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Chapter 3

The Genesis of IDARC and Advances in Macromodeling for Nonlinear Analysis of RC Structures

Sashi K. Kunnath

Abstract The development of IDARC (Park et al., IDARC: Inelastic Damage Analysis of Reinforced Concrete frame shear-wall structures. Technical report NCEER 87-0008, State University of New York, Buffalo, 1987) (Inelastic Damage Analysis of Reinforced Concrete structures) was initially motivated by the need for an efficient computer program to aid the design of shaking table experiments but then evolved into a tool for seismic assessment of reinforced concrete structures. The primary modeling technique employed in the IDARC computational platform is the representation of the overall behavior of components through macromodels. The effectiveness of the macro elements is enhanced through the introduction of distributed flexibility models that account for the effects of spread plasticity. Nonlinear material behavior is specified by means of a generic hysteretic force-deformation model that incorporates stiffness degradation, strength deterioration and pinching or bond-slip effects. Solution modules for nonlinear static, monotonic, quasi-static cyclic and transient seismic loads were implemented. The final response quantities are expressed in terms of damage indices that provide engineers with a qualitative interpretation of the structural response.

3.1 Introduction

The primary objective of any analysis, linear or nonlinear, is to estimate force and deformation demands at critical regions and to assess the likely performance of the structure due to the imposed loading. When the loading is deterministic and the system is expected to respond linearly, the modeling tasks are significantly simplified. However, for transient non-deterministic loads and structural systems that

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exhibit degrading inelastic behavior, the variables and uncertainties associated with selecting model variables or choosing an analysis method are more challenging. The consequences of choosing a simple model to carry out a complex nonlinear analysis or using a complex model to conduct a simple linear static analysis can be far-reaching. It is always important to ascertain the level of complexity required to achieve a desired solution.

The range of modeling options available to a designer is diverse. If the purpose of an analysis is to determine the state of stress or strain at a particular location in a structural component or connection, then it is necessary to resort to finite element models that incorporate detailed constituent material behavior. However, if the quantities of interest are more global in nature such as member rotation or inter-story drift in a building, then the use of approximate member models may be sufficient.

Since the inelastic behavior of RC structures, particularly under seismic loads, is mostly concentrated at known locations in an element, a macromodel approach can often be used with remarkable reliability. Macromodels are computationally efficient and offer a great deal of flexibility in modeling. Since it is possible to account for a variety of behavior patterns in an equivalent sense, they can be used to effectively and efficiently model the global response of reinforced concrete structures.

Material nonlinearities typically account for most of the inelastic behavior in reinforced concrete and is usually incorporated in one of the following ways: (i) through the use of constitutive stress-strain relations at a microscopic level (micromodels) as is done in the finite element or fiber model analyses; or (ii) a macromodel approach wherein the inelastic material behavior is described using force-deformation rules (also called hysteresis rules) which reflect member behavior as a whole.

3.2 Development of Force-Deformation Models

A typical load-displacement relationship of a reinforced concrete member is characterized by three stages: an elastic uncracked stage, a stage characterized by crack propagation, and a yielded or plastic stage. Of these, nonlinearity is evident only in the second and third stages and are a result of two major material effects: cracking of concrete and plasticity of the reinforcement. Other sources of non linearity also exist, such as bond-slip, aggregate interlock in cracked concrete and dowel action of the reinforcing steel, all of which are essentially independent of time, and long-term behavior such as creep and shrinkage which are time-dependent phenomena.

The primary difference between a macromodel approach and a finite element or fiber model representation is that, for macromodels, no constitutive equations are used. Instead, the inelastic behavior is described using force-deformation rules which reflect member behavior as a whole. The basis for development of force-deformation models is experimental testing.

