

# Lawrence Berkeley National Laboratory

## Recent Work

### Title

ON THE DYNAMICS OF HEMISPHERICAL PHASE GROWTH IN NONUNIFORM TEMPERATURE FIELDS

### Permalink

<https://escholarship.org/uc/item/9s61529n>

### Authors

Cheh, H.Y.

Tobias, Charles W.

### Publication Date

1967-03-01

**University of California**  
**Ernest O. Lawrence**  
**Radiation Laboratory**

**ON THE DYNAMICS OF HEMISPHERICAL PHASE  
GROWTH IN NONUNIFORM TEMPERATURE FIELDS**

**TWO-WEEK LOAN COPY**

*This is a Library Circulating Copy  
which may be borrowed for two weeks.  
For a personal retention copy, call  
Tech. Info. Division, Ext. 5545*

**Berkeley, California**

## **DISCLAIMER**

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

UNIVERSITY OF CALIFORNIA  
Lawrence Radiation Laboratory  
Berkeley, California  
AEC Contract No. W-7405-eng-48

ON THE DYNAMICS OF HEMISPHERICAL PHASE GROWTH IN  
NONUNIFORM TEMPERATURE FIELDS

H. Y. Cheh and Charles W. Tobias

March, 1967

ON THE DYNAMICS OF HEMISPHERICAL PHASE GROWTH IN  
NONUNIFORM TEMPERATURE FIELDS

H. Y. Cheh\* and Charles W. Tobias  
Inorganic Materials Research Division,  
Lawrence Radiation Laboratory, and  
Department of Chemical Engineering  
University of California, Berkeley

March, 1967

The dynamics of vapor phase growth yields important information to the understanding of nucleate boiling. The history of the growth of a vapor bubble may be divided into three stages known as the initial, transition and asymptotic stages. In the initial stage, the bubble is very small and the growth is slow due to the high surface force that arrests its radial expansion. The transition from this slow growth to the asymptotic growth occurs in a very short time (approximately  $10^{-2}$  sec).<sup>1</sup> In the asymptotic stage, the bubble has reached a size where the inertia, viscous and surface forces can all be neglected in comparison with the pressure force. This is the stage that provides practical interest.

Plesset and Zwick<sup>2</sup>, using the thin thermal boundary layer approximation, obtained a solution for the asymptotic bubble growth in a uniformly supersaturated liquid. Birkhoff, Margulies and Horning<sup>3</sup> and also Scriven<sup>4</sup> obtained an exact solution to this problem using the method of similarity transform. The exact solution agrees completely with Plesset and Zwick's calculation at large rate of growth. Skinner and Bankoff<sup>5</sup>, using a similar approach as that of Plesset and Zwick, developed a parametric solution for bubble growth in general temperature fields.

---

\* Present address: Bell Telephone Laboratories, Murray Hill, N. J.

In this paper, we shall apply this solution to two practical cases, i.e., the constant wall temperature and the constant heat flux cases<sup>6</sup>.

The parametric solution for the hemispherical bubble growth by Skinner and Bankoff can be expressed as

$$R^3(\zeta) = 3J \int_0^\infty g(m) \operatorname{erfc} \frac{m}{2\zeta^2} dm, \quad (1)$$

and

$$\tau(\zeta) = \frac{\alpha_l t}{l^2} = \int_0^\zeta \frac{d\zeta'}{R^4(\zeta')}, \quad (2)$$

where  $g(m)$  is a dimensionless initial condition,

$$g(m) = \frac{\int_0^\pi [f(m) - T_s] \sin \theta d\theta}{\int_0^\pi [f(0) - T_s] \sin \theta d\theta}, \quad (3)$$

and  $f(m)$  is the initial condition.

For the case of constant wall temperature, assuming no convection, the initial condition for bubble growth is<sup>7</sup>

$$f(r, \theta) = T_\infty - (T_w - T_\infty) \operatorname{erfc} \frac{r \cos \theta}{l}, \quad (4)$$

or,

$$g(m) = 1 - \omega \left[ \operatorname{erf}(3m)^{1/3} - \frac{(3m)^{-1/3}}{\sqrt{\pi}} (1 - e^{-(3m)^{2/3}}) \right], \quad (5)$$

where

$$\omega = \frac{T_w - T_\infty}{T_w - T_s}. \quad (6)^*$$

\*

$\omega$  can be expressed more generally by

$$\omega = \frac{\text{interface temperature} - \text{bulk temperature}}{\text{interface temperature} - \text{saturation temperature}}$$

Therefore,  $\omega=0$  signifies the uniform superheat case.  $0 < \omega < 1$  means that the liquid is superheated, however a gradient of temperature from the bulk to the interface exists.  $\omega=1$  means the bulk liquid is at its boiling temperature.  $\omega > 1$  is the case of subcooled boiling.

