THE EFFECT OF RATE OF RISE OF MAGNETIC FIELD ON THE ACCEPTANCE TIME OF THE BEVATRON

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ABSTRACT

The acceptance time of the Bevatron has been measured for three different rates of rise of magnetic field. These data indicate that the acceptance time varies inversely with rate of change of magnetic field.

An analysis is given of the variation of the probability of an injected proton's striking the inflector on successive turns. This analysis, which is idealized to a circular machine and assumes linear betatron oscillations, shows that the probability of missing the inflector varies slowly with the rate of change of magnetic field.

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I. INTRODUCTION

Protons are injected into the aperture of the Bevatron magnet from a 35° electrostatic analyzer or inflector located at a radius of 620 inches. The injection cycle is initiated by a peaking strip located in the fringing field of the northeast quadrant of the magnet. When the magnetic field reaches 297 gauss, a 200- to 350-microampere pulse of 9.8-Mev protons begins to spiral into the aperture. The injection interval, for an 8000-gauss/second rate of rise of magnetic field, is on the order of 250 microseconds. When the first turn of the spiral reaches the center of the aperture, the accelerating voltage is turned on and some of the protons are captured in phase-stable orbits and hence are accelerated. The number of protons accelerated to full energy depends upon (a) the quantity of charge injected into the aperture, (b) the fraction of the injected charge that is accepted in phase-stable orbits that does not strike the sides of the aperture, and (c) the fraction of the particles that is lost due to gas scattering, various resonances, errors in frequency tracking, and random phase errors in the radio-frequency accelerating system.

Considerable effort has been expended on item (c) in the development of the Bevatron, resulting in a total gain in beam intensity of the order of 10^6 or more. While a small increase in beam intensity may be obtained by further reduction of phase and frequency errors in the tracking system, it is clear that the greatest net gain in beam intensity over the past several months has been obtained by increasing the reliability of the present system components.

The injected charge was increased, shortly after the Bevatron start-up, by a factor of approximately three. This increase was mainly due to the installation of a drift-type buncher at the entrance to the linear accelerator. A continued ion-source development program is in progress which has an ultimate objective of increasing the injected charge by a factor of two.

There is another approach to increasing the injected charge with which this report is concerned. It consists of increasing the total acceptance time of the Bevatron. Since protons are injected into the aperture from a constant current source, it is desirable to make the injection time as long as possible. This can be effected by two means: (a) energy-modulating the injected beam, and (b) reducing the rate of rise of the magnetic field.

Energy modulation of the injected beam will increase the injected charge by a factor of approximately two.[1] This increase is obtained by causing the energy of the injected protons to increase in synchronism with the magnetic field. Thus protons injected late in the acceleration cycle do not have the 20- to 25-inch radial betatron oscillation amplitudes that result from constant-energy injection into a time-varying magnetic field. The actual increase in accepted charge may be greater than a factor of two, as gas scattering and phase losses are the only main factors contributing to beam loss. There is an additional factor of importance associated with this mode of operation which affects the utilization of the beam. For constant-energy injection the large radial amplitudes of the betatron oscillations, which damp as the inverse square root of the magnetic field, define the final size of the circulating beam at full energy at approximately 5 inches. If the energy of the injected beam is increased in synchronism with the pulsed magnetic field of the Bevatron, the radial width of the beam will be much narrower at full energy. Also, since the radial component of phase oscillation decays to a negligible amplitude by the time the magnetic field reaches 1000 gauss, the available radial width of magnetic field for errors in frequency tracking will be greater.

If the injected beam of the Bevatron were derived from a Van de Graaff accelerator, energy modulation could easily be effected by modulating the outer-shell electrode. As a linear accelerator is used in the Bevatron, the problem is somewhat complicated. Increasing the excitation voltage on the linear accelerator is not a practical method of modulating the energy, as this simply changes the phase-stable angle of the accelerated particles. In addition, any changes in the energy of the linear accelerator would require very precise tracking of the voltage of the buncher and the Cockcroft-Walton accelerator. A method has been outlined by Heard which circumvents these difficulties.[2] It consists of increasing energy of the 9.8-Mev protons after they leave the linear accelerator by adding a small postacceleration cavity. The inflector voltage must also be increased in synchronism with the energy gain of the protons from the post-acceleration cavity, but this is easily accomplished.