Fig. 3.1 Multi-parameter degrading hysteresis model

3.2.1 Uniaxial Models

Numerous models to reproduce a range of force-deformation behavior can be found in the literature. Prior to the development of stiffness degrading models, most nonlinear programs used simple bilinear force-deformation representations. Part of the reason for this was the fact that non-degrading bilinear models were adequate for analysis of steel structures and complexities associated with numerical implementation were minimal. The first notable improvement of the bilinear model was provided by Clough and Johnston (1966). The next significant contribution in RC analysis came from Takeda et al. (1970) that also laid the foundation of so-called “degrading” hysteresis models. Since then numerous enhancements have occurred in hysteresis model development with considerations of strength deterioration, pinching, bond-slip becoming rather commonplace in structural modeling.

A typical model with the versatility to incorporate most of the desired effects in reinforced concrete is shown in Fig. 3.1. This basic model, which is implemented in the IDARC series of programs, uses several control parameters to establish the rules under which inelastic loading reversals take place. A variety of hysteretic properties can be achieved through the combination of a non-symmetric trilinear curve and these control parameters which characterize stiffness degradation, strength deterioration and pinching, respectively. For example, α , which can be expressed as a function of the deformation, controls the amount of stiffness loss; ϕ and χ control the initiation and degree of pinching; and the slope s and the change in expected peak strength (M to M^*) controls the softening due to system deterioration. A sample simulation of observed behavior using this model is shown in Fig. 3.2. The hysteresis curves in this case was obtained from tests of a precast concrete connection with a hybrid combination of mild steel and post-tensioning steel (Cheok et al. 1998). However, the specification of hysteretic rules in an actual analysis is rather empirical and should be based on available experimental data. In the absence of such data, a parametric study to evaluate the sensitivity of these parameters is necessary.

Fig. 3.2 Simulation of observed hysteretic behavior

3.2.2 Bidirectional Models

Experimental evidence suggests that stiffness and strength degradation in reinforced concrete is accelerated in the presence of biaxial moments and varying axial loads. Yield surface models and fiber models are generally the only means of considering multiaxial material behavior in the inelastic range. However, in the context of macromodels, a convenient scheme for expressing bi-directional moments with possible interaction effects is needed for implementation in an analytical framework. One such scheme, which is an extension of the Bouc-Wen formulation (Bouc 1967; Wen 1976), was proposed by Kunnath and Reinhorn (1990). An expression between moment and curvature is derived, as follows:

$$\begin{Bmatrix} M_x \\ M_y \end{Bmatrix} = \alpha \begin{bmatrix} (EI)_x & 0 \\ 0 & (EI)_y \end{bmatrix} \begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix} + (1 - \alpha) \begin{bmatrix} M_x^y & 0 \\ 0 & M_y^x \end{bmatrix} \begin{Bmatrix} Z_x \\ Z_y \end{Bmatrix} \quad (3.1)$$

M_x , M_y , ϕ_x and ϕ_y are the moments and curvatures in the x and y directions respectively; M_x^y and M_y^x are the corresponding yield moments in the respective directions; α is the post-yield stiffness ratio; and Z_x , and Z_y are the non-dimensional hysteretic force components given by:

$$\begin{Bmatrix} \dot{Z}_x \\ \dot{Z}_y \end{Bmatrix} = (A [I] - B [\Omega]) [\Phi] \begin{Bmatrix} \dot{\phi}_x \\ \dot{\phi}_y \end{Bmatrix} \quad (3.2)$$

$$[\Phi] = \begin{bmatrix} 1/\phi_x^y & 0 \\ 0 & 1/\phi_y^x \end{bmatrix}; \quad [\Omega] = \begin{bmatrix} Z_x^2 [\text{sgn}(\dot{\phi}_x Z_x) + 1] & Z_x Z_y [\text{sgn}(\dot{\phi}_y Z_y) + 1] \\ Z_x Z_y [\text{sgn}(\dot{\phi}_x Z_x) + 1] & Z_y^2 [\text{sgn}(\dot{\phi}_y Z_y) + 1] \end{bmatrix} \quad (3.3)$$

$$\text{sgn}(\dot{\phi} Z) = 1 \quad \text{if} \quad \dot{\phi} Z > 0 \quad \text{or} \quad = -1 \quad \text{if} \quad \dot{\phi} Z < 0 \quad (3.4)$$

