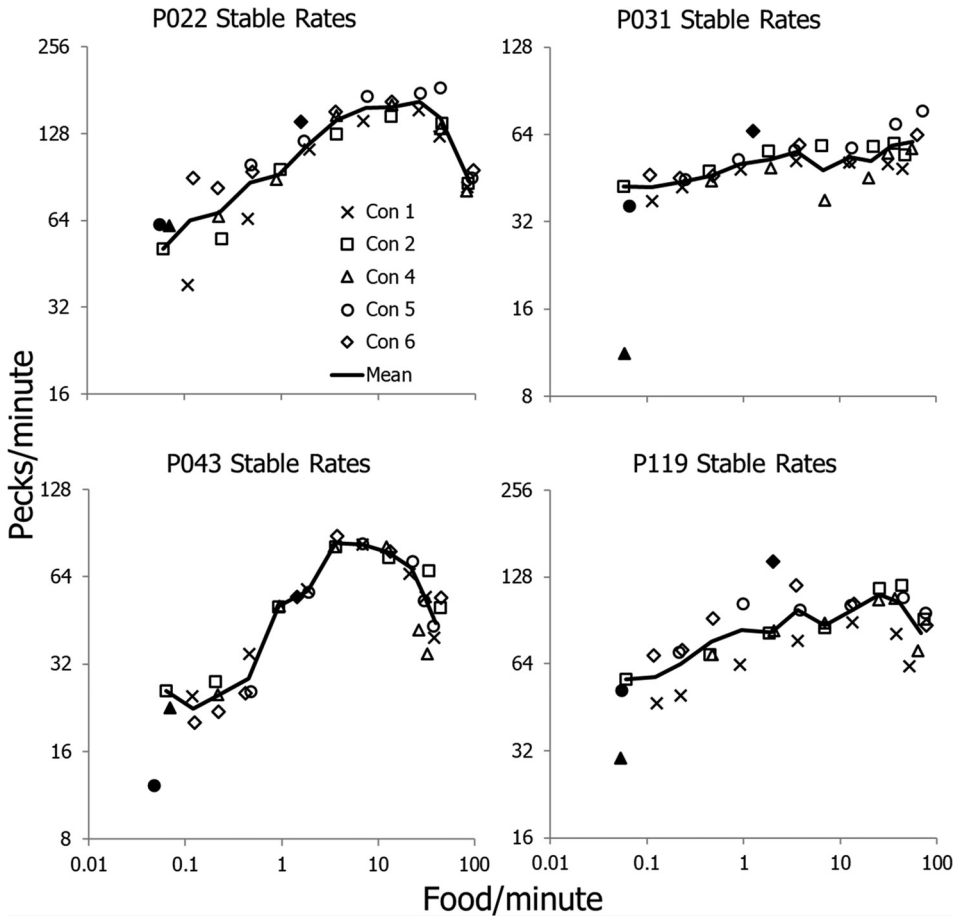

Fig. 4. Peck rates in the present experiment from Conditions 1 (VIs), 2 (VIs), 4 (VI + EXT), 5 (VI + EXT), and 6 (VI + RR). Solid lines connect the means across conditions. Filled circles and triangles show peck rates in the extinction components, plotted at the food rates for VI 512 s and VI 1024 s. Filled diamonds show peck rates maintained by the RR component. Note logarithmic axes. All vertical axes cover 4 log units (base 2), but they differ in range.

sessions (see numbers of food deliveries per component in Table 1). The peck rates for Condition 2 (VIs) from Figure 4 are included for comparison (diamonds). Except for P031, these rates were lower than in Condition 2 (rightmost points). Response rates in Condition 3, with no contingency, were lower than rates with contingency, but the pattern was generally preserved, with the partial exception of P119, in which peak responding occurred at a lower food rate. The biggest decreases were for the lowest food rates. Thus, the food continued to induce responding in the absence of contingency.

Figure 6 compares the peck rates in Condition 6, which included a RR component, with the rates in Condition 7, in which contingency was absent, except for FR 1. The data from Condition 7 are from the last five sessions of the 20 sessions. Peck rates and food rates were calculated for the latter two thirds of food deliveries in each component and averaged across the five sessions, as in Figure 5. The key light for the RR component was lit red in Condition 6 and also in the matched component in Condition 7 (VT). The filled points show peck rate in these signaled components. Only for P031 were these rates noticeably higher than for the unsignaled VI schedules. As in Figure 5, rates on FR 1 (unconnected squares) were equal to or less than those in Condition 6 (VI + RR). As in Figure 5, the absence of contingency lowered peck rate, but the pattern of responding across food rates remained.

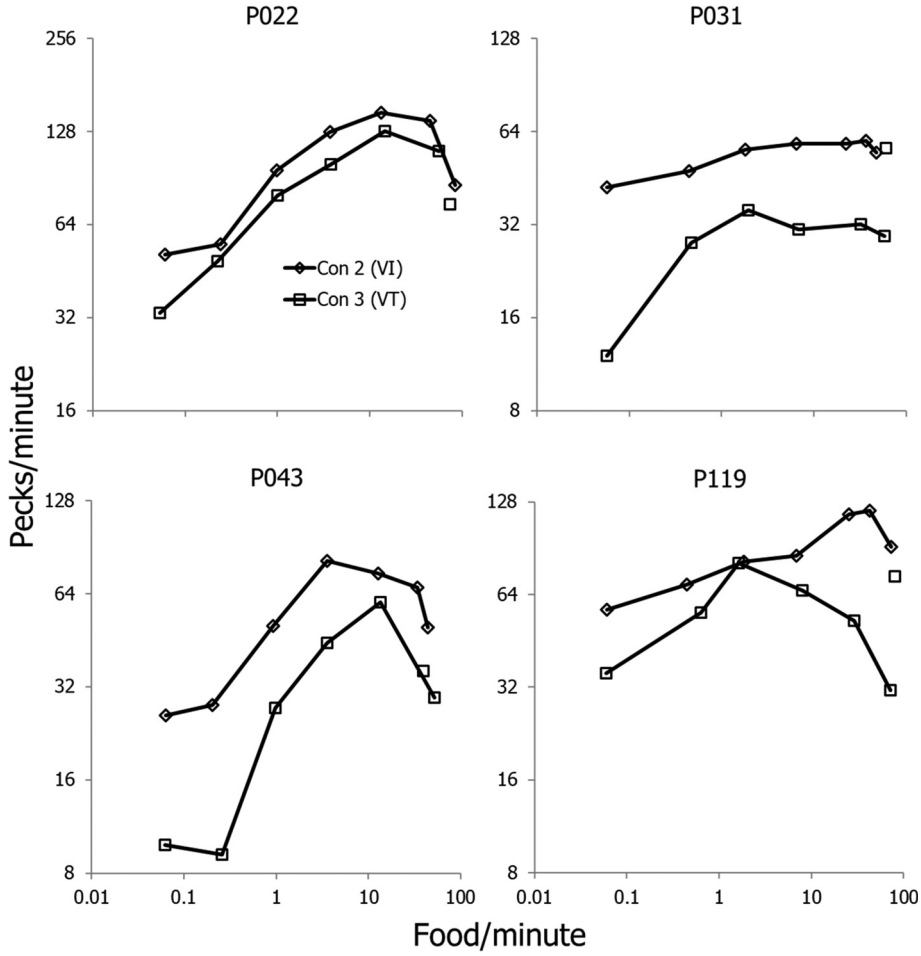

Fig. 5. Comparison of peck rates maintained in Condition 2, with peck–food contingency (VI schedules; diamonds), with peck rates maintained in Condition 3, without peck–food contingency (VT schedules; squares), except FR 1. Unconnected squares show peck rate for FR 1, with a contingency, in Condition 3. Note logarithmic axes. All vertical axes cover 4 log units (base 2), but vary in range.

The food continued to induce pecking, even after 20 sessions of no contingency.

One might expect that the peck rates in Conditions 3 (VT) and 7 (VT) would have continued to decrease beyond the 10 or 20 sessions in these conditions. To assess this possibility, we fitted a regression line to the base-2 logarithms of peck rates versus the last 10 sessions of Condition 7 for each component except FR 1. The slopes of these lines were usually negative, but sometimes positive (7 out of 24). Most slopes were close to zero, except for P022 and P043, for which the slope became more negative, the leaner the schedule. For VT 4 s, the slopes for P022, P031, P043, and P119 were: -0.023, 0.004, -0.003, and -0.067, all close to zero. The steepest, -0.067, corresponds to a decrease by a factor of 0.63, or 37% across the last 10 sessions. For VT 256 s, the slopes were: -0.053, 0.001, -0.286, and -0.069, comparatively steeper for P022 and P043. At least for three pigeons at high food rates, peck rates continued to be maintained even though food was noncontingent.

