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USER'S MANUAL FOR ANALYZE: A VARIABLE-RATE, MULTIPLE-WELL, LEAST SQUARES MATCHING ROUTINE FOR WELL-TEST ANALYSIS

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Publication Date

1981-07-01

LBL-10907

CONF-8006188--1

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

EARTH SCIENCES DIVISION

Presented at the Geothermal Computer Program
Workshop, Lawrence Berkeley Laboratory, University
of California, Berkeley, CA, June 17-19, 1980

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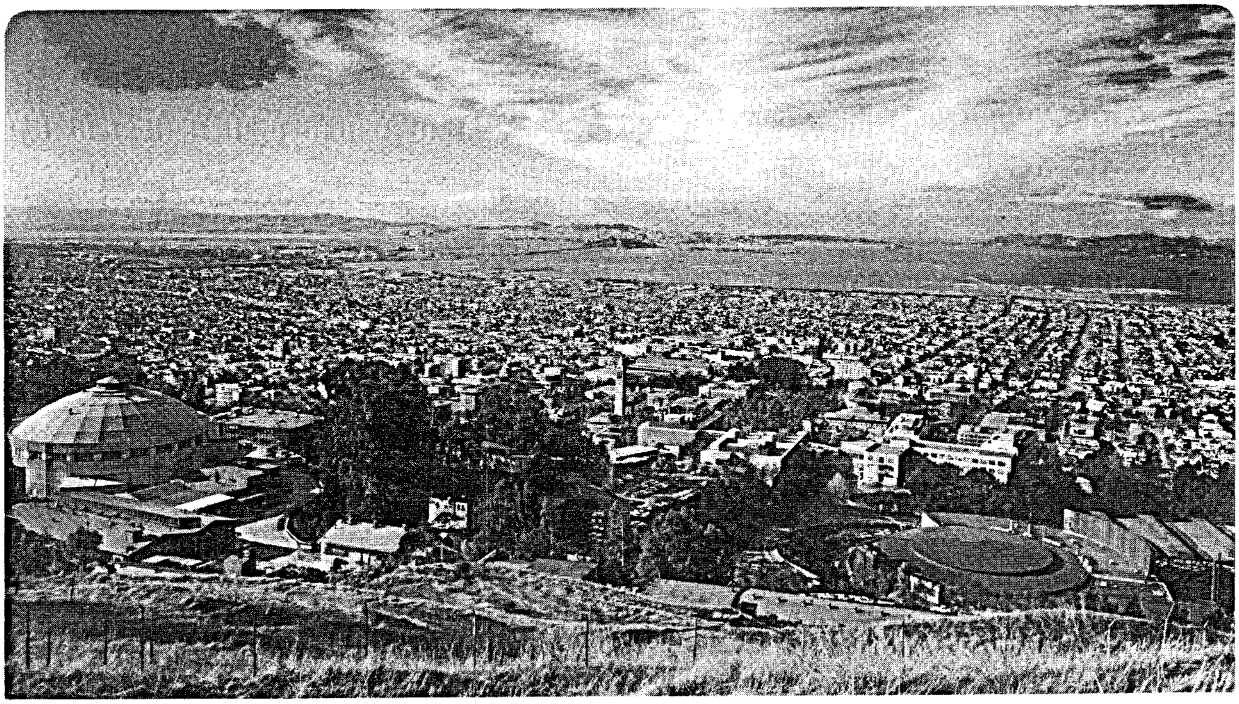

Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

DISCLAIMER

LBL-10907

USER'S MANUAL FOR ANALYZE:

A VARIABLE-RATE, MULTIPLE-WELL, LEAST-SQUARES MATCHING ROUTINE FOR WELL-TEST ANALYSIS

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This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Renewable Technology, Division of Geothermal and Hydropower Technologies of the U. S. Department of Energy under Contract No. W-7405-ENG-48.

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INTRODUCTION

ANALYZE is a history-matching program designed for pressure-transient analysis of well tests in single-phase, fluid-saturated reservoirs. Both interference tests and production tests can be analyzed to yield reservoir transmissivity (kh/μ), storativity (ϕch), and hydrologic boundaries. An analytic solution is used to calculate the pressure drawdown/buildup in an idealized reservoir system. Figure 1 shows a schematic of the basic reservoir/well model assumed by the computational algorithm. The reservoir is assumed to be an isothermal, isotropic, homogeneous, porous medium of constant thickness and infinite areal extent. The production well is modeled as a line source which fully penetrates the reservoir. The flow into the well is radial and uniformly distributed over the height of the well (gravity effects neglected).

The unique feature of the matching program is that the analytic solution which calculates the drawdown/buildup caused by arbitrarily variable flow rates

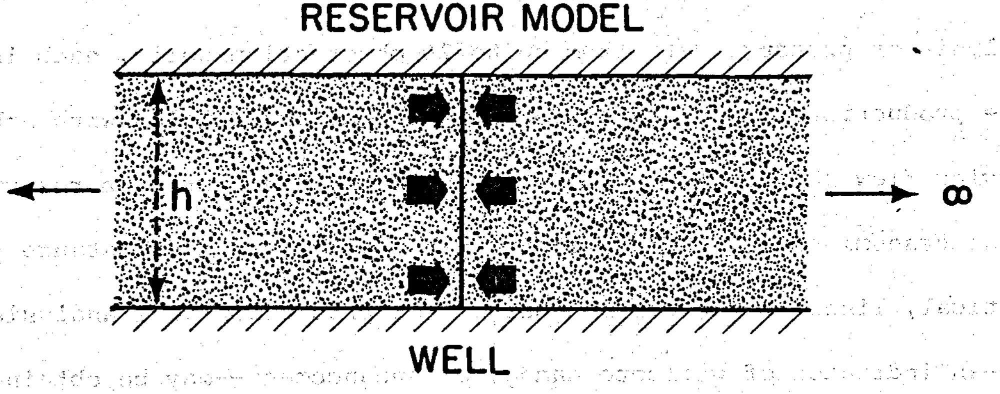


Figure 1. Schematic of well/reservoir model for ANALYZE.
[XBL 813-2722]

from one or more production wells. Any variable flow history can be modeled--to whatever accuracy is desired--by a sequence of straight line segments, or "production pulses," each of appropriate length and inclination (Figure 2). The flow rate during any production pulse may be constant or vary linearly with time as shown in Equation (1):

$$q(\tau) = A_k + B_k(\tau - \tau_k) \text{ for } \tau_k < \tau < \tau_{k+1}, \tag{1}$$

where $q(\tau)_k$ = the flow rate at time τ during the production pulse k,

τ_k = the time at which production pulse k begins,

τ_{k+1} = the time at which production pulse k ends,

A_k = the flow rate at time τ_k,

B_k = the slope of the production pulse,

which can be written

$$B_k = \frac{q(\tau_{k+1}) - q(\tau_k)}{\tau_{k+1} - \tau_k} .$$

Implementation of the technique presented herein allows for the simultaneous analysis of pressure data from up to 20 observation wells, each influenced by the production (and/or injection) of as many as 20 wells with arbitrarily varying flow rates. Pressure data can be analyzed for such reservoir properties as transmissivity (kh/μ), storativity (ϕch), and the distance to a single, vertical, linear reservoir boundary. For production-well analysis, a skin value--an indicator of wellbore damage or enhancement--may be obtained if the storativity is known.

In order to solve for the reservoir parameters, a nonlinear least-squares matching routine is used to minimize the sum.

$$\chi^2 = \sum_{n=1}^{N} \left(\frac{\Delta p_{calc} - \Delta p_{obs}}{\Delta p_{obs}} \right), \tag{2}$$

where N = the number of measured pressure observations,

Δp_{calc} = the calculated pressure change,

Δp_{obs} = the observed pressure change,

which is functionally dependent on the reservoir parameters, well configuration, and flow rates. By systematically varying the specified reservoir parameters, the χ^2 sum is reduced until the minimization requirements are satisfied. At this time the program assumes the correct reservoir parameters have been attained.

The program was written for use on a CDC 7600 and uses large-core memory (LCM). The memory requirements, however, are not unusually large, hence the code is easily transferable to other systems with a few minor modifications.

DESCRIPTION OF TECHNIQUE

VARIABLE FLOW RATE

To handle a variable flow rate $q(\tau)$, we assume that any production-rate history can be adequately represented by a sequence of straight line segments, each of appropriate length and inclination (Figure 2). We prescribe $q(\tau)$ to vary linearly within each interval τ_k to τ_{k+1}, so that, for the kth line segment, q_k can be written

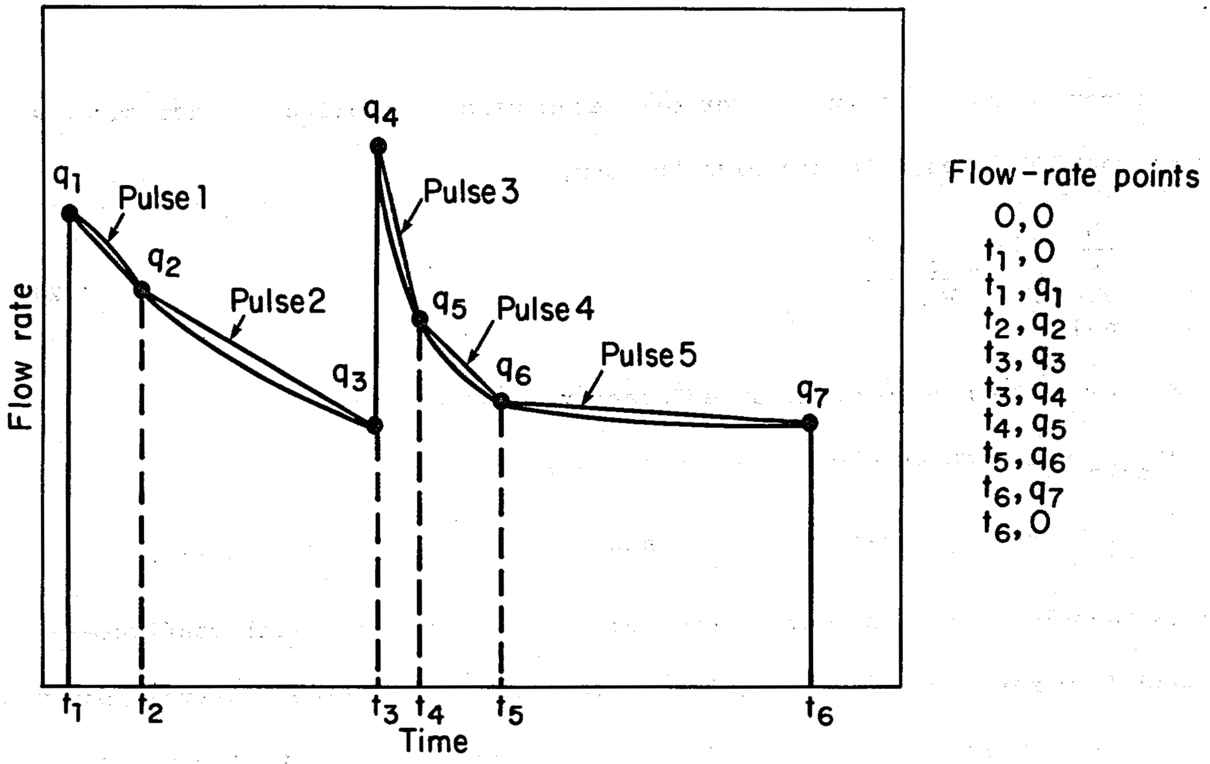


Figure 2. Representation of a variable flow rate as a sequence of straight line segments, or "production pulses." [XBL 815-3075]

$$q_k(\tau) = A_k + B_k(\tau - \tau_k). \tag{3}$$

(See Appendix C for nomenclature.)

The drawdown $\Delta P(t)$ caused by a variable production rate q_k can be calculated by the line-source solution (Carslaw and Jaeger, 1959):

$$\Delta P_k(t) = \frac{\mu}{4\pi kh} \int_{\tau_k}^{\tau_{k+1}} \frac{q(\tau)}{t - \tau} \exp\left[\frac{-r^2}{4\eta(t - \tau)}\right] dt. \tag{4}$$

It should be noted that the flow-rate schedule is assumed to be representative of the sandface flow rate, not the bulk flow rate recorded at the wellhead. In addition, volumetric flow rates are used--not mass flow rates.

Once $q(\tau)$ is substituted from Equation (3), the integral in Equation (4) can be evaluated. A closed-form solution of this integration is available, and a full development is given in Appendix A. The solution is

$$\Delta P_k(r,t) = \frac{\mu}{4\pi kh}\left\{[A_k + B_k(t - \tau_k)(1 + u_k)][W(u_k) - W(u_{k+1})] - B_k[(t - \tau_k)\exp(-u_k) - (t - \tau_{k+1})\exp(-u_{k+1})]\right\}, \tag{5}$$

where

$$u_k = \frac{\mu\phi cr^2}{4k(t - \tau_k)}, \quad u_{k+1} = \frac{\mu\phi cr^2}{4k(t - \tau_{k+1})}, \quad \text{and} \quad W(u) = \int_u^\infty \frac{\exp(-y)}{y}\,dy.$$

The pressure response at time t is obtained by summing the individual responses:

$$\Delta P(r,t) = \sum_{k=1}^{K} \Delta P_k(r,t), \tag{6}$$

where K is the number of production pulses before time t. This is the pressure response due to one production well.

SEVERAL PRODUCTION WELLS

To calculate the pressure response ($\Delta P(r,t)$) caused at any location in the reservoir by several production/injection wells, the drawdown/buildup attributable to each production/injection well is calculated using Equation (6); all of these values are then summed. Each production well may have a unique set of flow-rate data. Thus, using the principle of superposition,

$$\Delta P(r,t) = \sum_{j=1}^{J} \Delta P_j(r,t), \tag{7}$$

where J is the number of production/injection wells and ΔP_j is the drawdown caused by the production well j, as calculated by Equations (5) and (6). Equations (5), (6), and (7) form the basis of all pressure calculations in the program. With these fundamental assumptions and equations, multiple wells, multiple rates, and reservoir boundaries can be rigorously accounted for in the pressure calculations.

LEAST-SQUARES MINIMIZATION

The scalar χ^2 is a measure of how well the calculated pressure response agrees with the observed pressure response. The goal of the analysis technique is to determine the values of the reservoir parameters that minimize χ^2. This is accomplished by changing the values of the parameters from their initial values to the values which iteratively reduce the value of χ^2. The minimization process uses the least-squares program LSQVMT, a nonlinear fitting routine. For a more complete discussion of the minimization algorithm, see Beals (1966a,b).

The minimization statistic used for one observation well is shown in Equation (9).

$$\chi^2 = \frac{1}{I} \sum_{i=I}^{I} \left(\frac{\Delta P_{calc}(i) - \Delta P_{obs}(i)}{\Delta P_{obs}(i)} \right)^2 . \tag{8}$$

This formulation has an advantage over a logarithmic minimization statistic in that both positive and negative pressure changes can be considered simultaneously. This allows for the combination of injection data and production data in a single analysis. The difference between the calculated pressure and the observed pressure is normalized to the observed pressure change. This was done to give equal weight to all of the pressure data, regardless of the absolute magnitude of the response. This equal-weight feature is also implicit in conventional curve matching. The sum of the difference between the observed and calculated points is averaged over the total number of data points used for the analysis. This provides a means of comparing results from several analyses with various numbers of observation points. The absolute magnitude of χ^2 should not be used as the sole determinant for the success of a particular analysis, because the value of χ^2 is a function of the accuracy of the data as well as the correctness of the reservoir parameters. When an acceptable minimum has been reached, the calculated and observed pressure changes will be nearly equal. At this time it is reasonable to assume that the correct reservoir parameters have been obtained.

SEVERAL OBSERVATION WELLS

When there is more than one observation well, the program uses a minimization statistic which includes the data from each observation well. The calculated pressure response is obtained from Equation (7). The minimization statistic used is expressed in Equation (9).

$$x^2 = \frac{1}{H} \sum_{h=1}^{H} \frac{1}{I} \sum_{I=1}^{I(h)} \left(\frac{\Delta P_{calc}(h,i) - \Delta P_{obs}(h,i)}{\Delta P_{obs}(h,i)} \right)^2 . \qquad (9)$$

HYDROLOGIC BOUNDARIES

A single, fully penetrating, linear hydrologic boundary can be modeled using the method of images (Hantush, 1964b). Briefly stated, a boundary may be viewed (and modeled) as a line of bilateral symmetry about which image production wells are arrayed in one-to-one symmetric correspondence with real production wells (see Figure 3). To model an impermeable boundary, each image production well is assigned a flow-rate record identical to that of its symmetrically located real well. This results in zero pressure gradients perpendicular to the line of symmetry at the line of symmetry. A constant-potential

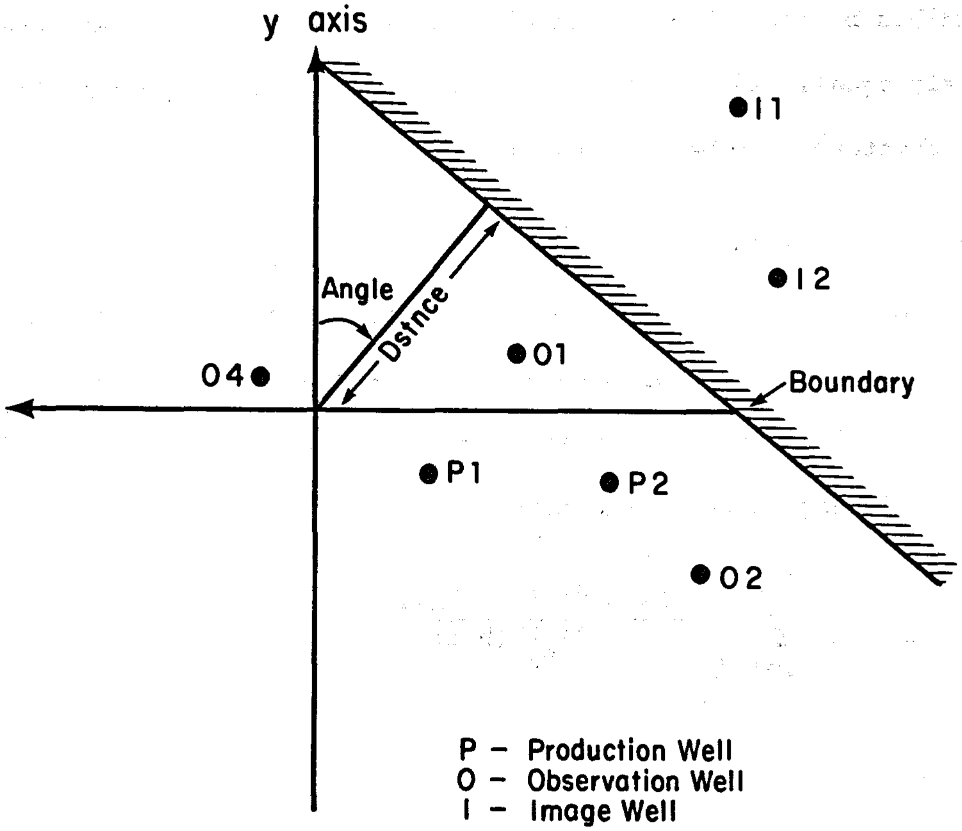


Figure 3. Image-well location schematic. [XBL 815-3073]

boundary is modeled using image wells whose production/injection records are identical to those of their symmetric counterparts but of opposite sign (production vs. injection). This situation results in zero pressure change along the line of symmetry, which is the necessary mathematical condition for a constant-potential boundary of infinite horizontal extent.

