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Park, So Eun

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**Essays on Bounded Rationality in Games and Markets**

by

So Eun Park

A dissertation submitted in partial satisfaction of the  
requirements for the degree of  
Doctor of Philosophy

in

Business Administration

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Teck Hua Ho, Chair  
Professor Ganesh Iyer  
Professor Minjung Park  
Professor Benjamin Handel

Spring 2014

**Essays on Bounded Rationality in Games and Markets**

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So Eun Park

## Abstract

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Doctor of Philosophy in Business Administration

University of California, Berkeley

Professor Teck Hua Ho, Chair

In economics, players are assumed to be rational: they exhibit self interested behavior and play equilibrium strategies. However, in laboratory games or actual markets, players often manifest behavior that is rather consistent with bounded rationality. This thesis consists of two chapters, which relax the standard assumptions on rationality and allow for bounded rationality of players.

The first essay weakens the assumption that players are self interested. In this essay, a retail market is empirically investigated under the relaxed assumption that firms may not be purely self interested or profit maximizing. Standard models of price competition stipulate that firms are pure profit maximizers; this assumption can be sensible and empirically useful in inferring product markups in a market with no direct government intervention. However, in markets for essential goods such as food and healthcare, a government may wish to address its consumer surplus concerns by imposing regulatory constraints or actively participating as a player in the market. As a consequence, some firms may have objectives beyond profit maximization and standard models may induce systematic biases in empirical estimation. This essay develops the structural model of price competition where some firms have consumer surplus concerns. Our model is applied in order to understand demand and supply behaviors in a retail grocery market where the dominant retailer publicly declares its consumer surplus objective. Our estimation results show that the observed low prices of this retailer arise indeed as a consequence of its consumer surplus concerns instead of its low marginal costs. The estimated degree of consumer surplus concerns suggests that the dominant retailer weighs consumer surplus to profit in a 1 to 7 ratio. The counterfactual analysis reveals that if the dominant retailer were to be profit maximizing as in the standard model, its prices would increase by 6.09% on average. As a consequence, its profit would increase by 1.16% and total consumer surplus would decrease by 7.18%. To the contrary, competitors lower their prices in response to the dominant retailers increased prices, i.e., become less aggressive as if they are strategic substitutes. Interestingly, even though profit of all firms increases, total social surplus would decrease by 3.21% suggesting that profit maximization by all firms induces an inefficient outcome for the market.

The second essay relaxes the rationality assumption that players exhibit equilibrium behavior, and develops a model that explains nonequilibrium behavior of players in laboratory games. In standard nonequilibrium models of iterative thinking, there is a fixed rule hierarchy and every player chooses a fixed rule level; nonequilibrium behavior emerges when some players do not perform enough thinking steps. Existing approaches however are inherently static. In this essay, we generalize models of iterative thinking to incorporate adaptive and sophisticated learning. Our model has three key features. First, the rule hierarchy is dynamic, i.e., the action that corresponds to each rule level can evolve over time depending on historical game plays. Second, players rule levels are dynamic. Specifically, players update beliefs about opponents rule levels in each round and change their rule level in order to maximize payoff. Third, our model accommodates a continuous rule hierarchy, so that every possible observed action can be directly interpreted as a real-numbered rule level  $r$ . The proposed model unifies and generalizes two seemingly distinct streams of nonequilibrium models (level- $k$  and belief learning models) and as a consequence nests several well-known nonequilibrium models as special cases. When both the rule hierarchy and players rule levels are fixed, we have a static level- $r$  model (which generalizes the standard level- $k$  model). When only players rule levels are fixed, our model reduces to a static level- $r$  model with dynamic rule hierarchy and captures adaptive learning. When only the rule hierarchy is fixed, our model reduces to a dynamic level- $r$  model and captures sophisticated learning. Since our model always converges to the iterative dominance solution, it can serve as a model of the equilibration process. Using experimental data on  $p$ -beauty contests, we show that our model describes subjects dynamic behavior better than all its special cases. In addition, we collect new experimental data on a generalized price matching game. The estimation results show that it is crucial to allow for both adaptive and sophisticated learning in predicting dynamic choice behaviors across games.

To my parents—who always support me for who I am with faith and love.

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# Chapter 1

## Consumer Surplus Moderated Price Competition

### 1.1 Introduction

*“Thirty nine years ago, NTUC FairPrice was formed for one social purpose—to share the load of rising costs with our customers. Everything we do is driven by this unique social mission of moderating the cost of living in Singapore.”*

*“We keep the prices of daily essentials stable to stretch the hard-earned money of our customers. [...] We have been able to consistently achieve excellence in both the business and social front.”*

— FairPrice Annual Report 2012.

Standard models of price competition assume that firms are driven solely by profit concerns. With no direct government intervention in a market, such assumption is realistic and powerful because one can then interpret observed market prices as equilibrium behaviors among profit maximizing firms. This equilibrium interpretation is empirically very useful because it allows one to systematically infer the product markup and hence the marginal cost of each product in the market.

This profit-maximization assumption however does not apply to every market. In fact, in markets for essential goods such as food, healthcare, and housing (i.e., products that satisfy physiological and safety needs in the Maslow’s hierarchy of needs), a government may wish to address its consumer surplus concerns by imposing regulatory constraints on price levels. Sometimes, the government may even take an additional step to actively participate in the market in order to have better market information and directly serve the consumers. In these markets, some firms will have different objectives than pure profit maximization and the nature of market competition may change dramatically. As a result, applying standard

models to these markets may induce systematic biases in empirical estimation.

There are many examples of consumer surplus moderated price competition. Surplus concerns can arise in at least 3 ways. First, there are countries where a significant portion of the enterprises are state-owned (e.g., China). China has moved from a communist country with no market prices to a regulated market where state-owned enterprises actively participate in many product markets from housing and food to energy and telecommunications.<sup>1</sup> Anecdotal evidence suggests that these state-owned enterprises are not pure profit maximizers since a significant portion of profit is used to increase public surplus and to stabilize cost of living for people.<sup>2</sup> Second, healthcare market in most countries is often heavily regulated and has active participation by a high number of nonprofit organizations. This is so because healthcare is considered a basic need to which every human being is entitled. For example, of the 3,900 nonfederal, short-term, acute care general hospitals in the United States in 2003, about 62 percent were nonprofit, 20 percent were government hospitals, and 18 percent were for-profit hospitals.<sup>3</sup> Non-profit hospitals are not investor-owned and hence often have different objectives than pure profit maximization. Third, government of countries with high income inequality may choose to participate in essential good markets in order to keep the cost of living low and stable. For example, Singapore government builds 85% of the apartments in the country in order to make housing affordable. In all three scenarios, one or more firms are likely to have a consumer surplus moderated objective and as a consequence will significantly change the nature of price competition.

Given this wide prevalence of consumer surplus moderated price competition, it is surprising that little research has investigated its equilibrium implications and that the existing research to date has been largely confined to the healthcare market. There exist a few works on consumer surplus concerned players in non-healthcare markets. Shiver and Srinivasan (2011) consider a duopoly market where one firm is profit maximizing while the other firm is consumer surplus maximizing given some constraint on its profit level. The competitive game is two-stage: firms sequentially decide on quality in the first stage and simultaneously decide on price in the second stage. Their key finding is that when the consumer surplus maximizing firm is the follower in the first stage, it can significantly improve consumer surplus by forgoing only small amounts of profit. Miravete, Seim and Thurk (2013) also investigate a government regulated market where the social planner (i.e., Pennsylvania state) is assumed to have consumer surplus concerns. Their work is based on the interesting observation that the Pennsylvania state imposes a statewide uniform markup policy on liquor, and one of its

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<sup>1</sup>Chinese government manages a total of 117 large state-owned conglomerates according to the State-owned Assets Supervision and Administration Commission of China. Each of these conglomerates owns hundreds of subsidiaries and they compete actively with non-state-owned enterprises in many markets.

<sup>2</sup>Keith Bradsher. "China's Grip on Economy Will Test New Leaders". The New York Times. November 9, 2012.

<sup>3</sup>GAO Testimony before Committee on Ways and Means, House of Representatives, by David M. Walker, Comptroller General of the United States, May 26 2005, p.4.

key findings is that the uniform markup policy induces cross-subsidization across customers compared with product-specific pricing scheme. Theoretically, investigating a consumer surplus moderated market is important because it allows a modeler to understand how the nature of competition changes as a result of some firms having consumer surplus concerns. Practically, it is relevant because it provides useful guidelines for both the policy makers and firms on how to compete in such markets.

This paper develops the structural model of retail price competition in which some firms (i.e., retailers) have consumer surplus concerns. We posit that if a firm has consumer surplus concerns, it optimizes a weighted average of its profit and total consumer surplus (i.e.,  $(1 - \alpha) \cdot (\text{Profit}) + \alpha \cdot (\text{Total Consumer Surplus})$ ), where  $\alpha$  measures the degree of the firm's consumer surplus concerns and may vary from firm to firm. When  $\alpha$  is set to 0 for all firms, the model reduces to the standard models of price competition. Hence our empirical model naturally nests standard models as special cases.

The total consumer surplus is modeled as the sum of the net utility of all consumers in the market, not just the consumers who are served by the firm itself. Unlike most existing research on healthcare markets, we do not resort to using a proxy for consumer welfare such as accessibility to patients (e.g., Newhouse, 1970; Frank and Salkever, 1991; Horwitz and Nichols, 2009).<sup>4</sup> Instead, we structurally derive a consumer surplus measure from first principles and hence, the derived measure is theoretically more sound and empirically more accurate.

We first investigate the theoretical properties of our model. We prove analytically that the total consumer surplus always increases when any firm decreases its price. In addition, we show that a firm's price and profit always decrease when its concerns for consumer surplus increase. Both results prove useful for estimating the model and interpreting the key results in the empirical estimation.

Before describing the main empirical results, let us illustrate how equilibrium prices in a single-product duopoly market competition may change as a result of one firm having consumer surplus concerns, and how our model can yield meaningful insight on such competition. *Ceteris paribus*, the consumer surplus moderated firm will wish to lower its price in order to increase the total consumer surplus. This lower price has a direct effect of increasing the firm's market share as well as an indirect effect on the price of the other firm who is a pure profit maximizer. This other firm may decrease or increase its price in response to the lower price set by the consumer surplus moderated firm, depending on whether it is a strategic complement or substitute. As a consequence, the total effect on total consumer

---

<sup>4</sup>An accessibility measure such as the number of beds and quantity of provided medical service is indeed relevant to the healthcare industry more than the consumer surplus measure because patients having insurance pay deductibles and as a consequence, their surplus does not significantly depend on the price level.

surplus becomes compounded. Our model is useful in empirically estimating this compound effect of a firm with consumer surplus concerns. Furthermore, upon observing market prices, a modeler can also infer the degree to which the firm is consumer surplus concerned, implying that our model can effectively disentangle the two forces causing low prices: consumer surplus concerns and price competition.

Is it empirically true, however, that a firm with consumer surplus concerns indeed lowers its equilibrium price? Let us compare prices of 2 dominant retailers in Singapore: *FairPrice* and *Dairy Farm*. *FairPrice* has 131 supermarket outlets and is the largest retailer with 49.04% total market share of consumer packaged goods. *FairPrice*, owned by the national labor union of Singapore (NTUC), has significantly deep ties with the government<sup>5</sup> and has openly stated its consumer surplus objectives as shown in the above quotations. On the other hand, *Dairy Farm*, the second largest retailer with a market share of 15.44%, is a pure profit maximizing firm.<sup>6</sup> Figure 1 shows the average prices of the most popular 3 national brands that are carried by both retailers in 2 food categories: rice and infant milk. We choose rice and infant milk because they represent the top 2 spending categories among the essential goods. As shown, in both categories, *FairPrice* has a systematically lower price than *Dairy Farm*.<sup>7</sup> This pattern of lower prices in essential goods is indeed consistent with *FairPrice*'s firm objective of "moderating the cost of living" for consumers. However, it is also consistent with an alternative explanation that *FairPrice*, as the dominant retailer in the market, may enjoy lower marginal costs than its competitor.

Figure 2 shows the average price of the most popular 3 national brands carried by both retailers in the chocolate category. We choose the chocolate category because it has the highest market share among the discretionary categories in terms of consumer expenditure (ranked 15th in dollar spending).<sup>8</sup> Unlike in Figure 1, *FairPrice* did not charge a systematically lower price than *Dairy Farm*.<sup>9</sup> If *FairPrice* indeed had lower marginal costs due to its higher market power, one is likely to see the same pattern of low prices in Figure 1 occur

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<sup>5</sup>*FairPrice* is owned by a cooperative of National Trades Union Congress (NTUC), which has close ties with the Singapore government. The head of the NTUC is always a cabinet minister. Also, the boards of the cooperatives owned by NTUC always have government representatives. See Appendix for the summary table of historical secretary generals and presidents of NTUC and their concurrently held government positions while incumbent at NTUC.

<sup>6</sup>*Dairy Farm*, the 2nd largest grocery retailer in the market, is a private company and is publicly listed on the Singapore stock exchange. In addition, *Dairy Farm*'s annual report in 2011 puts forward a slogan that their main goal is to "satisfy the appetites of Asian shoppers for wholesome food and quality consumer and durable goods at competitive prices" and it does not specifically mention their consumer surplus goal.

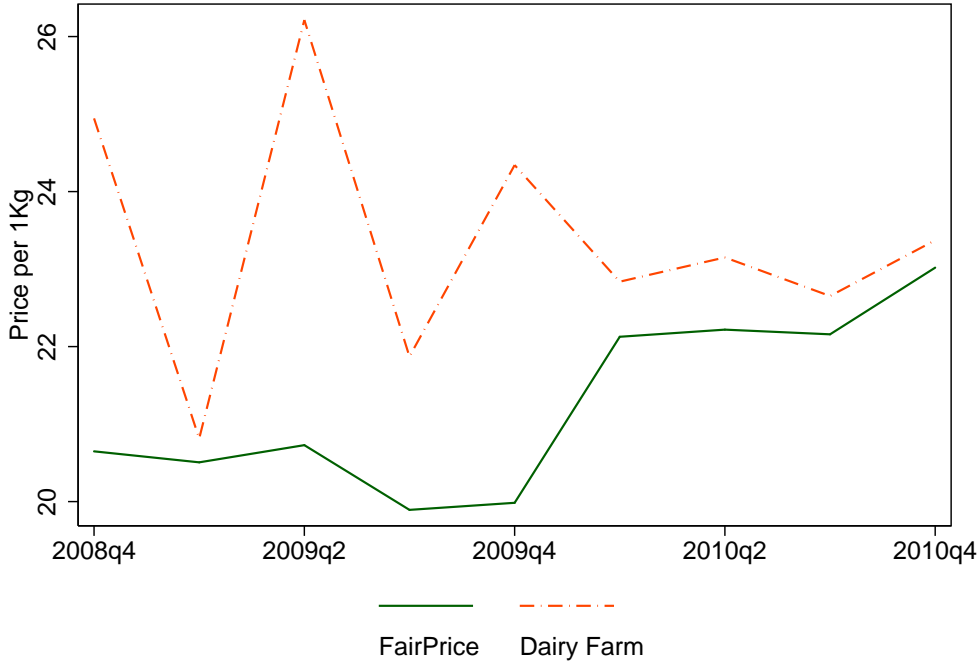
<sup>7</sup>We conducted Student's t-test on the quarterly average prices of the two retailers. In both of the two product categories, we rejected the null hypothesis that the means of price distributions of the two retailers are equal ( $p < 0.005$ ).

<sup>8</sup>The biscuit category is not considered despite higher expenditure because it is too differentiated over brands, flavor and types.

<sup>9</sup>We conducted Student's t-test on the quarterly average prices of the two retailers. We could not reject the null hypothesis that the mean of price distributions of the two retailers are equal ( $p > 0.10$ ).



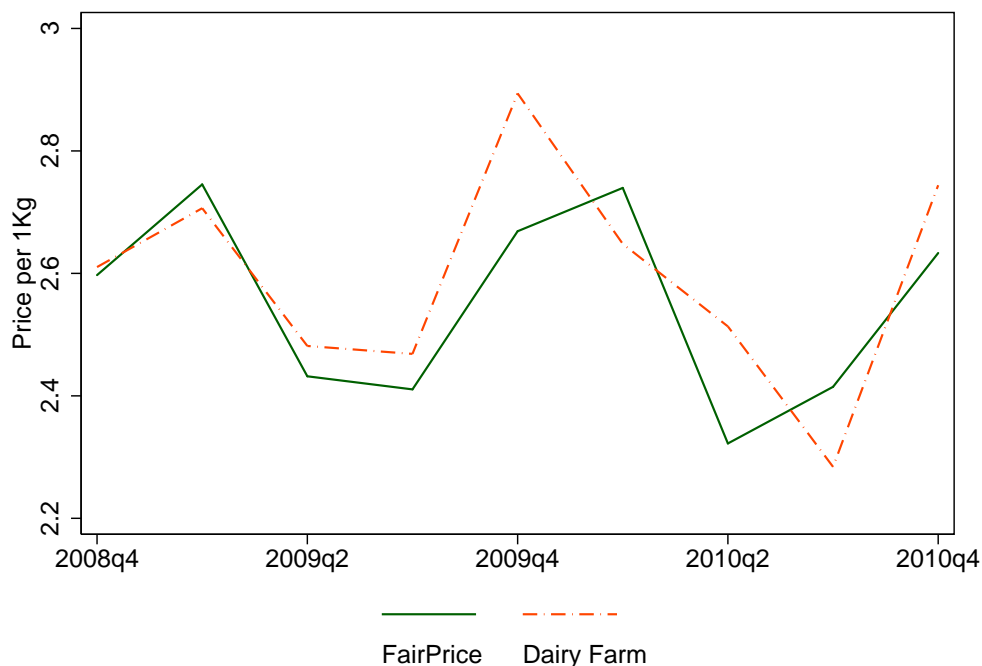
(a) Rice



(b) Infant Milk

Figure 1.1: Average Prices: Rice and Infant Milk

Figure 1.2: Average Prices: Chocolate



in the chocolate category as well. Thus, we conjecture that standard models of competition may not be able to account for the differing pattern of average prices between essential and nonessential food. To account for this differing pattern of prices, one must explicitly account for *FairPrice*'s customer surplus concerns in the model. In addition, it will be also interesting to investigate how *Dairy Farm*'s prices respond to *FairPrice*'s lower prices arising from consumer surplus concerns.

To empirically investigate whether *FairPrice* indeed has consumer surplus concerns and how such concerns affect the price competition, we apply our structural model to understand demand and supply behaviors in the Singaporean grocery market. Assuming that *FairPrice* possesses consumer surplus concerns while the other retailers do not, we empirically estimate *FairPrice*'s consumer surplus moderating parameter  $\alpha$ . If *FairPrice* does not have consumer surplus concerns, the model would empirically yield a corner solution  $\alpha = 0$ , suggesting that the standard model describes the data well. We obtain a panel dataset from a major marketing research firm, which contains grocery shopping data of 646 households from October 2008 to December 2010 in Singapore. Besides capturing a total of 190,959 shopping trips and 709,112 product purchase incidences on 118 consumer packaged good categories, the comprehensive dataset also contains 18 demographic variables including monthly income, size of the household, and the primary grocery buyer's age.



The estimation results and counterfactual analysis based on the rice category show that (the estimation on infant milk and chocolate categories are currently underway):

1. *FairPrice*'s low prices on rice are indeed a consequence of its consumer surplus concerns and its  $\alpha$  is estimated to be 0.13 averaged across all markets.
2. If the low prices were to maximize profit as in standard models, the estimated markups for *FairPrice* would be implausibly high (and hence their marginal costs would be implausibly low).
3. If *FairPrice* were to be profit maximizing, its profit would increase by 1.16% and the total consumer surplus would decrease by 7.18%. On the other hand, the profit of *Dairy Farm* would increase by 5.54%. Interestingly, the total social surplus would decrease by 3.21% suggesting that that profit maximization by all firms induces an inefficient outcome for the market.
4. The decrease in total consumer surplus due to all firms' profit maximization consists of two components: 1) the direct effect due to *FairPrice*'s higher prices under profit maximization objective and 2) the indirect effect due to price competition, i.e., competitors' response to such higher prices. The indirect effect is positive, suggesting that competitors respond to *FairPrice*'s price increase by lowering their prices (i.e. becoming less aggressive in price competition as if they are its strategic substitutes). Despite the positive indirect effect, the total consumer surplus loss is retained at 97.60% of the direct effect.

The remainder of paper is organized as follows. Section 2 describes the model of a consumer surplus moderated price competition. Section 3 describes data on Singapore's grocery retail market. Section 4 discusses the empirical results. Section 5 concludes.

## 1.2 The Model

### Notations

We consider  $M$  retail markets of a category of products served by  $I$  ( $I \geq 2$ ) firms. Firms are indexed by  $i$  ( $i = 1, 2, \dots, I - 1, I$ ) and markets are indexed by  $m$  ( $m = 1, 2, \dots, M - 1, M$ ). Each market  $m$  consists of  $K^m$  consumers and offers the set  $\mathcal{J}^m$  ( $\mathcal{J}^m = \{0, 1, \dots, J^m\}$ ) of products (i.e., choice menu) to consumers. Let  $\mathcal{J}_i^m$  be the set of products offered by firm  $i$  in market  $m$ . Consumers are indexed by  $k$  ( $k = 1, 2, \dots, K^m - 1, K^m$ ) and products are indexed by  $j \in \mathcal{J}^m$ .

Each firm  $i$  may or may not possess consumer surplus concerns and thus has a different objective  $\Pi_i(\cdot)$ . Firms choose retail prices simultaneously to maximize their respective objectives. Let  $\pi_i^m(\cdot)$  be firm  $i$ 's profit and  $\Phi^m(\cdot)$  be the total consumer surplus in market  $m$ ,

respectively. Specifically, we posit that firm  $i$  maximizes a weighted average of its profit and total consumer surplus in market  $m$ :

$$\Pi_i(\alpha_i) = (1 - \alpha_i) \cdot \pi_i^m(\mathbf{p}_i^m, \mathbf{p}_{-i}^m) + \alpha_i \cdot \Phi^m(\mathbf{p}_i^m, \mathbf{p}_{-i}^m).$$

where  $\alpha_i$  is the weight assigned to consumer surplus and  $\mathbf{p}_i^m$  is the price vector of all products of firm  $i$  and  $\mathbf{p}_{-i}^m$  is the price vector of all products of all other firms in market  $m$ . Price equilibrium is realized as a result of each firm's optimal pricing decision.

Each firm  $i$  offers multiple products in a retail market of a grocery category. Firms offer both national and store brands to compete with each other and firms' offerings of national brands may overlap. A product is defined as a brand-retailer combination and thus  $\mathcal{J}_i^m$  and  $\mathcal{J}_{i'}^m$  are mutually exclusive for all  $i \neq i'$ . A consumer in market  $m$  is assumed to choose a product out of her choice menu  $\mathcal{J}^m$ . We posit that the consumer choice process follows a random coefficient discrete choice model. We ignore choice dynamics over time in this paper.