The second method of increasing the acceptance time of the Bevatron is to reduce the rate of rise of magnetic field during the injected pulse. The amount by which the accepted charge will increase with the mode of operation is less simple in that somewhat more subtle factors must be considered. Clearly no net charge would be injected into the aperture if the magnetic field were constant during injection. All the particles would strike the inflector after one revolution in the aperture. As the rate of rise of the magnetic field increases, the probability of missing the inflector also increases, but the relation cannot be linear. The question naturally arises: How does the probability of missing the inflector vary with the rate of rise of magnetic field? This report treats the first-order theory in an attempt to answer this question. Measurements of the acceptance time of the Bevatron for different rates of rise of magnetic field, which are included in this report, show that the acceptance time increases by slightly more than a factor of

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[1] Coe, Ramm, and Vaughan, J. Sci. Instr. 33, 102 (1956).

[2] H.G. Heard, Energy Spread of the Injected Proton Beam of the Bevatron, Bev-171, University of California Radiation Laboratory, October 3, 1956.

of two if the rate of rise of field is halved. Preliminary measurements at approximately 1/10 the rate of rise of magnetic field have shown that circulating beam may be kept within the aperture for the extended injection time.[3] The practical problem of reducing the ripple amplitude associated with ignitron phase control of the magnet voltage may be solved by the ripple-feedback technique recently disclosed.[4]

II. VARIATION OF THE PROBABILITY OF MISSING THE INFLECTOR WITH RATE OF RISE OF MAGNETIC FIELD

It will be useful to have an analytical expression to indicate the effect of the rate of rise of the magnetic field on the probability of missing the inflector. In order to arrive at a solution capable of yielding a directly interpretable result, we can make several simplifying assumptions. Consider a circular magnet into which ions are injected while the magnetic field increases slowly with time. Assume that the injected ions oscillate about an instantaneous circle and experience a restoring force proportional to their displacement from the equilibrium position. Under these ideal conditions the ions execute linear betatron oscillations and their motion is describable by the system of differential equations,[5]

$$\frac{d^2x}{dt^2} = - \omega_r^2 x = - (1 - n)\, \omega_0^2\, x,$$

$$\frac{d^2z}{dt^2} = - \omega_z^2 z = - n\omega_0^2 z ,$$

where $x = r - r_0$ the radial displacement of the ion from the instantaneous orbit; z = the vertical displacement of the ion from the median plane; ω_r, ω_z = angular frequencies of the radial and vertical betatron oscillations; ω_0 = angular frequency of revolution of the ions within the magnet; and

$$-n = \frac{\text{fractional change in the magnetic induction}}{\text{fractional change in radius}} .$$

[3] H. G. Heard, Bevatron Operation and Development. VI, UCRL-3212, Nov. 1955.

[4] H. G. Heard, A New Method for Controlling the Magnetic Field in the Aperture of Synchrotrons, UCRL-3427, May 1956.

[5] Garren, Gluckstern, Henrich, and Smith, Theoretical Considerations in the Design of a Proton-Synchrotron, UCRL-547, Dec. 1949.

Harmonic solutions of these equations at injection will be of the form

$$x = \frac{p_x}{\omega_0\sqrt{1-n}} \cos \omega_0 t\sqrt{1-n} = A_x(n,t) \cos \omega_0 t\sqrt{1-n},$$

$$z = \frac{p_z}{\omega_0\sqrt{n}} \sin \omega_0 t\sqrt{n} = A_z(n,t) \sin \omega_0 t\sqrt{n},$$

where p_x, p_z are the associated momenta of the particles, and $\omega_0 t = 0$ corresponds to the injection time. After the kth turn ($\omega_0 t = 2\,nk$, $k = 1, 2, 3 \ldots$), the radial displacement of the ion from the point of injection will be just

$$\frac{p_z}{\omega_0\sqrt{1-n}} \left(1 - \cos 2\pi k\sqrt{1-n}\right),$$

and the vertical displacement from the median plane will be

$$\frac{p_z}{\omega_0\sqrt{n}} \left(\sin 2\pi k\sqrt{n}\right).$$

Let the radial distance from the point of injection--i.e., the center of the inflector gap--to the inner edge of the inflector be L_x, the vertical height of the inflector be L_z, and Δr be the radial motion of the instantaneous circle due to the rising magnetic field. Than a collision with the inflector on the kth turn is defined by the relations

$$L_x - k\Delta r = \frac{p_x}{\omega_0\sqrt{1-n}} \left(1 - \cos 2\pi k\sqrt{1-n}\right),$$

$$\frac{L_z}{2} = \frac{p_z}{\omega_0\sqrt{n}} \left(\sin 2\pi k\sqrt{n}\right).$$