The image-well distance $r^i_{h,j}$ is the distance from an observation well h to the image production well, which corresponds symmetrically to production well j on the opposite side of an assumed boundary line. The boundary location is specified in terms of a system of radial coordinates (angle α and perpendicular distance d) whose origin corresponds to that of the Cartesian coordinate system used to delineate the locations of the production and observation wells. The angle α is measured clockwise from the positive y axis. A more complete description of the boundary-location scheme is discussed in Appendix B.

The image wells that model a boundary contribute an additional component of pressure response that is simply added to that contributed by the real production wells. This additional pressure-response component is known particularly through the assumed values of kh/μ and ϕch (as before) and through the assumed values of angle α and perpendicular distance d from the boundary that are used to calculate the image-well distance $r^i_{h,j}$. The additional boundary effect is calculated using Equations (7) and (11). To see this more clearly, we may rewrite Equation (7) to include the image-well effect:

$$\Delta P(h,i) = \sum_{j=1}^{J} \sum_{k=1}^{K} \Delta P(h,i,j,k,r_{h,j}) \pm \Delta P(h,i,j,k,r^{i}_{h,j}), \tag{10}$$

where $\Delta P(h,i,j,k,r_{h,j})$ is the real-well contribution, and $\Delta P(h,i,j,k,r^{i}_{h,j})$ is the image-well contribution.

The plus sign denotes a barrier boundary and the minus sign denotes a constant-potential boundary. As before, the term $\Delta P_{calc}(h,i)$ is included in the scalar χ^2 summation (Eq. 8 or 9) for a comparison of parametrically calculated pressure responses and observed pressure responses. When an acceptable minimum value of χ^2 is reached, the four parameters kh/μ, ϕch, α, and d are assumed to be optimized results. When two or fewer observation wells are monitored, the location of the boundary is not unique. This is discussed further under Recommended Procedure for Analysis (p. 12).

PRODUCTION-WELL SKIN EFFECT

The calculated pressure response, $\Delta P_{calc}(h,i,j,k)$, may also include parametric, steady-state skin values for production-well tests. The degree of damage (or enhancement) that a production well may exhibit is characterized by its skin value. The skin effect is defined as the steady-state pressure change in the production well; it is directly proportional to the sandface flow rate, inversely proportional to the transmissivity, and directly proportional to s, the skin value,

$$\Delta P_{skin} = s \frac{q\mu}{2\pi kh} . \tag{11}$$

Positive skin values indicate a damaged well bore, and negative skin values indicate an enhanced well bore. This formulation models the degradation (or enhancement) of a region around the well as a film of resistance to fluid flow at the well-bore radius. Several authors (Agarwal et al., 1970; Wattenbarger and Ramey, 1970) have demonstrated an alternative interpretation of skin effect. They assume a finite, annular zone of damage (or enhancement) surrounding the well. The flow in this zone is assumed to be at steady state. The equation obtained is $s = [(k/k_1 - 1]\ell n(r_1/r_w)$, in which k_1 is the permeability of the zone and r_1 its radial extent. For a constant-bulk-rate production test in which the well-bore storage coefficient is less than $1000 \times 2\pi\phi chr^2_w$, Wattenbarger and Ramey (1970) have shown that for a damaged zone of radius greater than $100 \times r_w$, the steady-state skin effect and the composite reservoir system are not equivalent. The reason is that for storage coefficients smaller than specified above, the variation of the sandface flow rate is more acute, so that a transient pressure response is created in the large ($100 \times r_w$) annular region of damage, thus violating the steady-state assumption of the skin effect.

The validity of the steady-state skin effect is influenced by the rapidity of the change in sandface flow rate. Qualitatively, the smaller the change in sandface flow rate, the closer to steady-state flow conditions will be. The time required for the flow regime in the damaged region to approach steady state depends on the extent and permeability of the damaged zone and on the rapidity of the change in sandface flow rate. It is expedient, however, and in conformity with conventional production-test analysis, to assume that wellbore damage or enhancement may be represented by a steady-state skin effect.

The additional pressure change caused by the skin effect in a production well is taken into account by adding the response given by Equation (11) to Equation (7). Thus the calculated pressure is given by Equation (12):

$$\Delta P(h,i) = \sum_{i=1}^{J} \sum_{k=1}^{K} \Delta P(h,i) \pm \Delta P(h,i,j,k,r^{i}_{h,j}) + \frac{s\mu}{2\pi kh} q(t). \qquad (12)$$

RECOMMENDED PROCEDURE FOR ANALYSIS

The use of a fitting routine for well-test analysis is not completely automatic. Judgment must be used in choosing the initial values of the reservoir parameters and in gauging the reliability of any result. Furthermore, the technique presented here is simply an extension of conventional methods of well-test analysis: rules that apply to conventional analysis also apply here. This program is a tool to be used when the requirements of constant flow and/or a single production/injection well are not satisfied.

ANALYZE has been validated by comparison with many analytic and numerical models. For a discussion of the validation of the program, see McEdwards and Tsang (1978). Experience indicates that when initial values chosen for the reservoir parameters are within an order of magnitude of the correct values, the correct minimums are found. Several analyses should be performed, each with different initial guesses to ensure that the best possible fit is obtained. Different values for the reservoir parameters may be found for different guesses of the initial parameter values. This may result from (1) errors in the flow rate or pressure measurements or (2) difference in the character of the well/reservoir system being tested and the well/reservoir model implicit in the algorithm used by ANALYZE.

Judgment must be used to select the most reasonable result. If the minimum values are very large, then it is likely that the simple reservoir model assumed by the analytic solution is not valid for the reservoir being tested. In this case, another mathematical solution, dependent on a more realistic model, must be used to obtain the values for the reservoir parameters.

Well-test data can be subdivided into two categories: interference-test data and production-test data. Interference data consist of pressure records from one or more observation wells and flow-rate data from the production/ injection well(s). Data from interference tests are analyzed for reservoir transmissivity, storativity, and the location, existence, and type of linear reservoir boundary. Production-test data consist of pressure records in the production/injection well(s). These data are analyzed for reservoir transmissivity, skin, and the location, existence, and type of linear reservoir boundary.

A detailed procedure for analyzing interference test data is discussed below. Analysis of production well data is similar except that the skin value is obtained in addition to the reservoir parameters. However, in order to obtain a skin value for a well it is necessary to have a reliable estimate for ϕch. This is because ϕch and the skin value are mathematically nonunique. If a reliable estimate of ϕch is not available, the product ϕchr_e^2 may be obtained, where r_e is the effective wellbore radius.

The common element for all analyses is that the technique is not used to provide final values for all the parameters in just one computer run. Several analyses should be performed, each of which considers different effects (early time data, long-term data, boundary effects, skin effect, different initial values of the parameters). If this approach is not taken, spurious minimums may not be recognized as being nonphysical, although mathematical, solutions to the problem.

INTERFERENCE TEST DATA

In the case of two or more observation wells and one production well, pressure records should first be analyzed individually. The effects of reservoir heterogeneity may then be determined by comparing the values of the reservoir parameters found in each single well analysis. The effects of reservoir heterogeneity will be less evident in this comparison if two or more production wells affect the pressure records. The following analysis procedure consists of two steps and is generally applicable to each scenario. As experience is gained, the user may modify or simplify these procedures.

Step One

Purpose: (1) To determine if a boundary is present, and if so the type of boundary and approximate values of kh/μ and ϕch; (2) to determine values of kh/μ and ϕch if a boundary is not indicated; and (3) to determine the communication pattern between observation and production wells so that the final analysis of well-test data is not done using observation wells which do not communicate with one or more of the production wells.

Procedure: Perform sequential two-parameter (kh/μ, ϕch) interference analyses (no production pressures) for which later pressure data are progressively deleted. For example, in a series of three analyses, the first analysis will consider all of the pressure data, the second analysis will consider the earliest half of the data from each observation well, and the third analysis will consider the earliest quarter of the data from each observation well. Later data can be automatically ignored by use of the ERLYFT specification (see input). If, as the later data are progressively deleted, the values of kh/μ become progressively larger (or smaller), coinciding with a progressive decrease in the returned χ^2 values, a barrier (constant potential) boundary is indicated. If the kh/μ values and the χ^2 values do not change appreciably, then the data do not contain boundary information, and the values of kh/μ and ϕch are accepted as the analytical results. If a boundary is indicated, the values of kh/μ and ϕch from the earliest time may be used as initial guesses for a hydrologic boundary analysis. If two or more production wells influence the pressure data under consideration, the procedure may be repeated using different combinations of production wells and observation wells. Comparison of the χ^2 values obtained for the different observation and production well sets will indicate the communication pattern among wells. Thus the proper set of observation and production wells may be used in further analyses for boundary effects.

Step Two

Purpose. To determine simultaneously values of kh/μ, ϕch, and the angle and distance to a boundary.

Procedure. Perform a four-parameter (kh/μ, ϕch, angle, and distance) analysis using all the interference-pressure data and using as initial values of kh/μ and ϕch the values found for the earliest data fit in Step One. With initial values of angle and distance set to zero, implement the SEARCH option for the boundary type determined in Step One. If data from only one observation well and one production well are being analyzed, the SEARCH option should not be used because the boundary location is not uniquely determined. Only equivalent information about the distance to the image production well is known. In this case the initial guess for angle should be held fixed and the other parameters (kh/μ, ϕch, and distance) should be allowed to vary. In this manner, the distance to a boundary along a fixed angle will be known and the image-well distance may be calculated.

The boundary location, once determined, is theoretically unique if there are three or more observation wells. Two equivalent locations for the boundary are possible if there are two observation wells, and only an image-well distance may be found for one observation well.

INPUT DATA

INPUT DATA CATEGORIES

Parameter Unit Conversion Factors

All parameter input units are converted to SI units for internal use in the fitting program. At the end of the fitting procedure, the parameter units are converted again from SI units to the original input units. The required conversion-factor data are listed below.

IDIMEN	Flag to permit reading of input unit conversion factors (0 for default units, 1 to implement).

Input unit conversion factors read if IDIMEN = 1.

PAPRESS	Number of pascals per pressure input unit.
CMSFLOW	Number of cubic meters per second per flow-rate input unit.
SECTIME	Number of seconds per time input unit.
MLENGTH	Number of meters per length unit.
PASVISC	Number of pascal-seconds (Pa-s) per viscosity input unit.
SMPERM	Number of square millimeters per permeability input unit.

The default (IDIMEN = 0) conversion factors are all equal to 1. Thus, if the default option is chosen, the parameters are understood to be in SI units. If the IDIMEN = 1 option is chosen, the parameters are understood to be in the input units used to define the unit conversion factors.

Core Data Required for All Analyses

The input data that must be specified, regardless of the type of analysis performed, are:

NPAR	Number of parameters sought.
HH	Number of observation wells.
JJ	Number of production wells.
KHU	Initial guess of transmissivity, kh/μ.
PCH	Initial guess of storativity, ϕch.

Observation well data, H = 1, HH

OX(H), OY(H)	x and y coordinates of observation well H, length units.
YSTART(H)	Initial pressure at observation well H, pressure units.
YDATA (IH, H), DATA (IH,H)	Pressure and time coordinates for observation well H, pressure and time units

Production well data, J = 1, JJ

PX(J), PY(J)	x and y coordinates of production well J, length units.
AQ(KJ, J), TQ(KJ,J)	Flow and time coordinates for production well J, flow rate and time units

This core information is sufficient to do a two-parameter (kh/μ, ϕch) interference-test analysis involving several observation wells and several production wells. The additional data required for a boundary and/or skin-effect analysis are given below.

Boundary-Effect Analysis

LL	Boundary type specification. 0 for barrier boundary, 1 for constant-potential boundary.
ANGLE	Initial guess for azimuth to boundary, measured in degrees clockwise from the y axis of the well coordinate system.
DSTNCE	Initial guess for perpendicular distance to the boundary as measured from the coordinate system origin, length units.
ERLYFT	Latest observation time used in analysis. Observation times later than ERLYFT are not used in the analysis. The default value of zero is replaced automatically with the greatest input observation time.
SEARCH	An alphanumeric flag for the performance of a preminimization grid search for the location of a minimum χ^2 in angle-distance space. The final preliminary grid search values of angle and distance are used as initial guesses in the analysis. This option is implemented by setting SEARCH = 6HSEARCH. Any other character string is the default option.

Skin-Effect Analysis

JJS	Number of production wells which are also observation wells (dual nature wells) for which the skin effect is to be found.
X(I), I = 3, JJS + 2	Initial skin effect values for JJS production wells. Not read if JJS = 0.
LOBS(H)	Number (as read) of the production well which corresponds to observation well H. The default value is 0 and has no effect.

ERLYFT	Latest observation time used in analysis. Observation times later than ERLYFT are not used in the analysis. The default value of zero is replaced with the largest observation time.

Fixed-Parameter Analysis

The ability to hold some or all parameters constant during the fitting procedure is very useful and necessary for the full exploitation and proper application of the analysis technique. A parameter is held constant at its initial (input) value by internally multiplying the extrapolated change in the variable by zero. This is done by setting certain diagonal elements of the inverse of the second derivative matrix to zero.

IREAD	Flag to permit reading of diagonal elements of the H(I,J) matrix. Set IREAD = 1 to implement. The default value is 0.
H(I,I),I = 1, NPAR	diagonal elements of the H matrix. Set H(I,I) = 0.0 to hold the Ith parameter at its initial input value during the analysis. To include the Ith parameter in the analysis, set H(I,I) = 1.0. Up to NPAR-1 parameters may be held constant using the IREAD flag. To hold all parameters constant, use the IPLOT flag. The correspondence between I and the parameters is given below.

Parameter	Sequential number of Parameter, I
kh/μ	1
ϕch	2
Skin effect	3 to JJS+2 for JJS > 0
Angle	JJS+3
Distance	JJS+4

IPLOT	Flag to permit plotting of calculated pressures on the basis of input values of the parameters. No analysis is done; all parameters are held at their initial values. Set IPLOT = 1 to implement. The default value is 0.

INPUT DATA STRUCTURE

The sequence and format of the input data are given below. Figure 4 shows the structure of a typical input deck for ANALYZE. The definition of each input data item is given above but is repeated here for easy reference.

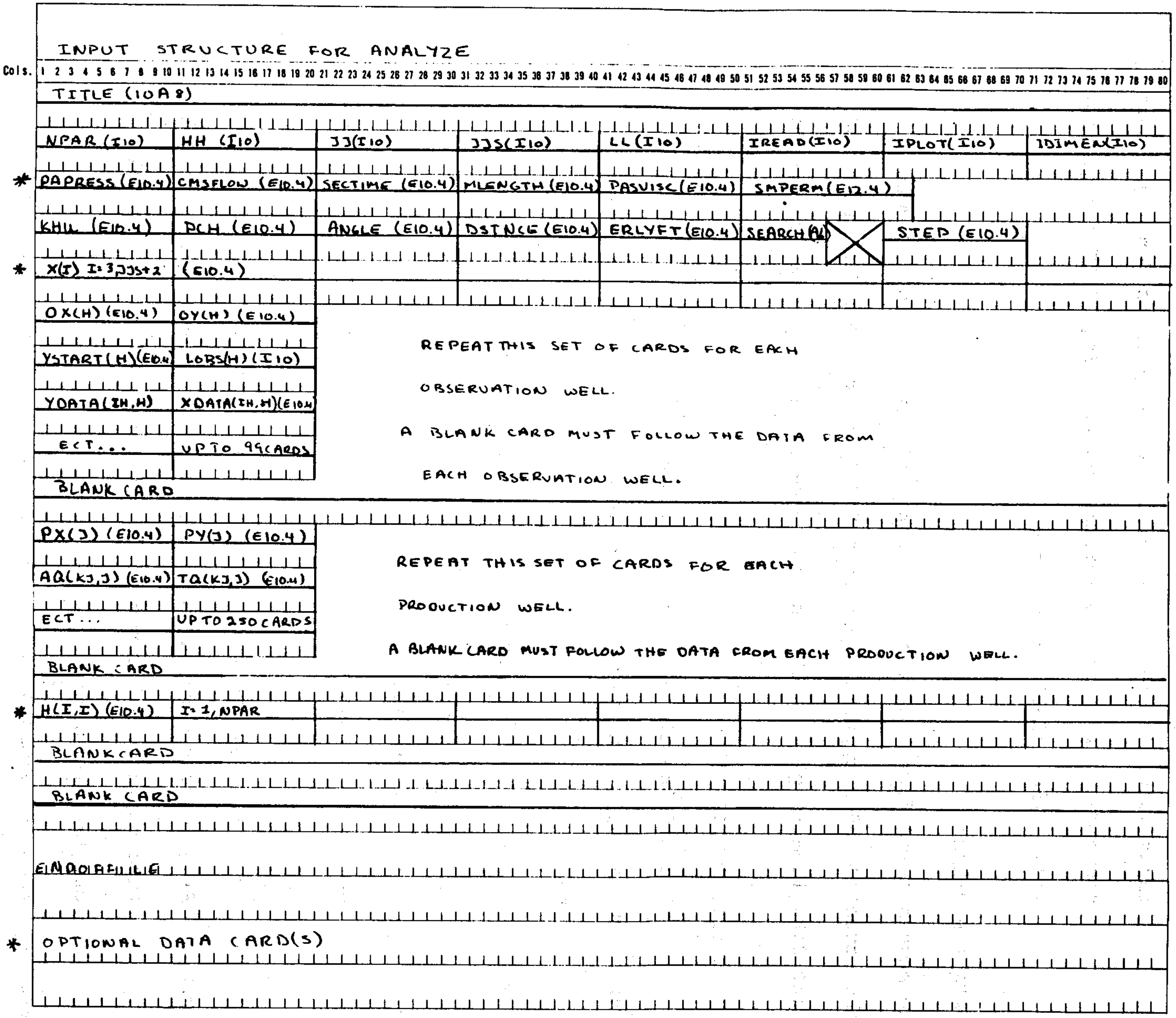
* Head (I), I=I,10 (10A8)

Identifying title information, one card.

* NPAR, HH, JJ, JJS, LL, IREAD, IPLOT, IDIMEN 8I10)

One card.

NPAR	Number of parameters used in analysis, 2 > NPAR > JJS+4.
HH	Number of observation well, 1 > HH > 20.
JJ	Number of production wells, 1 > JJ > 20.
JJS	Number of production wells for which the skin effect is to be found.
LL	Boundary type specifications, 0 for barrier, 1 for leaky.
IREAD	Flag to hold some parameters constant in analysis, 0 for all free to vary, 1 for some held constant at their input value. See last item, H(I,I).
IPLOT	Flag to permit plotting of calculated pressures on the basis of input values of parameters. No fitting analysis is done. 0 for no effect, 1 to implement.