As mentioned earlier, a possible theory that would explain the upturn in peck rate in Figure 2 (top) and Figure 4 might point to the increasingly ratio-like feedback functions of extremely short VI schedules, as depicted in Figure 3. Figure 7 shows feedback functions

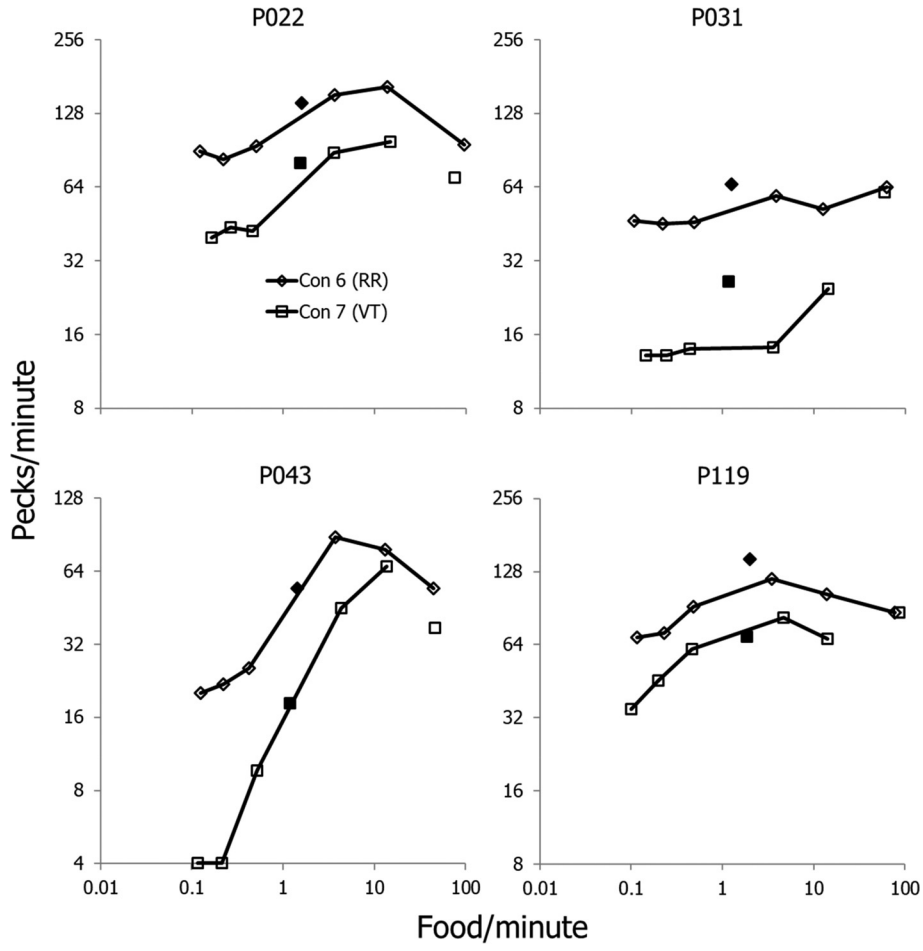

Fig. 6. Comparison of peck rates maintained in Condition 6, with peck–food contingency (VI schedules; diamonds), with peck rates maintained in Condition 7, without peck–food contingency (VT schedules; squares), except FR 1. Unconnected squares show peck rate for FR 1 in Condition 7. Filled diamonds show peck rates maintained by the RR components. Filled squares show peck rates maintained by the corresponding VT schedule. Note logarithmic axes. All vertical axes cover 5 log units (base 2), but vary in range.

for FR 1, VI 1 s, VI 2 s, and VI 4 s recovered from the data, and may be compared to Figure 3. The method for extracting these data was to calculate peck rate and food rate for each consecutive bin in components, and to plot the food rate against the peck rate. This method works only at relatively low response rates, because bins containing relatively high response rates include bursts of high-rate responding and few food deliveries, with the result that food rate starts decreasing as response rate increases. The points to the left in Figure 7 show the feedback function up to the response rate where food rate begins decreasing. To estimate food rate for higher response rates (circles), food rate was computed for the stable response rates in the components.

Figure 7, compared with Figure 3, verifies the conjecture that small VI schedules become ratio-like. The feedback function for VI 1 s (top two panels) follows the FR-1 line up to about 50 pecks/minute, whereas the feedback function for VI 2 s (lower left panel) falls away above about 25 pecks/minute, and the function for VI 4 s (lower right panel) falls away above about 15 pecks/minute and is largely flat. VI 1 s resembled FR 1 most, then VI 2 s, and VI 4 s, with its typical VI feedback function, resembled FR 1 least.

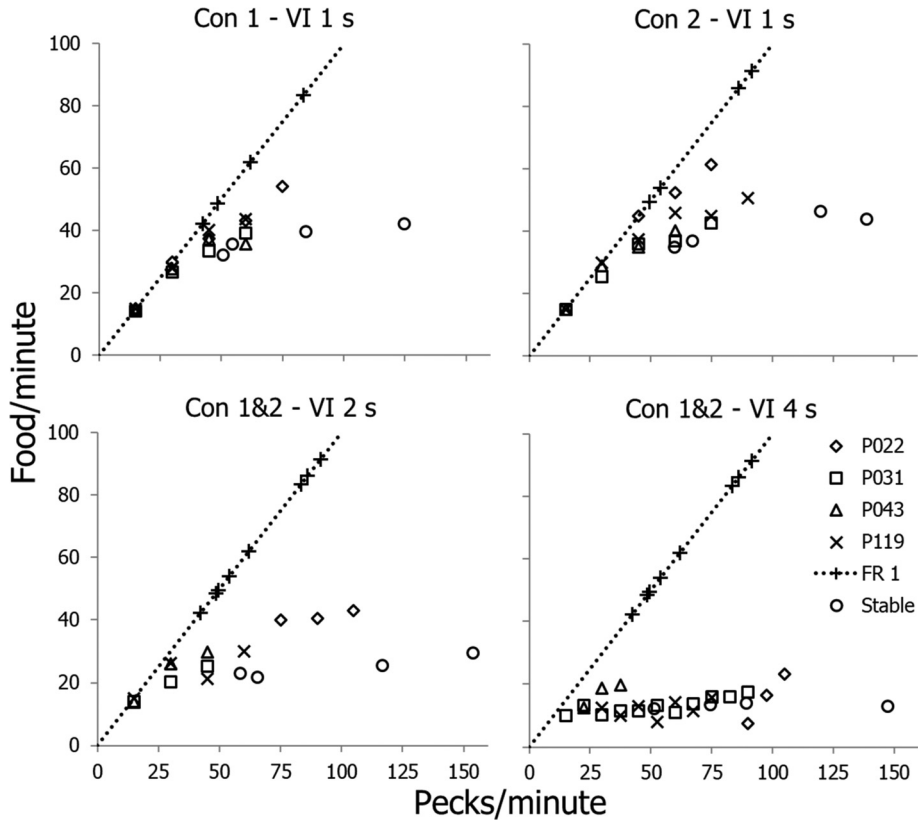

Fig. 7. Feedback functions recovered from the data. The dotted lines and crosses show feedback functions for FR 1. The top two graphs show feedback functions from Condition 1 (left) and Condition 2 (right) for FR 1 and VI 1 s. The bottom graphs show feedback functions from Conditions 1 and 2 combined for FR 1 and VI 2 s (left) and for FR 1 and VI 4 s (right). Except for the crosses (FR 1) and the circles (stable peck rates), the different symbols represent peck rates from different pigeons.

Figure 8 examines response rate immediately following food delivery across components. Squares show mean postfood pause (PFP), measured from end of food presentation until the first key operation, for all PFPs less than 2.5 s (to exclude long pauses). PFP was minimal at the highest food rates and increased as food rate decreased, presumably because other activities intervened between the end of food presentation and the initiation of pecking. Triangles show the mean of all interresponse times (IRTs) equal to 1 s or less (considering IRTs greater than 1.0 likely to include pauses). This mean IRT remained fairly stable across schedules and, except for P043, was at minimum for the highest food rates.

Baum (1992) speculated in modeling the feedback function for VI performance that an isolated food delivery may induce a burst of responding. Figure 9 explores this conjecture by plotting mean IRT in order from first to fifth and above for the four leanest schedules, VI 128 s, VI 256 s, VI 512 s, and VI 1024 s. The first through fourth IRTs are all shorter than the later (fifth and beyond), with VI 128 s showing the smallest difference, supporting the suggestion that when food is relatively rare, a burst of responding follows each food delivery. These bursts explain why VI feedback functions may approach a slope less than 1.0—the slope for FR 1—at extremely low response rates (Baum, 1992).