Details of the implementation of the above model which also incorporates stiffness and strength degradation are described in Kunnath and Reinhorn (1990). However, it must be mentioned that it is essential to select the base parameters of the model to eliminate rate-effects (Constantinou and Adnane 1987). The Bouc-Wen model and its derivatives also exhibit erroneous drift accumulation during constant-amplitude cycling in the same quadrant. The numerical implementation of this model, therefore, requires certain modifications to eliminate or minimize this error prior to use in a nonlinear analysis program.

3.3 Element Models

The basic objective in developing a member model for nonlinear analysis is to adequately characterize the varying stiffness properties of the element during a cyclic loading event.

3.3.1 Beam-Column Elements

To adequately represent spread plasticity due to distributed cracking in a beam-column element, IDARC utilizes a distributed flexibility approach wherein the flexibility matrix, including shear distortions, relating moments and rotations at the ends of the element is typically expressed as:

$$\begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix} = \begin{bmatrix} f_{AA} & f_{AB} \\ f_{BA} & f_{BB} \end{bmatrix} \begin{Bmatrix} M_A \\ M_B \end{Bmatrix} \quad (3.5)$$

In the above equation, θ_A and θ_B are the rotations at the ends, M_A and M_B are the moments at the ends of the element. The flexibility coefficients are obtained from:

$$f_{ij} = \int_0^L \left[\frac{m_i(x)m_j(x)}{EI(x)} \right] dx + \int_0^L \left[\frac{v_i(x)v_j(x)}{GA_Z} \right] dx \quad (3.6)$$

where $m_i(x)$ and $m_j(x)$ are the moment distributions due to a virtual unit moment at end “i” or “j”, respectively; $v_i(x)$ and $v_j(x)$ are the corresponding shear distributions. In such a formulation, the flexibility and resulting stiffness matrix is a function of the assumed curvature distribution across the member. Figure 3.3 shows the moment distribution in a typical beam element subjected to combined gravity and lateral loads. A pair of linear distributions (one which considers only yielding and the other which considers both cracking and yielding) is shown in Fig. 3.3. More complex nonlinear distributions can also be considered.

Fig. 3.3 Moment and curvature distribution in a flexural member bent in double curvature

3.3.2 Shear Wall Elements

Despite their enormous stiffness and strength, walls can respond in a stable ductile fashion under flexural conditions. In the past, walls were modeled as a single vertical element located at the center of the wall. If the wall was part of a building frame, this central vertical element was connected to adjoining beams by rigid links. An enhanced macro element model of a shear wall is the multiple-vertical-line-element (MVLE) model. A representation of this model is displayed in the general building model shown later in Fig. 3.5. It consists of a central vertical line to model flexural and shear behavior in the wall unit as a single entity and additional axial elements along the wall length to model axial stresses resulting from flexural deformations. Such a model was first used by Kabayesawa et al. (1983) in an attempt to capture the response of a seven story shear wall building in a full-scale pseudo-dynamic test conducted in Japan. Enhancements and variations of the model have also been proposed (Vulcano et al. 1989; Fajfar and Fischinger 1990). Stacking a set of MVLE to represent a single wall panel appears to produce the best results.

One drawback of the MLVE model lies in ascertaining the correct failure mode of the wall. The properties of the shear spring in the MVLE model are dependent on the flexural strength and axial force. Since the properties of the spring have to be assigned prior to the analysis, the potential interaction of these forces is not properly accounted. Separate formulations to compute the shear vs. strain characteristics of the wall panel can be derived from strut-and-tie modeling.