To estimate the probability of missing the inflector on all turns it will be necessary to express the probability that an ion will be found in the radial amplitude range between p and p + dp on the kth turn. For the linear harmonic motion of interest, this is just

$$Q_k = \frac{1}{\pi^2} \frac{L_z}{A_z} \left[2\,\frac{(L_x - k\Delta r)}{A_x}\right]^{1/2}.$$

If $P_k = 1 - Q_k$ represents the probability of missing the inflector on the kth turn, then the product of all the probabilities on all the k turns up to $k = \frac{L_x}{\Delta r}$ will represent the total probability of missing the inflector, that is,

$$P(A_x)dA_x = \prod_k P_k dA_x = \prod_{k=1}^{k=\frac{L_x}{\Delta r}} \left\{ 1 - \frac{1}{\pi^2} \frac{L_z}{A_z} \left[2 \frac{(L_x - k\Delta r)}{A_x} \right] \right\} dA_x$$

$$= \exp \left[- \frac{2\sqrt{2}}{3\pi^2} \frac{L_z L_x}{A_z A_x} \sqrt{\frac{L_x}{\Delta r}} \right] dA_x ,$$

so that

$$P(A_x)\, dA_x = \exp \left[- \frac{C}{A_x \sqrt{\Delta r}} \right] dA_x .$$

Now the equilibrium orbit contracts as

$$\Delta r = - \frac{2\pi r}{\omega_0 (1 - n)B} \left(\frac{dB}{dt} \right)$$

so that we have

$$\Delta r \propto \frac{dB}{dt} = \dot{B} .$$

Therefore one may express the probability of the ion's missing the inflector as

$$P(A_x)dA_x \sim \exp \left[- \frac{C}{A_x \sqrt{\dot{B}}} \right] dA_x ;$$

making the change of variable, we get

$$A_x = x \, A_x \max \quad \text{and} \quad \eta = \frac{C}{A_x \max \sqrt{\dot{B}}} ,$$

and on integration one has

$$P(\eta) = \int_0^1 e^{-\frac{\eta}{x}} dx .$$

Reference to $P(\eta)$, which is shown in Fig. 1, reveals the following significant features concerning the effect of $\dot{B}$ on the probability of missing the inflector. First, it is significant that η is a slowly varying function of $\dot{B}$. Therefore one should be able to make significant reductions in $\dot{B}$ without appreciably altering the probability of missing the inflector. Second, significant departures from a linear decrease in $P(\eta)$ begin to occur for $\eta > 0.05$. Finally, η varies inversely with the amplitude of betatron oscillations. Therefore, one should not vary the energy in the injected beam exactly in synchronism with the increasing field if optimum beam survival is expected.