INPUT STRUCTURE FOR ANALYZE

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	TITLE (10A8)							
	NPAR (I10)	HH (I10)	JJ(I10)	JJS(I10)	LL(I10)	IREAD(I10)	IPLOT(I10)	IDIMEN(I10)
*	PAPRESS (E10.4)	CMSFLOW (E10.4)	SECTIME (E10.4)	MLENGTH (E10.4)	PASVISC (E10.4)	SMPERM (E12.4)		
	KHL (E10.4)	PCH (E10.4)	ANGLE (E10.4)	DSTNCE (E10.4)	ERLYFT (E10.4)	SEARCH (A) ☒	STEP (E10.4)	
*	X(I) I=3,JJS+2	(E10.4)						
	OX(H) (E10.4)	OY(H) (E10.4)	REPEAT THIS SET OF CARDS FOR EACH OBSERVATION WELL. A BLANK CARD MUST FOLLOW THE DATA FROM EACH OBSERVATION WELL.					
	YSTART(H)(E10.4)	LOBS(H) (I10)						
	YDATA(IH,H)	XDATA(IH,H)(E10.4)						
	ECT...	UP TO 99 CARDS						
	BLANK CARD							
	PX(J) (E10.4)	PY(J) (E10.4)	REPEAT THIS SET OF CARDS FOR EACH PRODUCTION WELL. A BLANK CARD MUST FOLLOW THE DATA FROM EACH PRODUCTION WELL.					
	AQ(KJ,J) (E10.4)	TQ(KJ,J) (E10.4)						
	ECT...	UP TO 250 CARDS						
	BLANK CARD							
*	H(I,I) (E10.4)	I=1,NPAR						
	BLANK CARD							
	BLANK CARD							
	ENDOFFILE							
*	OPTIONAL DATA CARD(S)							

Figure 4. Data deck for ANALYZE. [XBL 818-11173]

IDIMEN	Flag to permit reading of input unit conversion factors if different from the SI system default input unit conversion factors. 0 for no effect, 1 to implement.

* PAPRESS, CMSFLOW, SECTIME, MLENGTH, PASVISC, SMPERM, (5E10.4, E12.4), input unit conversion factors. The default input units are SI units and all conversion factors. 0 for no effect, 1 to implement.

PAPRESS	Number of pascals per pressure input unit.
CMSFLOW	Number of cubic meters per second per flow rate input unit.
SECTIME	Number of seconds per time input unit. (Flow and pressure time must be in same units.)
MLENGTH	Number of meters per length input unit.
PASVISC	Number of pascal-seconds per viscosity input unit.
SMPERM	Number of square meters per permeability input unit.

* KHU, PCH, ANGLE, DSTNCE, ERLYFT, SEARCH, STEP (5E10.4, A6, 4X, E10.4).

KHU	Initial guess for transmissivity, kh/μ (permeability-length/viscosity units).
PCH	Initial guess for storativity, ϕch (length/pressure units).
ANGLE	Initial guess for azimuth to boundary, measured in degrees clockwise from y axis.
DSTNCE	Initial guess for perpendicular distance to boundary measured from origin (length units).
ERLYFT	Latest observation time to be used in analysis. Observation times later than ERLYFT are not used in analysis (time units).
SEARCH	Alphanumeric flag for performance of preliminary (preminimization) grid search for location of minimum χ^2 in ANGLE-DSTNCE

space. Final preliminary grid search values of angle and distance are used as initial guesses in minimization routine. Implement by setting SEARCH = 6HSEARCH, and leave blank for no effect.

STEP Increment of DSTNCE used in preminimization grid search. Eight steps taken, from DSTNCE + STEP to DSTNCE + 8*STEP. STEP must be greater than zero if search option is used (length units).

* X(I), I=3, JJS + 2, (8E10.3) JJS initial guesses of skin values for JJS dual wells (dimensionless); one or more cards. Omit if JJS = 0.

Observation-well data, H = 1, HH. Repeat next three items HH times.

* OX(H), OY(H), (2E10.3) x and y coordinates of observation well H measured in length units. Coordinate system may be of any orientation but it must be Cartesian.

* YSTART(H), LOBS(H), (E10.3, I10)

YSTART(H) = initial pressure of observation well H (pressure units).

LOBS(H) = number (as read) of the production well which corresponds to this observation well if this observation well is a dual well. If not a dual well, omit.

* YDATA (IH,H), XDATA (IH,H), (2E10.3), pressure and time data for observation well H, one data point per card. Pressure data are given in chronological order. Place blank card after last card. The program can handle up to 99 observation points per well.

Production-well data, J=1, JJ. Repeat next two items JJ times.

* PX(J), PY(J), (2E10.3) x and y coordinates of production well J.

* AQ(KJ,J), TQ(KJ,J), (2E10.3) coordinates of flow-rate history (flow rate, time) for production well J. One data point per card. For step-rate flow change, both points (rates) must be given for the time of the change. Flow-rate coordinates are given in chronological order. Place blank card after last card. The program can handle up to 250 flow-rate points per production well.

* H(I,I), I=1, NPAR (8E10.3) diagonal elements of inverse hessian matrix, one or more cards. Read if IREAD = 1. Set H(I,I) = 0 to hold the Ith parameter at its initial input value. The Ith parameter will then not be a fitting parameter. Set H(I,I) = 1.0 if Ith parameter is to be fitted. The correspondence between I and the parameters is given below.

Parameter	Sequential number of Parameter, I
kh/μ	1
ϕch	2
Skin effect	3 to JJS+2 for JJS > 0
Angle	JJS+3
Distance	JJS+4

Up to NPAR-1 parameters may be held constant. To hold all parameters constant use the IPLOT-1 option. Omit card(s) if IREAD = 0.

* Two blank cards.

Input data lists for four sample problems are shown along with the output for each example in the section entitled Sample Problems (p. 29).

OUTPUT INFORMATION

The following is a list of the output which the user will obtain with each computer run of the program. Items 1-8 pertain to input information and are printed to enable the user to check the input data. Item 9 includes information about the minimization process, and items 10-12 present analysis results and information about the minimum found.

Output Item List

Item Number	Description
1	Identifying title.
2	Number of observation and production wells in analysis.
3	The latest observation time used and the greatest observation time input.
4	Initial guesses of parameters, whether the analysis is for a leaky or a barrier boundary, and whether a preliminary search for the location of the boundary was done.
5	For each production well: its sequential number, its coordinates, the number of flow points input. For each production segment: the initial flow rate, the slope, and the beginning time.
6	For each observation well: its sequential number, the sequential production well number for which it corresponds if it also serves as a production well, its coordinates, the number of input data points, the initial pressure. For each data point: the drawdown, the time of observation, and the observed pressure.

Output Item List (continued)

Item Number	Description
7	The values of χ^2, $\partial\chi^2/\partial\chi_i$, and χ_i at the initial guesses of the parameters kh/μ, ϕch, etc. The χ_i values are internal fitting parameters and do not correspond in value to the input guesses. At the end of minimization, these internal parameters are converted to the user input units.
8	The values of the diagonal elements of the H_{ij} matrix. If a diagonal element is zero, the parameter corresponding to that diagonal element is held fixed at its initial value.
9	Internal fitting information, the detail of which depends on the IWRITE control variable discussed in the program description of VARMIT (Beals, 1966b). The recommended value of IWRITE is zero and is the value used in the present technique. Only final results, which are given in values internal to the fitting program, are printed for IWRITE = 0. These results are: (a) The number of random steps at which the best value of χ^2 (called F) was achieved. (b) The H_{ij} matrix that corresponds to the best minimum. (c) The values of χ^2, the fitted parameters, and the gradients of χ^2 (named FBEST, XBEST, and G respectively) that correspond to the best minimum. The fitting program takes a random step in parameter space from the first-found minimum and finds another minimum from this new starting

Output Item List (continued)

Item Number	Description
	position. The differences between the values of F and the fitted parameters found at each minimum are printed as plus (+) or minus (-) quantities and indicate qualitatively how well the minimum is defined. They do not constitute statistical information. More than one random step may be specified, but it is recommended that only one be specified (Beals, 1966a,b).
10	For each observation well, its sequential number. For each data point, the time, observed drawdown, calculated drawdown, the difference between the calculated and observed drawdown, the ratio between the calculated and observed drawdown, and a log-log plot of observed and calculated drawdowns versus time.
11	The final values of the parameters, in input units.
12	The value of χ^2 at the best minimum, the number of data points used in the analysis, and the number of input data points.

SAMPLE PROBLEMS

SAMPLE PROBLEM 1: MATCH OF THE THEIS CURVE

The Theis curve is the dimensionless solution to constant-rate production/injection from a line source which fully penetrates an areally infinite, isotropic, isothermal, porous medium of constant thickness. The algorithm used in ANALYZE reduces to the "Theis Solution," with one well flowing at a constant rate.

For this case,

$$\Delta P = \frac{Q\mu}{2\pi kh} \; P_D, \tag{S-1}$$

where

$$P_D = \frac{1}{2} \int_u^{\infty} \frac{e^{-u}}{u} \, du \tag{S-2}$$

and

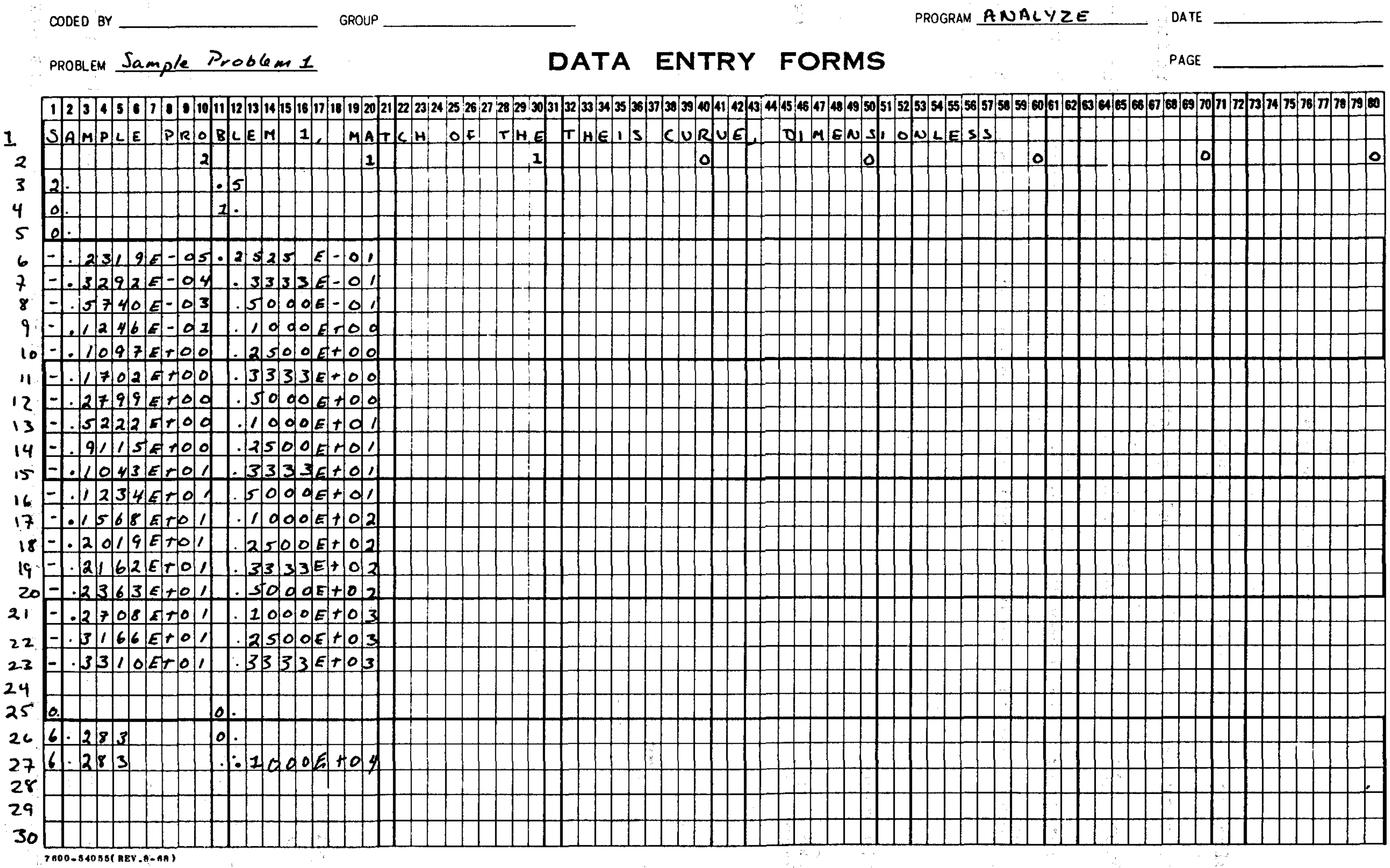
$$u = \frac{1}{4t_D} \tag{S-3}$$

and

$$t_D = \left(\frac{kh}{\mu}\right) \frac{1}{\phi ch} \frac{t}{r^2} \tag{S-4}$$

for any set of consistent units.

For the case where $kh/\mu = \phi ch$ and $r = 1$, Equation (S-4) becomes $t_D = t$, where t is the measured time from the beginning of constant flow. Also note that if $Q = 2\pi$ and $kh/\mu = 1$, then Equation (S-1) becomes $\Delta P = P_D$. This simple



CODED BY ____________ GROUP ____________ PROGRAM ANALYZE DATE ____________

PROBLEM Sample Problem 1

DATA ENTRY FORMS

PAGE ____________

```
   0        1         2         3         4         5         6         7         8
   12345678901234567890123456789012345678901234567890123456789012345678901234567890
1  SAMPLE PROBLEM 1, MATCH OF THE THEIS CURVE, DIMENSIONLESS
2           2         1         1         0         0         0         0         0
3  2.        .5
4  0.        1.
5  0.
6  -.2319E-05.2525 E-01
7  -.3292E-04 .3333E-01
8  -.5740E-03 .5000E-01
9  -.1246E-02 .1000E+00
10 -.1097E+00 .2500E+00
11 -.1702E+00 .3333E+00
12 -.2799E+00 .5000E+00
13 -.5222E+00 .1000E+01
14 -.9115E+00 .2500E+01
15 -.1043E+01 .3333E+01
16 -.1234E+01 .5000E+01
17 -.1568E+01 .1000E+02
18 -.2019E+01 .2500E+02
19 -.2162E+01 .3333E+02
20 -.2363E+01 .5000E+02
21 -.2708E+01 .1000E+03
22 -.3166E+01 .2500E+03
23 -.3310E+01 .3333E+03
24
25 0.        0.
26 6.283     0.
27 6.283      .1000E+04
28
29
30
```

7600-54055(REV.8-68)

Figure 5. Data deck for Sample Problem 1. [XBL 818-11171]

relationship is used as the basis for a demonstration of the program's matching capability. If the pairs (P_D, t_D) are used as observation points and Q is specified as 2π (6.283...), the program should yield $kh/\mu = \phi ch = 1$ for the reservoir parameters. The example is useful as a simple test of whether the program has been implemented correctly on another computer system.

The data deck for this problem is shown in Figure 5. As seen in line 2, NPAR = 2 (column 10), which specifies that only kh/μ and ϕch will be considered as fitting parameters. There is one observation well (column 20) and one production well (column 30). As no conversion units are required, IDIMEN (column 80) is set equal to zero. In line 3 the initial guesses of the parameters is arbitrary, but will affect the total number of iterations required to obtain a satisfactory match of the data.

The coordinates of the observation well (see line 4) are (0.0, 1.0), so that $r^2 = x^2 + y^2 = 1$. The initial pressure (YSTART) is set to zero, since the pressure data are input as pressure changes rather than absolute values. Lines 6 through 23 contain the observation points (P_D, t_D). The observation data are followed by a blank card (line 24).

The coordinates of the production well (line 25) are (0, 0). The flow rate (lines 26 and 27) is equal to 2π (6.283...). Two data cards are needed to input a single constant flow rate. The first card specifies the flow rate at time zero, and the second specifies the duration of that flow rate. The flow-rate data are followed by a blank card (line 28). The data deck is completed with two blank cards (lines 29 and 30).

The output from this problem is shown in Figures 6 and 7. The match of all the data points is close to perfect. As expected, the parameters kh/μ and ϕch are found to be equal to unity.

SAMPLE PROBLEM 2: VARIABLE-RATE, MULTI-OBSERVATION-WELL INJECTION TEST

The data for this sample problem were obtained from an injection test in a shallow groundwater aquifer under consideration for an aquifer thermal energy storage project. The aquifer tested is approximately 20 m thick and bounded above and below by clays of very low permeability. An array of observation wells (Figure 8) was drilled to monitor aquifer response to injection in an attempt to determine the aquifer parameters. Many difficulties were encountered during the test, resulting in a highly variable injection rate (Figure 9). For this reason, ANALYZE was used to interpret what could have been considered an unsuccessful test.

The data deck for this problem is shown in Figure 10. The deck is similar to that of Sample Problem 1, with a few exceptions. In this example, NPAR = 4 (line 2, column 10), indicating that the effects of a boundary will be included in the analysis. The zero in column 50 indicates that an impermeable hydrologic boundary should be considered. The 1 in column 80 of card 2 sets IDIMEN = 1 so that conversion units can be read in on the following card. Table 1 summarizes the conversion unit data found on card 3.

On card 4 the initial estimates of the reservoir parameters are given in terms of the units in Table 1. For example, the initial estimate of the reservoir transmissivity is 8.0 x 10^5 mD-m/cp. Cards 5 through 112 contain the

Table 1. Conversion Unit Data Found on Card 3.

	Input Unit	Conversion Factor
Pressure	Feet of water	1.984×10^3 Pa/ft of H_2O
Flow rate	Gallons per minute	6.31×10^{-5} m^3/s/gpm
Time unit	Hours	3600 s/hr
Length	Meters	1 m/m
Viscosity	Centipoises	1×10^{-03} Pa-s/cp
Permeability	Millidarcies	9.862×10^{-16} m^2/mD

water-level data from the four observation wells. The data for each well include the x and y coordinates of the well (meters) and the initial pressure (feet) followed by the water-level data and a blank card.

The injection rate for this problem was highly variable. To model the injection rate accurately, 46 data points were required. The rate consisted of both step functions and linearly varying injection pulses. Step functions are input by specifying an initial rate AQ at time t_1 and then specifying that same rate at a second time t_2. The flow rates are implemented as follows:

$$Q(t) = AQ(t_1) + BQ(t - t_1) \text{ for } t_1 < t < t_2$$

where

$$BQ = \frac{AQ(t_2) - AQ(t_1)}{(t_2 - t_1)} .$$

Thus, if $AQ(t_2) = AQ(t_1)$, then $BQ = 0$, so that $Q(t) = AQ(t_1)$.

