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The dynamics of vapor phase growth yields important information to the understanding of nucleate boiling. The history of the growth of a vapor bubble may be divided into three stages known as the initial, transition and asymptotic stages. In the initial stage, the bubble is very small and the growth is slow due to the high surface force that arrests its radial expansion. The transition from this slow growth to the asymptotic growth occurs in a very short time (approximately 10^{-2} sec).[1] In the asymptotic stage, the bubble has reached a size where the inertia, viscous and surface forces can all be neglected in comparison with the pressure force. This is the stage that provides practical interest.

Plesset and Zwick[2], using the thin thermal boundary layer approximation, obtained a solution for the asymptotic bubble growth in a uniformly supersaturated liquid. Birkhoff, Margulies and Horning[3] and also Scriven[4] obtained an exact solution to this problem using the method of similarity transform. The exact solution agrees completely with Plesset and Zwick's calculation at large rate of growth. Skinner and Bankoff[5], using a similar approach as that of Plesset and Zwick, developed a parametric solution for bubble growth in general temperature fields.

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In this paper, we shall apply this solution to two practical cases, i.e., the constant wall temperature and the constant heat flux cases[6].

The parametric solution for the hemispherical bubble growth by Skinner and Bankoff can be expressed as

$$\mathcal{R}^3(\zeta) = 3J \int_0^\infty g(m) \text{ erfc } \frac{m}{2\zeta^{\frac{1}{2}}} \, dm \, , \qquad (1)$$

and

$$\tau(\zeta) = \frac{\alpha_\ell t}{\ell^2} = \int_0^\zeta \frac{d\zeta'}{\mathcal{R}^4(\zeta')} \, , \qquad (2)$$

where g(m) is a dimensionless initial condition,

$$g(m) = \frac{\int_0^\pi [f(m)-T_s] \sin\theta \, d\theta}{\int_0^\pi [f(0)-T_s] \sin\theta \, d\theta} \, , \qquad (3)$$

and f(m) is the initial condition.

For the case of constant wall temperature, assuming no convection, the initial condition for bubble growth is[7]

$$f(r,\theta) = T_\infty - (T_w - T_\infty) \text{ erfc } \frac{r \cos\theta}{\ell} \, , \qquad (4)$$

or,

$$g(m) = 1-\omega\left[\text{erf}(3m)^{1/3} - \frac{(3m)^{-1/3}}{\sqrt{\pi}} \left(1-e^{-(3m)^{2/3}}\right)\right] , \qquad (5)$$

where

$$\omega = \frac{T_w - T_\infty}{T_w - T_s} \, . \qquad (6)^*$$

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* ω can be expressed more generally by

$$\omega = \frac{\text{interface temperature - bulk temperature}}{\text{interface temperature - saturation temperature}} \, .$$

Therefore, ω=0 signifies the uniform superheat case. $0<\omega<1$ means that the liquid is superheated, however a gradient of temperature from the bulk to the interface exists. ω=1 means the bulk liquid is at its boiling temperature. $\omega>1$ is the case of subcooled boiling.

For the case of constant heat flux with no convection, the initial condition is[7]

$$f(r,\theta) = T_\infty + T_q \ \mathrm{ierfc} \frac{r \cos \theta}{\ell}, \tag{7}$$

or, $$g(m) = 1-\omega \left[1-e^{-(3m)^{2/3}} + \frac{\sqrt{\pi}}{2} (3m)^{1/3} \mathrm{erfc}\ (3m)^{1/3} - \tfrac{1}{2}(3m)^{-1/3} \gamma\left(\tfrac{3}{2}\ (3m)^{2/3}\right)\right], \tag{8}$$

where $$\omega = \frac{T_q/\sqrt{\pi}}{T_\infty + \frac{T_q}{\sqrt{\pi}} - T_s}. \tag{9}$$

Eqs. (1) and (2) with the two initial conditions, Eqs. (5) and (8) are solved numerically. The results are given in Figure 1 and Figure 2.

The results show the same qualitative feature as those obtained by Skinner and Bankoff for the cases of a linear temperature profile and an exponential temperature profile. However, the present calculation should yield more precise information for vapor bubble growth in liquids.