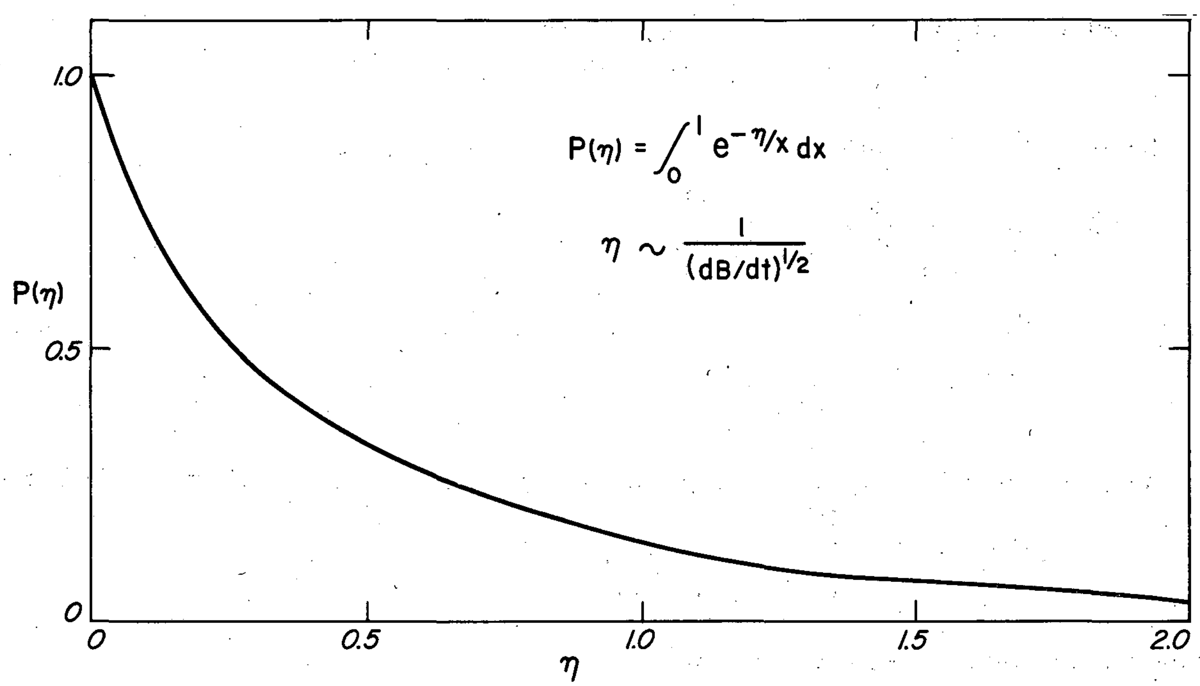

Fig. 1. Probability of an ion's missing the inflector as a function of the rate of rise of magnetic field.

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The value of $P(\eta)$ may be calculated for the Bevatron for 16-kv magnet excitation if the following constants are assumed to be typical of Bevatron operation:

$B = 297$ gauss,

$\dot{B} = 8000$ gauss/sec,

$\omega_0 = 2\pi f_0 = 2\pi \times 3.58 \times 10^5 = 2.25 \times 10^6$ rad/sec,

$r = 600$ inches,

$L_z = 4$ inches,

$L_p = 0.38$ inches,

$A_z = 3.5$ inches (assumes an average pressure of 6×10^{-5} mm Hg),[6]

$A_x = 9.5$ inches (assumes uniform distribution of betatron oscillation amplitudes over 38-inch aperture),

$n = 0.6$;

then η becomes

$$\eta = \left(\frac{2\sqrt{2}}{3\pi^2}\right)\left(\frac{L_z L_x}{A_z A_x}\right)\left(\frac{L_x \omega_0 (1-n) B}{2\pi r \dot{B}}\right)^{1/2}$$

$$= (0.096\left(\frac{(4)\ (0.38)}{(3.5)\ (9.5)}\right)\left(\frac{(0.38)\ (2.25 \times 10^6)\ (0.4)\ (297)}{(6.28)\ (600)\ (8000)}\right)^{1/2}$$

$$= (0.096)\ (0.046)\ (1.82) \cong 8 \times 10^{-3}\ .$$

Referring to Fig. 1, we see that the yields $P(\eta) = 0.975$, and very few particles are lost by collision with the inflector. Now if the rate of rise of field is reduced by an order of magnitude, and if (as assumed) the radial and vertical betatron amplitudes remain unchanged, then η may be obtained from the previous calculation and the relation

$$\eta \sim \frac{1}{\sqrt{\dot{B}}}\ ;$$

then $\eta = \dfrac{8 \times 10^{-3}}{800} \times 8000 = 2.5 \times 10^{-2}$,

so that $P(\eta) = 0.92$.

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[6] E. J. Lofgren and H. G. Heard, Bevatron Operation and Development III, UCRL-2822, Feb. 1955.

Now the actual value of $P(\eta)$ will be somewhat larger as η varies inversely with A_z and A_x. These latter variables will increase in amplitude, as the gas-scattering period is now ten times as long. To first order, then, the probability of missing the inflector is unchanged by the reduction in the rate of rise of magnetic field. In the Bevatron, the presence of straight sections, the spatial and temporal variations of n, the finite size of the inflector, the fundamentally nonlinear character of betatron oscillations, the angular divergence of the injected beam, and gas scattering all perturb the absolute phase of the betatron oscillations of each ion on successive turns. A numerical integration including all of these effects would be unnecessarily complicated, especially as these effects are already included in direct measurement on the Bevatron. Not all of the charge that misses the inflector may be available for acceleration. In addition to injection losses, one must account for the losses due to limited phase acceptance, errors in timing of the turn-on of accelerating voltage, and errors in the match of the frequency versus magnetic field relation for the central orbit, as well as coherent and incoherent phase errors in the frequency-tracking equipment. By varying the time of injection of a short pulse of charge of constant amplitude and energy, and determining the quantity of charge that remains after acceleration to more than ten times injection energy, one can determine the net effect of all the injection losses and thereby obtain a measure of the acceptance time of the accelerator. Acceleration to more than ten times injection energy assures that gas-scattering effects are included.