Figure 10 shows mean response rates from Figure 4 corrected by the same method as was used for Figure 2 (middle panel), using Equations 8 and 9. All show a ceiling on peck rate and, except for P022, an approximate S-shape

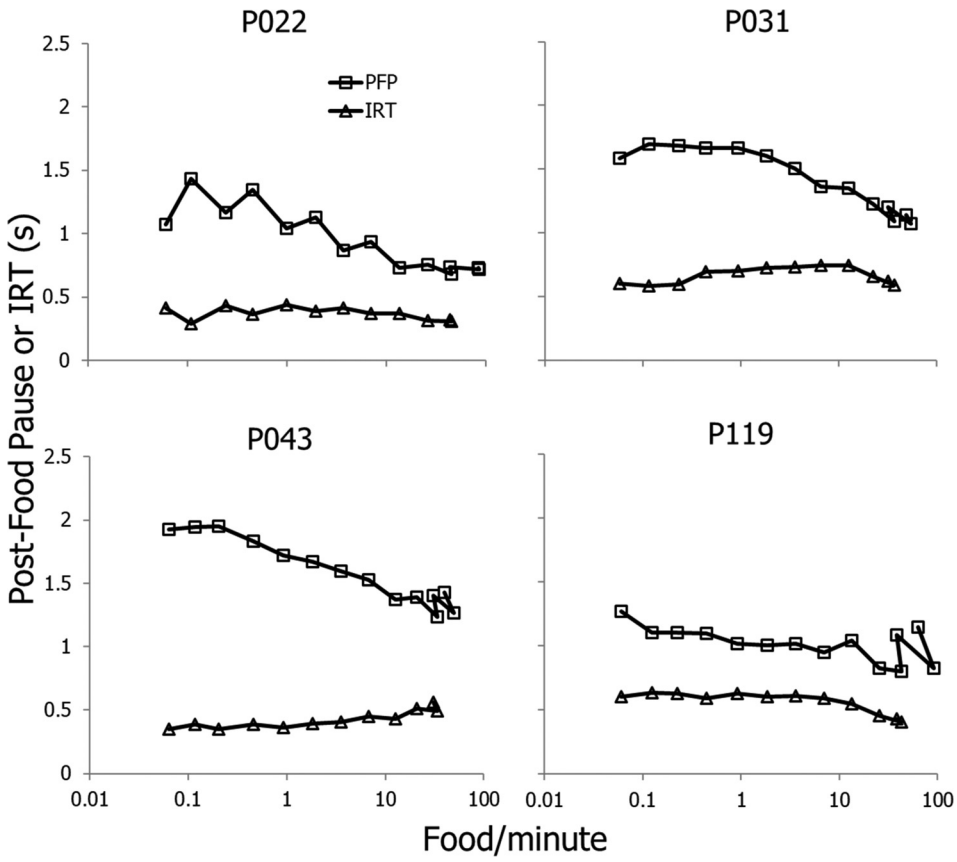

Fig. 8. Postfood pauses (PFP) and mean interresponse times (IRT) immediately following food as a function of food rate across components. Squares represent mean of PFPs less than 2.5 s. Triangles represent the mean of IRTs less than or equal to 1.0 s. Note the logarithmic x-axes.

that includes a leveling at relatively low food rates. P119 shows two S-shaped trends, because its peck rate flattens below about 0.2 and then flattens again in the higher range at about 2 – 10 fpm. A second S-shaped pattern appears in the high range of food rates, from about 10 to 70 fpm. The estimates of d (Eq. 9) were: P022, 0.39 s; P031, 0.0 s; P043, 0.75 s; P119, 0.32 s. The estimate is zero for P031 because P031 showed no downturn. Pecking at a low, almost invariant rate, P031 apparently shifted only slightly to the ratio-like topography (e.g., "swiping"), thus showing little upturn and no downturn.

One could estimate d in Equation 8 instead by subtracting the minimum IRT from the minimum PFP. Such estimates jibed with the estimates of d actually used for the corrected rates shown in Figure 10. We subtracted the mean IRT from the mean PFP in Figure 8 for VI 1 s in Condition 2 (VIs). Estimated this way, d equaled: 0.37 s (vs. 0.39 s) for P022; 0.50 s (vs. 0.0) for P031; 0.74 s (vs. 0.75 s) for P043; and 0.39 s (vs. 0.32 s) for P119. P031 represented an exception, because P031's rates showed no downturn. If the correction of 0.50 s were applied to P031's rates, they would continue to increase up to the highest food rates. P031's frequency distributions of IRTs (see Fig. 12 below) and PFPs showed high variance and skew, likely because they tended to include pausing (i.e., other activities). For VI 1 s (Condition 2), for which PFP was at a minimum, the mean of P031's shortest 10% of PFPs (0.62 s) was actually almost equal to the mean IRT less than or equal to 1.0 s (0.59 s). For no other pigeon did such a convergence occur.

Figure 11 shows the results of fitting Equation 12, as was done in Figure 2 (bottom). Table 3 shows the estimates of the

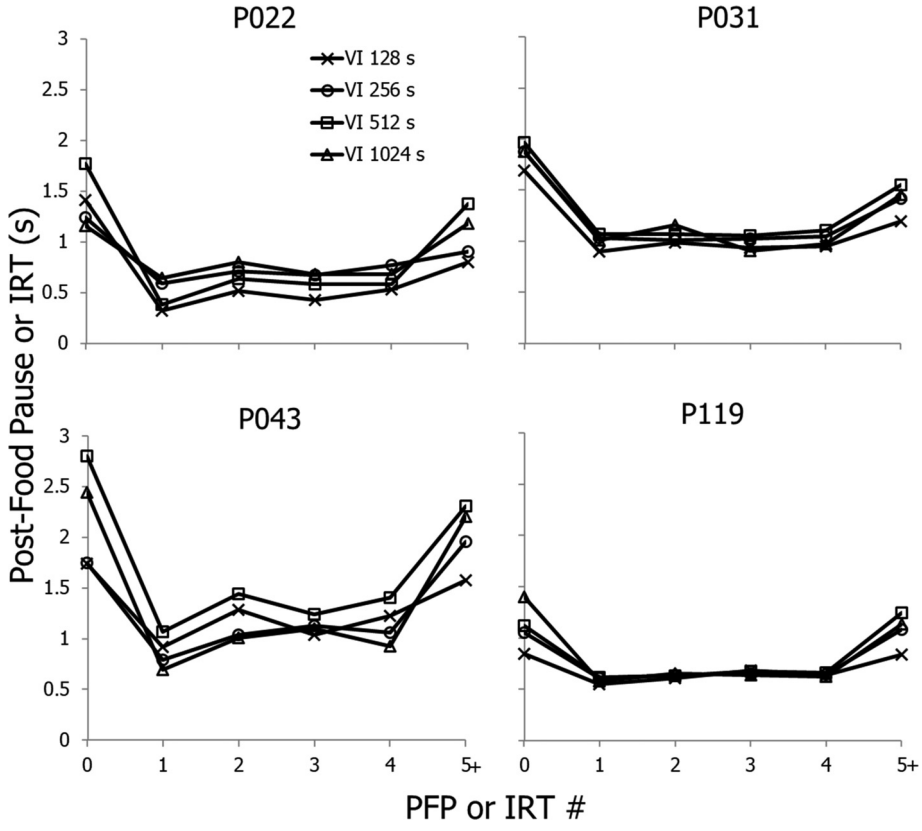

Fig. 9. Postfood pause (PFP) and interresponse times (IRT) immediately following food on VI 128 s (Xs), VI 256 s (circles), VI 512 s (squares) and VI 1024 s (triangles). Points at zero on the x-axis represent mean PFP from Fig. 8. The first through fourth IRTs (means) are represented along the x-axis, and the extreme right-hand points represent the means for all IRTs beyond the fourth IRT (denoted 5+).

parameters K, K_m, q, a, and h used in Figure 11. The parameters K and K_m were fixed while q, a, and h were permitted to vary freely in fitting with MS Excel® Solver. The threshold h is zero for P022, because no leveling off was discernable in the low range of food rates, as would be expected from experiments with conditions extended over many sessions (e.g., Fig. 1). P119 showed two leveling-off regions: one below 0.12 fpm, and one between roughly 1.0 and 10.0 fpm. In the present line of theorizing, the second upturn, from about 13 fpm onwards, reflects a second shift in topography to one that generated even higher peck rates than the first new topography. Thus, Table 3 shows P119's parameters for the low range of food rates and for the high range of food rates. As may be seen in Figure 11, the fits were close and residuals were unsystematic. The large number of parameters in relation to the number of data points makes these fits theoretically illuminating rather than statistically compelling, because the more parameters the more likely is a close fit. The variance accounted for (r^2) ranged from .94 to .99, except for P031, for which response rate varied over a more restricted range and variance accounted for equaled .81.