If $AQ(t_2) \neq AQ(t_1)$, the production pulse will be modeled with a linear variation in time.

```
 OBSERVATION WELL   1

         TIME       OBSERVED   CALCULATED  DIFFERENCE     RATIO


      .2525E-01  -.2319E-05  -.2317E-05   .1433E-08    .9994E+00
      .3333E-01  -.3292E-04  -.3292E-04  -.9755E-08    .1000E+01
      .5000E-01  -.5740E-03  -.5743E-03  -.2858E-06    .1000E+01
      .1000E+00  -.1246E-01  -.1246E-01  -.3853E-05    .1000E+01
      .2500E+00  -.1097E+00  -.1097E+00   .4610E-05    .1000E+01
      .3333E+00  -.1702E+00  -.1702E+00  -.2090E-04    .1000E+01
      .5000E+00  -.2799E+00  -.2799E+00   .1276E-04    .1000E+01
      .1000E+01  -.5222E+00  -.5221E+00   .1665E-04    .1000E+01
      .2500E+01  -.9115E+00  -.9114E+00   .1327E-04    .1000E+01
      .3333E+01  -.1043E+01  -.1043E+01   .5205E-04    .1000E+01
      .5000E+01  -.1234E+01  -.1234E+01   .4177E-04    .1000E+01
      .1000E+02  -.1568E+01  -.1568E+01   .5375E-04    .1000E+01
      .2500E+02  -.2019E+01  -.2019E+01   .6644E-04    .1000E+01
      .3333E+02  -.2162E+01  -.2161E+01   .8234E-04    .1000E+01
      .5000E+02  -.2363E+01  -.2363E+01   .1014E-03    .1000E+01
      .1000E+03  -.2708E+01  -.2708E+01   .9356E-04    .1000E+01
      .2500E+03  -.3166E+01  -.3166E+01   .1215E-03    .1000E+01
      .3333E+03  -.3310E+01  -.3309E+01   .1679E-03    .9999E+00


 SAMPLE PROBLEM 1,MATCH OF THEIS CURVE,DIMENSIONLESS

FINAL VALUES OF PARAMETERS

KHU =    .1000E+01  PERM.-LENGTH/VISC. UNITS

PCH =    .1000E+01  LENGTH/PRESSURE    UNITS


NORMALIZED CHI-SQUARED =    .47104E-07

FOR  18 DATA POINTS OUT OF  18 INPUT VALUES
```

Figure 6. Output for Sample Problem 1. [XBL 818-11172]

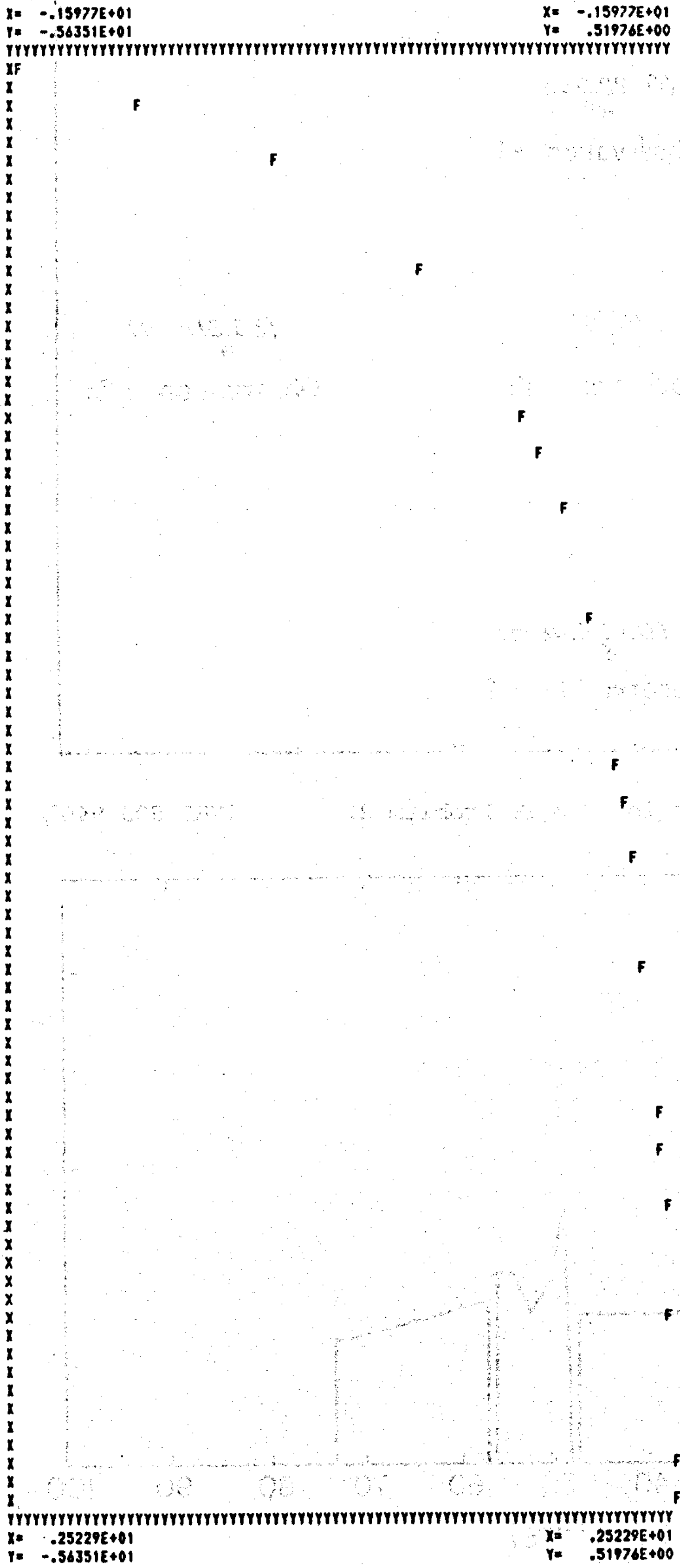


Figure 7. Graph of output for Sample Problem 1. [XBL 818-11160]

P REPRESENTS THE PREDICTED POINT
D REPRESENTS THE DATA POINT
F REPRESENTS BOTH THE DATA AND THE PREDICTED POINTS IF THEY LIE TOGETHER

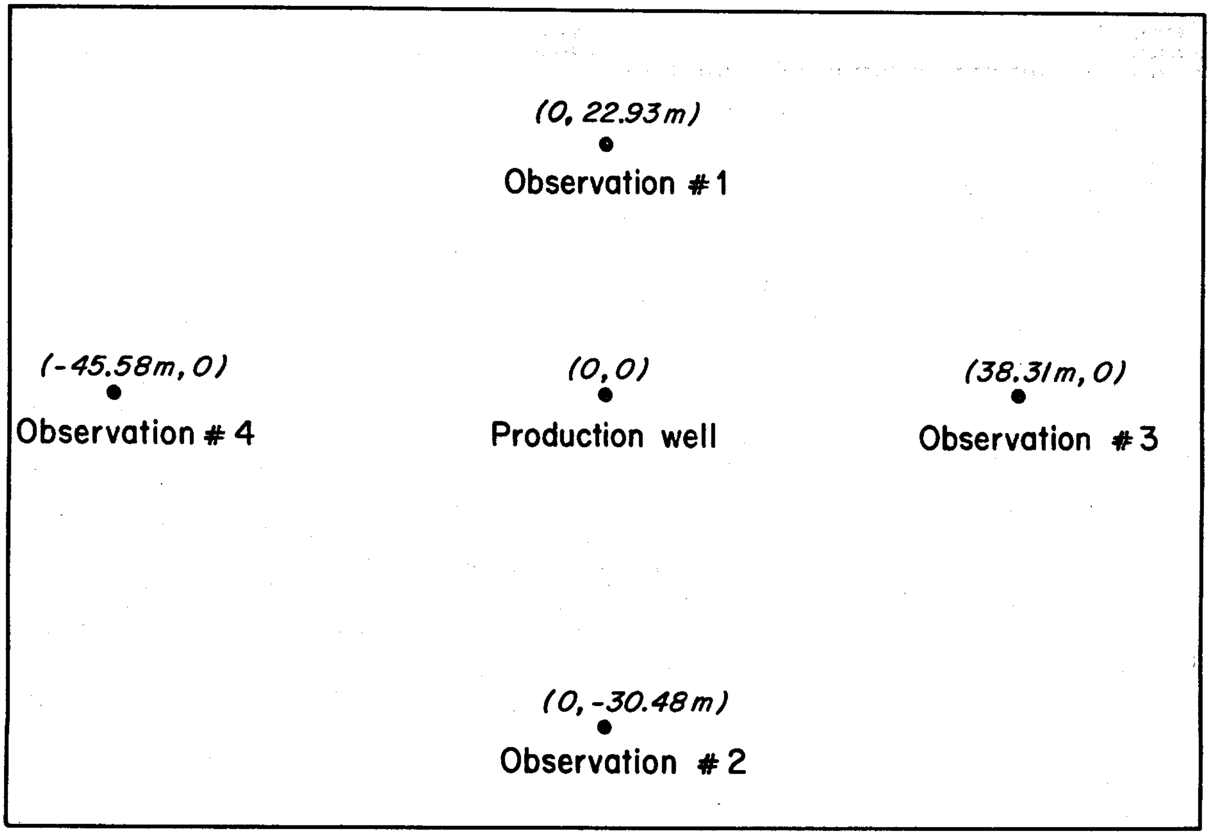


Figure 8. Observation well location for Sample Problem 2. [XBL 805-946]

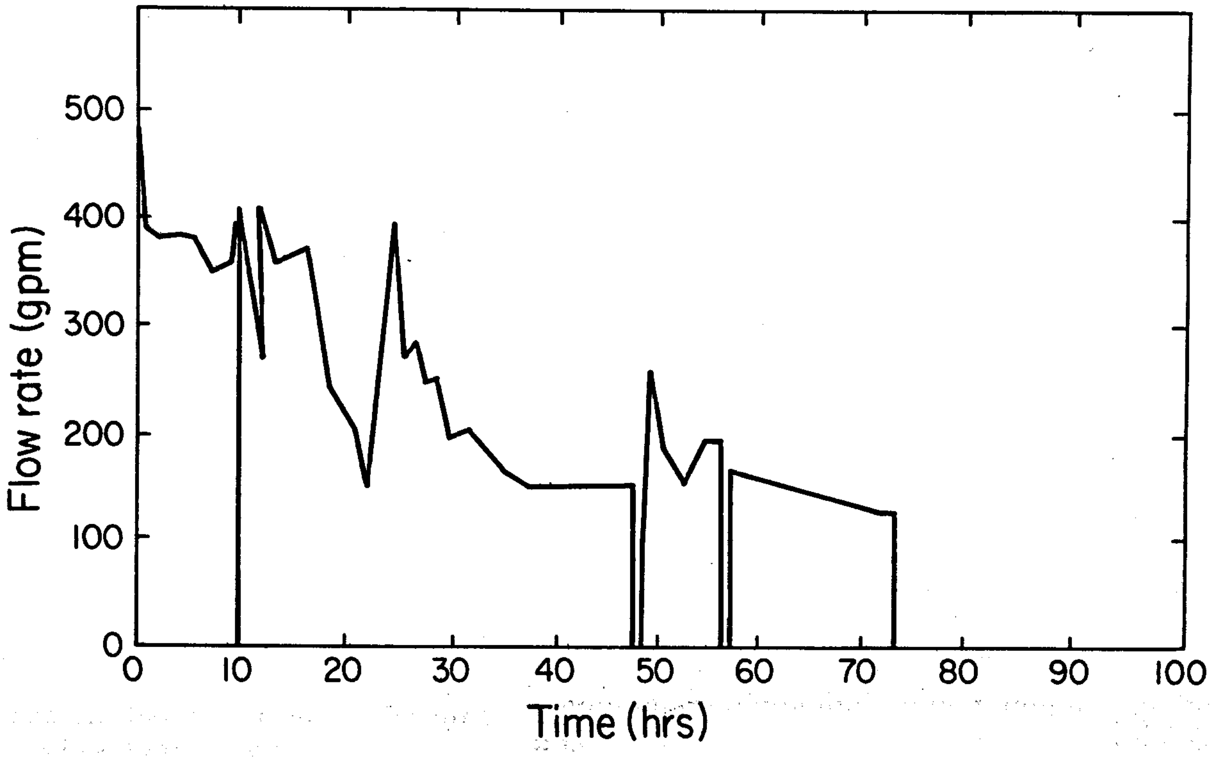


Figure 9. Variable injection rate for Sample Problem 2. [XBL 813-2723]

```
SAMPLE 2,4 OBS WELLS,1 PROD WELL,HIGHLY VARIABLE RATE
         4         4         1         0         0
2.984 E+03 6.31 E-05  3600.      1.       1.0 E-03  9.862 E-16
8.0    E05 4.7  E-04  320.     300.


0.        22.93                38.31     0.                   0.        0.
6.340                          6.33                           -485.     0.
10.96     1.3                  9.26      1.3                  -404.     .55
11.06     2.9                  9.77      2.8                  -392.     1.
11.45     4.6                  10.16     4.6                  -385.     2.
11.69     6.6                  10.44     6.6                  -383.     3.
11.5      8.7                  10.41     8.7                  -387.     4.
12.71     10.70                11.28     10.7                 -382.     5.4
12.85     13.53                11.41     13.53                -351.     7.
12.77     18.05                11.47     18.05                -360.     8.8
11.83     21.10                10.37     21.1                 -412.     9.6
12.5      25.1                 11.16     25.1                 -412.     11.43
11.58     28.                  10.7      28.                  0.        11.43
11.17     31.                  10.43     31.                  0.        11.55
10.67     37.                  10.15     37.                  -270.     11.55
10.32     49.4                 9.98      49.4                 -362.     13.13
10.54     52.3                 9.89      52.3                 -371.     16.6
9.55      73.6                 9.95      55.1                 -244.     18.25
8.57      74.30                9.53      72.                  -201.     20.67
8.35      75.4                 9.2       73.6                 -154.     22.
8.32      77.3                 8.58      74.3                 -391.     24.15
7.98      81.7                 8.37      75.4                 -275.     25.43
7.46      95.6                 8.2       77.30                -285.     26.2
7.28      103.4                7.99      81.7                 -247.     27.2
7.23      106.8                7.46      95.60                -245.     28.2
6.9       126.3                7.29      103.4                -199.     30.2
                               7.22      106.8                -201.     31.5
0.        -30.48                                              -167.     35.3
6.29                           -45.58    0.                   -149.     37.4
9.55      1.4                  6.41                           -96.      47.1
10.05     3.0                  9.44      1.5                  -96.      47.36
10.41     4.6                  9.9       2.9                  0.        47.36
10.69     6.6                  10.22     4.6                  0.        47.7
10.61     8.7                  10.52     6.6                  -257.     47.7
11.57     10.7                 10.46     8.7                  -190.     48.7
11.65     13.53                11.36     10.7                 -178.     50.4
11.69     18.05                11.54     13.53                -156.     52.3
10.64     21.10                11.46     18.05                -197.     54.7
11.42     25.10                10.45     21.1                 -197.     55.8
10.86     28.0                 11.23     25.1                 0.        55.8
10.55     31.                  10.76     28.                  0.        56.3
10.28     37.                  10.46     31.                  -164.     56.3
10.19     49.40                10.18     37.                  -123.     70.85
9.98      52.3                 10.1      49.4                 -104.     72.4
10.04     55.1                 9.92      52.3                 -104.     73.57
9.61      72.                  9.94      55.1                 0.        73.57
9.25      73.6                 9.56      7.2                  0.        293.2
8.53      74.3                 9.28      73.6
8.35      75.40                8.61      74.3
8.13      77.3                 8.43      75.4
7.93      81.7                 8.21      77.3                  EOR
7.42      95.60                8.04      81.7
7.25      103.4                7.51      95.6
7.21      106.8                7.36      103.4
                               7.29      106.8
```

Figure 10. Data deck for Sample Problem 2. [XBL 818-11164]

To model an instantaneous flow-rate change, set the time at which the rate change is initiated equal to the time at which the previous pulse ended, as in the following set of flow-rate points:

Flow rate	Time
-412	11.43
0	11.43

This set of points would allow an instantaneous rate change from -412 gpm to 0 gpm at 11.43 hr. The flow-rate data is followed by a blank card. Two blank cards complete the data deck.

This problem was analyzed using the procedure outlined in the section on Recommended Procedure for Analysis (p. 12). Analysis of data from individual wells indicated the influence of a barrier boundary. Early time data from each well were analyzed to obtain estimates of the reservoir transmissivity and storativity for use in a complete field analysis. Using the estimates

$kh/\mu = 8.0 \times 10^5$ mD-m/cp and $\phi ch = 4.7 \times 10^{-4}$ m/ft of H_2O

as the initial guesses, and using the water-level data from all four wells, a four-parameter analysis was made to obtain field-averaged reservoir parameters. The match of the calculated values and observed values obtained for each well are shown in Figures 11 and 12.

The final values of the reservoir parameters are:

$kh/\mu = 8.34 \times 10^5$ mD-m/cp,

$\phi ch = 5.16 \times 10^{-4}$ m/ft of H_2O,

$\alpha = 3.6$ degrees.

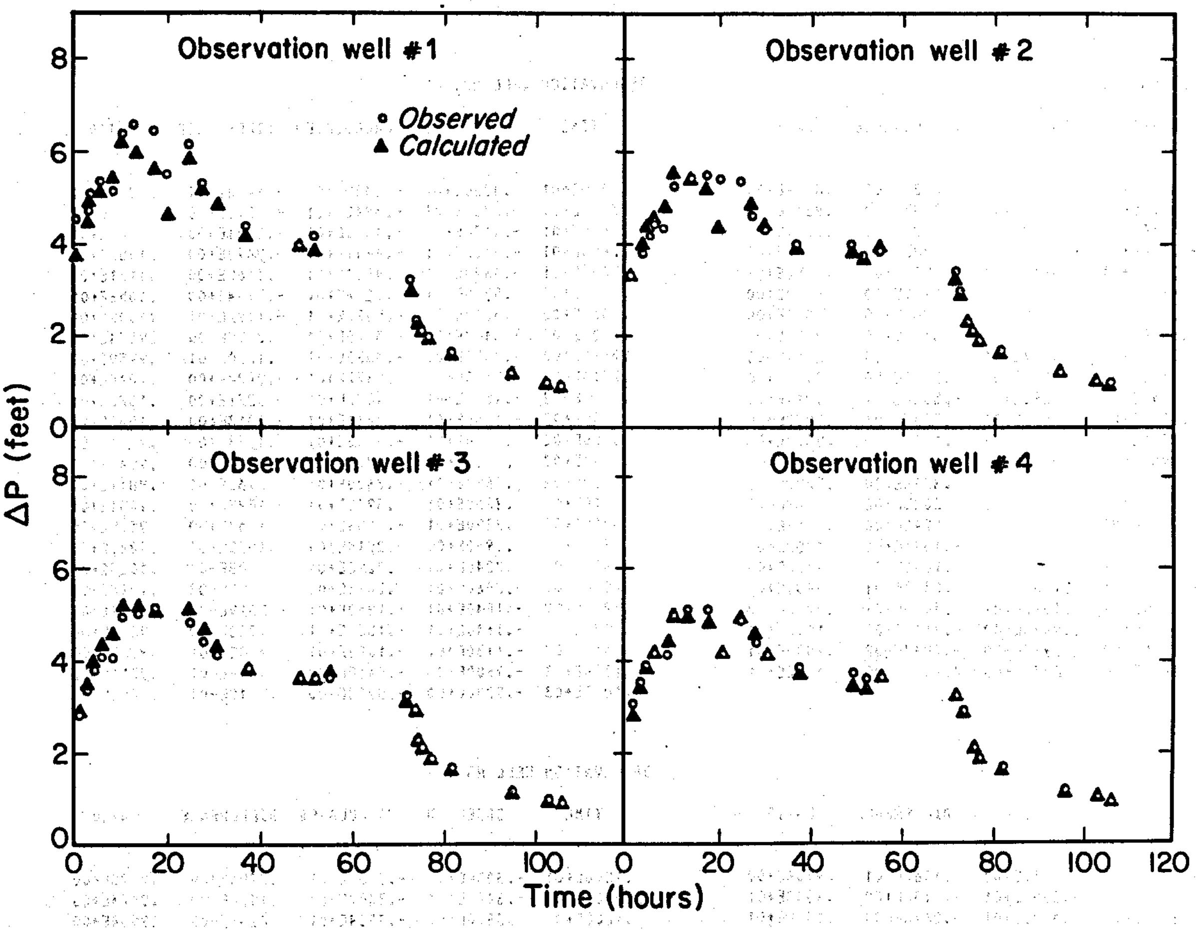


Figure 11. Graphed output for Sample Problem 2. [XBL 805-947]

Distance = 239 meters to the boundary in a direction along α, and

$\chi^2 = 0.44 \times 10^{-2}$.