Acceptance-Time Measurements

The acceptance time of the Bevatron was measured by varying the time of injection of a 10-microsecond constant-current pulse of protons and observing the amount of charge remaining in the accelerator above 100 Mev. To prevent fluctuations in injection parameters from affecting the measurements, the timing of the ion pulse from the Cockcroft-Walton accelerator was adjusted for maximum transmission through the linear accelerator and inflector. Then the timing of all the injection equipment was controlled by a precision time-delay chassis, which fixed the injection interval with respect to the peaking-strip signal. In effect, then, the injector consisted of a completely independent source of monoenergetic protons which produced a 10-microsecond pulse of 9.8-Mev protons at any desired value of magnetic field. The timing of the turn-on of accelerating voltage and the frequency of the accelerating voltage were optimized for a 500-microsecond beam pulse and maximum accelerated beam. These conditions correspond to turning on the accelerating voltage when the normal beam spiral reaches the 600-inch radius. The corresponding start frequency is 358 kc/s.

Measurements under the above conditions have been made on the Bevatron for rates of rise of magnetic field corresponding to 16.05 kv (normal), 14.20 kv, and 8.03 kv magnet excitation (see Figs. 2 and 3). The magnetic-field gradient was adjusted for the correct n-value by exciting the pole-face windings with appropriate currents. If one denotes the width of the curve at half value as the acceptance time of the Bevatron, the acceptance time for normal excitation is 222 microseconds, whereas at half magnet voltage the acceptance time increased to 450 microseconds.

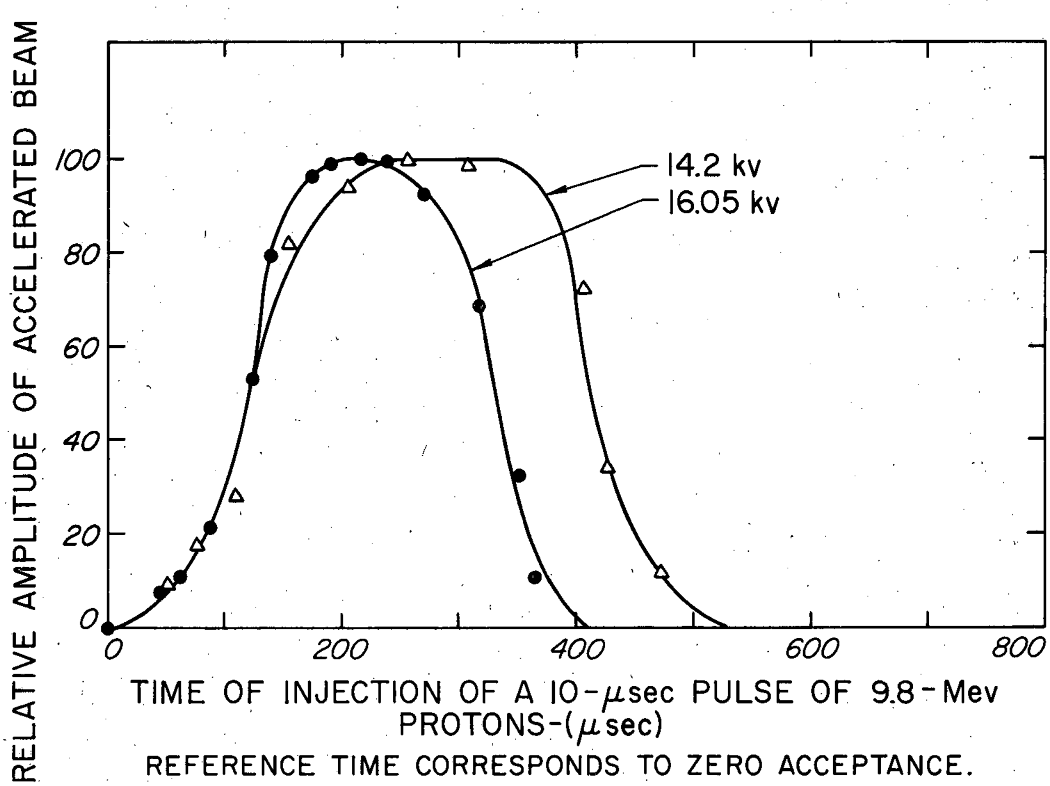

Fig. 2. Bevatron acceptance time for 16.05-kv and 14.2-kv magnet voltage.
Conditions:

1. Inflector at 621 inches. Normal high current in PFW 17-19.
2. Pole-face windings excited for n ~ 0.65 field gradient for 16.05-kv operation.
3. Two-turn loop connected for normal ripple cancellation. PFW 1-20 connected with 1024 μfd.