Table 3 shows typical individual variation. Low-rate tempo for pecking (K) ranged from 24 to 58 pecks/minute. For P119 in the high range of food rates, the lower-rate tempo was set equal to the higher-rate tempo in the low range of food rates: 100 pecks/minute. The coefficient q varied according to the differences in functional units defined by topography, being highest for P022, for which the difference between K and K_m was greatest. The exponent a was highest for

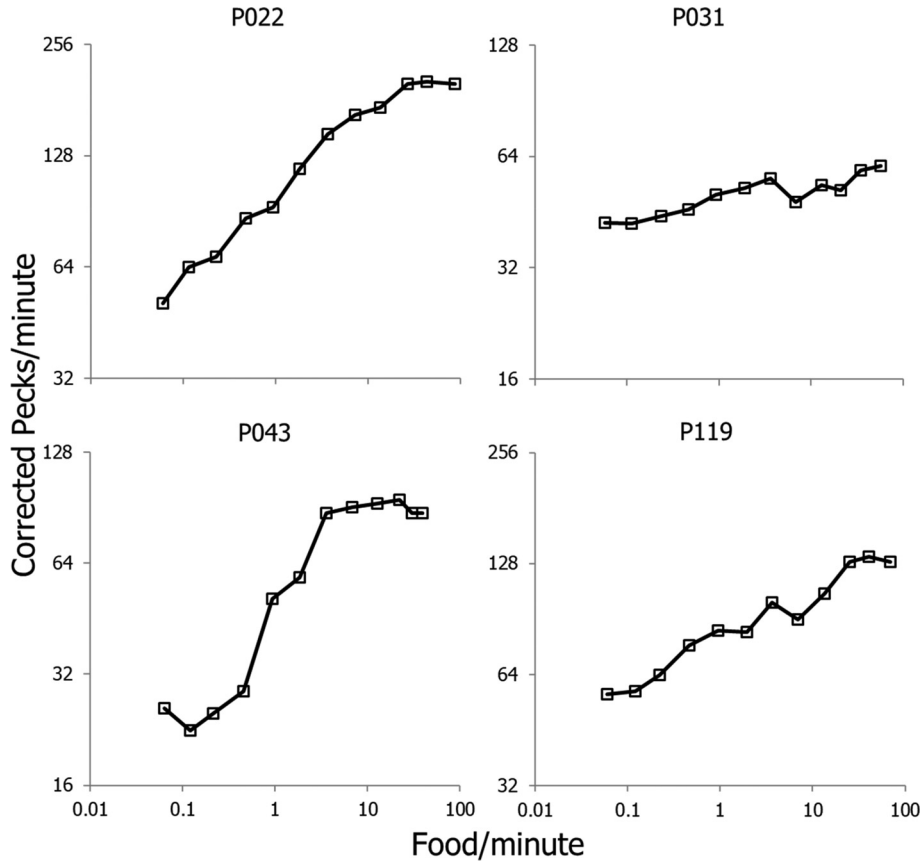

Fig. 10. Mean peck rates from Fig. 4 (Conditions 1, 2, 4, 5, and 6) corrected according to Eqs. 8 and 9 by subtracting the obligatory timeout following food. Note logarithmic axes. All vertical axes cover 3 log units (base 2) but vary in the range.

P043, reflecting high sensitivity to the contingent food.

The question arises: Can we find any support in our data for our speculations about changes in topography underlying the upturns in Figures 10 and 11? Previous studies of pigeons' topographies in interacting with response keys used photography (e.g., Jenkins & Moore, 1973; Smith, 1974). Without photography, another approach is to examine frequency distributions of IRTs. Figures 12 and 13 show IRT distributions across all the schedules (except FR 1) of Conditions 1 and 2 (VIs). The distributions show relative frequency and thus do not reflect response rate, which would depend on the time base, which increased as food rate decreased; sample sizes varied greatly across schedules, being larger for the leaner schedules. All distributions were smoothed with a 5-point running mean. From the bottom distribution (VI 1024 s) to top (VI 1 s), shifts are plainly visible. For P022, two modes appear, at about 0.08 s and 0.39 s. The second mode grows relatively more prominent as food rate increases. The single trend appears to be consistent with the speculation of a single topography shift in P022. P31's IRT distributions show two modes: one about 0.3 s and a poorly defined mode around 0.7 s to 0.9 s. These two modes shift in relative prominence, but in no systematic order, although the longer mode is clearly less prominent for the three leanest schedules (VI 256 s, VI 512 s, and VI 1024 s). Figure 13 shows the IRT distributions for P043 and P119 for the VI schedules in Conditions 1 and 2. Like P022, P043 shows a systematic shift from low to high food rate, as the first mode decreases and the second mode

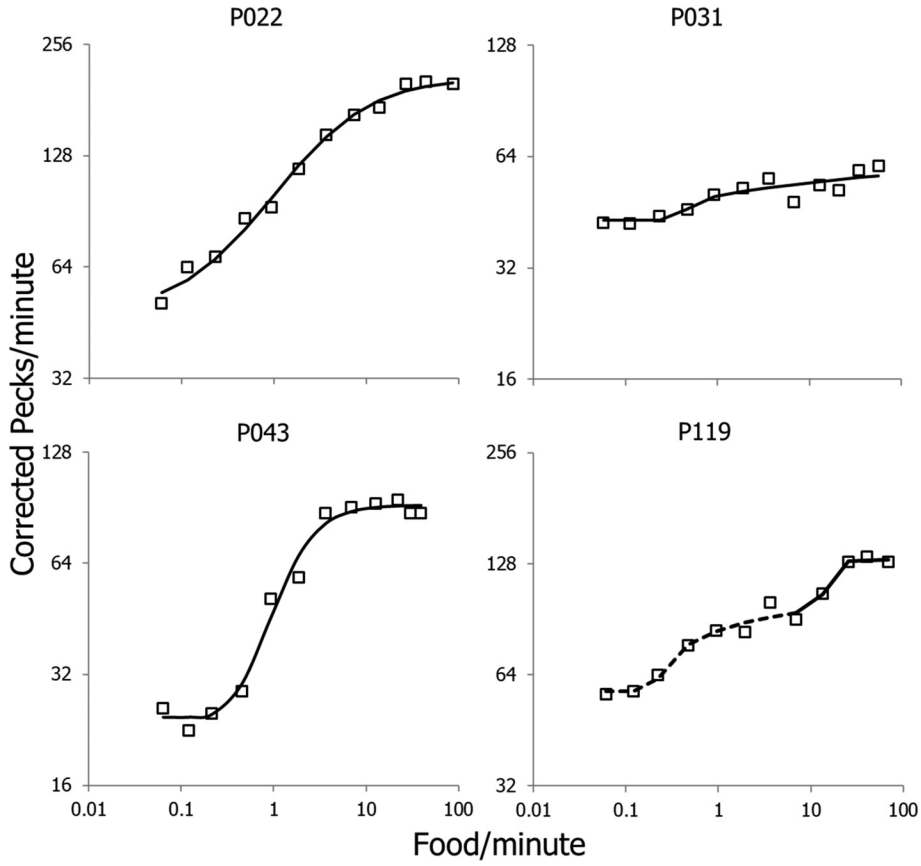

Fig. 11. Eq. 12 fitted to the corrected peck rates from Fig. 10. For P119, Eq. 12 is fitted twice, for the lower range of food rates (broken curve) and for the higher range of food rates (solid curve). Note logarithmic axes. All vertical axes cover 3 log units (base 2) but vary in range.

increases in prominence. A longer third mode appeared at VI 16 s and grew as food rate entered the highest levels, corresponding to the maximum rate in Figure 4. The shifts for P119 as food rate increased were more complex than for the other pigeons. The distribution for VI 1024 s shows three modes. As food rate increased, the short first mode grows less prominent and the longest mode grows more prominent up to VI 32 s. From VI 16 s onwards, the longest mode shifts to the left (i.e., grows shorter) and remains roughly the same in prominence as the second mode grows more and more prominent. For VI 1 s and VI 2 s, which maintained maximal response rates, the third mode all but

Table 3

Parameter estimates in Equation 12 used to fit corrected peck rates in Figure 11

Pigeon	Asymptote K	Asymptote K_m	Coefficient q	Exponent a	Threshold h
P022	48	208	2.09	0.88	0.0
P031	43	62	1.41	0.33	0.45
P043	24	92	1.45	1.69	0.15
P119 (low)	58	100	0.52	0.60	0.21
P119 (high)	100	133	0.08	0.48	13.4