A similar run made with only two parameters (kh/μ and ϕch) gave a χ^2 value of 0.77×10^{-2}. In a case like this where the χ^2 fitting parameter is not dramatically decreased, judgment must be used to determine which result is more physically plausible. In this specific example, a subsequent test with a constant flow rate was performed that confirmed both the values of the reservoir parameters and the existence of a boundary. Although this variable-rate formulation is a powerful tool for well-test analysis, it is by no means recommended that variable-rate tests be substituted for conventional single-rate tests.

```
OBSERVATION WELL NO.  1

    TIME       OBSERVED   CALCULATED  DIFFERENCE    RATIO


  .1300E+01  -.4620E+01  -.3737E+01   .8827E+00   .8089E+00
  .2900E+01  -.4720E+01  -.4412E+01   .3083E+00   .9347E+00
  .4600E+01  -.5110E+01  -.4932E+01   .1783E+00   .9651E+00
  .6600E+01  -.5350E+01  -.5172E+01   .1783E+00   .9667E+00
  .8700E+01  -.5160E+01  -.5419E+01  -.2586E+00   .1050E+01
  .1070E+02  -.6370E+01  -.6178E+01   .1919E+00   .9699E+00
  .1353E+02  -.6510E+01  -.5920E+01   .5903E+00   .9093E+00
  .1805E+02  -.6430E+01  -.5612E+01   .8182E+00   .8727E+00
  .2110E+02  -.5490E+01  -.4667E+01   .8231E+00   .8501E+00
  .2510E+02  -.6160E+01  -.5846E+01   .3144E+00   .9490E+00
  .2800E+02  -.5240E+01  -.5298E+01  -.5850E-01   .1011E+01
  .3100E+02  -.4830E+01  -.4801E+01   .2943E-01   .9939E+00
  .3700E+02  -.4330E+01  -.4148E+01   .1823E+00   .9579E+00
  .4940E+02  -.3980E+01  -.3999E+01  -.1864E-01   .1005E+01
  .5230E+02  -.4200E+01  -.3873E+01   .3268E+00   .9222E+00
  .7360E+02  -.3210E+01  -.3008E+01   .2025E+00   .9369E+00
  .7430E+02  -.2230E+01  -.2354E+01  -.1242E+00   .1056E+01
  .7540E+02  -.2010E+01  -.2115E+01  -.1045E+00   .1052E+01
  .7730E+02  -.1980E+01  -.1877E+01   .1030E+00   .9480E+00
  .8170E+02  -.1640E+01  -.1556E+01   .8373E-01   .9489E+00
  .9560E+02  -.1120E+01  -.1090E+01   .3000E-01   .9732E+00
  .1034E+03  -.9400E+00  -.9476E+00  -.7616E-02   .1008E+01
  .1068E+03  -.8900E+00  -.8979E+00  -.7947E-02   .1009E+01
  .1263E+03  -.5600E+00  -.6959E+00  -.1359E+00   .1243E+01
```

```
OBSERVATION WELL NO.  2

    TIME       OBSERVED   CALCULATED  DIFFERENCE    RATIO


  .1400E+01  -.3260E+01  -.3325E+01  -.6463E-01   .1020E+01
  .3000E+01  -.3760E+01  -.3942E+01  -.1818E+00   .1048E+01
  .4600E+01  -.4120E+01  -.4400E+01  -.2801E+00   .1068E+01
  .6600E+01  -.4400E+01  -.4649E+01  -.2493E+00   .1057E+01
  .8700E+01  -.4320E+01  -.4887E+01  -.5668E+00   .1131E+01
  .1070E+02  -.5280E+01  -.5576E+01  -.2964E+00   .1056E+01
  .1353E+02  -.5360E+01  -.5372E+01  -.1197E-01   .1002E+01
  .1805E+02  -.5400E+01  -.5170E+01   .2299E+00   .9574E+00
  .2110E+02  -.4350E+01  -.4332E+01   .1780E-01   .9959E+00
  .2510E+02  -.5130E+01  -.5377E+01  -.2474E+00   .1048E+01
  .2800E+02  -.4570E+01  -.4897E+01  -.3271E+00   .1072E+01
  .3100E+02  -.4260E+01  -.4461E+01  -.2013E+00   .1047E+01
  .3700E+02  -.3990E+01  -.3883E+01   .1073E+00   .9731E+00
  .4940E+02  -.3900E+01  -.3722E+01   .1782E+00   .9543E+00
  .5230E+02  -.3690E+01  -.3620E+01   .6962E-01   .9811E+00
  .5510E+02  -.3750E+01  -.3940E+01  -.1899E+00   .1051E+01
  .7200E+02  -.3320E+01  -.3156E+01   .1643E+00   .9505E+00
  .7360E+02  -.2960E+01  -.2914E+01   .4575E-01   .9845E+00
  .7430E+02  -.2240E+01  -.2296E+01  -.5608E-01   .1025E+01
  .7540E+02  -.2060E+01  -.2066E+01  -.6059E-02   .1003E+01
  .7730E+02  -.1840E+01  -.1842E+01  -.2319E-02   .1001E+01
  .8170E+02  -.1640E+01  -.1536E+01   .1038E+00   .9367E+00
  .9560E+02  -.1130E+01  -.1082E+01   .4802E-01   .9575E+00
  .1034E+03  -.9600E+00  -.9418E+00   .1818E-01   .9811E+00
  .1068E+03  -.9200E+00  -.8928E+00   .2719E-01   .9704E+00
```

```
OBSERVATION WELL NO.  3

    TIME       OBSERVED   CALCULATED  DIFFERENCE    RATIO


  .1300E+01  -.2930E+01  -.2919E+01   .1067E-01   .9964E+00
  .2800E+01  -.3440E+01  -.3554E+01  -.1138E+00   .1033E+01
  .4600E+01  -.3830E+01  -.4092E+01  -.2615E+00   .1068E+01
  .6600E+01  -.4110E+01  -.4376E+01  -.2657E+00   .1065E+01
  .8700E+01  -.4080E+01  -.4620E+01  -.5402E+00   .1132E+01
  .1070E+02  -.4950E+01  -.5266E+01  -.3163E+00   .1064E+01
  .1353E+02  -.5080E+01  -.5110E+01  -.3025E-01   .1006E+01
  .1805E+02  -.5140E+01  -.5004E+01   .1363E+00   .9735E+00
  .2110E+02  -.4040E+01  -.4226E+01  -.1864E+00   .1046E+01
  .2510E+02  -.4830E+01  -.5154E+01  -.3238E+00   .1067E+01
  .2800E+02  -.4370E+01  -.4737E+01  -.3673E+00   .1084E+01
  .3100E+02  -.4100E+01  -.4338E+01  -.2383E+00   .1058E+01
  .3700E+02  -.3820E+01  -.3792E+01   .2758E-01   .9928E+00
  .4940E+02  -.3650E+01  -.3586E+01   .6376E-01   .9825E+00
  .5230E+02  -.3560E+01  -.3517E+01   .4337E-01   .9878E+00
  .5510E+02  -.3620E+01  -.3800E+01  -.1802E+00   .1050E+01
  .7200E+02  -.3200E+01  -.3090E+01   .1100E+00   .9656E+00
  .7360E+02  -.2870E+01  -.2907E+01  -.3724E-01   .1013E+01
  .7430E+02  -.2250E+01  -.2328E+01  -.7808E-01   .1035E+01
  .7540E+02  -.2040E+01  -.2095E+01  -.5454E-01   .1027E+01
  .7730E+02  -.1870E+01  -.1863E+01   .6820E-02   .9964E+00
  .8170E+02  -.1660E+01  -.1548E+01   .1115E+00   .9328E+00
  .9560E+02  -.1130E+01  -.1087E+01   .4308E-01   .9619E+00
  .1034E+03  -.9600E+00  -.9454E+00   .1461E-01   .9848E+00
  .1068E+03  -.8900E+00  -.8960E+00  -.5977E-02   .1007E+01
```

```
OBSERVATION WELL NO.  4

    TIME       OBSERVED   CALCULATED  DIFFERENCE    RATIO


  .1500E+01  -.3030E+01  -.2751E+01   .2794E+00   .9078E+00
  .2900E+01  -.3490E+01  -.3307E+01   .1834E+00   .9474E+00
  .4600E+01  -.3810E+01  -.3804E+01   .6239E-02   .9984E+00
  .6600E+01  -.4110E+01  -.4101E+01   .8980E-02   .9978E+00
  .8700E+01  -.4050E+01  -.4345E+01  -.2946E+00   .1073E+01
  .1070E+02  -.4950E+01  -.4952E+01  -.2054E-02   .1000E+01
  .1353E+02  -.5130E+01  -.4830E+01   .2996E+00   .9416E+00
  .1805E+02  -.5050E+01  -.4789E+01   .2613E+00   .9482E+00
  .2110E+02  -.4040E+01  -.4070E+01  -.2996E-01   .1007E+01
  .2510E+02  -.4820E+01  -.4912E+01  -.9227E-01   .1019E+01
  .2800E+02  -.4350E+01  -.4541E+01  -.1910E+00   .1044E+01
  .3100E+02  -.4050E+01  -.4176E+01  -.1260E+00   .1031E+01
  .3700E+02  -.3770E+01  -.3667E+01   .1029E+00   .9727E+00
  .4940E+02  -.3690E+01  -.3444E+01   .2465E+00   .9332E+00
  .5230E+02  -.3510E+01  -.3392E+01   .1181E+00   .9664E+00
  .5510E+02  -.3530E+01  -.3647E+01  -.1174E+00   .1033E+01
  .7200E+01  -.3150E+01  -.4135E+01  -.9850E+00   .1313E+01
  .7360E+02  -.2870E+01  -.2854E+01   .1603E-01   .9944E+00
  .7430E+02  -.2200E+01  -.2315E+01  -.1149E+00   .1052E+01
  .7540E+02  -.2020E+01  -.2085E+01  -.6455E-01   .1032E+01
  .7730E+02  -.1800E+01  -.1856E+01  -.5626E-01   .1031E+01
  .8170E+02  -.1630E+01  -.1545E+01   .8547E-01   .9476E+00
  .9560E+02  -.1100E+01  -.1085E+01   .1465E-01   .9867E+00
  .1034E+03  -.9500E+00  -.9443E+00   .5742E-02   .9940E+00
  .1068E+03  -.8800E+00  -.8950E+00  -.1497E-01   .1017E+01
```

Figure 12. Output for Sample Problem 2. [XBL 818-11165]

SAMPLE PROBLEM 3: MULTIPLE-PRODUCTION-WELL INTERFERENCE TEST

The following data were obtained from a high-temperature, single-phase, liquid-dominated geothermal reservoir currently being used for electric power generation. The reservoir is a sedimentary deposit of sands, clays, and shales. The total reservoir thickness is not precisely known, but the central part of the reservoir is believed to be at least 1000 m thick. The wells tested range in depth from 2000 to 3000 m and have open intervals of 100 to 200 m. This interference test was conducted with four production wells that were being developed for the first time. For this reason, flow rates were gradually stepped up to the maximum rate, held constant for several days, and then shut down to a slow bleed. An accurate account of the production schedule was maintained for each well. The flow rate of each production well and the pressure response at the observation well are shown in Figure 13. The flow rates are modeled using the technique discussed in Sample Problem 2. The input deck for this problem is shown in Figure 14.

Figures 15 and 16 show the match of the calculated pressure drops and the real pressure drops. The fit, although not perfect, is acceptable. Separate analyses of both the drawdown data and the buildup data were done to determine if a reservoir boundary was influencing the pressure drops. Similar values of kh/μ and ϕch were obtained for both the drawdown and the buildup, indicating that no boundary was influencing the pressure response.

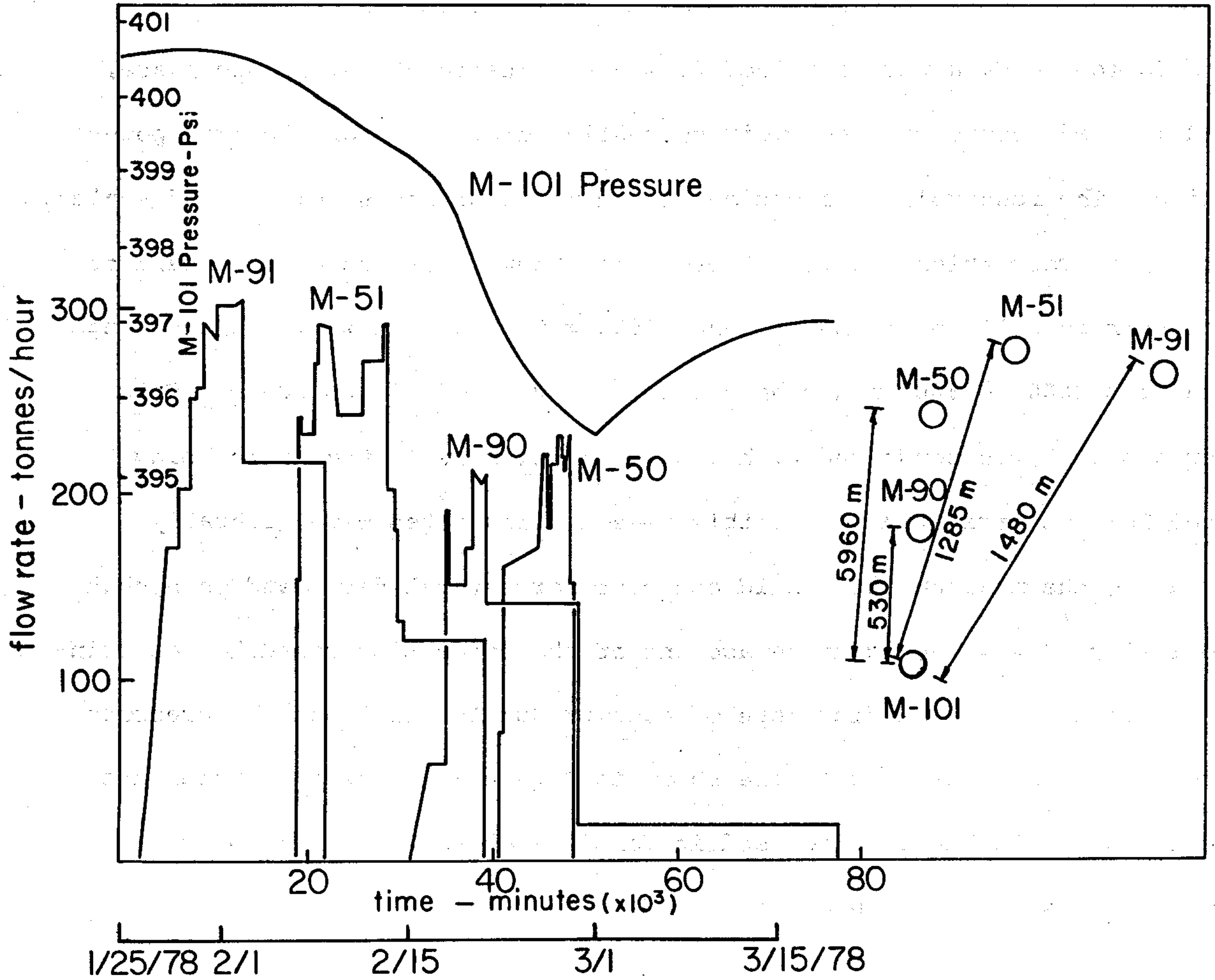


Figure 13. Flow rates and observation data for Sample Problem 3.