In following 3 subsections, we describe the 3 components of price equilibrium in a consumer surplus moderated market: 1) demand model, 2) consumer surplus, and 3) supply model.

## Demand

Consumer  $k$  ( $k = 1, 2, \dots, K^m$ ) chooses a product  $j \in \mathcal{J}^m$  ( $\mathcal{J}^m = \{0, 1, 2, \dots, J^m\}$ ), where  $\mathcal{J}^m$  is the entire product space of market  $m$  and  $j = 0$  refers to the outside product. Let  $u_{kj}^m$  be the indirect utility that consumer  $k$  obtains from consuming product  $j$  in market  $m$ . Then,

$$\begin{aligned} u_{kj}^m &= -\beta_k \cdot p_j^m + \mathbf{x}_j^m \cdot \gamma_k + \xi_j^m + \epsilon_{kj}^m \\ &= v_{kj}^m + \epsilon_{kj}^m \end{aligned} \tag{1.1}$$

where

$$\begin{aligned} v_{kj}^m &= -\beta_k \cdot p_j^m + \mathbf{x}_j^m \cdot \gamma_k + \xi_j^m \\ \begin{pmatrix} \beta_k \\ \gamma_k \end{pmatrix} &= \begin{pmatrix} \beta \\ \gamma \end{pmatrix} + \begin{pmatrix} \Omega_p \\ \Omega_x \end{pmatrix} D_k + \begin{pmatrix} \Sigma_p \\ \Sigma_x \end{pmatrix} v_k \end{aligned}$$

$p_j^m$  is the price of product  $j$  in market  $m$ ,  $\mathbf{x}_j^m$  is the vector of observed product characteristics of product  $j$  in market  $m$ ,  $D_k$  is a vector of consumer  $k$ 's observed demographic variables, and  $v_k$  are consumer  $k$ 's unobserved consumer characteristics. In addition,  $\xi_j^m$  is the product-market level disturbance.  $\epsilon_{kj}^m$  is an i.i.d shock which follows a type I extreme value distribution.  $\Omega_p$  and  $\Omega_x$  are the price and product characteristic coefficients that are interacted with observed demographic variables, respectively.  $\Sigma_p$  and  $\Sigma_x$  are the price and

product characteristic coefficients that are interacted with unobserved consumer characteristics, respectively. The mean of indirect utility for the outside product in any market  $m$ ,  $u_{k0}^m$ , is normalized to zero.

The above demand specification is the random coefficient discrete choice model, which is a generalization of the standard multinomial logit model (Berry, 1994; Berry et al., 1995; Nevo, 2000). The standard multinomial logit model is parsimonious because it expresses consumer  $k$ 's underlying utility for a product  $j$  in terms of its price and characteristics only, instead of those of all products in the consumer's choice menu (Luce, 1959; Luce and Suppes, 1965; Marschak, 1960; McFadden, 1974, 2001). This simplification dramatically reduces the number of parameters to estimate in empirical analyses. However, the standard multinomial logit model possesses the independence of irrelevant alternatives (IIA) property that makes a sharp prediction on price elasticities: if two products have the same market share, they should have an identical cross-price elasticity with respect to any other product in the choice menu. This prediction, however, frequently does not describe actual choice substitution well. The above random coefficient discrete choice model overcomes this inadequacy by allowing for a more flexible and realistic substitution pattern. This is accomplished by interacting consumers' demographic variables with price and product characteristics. Hence, if consumers with similar demographic variables have similar preferences for certain product characteristics, they will have similar choice and substitution patterns.

Let  $s_j^m$  be the market share of product  $j$  in market  $m$ ,  $s_{kj}^m$  be consumer  $k$ 's probability of choosing product  $j$  in market  $m$ , and  $A_{kj}^m$  be the region of i.i.d. shocks  $(\epsilon_{k0}^m, \dots, \epsilon_{kJ^m}^m)$  that lead to consumer  $k$ 's choosing product  $j$ . Then, by the random coefficient discrete choice model,  $s_j^m$  is given by:

$$\begin{aligned}
 s_j^m &= \int_D \int_v s_{kj}^m dF_v(v) dF_D(D) \\
 &= \int_D \int_v \left( \int_{A_{kj}^m} dF_\epsilon(\epsilon) \right) dF_v(v) dF_D(D) \\
 &= \int_D \int_v \frac{\exp(v_{kj}^m)}{\sum_{j \in \mathcal{J}^m} \exp(v_{kj}^m)} dF_v(v) dF_D(D)
 \end{aligned} \tag{1.2}$$

where  $F_\epsilon(\epsilon)$  is a joint distribution function of the consumer-level i.i.d. product shocks,  $F_D(D)$  is a joint distribution function of the population's observed demographic variables, and  $F_v(v)$  is a joint distribution function of the population's unobserved demographic shocks.

## Consumer Surplus

To derive consumer surplus, we need to define consumer  $k$ 's willingness to pay for product  $j$  in market  $m$ , denoted by  $\omega_{kj}^m$ . We posit that  $\omega_{kj}^m$  is the hypothetical price for product  $j$  that sets  $u_{kj}^m$  equal to  $u_{k0}^m$ , which is the utility of the outside product. As a consequence, the unit of consumer surplus is identical to that of profit. Since the mean of  $u_{k0}^m$  is normalized to zero, we have

$$-\beta_k \cdot \omega_{kj}^m + \mathbf{x}_j^m \cdot \gamma_k + \xi_j^m + \epsilon_{kj}^m = \epsilon_{k0}^m$$

and  $\omega_{kj}^m$  is given by

$$\omega_{kj}^m = \frac{\mathbf{x}_j^m \cdot \gamma_k + \xi_j^m + \epsilon_{kj}^m - \epsilon_{k0}^m}{\beta_k} \quad (1.3)$$

Consumer  $k$  purchases product  $j$  in market  $m$  only if her indirect utility  $u_{kj}^m$  from product  $j$  is greater than that from the outside product  $u_{k0}^m$ , suggesting that consumer  $k$  was willing to pay more for product  $j$  up to the price point where  $u_{kj}^m$  becomes equal to  $u_{k0}^m$ .

We posit that consumer  $k$ 's surplus  $\Phi_{kj}^m$  for product  $j$  she purchased in market  $m$  equals her willingness to pay for product  $j$  less the price she actually paid. That is,

$$\Phi_{kj}^m = \omega_{kj}^m - p_j^m$$

Plugging in equations (1.1) and (1.3), we obtain:

$$\begin{aligned} \Phi_{kj}^m &= \frac{\mathbf{x}_j^m \cdot \gamma_k + \xi_j^m + \epsilon_{kj}^m - \epsilon_{k0}^m}{\beta_k} - p_j^m \\ &= \frac{u_{kj}^m - \epsilon_{k0}^m}{\beta_k} \end{aligned}$$

Note that  $\Phi_{k0}^m = 0$ .  $\Phi_{kj}^m$  is specific to each purchase decision that consumer  $k$  makes. Consumer  $k$ 's total surplus  $\Phi_k^m$  is then defined as the expectation of this purchase decision specific surplus  $\Phi_{kj}^m$  over all possible purchase scenarios. Specifically,

$$\begin{aligned} \Phi_k^m &= \sum_{j \in \mathcal{J}^m} \left( \int_{A_{kj}^m} \Phi_{kj}^m dF_\epsilon(\epsilon) \right) \\ &= \sum_{j \in \mathcal{J}^m} \left( \int_{A_{kj}^m} \frac{u_{kj}^m - \epsilon_{k0}^m}{\beta_k} dF_\epsilon(\epsilon) \right) \end{aligned}$$

Recall that  $A_{kj}^m$  is the region of i.i.d. shocks  $(\epsilon_{k0}^m, \dots, \epsilon_{kJ^m}^m)$  that lead to consumer  $k$ 's choosing product  $j$ .

Finally, total consumer surplus  $\Phi^m$  in market  $m$  with market size  $K^m$  is defined as the consumer surplus of the entire population where each consumer  $k$  enjoys consumer-specific surplus  $\Phi_k^m$ . That is,

$$\begin{aligned} \Phi^m &= K^m \cdot \int_D \int_v \Phi_k^m dF_v(v) dF_D(D) \\ &= K^m \cdot \int_D \int_v \left( \sum_{j \in \mathcal{J}^m} \left( \int_{A_{kj}^m} \frac{u_{kj}^m - \epsilon_{k0}^m}{\beta_k} dF_\epsilon(\epsilon) \right) \right) dF_v(v) dF_D(D) \end{aligned} \quad (1.4)$$

If  $\epsilon_{kj}^m$  is distributed i.i.d Type I extreme value,  $\Phi_{kj}^m$  has a closed-form log-sum formula (Small and Rosen, 1981) and  $\Phi^m$  is expressed as:

$$\Phi^m = K^m \cdot \int_D \int_v \left( \frac{1}{\beta_k} \cdot \log \sum_{j \in \mathcal{J}^m} \exp(v_{kj}^m) \right) dF_v(v) dF_D(D) \quad (1.5)$$

**Theorem 1.1.** *In market  $m$ , ceteris paribus, total consumer surplus decreases in  $p_j^m$ ,  $\forall j \in \mathcal{J}^m$ , i.e.,  $\frac{\partial \Phi^m}{\partial p_j^m} < 0$ ,  $\forall j \in \mathcal{J}^m$ .*

*Proof of Theorem 1.1.* See Appendix. □

Theorem 1.1 demonstrates that if the price of *any* product offered in the market becomes lower, the total consumer surplus increases. This is because the total consumer surplus is defined as the sum of consumer surplus resulting from possible purchase scenarios of *all* products offered in the market, and not necessarily those products offered by the consumer surplus concerned firm.

## Supply

We consider an oligopoly retail market of a product category where each firm  $i$  offers multiple products. Specifically, firm  $i$  chooses prices  $\mathbf{p}_i^m$  that maximize a weighted average of its total profit  $\pi_i^m$  from all of its products, and total consumer surplus  $\Phi^m$  in market  $m$ . That is, firm  $i$ 's objective function is given by

$$\Pi_i^m(\alpha_i) = (1 - \alpha_i) \cdot \pi_i^m(\mathbf{p}_i^m, \mathbf{p}_{-i}^m) + \alpha_i \cdot \Phi^m(\mathbf{p}_i^m, \mathbf{p}_{-i}^m), \quad (1.6)$$

where

$$\pi_i^m(\mathbf{p}_i^m, \mathbf{p}_{-i}^m) = K^m \cdot \sum_{j \in \mathcal{J}_i^m} s_j^m \cdot (p_j^m - c_j^m)$$

and  $c_j^m$  is the marginal cost (i.e., wholesale price) of product  $j$  in market  $m$ . Recall that the total consumer surplus is given in equation (1.4) as:

$$\Phi^m = K^m \cdot \int_D \int_v \left( \frac{1}{\beta_k} \cdot \log \sum_{j \in \mathcal{J}^m} \exp(v_{kj}^m) \right) dF_v(v) dF_D(D)$$

Note that firm  $i$  considers the total consumer surplus in market  $m$  instead of surplus of only those consumers it serves.  $\alpha_i \in [0, 1]$  is exogenously given for each firm  $i$  and is the weight assigned to the total consumer surplus, capturing the degree to which firm  $i$  is consumer surplus concerned, i.e., the higher  $\alpha_i$  is, the bigger firm  $i$ 's concern is. The proposed weighted objective function necessarily nests the standard objective function. When  $\alpha_i = 0$ ,  $\forall i$ , equation (1.6) reduces to the standard objective function and gives rise to the standard price equilibrium solution. Thus, we set  $\alpha_{i'} = 0$  for any pure profit maximizing firm  $i'$  in our empirical estimation in the Section 2.3.

Price equilibrium is realized as a result of each firm's optimal pricing decision. Thus, for each firm  $i$ , each price  $p_j^m$ ,  $\forall j \in \mathcal{J}_i^m$ , must satisfy its first order condition:

$$0 = (1 - \alpha_i) \cdot \left( s_j^m + \sum_{j' \in \mathcal{J}_i^m} (p_{j'}^m - c_{j'}^m) \frac{\partial s_{j'}^m}{\partial p_j^m} \right) + \alpha_i \cdot \int_D \int_v (-s_{kj}^m) dF_v(v) dF_D(D)$$

## Theoretical Properties

**Theorem 1.2.** *Let  $p_j^m(\alpha_i)$  be the price of product  $j$  ( $j \in \mathcal{J}_i^m$ ) that optimizes the objective function of a consumer-surplus concerned firm  $i$  in market  $m$ , given the other firms' prices. Then,  $\forall j \in \mathcal{J}_i^m$ ,  $p_j^m(\alpha_i)$  decreases in  $\alpha_i$ . I.e.,  $\frac{\partial p_j^m(\alpha_i)}{\partial \alpha_i} < 0$ ,  $\forall j \in \mathcal{J}_i^m$ .*

*Proof.* See Appendix. □

**Theorem 1.3.** *Given the other firms' prices, the profit of a consumer surplus concerned firm  $i$  decreases in  $\alpha_i$ , i.e.,  $\frac{\partial \pi_i^m}{\partial \alpha_i} < 0$ .*

*Proof.* See Appendix. □

Theorem 1.2 suggests that the more a firm is concerned with consumer surplus, the lower the prices of *all* of its products are. It is noteworthy that prices of all products in the firm’s portfolio decrease unanimously as a result of increased consumer surplus concerns. As a result, theorem 1.3 shows that its profit decreases as well given that other retailers keep their prices unchanged. As will be shown in Section 1.4, theorems 1.2 and 1.3 provide useful insight on how the total gain on consumer surplus due to a firm’s consumer surplus concerns decomposes into the direct effect of these concerns and the indirect effect of competitors’ response to them.

Let us emphasize an interesting feature of this weighted objective function: prices can be strategic substitutes even under the Hotelling model-like demand. Consider a price competition between two firms. Firms are denoted by  $i$  ( $i = 1, 2$ ) and produce one product each at price  $p_i$  with zero marginal cost. These two products are horizontally differentiated (Hotelling, 1929) and each consumer buys only one product. In this horizontally differentiated market, demand of firm  $i$  can be expressed as  $D_i(p_i, p_{-i}) = 1 - p_i + p_{-i}$ . Firm  $i$  chooses  $p_i$  that maximizes its profit  $\pi_i(p_i, p_{-i}) = p_i \cdot D_i(p_i, p_{-i})$ . Then, it can be shown that  $\partial p_i^*(p_{-i})/\partial p_{-i} = 1/2 > 0$  and  $p_1$  and  $p_2$  are strategic complements of each other.

Now consider that firm 1 is consumer surplus concerned and optimizes a weighted objective function:

$$\Pi_1(p_1, p_2) = (1 - \alpha_1) \cdot \pi_1(p_1, p_2) + \alpha_1 \cdot (\text{Consumer Surplus})$$

where

$$\text{Consumer Surplus} = \frac{(1 - p_1 + p_2)^2}{2} + \frac{(1 - p_2 + p_1)^2}{2}$$

Then, firm 1’s best response function depending on  $\alpha_1$  can be summarized as:

$$\frac{\partial p_1^*(p_2)}{\partial p_2} \begin{cases} \geq 0, & \text{if } 0 \leq \alpha_1 \leq \frac{1}{3} \text{ or } \alpha_1 \geq \frac{1}{2} \\ < 0, & \text{if } \frac{1}{3} < \alpha_1 < \frac{1}{2} \end{cases}$$

This suggests that when firm 1’s level of consumer surplus concerns is too low ( $\alpha_1 \leq \frac{1}{3}$ ),  $p_1$  is a strategic complement of  $p_2$ , and firm 1 becomes more aggressive as firm 2 lowers its price. Note that when firm 1 weighs consumer surplus more than its profit ( $\alpha_1 \geq \frac{1}{2}$ ), firm 1 will price at marginal cost regardless of  $p_2$ , i.e.,  $\frac{\partial p_1^*(p_2)}{\partial p_2} = 0$ . On the other hand, when  $\alpha_1$  is moderate ( $\frac{1}{3} < \alpha_1 < \frac{1}{2}$ ), firm 1 becomes less aggressive in price competition and  $p_1$  increases (decreases) when  $p_2$  decreases (increases). This is because consumer surplus increases in  $(p_1 - p_2)^2$  and aggressive price competition (i.e., smaller price gap between  $p_1$  and  $p_2$ ) will cause reduction in consumer surplus.

## 1.3 Data

We use the household panel data in Singapore obtained from a major marketing research company. The company installed scanners at a representative sample of 646 households in the country and collected shopping basket data of each household for 9 quarters from October 2008 to December 2010.<sup>10</sup> The dataset contains households' purchasing history of a total of 118 consumer packaged goods.

The dataset also contains a total of 18 demographic variables for each household. Among those, we have the full name of the head of the household, household size, zip code, primary grocery buyer's age, household monthly income (one of the 11 income brackets), race, type of dwelling (private or subsidized public housing), work status (1 if primary grocery buyer works), maid (1 if the household has a maid), child below 4 (1 if the household has a child aged below 4), child between 5 and 14 (1 if the household has a child aged between 5 and 14), family (1 if the household is of family type and 0 if of singles/couples type), female below 9 (1 if the household has a female aged below 9), female between 10 and 19 (1 if the household has a female aged between 10 and 19), female between 20 and 29 (1 if the household has a female aged between 20 and 29), female between 30 and 39 (1 if the household has a female aged between 30 and 39), female between 40 and 49 (1 if the household has a female aged between 40 and 49), and female above 50 (1 if the household has a female aged above 50). Table 1.1 provides summary statistics for these variables. This rich set of demographic variables allows us to capture individual heterogeneity in product preferences and price sensitivities in the demand model. In our empirical estimation, we include household size, income, primary grocery buyer's age, work status, two race dummies (Chinese and Indian), child below 4, child between 5 and 14, and family in order to capture individual heterogeneity. The variable names used in empirical estimation and their corresponding description are listed in the Appendix.

The primary grocery buyer at each household was instructed to scan all grocery items after each shopping trip.<sup>11</sup> For each product scanned, the dataset contains the following 7 variables: 1) barcode, 2) date of scanning, 3) the name of retailer where the item was bought, 4) product category, 5) price, 6) quantity purchased, and 7) product description (a combination of brand, product name, and packaging size). From the product description, we have created 3 additional variables (brand, product name, and packaging size), yielding a total of 9 variables for each product. All expenditures in the summary statistics below are in Singaporean currency (SGD).

Table 1.2 shows the top 20 consumer packaged good categories by expenditures. As shown, the top 10 categories are infant milk (5.84%), rice (5.41%), liquid milk (4.52%), frozen food (4.38%), bread (3.29%), biscuit (2.73%), yoghurt (2.69%), facial care (2.65%), edible oil (3.26%), and detergent (2.51%). Note that most of these categories are food items. These top 10 categories accounted for 36.61% of the total spending on consumer packaged goods. Note that chocolate is ranked 15th in

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<sup>10</sup>The company started recruiting panelists in early 2008. We only include households who joined before October 1, 2008 and who have shopped at least once per month since joining.

<sup>11</sup>The company uses store-level data to check whether the recruited households scan regularly. It appears that a significant majority of them do scan their shopping baskets regularly.



Table 1.1: Summary of Demographic Variables

Number of households		646		
	Mean	Std. Dev.	Min	Max
Monthly income <sup>a</sup>	4552.63	3540.87	500	15000
Household size	3.82	1.38	1	12
Grocery buyer's age	50.29	8.96	30	81
Type of dwelling	0.86	0.35	0	1
Work status	0.67	0.47	0	1
Child below 4	0.94	0.29	0	1
Child between 5 and 14	0.36	0.48	0	1
Family	0.60	0.49	0	1
Maid	0.16	0.36	0	1
Female below 10	0.10	0.30	0	1
Female between 10 and 19	0.27	0.44	0	1
Female between 20 and 29	0.25	0.43	0	1
Female between 30 and 39	0.24	0.42	0	1
Female between 40 and 49	0.37	0.48	0	1
Female above 50	0.65	0.48	0	1

<sup>a</sup>Monthly income is in Singaporean dollars. Summary statistics are computed based on the median value of each of the 11 income brackets. Highest income bracket is "above \$10,000" and its median value is assumed to be \$15,000.

terms of expenditure.

Table 1.3 provides the summary statistics of households' shopping trips. In total, households spent \$4,348,076.54 over the entire period, among which \$2,195,455.72 (50.49%) was on consumer packaged goods. They made a total of 190,959 shopping trips to retailers and scanned 709,112 product purchase incidences. On average, a household made a total of 295.60 trips, spent \$22.77 per trip and \$249.29 per month, and recorded 3.71 purchase incidences on each trip. The average inter-shopping time was 4 days.

In the empirical estimation, we investigate top 2 nondiscretionary product categories (infant milk and rice), and 1 discretionary product category (chocolate).<sup>12</sup> Note that we determine whether a category is discretionary or nondiscretionary based on Classification of Individual Consumption According to Purpose (COICOP) provided by the United Nations statistics division.

Table 1.4 provides the distribution of total expenditure, total number of outlets, and total number

<sup>12</sup>Based on COICOP, we determine that categories such as facial care, laundry detergent and shampoo among top grossing categories fit more into the semi-discretionary categories, which consumers tend to downgrade instead of dispense with when facing financial restraint.

Table 1.2: Top 20 Grossing Categories of Consumer Packaged Goods

Product category	Expenditure (SGD)	Share of Expenditure <sup>a</sup>
Infant milk	128237.30	5.84%
Rice	118879.20	5.41%
Liquid milk	99204.29	4.52%
Frozen food	96098.58	4.38%
Bread	72285.25	3.29%
Biscuit	59828.20	2.73%
Yoghurt	59159.21	2.69%
Facial care	58261.02	2.65%
Edible oil	56691.50	2.58%
Laundry detergent	55013.77	2.51%
Coffee	53520.90	2.44%
Juices	47714.16	2.17%
Liquid soap	47685.44	2.17%
Shampoo	46477.65	2.12%
Chocolate	45692.51	2.08%
Instant noodles	43547.46	1.98%
Health food drink	42761.56	1.95%
Sauces	42729.48	1.95%
Toilet rolls	36659.53	1.67%
Diapers	35191.37	1.60%

<sup>a</sup>Share of expenditure on each product category out of the entire expenditure.