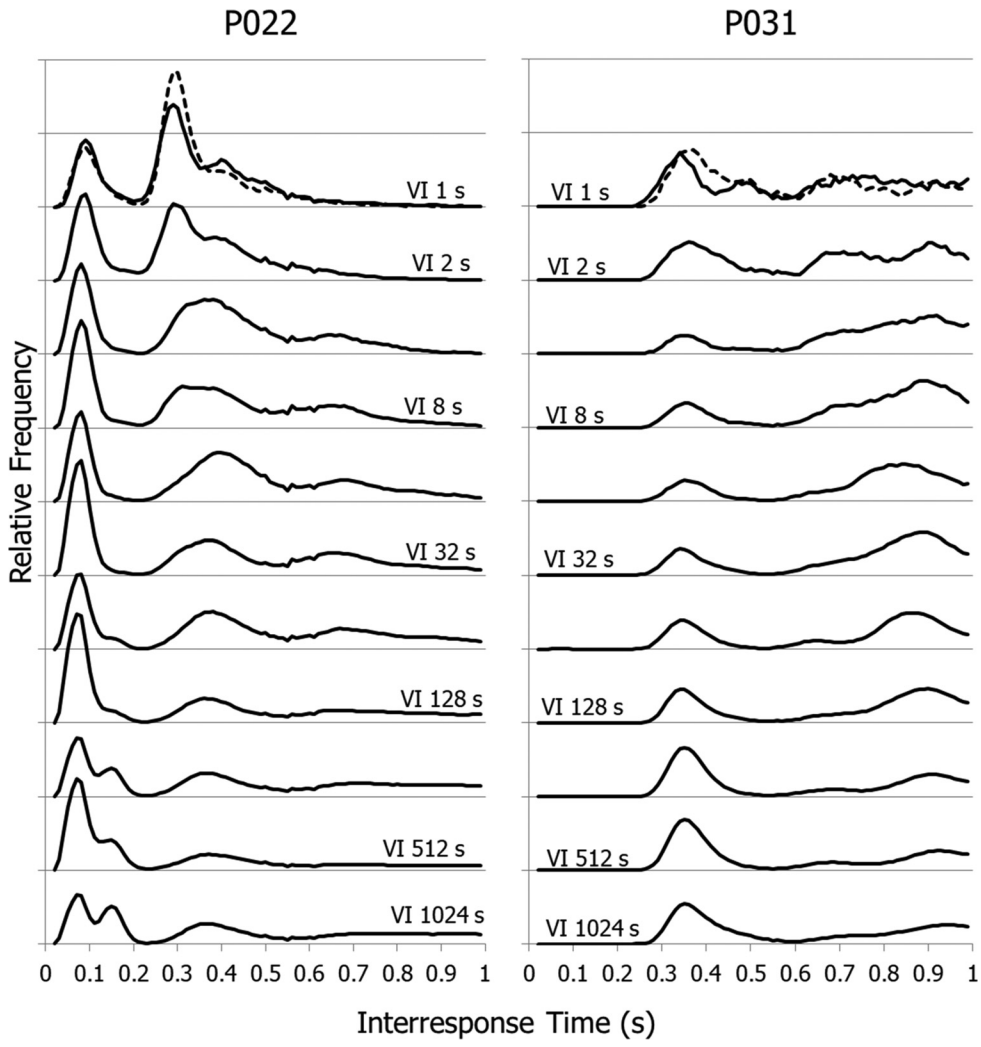

Fig. 12. Interresponse time frequency distributions across the VI schedules in Conditions 1 and 2 for P022 and P031. The vertical axis shows relative frequency out of all interresponse times collected. All distributions are plotted on the same vertical scale; the vertical distance between two horizontal axes represents 0.035 in relative frequency.

vanishes. The two trends seem consistent with the two shifts in response rate seen in Figure 11. The two sets of parameters in Table 3 reflect these shifts. The upper range, beginning with threshold h of 13.4 fpm, starts with a food rate between those maintained by VI 8 s and VI 4 s.

To clarify further, Figure 14 shows the ratios of relative frequencies between modes in Figures 12 and 13 as a function of food rate. P022 shows the increasing relative prominence of Mode 2, increasing slowly at lower food rates and faster at higher food rates. P031 shows no systematic trend. P043 shows a sudden increase at high food rates. To the extent that the increases in Mode 2 for P022 and P043 at the highest food rates represent changes in topography, the account of the downturn (Fig. 4) as due entirely to an obligatory delay following food may be incomplete.

Two lines appear in Figure 14 for P119. The diamonds show the relative frequency ratio for Mode 2 relative to Mode 1, as for the other pigeons. The squares show the relative frequency ratio for Mode 3 relative to Mode 2. This ratio remained about the same until

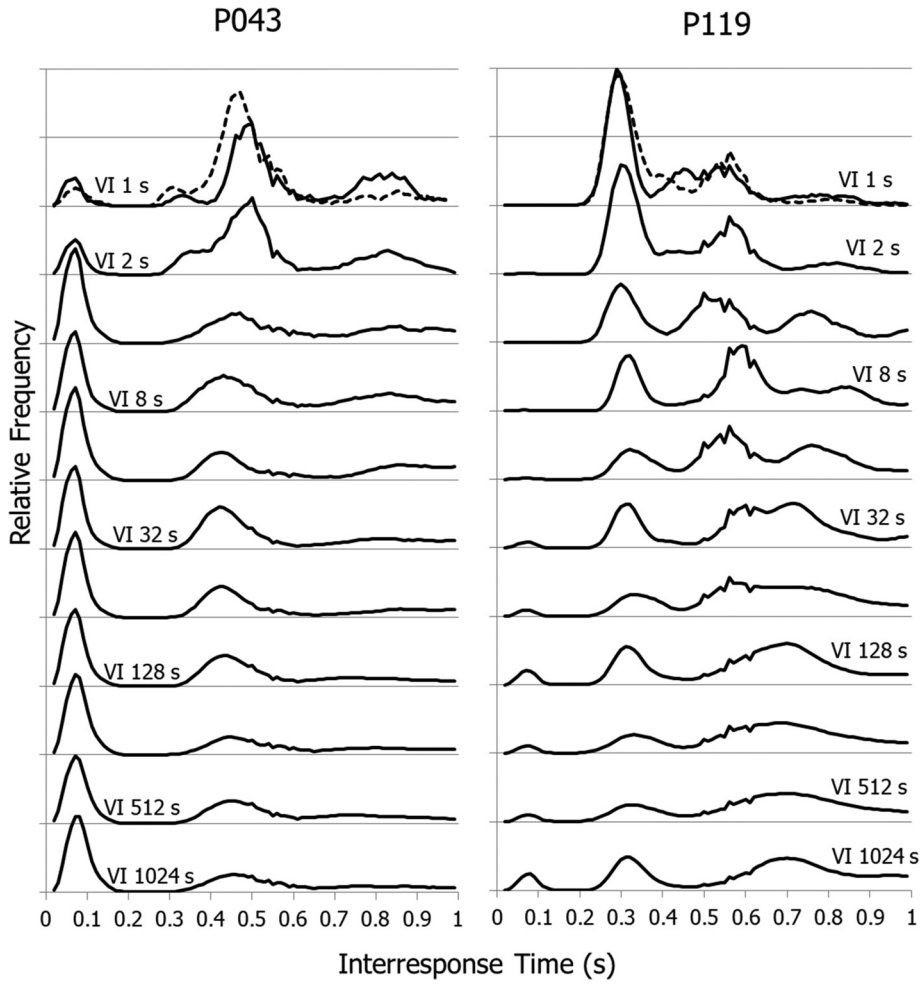

Fig. 13. Interresponse time frequency distributions across the VI schedules in Conditions 1 and 2 for P043 and P119. The vertical axis shows relative frequency out of all interresponse times collected. All distributions are plotted on the same vertical scale; the vertical distance between two horizontal axes represents 0.035 in relative frequency.

food rate increased above about four foods/minute, when Mode 3 began decreasing in prominence relative to Mode 2. These changes seem consistent with the two shifts seen in Figure 11.