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Nomenclature

f: An initial temperature field for bubble growth

g: A dimensionless temperature $= \dfrac{\int_0^\pi [f(m)-T_s] \sin\theta \, d\theta}{\int_0^\pi [f(0)-T_s] \sin\theta \, d\theta}$

J: Jakob numberfor heat transfer $= \frac{\kappa_\ell}{\rho_\ell L \alpha_\ell} \int_0^\pi [f(0,\theta)-T_s] \sin\theta \, d\theta$

L: Latent heat of vaporization

ℓ: A characteristic length

m: A dimensionless Lagrangian coordinate $= \frac{1}{3} \frac{r^3-R^3}{\ell^3}$

R: Radius of a bubble

$\mathcal{R}$: A dimensionless radius = R/ℓ

T: Temperature

t: Time

α: Thermal diffusivity of a liquid

ζ: A dimensionless Lagrangian coordinate $= \frac{\alpha_\ell}{t^6} \int_0^t R^4(t)\, dt$

κ: Thermal conductivity of a liquid

ρ: Density of a fluid

τ: A dimensionless time

ω: A dimensionless temperature $= \frac{\text{Interface temperature - bulk temperature}}{\text{Interface temperature - saturation temperature}}$

<u>Subscripts</u>

ℓ: Refers to properties in the liquid phase

q: Refers to a characteristic quantity for the constant heat flux case

s: Refers to quantities at equilibrium (or saturation)

w: Refers to quantities at the wall

∞: Refers to quantities in the bulk medium

<u>Functions</u>

$$\text{erfc } x = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} \, dt$$

$$\text{ierfc } x = \int_x^\infty \text{erfc } t \, dt$$

$$\gamma(a,x) = \int_0^x e^{-t} t^{a-1} \, dt$$

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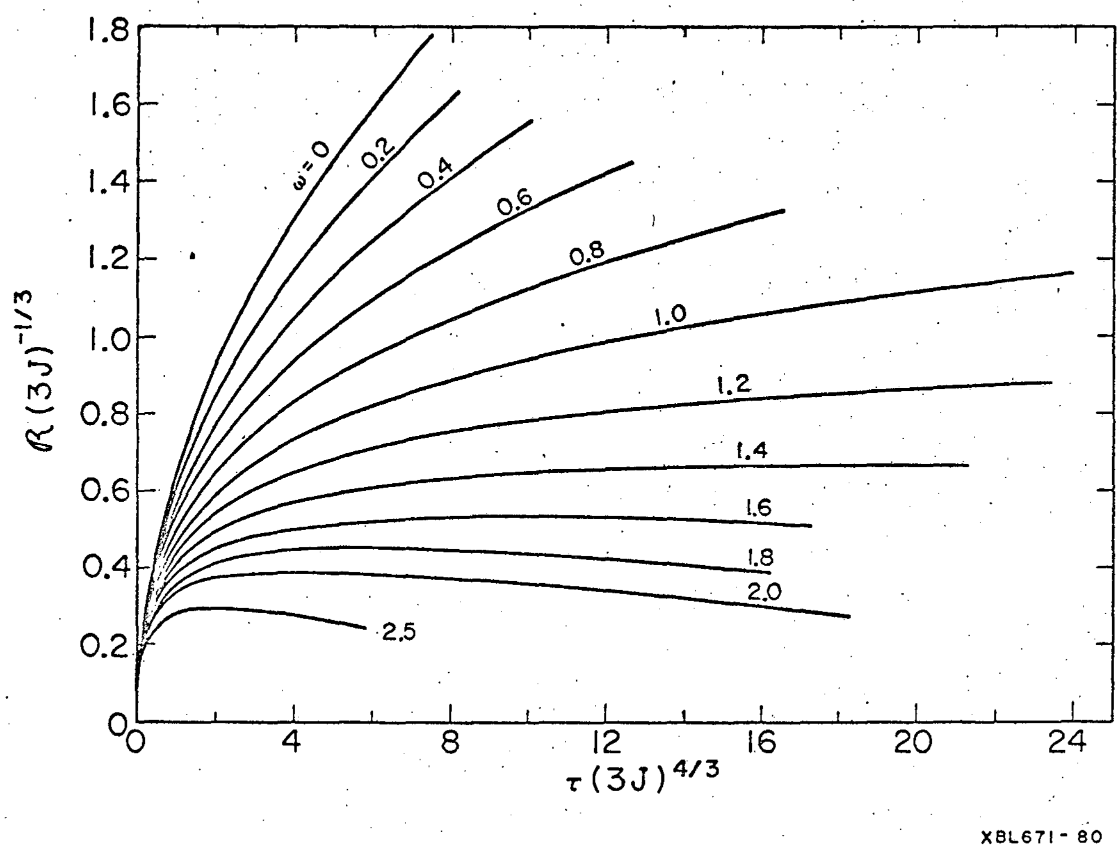

Figure 1. The Case of Constant Wall Temperature.