Table 1.3: Summary of Shopping Pattern

Number of households	646			
Number of total shopping trips	190,959			
Number of total scannings	709,112			
Total expenditure	\$4,348,076.54			
Total expenditure on consumer packaged goods	\$2,195,455.72			
	Mean	Std. Dev.	Min	Max
Average number of trips	295.60	197.09	63	1467
Average spending per trip	\$22.77	\$34.50	\$0.01	\$1598.62
Average spending per month	\$249.29	\$292.20	\$1.70	\$6462.13
Average number of purchase incidences per trip	3.71	3.32	1	52
Average inter-shopping days	3.64	3.67	1	54

Table 1.4: Expenditure and Shopping Trips by Top Three Retailers

Firm	Expenditure	Shopping Trips	Number of Outlets <sup>a</sup>
<i>FairPrice</i>	\$1,497,565	63,021	131
<i>Dairy Farm</i>	\$572,332	23,950	104
<i>Sheng Siong</i>	\$322,677	15,910	33
Others	\$1,955,501	88,078	N/A
Total	\$4,348,076	190,959	N/A

Firm	Rice	Infant milk	Chocolate
<i>FairPrice</i>	\$65,428.74	\$65,839.86	\$24,136.04
<i>Dairy Farm</i>	\$17,561.04	\$20,101.59	\$8,516.25
<i>Sheng Siong</i>	\$16,524.08	\$7,541.25	\$4,332.50
Others	\$19,365.34	\$34,754.60	\$8,707.73
Total	\$118,879.20	\$128,237.30	\$45,692.51

<sup>a</sup>This data is collected separately by visiting each retailer's website.

of shopping trips by retailers. The same table also shows the dollar share of the top 3 retailers for the 3 focused categories (infant milk, rice, and chocolate). The top three retailers are *FairPrice*, *Dairy Farm*, and *Sheng Siong*. These 3 retailers received 55.03% of the total expenditures where *FairPrice* accounted for 34.44%, *Dairy Farm* 13.16% and *Sheng Siong* 7.42% respectively. Similarly, the top 3 retailers accounted for 53.88% of the total number of shopping trips. In both total expenditure and total number of shopping trips, *FairPrice* is clearly the market leader.

The market leadership of *FairPrice* is as pronounced when we restrict ourselves to the 3 focused consumer packaged good categories. As shown, *FairPrice* is the market leader for all 3 categories and received 51.34%, 55.04%, and 52.82% from the category-specific total expenditure of infant milk, rice, and chocolate, respectively. *Dairy Farm* is the second largest retailer enjoying 15.68%, 14.77%, and 18.64% in the three categories respectively.

## 1.4 Empirical Results

### Estimation of Demand

A market for a product category is defined as a quarter of a year.<sup>13</sup> Since a purchase incidence contains combined information of total quantity purchased and packaging size, each purchase incidence is teased out by unit weight (e.g., 1kg for rice category). As a consequence, a consumer's choice problem reduces to the choice of a brand of unit weight. For example, if a household input

<sup>13</sup>Since we only have national level data and Singapore is a small, well-connected city country, whose population is 5.3 millions and size is 3.5 times Washington D.C. of the United States, we define the entire nation as one geographical market.

a purchase incidence of 2 bags of 5kg *Royal Umbrella* rice, such purchase incidence is considered as 10 separate choice incidences of 1kg *Royal Umbrella*. Note that a choice model posits that a consumer (i.e., household) makes only one choice out of her choice menu in each market. Thus, we treat those teased out 10 choice incidences as if 10 households of exactly same demographic characteristics purchased the same 1kg *Royal Umbrella* respectively.

Each household’s potential level of consumption is defined as the maximum quantity of unit weight it ever consumed in a market across all markets. For those households who never purchased the product category across all markets (but purchased other product categories and thus remain in the data), their potential level of consumption is defined as the bottom 1 percentile level of consumption of the households who ever purchased the product category.

Each product  $j \in \mathcal{J}^m$  in market  $m$  is defined as a combination of retailer and brand. In the rice category, if the brand *Royal Umbrella* is offered by both *FairPrice* and *Dairy Farm*, then *Royal Umbrella* by these two retailers are considered two different products. As a consequence,  $\mathcal{J}_i^m$  and  $\mathcal{J}_{i'}^m$  are mutually exclusive for any two firms  $i \neq i'$  in each market  $m$ . Consumers’ choice menu includes the top 19 products with highest market share and the outside product. *FairPrice*, *Dairy Farm* and *Sheng Siong* carry 7, 5, and 7 of these 19 products, respectively. We adjust prices by inflation using Singapore’s quarterly CPI data. We derive the representative unit price of each product in a market (corresponding to the unit weight) as the weighted average of prices that are input into the scanner by each individual household, where the weight is the quantity of unit weight.

In the full model, price, product dummies and market dummies enter the mean-level utility and correlation between price and product-market level disturbance ( $\xi_j^m$ ) is controlled for by these dummies.<sup>14</sup> Product characteristics that are interacted with demographic variables are: price, dummies of store brands by *FairPrice* and *Sheng Siong*, dummies of major national brands (*New Moon*, *Royal Umbrella*, and *Songhe*), and retailer dummies.<sup>15</sup> A total of 12 demographic variables interact with these product characteristics: household size, grocery buyer’s age, monthly income, work status, child below 4, child between 5 and 14, family, female, maid, government-housing (*HDB*) and race dummies of Chinese and Indian.

Since our dataset contains rich individual level purchase records, we use the simulated maximum likelihood estimation method to identify the demand model parameters, where the unobserved independent demographic shock  $v_k$  is the only variable to be simulated.<sup>16</sup> Note that the observed demographic variables  $D_k$  do not need to be simulated since we know exactly what these variables are for each household.

Since we do not observe  $v_k$ , we define the expected probability  $p_{kj}^m$  that household  $k$  purchases

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<sup>14</sup>More rigorous parameter estimation using supply-side cost shock data as an instrument is under way.

<sup>15</sup>The mean level utilities of these dummy variables are estimated by projecting estimated product dummies onto these variables.

<sup>16</sup>We searched over parameter values to maximize the simulated log-likelihood using unconstrained non-linear optimization in the MATLAB optimization toolbox.

product  $j$  given its observed demographic variables  $D_k$  as<sup>17</sup>:

$$\begin{aligned} p_{kj}^m &= \mathbb{E}_v [s_{kj}^m] \\ &= \int_v \frac{\exp(v_{kj}^m)}{\sum_{j \in \mathcal{J}^m} \exp(v_{kj}^m)} dF_v(v) \end{aligned}$$

where  $s_{kj}^m$  is defined in equation (1.2) as household  $k$ 's probability of purchasing product  $j$  given both its observed and unobserved demographic variables.

Let  $o_k^m \in \mathcal{J}^m$  and  $\mathbf{o}^m = (o_1^m, o_2^m, \dots, o_{K^m}^m)$  be household  $k$ 's observed product choice and the vector of observed product choices by all  $K^m$  households in market  $m$ , respectively. Then, the likelihood  $L(o_k^m)$  of observing choice  $o_k^m$  by household  $k$  is given by:

$$L(o_k^m) = \prod_{j \in \mathcal{J}^m} (p_{kj}^m)^{\mathbf{1}(j, o_k^m)}$$

where

$$\mathbf{1}(j, o_k^m) = \begin{cases} 1, & \text{if } j = o_k^m \\ 0, & \text{if } j \neq o_k^m \end{cases}$$

The total log-likelihood of observing entire data,  $LL(\mathbf{o}^1, \mathbf{o}^2, \dots, \mathbf{o}^M)$ , is then given by:

$$\begin{aligned} LL(\mathbf{o}^1, \mathbf{o}^2, \dots, \mathbf{o}^M) &= \sum_{m=1}^M \sum_{k=1}^{K^m} \log L(o_k^m) \\ &= \sum_{m=1}^M \sum_{k=1}^{K^m} \log \left( \prod_{j \in \mathcal{J}^m} (p_{kj}^m)^{\mathbf{1}(j, o_k^m)} \right) \end{aligned}$$

Table 1.5 list the parameter estimates of the full demand model. It has a total of 17 rows and 5 columns. The 17 rows are respectively labeled mean, standard deviation, each of the 12 demographic variables that are interacted with product characteristics, maximized log-likelihood, average price coefficient of the population, and the percentage of price coefficients that are positive in the model. The 5 columns are respectively labeled the 5 product characteristics the 12 demographic variables interact with: price, constant, store brand dummy, *FairPrice* dummy and *Dairy Farm* dummy.

The first row, mean, shows the mean level utility coefficient for each product characteristic variable, i.e, how the mean level utility responds to \$1 price increase or to each dummy variable. First

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<sup>17</sup>We assume that  $v_k$  follows the standard normal distribution and is independent between product characteristics it interacts with. Final estimation results are based on 100 random draws of  $v_k$ . Random draws are generated using the Halton sequence. We varied the number of draws up to 200 and found similar results.

column shows that the price coefficient for mean level utility is negative at -1.91 at a statistically significant level. We estimate the full model with product and market dummies and thus the last 4 product characteristics (constant, store brand dummy, *FairPrice* dummy and *Dairy Farm* dummy) are subsumed in product dummies in the mean-level utility estimation. Hence, we project the estimated product dummies onto these 4 product characteristic variables to estimate their mean level utilities. Estimates obtained this way are listed in the last 4 columns of the first row. The model estimates that baseline utility of any product is 0.73 higher than that of the outside product. Store brand *NTUC* induces 4.28 higher mean level utility than non-major national brands. This is because unlike in most other countries where store brands are usually not popular, store brand rice occupies highest market shares in Singapore.<sup>18</sup> Also, as will be shown in Table 1.5, own price elasticities of these two brands are lower than other high market share products, suggesting that the store brands increase households' utility and makes them less sensitive towards price change.

The second row, standard deviation, captures the effect of unobserved demographic variables. We see that the unobserved demographic effect is less significant for all 5 product characteristics than other parameter estimates. This implies that individual heterogeneity is effectively captured by the 12 observed demographic variables that are interacted in the full model.

The 12 demographic variables in the 3rd to the 14th row in general interact significantly with the 5 product characteristic variables. Quite a few parameter estimates are worth highlighting. Higher income level households are less price sensitive and prefer *FairPrice* more. They also less prefer *NTUC* brand. 16% of Singaporean households hire maids and interestingly, households with maids are more price sensitive. Chinese are less price sensitive than Indian or Malay (the base group).

The value of maximized log-likelihood is  $-158380.19$ , while that of the standard logit model is  $-163650.15$ . Log-likelihood test rejects the standard logit model in favor of the full model ( $p < 0.001$ ). About 97.21% of the entire population has negative price coefficients and price increase strictly reduces their utility.

Table 1.6 list the own- and cross-price elasticities of the top 4 brands offered by *FairPrice* and *Dairy Farm*. Price elasticities are estimated at the median price level. We see that own price elasticities range from -2.20 to -3.48 among top 4 brands of the two retailers, suggesting that for every 1 percent increase in the price of a major product, its own market share reduces by about 2.20 to 3.48 percent. Given the high market share of *FairPrice*, it is not surprising that cross-price elasticities of *FairPrice*'s products are bigger for the other *FairPrice*'s own products than for *Dairy Farm*'s products.

## Estimation of Supply

For notational simplicity, hereafter subscript  $i'$  refers to the profit maximizing firm (i.e., *Dairy Farm* and *Sheng Siong*) and subscript  $i$  refers to the consumer surplus concerned firm (i.e., *Fair-*

<sup>18</sup> *FairPrice*'s two major store brands, *NTUC* and *Golden Royal Dragon*, have a total of 22.68% of market share (including outside product), which is about 51.57% of market share conditional on rice consumption.

Table 1.5: Estimation Results in Rice Category: Demand

	Price	Constant	<i>NTUC</i>	<i>FairPrice</i>	<i>Dairy Farm</i>
Mean	-1.9190*** (0.7123)	0.7299*** (0.1044)	4.2844*** (0.3320)	-3.1280*** (0.1138)	-2.2120*** (0.1044)
Standard Deviation <sup>a</sup>	0.0006 (0.0014)	0.0011 (0.1044)	0.0023 (0.3320)	0.0017 (0.1138)	0.0046 (0.1044)
Demographic Variables					
<i>HHOLDSIZE</i>	-0.0480 (0.0350)	-0.1771** (0.0770)	0.0769*** (0.0289)	0.2320*** (0.0260)	0.1445*** (0.0275)
<i>INCOME</i>	0.0423*** (0.0139)	0.0747** (0.0308)	-0.1679*** (0.0115)	-0.0244** (0.0103)	-0.1191*** (0.0108)
<i>CHILD04</i>	0.1185 (0.1481)	-1.3423*** (0.3275)	1.0047*** (0.1166)	0.4550*** (0.1121)	0.6041*** (0.1167)
<i>CHILD514</i>	-0.5063*** (0.1064)	0.6258*** (0.2294)	-0.3709*** (0.0834)	0.7974*** (0.0698)	0.3981*** (0.0744)
<i>AGE</i>	0.0103 (0.0063)	-0.0193 (0.0136)	0.0286*** (0.0048)	-0.0095** (0.0038)	-0.0144*** (0.0039)
<i>HDB</i>	-0.8379*** (0.1212)	2.4885*** (0.2702)	-2.1051*** (0.1023)	0.7852*** (0.0846)	-0.1841** (0.0848)
<i>WORK</i>	-0.2924*** (0.0847)	0.7716*** (0.1852)	0.2572*** (0.0670)	-0.4885*** (0.0572)	-0.5401*** (0.0611)
<i>MAID</i>	-1.3321*** (0.0978)	3.0250*** (0.2088)	-1.5975*** (0.0775)	0.7637*** (0.0733)	0.9101*** (0.0764)
<i>CHINESE</i>	0.7632*** (0.1290)	-2.8738*** (0.2655)	-0.8130*** (0.1016)	1.6220*** (0.0875)	1.3200*** (0.0956)
<i>INDIAN</i>	-0.1857 (0.1875)	-0.0901 (0.3906)	-0.3401** (0.1718)	0.8266*** (0.1714)	-0.0444 (0.2184)
<i>FAMILY</i>	0.3613*** (0.1171)	0.4742* (0.2590)	-0.1259 (0.0963)	-0.6978*** (0.0796)	-0.7544*** (0.0850)
<i>FEMALE</i>	0.4363 (0.5824)	-2.5602** (1.1610)	-0.6295 (0.7226)	2.2131*** (0.6857)	2.9543*** (1.0220)
Maximized log-likelihood <sup>b</sup>	-158380.1894				
Average price coefficient	-1.4614				
% of price coefficient > 0	0.0279				

<sup>a</sup>Standard deviation parameters are exponentiated within the log-likelihood function, so that it can enter log-likelihood function positively and can be estimated unconstrained at the same time. Listed parameter estimates are transformed (i.e., exponentiated) values of those unconstrained estimates and standard errors are computed using the delta method.

<sup>b</sup>Standard logit model yields maximized log-likelihood of -163650.1475 and the log-likelihood test rejects it in favor of the full model ( $p < 0.001$ ).

Table 1.6: Price Elasticities of Top 4 Brands of *FairPrice* and *Dairy Farm*: Rice Category

Brand <sup>19</sup>	Double ( <i>F</i> )	Golden Royal Dragon ( <i>F</i> )	NTUC ( <i>F</i> )	Royal Umbrella ( <i>F</i> )	Golden Phoenix ( <i>D</i> )	New Moon ( <i>D</i> )	Royal Umbrella ( <i>D</i> )	Songhe ( <i>D</i> )
Double ( <i>F</i> )	-2.6487	0.1405	0.1490	0.1125	0.1261	0.1288	0.0990	0.1269
Golden Royal Dragon ( <i>F</i> )	0.1610	-2.3005	0.1618	0.1459	0.1577	0.1609	0.1440	0.1711
NTUC ( <i>F</i> )	0.3065	0.2904	-2.2043	0.2204	0.2513	0.2523	0.2260	0.2589
Royal Umbrella ( <i>F</i> )	0.0652	0.0738	0.0621	-2.8337	0.0736	0.0731	0.0768	0.0795
Golden Phoenix ( <i>D</i> )	0.0039	0.0042	0.0038	0.0039	-3.4843	0.0043	0.0038	0.0044
New Moon ( <i>D</i> )	0.0472	0.0515	0.0450	0.0462	0.0516	-2.6647	0.0474	0.0599
Royal Umbrella ( <i>D</i> )	0.0208	0.0263	0.0230	0.0278	0.0259	0.0271	-2.9533	0.0311
Songhe ( <i>D</i> )	0.0254	0.0299	0.0252	0.0274	0.0287	0.0327	0.0297	-3.4400



Price). Setting  $\alpha_{i'} = 0$  for both *Dairy Farm* and *Sheng Siong*,<sup>20</sup> we estimate in this section: 1) the consumer surplus moderating parameter  $\alpha_i$  for *FairPrice* and 2) the marginal costs and markups of all products offered in the market.

In the following 2 subsections, we describe the optimization problem of firms competing in the consumer surplus moderated market. The first subsection recaps the objective function and optimization problem of a profit maximizing firm (*Dairy Farm* and *Sheng Siong*). The second subsection formulates those of a consumer surplus concerned firm (*FairPrice*) and discusses how  $\alpha_i$  can be identified.

### Profit Maximizing Firm

A pure profit maximizing firm  $i'$  maximizes

$$\sum_{j \in \mathcal{J}_{i'}^m} s_j^m \cdot (p_j^m - c_j^m)$$

Thus, each product  $j \in \mathcal{J}_{i'}^m$  satisfies its first order condition:

$$0 = s_j^m + \sum_{j' \in \mathcal{J}_{i'}^m} (p_{j'}^m - c_{j'}^m) \frac{\partial s_{j'}^m}{\partial p_j^m}$$

Let  $\mathbf{c}_{i'}^m$  be the vector of marginal cost and  $\mathbf{s}_{i'}^m$  be the vector of market share of all products offered by firm  $i'$  in market  $m$ . Let  $\Gamma_{i'}^m$  be the own- and cross-price elasticity matrix of firm  $i'$  in market  $m$ , where  $\Gamma_{i'}(j, j') = \frac{\partial s_{j'}^m}{\partial p_j^m}$ ,  $j, j' \in \mathcal{J}_{i'}^m$ . Then, the marginal cost vector  $\mathbf{c}_{i'}^m$  solving the above set of first order conditions is given by

$$\mathbf{c}_{i'}^m = \mathbf{p}_{i'}^m + (\Gamma_{i'}^m)^{-1} \cdot \mathbf{s}_{i'}^m \tag{1.7}$$

Hence, upon observing market prices and correctly identifying the underlying demand model, we can structurally derive the marginal costs of all products offered by profit maximizing firms, i.e., *Dairy Farm* and *Sheng Siong*.

### Consumer Surplus Concerned Firm

We consider a consumer surplus concerned firm  $i$  with consumer surplus moderating parameter  $\alpha_i$ . Identifying marginal costs and  $\alpha_i$  for firm  $i$  is not as simple as the above. This is because there are more degrees of freedom than the number of first order conditions. Specifically, maximizing equation (1.6) is equivalent to solving:

$$\mathbf{c}_i^m = \mathbf{p}_i^m + (\Gamma_i^m)^{-1} \cdot \left( \mathbf{s}_i^m + \frac{\alpha_i}{(1 - \alpha_i)} \cdot \Lambda_i^m \right) \tag{1.8}$$

<sup>20</sup>The estimation of fully general model where  $\alpha_i$  of all three retailers are identified is under way.

where  $\Lambda_i^m$  is a  $|\mathcal{J}_i^m| \times 1$  vector<sup>21</sup> and

$$\Lambda_i^m(j, 1) = \frac{\partial \Phi^m}{\partial p_j^m}, \quad j \in \mathcal{J}_i^m.$$

Note that equation (1.8) is a system of first order conditions that are just as many as the number of products of firm  $i$ . However, what we wish to identify is all of its marginal costs as well as  $\alpha_i$ , the total number of which exceeds the number of first order conditions by 1. Thus, identification becomes infeasible without further information on the firm's marginal cost or its degree of consumer surplus concerns.

Empirically, we go about this issue by utilizing a separate dataset that we obtained from the highest market share national brand rice company: *Royal Umbrella*. The dataset from the *Royal Umbrella* company contains quarterly wholesale prices for all 3 retailers (*FairPrice*, *Dairy Farm*, and *Sheng Siong*) of its rice brand under the same name, *Royal Umbrella*. Availability of wholesale prices on *FairPrice*'s *Royal Umbrella* will effectively reduce 1 degree of freedom that needs be identified and will make feasible identification of the rest of *FairPrice*'s marginal costs as well as its  $\alpha_i$ .

The identification process of  $\alpha_i$  is formulated as follows. Let  $j$  denote the product subscript for *FairPrice*'s *Royal Umbrella*. From equation (1.8), we obtain

$$c_j^m = p_j + (\Gamma_i^m)_j^{-1} \cdot \left( \mathbf{s}_i^m + \frac{\alpha_i}{(1 - \alpha_i)} \cdot \Lambda_i^m \right) \quad (1.9)$$

where  $(\Gamma_i^m)_j^{-1}$  refers to the  $j$ -th row of  $(\Gamma_i^m)^{-1}$ . Inverting equation (1.9), we can identify  $\alpha_i$  as:

$$\hat{\alpha}_i = \frac{p_j^m - c_j^m + (\Gamma_i^m)_j^{-1} \cdot \mathbf{s}_i^m}{p_j^m - c_j^m + (\Gamma_i^m)_j^{-1} \cdot \mathbf{s}_i^m - (\Gamma_i^m)_j^{-1} \cdot \Lambda_i^m}$$

Table 1.7 lists the estimate of  $\alpha_i$  averaged across all markets and the prices and marginal costs of products of all three firms. Marginal costs are listed in the parentheses next to the prices. Out of the 19 products, 4 national brands overlap between *FairPrice* and *Dairy Farm*.<sup>22</sup> *FairPrice*'s consumer surplus moderating parameter  $\hat{\alpha}_i$  is estimated to be about 0.13 on average, suggesting that *FairPrice* weighs consumer surplus to profit in a 1 to 7 ratio. In other words, to *FairPrice*, every \$7 increase in consumer surplus is worth as much as \$1 increase in its profit.

The results show that prices and estimated marginal costs go hand in hand overall. *FairPrice*'s marginal costs are lower in general since its prices are lower. Given their high market share, it is not surprising to see that prices of *FairPrice*'s store brands (*Double*, *NTUC*, and *Golden Royal Dragon*) are cheaper than or very similar to the estimated marginal cost of some national brands such as *Royal Umbrella* and *Songhe*. Estimated marginal costs for the same overlapping brands are in general lower for *FairPrice*, suggesting that *FairPrice* enjoys lower wholesale prices due to its high market share in the grocery market.

<sup>21</sup> $|\mathcal{J}_i^m|$  refers to the cardinality of product space  $\mathcal{J}_i^m$ .

<sup>22</sup>All national brands except for the store brands are overlapping in the data but some are excluded from the empirical estimation (e.g., *FairPrice*'s *Golden Pineapple*) since its market share is negligible.