[XBL 792-7383]

```
SAMPLE  PROBLEM 3..FOUR PRODUCTION WELLS,VARIABLE RATE,SINGLE OBS
         2         1         4         0         0         0         0         1
6895.      6.31 E-05  60.       .3048     1.0 E-03  9.862 E-16
50000.    .002


0.         0.                   2220.     9727.                918.      35330.
0.                              2140.     10710.               1070.     35330.
-.29       15840.               2295.     10710.               1130.     35410.
-.49       18720.               2330.     12070.               1130.     35460.
-.62       21600.               1610.     12070.               1450.     35460.
-.89       24480.               2370.     12840.               1450.     35470.
-.98       25920.               2370.     13030.               1132.     35470.
-1.07      27360.               1645.     13030.               1132.     37490.
-1.31      30240.               1645.     22020.               1415.     37490.
-1.47      31680.               60.       22020.               1415.     38070.
-1.6       33120.                                              1530.     38070.
-1.82      34560.               -1350.    -4000.               1530.     38910.
-2.22      36000.               45.       0.                   1610.     39000.
-2.64      37440.               45.       19460.               1610.     39190.
-3.11      38880.               1148.     19460.               1570.     39200.
-3.54      40320.               1148.     19630.               1570.     39360.
-3.87      41760.               1836.     19680.               1610.     39710.
-4.12      43200.               1836.     19940.               1610.     39740.
-4.38      44640.               1760.     19940.               1070.     39470.
-4.6       46080.               1760.     20530.               1070.     49680.
-4.82      47520.               1760.     20870.               153.      49680.
-4.96      48960.               2066.     20870.
-5.11      50400.               2220.     20880.               0.        -3280.
-4.96      51840.               2220.     21410.               0.        0.
-4.78      53280.               2140.     21410.               0.        40780.
-4.69      54720.               2140.     23550.               540.      40780.
-4.60      56160.               2220.     23550.               540.      41190.
-4.42      57600.               2220.     23710.               1070.     41190.
-4.29      59040.               2140.     23710.               1224.     41560.
-4.16      60480.               2140.     23860.               1260.     44840.
-4.04      61920.               1836.     23860.               1450.     44840.
-3.91      63360.               1836.     26390.               1450.     45640.
-3.87      64800.               2070.     26390.               1683.     45650.
-3.8       66240.               2070.     28180.               1683.     45780.
-3.73      67680.               2220.     28180.               1450.     45780.
-3.64      69120.               2220.     28600.               1450.     45990.
-3.6       70560.               1530.     28600.               1640.     46000.
                                1530.     29260.               1640.     46630.
-3000.     -3750.               1380.     29260.               1607.     46630.
0.         0.                   1380.     29500.               1607.     46740.
0.         949.                 995.      29500.               1760.     46740.
1301.      5176.                995.      30270.               1760.     47030.
1301.      6340.                920.      30270.               1650.     47030.
1400.      6340.                920.      39600.               1650.     48120.
1400.      6595.                45.       39600.               1798.     48120.
1499.      6595.                                               1800.     48330.
1499.      6597.                110.      -1620.               1680.     48330.
1545.      6597.                40.       0.                   1680.     38350.
1545.      7760.                40.       31160.               1760.     48350.
1928.      7760.                420.      31160.               1760.     48440.
1928.      8025.                420.      35180.               1150.     48440.
2004.      8025.                765.      35180.               1150.     48590.
2004.      9100.                765.      35220.               40.       48590.
2220.      9100.                918.      35230.


                                                               EOR
```

Figure 14. Input deck for Sample Problem 3. [XBL 818-11166]

```
 OBSERVATION WELL NO.  1

      TIME      OBSERVED   CALCULATED  DIFFERENCE    RATIO


   .1584E+05   .2900E+00   .3020E+00   .1203E-01   .1041E+01
   .1872E+05   .4900E+00   .4547E+00  -.3529E-01   .9280E+00
   .2160E+05   .6200E+00   .6186E+00  -.1399E-02   .9977E+00
   .2448E+05   .8900E+00   .8026E+00  -.8745E-01   .9017E+00
   .2592E+05   .9800E+00   .9208E+00  -.5922E-01   .9396E+00
   .2736E+05   .1070E+01   .1054E+01  -.1604E-01   .9850E+00
   .3024E+05   .1310E+01   .1330E+01   .1996E-01   .1015E+01
   .3168E+05   .1470E+01   .1465E+01  -.5017E-02   .9966E+00
   .3312E+05   .1600E+01   .1680E+01   .8020E-01   .1050E+01
   .3456E+05   .1820E+01   .1934E+01   .1143E+00   .1063E+01
   .3600E+05   .2220E+01   .2172E+01  -.4756E-01   .9786E+00
   .3744E+05   .2640E+01   .2571E+01  -.6920E-01   .9738E+00
   .3888E+05   .3110E+01   .3037E+01  -.7305E-01   .9765E+00
   .4032E+05   .3540E+01   .3561E+01   .2103E-01   .1006E+01
   .4176E+05   .3870E+01   .3927E+01   .5717E-01   .1015E+01
   .4320E+05   .4120E+01   .4189E+01   .6882E-01   .1017E+01
   .4464E+05   .4380E+01   .4433E+01   .5324E-01   .1012E+01
   .4608E+05   .4600E+01   .4682E+01   .8240E-01   .1018E+01
   .4752E+05   .4820E+01   .4934E+01   .1139E+00   .1024E+01
   .4896E+05   .4960E+01   .5199E+01   .2387E+00   .1048E+01
   .5040E+05   .5110E+01   .5447E+01   .3374E+00   .1066E+01
   .5184E+05   .4960E+01   .5385E+01   .4253E+00   .1086E+01
   .5328E+05   .4780E+01   .5164E+01   .3837E+00   .1080E+01
   .5472E+05   .4690E+01   .4928E+01   .2382E+00   .1051E+01
   .5616E+05   .4600E+01   .4705E+01   .1049E+00   .1023E+01
   .5760E+05   .4420E+01   .4499E+01   .7863E-01   .1018E+01
   .5904E+05   .4290E+01   .4310E+01   .1959E-01   .1005E+01
   .6048E+05   .4160E+01   .4136E+01  -.2350E-01   .9944E+00
   .6192E+05   .4040E+01   .3978E+01  -.6224E-01   .9846E+00
   .6336E+05   .3910E+01   .3832E+01  -.7820E-01   .9800E+00
   .6480E+05   .3870E+01   .3697E+01  -.1728E+00   .9553E+00
   .6624E+05   .3800E+01   .3573E+01  -.2274E+00   .9402E+00
   .6768E+05   .3730E+01   .3457E+01  -.2730E+00   .9268E+00
   .6912E+05   .3640E+01   .3349E+01  -.2907E+00   .9201E+00
   .7056E+05   .3600E+01   .3249E+01  -.3512E+00   .9024E+00


 SAMPLE  PROBLEM 3..FOUR PRODUCTION WELLS,VARIABLE RATE,SINGLE OBS

FINAL VALUES OF PARAMETERS

KHU =   .1284E+07  PERM.-LENGTH/VISC. UNITS

PCH =   .2273E-01  LENGTH/PRESSURE    UNITS


NORMALIZED CHI-SQUARED =   .23373E-02

FOR  35 DATA POINTS OUT OF  35 INPUT VALUES
```

Figure 15. Output for Sample Problem 3.   [XBL 818-11167]

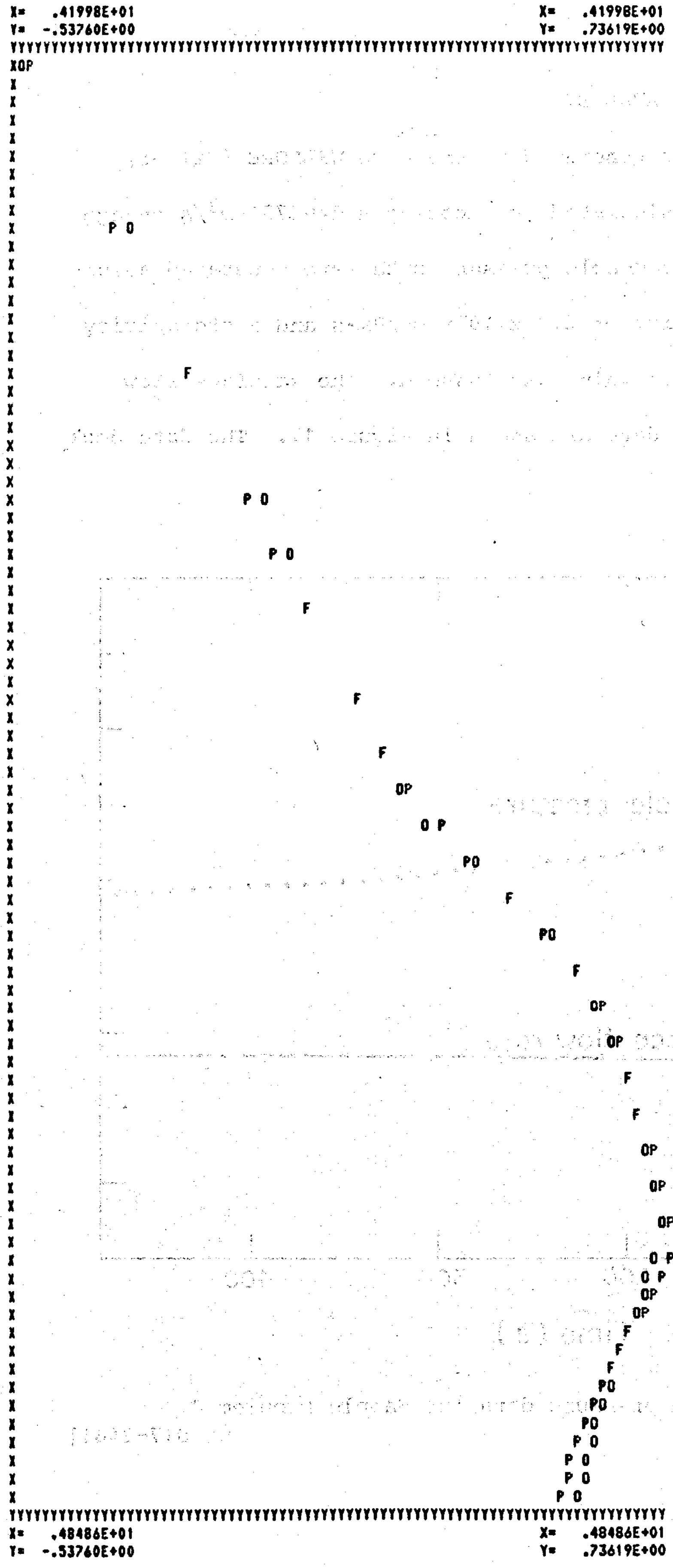


Figure 16. Graphed output for Sample Problem 3. [XBL 818-11161]

SAMPLE PROBLEM 4: PRODUCTION-WELL ANALYSIS

The data for this problem were generated by the code WELBORE (Miller, 1980). Sandface flow rates were calculated by imposing a 0.02752-m^3/s change in flow rate at the wellhead. Bottom-hole pressure data were generated assuming a reservoir with a transmissivity of 2.3×10^{-8} m^3/Pa-s and a storativity ϕch of 6.0×10^{-7} m/Pa. A zero skin value was assumed. The sandface flow rates and the bottom-hole pressure data are shown in Figure 17. The data deck is shown in Figure 18.

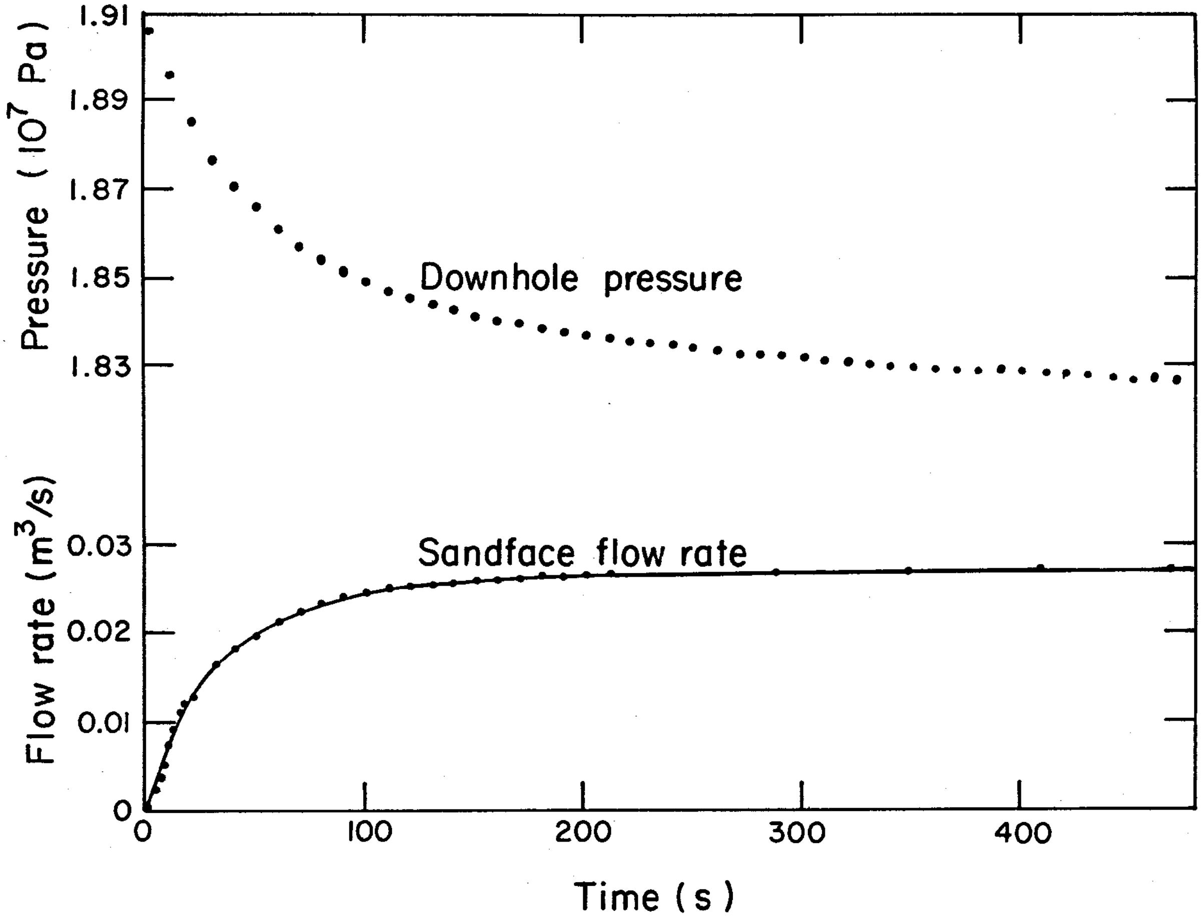


Figure 17. Sandface flow rate and pressure data for Sample Problem 4.
[XBL 817-2441]

```
SAMPLE PROBLEM 4  ANALYSIS OF SYNTHETIC PRODUCTION WELL DATA
         2         1         1         0         0         0         0
1.0   E-08 1.0  E-07                                800.

0.        .09
.19066E+08
.18968E+08.25000E+03
.18854E+08.26000E+03
.18776E+08.27000E+03
.18713E+08.28000E+03
.18660E+08.29000E+03
.18615E+08.30000E+03
.18577E+08.31000E+03
.18544E+08.32000E+03
.18517E+08.33000E+03
.18493E+08.34000E+03
.18473E+08.35000E+03
.18456E+08.36000E+03
.18441E+08.37000E+03
.18428E+08.38000E+03
.18416E+08.39000E+03
.18405E+08.40000E+03
.18396E+08.41000E+03
.18387E+08.42000E+03
.18379E+08.43000E+03
.18372E+08.44000E+03
.18365E+08.45000E+03
.18359E+08.46000E+03
.18353E+08.47000E+03
.18347E+08.48000E+03
.18342E+08.49000E+03
.18337E+08.50000E+03
.18332E+08.51000E+03
.18328E+08.52000E+03
.18324E+08.53000E+03
.18320E+08.54000E+03
.18316E+08.55000E+03
.18312E+08.56000E+03
.18309E+08.57000E+03
.18305E+08.58000E+03
.18302E+08.59000E+03
.18299E+08.60000E+03
.18296E+08.61000E+03
.18292E+08.62000E+03
.18290E+08.63000E+03
.18287E+08.64000E+03
.18284E+08.65000E+03
.18281E+08.66000E+03
.18279E+08.67000E+03
.18276E+08.68000E+03
.18273E+08.69000E+03
.18271E+08.70000E+03
.18269E+08.71000E+03
.18266E+08.72000E+03
.18264E+08.73000E+03
.18262E+08.74000E+03
.18260E+08.75000E+03
.18258E+08.76000E+03
.18256E+08.77000E+03
.18254E+08.78000E+03
.18251E+08.79000E+03
.18249E+08.80000E+03
.18247E+08.81000E+03
.18245E+08.82000E+03
.18243E+08.83000E+03
.18241E+08.84000E+03
.18240E+08.85000E+03
.18238E+08.86000E+03
.18237E+08.87000E+03
.18235E+08.88000E+03
.18233E+08.89000E+03
.18232E+08.90000E+03
.18230E+08.91000E+03
.18229E+08.92000E+03
.18227E+08.93000E+03
.18226E+08.94000E+03
.18224E+08.95000E+03
.18223E+08.96000E+03
.18221E+08.97000E+03
.18220E+08.98000E+03
.18219E+08.99000E+03
.18217E+08.10000E+04
.18210E+08.10600E+04
.18202E+08.11200E+04
.18196E+08.11800E+04
.18192E+08.12160E+04
.18192E+08.12180E+04
.18190E+08.12400E+04
.18184E+08.13000E+04
.18178E+08.13600E+04
.18173E+08.14200E+04
.18168E+08.14800E+04
.18163E+08.15400E+04
.18159E+08.16000E+04
.18154E+08.16600E+04
.18150E+08.17200E+04
.18146E+08.17800E+04
.18143E+08.18400E+04
.18139E+08.19000E+04
.18135E+08.19600E+04
.18132E+08.20200E+04
.18129E+08.20800E+04
.18126E+08.21400E+04
.18126E+08.21420E+04

0.        0.
.0        .24000E+03
.61500E-03.24200E+03
.22920E-02.24400E+03
.36110E-02.24600E+03
.58860E-02.24800E+03
.77580E-02.25000E+03
.92870E-02.25200E+03
.10519E-01.25400E+03
.11517E-01.25600E+03
.12246E-01.25800E+03
.12908E-01.26000E+03
.16356E-01.27200E+03
.18058E-01.28000E+03
.19905E-01.29000E+03
.21378E-01.30000E+03
.22508E-01.31000E+03
.23432E-01.32000E+03
.24148E-01.33000E+03
.24717E-01.34000E+03
.25104E-01.35000E+03
.25430E-01.36000E+03
.25695E-01.37000E+03
.25918E-01.38000E+03
.26107E-01.39000E+03
.26262E-01.40000E+03
.26372E-01.41000E+03
.26463E-01.42000E+03
.26539E-01.43000E+03
.26606E-01.44000E+03
.26670E-01.45000E+03
.26954E-01.52600E+03
.27063E-01.58600E+03
.27148E-01.64600E+03
.27211E-01.70600E+03
.27252E-01.76600E+03
.27340E-01.82600E+03
.27334E-01.88600E+03
.27348E-01.94600E+03
.27367E-01.10060E+04
.27376E-01.10660E+04
.27388E-01.11260E+04
.27401E-01.11860E+04
.27413E-01.12460E+04
.27423E-01.13060E+04
.27432E-01.13660E+04
.27439E-01.14260E+04
.27447E-01.14860E+04
.27453E-01.15460E+04
.27459E-01.16060E+04
.27464E-01.16660E+04
.27469E-01.17260E+04
.27474E-01.17860E+04
.27478E-01.18460E+04
.27482E-01.19060E+04
.27486E-01.19660E+04
.27489E-01.20260E+04
.27492E-01.20860E+04
.27495E-01.21460E+04
.27495E-01.22060E+04
.27517E-01.22660E+04
.27534E-01.23260E+04
.27520E-01.23860E+04

EOR
```

Figure 18. Data deck for Sample Problem 4. [XBL 818-11168]

The problem is presented to show that if a variable-rate analysis technique is used, pressure data influenced by wellbore effects can be accurately analyzed. It is not necessary to wait for 1 1/2 log cycles after wellbore storage is over to get an accurate analysis. If a variable-rate analysis is made, data from the transition period can be used for the pressure-transient analysis.

In this problem a wellhead change in flow rate of 0.02752 m^3/s was initiated at 240 s. Pressure points at 10-s intervals were used for the analysis. To input the sandface flow rate accurately, data points are used every 2 s when the sandface rate is changing rapidly and less frequently ($\pm$ 1 min) when it has been nearly stabilized.

Making use of the ERLYFT option, the data were analyzed using increasingly fewer pressure points. It was found that the same minimum was returned using as few as ten pressure points, or 100 s of pressure-transient data. Matches were made using several different initial guesses for the reservoir parameters. It should be noted that as the initial guesses begin to differ from the true values by more than one order of magnitude, computing times rise rapidly. Thus an attempt should be made to obtain reasonably good initial guesses for the reservoir parameters.