Figures 12,13, and 14 show changes in IRTs as VI schedules became more and more ratio-like, but the question remains as to what were the effects of adding an actual ratio schedule in Condition 6? Figure 15 shows the same analysis for Condition 6 (VI + RR) as appears in Figures 12 and 13. P022 shows two modes and a gradual increase in relative prominence of Mode 2 with increasing food rate, as in Figure 12. P031 shows a well-defined short mode and a less well-defined longer mode, also as in Figure 12. P043 shows a different pattern in Figure 15 from that in Figure 13: Two modes appear, and Mode 2 grows in relative prominence with increasing food rate, as in Figure 13, but, contrary to Figure 13, Mode 2 never becomes more prominent than Mode 1. This difference probably stems from the absence of VI 1 s and VI 2 s; the shift for Mode 2 occurred for those extremely rich schedules. Only for P119 did the IRT distribution for the ratio schedule appear different from the distributions for the VI schedules. The difference is due to the absence of VI 1 s from Condition 6, because across the longer VI schedules, the

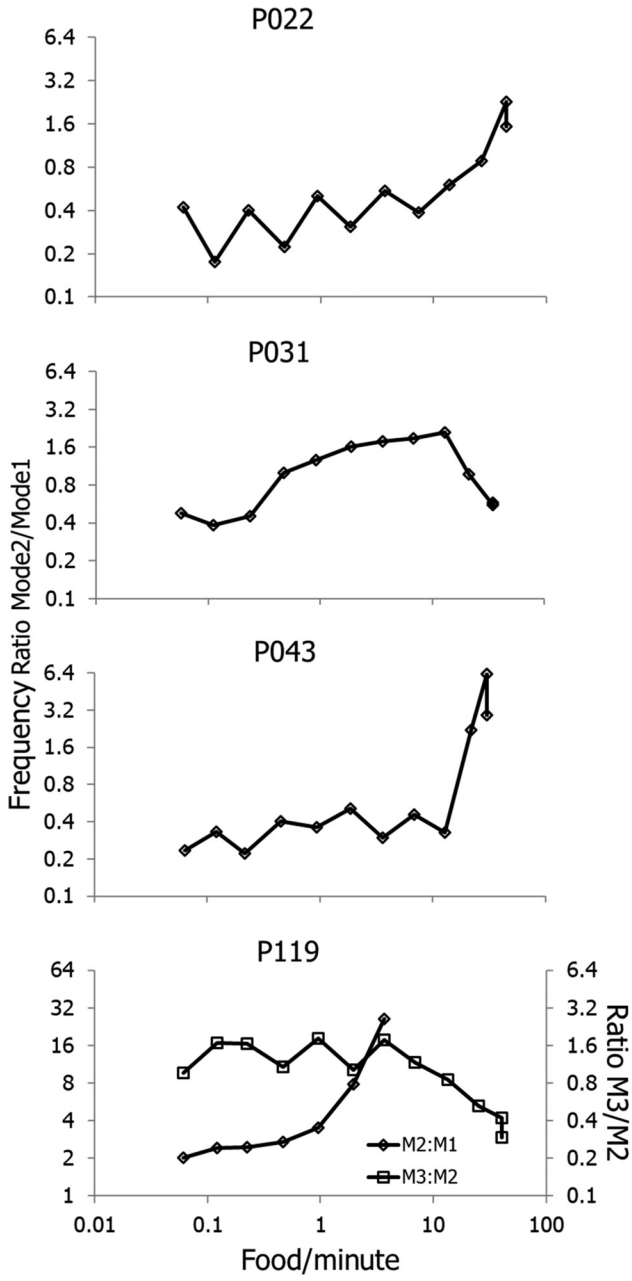

Fig. 14. Relative frequency ratios from Figs. 12 and 13. The left vertical axis shows the ratios of Mode 2 to Mode 1 (longer to shorter). The graph for P119 includes the ratio of Mode 3 to Mode 2 (squares; right-hand vertical axis). Note logarithmic axes. All vertical axes cover 6 log units (base 2) but vary in range.

shifts resemble those in Figure 13, and the ratio schedule shows an extremely prominent Mode 2, like VI 1 s in Figure 13. Thus, responding on the VI 1 s schedule resembled responding on the RR schedule.

Figure 16 shows the same relative frequency ratios for Figure 15 as Figure 14 showed for Figures 12 and 13. Compared with Figure 14, however, Figure 16 covers a smaller range of food rates, because the VI schedules varied

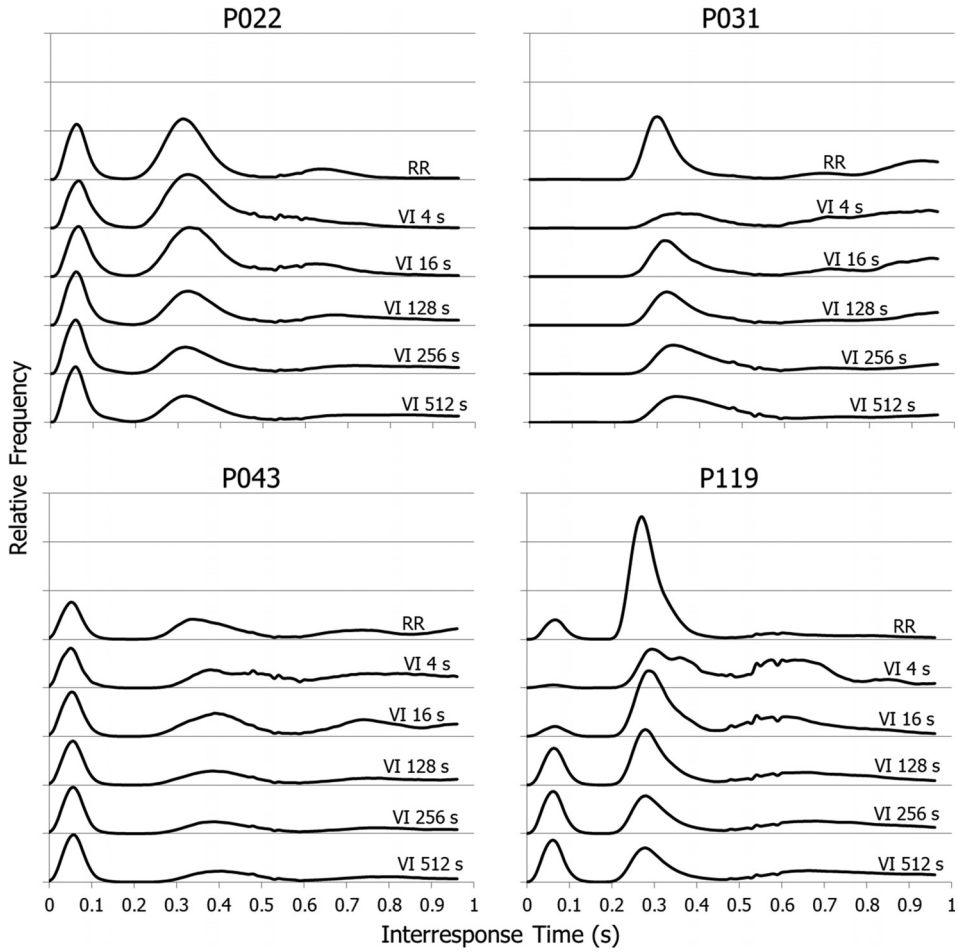

Fig. 15. Interresponse time frequency distributions across the VI and RR schedules in Condition 6. The vertical axis shows relative frequency out of all interresponse times collected. All distributions are plotted on the same vertical scale; the vertical distance between two horizontal axes represents 0.035 in relative frequency.

only from VI 512 s to VI 4 s. The filled symbols show ratios for the RR schedule, placing them at the intermediate food rates that the RR schedules delivered. As expected from Figure 15, and similar to Figure 14 for the same range of food rate, P022 shows increasing relative prominence of Mode 2, but with the point for the ratio schedule appearing a little above the trend. P031 shows no clear trend in relative prominence of Mode 2, as in Figure 14, but a point for the ratio schedule above the line for the VI schedules. P043 shows an increasing trend for the VI schedules similar to that in Figure 14 across the same range of food rates, and the relative prominence of Mode 2 not much different for the RR schedule. P119 shows increasing prominence of Mode 2 relative to Mode 1 (diamonds), as in Figure 14, and a ratio for the RR schedule a bit above the line for the VI schedules. Figure 16 departs from Figure 14 in the increase in prominence of Mode 3 relative to Mode 2 (P119; squares) for VI 4 s, the highest food rate. P119 differs from the other pigeons in showing a sharp reduction in relative prominence of Mode 3 for the RR schedule. This decrease resembles the decrease for VI 1 s and VI 2 s in Figure 14.

