Title
A Tool for the Incorporation of Non-Recurrent Congestion Costs of Freeway Accidents in Performance Management

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# A Tool for the Incorporation of Non-Recurrent Congestion Costs of Freeway Accidents in Performance Management 

Will Recker, Younshik Chung, Tom Golob<br>University of California, Irvine<br>California PATH Research Report<br>UCB-ITS-PRR-2005-30

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# A Tool for the Incorporation of Non-Recurrent Congestion Costs of Freeway Accidents in Performance Management 

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#### Abstract

In this research, we develop and apply an analytic procedure that estimates the amount of traffic congestion (vehicle hours of delay) that is caused by different types of accidents that occur on urban freeways in California. A key feature of this research is the development of a method to separate the non-recurrent delay from any recurrent delay that is present on the road at the time and place of a reported accident, in order to estimate the contribution of non-recurrent delay caused by the specific accident. Our analysis involves a case study of accidents that occurred on freeways in Orange County in 2001. The non-recurrent delay caused by the case study accidents is estimated based on inferred link speeds derived from loop data and a binary integer programming formulation to identify the temporal and spatial region affected by the accident. Computations of non-recurrent delay were successfully performed for 870 accidents that occurred on weekdays throughout the period of March through December 2001 on the six major Orange County non-toll freeways. A statistical model was estimated that describes non-recurrent delay as a function of day of week, time of day, weather, and the observable (e.g., from emergency calls and/or aerial or on-scene observation) characteristics of the accident.


Keywords: Non-recurrent delay, Accidents, Freeways, Congestion

## 1 Executive Summary

The objective of this research project is to develop and apply an analytic procedure that estimates the amount of traffic congestion (vehicle hours of delay) that is caused by different types of accidents that occur on urban freeways in California. Although it has been speculated that non-recurrent congestion caused by accidents, disabled vehicles, spills, weather events, and visual distractions accounts for one-half to three-fourths of the total congestion on metropolitan freeways, there are insufficient data to either confirm or deny this conjecture. A key feature of this research is the development of a method to separate the non-recurrent delay from any recurrent delay that is present on the road at the time and place of a reported accident, in order to estimate the contribution of nonrecurrent delay caused by the specific accident. The procedure provides a foundation for a forecasting model that will allow Caltrans to allocate resources in the most effective way to mitigate the effects of those accidents that are likely to result in the greatest amount of delay.

Our analysis involves a case study of accidents that occurred on freeways in Orange County in 2001. Two datasets were combined to accomplish the objective of the study: (1) accident data from the Traffic Accident Surveillance and Analysis System (TASAS), which covers all police-investigated accidents on the California State Highway System, and (2) traffic flow data from the Vehicle Detection System (VDS), received directly from the Caltrans District 12 front-end processor (FEP) using the UCI ATMIS Testbed Intertie with Caltrans District 12. Since the size of the traffic flow data set is several hundred gigabytes, a Database Management System (DBMS) is employed to efficiently manage and process the huge database. The non-recurrent delay caused by the case study accidents is estimated based on inferred link speeds derived from loop data and a binary integer programming formulation to identify the temporal and spatial region affected by the accident.

Using non-recurrent delay computed for a sufficient sample of accidents, a statistical model was estimated that describes non-recurrent delay as a function of day of week, time of day, weather, and the observable (e.g., from emergency calls and/or aerial or on-
scene observation) characteristics of the accident. These accident characteristics, which are available to Freeway Traffic Management Systems, include time of day, number of involved vehicles, whether a truck is involved, and collision location (by lane or side of road). This statistical model can be used to inform a manager as to the expected delay associated with an accident as soon as the accident is reported and its characteristics are observed. This can in turn be used in improving resource allocation.

Computations of non-recurrent delay were successfully performed for 870 accidents that occurred on weekdays throughout the period of March through December 2001 on the six major Orange County non-toll freeways.

The median total delay for these 870 accidents is 86 vehicle hours, the lower bound of the mean is 184 vehicle hours, and the lower bound of the standard deviation is 246 . As indicated by the difference between the median and the high standard deviation relative to the mean, the distribution of non-recurrent delay is highly skewed to the right (i.e., toward high values of delay), as expected. For regression purposes, it is useful to analyze the natural logarithm of delay, so that the regression residuals are approximately normally distributed. If delay values close to zero are ignored, the logarithm of delay is approximately normally distributed.

A regression model was developed that can forecast the expected amount of nonrecurrent delay for different types of accidents that occur at different times. The model uses the natural logarithm of delay as the dependent variable (resulting in residuals that are approximately normally distributed) and binary $(0,1)$ indicators of the various features of the accident. In order to present a model for forecasting delay once the accident is detected, we limited the descriptive variables in the model to those characteristics that would presumably be available to the Traffic Management Center shortly after the occurrence, as opposed to aspects that might only be known based on investigation and/or report. We assumed that the following characteristics would be known by direct observation, either from emergency calls, or aerial or on-scene observation: 1) day of week, 2) time of day, 3) location on roadway, 4) number of involved vehicles, and 5) whether or not a truck was involved. The regression was based on these five variables
and their first-order interaction terms. In the model, only those variables with effects that tested significantly different from zero at the $\mathrm{p}=0.05$ significance level were retained. The resulting model contained 14 variables that tested significant at the 0.05 level, and proportion of variance accounted for by the model is 0.134 .

Our results indicate that the following accident characteristics are crucial in identifying those accidents that are likely to cause the most delay: (a) how many vehicles are involved in any accident occurring during the weekday AM peak, and whether the accident is in the left lane or not, (b) how many vehicles are involved in an accident occurring in the midday period, and whether there is a truck involved in the accident, (c) which lane a PM peak period accident is located in, and whether or not it is a singlevehicle accident, (d) whether or not a truck is involved in any accident, and finally, (e) whether or not the accident occurs on Friday.

The most important predictor of delay is whether the accident involves two vehicles and occurs in the midday period. This combination indicates an accident that would generally occur in heavy traffic that is moving at relatively high speeds. It would also be an accident that occurs prior to buildup of the afternoon rush hour, so that lingering effects are typically likely to influence steadily increasing levels of traffic. The next most important indicators are whether a PM peak period accident was located in the interior or left lanes, and whether an AM peak period accident involved multiple vehicles.

Focusing on the magnitude of the effect of each indicator, the accident that is likely to cause the greatest delay is an AM peak period accident involving three or more vehicles, which multiplies the base level of delay by a factor of more than seven. Other indicators of extensive delay is whether a PM peak period accident is in the left lane or off-road left, whether a AM peak period accident involves two-vehicles, or whether a PM peak period accident is in the interior lane(s). Reduced levels of delay are expected for truckinvolved accidents and for AM peak-period accidents in the left lane. Truck involved accidents are less likely to be injury accidents ( $18 \%$ of truck-involved accidents in case study dataset were injury accidents, compared to more than $25 \%$ of non-truck-involved accidents). However, if a truck is involved in an accident that occurs in the midday
period (weekdays, 9:01 a.m. through 3:29 p.m.), more non-recurrent delay can be expected. Also, single-vehicle accidents that occur in the PM peak period will lead to more delay, because such accidents are typically more severe (over $30 \%$ of PM peak period single-vehicle accidents are injury accidents, compared to about $25 \%$ of PM peak period multiple-vehicle accidents).

Eventual application of the results reported here can give managers an estimate of the total non-recurrent delay due to each of these accidents, as soon as they are spotted and basic characteristics are known. These results can also be useful for the performance evaluation of accident management systems by quantifying accident congestion in terms of total delay to evaluate the benefit of accident management systems accrued from efficient traffic operations. Additionally, they can be used by public sector transportation With further testing and refinement, the modeling procedures developed could be incorporated as a layer in the ATMS map display that, once an accident and its essential characteristics are observed, would display the likely spatial and temporal extent of the congestion expected from the accident.

## 2 Background

### 2.1 Use of Accident Information

Although freeways comprise only a small fraction of metropolitan transportation network mileage, they form the backbone of the urban transportation network, carrying more than a third of all vehicular travel. It is therefore understandable that, when people speak of traffic congestion, their focus typically is on urban freeways. Most studies of freeway congestion have been focused on recurring congestion caused when traffic demand exceeds capacity. Since recurrent congestion typically follows a predictable pattern, such a "rush hour" pattern at a particular location that may be a bottleneck area, travelers can change their trips according to when and where recurring congestion will occur. As a result, demand control policies such as High-Occupancy Vehicle (HOV) lanes, congestion toll pricing, transit incentives, and ramp metering have been used to alleviate the congestion.

Non-recurring congestion due to such unpredictable events as traffic accidents (one of the main sources of non-recurrent congestion) requires immediate response. Thus, the specific field of Accident Management has become an important component of Freeway Traffic Management Systems (FTMS). For this sort of traffic management system to succeed requires basic information on how, where, why, and to what extent congestion occurs; the actions taken by traffic operators require a full understanding of the nature and tendencies of freeway accidents.

When considering the extreme difficulty of estimating accident likelihood due to the definitional properties of non-recurring congestion, the most important potentially soluble factor in the development of accident management strategies is to identify and to quantify the conditions affecting the total delay by accidents. The information regarding total delay caused by an accident and expected duration of the delay conditions is vital to the dispatching strategies as well as to persons traveling on a freeway congested by an accident. These can be useless if generated without real-time forecasting scheme. Realtime forecasting allows the traffic management system to enact control and management
strategies that are "one step ahead" rather than "one step behind" the onset of traffic conditions (Smith and Williams, 1999).

The kernel of Accident Management is clearing traffic accidents quickly and then minimizing the congestion effects on the traffic flow. Clearing an accident quickly involves managerial support among agencies, clear guidelines for action, and early identification of the accident. Minimizing congestion due to the accident involves the use of such traveler information systems as dynamic message signs and advisory radio, employing reversible direction lanes where available, and vehicle re-routing (Smith et al, 2001). Since there is a vicious cycle between traffic accidents and traffic congestion, these actions should be employed as soon as possible to avoid any secondary accidents that compound the congestion.

From the freeway manager's perspective, real-time accident information is directly useful in guiding decisions regarding the resources needed to clear and dispatch crews. Correspondingly, freeway travelers can alter their routes according to when the congestion will be cleared and where the congestion will be affected; the diversion of demand achieved by accident information leads to less congestion duration and delay.

There are an increasing number of deployment efforts by governmental organizations and professionals of advanced transportation technologies designed to mitigate traffic accidents. In general, these efforts need to be understood in terms of quantified costs in order to evaluate their benefits.

### 2.2 Previous Studies

Several methods have been proposed to estimate total non-recurrent congestion delay. They can be classified into three groups: 1) analytical methods using deterministic queuing diagrams (e.g., Morales, 1987; Khattak et. al, 1994; HCM, 1994; and Ozbay and Kachroo, 1999), 2) kinematic wave (i.e., shockwave) theory (e.g., Chow, 1974; Messer et. al, 1976; Wirasinghe, 1978; and Al-Deek et. al, 1995), and 3) statistical methods based
on probability distribution of delay (Skabardonis et. al, 2003; and Dowling et. al, 2004) or regression model (Golob et. al, 1986; Recker et. al, 1989; and Garib et. al, 1997).