3.3.3 Beam-Column Joints

Since seismic design requires the total column capacity to be in excess of 20 % of the total girder capacity at the joint, the intersection of large column cross-sections with beams results in a sizable joint region that can deform in shear and contribute to the overall joint rotations. Moreover, the shear capacity of the joint may be exceeded in certain inadequate or nonductile connections leading to additional inelastic distortion and rapid deterioration of the connection. When the joint shear forces are high, and the beam column panel zones are not adequately reinforced, panel-zone distortions can have measurable impact on the story drift.

Existing procedures in frame analysis assume either (a) the panel to be rigid in which case the angle between the panel zone and the adjacent members (beams and columns) are assumed to remain at right angles even after the panel zone has undergone severe shear deformation, or (b) a linear, elastic relationship to exist between the shearing forces and panel-zone distortions. In the former case, a single moment and associated joint rotation is used at the center of the panel. The latter approach recognizes the significance of joint deformations but does not account for large inelastic rotations that may occur if the shear capacity of the joint is exceeded. A simple macro-model that can be used effectively to represent joint deformations is shown in Fig. 3.4. A panel zone mechanism based on pure shear deformations form the basis of the model. In such a model, it is assumed that column moments can differ from beam moments. The panel zone connecting the beams and columns behave as rigid bars and can deform independent of one another. With reference to the figure, M_b , θ_b , M_c , θ_c are the moments and rotations of beams and columns, respectively. The shear distortion of the panel, γ_p is the relative change in the rotations of the beam and column element, as follows:

$$\gamma_p = (\theta_b - \theta_c) \quad (3.7)$$

A relationship of the following form can be derived:

$$\begin{Bmatrix} M_c \\ M_b \end{Bmatrix} = V_p G \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \theta_c \\ \theta_b \end{Bmatrix} \quad (3.8)$$

Here, V_p is the volume of the panel and G is the shear modulus of the material. The shear versus strain behavior is specified by means of a trilinear envelope with

Fig. 3.4 Modeling joint deformations in beam-column joints

degrading cyclic characteristics. This macromodel was developed and implemented in IDARC by Alath and Kunnath (1995) and has been successfully used by Bracci et al. (1998) to correlate the observed behavior of steel-concrete hybrid connections undergoing significant inelastic joint distortions.

3.4 Modeling RC Frame-Wall Buildings

DRAIN-2D (Kannan and Powell 1973) and SAKE (Otani 1974) were among the earliest non-commercial nonlinear two-dimensional frame analysis programs. DRAIN-2D was developed as a general-purpose frame analysis program with an array of element types (beam-column, panel, and truss) and used a two-component model to incorporate inelastic action. SAKE, on the other hand, was directed more towards reinforced concrete and was based entirely on a single component beam-column element with a Takeda-type hysteresis model.

A frame-wall system is modeled in IDARC as an assemblage of two-dimensional frames interconnected by transverse beams whose stiffness contributions in the two-dimensional plane of analysis is considered. Though torsional modes of vibration and bidirectional interaction are not accounted for, properly constructed two dimensional models can yield acceptable results for the large majority of regular structures. Three degrees-of-freedom (d-o-f) are used at each node however all lateral floor degrees-of-freedom are slaved to a floor master d-o-f based on the assumption that floor slabs act as rigid diaphragms.

Enhanced idealizations can be achieved by incorporating certain aspects of three-dimensional interaction within the two-dimensional framework. For example, the effect of transverse elements and in-plane floor flexibility can be included as shown in Fig. 3.5. The addition of other two-dimensional elements such as infill panels,

Fig. 3.5 Representation of a RC building as an assemblage of macro-elements

interstory braces or energy-dissipating elements, can provide the user with a variety of modeling options. These enhancements were incorporated into the later versions of IDARC.