For the case of constant heat flux with no convection, the initial condition is<sup>7</sup>

$$f(r, \theta) = T_{\infty} + T_q \operatorname{ierfc} \frac{r \cos \theta}{l}, \quad (7)$$

or,

$$g(m) = 1 - \omega \left[ 1 - e^{-(3m)^{2/3}} + \frac{\sqrt{\pi}}{2} (3m)^{1/3} \operatorname{erfc} (3m)^{1/3} - \frac{1}{2} (3m)^{-1/3} \gamma \left( \frac{3}{2}, (3m)^{2/3} \right) \right], \quad (8)$$

where

$$\omega = \frac{T_q / \sqrt{\pi}}{T_{\infty} + \frac{q}{\sqrt{\pi}} - T_s}. \quad (9)$$

Eqs. (1) and (2) with the two initial conditions, Eqs. (5) and (8) are solved numerically. The results are given in Figure 1 and Figure 2.

The results show the same qualitative feature as those obtained by Skinner and Bankoff for the cases of a linear temperature profile and an exponential temperature profile. However, the present calculation should yield more precise information for vapor bubble growth in liquids.

#### Acknowledgement

The authors wish to thank Mr. J. P. Earhart for his assistance during the numerical calculation for this work.

This work was performed under the auspices of the United States Atomic Energy Commission.

#### Nomenclature

f: An initial temperature field for bubble growth

g: A dimensionless temperature = 
$$\frac{\int_0^{\pi} [f(m) - T_s] \sin \theta \, d\theta}{\int_0^{\pi} [f(0) - T_s] \sin \theta \, d\theta}$$

J: Jakob number for heat transfer =  $\frac{\kappa_l}{\rho_l L \alpha_l} \int_0^\pi [f(0, \theta) - T_s] \sin \theta \, d\theta$

L: Latent heat of vaporization

l: A characteristic length

m: A dimensionless Lagrangian coordinate =  $\frac{1}{3} \frac{r^3 - R^3}{l^3}$

R: Radius of a bubble

R: A dimensionless radius =  $R/l$

T: Temperature

t: Time

$\alpha$ : Thermal diffusivity of a liquid

$\xi$ : A dimensionless Lagrangian coordinate =  $\frac{\alpha_l}{t} \int_0^t R^4(t) \, dt$

$\kappa$ : Thermal conductivity of a liquid

$\rho$ : Density of a fluid

$\tau$ : A dimensionless time

$\omega$ : A dimensionless temperature =  $\frac{\text{Interface temperature} - \text{bulk temperature}}{\text{Interface temperature} - \text{saturation temperature}}$

### Subscripts

l: Refers to properties in the liquid phase

q: Refers to a characteristic quantity for the constant heat flux case

s: Refers to quantities at equilibrium (or saturation)