Additionally, there are several approaches to predicting the duration by non-recurrent congestion. These approaches can be classified as being either statistical (Golob et. al, 1987; Giuliano, 1989; Recker et. al, 1989; Khattak, et al., 1995; Garib et. al, 1997; Sullivan, 1997; Jones et. al, 1991; Nam and Mannering, 2000; Smith, et al., 2001; and Stathopoulos and Karlaftis, 2002) or heuristic knowledge based expert system (Hobeika et. al, 1992; and Ozbay and Kachroo, 1999).

The results from these approaches have been applied alone or mixed. Since virtually all of these studies used a different source of incident data with different descriptive variables and reporting techniques, comparisons between different methods are difficult due to data issues (Smith et. al, 2001). Also, to date, there has been limited research into models that can predict how long a certain incident will affect traffic. This study proposes new methods for measuring freeway accident congestion effects based on ubiquitous single loop detector data.

### 2.3 Objectives

The objective of this study is to develop methods to quantify the accident delay and the temporal and spatial extent of accident-related congestion in real time. From these methods it will be possible to develop a performance measure to evaluate transportation policies and planning level analyses associated with design of transportation systems or preparation of operating plans for safety, as well as for evaluation of deployed transportation projects or technologies. Since the data set used in this study is a stream of one-year observations from single inductive loop detectors, the procedures of statistical analyses using huge database are addressed. In addition, this study presents a new approach to the quantification of the total delay caused by traffic accidents.

## 3 Methodology

### 3.1 Overview

This study uses data on approximately 9,200 crashes on six major freeways in Orange County during 2001 in combination with one-year historical inductive loop detector data from March 2001 to February 2002 as the base traffic dataset. Freeway traffic data for all Orange County are available from the UCI Testbed Research Laboratory. The original data include traffic counts and occupancy for each lane at each detector station every 30 seconds. Figure 3-1 shows loop detector stations of this study area. There are 499 mainline loop detector stations in the study area. However, many loop detector data were missing during the study period. Due to freeway construction and communication problems, a quarter of detectors were not functioning, and only a $61.3 \%$ of loop stations were able to provide more than $50 \%$ of traffic data. In this study, lane-by-lane traffic data were aggregated into 5-minute intervals at each detector station in order to obtain stable traffic data from each point. A simple method was applied in aggregating traffic measures.

Since the size of base data aggregated into 5-minute intervals is over 50,000,000 records, a Data Base Management System (DBMS) is employed to efficiently manage such a huge data set. From the available candidates (such as Oracle, MS-SQL server, Informix, Sybase, and MySQL), MySQL was selected and most analyses are processed using it and its application program interface (API) programs with C and $\mathrm{C}++$.


Figure 3-1. Loop Detector Stations of Study Area

Once the base data set is constructed appropriately, several procedures including statistical analyses and optimization programming are employed to find the accident impacts in terms of total delay. Moreover, a series of multivariate statistical methods are being employed to develop a prediction model for accident duration and delay in real time. Once developed, the model will be tested by using another data set from newly archived inductive loop detector dataset or simulated dataset from the PARAMICS microscopic traffic simulation model, widely used in California. Figure 3-2 depicts overall process for this study.


Figure 3-2. Overall process

### 3.2 Data Processing

### 3.2.1 Section Definition

A freeway section in this study corresponds to a mainline loop detector station. A section is discriminated in the middle of two detector stations as shown in Figure 3-3, implying that the estimated speed at the station is capable of being the representative speed for the corresponding section. Based on the sections and their corresponding detector stations, estimated speeds and densities for each section are calculated every 5-minute interval during the analysis one-year period.


Figure 3-3. Section definition and the corresponding detector location

### 3.2.2 Referencing Accident Events

Once freeway sections are defined as described above, it is necessary to identify the location of traffic accident. Although the traffic accident log has the location data with freeway postmile declared by Caltrans, linear referencing technique is useful to identify the accident locations from every section. All event features can be identified by a known measurement without $x-y$ coordinates, such as postmile on the linear feature (ESRI, 2001). Thus, every accident event can be identified to which section is related.

### 3.2.3 Speed Estimation using Single Loop Data

This study uses one-year historical traffic data from March 2001 to February 2002, and the original data includes traffic counts and occupancy for each lane at each detector station every 30 seconds. However, lane-by-lane traffic data were aggregated to 5minute intervals at each detector station through weighted average method in order to obtain stable traffic data from each point.

Loop detectors in Orange County are single loop detectors that provide only traffic counts and occupancies; thus, the travel speeds need to be estimated from these measures assuming a so-called "g-factor," the summation of the average vehicle length and effective detection length. The average speed, $\bar{s}$, can be calculated as:

$$
\begin{equation*}
\bar{s}=g \frac{q}{5280 \times o c c} \tag{3-1}
\end{equation*}
$$

where
$q=$ flow rate(veh/h)
occ $=$ occupancy
$g=g-$ factor

The most critical value in calculating the speed is the assumed g-factor. In fact, applying appropriate $g$-factors is a key in estimating speeds from single loop detector data. There are various ways to estimate g -factors. The simplest approach is to use a single g -factor for all loop detectors in the study area. Although this is very common in practice, the approximation from this approach is too rough to reach the objective of this study. An alternative is to use a single $g$-factor for each lane through the entire freeway system. This approach accounts for the fact that vehicle length varies by lane, but it assumes the same proportion of heavy vehicles on every section of the freeway. Thus it is very hard to capture the variation in the tuning of individual detectors. Another approach is to find a single $g$-factor for every loop in each lane. This is estimated by using flow and occupancy from some period that speed is known in order to calculate the $g$-factor. For example, flow and occupancy during 1:00AM-1:05AM, where the average speed can be assumed as a free flow speed ( 75 mph ), can be used to calculate the g-factor. This approach, however, does not account for the variation of the vehicle composition over time.

Zhanfeng et al. (2001) described that the g -factors for different loop detectors in the same district differ by as much as 100 percent, and the g-factors for the same loop can vary up to 50 percent over 24 -hour period. To consider spatial and temporal variations of the gfactors, in this study, a g-factor representing each hour for each loop detector station is calculated by the following assumptions and steps:

Step 1: Calculate initial g-factor by assuming the free-flow speed ( 75 mph ) when the occupancy is lower than 0.06

Step 2: Find an average g-factor representing each hour interval based on the identified initial g-factors over the 52 weeks

Step 3: Apply smoothing parameter (0.9) for the next time period

$$
\begin{equation*}
\hat{g}(t+1)=p \cdot \hat{g}(t)+(1-p) \cdot g(t+1) \tag{3-2}
\end{equation*}
$$

where

$$
\begin{aligned}
& \hat{g}(t+1)=g-\text { factor for time step } t+1 \\
& g(t+1)=\text { initial average } g-\text { factor for time step } t \\
& p=\text { smoothing parameter }(0.9)
\end{aligned}
$$

By applying the above procedure, representative $g$-factors for each hour of day are calculated for all loop detector stations. These g -factors provide basic information for speed calculation in this study. Based on these g-factors, speeds are calculated every 5 minutes for all 52 weeks.

### 3.2.4 Speed Distribution

For each section, for each day, for one year, for $t_{m}$ in 5-minute increments, flow $\left(q_{j}\left(t_{m}\right)\right)$ and occupancy $\left(\operatorname{occ}_{j}\left(t_{m}\right)\right)$ have been established; i.e., for every $j, t$, nominally 52 observations have constructed. For example, section $j$ on Monday from $t_{m}=08: 10$ to 08:15 for 52 weeks is composed of 52 samples. Thus the $n^{\text {th }}$ speed for any particular day-of-week/time interval/section combination can be estimated as following equation;

$$
\begin{equation*}
s_{j n}\left(t_{m}\right)=g_{j n}\left(t_{m}\right) \cdot \frac{q_{j n}\left(t_{m}\right)}{5280 \times o c c_{j n}\left(t_{m}\right)} ; n=1,2, \ldots, 52 \tag{3-3}
\end{equation*}
$$

Figure 3-4 shows the distribution of $s_{j n}\left(t_{m}\right)$ from 52 observations. Let $\Omega_{j m}=\Omega\left(\bar{s}_{j}\left(t_{m}\right), \sigma_{s_{j}\left(t_{m}\right)}\right)$ denote the set of parameters defining the distribution of the base case speeds $s_{j n}\left(t_{m}\right)$. Then, for $t_{m}>t_{o}, m=1,2, \ldots$ (i.e., time intervals after the accident that occurs in section $i$ at $t_{o}$ ), for all upstream sections that could possibly have been affected by the accident, we can compose a matrix of base case conditions (i.e.,
conditions in which there is no accident) that can be expected to prevail as described in Fable 3-1


Figure 3-4. Speed distribution from 52 observations

Table 3-1. Base case of distributional properties for non-accident speeds $s_{j n}\left(t_{m}\right)$

| Time | Freeway Section |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $i$ | $i-1$ | $i-2$ | $\ldots$ | 2 | 1 |
| $t_{1}$ | $\Omega_{i, 1}=\Omega\left(\bar{s}_{i}\left(t_{1}\right), \sigma_{s(t)}\right)$ | $\Omega_{i-1,1}=\Omega\left(\bar{s}_{i-1}\left(t_{i}\right), \sigma_{s+1(t)}\right)$ | ... | $\ldots$ | $\ldots$ | $\Omega_{1,1}=\Omega\left(\bar{s}_{1}\left(t_{1}\right), \sigma_{\sigma_{\text {S }}(1,)}\right)$ |
| $t_{2}$ | $\Omega_{\mathrm{i} 2}=\Omega\left(\bar{s}_{i}\left(t_{2}\right), \sigma_{s(2)}\right)$ | $\Omega_{i-1,2}=\Omega\left(\bar{s}_{i-1}\left(t_{2}\right), \sigma_{\text {Si-1 }}(2)\right.$ | $\ldots$ | $\ldots$ | ... | $\Omega_{1,2}=\Omega\left(\bar{s}_{1}\left(t_{2}\right), \sigma_{s(t)}\right)$ |
| $t_{3}$ | $\Omega_{i, 3}=\Omega\left(\bar{s}_{i}\left(t_{3}\right), \sigma_{s,(, 3)}\right)$ | $\Omega_{i-1,3}=\Omega\left(\bar{s}_{-1-1}\left(t_{3}\right) \sigma_{\text {S }}\left(t_{3}\right)\right.$ | $\ldots$ | $\ldots$ | ... | $\Omega_{1,3}=\Omega\left(\bar{s}_{1}\left(t_{3}\right), \sigma_{s_{i(t, 3)}}\right)$ |
| $\vdots$ | ! | ! | ! | ! | : | $\vdots$ |
| $t_{M-1}$ | $\Omega_{i, w-1}=\Omega\left(\bar{s}_{i}\left(t_{w-1}\right), \sigma_{s,(t y-1}\right)$ | $\Omega_{i-1, w-1}=\Omega\left(\bar{s}_{i-1}\left(t_{w-1}\right) \sigma_{s+1}\left(t_{(x-1)}\right)\right.$ | ... | ... | ... | $\Omega_{1, w-1}=\Omega\left(\bar{s}_{1}\left(t_{M-1}\right), \sigma_{5\left(y_{(w-1)}\right)}\right)$ |
| $t_{\mu}$ | $\Omega_{i, M}=\Omega\left(\bar{s}_{i}\left(t_{\mu}\right), \sigma_{s,\left(t, t^{\prime}\right.}\right)$ | $\Omega_{i-1, M}=\Omega\left(\bar{s}_{i-1}\left(t_{M}\right), \sigma_{s-1}\left(t_{M}\right)\right.$ | ... | ... | ... | $\Omega_{1, u}=\Omega\left(\bar{s}_{1}\left(t_{M}\right), \sigma_{s,\left(t_{w}\right.}\right)$ |