3.5 Damage Modeling

The current release of IDARC incorporates three models for damage quantification:

Modified Park-Ang Damage Model: The Park-Ang damage model (Park et al. 1984) was incorporated in the original release of the program (Park et al. 1987). Furthermore, the Park and Ang damage model is also an integral part of the three parameter hysteretic model since the rate of strength degradation is directly related to the parameter β described below. The Park-Ang damage index for a structural element is defined as:

$$DI_{P\&A} = \frac{\delta_m}{\delta_u} + \frac{\beta}{\delta_u P_y} \int dE_h \quad (3.9)$$

where δ_m is the maximum experienced deformation; δ_u is the ultimate deformation of the element; P_y is the yield strength of the element; $\int dE_h$ is the hysteretic energy absorbed by the element during the response history; and β is a model constant parameter. Three damage indices are computed using this

damage model: (1) Element damage index; (2) Story damage index for vertical and horizontal components; and (3) Overall building damage. The combination of damage indices is accomplished through weighting factors which are presently expressed as a function of the dissipated hysteretic energy. Since the above model returns a finite damage value for elastic response, the model was modified to subtract the recovered elastic deformations during enhancements added to Version 3 of the program (Kunnath et al. 1992).

Fatigue Based Damage Model: The fatigue based damage model was introduced by Reinhorn et al. (2009). The damage model was developed based on basic structural response considerations, and a low-cycle fatigue rule. The damage index is:

$$DI = \frac{\delta_a - \delta_y}{\delta_u - \delta_y} \frac{1}{\left(1 - E_h / \{4 (\delta_u - \delta_y) F_y\}\right)} \quad (3.10)$$

where δ_a is the maximum experienced deformation, rotation, or curvature; δ_y is the yield deformation capacity; δ_u is the ultimate deformation capacity; F_y is the yield force capacity; and E_h is the cumulative dissipated hysteretic energy. Note that simplifying the above damage model for the case when the ratio $(\delta_a - \delta_y)/(\delta_u - \delta_y)$ is close to unity, the above expression simplifies to:

$$DI = \frac{\delta_a - \delta_y}{\delta_u - \delta_y} + \frac{E_h}{4 (\delta_u - \delta_y) F_y} \quad (3.11)$$

This is similar to the Park-Ang formulation for $\beta = 0.25$.

Global Damage Model: Another measure of structural damage is the variation in the fundamental period of vibration of the structure. This history is related to the overall stiffness loss in the structure due to inelastic behavior. DiPasquale and Cakmak (1988) defined the softening in the structure as:

$$DI = 1 - \frac{(T_0)_{initial}}{(T_0)_{equivalent}} \quad (3.12)$$

A possible interpretation of the computed damage estimates is summarized in Table 3.1.

3.6 Sample Simulation Using IDARC

Next, the response of a seven-story reinforced concrete building that was instrumented during the 1994 Northridge earthquake is analyzed. The objective herein is to ascertain if the two-dimensional modeling framework and analysis techniques, as presented in earlier sections, are adequate to predict the inelastic response of concrete structures under strong ground motions.

Table 3.1 Damage classification

Damage state ^a	Usability	Damage description ^a	Park-Ang damage index	Inter-story drift demand (%)	
				Flexure	Shear
None		No visible damage, cosmetic or structural	0.0	<0.25	<0.05
Insignificant	Usable	Only cosmetic repair required; no structural repair	0.0–0.2	0.25–0.50	0.05–0.20
Moderate	Temporarily unusable	Repairable structural damage	0.2–0.4	0.5–2.0	0.25–0.50
Heavy	Unusable	Extensive damage requiring demolition or replacement	>0.5	>2.0	>1.0

^aATC-33 classification

Fig. 3.6 South perimeter frame of the Holiday Inn building

Detailed building information, damage data and estimates of section properties is reported in Islam (1996). The structural damage was primarily confined to the longitudinal perimeter frames with the most severe damage between fourth and fifth floors of the south perimeter frame (Fig. 3.6). The south perimeter frame, which sustained the most severe damage, was selected for evaluation. Material and section property data were adjusted to include the effects of concrete aging and probable damage from the 1971 earthquake. These are reported in Islam (1996).