Output data from two runs are shown in Figures 19 to 22. In the first run (see Figure 19), 560 s of pressure data and the the sandface flow rate are used. The calculated values of the reservoir parameters are

$$kh/\mu = 2.37 \times 10^{-8} \ m^3/Pa\text{-}s \text{ and } \phi ch = 4.3 \times 10^{-7} \ m/Pa.$$

The discrepancy between the storativity used to generate the problem and the calculated storativity is due to the finite-difference implementation of the wellbore-reservoir connection in the program WELBORE. For comparison a similar run was made in which the wellhead flow rate was used instead of the sandface flow rate. As seen in Figures 20 and 21, the match is not satisfactory and the reservoir parameters returned ($kh/\mu = 1.0 \times 10^{-8}$ and $\phi ch = 2.6 \times 10^{-5}$) are not correct.

For this particular example, a zero skin value was used. If a nonzero skin value had been used, the value of LOBS would be set equal to 1. For a three-parameter analysis, (kh/μ, ϕch, and skin), the value of the storativity and the skin are coupled in such a way as to make them nonunique. If it is desired to obtain a value of the skin, it is necessary to obtain an accurate value of ϕch from a previous interference production test. If an accurate value of ϕch is available, an analysis including skin effects may be made by holding the parameter ϕch constant during the fitting procedure. This is achieved by setting H(2,2) = 0 as discussed in the section entitled Input Data (p. 17).

```
    TIME      OBSERVED   CALCULATED  DIFFERENCE    RATIO


 .2500E+03   .9800E+05   .1004E+06   .2382E+04   .1024E+01
 .2600E+03   .2120E+06   .2132E+06   .1231E+04   .1006E+01
 .2700E+03   .2900E+06   .2869E+06  -.3103E+04   .9893E+00
 .2800E+03   .3530E+06   .3490E+06  -.3987E+04   .9887E+00
 .2900E+03   .4060E+06   .4015E+06  -.4426E+04   .9891E+00
 .3000E+03   .4510E+06   .4468E+06  -.4224E+04   .9906E+00
 .3100E+03   .4890E+06   .4849E+06  -.4111E+04   .9916E+00
 .3200E+03   .5220E+06   .5177E+06  -.4269E+04   .9918E+00
 .3300E+03   .5490E+06   .5458E+06  -.3249E+04   .9941E+00
 .3400E+03   .5730E+06   .5698E+06  -.3156E+04   .9945E+00
 .3500E+03   .5930E+06   .5898E+06  -.3197E+04   .9946E+00
 .3600E+03   .6100E+06   .6072E+06  -.2819E+04   .9954E+00
 .3700E+03   .6250E+06   .6225E+06  -.2514E+04   .9960E+00
 .3800E+03   .6380E+06   .6361E+06  -.1855E+04   .9971E+00
 .3900E+03   .6500E+06   .6484E+06  -.1554E+04   .9976E+00
 .4000E+03   .6610E+06   .6595E+06  -.1485E+04   .9978E+00
 .4100E+03   .6700E+06   .6693E+06  -.7244E+03   .9989E+00
 .4200E+03   .6790E+06   .6781E+06  -.9202E+03   .9986E+00
 .4300E+03   .6870E+06   .6861E+06  -.8833E+03   .9987E+00
 .4400E+03   .6940E+06   .6935E+06  -.4548E+03   .9993E+00
 .4500E+03   .7010E+06   .7005E+06  -.4790E+03   .9993E+00
 .4600E+03   .7070E+06   .7068E+06  -.2048E+03   .9997E+00
 .4700E+03   .7130E+06   .7127E+06  -.3352E+03   .9995E+00
 .4800E+03   .7190E+06   .7182E+06  -.7573E+03   .9989E+00
 .4900E+03   .7240E+06   .7236E+06  -.4256E+03   .9994E+00
 .5000E+03   .7290E+06   .7287E+06  -.3089E+03   .9996E+00
 .5100E+03   .7340E+06   .7336E+06  -.3828E+03   .9995E+00
 .5200E+03   .7380E+06   .7384E+06   .3721E+03   .1001E+01
 .5300E+03   .7420E+06   .7429E+06   .8908E+03   .1001E+01
 .5400E+03   .7460E+06   .7470E+06   .1037E+04   .1001E+01
 .5500E+03   .7500E+06   .7510E+06   .1003E+04   .1001E+01
 .5600E+03   .7540E+06   .7548E+06   .8225E+03   .1001E+01
 .5700E+03   .7570E+06   .7585E+06   .1514E+04   .1002E+01
 .5800E+03   .7610E+06   .7621E+06   .1088E+04   .1001E+01
 .5900E+03   .7640E+06   .7655E+06   .1540E+04   .1002E+01
 .6000E+03   .7670E+06   .7688E+06   .1845E+04   .1002E+01
 .6100E+03   .7700E+06   .7720E+06   .2049E+04   .1003E+01
 .6200E+03   .7740E+06   .7752E+06   .1162E+04   .1002E+01
 .6300E+03   .7760E+06   .7782E+06   .2192E+04   .1003E+01
 .6400E+03   .7790E+06   .7811E+06   .2146E+04   .1003E+01
 .6500E+03   .7820E+06   .7840E+06   .2011E+04   .1003E+01
 .6600E+03   .7850E+06   .7868E+06   .1765E+04   .1002E+01
 .6700E+03   .7870E+06   .7894E+06   .2445E+04   .1003E+01
 .6800E+03   .7900E+06   .7921E+06   .2059E+04   .1003E+01
 .6900E+03   .7930E+06   .7946E+06   .1614E+04   .1002E+01
 .7000E+03   .7950E+06   .7971E+06   .2111E+04   .1003E+01
 .7100E+03   .7970E+06   .7995E+06   .2540E+04   .1003E+01
 .7200E+03   .8000E+06   .8019E+06   .1873E+04   .1002E+01
 .7300E+03   .8020E+06   .8041E+06   .2149E+04   .1003E+01
 .7400E+03   .8040E+06   .8064E+06   .2375E+04   .1003E+01
 .7500E+03   .8060E+06   .8086E+06   .2553E+04   .1003E+01
 .7600E+03   .8080E+06   .8107E+06   .2687E+04   .1003E+01
 .7700E+03   .8100E+06   .8128E+06   .2812E+04   .1003E+01
 .7800E+03   .8120E+06   .8150E+06   .2992E+04   .1004E+01
 .7900E+03   .8150E+06   .8172E+06   .2154E+04   .1003E+01
 .8000E+03   .8170E+06   .8193E+06   .2292E+04   .1003E+01
```

```
 SAMPLE PROBLEM 4  ANALYSIS OF SYNTHETIC PRODUCTION WELL DATA


    1 OBSERVATION WELLS                 1 PRODUCTION WELLS

LATEST OBSERVATION TIME USED IS   .80000E+03 TIME UNITS

GREATEST OBSERVATION TIME DATA IS   .21420E+04 TIME UNITS


INITIAL GUESSES OF PARAMETERS

KHU =     .1000E-07  PRM-LGTH/VISC    PCH =    .1000E-06  LGTH/PRESS


FINAL VALUES OF PARAMETERS

KHU =   .2375E-07  PERM.-LENGTH/VISC. UNITS

PCH =   .4329E-06  LENGTH/PRESSURE    UNITS


NORMALIZED CHI-SQUARED =   .27669E-04

FOR  56 DATA POINTS OUT OF  98 INPUT VALUES
```

Figure 19. Output for Sample Problem 4 using the sandface flow rate. [XBL 818-11170]

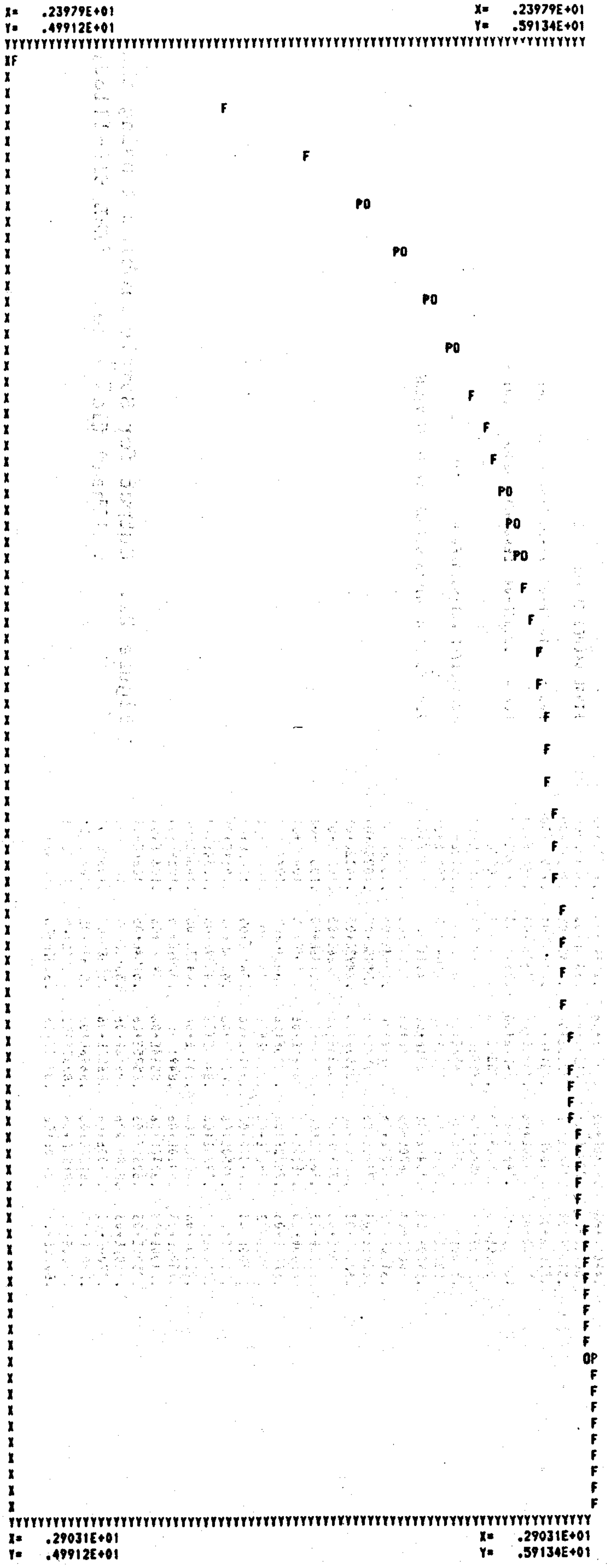


Figure 20. Graphed output for Sample Problem 4 using the sandface flow rate. [XBL 818-11162]

```
OBSERVATION WELL NO.  1

    TIME      OBSERVED   CALCULATED  DIFFERENCE    RATIO


  .2500E+03   .9800E+05   .1141E+06   .1609E+05   .1164E+01
  .2600E+03   .2120E+06   .2160E+06   .3979E+04   .1019E+01
  .2700E+03   .2900E+06   .2859E+06  -.4121E+04   .9858E+00
  .2800E+03   .3530E+06   .3387E+06  -.1430E+05   .9595E+00
  .2900E+03   .4060E+06   .3811E+06  -.2490E+05   .9387E+00
  .3000E+03   .4510E+06   .4165E+06  -.3450E+05   .9235E+00
  .3100E+03   .4890E+06   .4469E+06  -.4213E+05   .9138E+00
  .3200E+03   .5220E+06   .4735E+06  -.4853E+05   .9070E+00
  .3300E+03   .5490E+06   .4971E+06  -.5188E+05   .9055E+00
  .3400E+03   .5730E+06   .5184E+06  -.5458E+05   .9047E+00
  .3500E+03   .5930E+06   .5378E+06  -.5522E+05   .9069E+00
  .3600E+03   .6100E+06   .5555E+06  -.5446E+05   .9107E+00
  .3700E+03   .6250E+06   .5719E+06  -.5307E+05   .9151E+00
  .3800E+03   .6380E+06   .5872E+06  -.5084E+05   .9203E+00
  .3900E+03   .6500E+06   .6014E+06  -.4863E+05   .9252E+00
  .4000E+03   .6610E+06   .6147E+06  -.4630E+05   .9300E+00
  .4100E+03   .6700E+06   .6272E+06  -.4276E+05   .9362E+00
  .4200E+03   .6790E+06   .6391E+06  -.3991E+05   .9412E+00
  .4300E+03   .6870E+06   .6503E+06  -.3668E+05   .9466E+00
  .4400E+03   .6940E+06   .6610E+06  -.3302E+05   .9524E+00
  .4500E+03   .7010E+06   .6711E+06  -.2986E+05   .9574E+00
  .4600E+03   .7070E+06   .6808E+06  -.2616E+05   .9630E+00
  .4700E+03   .7130E+06   .6901E+06  -.2289E+05   .9679E+00
  .4800E+03   .7190E+06   .6990E+06  -.2000E+05   .9722E+00
  .4900E+03   .7240E+06   .7075E+06  -.1646E+05   .9773E+00
  .5000E+03   .7290E+06   .7157E+06  -.1326E+05   .9818E+00
  .5100E+03   .7340E+06   .7236E+06  -.1035E+05   .9859E+00
  .5200E+03   .7380E+06   .7313E+06  -.6732E+04   .9909E+00
  .5300E+03   .7420E+06   .7386E+06  -.3373E+04   .9955E+00
  .5400E+03   .7460E+06   .7457E+06  -.2602E+03   .9997E+00
  .5500E+03   .7500E+06   .7526E+06   .2624E+04   .1003E+01
  .5600E+03   .7540E+06   .7593E+06   .5293E+04   .1007E+01
  .5700E+03   .7570E+06   .7658E+06   .8760E+04   .1012E+01
  .5800E+03   .7610E+06   .7720E+06   .1104E+05   .1015E+01
  .5900E+03   .7640E+06   .7781E+06   .1413E+05   .1019E+01
  .6000E+03   .7670E+06   .7841E+06   .1706E+05   .1022E+01
  .6100E+03   .7700E+06   .7898E+06   .1983E+05   .1026E+01
  .6200E+03   .7740E+06   .7954E+06   .2145E+05   .1028E+01
  .6300E+03   .7760E+06   .8009E+06   .2492E+05   .1032E+01
  .6400E+03   .7790E+06   .8063E+06   .2726E+05   .1035E+01
  .6500E+03   .7820E+06   .8115E+06   .2946E+05   .1038E+01
  .6600E+03   .7850E+06   .8165E+06   .3155E+05   .1040E+01
  .6700E+03   .7870E+06   .8215E+06   .3451E+05   .1044E+01
  .6800E+03   .7900E+06   .8264E+06   .3636E+05   .1046E+01
  .6900E+03   .7930E+06   .8311E+06   .3810E+05   .1048E+01
  .7000E+03   .7950E+06   .8357E+06   .4074E+05   .1051E+01
  .7100E+03   .7970E+06   .8403E+06   .4329E+05   .1054E+01
  .7200E+03   .8000E+06   .8447E+06   .4473E+05   .1056E+01
  .7300E+03   .8020E+06   .8491E+06   .4709E+05   .1059E+01
  .7400E+03   .8040E+06   .8534E+06   .4936E+05   .1061E+01
  .7500E+03   .8060E+06   .8575E+06   .5154E+05   .1064E+01
  .7600E+03   .8080E+06   .8616E+06   .5365E+05   .1066E+01
  .7700E+03   .8100E+06   .8657E+06   .5568E+05   .1069E+01
  .7800E+03   .8120E+06   .8696E+06   .5763E+05   .1071E+01
  .7900E+03   .8150E+06   .8735E+06   .5851E+05   .1072E+01
  .8000E+03   .8170E+06   .8773E+06   .6032E+05   .1074E+01
```

```
 SAMPLE PROBLEM 4  ANALYSIS OF SYNTHETIC PRODUCTION WELL DATA (SURFACE FLOW RATE)


    1 OBSERVATION WELLS                1 PRODUCTION WELLS

LATEST OBSERVATION TIME USED IS   .80000E+03 TIME UNITS

GREATEST OBSERVATION TIME DATA IS   .21420E+04 TIME UNITS


INITIAL GUESSES OF PARAMETERS

KHU =    .1000E-07  PRM-LGTH/VISC    PCH =    .1000E-06  LGTH/PRESS


FINAL VALUES OF PARAMETERS

KHU =   .1022E-07  PERM.-LENGTH/VISC. UNITS

PCH =   .2633E-04  LENGTH/PRESSURE    UNITS


NORMALIZED CHI-SQUARED =   .34183E-02

FOR  56 DATA POINTS OUT OF  98 INPUT VALUES
```

Figure 21. Output for Sample Problem 4 using the wellhead flow rate. [XBL 818-11163]

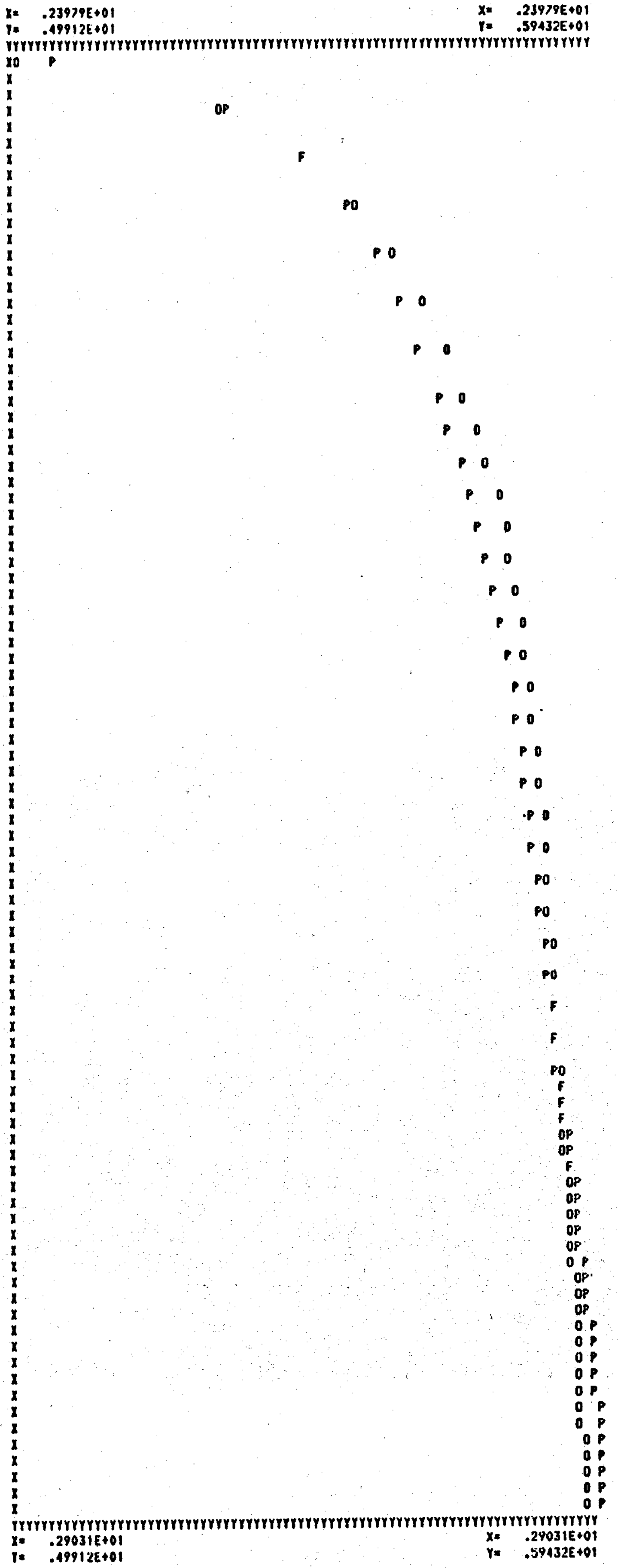


Figure 22. Graphed output for Sample Problem 4 using the wellhead flow rate. [XBL 818-11169]