Similarly, the speed distribution under each traffic accident can be described as in Table 3-2. For example, suppose that an accident occurred on freeway section $i$ at time $t=t_{o}$. Then, we can observe the corresponding measurements for the accident conditions, $\hat{q}_{j}\left(t_{m}\right), \hat{k}_{j}\left(t_{m}\right) ; j=i, i-1, i-2, \ldots ; m=1,2, \ldots$, from which we can calculate:

$$
\begin{equation*}
\hat{s}_{j}\left(t_{m}\right)=g_{j}\left(t_{m}\right) \cdot \frac{\hat{q}_{j}\left(t_{m}\right)}{\hat{k}_{j}\left(t_{m}\right)} ; j=i, i-1, i-2, \ldots ; m=1,2, \ldots \tag{3-4}
\end{equation*}
$$

We can then compose a matrix of accident case conditions (i.e., conditions that were observed to prevail following the accident) as:

Table 3-2. Observed accident speeds $\hat{s}_{j}\left(t_{m}\right)$

| Time | Freeway Section |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $i$ | $i-1$ | $i-2$ | $\cdots$ | 2 | 1 |
| $t_{1}$ | $\hat{s}_{i}\left(t_{1}\right)$ | $\hat{s}_{i-1}\left(t_{1}\right)$ | $\cdots$ | $\cdots$ | $\ldots$ | $\hat{s}_{1}\left(t_{1}\right)$ |
| $t_{2}$ | $\hat{s}_{i}\left(t_{2}\right)$ | $\hat{s}_{i-1}\left(t_{2}\right)$ | $\cdots$ | $\cdots$ | $\cdots$ | $\hat{s}_{1}\left(t_{2}\right)$ |
| $t_{3}$ | $\hat{s}_{i}\left(t_{3}\right)$ | $\hat{s}_{i-1}\left(t_{3}\right)$ | $\cdots$ | $\cdots$ | $\cdots$ | $\hat{s}_{1}\left(t_{3}\right)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $t_{M-1}$ | $\hat{s}_{i}\left(t_{M-1}\right)$ | $\hat{s}_{i-1}\left(t_{M-1}\right)$ | $\cdots$ | $\cdots$ | $\cdots$ | $\hat{s}_{1}\left(t_{M-1}\right)$ |
| $t_{M}$ | $\hat{s}_{i}\left(t_{M}\right)$ | $\hat{s}_{i-1}\left(t_{M}\right)$ | $\cdots$ | $\cdots$ | $\cdots$ | $\hat{s}_{1}\left(t_{M}\right)$ |

Relative to the display of information in Fable 3-2 we can describe the negative effects (i.e., speed reduction) of the accident schematically as shown in Figure 3-5. The negative effect by the accident will be propagated from the accident section to upstream sections. Such a distinct discontinuity between non-congested and congested flow is known as a shock wave (May, 1990). If the dot-shaded area affected by the shock wave in Figure 3-5 is identified, then the temporal and spatial impacts of the accident will be also determined. The following section describes the method for discriminating the regions between non-congested and congested area due to traffic accidents.

|  | Time | Freeway Section |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $i$ | $i-1$ | $i-2$ | $i-3$ | i-4 | $i-5$ | $i-6$ | $i-7$ | $i-8$ | $\ldots$ | 1 |
|  | $t_{1}$ | $\hat{s}\left(t_{1}\right)$ | $\hat{s}_{i-1}\left(t_{1}\right)$ | $\hat{s}_{i-2}\left(t_{1}\right)$ | $\hat{s}_{i-3}\left(t_{1}\right)$ | $\hat{s}_{i-4}\left(t_{1}\right)$ | $\hat{s}_{i-5}\left(t_{1}\right)$ | $\hat{s}_{i-6}\left(t_{1}\right)$ | $\hat{s}_{i-7}\left(t_{1}\right)$ | $\hat{s}_{i-8}\left(t_{1}\right)$ | $\ldots$ | $\hat{s}_{1}\left(t_{1}\right)$ |
|  | $t_{2}$ | $\hat{s}\left(t_{2}\right)$ | $S_{1}\left(t_{2}\right)$ | $\hat{s}_{i-2}\left(t_{2}\right)$ | $\hat{s}_{i-3}\left(t_{2}\right)$ | $\hat{s}_{i-4}\left(t_{2}\right)$ | $\hat{s}_{i-5}\left(t_{2}\right)$ | $\hat{s}_{i-6}\left(t_{2}\right)$ | $\hat{s}_{i-7}\left(t_{2}\right)$ | $\hat{S}_{i-8}\left(t_{2}\right)$ | $\ldots$ | $\hat{S}_{1}\left(t_{2}\right)$ |
|  | $t_{3}$ | $\hat{s}\left(r_{3}\right)$ | $\hat{s}_{-1}\left(t_{5}\right)$ | $\hat{s}^{2}\left(t_{3}\right)$ | $\hat{S}_{i-3}\left(t_{3}\right)$ | $\hat{s}_{i-4}\left(t_{3}\right)$ | $\hat{S}_{i-5}\left(t_{3}\right)$ | $\hat{s}_{i-6}\left(t_{3}\right)$ | Set of freeway sections that do not have data relevant to the accident |  |  |  |
|  | $t_{4}$ | $\hat{S}\left(t_{4}\right)$ | $\hat{s}_{\text {H- }}\left(t_{4}\right)$ | $\hat{S}_{-2}\left(t_{4}\right)$ | $\hat{s}_{i-3}\left(t_{4}\right)$ | $\hat{s}_{i-4}\left(t_{4}\right)$ | $\hat{s}_{i-5}\left(t_{4}\right)$ | $\hat{s}_{i-6}\left(t_{4}\right)$ | $\hat{s}_{i-7}\left(t_{4}\right)$ | $\hat{S}_{i-8}\left(t_{4}\right)$ | ... | $\hat{s}_{1}\left(t_{4}\right)$ |
|  | $t_{5}$ | $S$ St $\left(t_{r}\right)$ | $\hat{s}_{\text {Le }}\left(t_{s}\right)$ | $\hat{S}_{-2}\left(l_{5}\right)$ | $\hat{s}_{i-3}\left(t_{5}\right)$ | $\hat{s}_{i-4}\left(t_{5}\right)$ | $\hat{s}_{i-5}\left(t_{s}\right)$ | $\hat{s}_{i-6}\left(t_{s}\right)$ | $\hat{s}_{i-7}\left(t_{5}\right)$ | $\hat{s}_{i-8}\left(t_{5}\right)$ | ... | $\hat{s}_{1}\left(t_{5}\right)$ |
| 4 | $t_{6}$ | $\hat{s}_{i}\left(t_{6}\right)$ | $\hat{S}_{-1}\left(t_{6}\right)$ | $\hat{S}_{-2}\left(t_{0}\right)$ | $\hat{s}_{i}-\left(t_{6}\right)$ | $\hat{S}_{i-4}\left(t_{6}\right)$ | $\hat{S}_{i-5}\left(t_{6}\right)$ | $\hat{S}_{i-6}\left(t_{6}\right)$ | $\hat{S}_{i-7}\left(t_{6}\right)$ | $\hat{s}_{i-8}\left(t_{6}\right)$ | $\cdots$ | $\hat{S}_{1}\left(t_{6}\right)$ |
|  | $t_{7}$ | $\hat{S}_{i}\left(t_{7}\right)$ | $\hat{S}_{1}\left(t_{1}\right)$ | $\hat{S}_{-2}\left(t_{1}\right)$ | $\hat{S}_{-3}\left(t_{7}\right)$ | $\hat{S}_{i-4}\left(t_{i}\right)$ | $\hat{S}_{i-5}\left(t_{7}\right)$ | $\hat{s}_{i-6}\left(t_{7}\right)$ | $\hat{s}_{i-7}\left(t_{7}\right)$ | $\hat{S}_{S_{i-8}\left(U_{7}\right)}$ Accident shock wave |  |  |
|  | $t_{8}$ | $\hat{s}_{i}\left(t_{8}\right)$ | $\hat{S}_{i-1}\left(t_{8}\right)$ | $\hat{S}_{-2}\left(t_{\text {d }}\right.$ | Set of freew | way section | $\hat{S}_{s,}\left(t_{s}\right)$ | $S_{i-6}\left(t_{r}\right)$ | $\widehat{s}_{i-7}\left(t_{8}\right)$ | $\hat{s}_{i-8}\left(t_{p}\right)$ | $\ldots$ | $\hat{S}_{1}\left(t_{8}\right)$ |
|  | $t_{9}$ | $\hat{s}_{i}\left(t_{9}\right)$ | $\hat{s}_{i-1}\left(t_{9}\right)$ | $\hat{S}_{-2}(t$, | impacted by | by accident |  | $\hat{S}_{-c}\left(t_{0}\right)$ | $\hat{S}_{i-7}\left(t_{9}\right)$ | $\hat{s}_{-8}\left(t_{9}\right)$ | $\ldots$ | $\hat{S}_{1}\left(t_{9}\right)$ |
|  | $t_{10}$ | $\hat{s}_{i}\left(t_{10}\right)$ | $\hat{s}_{i-1}\left(t_{10}\right)$ | $S_{i-2}\left(t_{10}\right)$ | ${ }^{-3}{ }^{(410)}$ | (lia) | $\hat{S}_{\text {i- }}\left(t_{\text {fo }}\right)$ | $\hat{S}_{\text {F-0 }}\left(t_{10}\right)$ | $\hat{s}_{i-7}\left(t_{10}\right.$ | $\hat{S}_{i-8}\left(t_{10}\right)$ | $\ldots$ | $\hat{s}_{1}\left(t_{10}\right)$ |
|  | $t_{11}$ | Set of freeway sections that could have possibly been impacted by accident $\qquad$ |  |  |  |  |  | $\hat{S}_{-s}\left(t_{1}\right)$ | $\hat{s}_{\text {i, }}\left(t_{1}\right)$ | $\hat{S}_{s-8}\left(t_{1}\right)$ | $\ldots$ | $\hat{S}_{1}\left(t_{11}\right)$ |
|  | $t_{12}$ |  |  |  |  |  | $\hat{s}_{i-5}\left(t_{12}\right)$ |  | $S_{+}\left(t_{12}\right)$ | $\hat{s}_{\text {cs }}\left(t_{12}\right)$ | $\ldots$ | $\hat{S}_{1}\left(t_{12}\right)$ |
|  | ! | $\vdots$ | ! | ! | Clearing wave |  |  |  |  | ! | $\ldots$ | $\vdots$ |
|  | $t_{M-1}$ | $\hat{s}_{i}\left(t_{M-1}\right)$ | $\hat{s}_{i-1}\left(t_{M-1}\right)$ | $\hat{s}_{i-2}\left(t_{M-1}\right)$ | $\hat{s}_{i-3}\left(t_{M-1}\right)$ | $\hat{s}_{i-4}\left(t_{M-1}\right)$ | $\hat{s}_{i-5}\left(t_{M-1}\right)$ | $\hat{s}_{i-6}\left(t_{M-1}\right)$ | $\hat{s}_{i-7}\left(t_{M-1}\right)$ | $\hat{s}_{i-8}\left(t_{M-1}\right)$ | ... | $\hat{s}_{1}\left(t_{M-1}\right)$ |
|  | $t_{M}$ | $\hat{s}_{i}\left(t_{M}\right)$ | $\hat{s}_{i-1}\left(t_{M}\right)$ | $\hat{s}_{i-2}\left(t_{M}\right)$ | $\hat{s}_{i-3}\left(t_{M}\right)$ | $\hat{S}_{i-4}\left(t_{M}\right)$ | $\hat{s}_{i-5}\left(t_{M}\right)$ | $\hat{S}_{i-6}\left(t_{M}\right)$ | $\hat{s}_{i-7}\left(t_{M}\right)$ | $\hat{s}_{i-8}\left(t_{M}\right)$ | ... | $\hat{S}_{1}\left(t_{M}\right)$ |