Discussion

The results indicate that power-function induction and competition among activities (the "matching law"; Eqs. 1 and 2) may

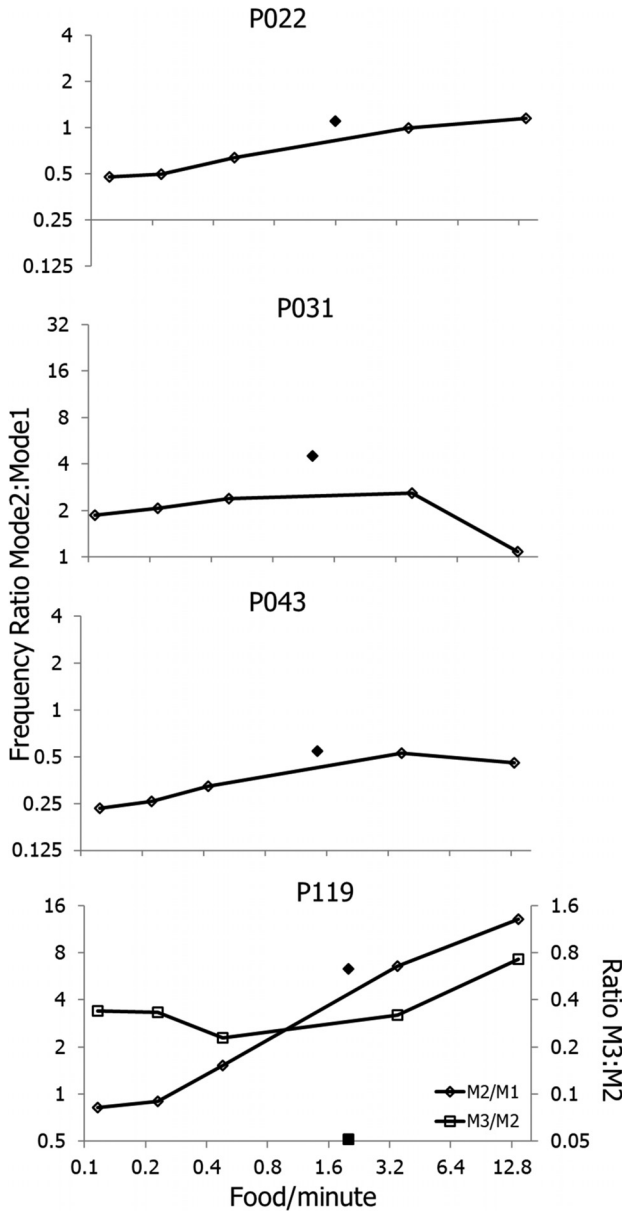

Fig. 16. Relative frequency ratios from Fig. 15. The left vertical axis shows the ratios of Mode 2 to Mode 1 (longer to shorter). The graph for P119 includes the ratio of Mode 3 to Mode 2 (squares; right-hand vertical axis). Note logarithmic axes. All vertical axes cover 5 log units (base 2) but vary in range.

explain response rates on VI schedules as food rate varies. Equations 5, 6, and 7 reformulated the basic matching equations for single schedules in terms of power-function induction, and those equations, combined with a threshold for a shift in topography at high food rates (Eqs. 10, 11, and 12), explained response rates from earlier research (Figs. 1 and 2; Baum, 1993; Catania & Reynolds, 1968) and the present experiment (Fig. 11).

The phenomena in performance on VI schedules that require explanation are, as food rate increases: (a) the negatively accelerated increase in response rate up to

moderately high food rates (about 2 fpm); (b) the upturn in response rate from moderately high to high food rates; and (c) the downturn in response rate for the extreme food rates. Figure 4 shows that we replicated these three phenomena, reported by Baum (1993), in our study. The only exceptions were: no downturn at extreme food rates for P031; and no clear flattening at the low end of food rates for P022. These exceptions had no effect on the conclusions to be drawn from the overall results, because previous research indicates that P022's response rate would flatten out at food rates lower than the ones included in the present study (e.g., Herrnstein, 1970), and P031 failed to make the required discrimination among the component schedules.

The negatively accelerated increase in the low-moderate range of food rates (Fig. 1), originally described by Equation 4, is better explained by Equation 7, derived from Equation 6, which incorporates competition between activities as they are induced by the food. Equation 4 hinted at competition, but did not incorporate induction specifically.

The upturn in the moderate-high range of food rates reflects a change in the topography of the operant activity from pecking to swiping or nibbling at the key. This change is a shift from responding typical of longer interval schedules toward responding more typical of ratio schedules. As VI schedules get shorter and shorter, the VI starts more and more to resemble a ratio schedule, as illustrated by the feedback functions in Figure 3. We confirmed this hypothesis in Figure 7 with feedback functions derived from the data, and, consistent with our prediction, from VI 4 s to VI 2 s to VI 1 s, the feedback functions for short VI schedules more and more approximated the linear feedback function for FR 1. The more ratio-like is the VI schedule, the more it selects and induces activities with topography beyond pecking, such as swiping or nibbling at the key. As the food rate increases from moderate to high, these activities replace simple pecking, until the rate of operating the key equals a rate typical of that maintained by ratio schedules (Baum, 1993). P119 (Fig. 11) presents a particularly illuminating example, because topography appeared to shift twice. As food rate increased from moderate to high, a faster-switch-operating activity replaced pecking, but at still higher food rates, that activity also gave way to another even more efficient activity that operated the key at a still higher rate. Baum and Davison (2014) found evidence of similar shifts in topography in the lever pressing of rats in an experiment by Soto, McDowell, and Dallery (2005). The hypothesized shifts in topography found empirical support here in corresponding shifts of IRT frequency distributions (Figs. 12 – 16).

As a test of the theory that peck rates induced by the short VI schedules reached levels characteristic of ratio schedules, we inserted a RR component in Condition 6 (filled diamonds in Figs. 4 and 6). For three pigeons, the RR schedule maintained peck rates no higher than the short VI schedules. The exception was P119, for which the RR peck rate was higher than any of the VI peck rates. The difference suggests that still another shift in topography was possible for P119 and occurred when the schedule was a RR.

We explained the downturn at extremely high food rates as an artifact due to the physical arrangements of the apparatus. A pigeon requires a small amount of time to remove its head from the food hopper and move up to peck the key. This obligatory activity is brief, but becomes significant at extremely high food rates. This explanation follows a similar proposal by Wetherington (1979) in a study of food-induced drinking in rats. Baum and Davison (2014) treated the downturn as the result of competition from induced activity. The present account equates with theirs if one assumes that the activity that competes with the operant activity is the obligatory activity of moving from food hopper to key. Their explanation and the present explanation resemble each other functionally, but conceiving of the downturn as competition suffers from two defects. First, Baum and Davison had to make additional assumptions that might have little empirical support. They had to assume, for example, that B_O increases even as B hits a maximum, but one may wonder why B_O should increase slower than B at low food rates and faster than B at high food rates. Second, the activity of moving from hopper to key is obligatory. It does not compete with pecking the way other activities compete with pecking, because time taken by other activities is time that could be taken by pecking, whereas time taken moving from hopper to key is time when

pecking cannot occur; it is functionally a timeout. Routine practice subtracts the time spent eating—hopper duration—from the time base for this reason, and subtracting the hopper-to-key time makes sense for the same reason. Accordingly, subtracting delay d from the time base is both simpler and more logical.

Figure 8 supports the rationale for correcting the downturn at extreme food rates (Eqs. 8 and 9). The reasoning behind Equations 8 and 9 was that the PFP consists of three components: pausing (i.e., other activities than key pecking); the time required to peck the key; and the time required for the pigeon to lift its head out of the food hopper. Figure 8 (squares) shows that mean PFP increased as food rate decreased, indicating that other activities besides pecking crept into the PFP as food rate decreased. The method of programming the VI schedules by multiplying intervals by the mean required may explain the pausing (i.e., intrusion of other activities), because the shortest interval across schedules varied directly with the mean interval. The decrease in shortest interval for the short VI schedules may have contributed to the increase in prominence of Mode 2 in IRT distributions for P022 and P043 (Figs. 12 and 13). The triangles in Figure 8 show that mean IRT ($\leq$ 1.0 s) was substantially shorter than the PFP, as expected. That this mean IRT varied little with food rate suggests that variation in peck rate depended primarily on pausing (i.e., interjection of other activities).

In Conditions 3 and 7, we tested the role of contingency—requiring operation of the response key for producing food. Figures 5 and 6 show that removing the contingency by switching the VI schedules to VT schedules lowered response rates, demonstrating that the contingency adds to the peck rates. Two points are noteworthy. Firstly, rates remained substantial and the pattern of rates remained except at low food rates. The maintained rates, after as many as 20 sessions without contingency, indicate that food continued to induce pecking. The high peck rates at the red key light (Fig. 6; filled diamonds) further support the conclusion that pecking was induced by the food and its key-light proxies (Baum, 2012a, 2018a, b).