An incremental time-history analysis was performed on this frame using the recorded accelerations at the base level of the building as the input motion. Figure 3.7 shows the comparison of recorded and calculated roof displacement time-history. The recorded peak displacement at the roof level in this direction

Fig. 3.7 Comparison of observed response with numerical prediction

was 9.20" @ 9.34 s compared to approximately 9.0 in. predicted by the computer program at almost the same instant. The building period, based on the recorded roof displacement response, appears to be about 1.5 s during the early part of the earthquake, 2.1 s between 10 and 20 s and about 2.4 s thereafter. The program-generated time history compares well with the observed response.

3.7 Some Modeling and Numerical Implementation Issues

All nonlinear computer programs are based on "incremental" or "iterative" linear methods. Nonlinearity is a consequence of stiffness changes occurring during (as in iterative schemes) or at the end (as in incremental schemes) of each linear step. Nonlinear constitutive models (either stress-strain or force-deformation) are specified as continuous or multi-linear paths. For continuously varying stiffness properties, it is necessary to carry out an iterative analysis to ensure equilibrium at the end of each step.

The event-to-event strategy is suitable for multi-linear models since it is possible to predict the required force (or deformation) to cause the next change in state. However, for degrading models with numerous stiffness changes in a single pass (loading or unloading) such as the incorporation of pinching, bond-slip, strength and/or stiffness deterioration, the number of events that need to be checked may become excessive, particularly for large structural systems.

The stiffness (or flexibility) matrix for a member needs to be updated for one or more of the following reasons: (a) a transition in stiffness as prescribed by the hysteresis model or the stress-strain curve; (b) a shift in the contraflexure point; and (c) a change in the plastic hinge length. All such changes lead to unbalanced forces between two solution steps. Item (a) can be dealt with using an event-to-event strategy. The changes resulting from either varying contraflexure location or

hinge length are not associated with any predefined event change. Hence, an iterative approach to ensure stability of the final solution is necessary.

Given the complexity of carrying out an iterative or event-to-event analysis, it is sometimes prudent to consider approximate ways to treat convergence issues. A compensation procedure, for example, can be adopted to minimize equilibrium errors. The unbalanced force in any given time step is applied as a corrective force in the subsequent time step. Alternatively, the velocities or accelerations can be adjusted to satisfy dynamic equilibrium at the end of each step. Either approach is not physically accurate but serves to limit numerical errors from increasing without bounds.

3.8 Concluding Remarks

The release of IDARC was a significant milestone in nonlinear analysis of RC structures. Among other features it was the first non-commercial special-purpose computer program to introduce spread plasticity formulations for beam-column elements, the first to include a shear-wall element, the first to consider the effects of transverse (orthogonal) elements within a two-dimensional framework, the first to incorporate a versatile hysteretic model that was capable of simulating stiffness degradation, strength deterioration and pinching behavior, the first to provide a pushover curve as a standard nonlinear static analysis option, and the first to incorporate a damage model to post-process the results of the seismic analysis.

Since the first release of IDARC in 1987, the program has seen numerous upgrades and enhancements and is currently distributed as Version 7.0. Nonlinear analysis of reinforced concrete structures in general has also seen significant advances in the past two decades with fiber-based modeling becoming more commonplace. Ultimately, the efforts of modeling must find their way into computational tools. Currently, there are several available computer programs which, to varying degrees, are capable of performing nonlinear analysis of RC structures. Of these, OpenSees (2013) is the most widely used program within the seismic research community.

It is conceivable that high-fidelity finite-element based simulations will become more reliable and used more routinely as computational facilities become even more powerful and affordable. However, instrumentation of existing buildings and bridges and full-scale structural testing must continue so that computational tools used in sophisticated evaluation are continually validated and enhanced.

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