Figure 3-5. Schematic accident effect

### 3.3 Quantifying Accident Duration and Total Delay

### 3.3.1 Determining Maximum Extent of Accident Influence

Since data on either the severity (such as the number of closed lanes by the accident) or when the accident was cleared is not directly obtainable from loop data, we first estimate the maximum possible extent of the shock wave by assuming the worst possible conditions-total blockage for some pre-specified time period. Thus, for any given accident occurring at section $i$ at time $t_{1}$, we compute the maximum number of upstream sections that could be affected by the assumed persistent total blockage at section $i$ at time $t_{1}$. Table 3-3 provides one such set of calculations for an accident on SR22.

Table 3-3. An example of maximum possible extent of shock wave

| Acc <br> ID | Acc time | Time lapse | SW <br> length | Acc <br> section | Affected sections |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 9 | $2001-03-0114: 15: 00$ | $2001-03-0114: 15: 00$ | 0.000 | 11 |  |
| 9 | $2001-03-0114: 15: 00$ | $2001-03-0114: 20: 00$ | -0.787 | 11 |  |
| 9 | $2001-03-0114: 15: 00$ | $2001-03-0114: 25: 00$ | -1.517 | 11 | 10,9 |
| 9 | $2001-03-0114: 15: 00$ | $2001-03-0114: 30: 00$ | -2.179 | 11 | $10,9,8,7$ |
| 9 | $2001-03-0114: 15: 00$ | $2001-03-0114: 35: 00$ | -2.945 | 11 | $10,9,8,7$ |
| 9 | $2001-03-0114: 15: 00$ | $2001-03-0114: 40: 00$ | -3.501 | 11 | $10,9,8,7,6,5$ |
| 9 | $2001-03-0114: 15: 00$ | $2001-03-0114: 45: 00$ | -4.081 | 11 | $10,9,8,7,6,5,4$ |
| 9 | $2001-03-0114: 15: 00$ | $2001-03-0114: 50: 00$ | -4.593 | 11 | $10,9,8,7,6,5,4,3$ |
| 9 | $2001-03-0114: 15: 00$ | $2001-03-0114: 55: 00$ | -5.264 | 11 | $10,9,8,7,6,5,4,3,2$ |
| 9 | $2001-03-0114: 15: 00$ | $2001-03-0115: 00: 00$ | -5.813 | 11 | $10,9,8,7,6,5,4,3,2,1$ |
| 9 | $2001-03-0114: 15: 00$ | $2001-03-0115: 05: 00$ | -6.376 | 11 | $10,9,8,7,6,5,4,3,2,1$ |
| 9 | $2001-03-0114: 15: 00$ | $2001-03-0115: 10: 00$ | -6.750 | 11 | $10,9,8,7,6,5,4,3,2,1$ |

Using this sort of data, we can schematically construct the "maximum area of interest" for any accident occurring at section $i$ at time $t_{1}$ as the shaded (green) area in Figure 3-6. Based on this interpretation, the only data relevant to the current example of an accident occurring at section $i$ at time $t_{1}$ is restricted to cells in the shaded (green) area in Figure 3-6. That is, the region can be depicted as shown in Figure 3-7.

|  | Time | Freeway Section |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $i$ | $i-1$ | $i-2$ | $i-3$ | $i-4$ | $i-5$ | $i-6$ | $i-7$ | $i-8$ | $\ldots$ | 1 |
|  | $t_{1}$ | $\hat{s}_{( }\left(t_{1}\right)$ | $\hat{s}_{i-1}\left(t_{1}\right)$ | $\hat{s}_{i-2}\left(t_{1}\right)$ | $\hat{s}_{i-3}\left(t_{1}\right)$ | $\hat{s}_{i-4}\left(t_{1}\right)$ | $\hat{S}_{i-5}\left(t_{1}\right)$ | $\hat{s}_{i-6}\left(t_{1}\right)$ | $\hat{s}_{i-7}\left(t_{1}\right)$ | $\hat{s}_{i-8}\left(t_{1}\right)$ | $\ldots$ | $\hat{s}_{1}\left(t_{1}\right)$ |
|  | $t_{2}$ | $\hat{S}\left(t_{2}\right)$ | $S_{1}\left(t_{2}\right)$ | $\hat{s}_{i-2}\left(t_{2}\right)$ | $\hat{S}_{i-3}\left(t_{2}\right)$ | $\hat{s}_{i-4}\left(t_{2}\right)$ | $\hat{S}_{i-5}\left(t_{2}\right)$ | $\hat{S}_{i-6}\left(t_{2}\right)$ | $\hat{s}_{i-7}\left(t_{2}\right)$ | $\hat{s}_{i-8}\left(t_{2}\right)$ | $\ldots$ | $\hat{s}_{1}\left(t_{2}\right)$ |
|  | $t_{3}$ | $\hat{s}\left(t_{3}\right)$ | $\hat{S}_{-1}\left(t_{5}\right)$ | ${ }_{2}\left(t_{3}\right)$ | $\hat{s}_{i-3}\left(t_{3}\right)$ | $\hat{S}_{i-4}\left(t_{3}\right)$ | $\hat{s}_{i-5}\left(t_{3}\right)$ | $\hat{S}_{i-6}\left(t_{3}\right)$ | Set of freeway sections that do not have data relevant to the accident |  |  |  |
|  | $t_{4}$ | $\hat{s}\left(t_{4}\right)$ | $\hat{S}_{-1}\left(t_{4}\right)$ | $\hat{S}_{-2}\left(t_{4}\right)$ | $\hat{s}_{i-3}\left(t_{4}\right)$ | $\hat{S}_{i-4}\left(t_{4}\right)$ | $\hat{S}_{i-5}\left(t_{4}\right)$ | $\hat{S}_{i-6}\left(t_{4}\right)$ | $\hat{s}_{i-7}\left(t_{4}\right)$ | $\hat{s}_{i-8}\left(t_{4}\right)$ | ... | $\hat{s}_{1}\left(t_{4}\right)$ |
|  | $t_{5}$ | $\hat{S}\left(t_{s}\right)$ | $\hat{S}_{\text {H }}\left(t_{s}\right)$ | $\hat{S}_{-2}\left(t_{5}\right)$ | $\hat{s}_{i-3}\left(t_{5}\right)$ | $\hat{S}_{i-4}\left(t_{5}\right)$ | $\hat{s}_{i-5}\left(t_{s}\right)$ | $\hat{s}_{i-6}\left(t_{s}\right)$ | $\hat{s}_{i-7}\left(t_{5}\right)$ | $\hat{s}_{i-8}\left(t_{5}\right)$ | $\ldots$ | $\hat{s}_{1}\left(t_{5}\right)$ |
| 4 | $t_{6}$ | $\hat{s}_{i}\left(t_{6}\right)$ | $\hat{S}_{-1}\left(t_{6}\right)$ | $\hat{S}_{-2}\left(t_{6}\right)$ | $\hat{s}_{i \rightarrow( }\left(t_{6}\right)$ | $\hat{S}_{i-4}\left(t_{6}\right)$ | $\hat{S}_{i-5}\left(t_{6}\right)$ | $\hat{s}_{i-6}\left(t_{6}\right)$ | $\hat{s}_{i-7}\left(t_{6}\right)$ | $\hat{S}_{i-8}\left(t_{6}\right)$ | $\cdots$ | $\hat{s}_{1}\left(t_{6}\right)$ |
|  | $t_{7}$ | $\hat{s}_{i}\left(t_{7}\right)$ | (t.) | $\hat{S}_{-2}\left(t_{1}\right)$ | $\hat{S}_{-, 3}\left(t_{r}\right)$ | $S_{i-4}\left(T_{i}\right)$ | $\hat{s}_{\text {i-s }}\left(t_{7}\right)$ | $\hat{s}_{i-6}\left(t_{7}\right)$ | $\hat{s}_{i-7}\left(t_{7}\right)$ | Accident shock wave |  |  |
|  | $t_{8}$ | $\hat{s}_{i}\left(t_{8}\right)$ | $\overline{\hat{s}_{i-1}\left(t_{8}\right)}$ | $\hat{S}_{\text {-2, }} \mathrm{t}_{\text {d }}$ | Set of freew | ay section | $\hat{S}_{s}\left(t_{0}\right)$ | $\hat{S}_{i=\sigma}\left(t_{s}\right)$ | $\hat{s}_{i-7}\left(t_{8}\right)$ | $\hat{s}_{\text {i-s }}\left(t_{r}\right)$ | $\ldots$ | $\hat{S}_{1}\left(t_{8}\right)$ |
|  | $t$, | $\hat{s}_{i}\left(t_{9}\right)$ | $\hat{S}_{i-1}\left(t_{9}\right)$ | (t, | impacted | y accident | $\hat{S}_{-,}\left(t_{t}\right)$ | $\hat{S}_{\text {Sol }}\left(t_{0}\right)$ | $\hat{s}_{i-7}\left(t_{9}\right)$ | $\hat{s}_{5} /{ }_{8}\left(t_{9}\right)$ | $\ldots$ | $\hat{s}_{1}\left(t_{9}\right)$ |
|  | $t_{10}$ | $\hat{s}_{i}\left(t_{10}\right)$ | $\hat{s}_{i-1}\left(t_{10}\right)$ | ( $t_{10}$ ) | $S_{i-3}(4)$ |  | $\hat{S}_{\text {is }}\left(t_{10}\right)$ | $\hat{S}_{\text {c- }}\left(t_{10}\right)$ | $\hat{s}_{i-7}(t$ | $i_{i-8}\left(t_{10}\right)$ | ... | $\hat{s}_{1}\left(t_{10}\right)$ |
|  | $t_{11}$ |  |  |  |  |  |  | $\hat{S}_{\text {S }}^{\sim}$ ( $t_{11}$ ) | $\hat{s}_{\text {i- }}\left(t_{11}\right)$ | $\hat{S}_{-s}\left(t_{1}\right)$ | ... | $\hat{s}_{1}\left(t_{11}\right)$ |
|  | $t_{12}$ |  |  |  |  |  | $\hat{s}_{i-5}\left(t_{12}\right)$ |  | ${ }_{5}+\left(t_{12}\right)$ | $\hat{S}_{\text {s }}\left(t_{12}\right)$ | $\ldots$ | $\hat{S}_{1}\left(t_{12}\right)$ |
|  | ! | : | : |  | Clearing wave |  |  |  |  |  | ... |  |
|  | $t_{M-1}$ | $\hat{s}_{i}\left(t_{M-1}\right)$ | $\hat{S}_{i-1}\left(t_{M-1}\right)$ | $\hat{s}_{i-2}\left(t_{M-1}\right)$ | $\hat{s}_{i-3}\left(t_{M-1}\right)$ | $\hat{S}_{i-4}\left(t_{M-1}\right)$ | $\hat{S}_{i-5}\left(t_{M-1}\right)$ | $\hat{S}_{i-6}\left(t_{M-1}\right)$ | $\hat{s}_{i-7}\left(t_{M-1}\right)$ | $\hat{S}_{i-8}\left(t_{M-1}\right)$ | ... | $\hat{s}_{1}\left(t_{M-1}\right)$ |
|  | $t_{M}$ | $\hat{s}_{i}\left(t_{M}\right)$ | $\hat{s}_{i-1}\left(t_{M}\right)$ | $\hat{s}_{i-2}\left(t_{M}\right)$ | $\hat{s}_{i-3}\left(t_{M}\right)$ | $\hat{s}_{i-4}\left(t_{M}\right)$ | $\hat{s}_{i-5}\left(t_{M}\right)$ | $\hat{s}_{i-6}\left(t_{M}\right)$ | $\hat{s}_{i-7}\left(t_{M}\right)$ | $\hat{s}_{i-8}\left(t_{M}\right)$ | $\ldots$ | $\hat{s}_{1}\left(t_{M}\right)$ |