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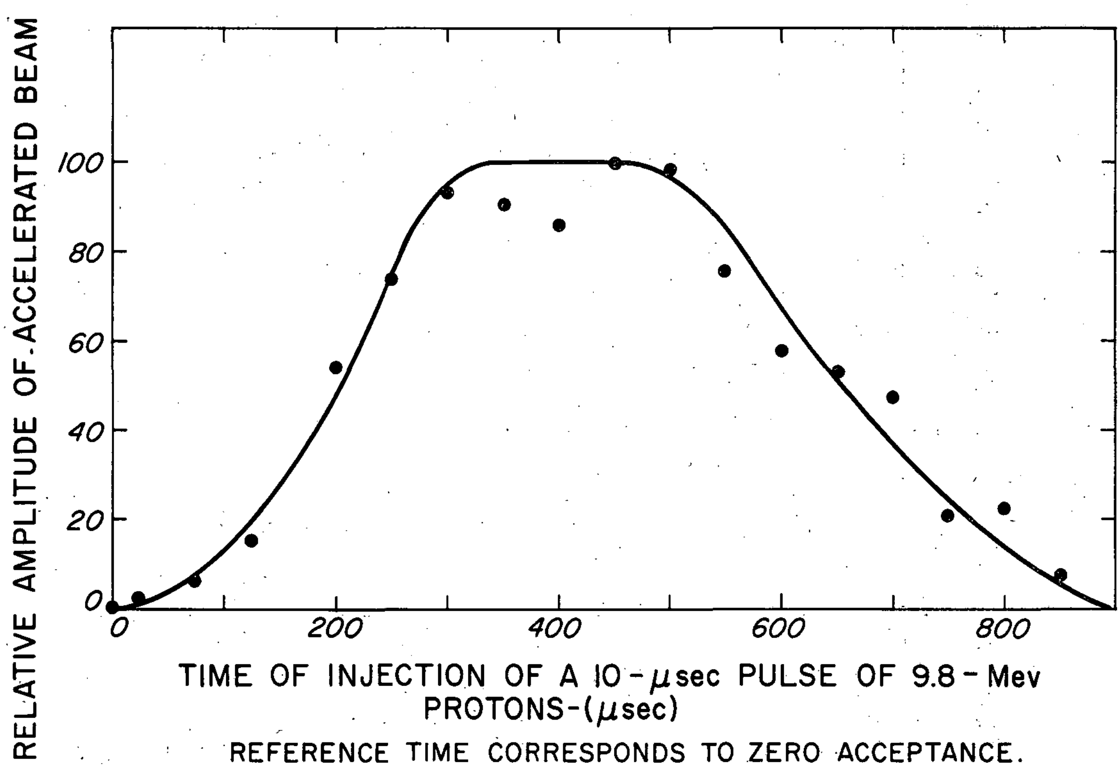

Fig. 3. Bevatron acceptance time for 8.03-kv magnet voltage. Conditions:

1. East motor generator excited.
2. Pole-face windings excited for one machine operation.
3. Series 10-microhenry choke installed.
4. Inflector radial position 619 inches.
5. Ripple cancellation filter with 2-turn loop and 512 μfd connected between PFW 1-19 and 2-20.

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If the acceptance-time curves are normalized at their peak values and compared on the basis of integrated areas, their areas are in the ratios of 1 : 1. 4: 2.1. Although these results indicate a net gain in acceptance time for lower rates of rise of magnetic field, they do not represent an increase in the net charge. Whether or not the increased acceptance time will increase the total beam has not been demonstrated. Preliminary experimental results show that the total beam is in fact decreased for lower rates of rise of magnetic field. This observed reduction in beam may be explained by uncompensated errors in the frequency-tracking system at the lower magnet excitation voltage. More definitive measurements are in progress.

III. CONCLUSIONS

1. An experimental verification of the indicated increase of acceptance of beam charge is needed.
2. The present results tend to indicate that one may expect to increase the injected charge by factors of 3 to 5 before inflector losses and gas scattering become limitations.

IV. ACKNOWLEDGMENTS

The author wishes to thank Dr. Lloyd Smith for illuminating discussions on the feasibility of this method of increasing the acceptance time. The cooperation of Dr. Edward J. Lofgren and the operating staff of the Bevatron have greatly aided in the assembly of equipment and gathering of necessary data.

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