Table 1.7: Estimation Results in Rice Category: Supply

Consumer Surplus Moderating Parameter ( $\hat{\alpha}_i$ )			
	<i>FairPrice</i>		
	0.1262***		
	(0.0168)		
Prices (Marginal Costs) <sup>a</sup>			
Brand <sup>b</sup>	<i>FairPrice</i>	<i>Dairy Farm</i>	<i>Sheng Siong</i>
Double	2.0027 (1.0906)	–	–
Golden Royal Dragon	1.7284 (0.8451)	–	–
NTUC	1.6685 (0.8192)	–	–
Happy	–	–	1.4970 (0.9282)
Nangrum	–	–	1.7437 (1.1302)
Royal Golden Grain	–	–	2.1726 (1.4673)
Golden Phoenix	2.3390 (1.4838)	2.7500 (1.9146)	–
New Moon	2.3573 (1.3561)	2.1211 (1.2950)	2.3823 (1.4735)
Royal Umbrella	2.4813 (1.4906)	2.5497 (1.6311)	2.5117 (1.5597)
Songhe	2.4917 (1.5884)	2.5104 (1.7340)	2.4943 (1.6222)
Golden Pineapple	–	2.0867 (1.4642)	1.9384 (1.2993)

<sup>a</sup>Prices and marginal costs are with respect to 1kg.

<sup>b</sup>First 6 rows list storebrands; the rest are national brands.

## Counterfactual Analysis

In this section, we conduct a counterfactual analysis of our model setting  $\alpha_i = 0$ , i.e., *FairPrice* is purely profit driven like other firms and our model reduces to the standard model of price competition. We study 3 aspects of this counterfactual analysis: 1) counterfactual equilibrium prices, 2) counterfactual profit level of all firms, and 3) counterfactual consumer surplus level and the decomposition of surplus gain due to consumer surplus concerns into the direct effect of  $\alpha_i$  and indirect effect of price competition among firms.

Let us first briefly investigate why the standard price competition model is not likely to correctly describe the pricing pattern of *FairPrice* observed in the data. We test this by artificially setting  $\alpha_i$  to zero and estimate *FairPrice*'s marginal costs. Table 1.8 juxtaposes the marginal costs and product markups of *FairPrice* predicted by the standard model with those predicted by our model. As shown, the marginal costs of the top 2 brands, *NTUC* and *Golden Royal Dragon*, are estimated to be really low at 0.93 and 0.68 respectively, yielding unreasonably high markups of 133.00% and 171.58% respectively. Note that when the same standard model is applied to *Dairy Farm* and *Sheng Siong*, the estimated marginal costs and markups are in a sensible range. Their markups from 43.49% to 75.79%. Were the standard model to predict the data well, it would not give such distinctively different ranges of estimated markups between *FairPrice* and other firms. Thus, the standard price competition model may not be adequate to capture the underlying pricing behavior

Table 1.8: Estimated Marginal Costs and Markups setting  $\alpha_i = 0$ .

Brand	$\alpha_i = 0$	$\alpha_i = 0.13$		
	<i>FairPrice</i>	<i>FairPrice</i>	<i>Dairy Farm</i>	<i>Sheng Siong</i>
Double	0.9283 (133.00%)	1.0906 (84.21%)	–	–
Golden Royal Dragon	0.6850 (171.58%)	0.8451 (106.33%)	–	–
NTUC	0.6657 (168.34%)	0.8192 (105.04%)	–	–
Golden Phoenix	1.3298 (84.65%)	1.4838 (57.93%)	1.9146 (43.49%)	–
New Moon	1.1757 (115.20%)	1.3561 (74.55%)	1.2950 (63.87%)	1.4735 (61.69%)
Royal Umbrella	1.3152 (102.54%)	1.4906 (66.71%)	1.6311 (56.34%)	1.5597 (61.06%)
Songhe	1.4275 (84.22%)	1.5884 (57.04%)	1.7340 (44.85%)	1.6222 (53.89%)

Table 1.9: Counterfactual Equilibrium Prices setting  $\alpha_i = 0$ 

Brand	$\alpha_i = 0$			$\alpha_i = 0.13$		
	<i>FairPrice</i>	<i>Dairy Farm</i>	<i>Sheng Siong</i>	<i>FairPrice</i>	<i>Dairy Farm</i>	<i>Sheng Siong</i>
Double	2.1246*	–	–	2.0027	–	–
Golden Royal Dragon	1.8508*	–	–	1.7284	–	–
NTUC	1.7734*	–	–	1.6685	–	–
Happy	–	–	1.4968	–	–	1.4970
Nangrum	–	–	1.7423	–	–	1.7437
Royal Golden Grain	–	–	2.1679	–	–	2.1726
Golden Phoenix	2.4457*	2.7315	–	2.3390	2.7500	–
New Moon	2.5179*	2.1070	2.3615	2.3573	2.1211	2.3823
Royal Umbrella	2.6427*	2.5315	2.4940	2.4813	2.5497	2.5117
Songhe	2.6124*	2.5013	2.4844	2.4917	2.5104	2.4943
Golden Pineapple	–	2.0790	1.9345	–	2.0867	1.9384

of firms shown in the data.

We now describe each of the 3 aspects of the counterfactual analysis of our model. First, Table 1.9 shows the counterfactual equilibrium prices of all firms when  $\alpha_i$  is set to zero (i.e., *FairPrice* only maximizes its profit) as well as the observed market prices where  $\alpha_i = 0.13$ . Prices that increase under the counterfactual analysis are marked with asterisks. The counterfactual analysis reveals that if *FairPrice* were to be profit maximizing, prices of all of its products would increase by 6.09% on average. That the store brands' prices would increase the most suggests that *FairPrice* would enjoy much higher markups for store brands but it is letting them go due to its consumer surplus concerns. On the other hand, prices of *Dairy Farm* and *Sheng Siong* all decrease in response to *FairPrice*'s increased prices. As a consequence, price dispersion among firms has widened.

Next, we investigate percentage changes in all firms' profit, consumer surplus and total surplus. Table 1.10 compares profits when  $\alpha_i = 0$  with those when  $\alpha_i = 0.13$ . As shown, if *FairPrice* were profit maximizing, the profit of *FairPrice*, *Dairy Farm* and *Sheng Siong* would all increase by 1.16%, 5.54% and 6.47% respectively. It is interesting that these profits increase for quite different reasons: *FairPrice*'s profit increases because it increases prices for almost all of its products as in Table 1.9. Note that higher prices induce two opposite effects on the profit: the *market share effect* and *surplus extraction effect*. First, the *market share effect* means that higher prices decrease market share. Second, the *surplus extraction effect* means that the firm extracts more surplus for each

Table 1.10: Counterfactual Analysis: Profit, Consumer Surplus and Total Surplus

	$\alpha_i = 0.13$	$\alpha_i = 0$	% Change
Profit <sup>a</sup>			
<i>FairPrice</i>	0.3014	0.3049	+1.16%
<i>Dairy Farm</i>	0.0325	0.0343	+5.54%
<i>Sheng Siong</i>	0.0433	0.0461	+6.47%
Consumer Surplus <sup>b</sup>	0.5091	0.4725	-7.18%
Total Surplus <sup>c</sup>	0.8862	0.8578	-3.21%

<sup>a</sup>Expected per capita profit for unit weight (1kg) products in dollar terms is listed.

<sup>b</sup>Expected per capita consumer surplus resulting from consumption of a unit weight (1kg) product is computed. The unit is in dollar terms, the same as profit.

<sup>c</sup>Total surplus is defined as the sum of producer surplus and consumer surplus. The quantity sold at each retailer under the counterfactual policy scenario remains unchanged. Robustness checks where quantities change according to quantity discount scheme yielded similar results.

product sold due to higher price. Under the profit maximization objective, *FairPrice* increases its prices so that the surplus extraction effect overrides the market share effect, which in turn increases its profit. On the contrary, the other two firms' profit increases despite their lower prices due to the market share effect.

In addition, Table 1.10 shows that consumer surplus would decrease by 7.18% and the total surplus, which is defined as the sum of profit (i.e., retailer surplus) and consumer surplus, would decrease by 3.21%. The latter is particularly worth highlighting since it suggests that profit maximization by all three retailers decreases total surplus in the market in spite of increase profit level of all retailers, and thus induces an inefficient outcome for the market.

Lastly, we decompose the loss in consumer surplus into two effects: direct and indirect effects. In a nutshell, the direct effect captures the sole effect of profit maximizing behavior of *FairPrice*. That is, it captures how much the consumer surplus would decrease due to loss of consumer surplus concerns by *FairPrice*. As shown in theorem 1.1, *FairPrice* would increase prices of *all* of its products as it becomes more profit concerned. Note that this will then be followed by price competition among firms and the new price equilibrium will be reached ultimately. The indirect effect captures the effect of such price competition that follows.

Let us formulate the direct and indirect effect. Let  $\mathbf{p}_i^m(\alpha_i)$  and  $\mathbf{p}_{-i}^m(\alpha_i)$  be respectively the equilibrium prices of *FairPrice* and those of the other firms in market  $m$  when *FairPrice*'s consumer surplus moderating parameter is  $\alpha_i$ . Further, let  $BR_i(\mathbf{p}_{-i}^m | \alpha_i)$  be *FairPrice*'s prices that best respond to  $\mathbf{p}_{-i}^m$  given  $\alpha_i$ , i.e.,  $BR_i(\mathbf{p}_{-i}^m | \alpha_i)$  optimizes its objective function given  $\mathbf{p}_{-i}^m$  and  $\alpha_i$ . Then,

$$\begin{aligned} \text{Direct Effect} &= \Phi^m(BR_i(\mathbf{p}_{-i}^m(\alpha_i) | 0), \mathbf{p}_{-i}^m(\alpha_i)) - \Phi^m(\mathbf{p}_i^m(\alpha_i), \mathbf{p}_{-i}^m(\alpha_i)) \\ \text{Indirect Effect} &= \Phi^m(\mathbf{p}_i^m(0), \mathbf{p}_{-i}^m(0)) - \Phi^m(BR_i(\mathbf{p}_{-i}^m(\alpha_i) | 0), \mathbf{p}_{-i}^m(\alpha_i)) \end{aligned}$$

Table 1.11: Decomposition of Consumer Surplus

Total Effect	-0.0366
Direct Effect	-0.0375
Indirect Effect	+0.0009

Table 1.12: Validity Check of Consumer Surplus Moderated Model

(Unit: SGD per 1kg)	2008	2009				2010			
	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Wholesaler’s marginal cost	1.17	1.14	1.17	1.17	1.15	1.20	1.11	1.04	1.06
Retailer’s marginal cost (from standard model)	0.95	1.03	0.90	1.01	1.09	1.11	1.11	1.05	1.07
Retailer’s marginal cost (from our model)	1.39	1.25	1.27	1.39	1.44	1.42	1.44	1.19	1.30

Table 1.11 shows the direct and indirect effect of decreasing in  $\alpha_i$  from 0.13 to 0. Two aspects are worth highlighting. First, the size of indirect effect (0.0009) is marginal so that the total surplus loss at the new price equilibrium (0.0375) is 97.60% of the direct effect (0.0366), which is what *FairPrice* would achieve due to  $\alpha_i$  decreasing to 0 were other firms’ prices to remain unchanged, i.e., no subsequent price competition. Second, we see that the indirect effect is positive at +0.0009. This is because *Dairy Farm* and *Sheng Siong* decrease its prices as shown in Table 1.9, and behave as if they are strategic substitutes of *FairPrice*.

### Validity Check

In this section, we conduct a validity check using a separate dataset on the quarterly aggregate import price of rice obtained from the International Enterprise of Singapore.<sup>23</sup> Through this validity check, we confidently reject the standard model of price competition in favor of the consumer surplus moderated model.

Singapore is a small city country that does not have enough land to produce rice. Thus, it imports its entire rice from other countries, mostly from Thailand. As a consequence, the import price of rice represents the (minimum) marginal cost for wholesalers, which is listed in the 1st row of Table 1.12. On the other hand, the marginal costs of retailers recovered from the standard model (listed in the 2nd row) or our model (listed in the 3rd row) represent the (maximum) price at which the wholesalers may sell rice to the retailers. Considering the wholesalers’ positive markup, these recovered marginal costs of retailers must be higher than the import price of rice.

<sup>23</sup>International Enterprise of Singapore is a statutory board under the Ministry of Trade and Industry of the Singapore Government that facilitates the overseas growth of Singapore-based companies and promotes international trade. All prices and quantities of rice imported into Singapore should be reported to International Enterprise of Singapore.

Nonetheless, as shown in Table 1.12, the marginal costs recovered from the standard model are lower than the import price of rice in almost all quarters, or only marginally bigger. To the contrary, marginal costs recovered from our consumer surplus moderated model are reasonably bigger than the import price of rice, likely reflecting the wholesalers' markup. As a consequence, we confidently reject the standard model of price competition in favor of our model and validate that our model better explains the retailers' pricing pattern of rice in the Singaporean grocery retail market.

## 1.5 Conclusion

The assumption that firms are interested only in maximizing their own profit has been the pillar of standard price competition models. However, when government actively intervenes or participates in a market, this assumption may not capture the behavior of firms well because some firms may choose prices to maximize a weighted sum of profit and consumer surplus. In such markets, standard models may wrongly predict the outcome of competition or produce systematic biases in parameter estimates.

This paper develops a new structural model of consumer surplus moderated price competition. Since the measure for consumer surplus is explicitly derived as the sum of consumers' net utility from all possible purchase scenarios, it is theoretically more sound and empirically more accurate than other surplus measures used in prior literature. Our model nests standard price competition models as special cases. It allows one to empirically estimate not only the degree of consumer surplus concerns a firm has but also the associated gain in total consumer surplus. Theoretically, our model predicts that total consumer surplus increases whenever the price of any product decreases, and *ceteris paribus*, a firm would always decrease all of its product prices as its concerns for consumer surplus increase. The competitive response by other firms may be either more or less aggressive.

We also apply our model to the Singapore grocery retail market data, where the dominant retailer, *FairPrice*, publicly commits to consumer surplus concerns. Particularly, we investigate the rice category, which is one of the primary nondiscretionary categories within the country. We find that 1) *FairPrice*'s consumer surplus concern is estimated to be about 0.13, 2) standard price competition model would predict *FairPrice*'s product marginal costs to be implausibly low compared with those of its competitors, 3) under the profit maximization objective, *FairPrice*'s profit would increase by 1.16% and the total consumer surplus would decrease by 7.18%. 4) the indirect effect of *FairPrice*'s profit maximization is positive, suggesting that competitors respond to *FairPrice*'s higher price level by lowering their prices, i.e., behave as *FairPrice*'s strategic substitutes. Even though the total consumer surplus gain is mitigated by the negative indirect effect, the total consumer surplus gain is retained at 97.60% of the direct effect.

## Chapter 2

# Level- $r$ Model with Adaptive and Sophisticated Learning<sup>1</sup>

### 2.1 Introduction

In dominance solvable games, standard game theory prescribes that players should iteratively remove dominated strategies. At each iteration, players' strategy space becomes smaller because some strategies are eliminated. This iterative process continues until each player's strategy space is left with only one action, and this surviving action is the unique iterative dominance solution that every player should choose. Such a prescription, however, hinges on the crucial assumption that *every* player is certain that *all* other players will remove dominated strategies at *each* step of the iterative process (Bernheim 1984; Pearce 1984). If players believe that some player may stop this iterative process prematurely, then the unique iterative dominance solution may poorly predict players' actual behavior.

Indeed, there is accumulating experimental evidence that casts doubt on this sharp iterative dominance prediction (Nagel, 1995; Stahl and Wilson, 1994, 1995; Ho et al., 1998; Costa-Gomes et al., 2001; Bosch-Doménech et al., 2002; Costa-Gomes and Crawford, 2006; Costa-Gomes and Weizsäcker, 2008). Despite nontrivial financial incentives, subjects frequently do not play the unique iterative dominance solution. A well-studied example is the  $p$ -beauty contest game. In this game, players simultaneously choose numbers ranging from 0 to 100, and the winner is the player who chose the number closest to the *target number*, defined as  $0 < p < 1$  times the average of all players' choices. A fixed reward goes to the winner, and is divided evenly among winners in the case of a tie. It is straightforward to show that the only number that survives the iterative elimination process is 0, which is the unique iterative dominance solution.<sup>2</sup> In laboratory experiments,

<sup>1</sup>This chapter is a joint work with Teck Hua Ho and Xuanming Su.

<sup>2</sup>To illustrate, let us assume  $p = 0.7$ . Strategies between 70 and 100 are weakly dominated by 70 and thus are eliminated at the first step of elimination of dominated strategies. Likewise, strategies between 49 and 70 are eliminated at the second step of elimination as they are weakly dominated by 49. Ultimately, the only strategy that survives the iterative elimination of dominated strategies is zero, which is the unique iterative dominance solution of the game.



however, subjects' choices are initially far from equilibrium but move closer to it over time (Ho et al., 1998).<sup>3</sup>

Cognitive hierarchy (Camerer et al., 2004) and level- $k$  (Nagel, 1995; Stahl and Wilson, 1994, 1995; Costa-Gomes et al., 2001; Costa-Gomes et al., 2009; Costa-Gomes and Crawford, 2006; Crawford, 2003; Crawford and Iriberri, 2007b) models have been proposed and used to explain nonequilibrium behaviors in experimental games (for a comprehensive review, see Crawford et al., 2013). Both models assume players choose rules from a well-defined discrete rule hierarchy. The rule hierarchy is defined iteratively by assuming that a level- $k$  rule best-responds to lower level rules (e.g., level- $(k - 1)$ , where  $k \geq 1$ ). The level-0 rule can be specified a priori either as uniform randomization among all possible strategies or as the most salient action derived from the payoff structure.<sup>4</sup>

Let us illustrate how this class of nonequilibrium models works in the  $p$ -beauty contest game with  $p = 0.7$  and  $n = 3$  players, using the standard level- $k$  model where a level- $k$  rule best-responds only to the level- $(k - 1)$  rule. Assume that the level-0 rule randomizes between all possible choices between 0 and 100 and thus chooses 50 on average. Best-responding to the level-0 opponents, the level-1 rule seeks to hit the exact target number in order to maximize her payoff. Specifically, the level-1 rule's choice  $x^*$  solves:

$$x^* = p \cdot \frac{x^* + (n - 1) \cdot 50}{n}.$$

Hence,

$$x^* = \frac{p \cdot (n - 1)}{n - p} \cdot 50 = \frac{0.7 \cdot 2}{3 - 0.7} \cdot 50 = 30.4$$

and the level-1 rule chooses  $x^* = 30.4$  as a best response to its level-0 opponents, taking into account her own influence on the group mean.<sup>5</sup> In general, a level- $k$  rule will choose  $\left(\frac{p \cdot (n - 1)}{n - p}\right)^k \cdot 50 = \left(\frac{0.7 \cdot 2}{3 - 0.7}\right)^k \cdot 50$ .

As a consequence, standard level- $k$  models predict that we should see spikes in the distribution of choices at 50, 30.4, 18.5, 11.3, 6.9, 4.2, and so forth if the level-0 rule chooses 50. That is, only a limited number of numbers will be chosen by subjects. Figure 2.1 shows the data of first-round choices from Ho et al. (1998). As opposed to the prediction, we do not see clear spikes in the first round choices except for the single peak around 50. In fact, the proportion of choices falling into

<sup>3</sup>Specifically, 2% of the choices were iterative dominance solution plays in the first round and the proportion grew to 13% in the last (10th) round in Ho et al. (1998).

<sup>4</sup>The level- $k$  and cognitive hierarchy models have also been applied to study games with asymmetric information such as zero-sum betting or auctions or strategic information disclosure (e.g., Camerer et al., 2004; Crawford and Iriberri, 2007a.; Brocas et al., 2010; Brown et al., 2010; Östling et al., 2011). Crawford et al. (2009) employ level- $k$  models to determine the optimal design for auction when bidders are boundedly rational. Goldfarb and Yang (2009) and Goldfarb and Xiao (2011) apply the cognitive hierarchy model to capture heterogeneity in firms' or managers' ability and use it to predict a firm's long-term success in oligopolistic market.

<sup>5</sup>Some prior research (e.g., Nagel, 1995) ignores one's own influence on the group average by simply assuming that level-1 chooses  $35 = 0.7 \cdot 50$  and level  $k$  chooses  $p^k \cdot 50$  in general. This approximation is good only when  $n$  is large. For example, when  $n = 100$ , level-1's exact best response is 34.89, which is close to 35.

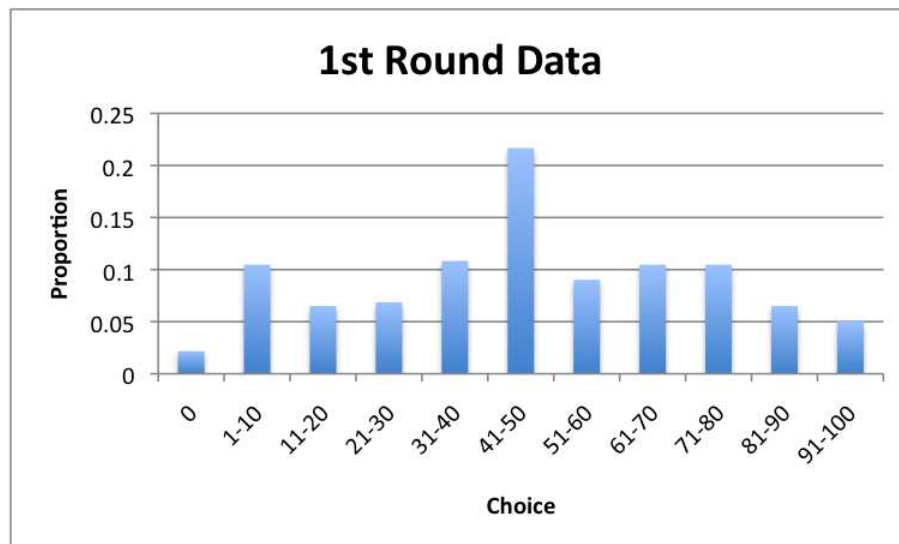


Figure 2.1: First Round Data

rule levels 0 and above including the unique iterative dominance solution (i.e., choices in the intervals,  $[49.5, 50.5]$ ,  $[29.9, 30.9]$ ,  $[18.0, 19.0]$ ,  $[10.8, 11.8]$ ,  $[6.4, 7.4]$ ,  $[3.7, 4.7]$  and so forth, down to  $[0, 0.5]$ ) is only 25.0%.<sup>6</sup> Put differently, the distribution of choices appears rather continuous with choices sprinkled around all possible numbers in the strategy space. The only way to account for numbers which do not correspond to discrete rules (e.g., choice such as 20) is to assume error in players' choices. It would however require a high error rate in subjects' choices and one must allow for different error rates for different rule levels in order to match the data well.