Figure 3-6. Maximum set of freeway sections impacted by accident


Figure 3-7. Representation of maximum set of relevant data

### 3.3.2 Determining Congested Region

Figure 3-7 shows two shaded area; dot-shaded ("yellow") area and generally shaded ("green") area. The cells in the dot-shaded area represent speeds, $\hat{s}_{j}\left(t_{m}\right)$, that have been lowered due to the accident (i.e., affected by accident). Other shaded cells represent speeds that are not significantly different from non-accident conditions. That is, they are speeds that are determined from observation not to be affected by the pertinent accident, but within the maximum possible affected congestible sections by an accident of greatest consequence (when all lanes are blocked, as assumed above).

Considering that the speed of traffic in sections adversely affected by the traffic accident will be lowered, the basic idea behind discriminating between these two regions is to compare the accident speed, $\hat{s}_{j}\left(t_{m}\right)$, to the distribution of the non-accident speeds $s_{j n}\left(t_{m}\right) ; n=1,2, \cdots, n_{\text {obs }} ; n_{\text {obs }} \leq 52$ and assign some level of confidence that any particular $\hat{s}_{j}\left(t_{m}\right)$ was not drawn from the distribution of $s_{j n}\left(t_{m}\right)$. One simple way to carry out this idea is to calculate the proportion of $s_{j n}\left(t_{m}\right)<\hat{s}_{j}\left(t_{m}\right)$ as the probability that $\hat{s}_{j}\left(t_{m}\right)$ belongs to the distribution of $s_{j n}\left(t_{m}\right)$ as shown in Figure 3-8.


Figure 3-8. Probability of accident speed

In this example, it is concluded that there is only a 0.13 chance that $\hat{s}_{j}\left(t_{m}\right)$ belongs to the distribution of $s_{j n}\left(t_{m}\right)$. Likewise, if this approach is applied to $\hat{s}_{j}\left(t_{m}\right)$ of all shaded cells in Figure 3-7, a matrix can be constructed as shown in Figure 3-9.

|  | Time | Freeway Section |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\downarrow$ |  | $i$ | $i-1$ | $i-2$ | $i-3$ | $i-4$ | $i-5$ | $i-6$ | $i-7$ | $i-8$ | $\ldots$ | 1 |
|  | $t_{1}$ | 0.02 | 0.54 |  |  |  |  |  |  |  |  |  |
|  | $t 2$ | 0.04 | 0.52 | 0.62 |  |  |  |  |  |  |  |  |
|  | $t_{3}$ | 0.10 | 0.38 | 0.56 |  |  |  |  |  |  |  |  |
|  | $t_{4}$ | 0.06 | 0.06 | 0.29 | 0.56 |  |  |  |  |  |  |  |
|  | $t_{5}$ | 0.23 | 0.17 | 0.21 | 0.52 |  |  |  |  |  |  |  |
| 4 | $t_{6}$ | 0.62 | 0.15 | 0.38 | 0.77 | 0.62 | 0.87 |  |  |  |  |  |
|  | $t_{7}$ | 0.63 | 0.19 | 0.02 | 0.10 | 0.63 | 0.90 | 0.96 | 0.88 |  |  |  |
|  | $t_{8}$ | 0.87 | 0.58 | 0.37 | 0.29 | 0.31 | 0.10 | 0.87 | 0.52 |  |  |  |
|  | $t_{9}$ | 0.90 | 0.62 | 0.44 | 0.23 | 0.17 | 0.08 | 0.31 | 0.90 |  |  |  |
|  | $t_{10}$ | 0.85 | 0.85 | 0.85 | 0.85 | 0.33 | 0.35 | 0.29 | 0.85 | 0.37 | 0.63 |  |
|  | $t_{11}$ | 0.77 | 0.60 | 0.77 | 0.56 | 0.63 | 0.79 | 0.25 | 0.19 | 0.21 | 0.92 |  |
|  | $t_{12}$ | 0.88 | 0.56 | 0.87 | 0.63 | 0.21 | 0.62 | 0.88 | 0.08 | 0.38 | 0.87 | 0.87 |
|  | : | 0.75 | 0.92 | 0.62 | 0.52 | 0.77 | 0.75 | 0.52 | 0.62 | 0.75 | 0.90 | 0.90 |
|  | $t_{M-1}$ |  |  |  |  |  |  |  |  |  |  |  |
|  | $t_{M}$ |  |  |  |  |  |  |  |  |  |  |  |

Figure 3-9. Probability matrix for accident speed

Since the probability matrix is formed nominally from 52 observations, each cell of the probability matrix composes the range from 0.00 (i.e., the case of $\left.\operatorname{prob}\left(s_{j n}\left(t_{m}\right)\right)<1 / 52\right)$ to 0.98 (i.e., the case of $\operatorname{prob}\left(s_{j n}\left(t_{m}\right)\right)<52 / 52$ ). Thus the ideal case for the example above would be as shown in Figure 3-10.

|  | Time | Freeway Section |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $i$ | $i-1$ | $i-2$ | $i-3$ | $i-4$ | $i-5$ | $i-6$ | $i-7$ | $i-8$ | ... | 1 |
|  | $t_{1}$ | 0.00 | 0.98 |  |  |  |  |  |  |  |  |  |
|  | $t_{2}$ | 0.00 | 0.98 | 0.98 |  |  |  |  |  |  |  |  |
|  | $t_{3}$ | 0.00 | 0.00 | 0.98 |  |  |  |  |  |  |  |  |
|  | $t_{4}$ | 0.00 | 0.00 | 0.00 | 0.98 |  |  |  |  |  |  |  |
|  | $t_{5}$ | 0.00 | 0.00 | 0.00 | 0.98 |  |  |  |  |  |  |  |
|  | $t_{6}$ | 0.98 | 0.00 | 0.00 | 0.98 | 0.98 | 0.98 |  |  |  |  |  |
|  | $t_{7}$ | 0.98 | 0.00 | 0.00 | 0.00 | 0.98 | 0.98 | 0.98 | 0.98 |  |  |  |
|  | $t_{8}$ | 0.98 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.98 | 0.98 |  |  |  |
|  | $t$, | 0.98 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.98 |  |  |  |
|  | $t_{10}$ | 0.98 | 0.98 | 0.98 | 0.98 | 0.00 | 0.00 | 0.00 | 0.98 | 0.98 | 0.98 |  |
|  | $t_{11}$ | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.00 | 0.00 | 0.00 | 0.98 |  |
|  | $t_{12}$ | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.00 | 0.00 | 0.98 | 0.98 |
|  | ! | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 |
|  | $\frac{t_{M-1}}{t_{m}}$ |  |  |  |  |  |  |  |  |  |  |  |

Figure 3-10. Probability matrix of accident speed for ideal case

Using this procedure, the problem of determining the "best" set of "yellow" cells can be formulated as following statement.

$$
\begin{equation*}
\sum_{\forall d o t-s h a d e d} P_{\text {cells }} P_{j m}+\sum_{\forall s h a d e d}\left(1-P_{j m}\right)=\text { Minimum } \tag{3-5}
\end{equation*}
$$

where $P_{j m}$ is the proportion of $s_{j m}\left(t_{m}\right) ; n=1,2, \cdots, n_{\text {obs }} ; n_{\text {obs }} \leq 52$ that are less than $\hat{s}_{j}\left(t_{m}\right)$.

Let $\delta_{j m}=\left\{\begin{array}{ll}1, & \text { if cell is dot-shaded } \\ 0, & \text { if cell is shaded }\end{array}\right.$.