Historically, the maintenance of pecking on the VT schedules in Figures 5 and 6 might have been attributed to "adventitious reinforcement," on the theory that pecking might be reinforced by accidental coincidences with food delivery. Skinner (1948) proposed such maintenance with his "superstition" experiment. Since the replication of Skinner's procedure by Staddon and Simmelhag (1971), and Segal's (1972) introduction of the concept of induction, that explanation has become less and less tenable. Staddon and Simmelhag pointed out that adventitious reinforcement is unlikely from an evolutionary perspective, because organisms easily misled by such coincidences would be less fit than others not so easily misled. Several empirical studies discredit adventitious reinforcement. Rachlin and Baum (1972) found that when contingent and noncontingent food are mixed, pigeons' responding discriminates between the two sources of food in the same way as if they were both contingent from two different keys (see also Kuroda et al., 2013). Killeen (1977) found that pigeons in a detection task accurately distinguished contingent from noncontingent food. Baum (1981b) showed that pigeons readily distinguish positive from negative temporally extended contingencies. Kuroda and Lattal (2018) found that pigeons distinguish extended contingency from noncontingency even when local contingencies are equated.

Instead of attributing the maintenance of VT pecking to adventitious reinforcement, we might speculate that the reduction in pecking results from competition with other induced behavior once the contingency is removed. Apparently, noncontingent food brings back B_O (Eqs. 4, 5, and 6) to compete with the induced pecking, particularly at low food rates. The contingency ensures that pecking is induced selectively, enhancing competition against other induced activities (Baum, 2012a, 2018a, b). The selection and enhanced competition may explain why B_O in Equation 6 might decrease as B increases. That is, because r represents food contingent on pecking, increasing r actually suppresses other behavior induced by food (B_O). If s_I and s_O were constant, however, for B_O to decrease s_O must be negative, a conclusion that seems improbable. Another possibility would be that s_I and s_O are neither constant nor independent—that the removal of the contingency decreases s_I and increases s_O. A systematic study of the effects of contingency remains to be done.

Another possible explanation of the maintained VT peck rates in Figures 5 and 6 might be that the continued contingency in the FR 1 component somehow supported pecking in the other components. An effect of FR 1, however, would fail to explain the drop in peck rate due to the removal of the contingency. Pecking discriminated the lack of contingency in the VT components.

The line of research and theory developed in the present paper elaborates Segal's (1972) concept of induction in at least two ways. Firstly, we quantify induction with power functions. This is probably no accident. Power functions find their way into the study of behavior when the variables being related are measured on ratio scales—that is, magnitude scales with equal units and a nonarbitrary zero. A notable example is the psychophysical power law (Stevens, 1975). Stevens (1960) found many sensory continua to conform to power-law sensitivity and pointed out the ratio scales make power functions likely. Power functions also describe many natural phenomena, because they are timescale-invariant and common in self-similar dynamical systems (Gisiger, 2001). Secondly, we take the concept of induction to its logical conclusion: that, not only adjunctive behavior, but also operant behavior is induced by the so-called "reinforcers" and other PIEs (Baum, 2012a, 2015). A contingency, say, between food and lever pressing, establishes covariance between pressing rate and food rate, and this covariance converts the operant activity (pressing) into food-induced activity (Baum, 2012a, 2018a, 2018b), Experimental results with food-induced drinking, for example, indicate that food induces whole patterns of behavior, including operant activities and induced drinking (Ruiz, López-Tolsa, & Pellón, 2016), as Staddon (1977) suggested. The maintenance of pecking seen in Figures 5 and 6, for example, supports this extension. These two elaborations allow induction to be integrated with matching theory to provide a powerful theoretical framework for understanding behavioral allocation.

In this paper, we adopted a "molar" (multiscale) view of behavior. We treated behavior as composed of temporally extended activities instead of discrete momentary responses. Baum (2013, 2017, 2018b) pointed out several reasons, both ontological and epistemological, why the units of behavior must be temporally extended. The molecular view of behavior that assumes discrete responses and momentary causation by response–reinforcer contiguity fails to provide explanations of even the most basic phenomena of behavior in the laboratory—for example, the characteristic response rates maintained by ratio versus interval schedules (Baum, 2018b). A theory of behavior that explains laboratory phenomena, to be useful, must also explain phenomena in everyday behavior. A theory that explained only phenomena in the laboratory would be pointless.

Have we explained performance on VI schedules at all, let alone in a way that might be applied to everyday behavior? Does our model explain the phenomena, or does it merely describe them? The physicist Ernst Mach (1960/1933) pointed out that explanation *is* description—just description in terms more general than the original observation. Not all descriptions are explanations, but all explanations are descriptions. For example, we explain lightning by describing it as static electricity. Some philosophers of science add that the description should be causal, though others disagree. Whether necessary or not, our model passes this test too, because food rate is a causal variable.

Conclusion

At the time that Baum and Rachlin (1969) proposed their generalized version of matching as time allocation, they had not incorporated the concept of induction nor turned to power functions as the functional form that would offer more generality. In hindsight, we now can see that power-function induction is the heart of the matching law. Equations 5, 6, and 7 embody this insight.

These equations, combining matching theory with induction, allowed us to explain the performances we observed on the wide range of VI schedules, from lean to the extreme of FR 1. The only modification required was to insert a threshold food rate at which ratio-like interaction with the response key began to replace interval-like "pecking" as food rate increased from moderate to high. This explained one key observation: the upturn in peck rates as they went from moderate to high.

Another observation that required explaining was the downturn in peck rate at extremely high

food rate. We showed that this downturn is an artifact of the apparatus, due to the fraction of a second required for a pigeon to lift its head out of the food hopper.

Those two phenomena, reported by Baum (1993), among others, were relatively new, in comparison with performance on lean-moderate VI schedules, which was accounted for well by Herrnstein's (1970) proposal—Equation 4 in this paper. Equation 7 is an update of Herrnstein's equation based on power-function induction. By incorporating induction of other activities than the operant activity, the update explicitly introduces the concept of competition—that is, competition among activities for the time available.

Combining matching theory with induction affords an elegant foundation for understanding all behavior—operant and otherwise—in a single conceptual framework. Future research may build on this foundation and develop the theory further.

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Appendix

The method we used for estimating stable response rate is easy to use in a spreadsheet program. One has a series of data (rates here) that change over time and come to equilibrium. Variation never ceases, but may cease to exhibit a trend. The top curves in Figure A1 show two examples. To find the points to average, one fits a least-squares regression line to all the points, then a line excluding the first point, then excluding the first two points, then excluding the first three points, and so on. One examines the slopes of these fitted lines in sequence, as shown in the lower graph in Figure A1. The slope decreases in absolute value toward zero. At some point, the slope approaches closest to zero, and one averages the rates from there onwards (filled symbols).

As the number of points included in the fit decreases, the slope may become highly variable, because the slope's estimation depends on too few points. One may then choose a slope close to zero but including an adequate number of data points. This method was used by Bell and Baum (2017) to estimate stable response rates across sessions.

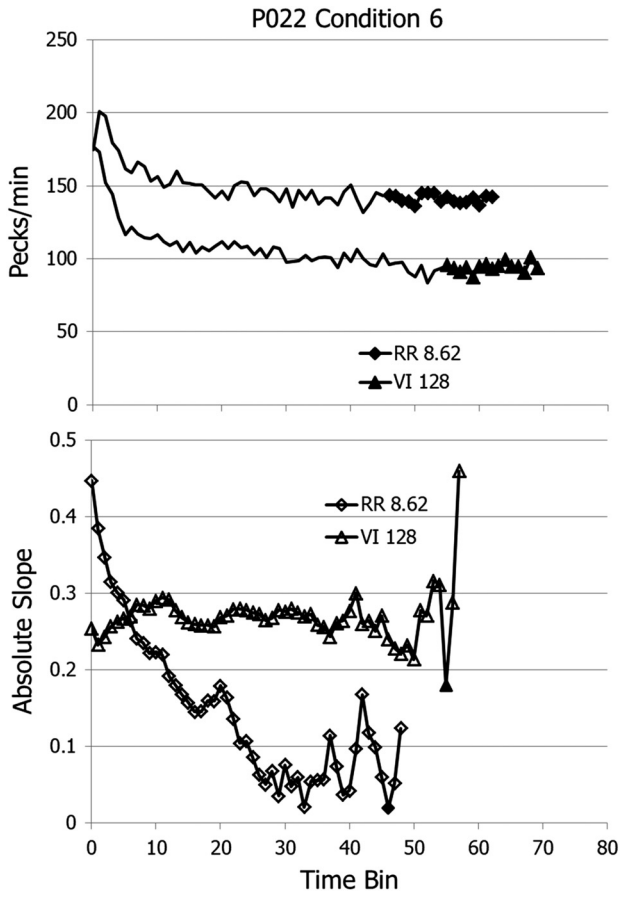

Fig. A1. Illustration of the method for estimating stable response rates. Results for P022 are shown for two schedules in Condition 6: VI 128 s and RR 8.62. Top graphs show response rates in order during components. The filled points show the bins that were averaged to estimate stable rate. Bottom graphs show absolute value of least-squares slopes from each bin to the last bin. The filled points show the minimum slopes.