An alternative way to account for the observed data pattern is to allow players to choose from a continuous, instead of discrete, rule hierarchy. Under the continuous rule hierarchy, a level- $(r + 1)$  rule best-responds to a level- $r$  rule, where  $r \in \mathbb{R}_0^+$ . That is, a level- $r$  rule will choose  $\left(\frac{p \cdot (n-1)}{n-p}\right)^r \cdot 50$  if the level-0 rule is assumed to choose 50. As a consequence, any chosen number that is less than the level-0 rule will always correspond to a rule in the continuous rule hierarchy. For example, 20 corresponds to rule level  $\log_{p \cdot (n-1)/(n-p)}\left(\frac{20}{50}\right) = 1.85$  in the 3-player  $p$ -beauty contest game with  $p = 0.7$  (note that number 20 would not be admissible if players are restricted to choose from a discrete rule hierarchy). The continuous rule hierarchy nests the discrete rule hierarchy as a special case and makes the level- $k$  model better suited for games with a continuous strategy space. Also, the distribution of choices is interpreted more as heterogeneity in players' rule levels rather than as error rates in their choices. For these reasons, we employ a continuous rule hierarchy throughout this paper.

The static rule hierarchy models, either continuous or discrete, face another challenge in explaining

<sup>6</sup>If we maximize the proportion of choices that fall within integral rule levels by varying the level-0 rule, we find that the optimal level-0 rule is 89.8 and the maximized proportion is 28.6%.

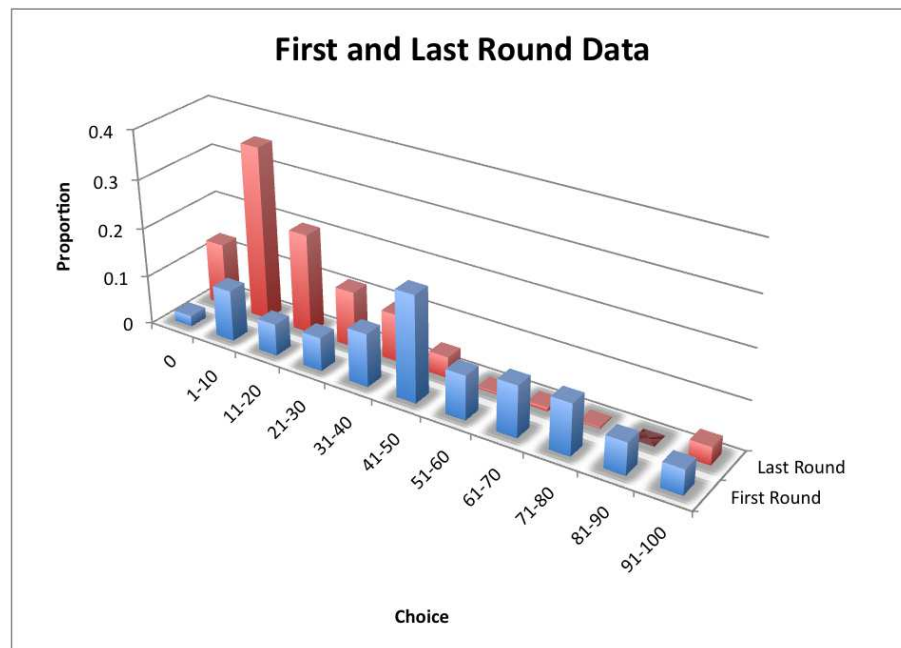


Figure 2.2: First and Last Round Data

the  $p$ -beauty contest game data. All static rule hierarchy models posit that players' rule levels are fixed. This implies that if a player is level- $r$ , she will choose the same rule in every round. As a result, these models predict that the distribution of choices should remain fixed as the proportion of players choosing each rule level remains unchanged over time. Figure 2.2 compares the data of the first round with that of the last round from Ho et al. (1998). As shown, subjects' choices move much closer to equilibrium over time. A Kolmogorov-Smirnov test rejects the null hypothesis that the two distributions of choices are identical ( $p < 0.001$ ). As a consequence, any model seeking to capture this shift in behavior over time needs to be dynamic in nature.

One way to capture subjects' dynamic behavior is to allow the level-0 players to change their choices over time by adapting to their environment. Formally, level-0 players in each round are assumed to choose a weighted average of past *ex post* best responses so that level-1 players will follow an adaptive learning process similar to that of the weighted fictitious play model (Fudenberg and Levine, 1998). As a result of level-0 players' adaptation, the choices of level-1 and higher rule levels change over time, so we have an *adaptive* rule hierarchy mapping that adapts to historical game plays. Specifically, in the  $p$ -beauty contest game, the *ex post* best responses are the target numbers. As long as the target number becomes smaller over time, this static level- $r$  model with an adaptive rule hierarchy predicts that choices will converge to 0 over time, explaining the main feature of the data.

Nevertheless, the static level- $r$  model with an adaptive rule hierarchy continues to assume that players' rule levels remain fixed over time. The fixed rule levels constrain level- $r$ 's adaptation over time in a specific way. Let us consider an example. Suppose that the level-0 rule's choice was 50 in

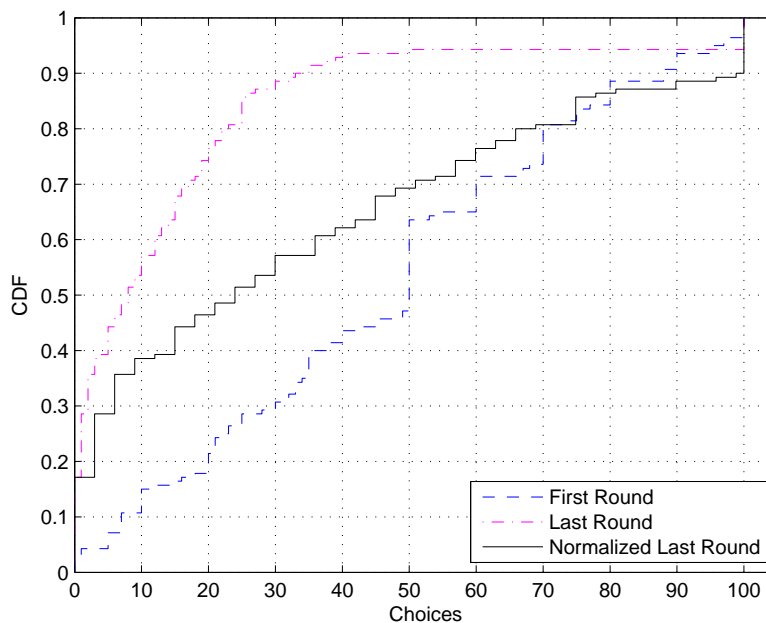


Figure 2.3: Distribution of First, Last, and Normalized Last Round Data

the first round and it became 25 in the last round. Hence, the level- $r$  players choose  $\left(\frac{p \cdot (n-1)}{n-p}\right)^r \cdot 50$  in the first round and  $\left(\frac{p \cdot (n-1)}{n-p}\right)^r \cdot 25$  in the last round. If we normalize level- $r$ 's choices in the last round by  $\frac{50}{25}$  (i.e., ratio of level-0 choices in the first and last rounds), we observe that level- $r$ 's first-round choices and normalized last-round choices become identical. Put differently, the static level- $r$  model with an adaptive rule hierarchy makes a sharp prediction: the distribution of first-round choices and the *normalized* last-round choices are identical. We can readily test this sharp prediction using data from Ho et al. (1998). A Kolmogorov-Smirnov test rejects the null hypothesis that data from the first round and the normalized data from the last round come from the same distribution ( $p < 0.001$ ). Figure 2.3 shows the distributions of the first-round choices, the last-round choices, and the normalized last-round choices. Contrary to the prediction of the static level- $r$  model with an adaptive rule hierarchy, the distribution of the normalized last-round choices is to the left of that of the first-round choices, suggesting that players' rule levels are higher in the last round than in the first round and that weighted fictitious play models under-predict the speed of convergence towards dominance solvable solution.

A sensible way to account for the systematic shift in players' normalized choices is to allow players to become more sophisticated (i.e., increase their rule levels) over time. Specifically, players are assumed to form beliefs about what rules opponents are likely to use, update their beliefs after each round based on their observations, and best-respond to their updated belief in the following round. Consequently, the distribution of players' rule levels change over time. To the extent that players

move up the rule hierarchy over time, this dynamic level- $r$  model with an adaptive rule hierarchy will be able to explain the leftward shift of the normalized last round choices.<sup>7</sup>

In this paper, we propose a model that generalizes the standard level- $k$  model in 3 significant ways:

1. Our model utilizes a continuous rule hierarchy. This rule hierarchy is a natural extension of the integral rule levels to all of  $\mathbb{R}_0^+$ . As before, rule levels are iteratively defined such that a level- $(r + 1)$  rule is a best response to a level- $r$  rule, where  $r \in \mathbb{R}_0^+$ .
2. We define the level-0 rule's choice in each round as the weighted average of *ex post* best responses in previous rounds. Since the level-0 rule's choice evolves according to historical game play in each round, the entire rule hierarchy mapping becomes dynamic in an adaptive manner.
3. Players' rule levels are dynamic and vary over time. Specifically, players update their beliefs about opponents' rule levels based on past observations and best-respond to their beliefs in each round. Allowing for dynamic rule levels also fixes a drawback of weighted fictitious play models, which often predict a slower convergence than actual behaviors observed in experiments.

The proposed model unifies and generalizes two seemingly distinct streams of research on nonequilibrium models: 1) level- $k$  models and 2) belief-based learning models. As a consequence, it is quite general, nesting many well-known nonequilibrium models as special cases. For instance, when players' rule levels are static and the rule hierarchy mapping is fixed over time, we have a static level- $r$  model (which generalizes the standard level- $k$  model). When the rule hierarchy is allowed to change but players' rule levels remain fixed, our model captures the notion of adaptive learning; in particular, when all players are level-1 and best-respond to level-0 players, the model becomes the weighted fictitious play model. When the level-0 rule is fixed, our model reduces to a dynamic level- $r$  model with a fixed rule hierarchy (which generalizes a dynamic level- $k$  model developed by Ho and Su, 2012). This nested model captures the notion of sophisticated learning, i.e., players learn about opponents' rule levels and change their own rules over time.<sup>8</sup> Moreover, our model always converges to the unique iterative dominance solution through adaptive or sophisticated learning process. Hence, the proposed model can be considered as a model of equilibration process in dominance solvable games.

We apply our model to explain players' dynamic behavior in the  $p$ -beauty contest game, and show that the general model describes subjects' behavior better than all its special cases. In addition, the estimation results reveal that prior adaptive learning models (which ignore sophisticated learning)

<sup>7</sup>Ho and Su (2012) is the first to develop a dynamic level- $k$  model to account for subjects' dynamic behavior in repeated games. In their model, however, the rule hierarchy is discrete and fixed over time. They show that their model can explain learning behavior in the centipede game well.

<sup>8</sup>Camerer, Ho and Chong (2002) study 'sophistication and strategic teaching' in games. Their notion of sophistication is very different from the notion of sophisticated learning discussed here. Sophisticated players in their paper do not learn and change their rule level over time. They behave like equilibrium players in that they take into account the fact that one segment of players are adaptive and another segment of players are sophisticated like themselves.

might have produced systematically biased parameter estimates, which would have characterized the underlying adaptive learning dynamics wrongly. We further examine the robustness of our results by applying our model to a generalized price matching game. We fit our model to freshly generated experimental data of this game, and show that our general model again fits the data best.

The rest of the paper is organized as follows. Section 2 formulates the general model and describes its well-known special cases. Section 3 applies the model to the  $p$ -beauty contest game and presents empirical results of the best fitted model and its special cases. Section 4 applies the model to a generalized price matching game and explains players' dynamic behavior in the game. Section 5 concludes.

## 2.2 Model

### Notations

We consider the class of  $n$ -player  $p$ -beauty contest games where  $p \in (0, 1)$ . Players are indexed by  $i$  ( $i = 1, \dots, n$ ), and their strategy space is a closed interval denoted by  $S = [L, U]$  where  $0 \leq L < U$ . Players play the game repeatedly for a total of  $T$  rounds. Player  $i$ 's choice at time  $t$  is denoted by  $x_i(t) \in S$ . The vector of all players' choices at time  $t$  excluding player  $i$ 's is denoted by  $\mathbf{x}_{-i}(t) = (x_1(t), \dots, x_{i-1}(t), x_{i+1}(t), \dots, x_n(t))$ .

### Rule Hierarchy Mapping

Players choose rules from a well-defined rule hierarchy. The *rule hierarchy mapping* at round  $t$  associates player  $i$ 's rule level  $r_i(t) \in \mathbb{R}_0^+$  with her choice  $x_i(t) \in S$ . The rule hierarchy mapping is defined iteratively and begins with specifying the level-0 rule's corresponding choice  $x_0(t)$ . Formally, given the level-0 player's choice  $x_0(t)$ , the rule hierarchy mapping is defined as a function  $M(\cdot|x_0(t)) : \mathbb{R}_0^+ \rightarrow S$ , where  $x_i(t) = M(r_i(t)|x_0(t))$ . Hence, if player  $i$  chooses a rule level  $r_i(t)$  at round  $t$ , her choice implied by the rule hierarchy mapping is  $x_i(t) = M(r_i(t)|x_0(t))$ .

In the rule hierarchy mapping, rule level  $r(t) + 1$  best-responds rule level  $r(t)$  (Stahl and Wilson, 1994; Nagel, 1995; Crawford et al., 2013)<sup>9</sup>. Therefore, in the  $n$ -player  $p$ -beauty contest game, rule  $r(t) + 1$  is defined iteratively as a function of rule  $r(t)$  as follows<sup>10</sup>:

$$M(r(t) + 1|x_0(t)) = p \cdot \frac{M(r(t) + 1|r_0(t)) + (n - 1) \cdot M(r(t)|r_0(t))}{n}.$$

Hence,

$$M(r(t) + 1|x_0(t)) = \frac{p \cdot (n - 1)}{n - p} \cdot M(r(t)|r_0(t)).$$

<sup>9</sup>Stahl (1996 and 2000) posits a rule learning theory where players adjust rules over time based on past experience. His model does not distinguish between adaptive and sophisticated learning and does not nest the standard static level- $k$  model as a special case.

<sup>10</sup>Like standard level- $k$  models, our rule  $r(t) + 1$  assumes all other players are of rule  $r(t)$ .

Thus, defining rule levels as iterative best responses, we posit the following rule hierarchy mapping:

$$M(r(t)|x_0(t)) = \left( \frac{p \cdot (n-1)}{(n-p)} \right)^{r(t)} \cdot x_0(t)$$

Note the following:

1. This rule hierarchy mapping is a one-to-one function from  $\mathbb{R}_0^+$  to  $S$ .<sup>11</sup> As a result, any rule in the continuous rule space is always mapped to a unique action in the strategy space.
2. The standard level- $k$  rule hierarchy as iterative best responses is typically defined similarly as  $M(k|x_0(t)) = \left( \frac{p \cdot (n-1)}{(n-p)} \right)^k \cdot x_0(t)$  where  $k$  are nonnegative integers. As a consequence, the standard level- $k$  rule hierarchy is a special case of the level- $r$  rule hierarchy. While the former only admits discrete rule values, the latter admits all possible rule values in the continuous space.
3. Unlike the standard level- $k$  rule hierarchy (where the level-0 rule is the only anchor), the level- $r$  rule hierarchy has any rule  $r(t) \in [0, 1)$  as a possible anchor to generate the rules  $r(t) + 1, r(t) + 2, r(t) + 3$ , and so on. For instance, if  $x_0(t) = 50$ ,  $p = 0.7$  and  $n = 3$ , then  $M(0.5|x_0(t)) = \left( \frac{0.7 \cdot 2}{3-0.7} \right)^{0.5} \cdot 50 = 39.01$ . Then, one can easily generate actions corresponding to rules 1.5, 2.5, 3.5, and so forth through iterative best responses by using rule level 0.5 as the anchor.

### Payoff-Relevant Statistic

In the  $p$ -beauty contest game, player  $i$  does not need to know the entire vector of  $\mathbf{x}_{-i}(t)$  in order to formulate her best response that maximizes her payoff in round  $t$ . Instead, player  $i$  only needs to know a *payoff-relevant statistic*  $z_{-i}(t)$ , which is a function of  $\mathbf{x}_{-i}(t)$ , in order to determine her best response. As a consequence, player  $i$  can derive her best response from simply knowing  $z_{-i}(t)$ . Conceptually, we can think of this as if each player groups her opponents into a single *aggregate opponent*. The aggregate opponent of player  $i$  chooses an action  $z_{-i}(t) \in S$  at time  $t$ , which contains all the payoff-relevant information for player  $i$ .

In the  $p$ -beauty contest game, each player seeks to win by exactly hitting the target number that is equal to  $p \cdot \frac{\sum_{i=1}^n x_i(t)}{n}$ . Hence, player  $i$ 's best response  $x_i^*(t)$  to her opponents' choices solves:

$$x_i^*(t) = p \cdot \frac{x_i^*(t) + \sum_{j \neq i} x_j(t)}{n}.$$

<sup>11</sup>Here we slightly abuse the term ‘‘one-to-one’’ meaning that the rule hierarchy mapping is one-to-one in the subspace  $[0, r_{max}] \in \mathbb{R}_0^+$  where the level- $r_{max}$  rule is the lowest rule level corresponding to the iterative dominance solution. Note that if  $L = 0$  and  $S = [0, U]$ , the iterative dominance solution is 0 and  $r_{max} = \infty$ , so the rule hierarchy mapping is indeed one-to-one in  $\mathbb{R}_0^+$ . However, if  $L > 0$ , the rule hierarchy mapping is one-to-one in  $[0, r_{max}]$  where  $r_{max} < \infty$ , and the rule hierarchy mapping maps all higher rule levels  $r > r_{max}$  to the iterative dominance solution as well.

Then, player  $i$ 's payoff maximizing strategy<sup>12</sup> is:

$$x_i^*(t) = \frac{p \cdot (n-1)}{n-p} \cdot \frac{\sum_{j \neq i} x_j(t)}{n-1},$$

which is a function of  $\frac{1}{n-1} \cdot \sum_{j \neq i} x_j(t)$ . As a consequence, we define the payoff-relevant statistic as the average of the opponents' choices:

$$z_{-i}(t) = \frac{1}{n-1} \cdot \sum_{j \neq i} x_j(t).$$

In other words, in order to figure out their best responses that maximize the payoff in the  $p$ -beauty contest game, players only need to know their opponents' average choice. As a result, players only need to predict the likely payoff-relevant statistic or the likely average choice of their opponents in each round in order to maximize their payoff (how this prediction is modeled will be described shortly). Conceptually, we can think of the payoff-relevant statistic as an action coming from an *aggregate opponent*, which summarizes the behavior of all opponents.

### Belief and Best Response

Players form beliefs about what rules their aggregate opponents choose and use these beliefs to develop a best response in each round. Let player  $i$ 's belief about the aggregate opponent's rule level at the end of time  $t$  be  $b_i(t)$ . Based on this belief, player  $i$  predicts that her aggregate opponent at time  $t+1$  will choose

$$\hat{z}_{-i}(t+1) = M(b_i(t)|x_0(t+1)) = \left( \frac{p \cdot (n-1)}{(n-p)} \right)^{b_i(t)} \cdot x_0(t+1).$$

Then, player  $i$ 's best response is given by

$$x_i(t+1) = \frac{p(n-1)}{n-p} \cdot \hat{z}_{-i}(t+1) = \left( \frac{p \cdot (n-1)}{(n-p)} \right)^{b_i(t)+1} \cdot x_0(t+1).$$

Closely resembling the iterative nature of the rule hierarchy mapping, player  $i$ 's adopted rule level at time  $t+1$  is exactly 1 rule level higher than her belief at the end of time  $t$  (i.e.,  $r_i(t+1) = b_i(t)+1$ ).

In the next few subsections, we shall consider 3 increasingly general versions of the standard level- $k$  model. First, we restrict  $x_0(t) \equiv x_0(1)$  and  $b_i(t) \equiv b_i(0)$  and consider the static level- $r$  model with a fixed but continuous rule hierarchy. Note that this is a generalization of the standard level- $k$  model. Next, we extend this static level- $r$  model by allowing  $x_0(t)$  to adapt to past observations so that we now have a rule hierarchy mapping function that varies over time. Finally, we extend the model further by allowing players to learn and update their beliefs  $b_i(t)$  so that players may use different rules over time. We term this most general model the *dynamic level- $r$  model with adaptive rule hierarchy*.

<sup>12</sup>If this payoff maximizing strategy is in a tie with some other player's strategy, player  $i$  always loses by deviating and hence winning in a tie is indeed player  $i$ 's payoff maximizing outcome.



### Static Level- $r$ with Fixed Rule Hierarchy ( $L_r$ Model)

The static level- $r$  model with fixed rule hierarchy serves as the first step to generalize the standard level- $k$  model. The former utilizes a continuous rule hierarchy, whereas the latter uses a discrete rule hierarchy. The continuous rule hierarchy is iteratively defined in the same manner as the discrete rule hierarchy, with a level- $(r + 1)$  rule best-responding to a level- $r$  rule, but the rule level  $r$  is defined over the continuous space of nonnegative real numbers  $\mathbb{R}_0^+$  (i.e.,  $r \in \mathbb{R}_0^+$ ). Note that the rule space is  $\mathbb{Z}_0^+$  in the standard level- $k$  model.

The continuous level- $r$  rule hierarchy has 2 benefits. First, since  $\mathbb{R}_0^+$  is a strict superset of  $\mathbb{Z}_0^+$ , more choices in the continuous strategy space  $S$  will be associated with their corresponding rule levels in  $\mathbb{R}_0^+$ . As a consequence, the distribution of players' observed choices can be interpreted as heterogeneity in rule levels rather than as error rates in players' choices. Second, the continuous rule hierarchy conceptually nests the discrete rule hierarchy as a special case. We shall show below that allowing continuous rule hierarchy indeed helps to better predict observed behaviors.

In the static level- $r$  model with fixed rule hierarchy, both the players' beliefs about the aggregate opponent's rule level and the rule hierarchy mapping are fixed over time. As a consequence, both the players' rule levels and their implied choices remain unchanged over time. Formally, the static level- $r$  model with fixed rule hierarchy posits:

1. Player  $i$ 's belief about her aggregate opponent's rule level at the end of time  $t$  is static and given by  $b_i(t) = b_i(0)$ ,  $\forall t$ ;
2. Rule hierarchy mapping is static at  $M(\cdot|x_0(t)) = M(\cdot|x_0(1))$ ,  $\forall t$ ;
3. Player  $i$  will choose the payoff maximizing strategy that best-responds to her aggregate opponent's predicted choice, i.e., the predicted payoff-relevant statistic  $\hat{z}_{-i}(t) = M(b_i(t - 1)|x_0(t)) = M(b_i(0)|x_0(1)) = \left(\frac{p(n-1)}{(n-p)}\right)^{b_i(0)} \cdot x_0(1)$ ,  $\forall t$ . Specifically, player  $i$  will adopt rule level  $b_i(0) + 1$ , and choose  $x_i(t) = \frac{p(n-1)}{(n-p)} \cdot \hat{z}_{-i}(t) = \left(\frac{p(n-1)}{(n-p)}\right)^{b_i(0)+1} \cdot x_0(1)$ ,  $\forall t$ .