Then, equation (3-5) can be written as:

$$
\begin{equation*}
\sum_{\forall j, m} P_{j m} \cdot \delta_{j m}+\left(1-P_{j m}\right) \cdot\left(1-\delta_{j m}\right)=\text { Minimum } \tag{3-6}
\end{equation*}
$$

As described above, the subset of cells for which the accident speeds are significantly different from the non-accident speeds comprise a region that theoretically must obey certain properties. Specifically, there are four impossible local shape configurations for the subset of spatio-temporal cells congested by the accident. The first such case is a region that contains any holes, as shown in Figure 3-11. Moreover, the vertical position $(t)$ of any dot-shaded ("yellow") section $j$ must be either lower or same (i.e., $\leq$ ) vertical position of the neighboring shaded ("yellow") section $j-n$, as shown in Figure 3-12. Likewise, Figure 3-13 and Figure 3-14 represent other impermissible configurations.

|  | Time | Freeway Section |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\downarrow$ |  | $i$ | $i-1$ | $i-2$ | $i-3$ | $i-4$ | $i-5$ | $i-6$ | $i-7$ | $i-8$ | $\ldots$ | 1 |
|  | $t_{1}$ | $\hat{s}(t, t)$ | $\hat{S}_{i-1}\left(t_{1}\right)$ |  |  |  |  |  |  |  |  |  |
|  | $t 2$ | $\hat{s}\left(t_{2}\right)$ | $\hat{S}_{i-1}\left(t_{2}\right)$ | $\hat{s}_{i-2}\left(t_{2}\right)$ |  |  |  |  |  |  |  |  |
|  | $t_{3}$ | $\hat{s}\left(t_{3}\right)$ | $\hat{S}_{1+1}\left(t_{3}\right)$ | $\hat{s}_{i-2}\left(t_{3}\right)$ |  |  |  |  | Not po |  |  |  |
|  | $t_{4}$ | $\hat{S}\left(t_{t}\right)$ | $\hat{S}_{1}\left(t_{4}\right)$ | $\hat{S}_{-2}\left(t_{4}\right)$ | $\hat{S}_{i-3}\left(t_{4}\right)$ |  |  |  |  |  |  |  |
|  | $t_{5}$ | $\hat{s}\left(t_{s}\right)$ | $\hat{S}_{-1}\left(t_{5}\right)$ | $\hat{S}_{-2}\left(t_{5}\right)$ | $\hat{s}_{i-3}\left(t_{5}\right)$ |  |  |  |  |  |  |  |
| 4 | $t_{6}$ | $\hat{s}_{i}\left(t_{6}\right)$ | $\hat{S}_{1}\left(t_{\text {f }}\right)$ | $\hat{S}_{-2}\left(t_{0}\right)$ | $\hat{S}_{i-3}\left(t_{6}\right)$ | $\hat{s}_{i-4}\left(t_{6}\right)$ | $/ \hat{s}_{i-5}\left(t_{6}\right)$ |  |  |  |  |  |
|  | $t_{7}$ | $\hat{s}_{i}\left(t_{7}\right)$ | $\hat{S}_{1,}\left(t_{1}\right)$ | $\hat{S}_{-2}\left(t_{1}\right)$ | $\hat{S}_{\text {L, }}\left(t_{2}\right)$ | $\hat{S}_{i-4}\left(t_{7}\right)$ | $\hat{S}_{i-5}\left(t_{7}\right)$ | $\hat{S}_{i-6}\left(t_{7}\right)$ | $\hat{s}_{i-7}\left(t_{7}\right)$ |  |  |  |
|  | $t_{8}$ | $\hat{s}_{i}\left(t_{8}\right)$ | $\hat{S}_{i-1}\left(t_{8}\right)$ | $\hat{S}_{-2}\left(t_{s}\right)$ | $\hat{S}_{\text {S }}\left(r_{\text {k }}\right)$ | $\hat{S}_{1, ~}(t)$ | $\hat{S}_{\text {. }, ~(~}^{\text {a }}$ ) | $\hat{S}_{i-6}\left(t_{8}\right)$ | $\hat{s}_{i-7}\left(t_{8}\right)$ |  |  |  |
|  | $t$, | $\hat{s}_{i}\left(t_{9}\right)$ | $\hat{s}_{i-1}\left(t_{9}\right)$ | $\hat{S}_{-2}\left(t_{5}\right)$ | $\hat{S}_{-3}\left(t_{i}\right)$ | $\hat{S}_{i-4}\left(t_{9}\right)$ | $\hat{s}_{i-5}\left(t_{9}\right)$ | $\hat{s}_{-0}\left(t_{0}\right)$ | $\hat{s}_{i-7}\left(t_{9}\right)$ |  |  |  |
|  | $t_{10}$ | $\hat{S}_{i}\left(t_{10}\right)$ | $\hat{s}_{i-1}\left(t_{10}\right)$ | $\hat{s}_{i-2}\left(t_{10}\right)$ | $\hat{s}_{i-3}\left(t_{10}\right)$ | $\hat{S} \hat{S}^{( },\left(t_{10}\right)$ | $S_{\text {S }}^{5}\left(t_{10}\right)$ | $\hat{S}_{-6}\left(t_{10}\right)$ | $\hat{s}_{i-7}\left(t_{10}\right)$ | $\hat{S}_{i-8}\left(t_{10}\right)$ | $\ldots$ |  |
|  | $t_{11}$ | $\hat{s}_{i}\left(t_{11}\right)$ | $\hat{s}_{i-1}\left(t_{11}\right)$ | $\hat{s}_{i-2}\left(t_{11}\right)$ | $\hat{s}_{i-3}\left(t_{11}\right)$ | $\hat{s}_{i-4}\left(t_{11}\right)$ | $\hat{s}_{i-5}\left(t_{11}\right)$ | $\hat{S}_{-6}\left(t_{11}\right)$ | $\hat{S}_{-( }\left(t_{1 H}\right)$ | $\hat{S}_{-S}\left(t_{11}\right)$ | $\ldots$ |  |
|  | $t 12$ | $\hat{s}_{i}\left(t_{12}\right)$ | $\hat{s}_{i-1}\left(t_{12}\right)$ | $\hat{s}_{i-2}\left(t_{12}\right)$ | $\hat{s}_{i-3}\left(t_{12}\right)$ | $\hat{s}_{i-4}\left(t_{12}\right)$ | $\hat{s}_{i-5}\left(t_{12}\right)$ | $\hat{s}_{i-6}\left(t_{12}\right)$ | $\hat{s}_{-,\left(t_{12}\right)}$ | $\hat{S}_{-8}\left(t_{12}\right)$ | $\ldots$ | $\hat{s}_{1}\left(t_{12}\right)$ |
|  | ! | ! | ! | $\vdots$ | ! | ! | ! | : | ! | ! | ... | $\vdots$ |
|  | $t_{M-1}$ $t_{M}$ |  |  |  |  |  |  |  |  |  |  |  |

Figure 3-11. Impossible shape I of congested region


Figure 3-12. Impossible shape II for the shape of congested region


Figure 3-13. Impossible shape III for the shape of congested region


Figure 3-14. Impossible shape IV for the shape of congested region

These conditions can be enforced by the following relationships:

$$
\begin{align*}
& \delta_{j+k, m} \leq\left[1-\left(\delta_{j, m}-\delta_{j+1, m}\right)\right] \cdot R ; \forall j, m ; \forall k \leq J-j  \tag{a}\\
& \delta_{j, m+r} \leq\left[1-\left(\delta_{j, m}-\delta_{j, m+1}\right)\right] \cdot R ; \forall j, m ; \forall r \leq M-m  \tag{b}\\
& \delta_{j, m+k} \leq\left[1-\left(\delta_{j, m}-\delta_{j+1, m}\right)\right] \cdot R ; \forall j, m ; \forall k \leq M-m  \tag{c}\\
& \sum_{\substack{k=j \\
\forall r \leq M-m}}^{J} \delta_{k, m+r} \leq\left[3-\left\{\left(\delta_{j, m}-\delta_{j+1, m}\right)+\left(\delta_{j, m}-\delta_{j, m+1}\right)+\left(\delta_{j, m}-\delta_{j+1, m+1}\right)\right\}\right] \cdot R ; \forall j, m \tag{d}
\end{align*}
$$

where $R$ is a large number.

Formulae (3-6) and (3-7) are in the form of the objective function and constraint, respectively, of "Binary Integer Programming (BIP)." Thus, the determination problem described above can be represented in the form of the following BIP problem.
$\underset{\delta_{j m}}{\operatorname{Min}} Z=\sum_{\forall j, m}\left\{P_{j m} \cdot \delta_{j m}+\left(1-P_{j m}\right) \cdot\left(1-\delta_{j m}\right)\right\}$
s.t:

$$
\begin{align*}
& \delta_{j+k, m} \leq\left[1-\left(\delta_{j, m}-\delta_{j+1, m}\right)\right] \cdot R ; \forall j, m ; \forall k \leq J-j \\
& \delta_{j, m+r} \leq\left[1-\left(\delta_{j, m}-\delta_{j, m+1}\right)\right] \cdot R ; \forall j, m ; \forall r \leq M-m  \tag{3-8}\\
& \delta_{j, m+k} \leq\left[1+\left(\delta_{j, m}-\delta_{j+1, m}\right)\right] \cdot R ; \forall j, m ; \forall k \leq M-m \\
& \sum_{\substack{k=j \\
\forall r \leq M-m}}^{J} \delta_{k, m+r} \leq\left[3-\left\{\left(\delta_{j, m}-\delta_{j+1, m}\right)+\left(\delta_{j, m}-\delta_{j, m+1}\right)+\left(\delta_{j, m}-\delta_{j+1, m+1}\right)\right\}\right] \cdot R ; \forall j, m \\
& \delta_{j m}=\left\{\begin{array}{l}
0 \\
1
\end{array}\right.
\end{align*}
$$

where $R$ is an arbitrary large number.

This problem can be solved by commercial optimization tools such as CPLEX, LINDO, LINGO, and GAMS. In this study, CPLEX is employed to solve the problem.