w: Refers to quantities at the wall

$\infty$ : Refers to quantities in the bulk medium

### Functions

erfc x =  $\frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} \, dt$

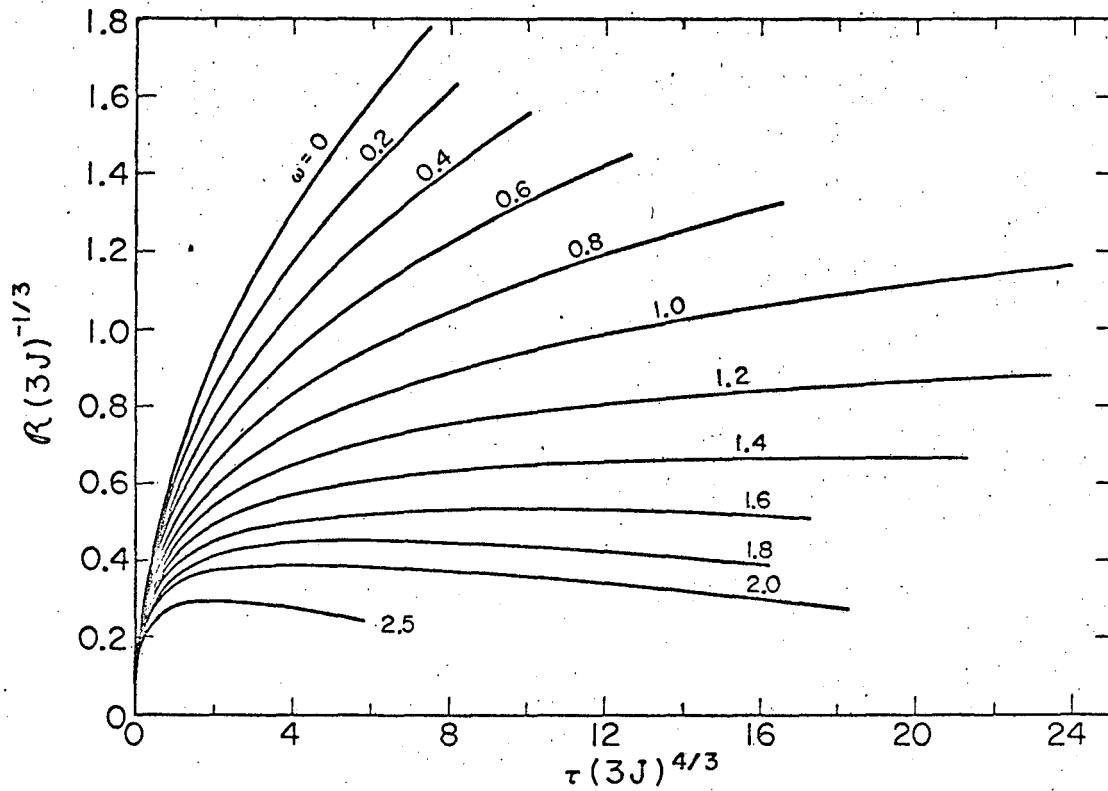


$$\text{ierfc } x = \int_x^{\infty} \text{erfc } t \, dt$$

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} \, dt$$

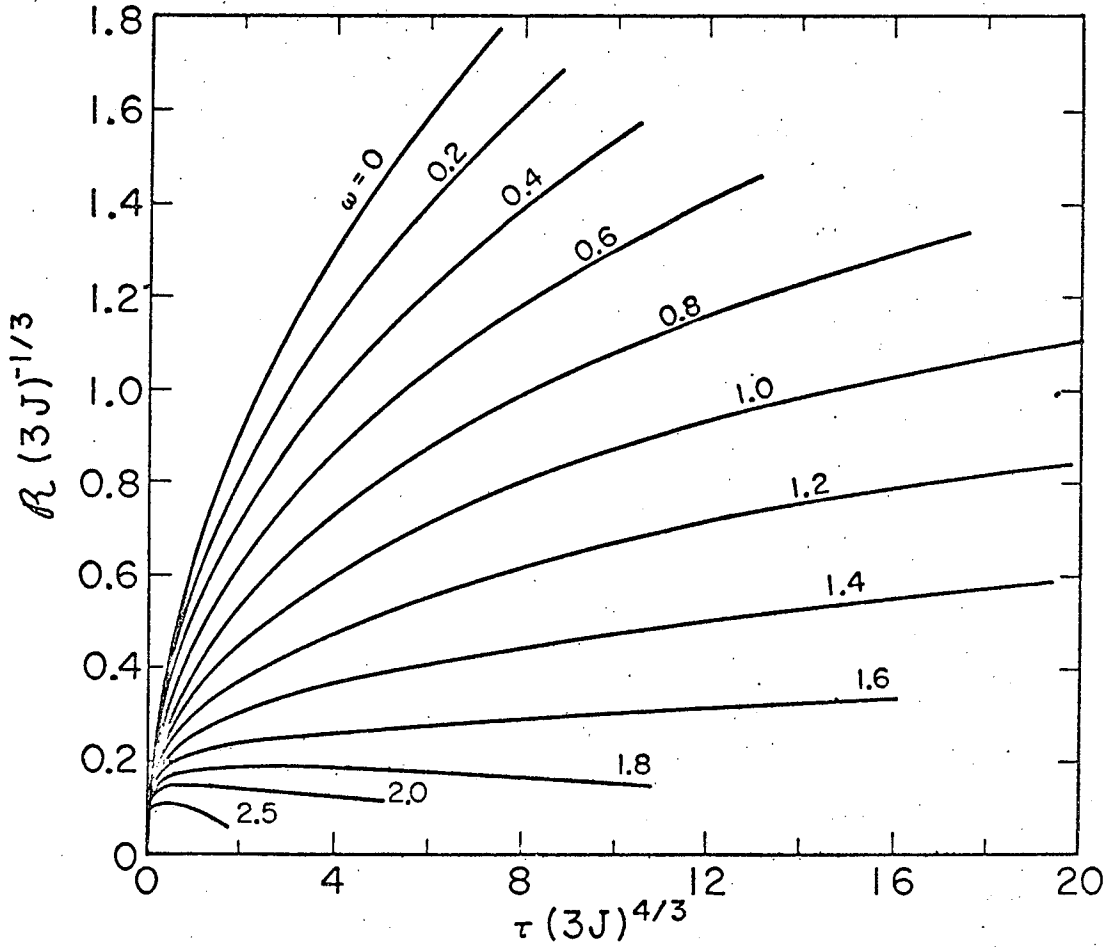
References

1. L. A. Waldman and G. Houghton, Chem. Eng. Sci. 20, 625 (1965).
2. M. S. Plesset and S. A. Zwick, J. Appl. Phys. 25, 493 (1954).
3. G. Birkhoff, R. S. Margulies and W. A. Horning, Phys. Fluids 1, 201 (1958).
4. L. E. Scriven, Chem. Eng. Sci. 10, 1 (1959).
5. L. A. Skinner and S. G. Bankoff, Phys. Fluids 8, 1417 (1965).
6. H. Y. Cheh, "On the Mechanism of Electrolytic Gas Evolution," Ph.D. Thesis, UCRL-17324, University of California, Berkeley, 1967.
7. H. S. Carslaw and J. C. Jaeger, "Conduction of Heat in Solids," 2nd ed., Oxford, 1959.



XBL671-80

Figure 1. The Case of Constant Wall Temperature.



XBL671-81

Figure 2. The Case of Constant Heat Flux.

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