Note that all players are strategic thinkers in our model. Specifically, players choose their best response rules based on their beliefs about the aggregate opponent's rule level and as a consequence, the lowest level rule they can adopt is a level-1 rule (as a best-response to the level-0 belief). On the contrary, the standard level- $k$  rule hierarchy allows the existence of non-strategic level-0 players.<sup>13</sup>

### Static Level- $r$ with Adaptive Rule Hierarchy ( $L_r(t)$ Model)

As discussed above, players' behaviors change over time. As a consequence, the  $L_r$  model cannot capture the dynamics of players' choices. In this subsection, we extend the  $L_r$  model to the  $L_r(t)$

<sup>13</sup>Ho and Su (2013) makes the similar assumption. Studies which allow the proportion of level-0 players to be independently estimated often find the level-0 proportion to be negligible (Crawford et al., 2013).

model by considering an adaptive rule hierarchy mapping. Specifically, we allow the level-0 rule's choice  $x_0(t)$  to change over time. Consequently, the entire rule hierarchy mapping changes because the mapping  $M(\cdot|x_0(t))$  is uniquely determined given  $x_0(t)$ .

The  $L_r(t)$  model posits that the level-0 rule's choice in each round is the weighted average of the *ex post* best responses in the preceding rounds. The *ex post* best response in a round is defined as the *ex post payoff maximizing strategy* given all players' choices in that round. For example, in the  $p$ -beauty contest game, the *ex post* best response in a round is the announced target number because the player who had chosen the target number (i.e.,  $p \cdot \frac{\sum_{i=1}^n x_i(t)}{n}$ ) would have been the winner in that round.

Let the *ex post* best response at time  $t$  be denoted by  $\omega(t)$ . We assume that the *ex post* best response is known to all players after each round.<sup>14</sup> Let  $x_0(1) = \omega(0)$ , which is the hypothetical *ex post* best response in round 0 (i.e., level-0 player's predicted choice in round 1) and it is to be estimated empirically. Then, the level-0 rule's choice  $x_0(t)$  at time  $t$  is specified as :

$$x_0(t) = \frac{\rho_0 \cdot \phi_0^{t-1} \cdot \omega(0) + \phi_0^{t-2} \cdot \omega(1) + \dots + \phi_0 \cdot \omega(t-2) + \omega(t-1)}{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + 1}, \quad (2.1)$$

where  $0 \leq \rho_0$  and  $0 \leq \phi_0 \leq 1$  are the initial prior strength and the memory decay factor, respectively. Therefore, the level-0 rule's choice at time  $t$  is the weighted average of the past *ex post* best responses from time 0 up to  $t-1$ , where  $\omega(0)$  is to be estimated and  $\omega(1), \omega(2), \dots$ , and  $\omega(t-1)$  are observed. Note that level-0 rule's updating rule is similar to the belief updating rule of the weighted fictitious play model (see Fudenberg and Levine, 1998), and higher level rules iteratively best-respond to the level-0 rule.<sup>15</sup>

The dynamics of the level-0 rule's choice makes the entire rule hierarchy mapping adaptive and it has 3 distinct benefits:

1. In the  $p$ -beauty contest game, choices converge to the iterative dominance solution over time (i.e., they get smaller over time). Allowing the rule level 0's choice to adapt to that trend seems a sensible way to capture such choice dynamics. Note that higher level rules will best-respond to lower level rules and hence will also generate smaller choices over time. As a result, as long as average choices get smaller over time, all rule level players will iteratively choose smaller numbers, capturing the key feature of the data that choices converge to the iterative dominance solution.
2. The model nests familiar cases of adaptive learning in the literature. When  $\rho_0 = 1$  and  $\phi_0 = 1$ , level-0 players weight all past *ex-post* best responses equally to determine their choice in each

<sup>14</sup>In most  $p$ -beauty contest experiments, the target number is always announced publicly in order to determine individual player's payoff after every round. Similarly, in the generalized price matching game to be discussed later, the *ex post* best response can be inferred easily from the minimum of opponents' choice provided to each individual player.

<sup>15</sup>In particular, the level-1 rule best-responds to the adaptively generated level-0 rule; this process is similar to Selten's (1991) model of anticipatory learning.

round. As a result, level-1 players can be naturally interpreted as the simple fictitious play learners who believe that level-0 players simply mimic past *ex post* best responses with equal probability (Brown, 1951). When  $\phi_0 = 0$ , level-0 players simply repeat the latest *ex post* best response. Here, the level-1 players can be viewed as following Cournot best-response dynamics.

3. In the next subsection, we will also allow players to change their rule levels over time (i.e., players engage in sophisticated learning). To study the extent of sophisticated learning empirically, we need to control for the potential existence of adaptive learning. We choose the weighted fictitious play model (Fudenberg and Levine, 1998) to model adaptive learning because of its analytical tractability. We shall show later that players empirically exhibit both adaptive and sophisticated learning in games.

In the  $L_r(t)$  model, the rule hierarchy mapping adapts to game history but players' beliefs about aggregate opponents' rule levels remain fixed over time. As a result, even though players' rule levels remain the same, these same rule levels may imply different choices over time. In sum, the  $L_r(t)$  model posits:

1. Player  $i$ 's belief about her aggregate opponent's rule level at the end of time  $t$  is static and given by  $b_i(t) = b_i(0), \forall t$ ;
2. Level-0 rule's corresponding choice  $x_0(t)$  is the weighted average of the *ex post* best responses in the preceding rounds, and as a consequence, the rule hierarchy mapping  $M(\cdot|x_0(t))$  is adaptive;
3. Player  $i$  will choose the payoff maximizing strategy that best-responds to her aggregate opponent's predicted choice, i.e., the predicted payoff-relevant statistic  $\hat{z}_{-i}(t) = M(b_i(0)|x_0(t)) = \left(\frac{p(n-1)}{(n-p)}\right)^{b_i(0)} \cdot x_0(t)$ . Specifically, player  $i$  will adopt rule level  $b_i(0) + 1$  and choose  $x_i(t) = \frac{p(n-1)}{(n-p)} \cdot \hat{z}_{-i}(t) = \left(\frac{p(n-1)}{(n-p)}\right)^{b_i(0)+1} \cdot x_0(t)$ .

## Dynamic Level- $r$ with Adaptive Rule Hierarchy ( $DL_r(t)$ Model)

In this subsection, we extend the  $L_r(t)$  model to the  $DL_r(t)$  model by allowing players to dynamically change their rule levels in response to the feedback they receive. Besides allowing for an adaptive rule hierarchy, the  $DL_r(t)$  model posits that players infer their aggregate opponent's rule level from the observed payoff-relevant statistic after each round, and use it to update their belief about what rule the aggregate opponent is likely to choose in the next round. Specifically, players update their belief after each round by forming a *weighted sample average* of all past inferred rule levels up to that round. Using these individually updated beliefs, players subsequently predict the likely choice of their aggregate opponent (i.e., the payoff-relevant statistic) in the next round. They then best-respond to it by choosing a payoff-maximizing strategy, which is a function of the payoff-relevant statistic.

Denote player  $i$ 's belief about her aggregate opponent's rule level at the end of time  $t$  by  $b_i(t)$  and player  $i$ 's aggregate opponent's rule level inferred from her payoff-relevant statistic  $z_{-i}(t)$  at time  $t$  by  $r(z_{-i}(t))$ . Formally,  $z_{-i}(t) = M(r(z_{-i}(t))|x_0(t))$  and hence  $r(z_{-i}(t)) = M^{-1}(z_{-i}(t)|x_0(t))$ . Since  $M(\cdot|x_0(t))$  is a one-to-one function, its inverse  $M^{-1}(z_{-i}(t)|x_0(t))$  always exists. We posit that player  $i$ 's updated belief at the end of time  $t$  is given by:

$$b_i(t) = \frac{\rho_d \cdot \phi_d^t \cdot b_i(0) + (n-1) \cdot \phi_d^{t-1} \cdot r(z_{-i}(1)) + \dots + (n-1) \cdot r(z_{-i}(t))}{\rho_d \cdot \phi_d^t + (n-1) \cdot \phi_d^{t-1} + \dots + (n-1)} \quad (2.2)$$

where  $b_i(0)$  is player  $i$ 's initial belief about her aggregate opponent's rule level. We assume the initial belief is heterogenous among players.<sup>16</sup> Next, the aggregate opponent's inferred rule level  $r(z_{-i}(t))$  at each time  $t$  is multiplied by  $(n-1)$  to capture the fact that the inference is based on aggregate actions of  $(n-1)$  individual opponents. Finally, the parameters  $\rho_d$  and  $\phi_d$  are the initial belief strength and the memory decay factor, respectively.

Given the updated belief  $b_i(t)$  at the end of time  $t$ , player  $i$  predicts the payoff-relevant statistic at round  $t+1$  to be  $\hat{z}_{-i}(t+1) = M(b_i(t)|x_0(t+1))$ , and best-responds to it by choosing a payoff-maximizing strategy. Specifically, at time  $t+1$ , player  $i$  will choose  $x_i(t+1) = \frac{p(n-1)}{(n-p)} \cdot \hat{z}_{-i}(t+1) = M(b_i(t) + 1|x_0(t+1))$ , which corresponds to the level- $(b_i(t) + 1)$  rule.

In sum, the  $DL_r(t)$  model posits:

1. Player  $i$ 's belief about her aggregate opponent's rule level at the end of time  $t$  is dynamic and given by  $b_i(t)$ ;
2. Level-0 rule's corresponding choice  $x_0(t)$  is the weighted average of the *ex post* best responses in the preceding rounds and as a consequence, the rule hierarchy mapping  $M(\cdot|x_0(t))$  is adaptive;
3. Player  $i$  predicts her aggregate opponent's choice (i.e., the payoff-relevant statistic) to be  $\hat{z}_{-i}(t+1) = M(b_i(t)|x_0(t+1))$  at time  $t+1$ .
4. Player  $i$  chooses  $x_i(t+1)$  in order to maximize her payoff given  $\hat{z}_{-i}(t+1)$ . Formally,  $x_i(t+1)$  is uniquely determined from  $\hat{z}_{-i}(t+1)$  as follows:

$$x_i(t+1) = \frac{p \cdot (n-1)}{n-p} \cdot \hat{z}_{-i}(t+1) = \left( \frac{p \cdot (n-1)}{n-p} \right)^{b_i(t)+1} \cdot x_0(t+1).$$

<sup>16</sup>Empirically, we assume that  $b_i(0)$  follows a beta distribution,  $\text{Beta}(\alpha, \beta)$ , scaled by a constant factor. Since the beta distribution has continuous support  $[0,1]$ , scaling it by a multiplicative factor implies that  $b_i(0)$  may range between 0 and the scaling factor. In other words, the scaling factor can be interpreted as the maximum rule level a player may adopt (i.e., it corresponds to the iterative dominance solution play). In the estimation, we set this maximum rule level large enough so that the corresponding choice approximates the level- $\infty$  choice well.

Hence,  $x_i(t+1)$  changes over time due to dynamics in  $\hat{z}_{-i}(t)$ , which arises from both belief dynamics (i.e.,  $b_i(t)$ ) and the adaptive rule hierarchy (i.e.,  $x_0(t)$ ).

The  $DL_r(t)$  model can be thought of as a unified framework that captures both adaptive and sophisticated learning (Milgrom and Roberts, 1991) simultaneously. Adaptive learning is modeled through dynamics in the rule hierarchy. Specifically, the level-0 rule adapts to historical game outcomes, and higher level rules are defined iteratively. In this way, the entire rule hierarchy mapping shifts over time. Hence, a player choosing the same rule level exhibits adaptive learning and adjusts actions according to historical game play. On the other hand, sophisticated learning is modeled through dynamics in players' rule levels. Specifically, players observe their aggregate opponents choose different rule levels over time, update their beliefs about opponents' rule levels, and best-respond to their beliefs. Consequently, players may move up and down the rule hierarchy (i.e., choose different rule levels) depending on how sophisticated they think their opponents are and how sophisticated they think they should optimally be. In the empirical analysis that follows, we show that both adaptive and sophisticated learning are crucial in capturing subjects' dynamic behaviors in  $p$ -beauty contest games.

## Special Cases

### Level- $k$ Models

The standard level- $k$  model has been used to explain nonequilibrium behaviors in applications including auctions (Crawford and Iriberri, 2007b), matrix games (Costa-Gomes et al., 2001), and signaling games (Brown et al., 2012). For a comprehensive review, see Crawford et al. (2013). If  $x_0(t) = x_0(1)$ ,  $\forall t$  and  $b_i(t) = b_i(0)$ ,  $\forall t$ , then the  $DL_r(t)$  model becomes the  $L_r$  model. The  $L_r$  model becomes standard level- $k$  models if  $b_i(0)$  is restricted to have probability masses only at discrete nonnegative integers. As a result, our model will naturally capture the empirical regularities in the above games as well.

Recently, Ho and Su (2012) generalize the standard level- $k$  model to allow players to dynamically adjust their rule level. Their discrete model is shown to explain the dynamic behaviors in the centipede game and sequential bargaining game well. If  $x_0(t) = x_0(1)$ ,  $\forall t$  and  $b_i(t)$  is allowed to vary over time, the resulting model (which we call  $DL_r$ ) captures this dynamic change in rule levels and as a result can explain behaviors in these games too.

### Weighted Fictitious Play

The weighted fictitious play model (which includes Cournot best-response dynamics and simple fictitious play as special cases) has been used to explain learning behavior in a wide variety of games (Ellison and Fudenberg, 2000; Fudenberg and Kreps, 1993; Fudenberg and Levine, 1993).<sup>17</sup> If  $b_i(t) = 0$ ,  $\forall t$  and  $\forall i$ , but  $x_0(t)$  is allowed to adapt as specified in our model, then the  $DL_r(t)$

<sup>17</sup>Note that weighted fictitious play is a special case of experienced-weighted attraction (EWA) learning (Camerer and Ho, 1999; Ho et al. 2007). To choose the former is to keep the model analytically tractable. As a consequence,  $DL_r(t)$  model does not nest EWA learning as a special case.

model becomes weighted fictitious play models because all players are restricted to have level-0 beliefs and thus they use only the level-1 rule. Note that all players learn the same way because they share the common belief about the level-0 rule's choice (i.e., see the same target number in each round).

### Other Nonequilibrium Models

Ho et al. (1998) propose a model of iterative best responses to explain dynamic behaviors in  $p$ -beauty contest games. In their model, the level-0 rule adapts over time (which induces an adaptive rule hierarchy) but players do not adjust their rule levels. Moreover, the model uses a discrete rule hierarchy. As a consequence, their model is analogous to our  $L_r(t)$  model. Put differently, their model only captures adaptive learning but does not allow for sophisticated learning. As a result, their model will predict slower learning than the  $DL_r(t)$  model. Empirically, the  $DL_r(t)$  model allows one to assess the importance of incorporating sophisticated learning in explaining players' choice dynamics.

### Iterative Dominance Solution

The  $DL_r(t)$  model nests the iterative dominance solution as a special case when  $b_i(t) = \infty$ ,  $\forall i$  and  $\forall t$ . As a result, we can use our model to evaluate the iterative dominance solution empirically. Besides, as the next result shows, the  $DL_r(t)$  model always converges to the iterative dominance solution as long as either adaptive learning or sophisticated learning is present.

**Theorem 2.4.** (*Repetition Unraveling*)  $\mathbb{E}[x_i(t)] \rightarrow L$  as  $t \rightarrow \infty$  if  $\rho_0 < \infty$  or  $\rho_d < \infty$ .

*Proof.* See Appendix. □

In the experimental  $p$ -beauty contest game, players do not choose the unique iterative dominance solution initially, but their behavior converges toward it over time, a behavioral regularity we call *repetition unraveling*. Hence, Theorem 2.4 states that the  $DL_r(t)$  model can explain the repetition unraveling property in the experimental game through either adaptive or sophisticated learning of players. Below we will empirically assess the relative importance of these two forms of learning.

One intuitive way to interpret Theorem 1 is to contrast it with the iterative dominance solution, which requires many levels of iterative reasoning instantly. The  $DL_r(t)$  model recognizes that some players may stop this iterative reasoning prematurely, but predicts that players will ultimately reach the limiting point or equilibrium if the game is played repeatedly. In this regard,  $DL_r(t)$  model can be considered as a tracing procedure for the equilibrium and thus serves as a structural model of the equilibration process.

## 2.3 Estimation and Results

### Experimental Data

We fit the  $DL_r(t)$  model and its special cases using the  $p$ -beauty contest data collected by Ho et al. (1998). The experiment consisted of a  $2 \times 2$  factorial design with  $p = 0.7$  or  $0.9$  and  $n = 3$  or  $7$ . Players chose any numbers from  $[0, 100]$ . The prize in each round was \$1.5 for groups of size 3 and \$3.5 for groups of size 7, keeping the average prize at \$0.5 per player per round. A total of 277 subjects participated in the experiment. Each subject played the same game for 10 rounds with a fixed matching protocol.<sup>18</sup> There were 14 groups of size 7 and 14 groups of size 3 with  $p = 0.7$ ; and there were 14 groups of size 7 and 13 groups of size 3 with  $p = 0.9$ . As a consequence, we have a total of 55 groups and 2770 observations.

### Likelihood Function

Let  $\hat{x}_i(t)$  be player  $i$ 's predicted choice, and  $x_i(t)$  be player  $i$ 's actual choice. We assume that the actual choice occurs with a normally distributed *i.i.d.* error so that  $x_i(t) = \hat{x}_i(t) + \epsilon_i(t)$  where  $\epsilon_i(t) \sim N(0, \sigma^2)$ .

Let  $\mathbf{x}_i(t) = (x_i(1), x_i(2), \dots, x_i(t))$  be player  $i$ 's choice vector and  $\mathbf{z}_{-i}(t) = (z_{-i}(1), z_{-i}(2), \dots, z_{-i}(t-1), z_{-i}(t))$  be player  $i$ 's history of payoff-relevant statistics for the first  $t$  rounds of the game plays. Let  $\boldsymbol{\omega}(t) = (\omega(0), \omega(1), \dots, \omega(t))$  be the history of ex post best responses for the first  $t$  round (including the level-0 player's predicted choice in round 1  $\omega(0)$ ) that is common to all players. Furthermore, let  $f_{it}(x_i(t) | \boldsymbol{\omega}(t-1), \mathbf{z}_{-i}(t-1), b_i(0))$  be the probability density of player  $i$ 's choosing  $x_i(t)$  at time  $t$ , given the history of ex post best responses up to time  $t-1$ , the history of her payoff-relevant statistics up to time  $t-1$  and her initial belief  $b_i(0)$ . The likelihood of observing player  $i$ 's vector of choices for the entire experiment (i.e.,  $\mathbf{x}_i(10)$ ) is denoted by  $L_i(\mathbf{x}_i(10))$  and given as follows<sup>19</sup>:

$$L_i(\mathbf{x}_i(10)) = \int \left( \prod_{t=1}^{10} f_{it}(x_i(t) | \boldsymbol{\omega}(t-1), \mathbf{z}_{-i}(t-1), b_i(0)) \right) dG(b_i(0)),$$

where  $G$  is a well-defined distribution function of the initial belief. In the estimation,  $G$  is assumed to be a scaled beta distribution, described in more detail below.

As a consequence, the total log-likelihood of observing all players' choice vectors is:

$$\begin{aligned} LL &= \sum_{i=1}^N \log(L_i(\mathbf{x}_i(10))) \\ &= \sum_{i=1}^N \log \left( \int \left( \prod_{t=1}^{10} f_{it}(x_i(t) | \boldsymbol{\omega}(t-1), \mathbf{z}_{-i}(t-1), b_i(0)) \right) dG(b_i(0)) \right). \end{aligned}$$

<sup>18</sup>Reputation building is unlikely because the  $p$ -beauty contest game is a constant-sum game where players' interests are strictly opposite.

<sup>19</sup>Note that  $f_{it}$  is truncated at 0 and 100.

## Estimated Models

The  $DL_r(t)$  model consists of 8 parameters: 1) initial belief parameters  $\alpha$  and  $\beta$  for the scaled beta distribution; 2) level-0 player's predicted choice in round 1,  $\omega(0)$ ; 3) updating parameters for the adaptive rule hierarchy,  $\rho_0$  and  $\phi_0$ ; 4) dynamic rule level belief updating parameters,  $\rho_d$  and  $\phi_d$ ; and 5) standard deviation of the error term,  $\sigma$ .

We estimated a total of five models. Besides the full model  $DL_r(t)$ , we estimated the following models:

1. Weighted fictitious play model ( $L_1(t)$ ): This is a special case of  $L_r(t)$  in which all players are of rule level 1. We obtain the model by setting  $\alpha = 1$ ,  $\beta = \infty$ ,  $\rho_d = \infty$ , and  $\phi_d = 1$ . This model captures the standard adaptive learning in the absence of a rule hierarchy.
2. Static level- $r$  with fixed rule hierarchy ( $L_r$  model): This is the continuous rule hierarchy version of the standard level- $k$  model. It is a nested case of  $DL_r(t)$  model obtained by setting  $\rho_0 = \infty$ ,  $\phi_0 = 1$ ,  $\rho_d = \infty$ , and  $\phi_d = 1$ . This model captures neither adaptive nor sophisticated learning dynamics.
3. Static level- $r$  with adaptive rule hierarchy ( $L_r(t)$  model): This model allows the level-0 player's choice to be adaptive and vary over time. It is a nested case of  $DL_r(t)$  with  $\rho_d = \infty$  and  $\phi_d = 1$ . The model adds adaptive learning to the level- $r$  model by allowing the level-0 player to learn over time.
4. Dynamic level- $r$  with fixed rule hierarchy ( $DL_r$  model): This model allows players to change their rule levels over time. It is a nested case of  $DL_r(t)$  obtained by setting  $\rho_0 = \infty$  and  $\phi_0 = 1$ . The model captures only sophisticated learning.

## Estimation Method

In the estimation, we assume that players' initial belief follows a beta distribution  $Beta(\alpha, \beta)$  scaled by a multiplicative factor of 50. Hence, the maximum belief a player can have about the aggregate opponents' rule level is 50. Note that  $(0.7)^{50} \cdot 100 = 1.8 \times 10^{-8} \approx 0$  and  $(0.9)^{50} \cdot 100 = 0.52 \approx 0$ , so rule levels higher than 50 approximate the iterative dominance solution well.<sup>20</sup>

We searched over parameter values to maximize the simulated log-likelihood using constrained nonlinear optimization in the MATLAB optimization toolbox. At each iteration, a new vector of parameter values was tried, and 1500 random draws were drawn from the corresponding scaled beta distribution of initial belief in order to evaluate the simulated log-likelihood.<sup>21</sup> To make sure

<sup>20</sup>We conducted robustness checks by varying the scale factor up to 100 and found that parameter estimates varied only slightly. An alternative way to formulate initial belief is to use the 2-parameter gamma distribution. Using the gamma distribution to model initial belief did not affect the parameter estimates much. For example, when the gamma distribution was assumed for  $DL_r(t)$  model, parameter estimates for  $\omega(0)$ ,  $\rho_0$ ,  $\phi_0$ ,  $\rho_d$ ,  $\phi_d$ , and  $\sigma$  were 79.90, 8.29, 0.88, 2.09, 0.04, and 19.99, respectively, which were close to the corresponding parameter estimates reported in Table 2.1.