### 3.3.3 Screening the Results

If each loop detector in the affected region has no error and is correctly working, the result should be represented as shown in Figure 3-15. However, many loop detectors are not temporarily working, usually due to facility checking or road rehabilitation work. On the Orange County freeway system, there are many such cases of missing data caused by these reasons.

|  | Freeway Section |  |
| :---: | :---: | :---: |
|  | Accident region | Probability of speed affected by maximum shockwave |
|  | 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 | 0.0769 |
|  | $1,0,0,0,0,0,0,0,0,0,0$ | 0.1538 |
|  | $1,1,0,0,0,0,0,0,0,0,0$ | 0.2500, 0.1154 |
|  | $1,1,1,0,0,0,0,0,0,0,0$ | $0.2115,0.1346,0.0577$ |
|  | $1,1,1,1,1,0,0,0,0,0,0$ | 0.2500, 0.1731, 0.1154, 0.0000, 0.0769 |
|  | 1, 1, 1, 1, 1, 1, 0, 0, 0, 0,0 | 0.3077, $0.2115,0.0000,0.2115,0.0769,0.0577,0.0769$ |
|  | 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0 | $0.2885,0.1923,0.1154,0.1154,0.0577,0.0385,0.9615,0.9808,0.9615,0.9423,0.0385$ |
|  | $1,1,1,1,1,1,0,0,0,0,0$ | $0.2500,0.1923,0.0000,0.0577,0.0962,0.0577,0.7885,0.8269,0.8462,0.7885,0.0577$ |
|  | 1, 1, 1, 1, 1, 1, 1, 0, 0, 0,0 | $0.1154,0.0962,0.1346,0.0577,0.0192,0.0385,0.0192,0.8077,0.9038,0.9231,0.0962$ |
|  | $1,1,1,1,1,1,1,0,0,0,0$ | $0.2500,0.1731,0.1731,0.0962,0.1154,0.1154,0.7500,0.9423,0.9615,0.9615,0.0577$ |
|  | $1,1,1,1,1,1,1,1,0,0,0$ | $0.2308,0.1538,0.1731,0.1538,0.0000,0.0577,0.0385,0.0000,0.8462,0.6923,0.0769$ |
|  | $1,1,1,1,1,1,1,1,0,0,0$ | $0.2692,0.1731,0.1346,0.0962,0.0962,0.0769,0.0192,0.0000,0.9231,0.8846,0.1154$ |
| $\nabla$ | $1,1,1,1,1,1,1,1,0,0,0$ | $0.1731,0.1346,0.0962,0.0577,0.0769,0.0769,0.0385,0.0000,0.9423,0.8654,0.0385$ |
|  | 1, 1, 1, 1, 1, 1, 1, 0, 0, 0,0 | $0.0769,0.2115,0.0577,0.0385,0.0000,0.0577,0.0000,0.6154,0.9231,0.8846,0.0385$ |
|  | 1, 1, 1, 1, 1, 1, 0, 0, 0, 0,0 | $0.0769,0.0000,0.1154,0.0000,0.1154,0.0769,0.7115,0.8462,0.9231,0.7115,0.1538$ |
|  | 1, 1, 1, 1, 1, 1, 0, 0, 0, 0,0 | $0.0577,0.1538,0.0962,0.1346,0.0192,0.0000,0.9231,0.9231,0.9423,0.7308,0.0962$ |
| 亏3 | 1, 1, 1, 1, 1, 1, 0, 0, 0, 0,0 | $0.1346,0.2308,0.1346,0.0962,0.0962,0.0000,0.7885,0.8269,0.9423,0.7500,0.0385$ |
|  | $1,1,1,1,1,1,0,0,0,0,0$ | $0.1154,0.1154,0.0577,0.0192,0.0000,0.0192,0.9808,0.9423,0.9423,0.8269,0.1346$ |
|  | 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0 | $0.0962,0.0192,0.1346,0.1154,0.6923,0.9231,0.9808,0.9423,0.9615,0.9038,0.1154$ |
|  | 1, 1, 1, 1, 0, 0, 0, 0, 0, 0,0 | $0.1346,0.0577,0.0192,0.4615,0.9808,0.9038,0.8462,0.9231,0.9231,0.8269,0.1346$ |
|  | $0,1,1,0,0,0,0,0,0,0,0$ | $0.5192,0.4423,0.0385,0.5962,0.9231,0.8462,0.9808,0.9423,0.1538,0.8846,0.0577$ |
|  | $0,0,0,0,0,0,0,0,0,0,0$ | $0.8269,0.9423,0.9615,0.9808,0.9231,0.8654,0.9808,0.9423,0.8077,0.8654,0.0769$ |

Figure 3-15. BIP result for the delay region affected by accident

Thus, we need to discriminate among accident observations those for which sufficient data are not available, as well as certain cases for which the observed data are inconsistent, including unreasonable congestion regions. As mentioned above, unacceptable results can be classified into two types; (1) only a portion of congested region is formed by missing data (Figure 3-16) and (2) unrealistic congestion region is formed by erroneous detector data ( Figure 3-17 and Figure 3-18).

|  | Freeway Section |  |  |
| :---: | :---: | :---: | :---: |
|  | Accident region | Probability of spd affected by max. shockwave | Section ID |
| $\downarrow$ | 1, 0, 0, 0, 0, 0, 0 | 0.0385 | 36 |
|  | 1, 1, 0, 0, 0, 0, 0 | 0.0000, 0.0385 Not end section | 36, 35 |
|  | 1, 1, 1, 0, 0, 0, 0 | 0.0577, 0.0385, 0.0192 | 36, 35, 34 |
|  | 1, 1, 1, 1, 0, 0, 0 | 0.0192, 0.0385, $0.0962,0.0577$ | 36, 35,34, 33 |
|  | 1, 1, 1, 1, 1, 0,0 | 0.0385, 0.0192, 0.0769, 0.0769, 0.1346 | 36, 35, 34, 33, |
|  | 1, 1, 1, 1, 1, 0,0 | 0.0192, 0.0577, 0.0385, 0.0577, 0.0385 | 36, 35, 34, 33, 32 |
|  | 1, 1, 1, 1, 1, 0,0 | 0.0769, 0.0000, 0.0769, 0.0769, 0.1346 | 36, 35, 34, 33, 32 |
|  | 1, 1, 1, 1, 1, 1, 1 | 0.0962, $0.0192,0.0192,0.0769,0.0577,0.0577,0.1346$ | 36, 35, 34, 33, 32, 29, 28 |
|  | 1, 1, 1, 1, 1, 1, 1 | $0.0577,0.0577,0.0192,0.0000,0.0577,0.1154,0.1731$ | 36, 35, 34, 33, 32, 29, 28 |
|  | 1, 1, ok 1, 1,1 | $0.0385,0.0385,0.0385,0.0385,0.0385,0.1154,0.1923$ | 36, 35, 34, 33, 32, 29, 28 |
|  | 1, 1, 1, $4,1,1,1$ | $0.0385,0.0769,0.0385,0.0192,0.0577,0.0577,0.1731$ | 36, 35, 34, 33, 32, 29, 28 |
|  | 1, 1, 1, 1, 1, 1, 1 | 0.0192, 0.0577, 0.0192, $0.0000,0.0000,0.0769,0.1154$ | 36, 35, 34, 33, 32, 29, 28 |
|  | 1, 1, 1, 1, 1, 1, 1 | $0.4615,0.5962,0.5385,0.5192,0.9038,0.0769,0.1154$ | 36, 35, 34, 33, 32, 29, 28 |
|  | 1, 1, 1, 1, 1, 1, 1 | $0.5192,0.3846,0.0385,0.0769,0.7115,0.0385,0.2692$ | 36, 35, 34, 33, 32, 29, 28 |
| $\underset{\substack{1}}{\substack{1}}$ | 1, 1, 1, 1, 1, 1, 1 | $0.1154,0.0000,0.0000,0.0192,0.6731,0.0577,0.1731$ | 36, 35, 34, 33, 32, 29, 28 |
|  | $0,0,0,0,0,1,1$ | 0.4231, 0.8269, 0.6346, 0.7692, 0.9231, 0.1731, 0.2308 | 36, 35, 34, 33, 32, 29, 28 |
|  | $0,0,0,0,0,1,1$ | $0.5000,0.4615,0.4615,0.4231,0.8462,0.2500,0.2692$ | 36, 35, 34, 33, 32, 29, 28 |
|  | $0,0,0,0,0,1,1$ | $0.7308,0.8077,0.7692,0.5769,0.9423,0.2115,0.2500$ | 36, 35, 34, 33, 32, 29, 28 |
|  | $0,0,0,0,0,1,1$ | 0.8077, 0.2885, 0.0577, 0.0385, 0.8077, 0.3077, 0.3846 | 36, 35, 34, 33, 32, 29, 28 |
|  | $0,0,0,0,0,1,1$ | $0.2500,0.1346,0.2500,0.2500,0.7885,0.3077,0.3654$ | 36, 35, 34, 33, 32, 29, 28 |

Figure 3-16. Missing data section in a portion of congested region

|  | Freeway Section |  |  |
| :---: | :---: | :---: | :---: |
|  | Accident region | Probability of speed affected by maximum shockwave |  |
| © | 0,0, 0, 0 | 0.9808 |  |
|  | 0, D, 0, 0 | 0.8077, 0.1731, 0.6346 |  |
|  | 0, , , 2, 0 | 0.8846, 0.1346, 0.6538, 0.173 |  |
|  | 0, ,0, 0 | $0.2308,0.2500,0.7885,0.0962$ |  |
|  | 0, p, 0, 0 | O.9808, 0.7115, 0.9808, 0.1154 |  |
|  | 0 1, 1, , 1,1 | 0.9423, 0.2115, 0.9615, 0.0769 | Not congested for 30 minutes |
|  | 1, 1, 1, 1 | 0.8269, 0.1346, 0.6923, 0.0962 | after accident occurrence ??? |
|  | 1, 1, 1, 1 | 0.1346, 0.0192, 0.0385, 0.0577 |  |
|  | 1, 1, 1, 1 | 0.1538, 0.0192, 0.0769, 0.1731 |  |
|  |  | $\ldots$ |  |
|  | 1, 1, 1, 1 | 0.1538, 0.0769, 0.6731, 0.0769 |  |
|  | 1, 1, 1, 1 | 0.2308, 0.1923, 0.7500, 0.0192 |  |
|  | 0, 1, 1, 1 | 0.5577, 0.1538, 0.5962, 0.0385 |  |
|  | 0, 1, , , 1 | 0.0962, 0.0962, 0.6346, 0.2115 |  |
|  | 0, 1, 1, 1 | 0.6923, 0.1154, 0.9423, 0.1731 |  |
|  | 0, 1, 1, 1 | 0.1923, 0.0577, 0.1346, 0.1923 |  |
|  | 0, 1, 1, 1 | 0.0962, 0.0577, 0.1923, 0.2500 |  |
|  | 0, 1, 0, 0 | 0.8269, 0.1731, 0.8462, 0.2692 |  |
|  | 0, 1, 0, 0 | 0.9808, 0.2308, 0.8846, 0.2115 |  |
|  | 0, 1, 0, 0 | 0.8846, $0.1538,0.8269,0.2308$ |  |

Figure 3-17. Unrealistically formed congestion region I


Figure 3-18. Unrealistically formed congestion region II


Figure 3-19. Unrealistically formed congestion region III

Additionally, an upper limit of four hours after accident occurrence was applied in the determination the spatio-temporal extent of the congestion region. However, in some
cases the congestion was observed to remain after four hours (Figure 3-19). Such cases may include fatalities, secondary accidents that occurred before the first accident was cleared, when the accident was related to hazardous materials, etc. In such cases, since the congestion caused by accident is not cleared, the procedure for total delay estimation by accident would yield erroneous results; these cases were excluded from the analysis.