APPENDIX A

PRESSURE RESPONSE OF A LINEARLY VARYING PRODUCTION PULSE

The pressure change around a line source of instantaneous strength q^* is given by

$$\Delta P(r,t) = \frac{q^*}{4\pi\eta t} \exp\left[\frac{-r^2}{4\eta t}\right], \tag{A-1}$$

where $\eta = k/\phi\mu c$ is the hydraulic diffusivity and the strength q^* is defined as the drawdown per unit length line source, caused by the instantaneous release of a given amount of fluid per unit length line source. With q as the amount of fluid released instantaneously per unit length line source, we may write

$$q' = q^*\phi c \quad \text{or} \quad q^* = \frac{q'}{\phi c} . \tag{A-2}$$

Further, with h as the length of the line source (thickness of the aquifer with a fully penetrating well) the total amount of fluid released instantaneously by the line source is

$$q = q'h = q^*\phi ch \quad \text{or} \quad q^* = \frac{q}{\phi ch} . \tag{A-3}$$

Substituting (A-3) into (A-2) and noting that $\eta = kh/\mu\phi ch$, we obtain the pressure change caused at any time and at any radial distance by the instantaneous release of a volume of fluid q from the line source

$$\Delta P(r,t) = \frac{q}{4\pi kh} \frac{\exp(-r^2/4\eta t)}{t} . \tag{A-4}$$

We may think of a time-varying flow rate as a sequence of instantaneous Q's whose magnitudes correspond in time to the flow-rate value. With $q_k(\tau)$ representing the flow rate from time τ_k to τ_{k+1}, we integrate the instantaneous response (A-4) in time to get

$$\Delta P_k(r,t) = \frac{\mu}{4\pi kh} \int_{\tau_k}^{\tau_{k+1}} q_k(\tau) \frac{\exp\left[\dfrac{-r^2}{4\eta(t-\tau)}\right]}{t-\tau} \, d\tau, \tag{A-5}$$

which is the pressure response due to a line source active from τ_k to τ_{k+1} with flow rate $q_k(\tau)$.

To handle the variable flow rate q(t), we assume that any production-rate history can be represented by a sequence of straight line segments, each of necessary length and inclination (Figure 2). We prescribe q(t) to vary linearly within the interval τ_k to τ_{k+1} as

$$q_k(\tau) = A_k + B_k(\tau - \tau_k). \tag{A-6}$$

In light of (A-6), (A-5) becomes:

$$\Delta P_k(r,t) = \frac{\mu}{4\pi kh} \int_{\tau_k}^{\tau_{k+1}} \frac{\left[A_k + B_k(\tau - \tau_k)\right] \exp\left[\dfrac{r^2}{4\eta(t-\tau)}\right]}{t-\tau} \, d\tau, \tag{A-7}$$

which may be written as:

$$\Delta P_k(r,t) = \frac{\mu}{4\pi kh} \int_{\tau_k}^{t} \frac{\left[A_k + B_k(\tau - \tau_k)\right] \exp\left[\dfrac{r^2}{4\eta(t-\tau)}\right]}{t-\tau} \, d\tau$$

$$- \frac{\mu}{4\pi kh} \int_{\tau_{k+1}}^{t} \frac{\left[A_k + B_k(\tau - \tau_k)\right] \exp\left[\dfrac{-r^2}{4\eta(t-\tau)}\right]}{t-\tau} \, dt. \tag{A-8}$$

Because the two integrals in (A-8) are identical except for sign and the lower limit of integration, we operate only on the first integral and then, by exact analog, extend all analytical results to the second integral.

We first note that by adding and subtracting $B_k(t - \tau)$, the flow rate $q_k(t)$ may be written as

$$A_k + B_k\,(\tau - \tau_k) = A_k + B_k\,(t - \tau_k) - B_k\,(t - \tau). \tag{A-9}$$

Substituting (A-9) into the first integral of (A-8) and simplifying, we obtain

$$\int_{\tau_k}^{t} [A_k + B_k(t - \tau_k) - B_k(t - \tau)]\,\frac{\exp\left[\dfrac{-r^2}{4\eta(t - \tau)}\right]}{t - \tau}\,d\tau$$

$$= [A_k + B_k(t - \tau_k)] \int_{\tau_k}^{t} \frac{\exp\left[\dfrac{-r^2}{4\eta(t - \tau)}\right]}{t - \tau}\,d\tau$$

$$- B_k \int_{\tau_k}^{t} \exp\left[\frac{-r^2}{4\eta(t - \tau)}\right] d\tau. \tag{A-10}$$

To evaluate the integrals in (A-10) we let

$$u = \frac{r^2}{4\eta(t - \tau)}, \tag{A-11}$$

from which

$$du = \frac{r^2}{4\eta(t - \tau)^2}\,d\tau \quad \text{and} \quad \frac{d\tau}{t - \tau} = \frac{1}{u}\,du.$$

With $u_k = r^2/4\eta(t - \tau_k)$, the first integral in (A-10) becomes

$$\int_{\tau_k}^{t} \frac{\exp\left[\dfrac{-r^2}{4\eta(t - \tau)}\right]}{t - \tau}\,d\tau = \int_{u_k}^{\infty} \frac{\exp\,[-u]}{u}\,du = W(u_k), \tag{A-12}$$

where $W(u_k)$ is the well function (Theis, 1935) of the argument u_k.

Noting that $d\tau = r^2/(4\eta u^2)$, the second integral in (A-10) becomes

$$\int_{\tau_k}^{t} \exp\left[\frac{-r^2}{4\eta(t-\tau)}\right] d\tau = \frac{r^2}{4\eta}\int_{u_k}^{\infty} \frac{\exp[-u]}{u^2}\,du. \qquad \text{(A-13)}$$

It may be shown by integration by parts that

$$\frac{\exp[-u]}{u^2}\,du = \frac{-\exp[-u]}{u} - \int \frac{\exp[-u]}{u}\,du. \qquad \text{(A-14)}$$

Using (A-14) to expand (A-13), the second integral in (A-10) becomes

$$\int_{\tau_k}^{t} \exp\left[\frac{-r^2}{4\eta(t-\tau)}\right] d\tau = \left\{\frac{r^2}{4\eta}\;\frac{\exp[-u_k]}{u_k} - \int_{u_k}^{\infty} \frac{\exp(-u)}{u}\,du\right\}$$

$$= (t - \tau_k)\exp[-u_k] - \frac{r^2}{4\eta}W(u_k). \qquad \text{(A-15)}$$

Substituting (A-12) and (A-15) into (A-10), we obtain for the first integral in (A-8)

$$\int_{\tau_k}^{t} [A_k + B_k(t-\tau_k)]\,\frac{\exp\left[\frac{-r^2}{4\eta(t-\tau)}\right]}{t-\tau}\,d\tau$$

$$= [A_k + B_k(t-\tau_k)]W(u_k) - B_k[(t-\tau_k)\exp[-u_k] - \frac{r^2}{4\eta}W(u_k)]$$

$$= \left[A_k + B_k\left(\frac{r^2}{4\eta} + t - \tau_k\right)\right]W(u_k) - B_k(t-\tau_k)\exp[-u_k]. \qquad \text{(A-16)}$$

By analogy with the preceding development and with $u_{k+1} = r^2/4\eta(t - \tau_{k+1})$, we may write the second integral in (A-8) as

$$\int_{\tau_{k+1}}^{t} \frac{[A_k + B_k(t - \tau)] \exp\left(\frac{-r^2}{4\eta(t - \tau)}\right)}{t - \tau} d\tau = \left[A_k + B_k\left(\frac{r^2}{4\eta} + t - \tau_k\right)\right] W(u_{k+1})$$

$$- B_k(t - \tau_{k+1}) \exp [-u_{k+1}]. \qquad \text{(A-17)}$$

Finally, substituting (A-16) and (A-17) into (A-8), we obtain

$$\Delta P(k) = \frac{\mu}{4\pi kh}\Bigg\{\left[A_k + B_k\left(\frac{r^2}{4\eta} + t - \tau_k\right)\right] [W(u)_k - W(u_{k+1})]$$

$$- B_k[(t - \tau_k) \exp (-u_k) - (t - \tau_{k+1}) \exp (-u_{k+1})]\Bigg\}, \qquad \text{(A-18)}$$

which becomes identical to Equation 5 upon making the substitution

$$(t - \tau_k)(1 + u_k) = \frac{r^2}{4\eta} + t - \tau_k.$$

For the assumption of constant flow rate, $A_k - q$, $B_k = 0$, $\tau_k = 0$, and $\tau_{k+1} = t$, Equation (A-18) becomes

$$\Delta P = \frac{\mu}{4\pi kh} [qW(u)] = \frac{q\mu}{4\pi kh} \int_{u}^{\infty} \frac{\exp (-y)}{y} dy, \qquad \text{(A-19)}$$

which is the conventional constant-strength, line-source solution.

APPENDIX B

CALCULATION OF IMAGE-WELL DISTANCES

The location of a linear, vertical hydrologic boundary is calculated in terms of an azimuthal angle α and a distance d. The angle is measured clockwise from the y axis around the origin of the Cartesian coordinate system that defines the well locations. The distance d is measured along the direction from the origin and represents the horizontal perpendicular distance to a vertical boundary. Image-well distances are calculated using the locations, in a second coordinate system, of the real wells and the image wells. Figure 23 depicts this arrangement.

For a given value of α, a second Cartesian coordinate system is defined whose ordinate axis y' is parallel to the linear boundary. The transformation equations are

$$x' = x\sin\alpha + y\cos\alpha,$$

$$y' = y\sin\alpha - x\cos\alpha. \qquad \text{(B-1)}$$

The image-well distance corresponding to observation well h and production well j is given in terms of the transformed coordinates of these two wells. With PX(j) and PY(j) denoting the x and y coordinates of production well j and OX(h) and OY(h) denoting the x and y coordinates of the observation well, their coordinates in the transformed coordinate system are

$$RPX(j) = PX(j)\sin\alpha + PY(j)\cos\alpha,$$

$$RPY(j) = PY(j)\sin\alpha - PX(j)\cos\alpha, \qquad \text{(B-1a)}$$

and

$$ROX(h) = OX(h)\sin\alpha + OY(h)\cos\alpha,$$

$$ROY(h) = OY(h)\sin\alpha - OX(h)\cos\alpha. \qquad \text{(B-1b)}$$

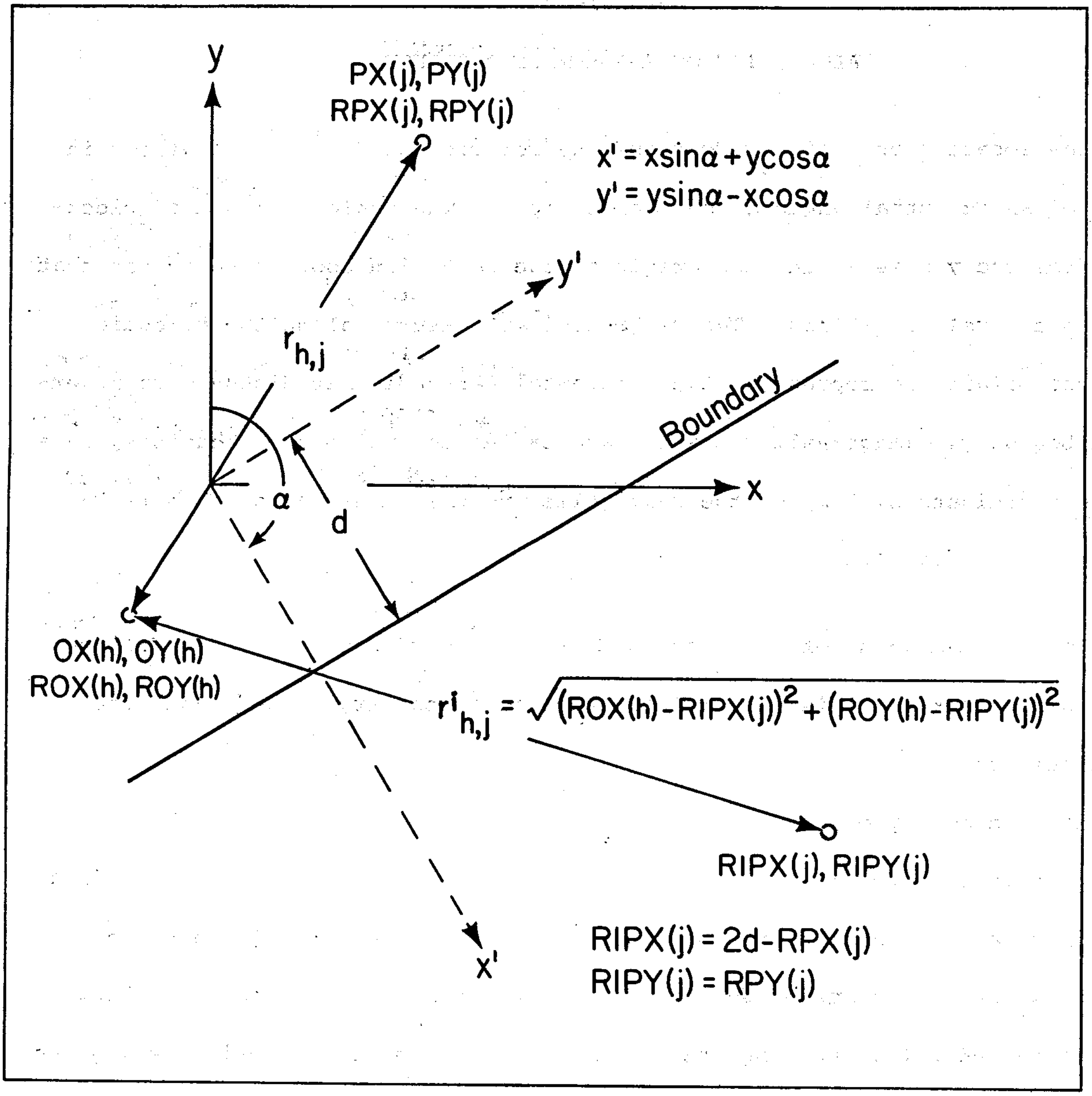


Figure 23. Schematic for the calculation of image-well distances.

[XBL 792-496]

The x' image-well coordinate

$$\mathrm{RIPX}(j) = 2d - \mathrm{RPX}(j). \tag{B-2}$$

Because the y' axis is parallel to the boundary, the y' coordinates of the image well and the production well are the same, i.e.,

$$\mathrm{RIPY}(j) = \mathrm{RPY}(j). \tag{B-3}$$

The image-well distance corresponding to observation well h and production well j is calculated using the transformed observation well coordinates given by (B-1b) and the transformed image-well coordinates given by (B-2) and (B-3):

$$\mathrm{RI}(h,j) = [\mathrm{ROX}(h) - \mathrm{RIPX}(j)]^2 + [\mathrm{ROY}(h) - \mathrm{RIP}(j)]^2. \tag{B-4}$$

Since the calculation of the component of pressure change due to an image well requires only the square of the distance to the image well, it is calculated directly:

$$\mathrm{RI2}(h,j) = [\mathrm{ROX}(h) - \mathrm{RIPX}(j)]^2 + [\mathrm{ROY}(h) - \mathrm{RIPY}(j)]^2. \tag{B-5}$$

Thus for any number of observation wells and production wells, the effect of a linear boundary is modeled in terms of α and d, from which values of RI2(h,j) are calculated.

APPENDIX C

NOMENCLATURE

α	Angle to hydrologic boundary	[deg]
A_k	Flow rate at beginning of production pulse k	$[L^3/T^2]$
B_k	Linear rate of change of flow rate during production pulse k	$[L^3/T^2]$
c	Combined aquifer matrix and fluid compressibility	$[LT^2/M]$
χ^2	Statistic minimized in present technique	[-]
$\chi^2(h)$	χ^2 statistic for observation well h	[-]
d	Distance to hydrologic boundary	[L]
η	Hydraulic diffusivity	$[L^2/T]$
F	Alternate symbol for χ^2	[-]
F_i	Derivative of F with respect to parameter i or the gradient vector of F	[variable]
F_{ij}	Derivative of F_i with respect to the parameter j or the second derivative matrix (Hessian) of F	[variable]
h	Height of reservoir or height of open interval of production well	[L]
	Index of observation wells	[-]
H	Number of observation wells considered	[-]
H_{iu}	Inverse of F_{ij} matrix	[variable]
i	Index of pressure changes or index of parameters	[-]
I	Number of observed pressures	[-]
I(h)	Number of observed pressures of observation well h	[-]
j	Index of production wells	[-]
J	Number of production wells considered	[-]
k	Permeability	$[L^2]$
	Index of production pulses	[-]
K	Number of production pulses	[-]
K(j)	Number of production pulses of production well j	[-]

kh/μ	Transmissivity	$[L^4T/M]$
μ	Dynamic viscosity	$[M/LT]$
n	Number of parameters considered	$[-]$
ϕ	Porosity	$[-]$
ΔP_c	Observed pressure change	$[M/LT^2]$
P_i	Initial production well pressure	$[M/LT^2]$
ΔP_o	Observed pressure change	$[M/LT^2]$
ΔP_{skin}	Pressure change due to skin effect	$[M/LT^2]$
q^*	Strength of line source	$[M/LT^2]$
q_{bulk}	Wellhead flow rate	$[L^3/T]$
q_k	Flow rate that obtains during production segment k	$[L^3T]$
q_{sf}	Sandface flow rate	$[L^3/T]$
r_e	Effective wellbore radius	$[L]$
r	Distance between the observation and the production well	$[L]$
r_i	Distance between the observation well and the image production well	$[L]$
$r_{h,j}$	Distance between observation well h and image well of production well j	$[L]$
r_w	Wellbore radius	$[L]$
s	Skin effect value	$[-]$
ϕch	Storativity	$[L^2T^2/M]$
τ	Production time	$[T]$
t	Observation time	$[T]$
u	Argument of well function $\int_u^\infty \left(\frac{\exp^{-y}}{y}\right) dy$	$[-]$
$W(u)$	Well function or negative exponential integral of argument $-u$	$[-]$
x_i	Parameter i or vector of parameters	[variable]
x,y	Axes of well-field coordinate system	$[L]$
x', y'	Axes of image-well coordinate system	$[L]$

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