<sup>21</sup>We also varied the number of random draws up to 3000 and found similar results.



that the maximized log-likelihood is indeed the global maximum, we tried optimization with many starting points. The constraints imposed on the parameters based on theoretical considerations were:  $0 < \alpha, \beta$ ;  $0 \leq \omega(0) \leq 100$ ;  $0 < \rho_0, \rho_d$ ; and  $0 \leq \phi_0, \phi_d \leq 1$ . As in Camerer and Ho (1999), we restrict  $\rho_0 \leq \frac{1}{1-\phi_0}$  and  $\rho_d \leq \frac{n-1}{1-\phi_d}$  so that both the adaptive learning and belief updating obey the law of diminishing effect that players weight new observations less over time. We also normalize each round's data by each round's sample mean and sample standard deviation as in Ho et al. (1998) so that observations were weighted evenly across rounds. Standard errors of parameters were estimated using bootstrapping. In Table 2.1, standard errors are reported in parentheses below parameter estimates.

## Results and Parameter Estimates

Table 2.1 presents the parameter estimates and the corresponding maximized log-likelihoods for the five models. As shown, Table 1 has 6 columns: the first column indicates the parameter names, the second column the weighted fictitious play model ( $L_1(t)$ ), the third column continuous level- $r$  model ( $L_r$ ), the fourth column static level- $r$  with adaptive rule hierarchy model ( $L_r(t)$ ), the fifth column dynamic level- $r$  model with fixed rule hierarchy model ( $DL_r$ ), and the last column dynamic level- $r$  model with adaptive rule hierarchy model ( $DL_r(t)$ ). There are five sets of parameters: 1) beta distribution parameters ( $\alpha, \beta$ ) for players' initial belief; 2) initial choice  $\omega(0)$  for the level-0 player; 3) adaptive learning parameters ( $\rho_0, \phi_0$ ); 4) sophisticated learning parameters ( $\rho_d, \phi_d$ ); and 5) standard deviation of choice parameter  $\sigma$ .

Finally, the standard errors of parameter estimates were estimated using bootstrapping and they are reported in parentheses.

Since  $DL_r(t)$  is a generalized version of the other 4 models, it will necessarily fit better than its special cases in terms of log-likelihoods. Therefore, we also provide  $\chi^2$  and Akaike Information Criterion (AIC) in order to test whether more complex models indeed fit better than simpler models.

Note that the full model  $DL_r(t)$  fits the the data the best, having the highest maximized log-likelihood of -2939.87. The  $\chi^2$  as well as AIC statistics suggest that the four special cases are rejected in favor of the full model. For example, the static level- $r$ , which has a maximized log-likelihood of -3403.13, is rejected with a  $\chi^2$  value of 926.52. Similarly, the weighted fictitious play model, which has a maximized likelihood of -2992.88, is rejected with a  $\chi^2$  value of 106.02.

All estimated parameters are quite reasonable. The initial belief parameter estimates ( $\hat{\alpha} = 0.05$  and  $\hat{\beta} = 0.27$ ) suggest that about 60% of the players believe that their opponents are close to level 0 (see Figure 2.4). The level-0 player's initial choice is estimated to be around 81.<sup>22</sup> The adaptive

<sup>22</sup>To get a better sense of how initial belief parameters  $\alpha$  and  $\beta$  are jointly estimated, we compared the proportion of level-0 players and the average rule level resulting from each set of estimates from 120 bootstrap runs with those resulting from the full-sample estimates. The standard deviations over those 120 estimates are in the parentheses. In  $DL_r(t)$ , the average proportion of level-0 players and the average rule level are 61.53% (10.94%) and 7.86 (2.04) respectively, while those based on full-sample estimates are 62.35% and 7.94; in  $DL_r$ , the average proportion of level-0 players and the average rule level are 57.97% (13.65%) and 7.87 (2.33) respectively, while those based on full-sample estimates are 61.96% and 8.18; in  $L_r(t)$ , the average

Table 2.1: Parameter Estimates and Maximized Log-likelihood for  $DL_r(t)$  Model and its Special Cases

Parameters/Models	$L_1(t)$	$L_r$	$L_r(t)$	$DL_r$	$DL_r(t)$
Prior Belief of Rule (Beta( $\alpha, \beta$ ))					
$\hat{\alpha}$	1	0.48 (0.39)	0.00 (0.02)	0.05 (0.06)	0.05 (0.04)
$\hat{\beta}$	$\infty$	1.04 (1.90)	0.02 (1.29)	0.26 (0.56)	0.27 (0.40)
Level-0's Choice at $t = 1$					
$\hat{\omega}(0)$	63.26 (2.81)	59.24 (8.53)	68.66 (3.77)	82.44 (6.54)	81.25 (5.81)
Adaptive Learning					
$\hat{\rho}_0$	0.36 (0.27)	$\infty$	1.08 (0.55)	$\infty$	8.14 (15.55)
$\hat{\phi}_0$	0.00 (0.05)	1	0.07 (0.08)	1	0.88 (0.08)
Sophisticated Learning					
$\hat{\rho}_d$	$\infty$	$\infty$	$\infty$	2.18 (0.59)	2.16 (0.31)
$\hat{\phi}_d$	1	1	1	0.08 (0.04)	0.08 (0.04)
Std. Dev. of Error					
$\hat{\sigma}$	21.84 (0.99)	29.13 (1.44)	21.56 (1.05)	19.61 (0.98)	19.98 (1.06)
$LL$	-2992.88	-3403.13	-2972.61	-2947.01	-2939.87
$\chi^2$	106.02	926.52	65.48	14.28	–
( $p$ -value, dof)	(0.000, 4)	(0.000, 4)	(0.000, 2)	(0.001, 2)	–
AIC	5993.77	6814.27	5957.21	5906.03	5895.74*

learning parameters ( $\hat{\rho}_0 = 8.14$  and  $\hat{\phi}_0 = 0.88$ ) suggest that players weight their initial prior of the level-0 rule  $\omega(0)$  quite heavily (equivalent to 8 observations of opponents' choices) and the influence of an observation drops by half after every 5 rounds ( $0.88^5 \approx 0.5$ ). Next, the sophisticated learning parameters suggest that belief updating for aggregate opponent's rule level resembles Cournot dynamics ( $\hat{\phi}_d = 0.08 \approx 0$ ). Moreover, players give their initial belief about the aggregate opponent's rule level a weight of 2.16, which declines quickly after the first round. Finally, the standard deviation of the error term is estimated to be 19.98, which seems reasonable given the size of strategy space  $[0, 100]$ .

proportion of level-0 players and the average rule level are 89.92% (5.51%) and 3.67 (1.72) respectively, while those based on full-sample estimates are 90.28% and 4.44; in  $L_r$ , the average proportion of level-0 players and the average rule level are 8.64% (7.97%) and 17.36 (7.58) respectively, while those based on full-sample estimates are 5.34% and 15.65. Overall, the pattern of initial belief distribution from bootstrapping appears to be quite similar on average to that based on full-sample estimates.

Comparing the log-likelihoods of the last 3 columns, we note that the  $L_r(t)$  and  $DL_r$  models are rejected in favor of the full  $DL_r(t)$  model. This suggests that both an adaptive rule hierarchy and dynamic rule levels are crucial in explaining subjects' behaviors over time. Put differently, players engage in both adaptive and sophisticated learning. Note that although  $L_r(t)$  and  $DL_r$  have the same number of parameters,  $L_r(t)$  fits worse than  $DL_r$ . Hence, sophisticated learning appears more important than adaptive learning in describing  $p$ -beauty contest data. This observation is further confirmed by the lower estimated standard deviation of the error term (i.e.,  $\hat{\sigma}$ ) in the  $DL_r$  model.

We also fit the  $L_k$  model with a discrete Poisson initial belief (with mean  $\lambda$ ) and compare it with  $L_r$  model. For the  $L_k$  model, the maximized log-likelihood is -3459.62.<sup>23</sup> The  $L_k$  model is mathematically not a nested case of  $L_r$  model because one cannot express the discrete Poisson distribution as a special case of the continuous beta distribution. As a consequence, we use the AIC to compare their relative goodness of fit.<sup>24</sup> The AICs are 6925.25 and 6814.27 for  $L_k$  and  $L_r$  models, respectively. Clearly,  $L_r$  dominates  $L_k$  in terms of goodness of fit. This result suggests that having a continuous rule hierarchy helps to explain the data.

Three models (i.e.,  $L_1(t)$ ,  $L_r(t)$ , and  $DL_r(t)$ ) allow for adaptive learning of players by varying the rule hierarchy mapping through time. Comparing these three models, we note that the adaptive learning dynamics is of Cournot type in  $L_1(t)$  and  $L_r(t)$  (i.e.,  $\hat{\phi}_0 \approx 0$ ) whereas it is of weighted fictitious play type (i.e.,  $\hat{\phi}_0 = 0.88$ ) in  $DL_r(t)$ . That is, the adaptive learning model was misspecified as Cournot type in the absence of sophisticated learning. In other words, by allowing players to update their rule levels, the adaptive learning models would be less fickle and become more fictitious play like. This is an important result because it suggests that previously estimated parameters of adaptive learning models could be biased if they neglect sophisticated learning. Interestingly, when we compare  $DL_r(t)$  with  $DL_r$ , we find that belief updating (i.e., rule level dynamics) in both models is of Cournot type (i.e.,  $\hat{\phi}_d = 0.08$  in both models). This suggests that our proposed structural model of sophisticated learning is not misspecified in the absence of adaptive learning.

Figure 2.4 presents the initial belief distributions of  $L_r$ ,  $L_r(t)$ ,  $DL_r$ , and  $DL_r(t)$ . Three features are worth noting:

1. Initial belief distributions of both  $DL_r(t)$  and  $DL_r$  exhibit a rapid increase at both 0 and 50, suggesting that in these models, players are of either very low or very high rule levels when the game starts. The fact that these distributions are remarkably close to each other corroborates the point made earlier that the structural model of belief updating in the  $DL_r(t)$  model is not misspecified in absence of adaptive learning.
2. In  $L_r(t)$ , the proportion of low level players is highest with more than 90% of players having initial belief near 0. On the contrary, in  $L_r$ , the proportion of low level players is less than 10% and the distribution of player's rule levels is evenly distributed between 0 and 50. Note that as soon as adaptive learning is introduced (i.e., as we move from  $L_r$  to  $L_r(t)$ ), the

<sup>23</sup>Parameter estimates of the  $L_k$  model are:  $\hat{\lambda} = 11.32$ ,  $\hat{\omega}(0) = 100.00$  and  $\hat{\sigma} = 31.47$  where  $\lambda$  is the parameter of Poisson distribution.

<sup>24</sup>The Akaike criterion (AIC) is  $2 \cdot k - 2 \cdot LL$  where  $LL$  is the maximized log-likelihood and  $k$  is the number of parameters.

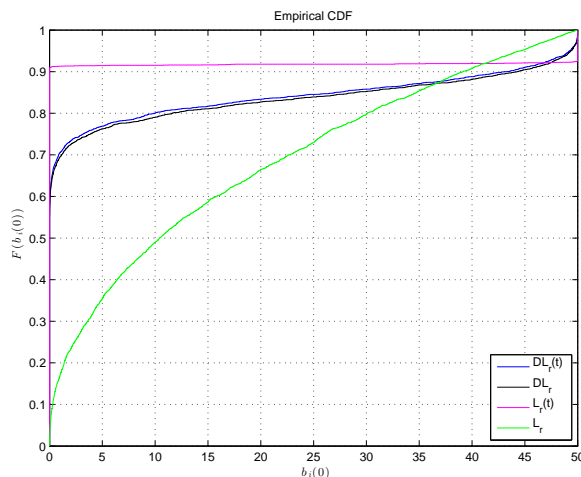


Figure 2.4: Distribution of Initial Beliefs of  $L_r$ ,  $L_r(t)$ ,  $DL_r$ , and  $DL_r(t)$  Models

initial belief is immediately clustered around 0, making  $L_r(t)$  resemble the weighted fictitious model. Consequently, we are not surprised to see that  $L_1(t)$  and  $L_r(t)$  models have similar log-likelihoods and their adaptive learning parameters (i.e.,  $\hat{\omega}(0)$ ,  $\hat{\rho}_0$ ,  $\hat{\phi}_0$ ) in Table 2.1 are close to each other. When adaptive learning is present, merely introducing higher level players does not improve the fit much.

3. The proportion of low level players of  $DL_r(t)$  or  $DL_r$  is smaller than that of  $L_r(t)$  but larger than that of  $L_r$ . This implies that  $L_r$  overestimates players' initial rule levels while  $L_r(t)$  underestimates them.  $L_r$  overestimates players' initial rule levels because it assumes they are static and hence need to be high in order to account for smaller choices that are closer to 0 in later rounds.  $L_r(t)$  underestimate players' initial rule levels because it over-fits the data by using adaptive learning dynamics alone. In other words,  $L_r(t)$  suppresses the rule level distribution as much as possible by endowing most players with a belief of 0. Interestingly,  $DL_r(t)$  strikes a compromise between these two polar cases by having players' beliefs fall between these two extreme distributions, suggesting that players' initial beliefs are neither concentrated at level-0 nor clustered around high rule levels if players are allowed to change their rule levels over time.

Figures 2.5 through 2.10 present the three-dimensional bar graphs of choice frequency of observed data and model predictions of  $DL_r(t)$ ,  $DL_r$ ,  $L_r(t)$ ,  $L_r$  and  $L_1(t)$ , respectively. Since the experimental data consists of a 2x2 factorial design with  $p = 0.7$  or  $0.9$  and  $n = 3$  or  $7$ , the model prediction for each possible game was first determined and then the final model prediction is derived as the weighted average of these 4 model predictions where the weight equals the relative proportion of each game's observations in the experimental data.

Note the following:

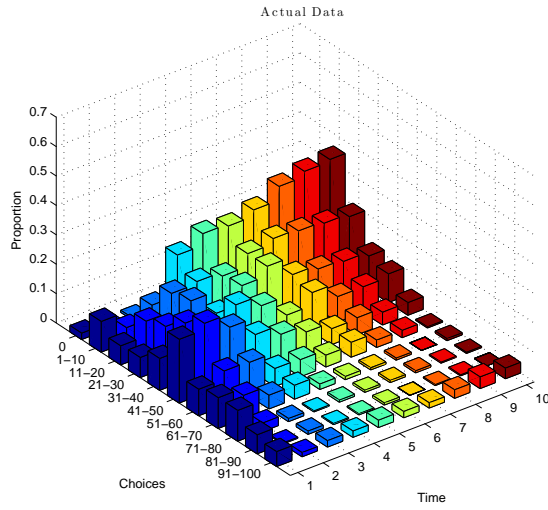


Figure 2.5: Actual Data

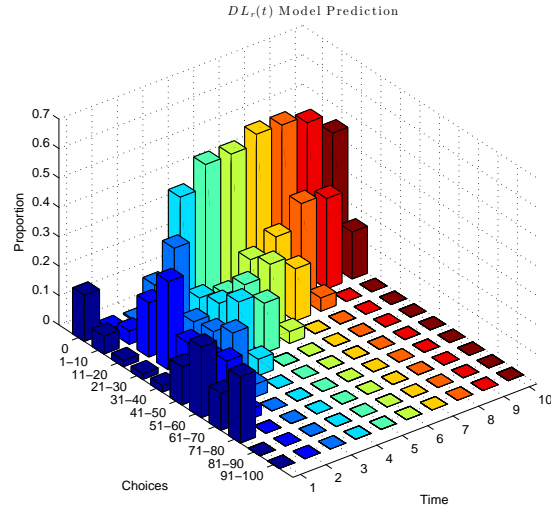


Figure 2.6:  $DL_r(t)$  Model Prediction

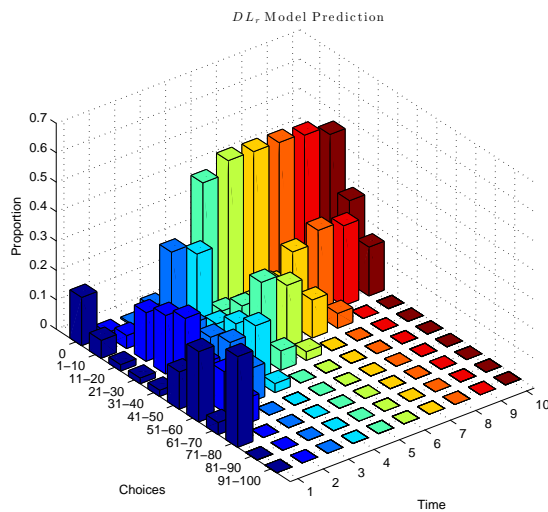


Figure 2.7:  $DL_r$  Model Prediction

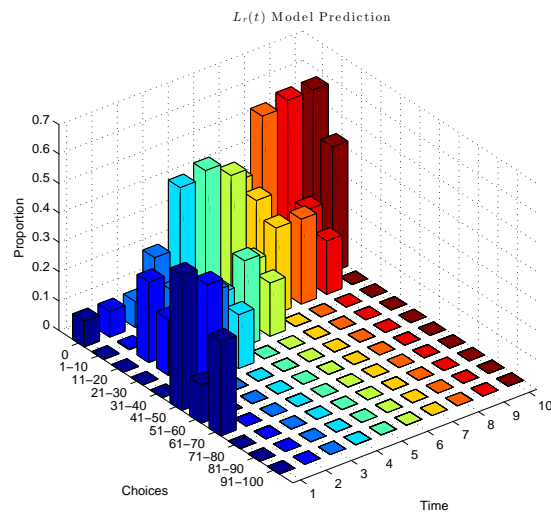


Figure 2.8:  $L_r(t)$  Model Prediction

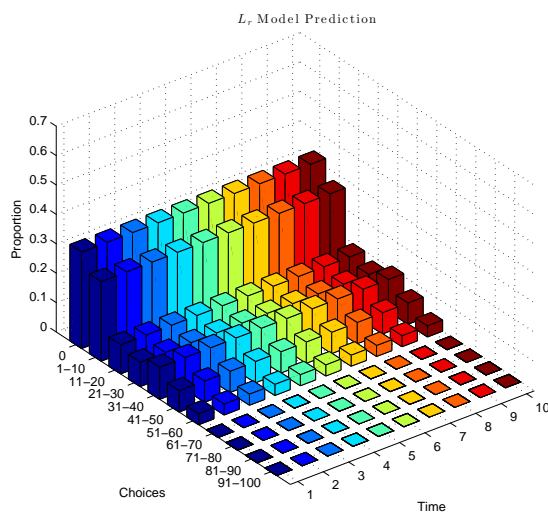


Figure 2.9:  $L_r$  Model Prediction

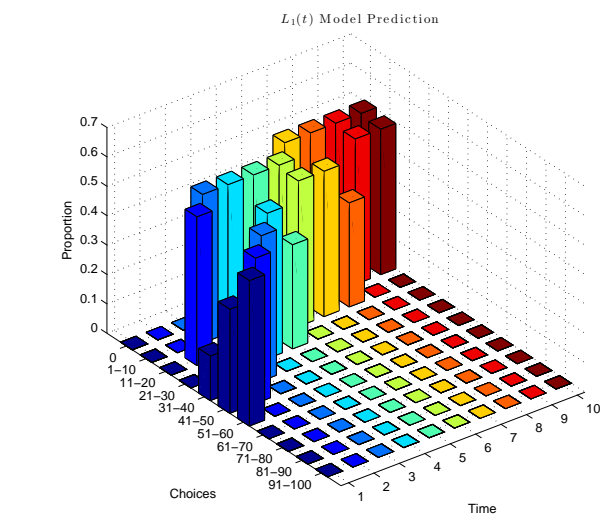


Figure 2.10:  $L_1(t)$  Model Prediction

1. Having similar sophisticated learning parameters,  $DL_r(t)$  and  $DL_r$  show a similar convergence pattern toward equilibrium overall. However,  $DL_r(t)$  predicts that more choices are closer to equilibrium in rounds 1–5 than  $DL_r$ . This is because  $DL_r(t)$  permits choices to shift dynamically by allowing for both adaptive learning (through an adaptive rule hierarchy) and sophisticated learning (through belief updating). Thus,  $DL_r(t)$  predicts that players choose smaller choices in earlier rounds due to adaptive learning (by adapting to the environment and sequentially best-responding to ex post best responses in previous rounds) even though their updated belief about aggregate opponent is not yet high enough. Note that if players' belief itself is high enough, they would choose smaller numbers regardless of level-0 player's choice  $x_0(t)$ . Indeed, this is the case for both model predictions in later rounds. Players' updated beliefs in later rounds become high enough that even in the absence of adaptive learning, the  $DL_r$  model yields predictions that match those of the  $DL_r(t)$  model. In sum,  $DL_r(t)$  distinguishes itself from  $DL_r$  mostly in its power to better explain earlier rounds' data.
2. As a model with the third best log-likelihood fit,  $L_r(t)$  shows slower convergence than  $DL_r(t)$  and  $DL_r$ . The distribution of choices in each round for  $L_r(t)$  is also more clustered than that of  $DL_r(t)$  or  $DL_r$ . Moreover,  $L_r(t)$  yields predictions that closely resemble those from  $L_1(t)$ . This is because the distribution of initial beliefs in  $L_r(t)$  is mostly concentrated around rule level 0 and thus the model approximates a weighted fictitious play model.
3.  $L_r(t)$  surpasses  $L_1(t)$  in the model fit mostly due to the continuous spread of players' rule levels. Comparing Figure 2.8 and 2.10, we note that  $L_r(t)$  predictions in each round are more sprinkled around all possible choices because about 10% of players have initial beliefs higher than 0 and choose smaller choices than in  $L_1(t)$ .
4.  $L_r$  model prediction is fixed over time because it lacks either adaptive rule hierarchy or dynamic rule levels. To account for the fast convergence of data,  $L_r$  predicts that players are concentrated at high rule levels or small choices throughout the game.<sup>25</sup>

## 2.4 Generalized Price Matching Game

To check the generalizability of the above results, we investigate the dominance-solvable traveler's dilemma game (Basu, 1994). In the original traveler's dilemma game, two travelers ( $i = 1, 2$ ) whose antiques were damaged during travel at the airline's fault were promised adequate compensation by the airline manager. Not knowing the true value of the two antiques but exploiting the fact that they were identical, the airline manager offers the following scheme for compensation. He asks both travelers to privately write down the cost of the antique on a piece of paper, where the cost can be one of  $\{2, 3, \dots, 100\}$  units of money. Upon seeing the two numbers  $x_i$  ( $i = 1, 2$ ) that the travelers wrote down, the manager then compensates traveler  $i$   $1) \min\{x_1, x_2\} - 2$  if  $x_i > x_j$ ,  $2)$

<sup>25</sup>Using parameter estimates of the  $L_k$  model in Section 2.3, we also compare the model predictions of  $L_r$  and  $L_k$ . We observe that  $L_r$  predicts more evenly placed choices than  $L_k$ , while predictions for smaller choices ( $< 10$ ) of both models are very similar. This is because  $L_r$  utilizes a continuous rule hierarchy and admits more numbers as possible choices.