### 3.3.4 Total Delay Estimation

Having completed the above steps that determine the region (in time and space) that is negatively affected by any particular accident, we can calculate the total delay caused by the accident as:

$$
\begin{equation*}
\text { Total Delay }=\sum_{\forall m, j \in \text { dot-shaded cells }} L_{j} \cdot\left[1 / \hat{s}_{j}\left(t_{m}\right)-1 / \bar{s}_{j}\left(t_{m}\right)\right] \cdot V_{j m} \tag{3-9}
\end{equation*}
$$

where
$L_{j}=$ Length of freeway segment $j$
$V_{j m}=$ Volume (count) of vehicles in segment $j$ during time interval $m$.

## 4 Results

### 4.1 Descriptive Statistics

Computations of non-recurrent delay were successfully performed for 870 accidents that occurred on weekdays throughout the period of March through December 2001 on the six major Orange County non-toll freeways. The breakdown by freeway is: I-5 (222 accidents), I-405 (157 accidents). SR-22 (153 accidents), SR-55 (94 accidents), SR-57 (138 accidents), and $\operatorname{Sr}-91$ (106 accidents). Calculations for some of the accidents were cut off in terms of space and/or time, due to data availability; thus, total non-recurrent delay must be regarded as a lower bound. This truncation was due to spatial boundary conditions (e.g., effects reaching county lines or roadway ends) and to data limitations.

However, the distribution of delay among accident types, times, and places is considered to be accurate.

The median total delay for these 870 accidents is 86 vehicle hours, the lower bound of the mean is 184 vehicle hours, and the lower bound of the standard deviation is 246 . As indicated by the difference between the median and the high standard deviation relative to the mean, the distribution of non-recurrent delay is highly skewed to the right (i.e., toward high values of delay), as expected. For regression purposes, it is useful to analyze the natural logarithm of delay, so that the regression residuals are approximately normally distributed. As shown in the histogram of Figure 4-1, if delay values close to zero are ignored, the logarithm of delay is approximately normally distributed, especially considering that high values are probably truncated well below their true values. Without this truncation, the tail of the distribution would extend further to the right.


Figure 4-1. Histogram of Natural Logarithm of Total Delay for 870 Weekday Accidents

Average delay per accident as a function of the time period of the accident is shown in Figure 4-2. Accidents occurring during the weekday afternoon peak hours (3:30 through 6:30 p.m.) lead to the most delay, followed by mid-day accidents (9:01 a.m. through 3:29 p.m.). As expected, accidents either after $6: 30$ p.m. or before 6 a.m. result in the least delay. These differences in means by time period are statistically significant at all of the usual confidence intervals.


Figure 4-2. Average Total Delay per Accident by Time Period of Occurrence.

Average delay by day of the week is shown in Figure 4-3. The worst day is Friday, followed by Tuesday, Thursday and Wednesday. Accidents that occur on Monday contribute the least to total non-recurrent delay. The differences are statistically significant.


Figure 4-3. Average Total Delay per Accident by Day of Week of Occurrence.

The effects of the location of the primary collision on average delay are shown in Figure 4-4. Accidents that occur on the roadway or off-road to drivers' left are more serious in terms of induced delay than those that occur off road to drivers' right. Due to high standard deviations associated with the means, these differences according to location are not statistically significant. However, it will be shown in the next section that the combination of time period and accident location leads to many important differences in non-recurrent delay.


Figure 4-4. Average Total Delay per Accident by Location of Primary Collision.

Average delay per accident as a function of the number of involved vehicles is shown in Figure 4-5. Delay is greater for multi-vehicle accidents than for accidents involving a single vehicle, and within multi-vehicle accidents, slightly greater for two-vehicle accidents than for accidents involving more than two vehicles. This latter result is consistent with the pattern that multi-vehicle rear-end collisions typically occur in highly congested conditions, where speeds are already low (Golob, Recker and Alvarez, 2004). These differences per number of involved vehicles are statistically significant.


Figure 4-5. Average Total Delay per Accident by Number of Involved Vehicles.

### 4.2 A Non-recurrent delay Forecasting Model

A regression model was developed that can forecast the expected amount of nonrecurrent delay for different types of accidents that occur at different times. As discussed previously, the natural logarithm of delay was found to be approximately normal distributed, indicating a model of the form:

Delay $=\exp \left(B_{0}+\sum_{j} B_{j} \delta_{j}\right)$
or
$\ln ($ Delay $)=B_{0}+\sum_{j} B_{j} \delta_{j}$
in which the dependent variable is the natural logarithm of delay, resulting in residuals that are approximately normally distributed. In the above, the $B_{j}$ are the regression coefficients and the $\delta_{j}$ are binary $(0,1)$ indicators of the various features of the accident.

In order to present a model for forecasting delay once the accident is detected, we limited the descriptive variables in the model to those characteristics that would presumably be available to the Traffic Management Center shortly after the occurrence, as opposed to aspects that might only be known based on investigation and/or report. We assumed that the following characteristics would be known by direct observation, either from emergency calls, or aerial or on-scene observation: 1) day of week, 2) time of day, 3) location on roadway, 4) number of involved vehicles, and 5) whether or not a truck was involved. The regression was based on these five variables and their first-order interaction terms. In the model, only those variables with effects that tested significantly different from zero at the $p=0.05$ significance level were retained. The result of the estimation is listed in Table 4-1. The proportion of variance accounted for (model $\mathrm{R}^{2}$ ) is 0.134 .

Table 4-1. Linear Regression Model of Logarithm of Delay in Terms of Observable Accident Characteristics

|  | B | $\exp (\mathrm{B})$ | Beta | t | 1-tail sig. |
| :--- | ---: | ---: | ---: | ---: | ---: |
| (Constant) | 2.912 | 18.40 |  | 20.187 | 0.000 |
| Truck involved | -0.597 | 0.55 | -0.098 | -2.196 | 0.014 |
| Friday | 0.248 | 1.28 | 0.059 | 1.824 | 0.034 |
| AM peak and 2 vehicles | 1.625 | 5.08 | 0.262 | 6.599 | 0.000 |
| AM peak and 3+ vehicles | 1.964 | 7.13 | 0.282 | 6.398 | 0.000 |
| AM peak and left lane | -0.753 | 0.47 | -0.102 | -2.457 | 0.007 |
| Midday and single vehicle | 0.811 | 2.25 | 0.092 | 2.620 | 0.005 |
| Midday and 2 vehicles | 1.478 | 4.38 | 0.336 | 7.613 | 0.000 |
| Midday and 3+ vehicles | 1.503 | 4.49 | 0.255 | 6.549 | 0.000 |
| Midday and truck involved | 0.981 | 2.67 | 0.115 | 2.488 | 0.007 |
| PM peak and off-road left | 1.644 | 5.18 | 0.152 | 4.478 | 0.000 |
| PM peak and left lane | 1.826 | 6.21 | 0.284 | 7.576 | 0.000 |
| PM peak and interior lane(s) | 1.603 | 4.97 | 0.290 | 7.382 | 0.000 |
| PM peak and right lane | 1.554 | 4.73 | 0.210 | 5.791 | 0.000 |
| PM peak and single vehicle | 1.403 | 4.07 | 0.053 | 1.634 | 0.052 |

Fourteen accident descriptors proved important in explaining differences among accidents in terms of the non-recurrent delay they imposed. The importance of each variable is measured by the beta coefficients, which are the regression coefficients of the variables in standardized form (converted to unity standard deviation to eliminate scale differences). The most important predictor is whether the accident involves two vehicles and occurs in the midday period. This combination indicates an accident that would generally occur in heavy traffic that is moving at relatively high speeds. It would also be an accident that occurs prior to buildup of the afternoon rush hour, so that lingering effects are typically likely to influence steadily increasing levels of traffic. The next most important indicators are whether a PM peak period accident was located in the interior or left lanes, and whether an AM peak period accident involved multiple vehicles.

Focusing on the magnitude of the effect of each indicator, the accident that is likely to cause the greatest delay is an AM peak period accident involving three or more vehicles, which multiplies the base level of delay by a factor of more than seven (i.e., $\exp (B)=7.13$, Table 4-1). Other indicators of extensive delay is whether a PM peak period accident is in the left lane or off-road left, whether a AM peak period accident involves two-vehicles, or whether a PM peak period accident is in the interior lane(s). Reduced levels of delay are expected for truck-involved accidents and for AM peakperiod accidents in the left lane. Truck involved accidents are less likely to be injury accidents ( $18 \%$ of truck-involved accidents in case study dataset were injury accidents, compared to more than $25 \%$ of non-truck-involved accidents). However, if a truck is involved in an accident that occurs in the midday period (weekdays, 9:01 a.m. through 3:29 p.m.), more non-recurrent delay can be expected. Also, single-vehicle accidents that occur in the PM peak period will lead to more delay, because such accidents are typically more severe (over $30 \%$ of PM peak period single-vehicle accidents are injury accidents, compared to about $25 \%$ of PM peak period multiple-vehicle accidents).

## 5 Continuation of the study

The results reported here constitute progress in a continuing study of accident effects and their relationship to traffic. Work is continuing on the calculation of delay for the remaining accident cases within the data set. It is anticipated that there will be sufficient valid cases to conduct further statistical analyses on the nature of delay as it relates to accident type and traffic flow characteristics. All of the models will be developed based on year 2001 data collected from loop detectors on major freeways in Orange County, and then tested for reasonableness and applicability with application to year 2002 data set.

The calculation of congestion delay as outline above can serve as input to other modeling efforts designed to provide insight to the nature of traffic-safety interaction. For example, to find the key factors affecting the total delay by accidents, multivariate statistical methods can be applied. Golob et. al (2003) used a series of multivariate statistical methods that determine the relationship between accident characteristics and traffic flow characteristics. These models can easily be extended to include the congestion effects described by this study.

The amount of delay caused by an accident depends on the nature of the accidents, roadway conditions, and execution of accident clearance (Hall, 2002). The duration of accident effects is directly related to its detection, dispatching and removal, and prevailing traffic conditions. Minimizing the duration relies principally on the dispatching time of emergency crews such as police officers and freeway-service-patrol trucks and removal time of debris-how fast they arrive at the accident location and remove the accident debris. Usually, major accidents such as truck-involved accidents, hazardous material spills, and multi-car crashes require a specialized assistance team for removal from the freeway. Thus they take much more time to clear as compared to common traffic accidents.

Dispatching processes and response times for freeway-service-patrols is a well-studied topic in the field of operations research. Dispatching processes are typically modeled as a spatial queuing system, and the response time depends on the density of servers (cars per square mile), their overall utilization (ratio of demand to capacity) and the policy for
dispatching officers (Hall, 2002). The time to clear the accident can be determined by using these theories. Once the time to clear the accident is identified, the recovery time to general traffic flow can be estimated by kinematic wave theory with loop detector data.

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