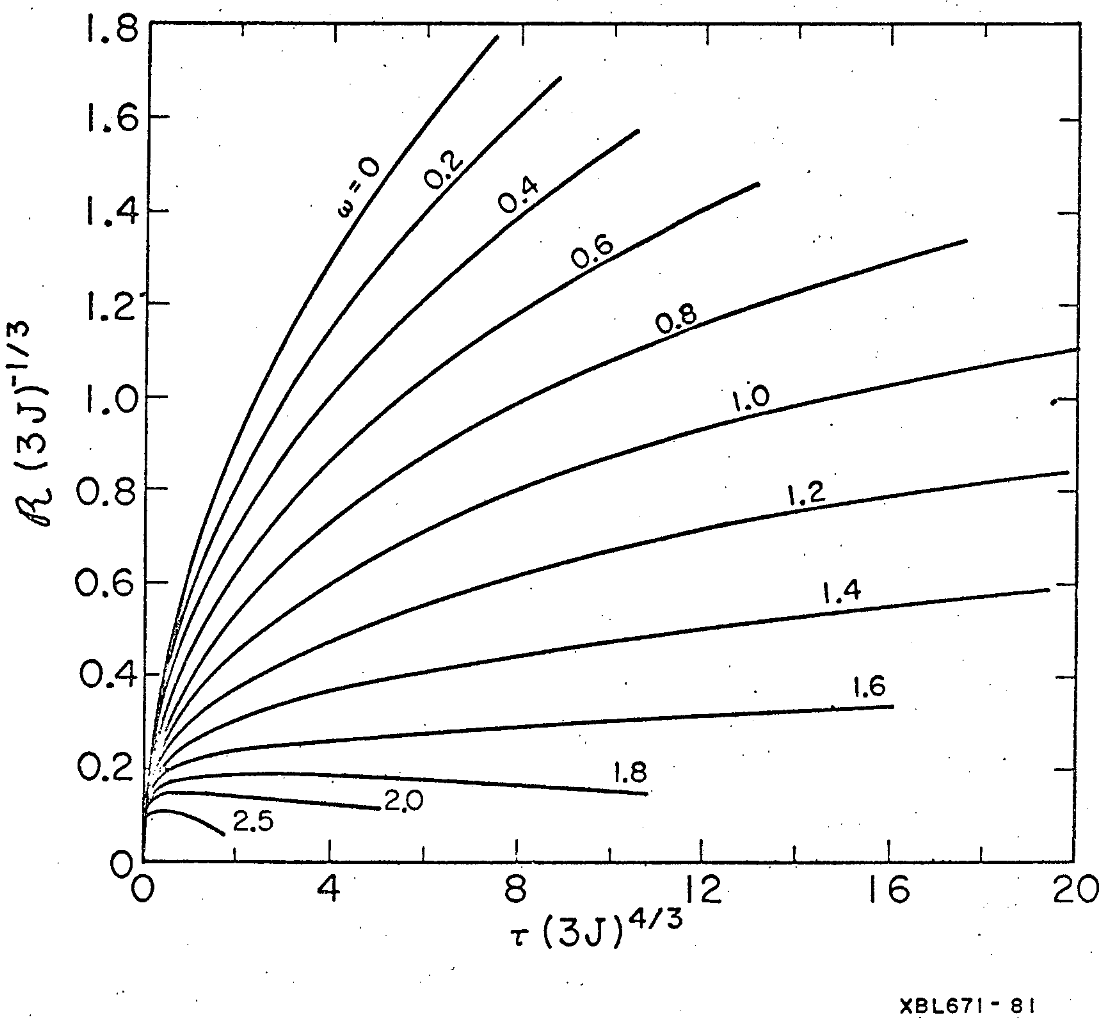

Figure 2. The Case of Constant Heat Flux.