$\min\{x_1, x_2\} + 2$  if  $x_i < x_j$ , or 3)  $x_i$  if  $x_i = x_j$ . That is, the traveler who wrote down the smaller number is rewarded for being honest while the one who wrote down the bigger number is penalized for lying. If both travelers' numbers were the same, they are thought to be telling the truth and receive neither reward nor penalty.

This traveler's dilemma game is dominance solvable and its unique iterative dominance solution is 2 (in general, it is the lower corner of the strategy space).<sup>26</sup> However, as is the case for the experimental  $p$ -beauty contest game, subjects do not choose the unique iterative dominance solution instantly but tend to converge to it over time (Capra et al., 1999; Goeree and Holt, 2004). Therefore, the standard game theory solution is a poor predictor of subjects' behavior in this game, failing to capture their learning dynamics.

An alternative interpretation of the traveler's dilemma game is to see it as a price matching (i.e., lowest price guarantee) game between two firms selling a homogenous product. As a result of price matching, both firms effectively charge only the minimum price in the market. In addition, the firm with a lower price will be rewarded with a positive customer goodwill while the one with a higher price will be penalized with a negative customer goodwill.<sup>27</sup>

We generalize this price matching game in 3 ways. First, we allow for more than 2 firms (i.e.,  $n \geq 2$ ) in this competition. Second, we allow for a continuous strategy space, while the standard price matching game permits only discrete strategy choices. Third, since the standard price matching game is no longer dominance solvable if it is defined over a continuous strategy space, we generalize its payoff structure so that the generalized game remains to be dominance solvable even with a continuous strategy space. We show below that our generalized price matching game nests and converges to the standard price matching game.

In following subsections, we describe the generalized price matching game, apply the  $DL_r(t)$  model to the game, structurally estimate the model using freshly generated experimental data, and interpret our empirical findings.

## Payoff Structure of Generalized Price Matching Game

In the generalized price matching game,  $n$  firms (players) engage in price competition in order to attract customers. They simultaneously choose prices in  $[L, U]$  for a homogenous good. Firms are indexed by  $i$  ( $i = 1, 2, \dots, n$ ). Firm  $i$ 's choice of price at time  $t$  is denoted by  $x_i(t)$  and the choice vector of all firms excluding firm  $i$  is denoted by  $\mathbf{x}_{-i}(t) = (x_1(t), \dots, x_{i-1}(t), x_{i+1}(t), \dots, x_n(t))$ .

<sup>26</sup>At the first step of elimination of dominated strategies, 99 and 100 are eliminated since they are dominated by 98. At the second step of elimination, 97 and 98 are eliminated being dominated by 96. Hence, the strategy that ultimately survives the iterative elimination of dominated strategies is 2.

<sup>27</sup>Price matching is common in retail industry. Marketing researchers (Jain and Srivastava, 2000; Chen et al., 2001; Srivastava and Lurie, 2001) suggest that firms who promise to charge the lowest price are perceived negatively by customers when their prices are found to be not the lowest in the market.

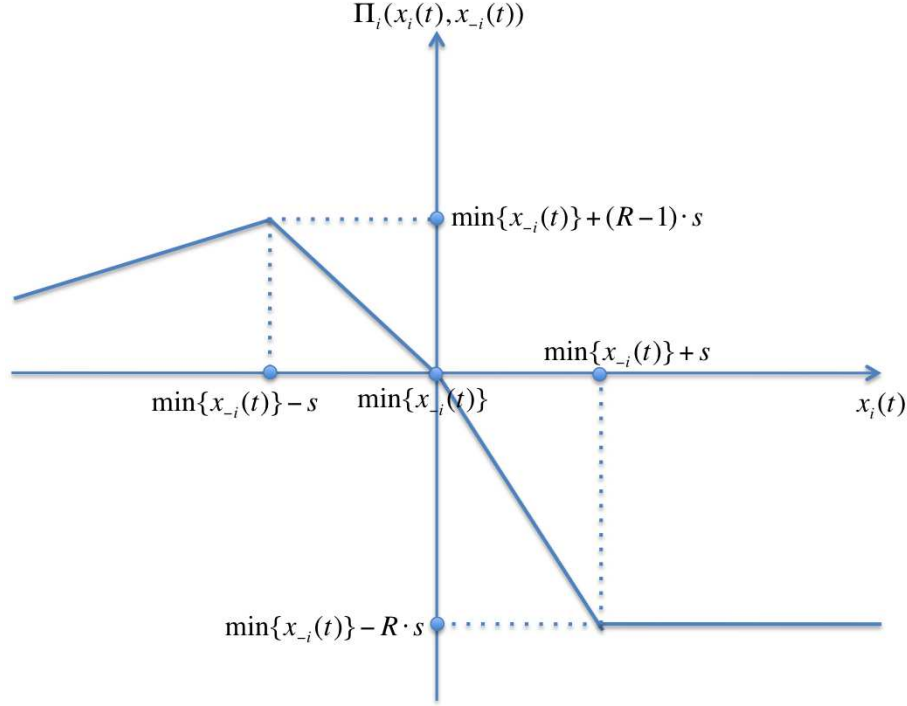


Figure 2.11: Generalized Price Matching Game: Payoff Function

The payoff function of the generalized price matching game at time  $t$  is given by:

$$\Pi_i(x_i(t), \mathbf{x}_{-i}(t)) = \begin{cases} \min\{\mathbf{x}_{-i}(t)\} - R \cdot s & \text{if } \min\{\mathbf{x}_{-i}(t)\} + s \leq x_i(t) \\ \min\{\mathbf{x}_{-i}(t)\} - R \cdot |x_i(t) - \min\{\mathbf{x}_{-i}(t)\}| & \text{if } \min\{\mathbf{x}_{-i}(t)\} < x_i(t) < \min\{\mathbf{x}_{-i}(t)\} + s \\ x_i(t) & \text{if } x_i(t) = \min\{\mathbf{x}_{-i}(t)\} \\ x_i(t) + R \cdot |x_i(t) - \min\{\mathbf{x}_{-i}(t)\}| & \text{if } \min\{\mathbf{x}_{-i}(t)\} - s < x_i(t) < \min\{\mathbf{x}_{-i}(t)\} \\ x_i(t) + R \cdot s & \text{if } x_i \leq \min\{\mathbf{x}_{-i}(t)\} - s \end{cases} \quad (2.3)$$

where  $R > 1$  and  $s > 0$  are constant parameters.

Figure 2.11 illustrates how each player's payoff (on the y-axis) depends on the minimum of her opponents' choices (on the x-axis). As (2.3) indicates, there are five possible cases. In all cases, each player's payoff is the lowest price offered by all players (due to price matching) as well as an additional goodwill term (which may be positive or negative depending on the relative position of the player's price). If the player's price is smaller than the minimum of opponents' choices, the player receives positive customer goodwill because she uniquely offers the lowest price to the customers. This positive customer goodwill increases linearly at rate  $R$  as the player's own choice gets smaller. Since  $R > 1$ , payoff increases as the player's choice decreases because the increase in



goodwill surpasses for the decrease in price. However, once the goodwill reaches its cap of  $R \cdot s$ , it stops increasing and stays constant. Thus, if the player's price continues to decrease, payoff will start to decrease. On the other hand, if the player's own choice is larger than the minimum of her opponents' choices, she receives negative customer goodwill because other firms are charging a lower price. The magnitude of negative customer goodwill also increases linearly as the player's own choice gets larger. However, once the goodwill reaches its cap of  $-R \cdot s$ , it stops increasing and stays constant. If the player's own choice and the minimum of her opponents' choices are identical, she receives neither positive nor negative customer goodwill and the payoff is simply the lowest price she offers.

There are several implications of the payoff structure of this game. Note that player  $i$ 's maximum possible payoff is  $\min\{\mathbf{x}_{-i}(t)\} + (R-1) \cdot s$ , which is attained when  $x_i(t) = \min\{\mathbf{x}_{-i}(t)\} - s$ , assuming that the latter is above the lowest possible choice  $L$ . In other words, if her opponents choose  $\mathbf{x}_{-i}(t)$ , player  $i$ 's best response is

$$x_i^*(t) = \max\{L, \min\{\mathbf{x}_{-i}(t)\} - s\}.$$

Therefore, we may interpret  $s > 0$  as the *step size* that a best-responding player will take to undercut her opponents, and  $R > 1$  as the *rate of change* of customer goodwill around the lowest price offered by competitors in the market.

It is quite simple to show that this game is dominance solvable and has a unique iterative dominance solution where all players choose the lower corner of strategy space (i.e.,  $L$ ).<sup>28</sup> Note also that when  $s \rightarrow 0$ ,  $R \rightarrow \infty$ , and  $R \cdot s$  stays constant, the game converges to the standard price matching game where the size of goodwill is always fixed regardless of the relative magnitude among choices.

## $DL_r(t)$ Model

Now, we apply the  $DL_r(t)$  model to the generalized price matching game. We begin by describing three building blocks: the rule hierarchy mapping, the payoff-relevant statistic, and players' best responses to their beliefs.

First, we describe the rule hierarchy mapping. Let  $x_0(t)$  be the choice of the level-0 player in round  $t$ , and  $x_i(t) = M(r_i(t)|x_0(t))$  be player  $i$ 's choice with rule level  $r_i(t)$  conditional on the level-0 player's choice  $x_0(t)$ . Since rule level  $r_i(t) + 1$  is a best response to rule level  $r_i(t)$ , the rule hierarchy mapping satisfies

$$M(r_i(t) + 1|x_0(t)) = M(r_i(t)|x_0(t)) - s,$$

assuming that the right-hand side remains above the lowest possible choice  $L$ . Therefore, we adopt the following rule hierarchy mapping:

$$M(r_i(t)|x_0(t)) = \max\{L, x_0(t) - s \cdot r_i(t)\}.$$

<sup>28</sup>Suppose  $L = 80$ ,  $U = 200$ , and  $s = 10$ . The choices above  $U - s = 190$  are dominated by 190 and are eliminated after the first step of iterated elimination of dominated strategies. In the second step, choices above  $190 - s = 180$  are eliminated. Proceeding in this fashion, only the choice  $L = 80$  remains after 12 steps.

Second, we note that the payoff-relevant statistic is

$$z_{-i}(t) = \min\{\mathbf{x}_{-i}(t)\}$$

since each player's payoff depends on her opponents' choices  $\mathbf{x}_{-i}(t)$  only through their minimum value. Therefore, it is as if each player faces an aggregate opponent who chooses the minimum competitive price.

Third, we describe how a player best-responds to her belief in the generalized price matching game. Let  $b_i(t)$  denote player  $i$ 's belief about the aggregate opponent's rule level at the end of round  $t$ . In other words, player  $i$  expects that her aggregate opponent's choice in round  $t + 1$  will be

$$\hat{z}_{-i}(t + 1) = M(b_i(t)|x_0(t + 1)) = \max\{L, x_0(t + 1) - s \cdot b_i(t)\}.$$

Accordingly, player  $i$  best-responds by adopting level- $(b_i(t) + 1)$  rule and thus chooses

$$x_i(t + 1) = M(b_i(t) + 1|x_0(t + 1)) = \max\{L, x_0(t + 1) - s \cdot (b_i(t) + 1)\}.$$

With the three building blocks in place, we now proceed to complete the description of the  $DL_r(t)$  model by describing two separate sets of dynamics in the model, i.e., how the rule hierarchy mapping changes adaptively and how players' beliefs are updated based on observed game plays.

First, the rule hierarchy mapping varies over time because the level-0 rule's choice changes dynamically, adapting to the history of game play. Since the mapping  $M(\cdot|x_0(t))$  is uniquely determined given  $x_0(t)$ , it suffices to characterize the dynamic behavior of  $x_0(t)$ . As in (2.1), recall that the level-0 rule's choice  $x_0(t)$  at time  $t$  is the weighted average of ex post best responses in previous rounds. The ex post best response at time  $t$  is

$$\omega(t) = \max\{L, \min\{\mathbf{x}(t)\} - s\}$$

because it is the choice that would maximize a player's payoff in response to realized choices at time  $t$  in the ex post sense. (Note that the above minimum is taken from all players' choices.) Then, the level-0 rule's choice  $x_0(t)$  at time  $t$  is given by

$$x_0(t) = \frac{\rho_0 \cdot \phi_0^{t-1} \cdot \omega(0) + \phi_0^{t-2} \cdot \omega(1) + \dots + \phi_0 \cdot \omega(t-2) + \omega(t-1)}{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + 1},$$

where  $0 \leq \rho_0, 0 \leq \phi_0 \leq 1$ , and  $L \leq \omega(0) \leq U$  are respectively the initial prior strength, the memory decay factor, and the hypothetical ex post best response in round 0. These three parameters are to be estimated and they capture the nature of adaptive learning in the data.

Second, players' beliefs about the aggregate opponent's rule level change dynamically because beliefs are updated after each round based on historical information about observed game plays. Let us consider what happens at the end of a particular round  $t$ . Let the payoff-relevant statistic observed by player  $i$  in this round  $t$  be  $z_{-i}(t)$ . Based on this statistic, player  $i$  infers that her aggregate

opponent was of rule level  $r(z_{-i}(t))$ , where this rule level is inferred using the inverse rule hierarchy mapping:  $r(z_{-i}(t)) = M^{-1}(z_{-i}(t)|x_0(t))$ . Therefore, the inferred rule level satisfies

$$r(z_{-i}(t)) = \frac{x_0(t) - z_{-i}(t)}{s}.$$

Hence, as in (2.2), player  $i$ 's updated belief at the end of round  $t$  is given by

$$b_i(t) = \frac{\rho_d \cdot \phi_d^t \cdot b_i(0) + (n-1) \cdot \phi_d^{t-1} \cdot r(z_{-i}(1)) + \dots + (n-1) \cdot r(z_{-i}(t))}{\rho_d \cdot \phi_d^t + (n-1) \cdot \phi_d^{t-1} + \dots + (n-1)}$$

where  $0 \leq \rho_d$  and  $0 \leq \phi_d \leq 1$  are respectively the initial belief strength and the memory decay factor, which are parameters to be estimated. Further, recall that the initial belief  $b_i(0)$  is heterogeneous among players and follows a scaled beta distribution with parameters  $\alpha$  and  $\beta$  to be estimated. The above rule dynamics capture the nature of sophisticated learning in the data.

For the generalized price matching game, we again establish the repetition unraveling result.

**Theorem 2.5.** (*Repetition Unraveling*)  $\mathbb{E}[x_i(t)] \rightarrow L$  as  $t \rightarrow \infty$  if  $\rho_0 < \infty$  or  $\rho_d < \infty$ .

*Proof.* See Appendix. □

## Experimental Data

To test the  $DL_r(t)$  model described above, we generate data on the generalized price matching game by conducting another experiment. The experiment comprises two game settings varying the group size. Specifically, we set  $n = (3, 7)$ ,  $s = 10$  and  $R = 1.5$ . A total of 77 college students from a major university in Southeast Asia participated in the experiment, consisting of 7 groups of size 3 and 8 groups of size 7. Subjects chose any number from  $[80, 200]$ . Each subject played the same game for 10 rounds with a random matching protocol.<sup>29</sup> Subjects accumulated points in each round according to the payoff function in (2.3). After all 10 rounds, these points are then converted to the dollar amount at a rate of \$0.01 per point. The equilibrium payoff in all game settings was fixed at \$8. In addition, all participants were paid an additional \$5 show-up fee.<sup>30</sup> The full set of instructions is given in the Appendix.

The experimental data indicates that players' choices vary dramatically over time. Figure 2.12 compares the distributions of subjects' choices in the first round and in the last round. It is clear that subjects' choices are closer to the iterative dominance solution of  $L = 80$  in the last round. A Kolmogorov-Smirnov test rejects the null hypothesis that the distributions of choices in the first and last rounds are equal ( $p < 0.001$ ). In addition, Figure 2.13 shows that the proportion of subjects who chose the iterative dominance solution  $L = 80$  steadily increases over time. Overall, 63.77% of choices were 80. Among those choices, 33.40% occurred in rounds 1-5 and 66.60% occurred in rounds 6-10, suggesting that twice as many players chose the iterative dominance solution in the

<sup>29</sup>We use a random matching protocol to avoid reputation building. Subjects did not seem to engage in reputation building in our experiments.

<sup>30</sup>The prevailing exchange rate at the time of the experiment was US\$1 = \$1.20.

later rounds of the game. In the last round, 92.21% of choices were iterative dominance solution plays. Thus, the data shows that subjects' choices converge to the iterative dominance solution over time, consistent with the repetition unraveling result of Theorem 2.5.

## Results and Parameter Estimates

We fit the  $DL_r(t)$  model to our data on the generalized price matching game. The structural estimation procedures are similar to those described in Section 2.3 for the  $p$ -beauty contest game.<sup>31</sup> As before, there are five sets of parameters to be estimated: 1) beta distribution parameters  $(\alpha, \beta)$  for players' initial belief; 2) initial choice  $\omega(0)$  for the level-0 player; 3) adaptive learning parameters  $(\rho_0, \phi_0)$ ; 4) sophisticated learning parameters  $(\rho_d, \phi_d)$ ; and 5) standard deviation of choice parameter  $\sigma$ .

The imposed constraints on the parameters based on theoretical considerations were:  $0 < \alpha, \beta$ ;  $80 \leq \omega(0) \leq 200$ ;  $0 < \rho_0, \rho_d$ ; and  $0 \leq \phi_0, \phi_d \leq 1$ . Our results are shown below in Table 2.2, which is presented in the same format as Table 2.1 above. Standard errors of parameters were estimated using bootstrapping. In Table 2.2, standard errors are reported in parentheses below parameter estimates.

The log-likelihood scores in Table 2.2 show that the  $DL_r(t)$  model fits the data best. Based on the  $\chi^2$  test statistics and the AIC, we find that all four special cases are rejected in favor of the full  $DL_r(t)$  model, as in Section 2.3. We make the following observations:

1. The adaptive learning parameters are estimated to be  $\hat{\rho}_0 = 3.73$  and  $\hat{\phi}_0 = 0.90$ . That is, players give the level-0 player's initial choice a weight of 3.73 (equivalent to about 4 observations) and the influence of a past ex-post best response drops by half after every 6 rounds ( $0.90^6 = 0.53$ ). Recall that in the  $p$ -beauty contest game, past observations are discounted at a similar rate ( $\hat{\phi}_0 = 0.88$ ), but the weight on the level-0 player's initial choice is twice as high ( $\hat{\rho}_0 = 8.14$ ). As a result, adaptive learning is more pronounced (i.e., subjects adapt more rapidly) in generalized price matching than in  $p$ -beauty contest game.
2. The sophisticated learning parameters are  $\hat{\rho}_d = 59.55$  and  $\hat{\phi}_d = 0.98$ . The high estimate for  $\hat{\rho}_d$  shows that players weight their initial belief heavily (equivalent to about 60 observations). The estimate for  $\hat{\phi}_d$  suggests that the influence of past observed rule levels of aggregate opponent remains strong over time. Recall that the sophisticated learning parameters are  $\hat{\rho}_d = 2.16$  and  $\hat{\phi}_d = 0.08$  in the  $p$ -beauty contest game. In combination, these parameter estimates suggest that sophisticated learning is less pronounced (i.e., subjects update their rule levels more slowly) in generalized price matching game than in  $p$ -beauty contest game.

Consistent with the parameter estimates of  $DL_r(t)$  discussed above, the  $L_r(t)$  model fits better than the  $DL_r$  model on the data for the generalized price matching game. Both models have the

<sup>31</sup>One minor difference is that we assume that players' initial belief of opponents' rule levels follows a beta distribution scaled by a factor of 12 (instead of 50 in the  $p$ -beauty contest game). Since  $200 - 12 \cdot 10 = 80$ , we do not need to consider rule levels higher than 12 as they approximate the iterative dominance solution well irrespective of the level-0 player's choice  $x_0(t)$ . We varied the scale factor up to 30 and found that parameter estimates varied only slightly.

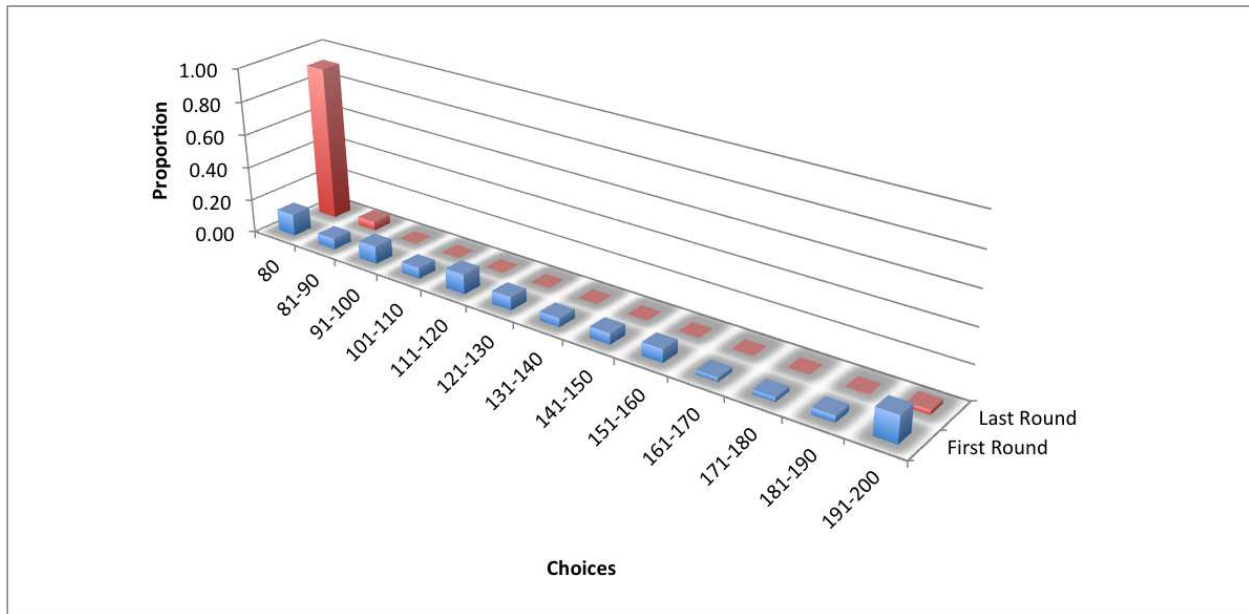


Figure 2.12: First and Last Round Data

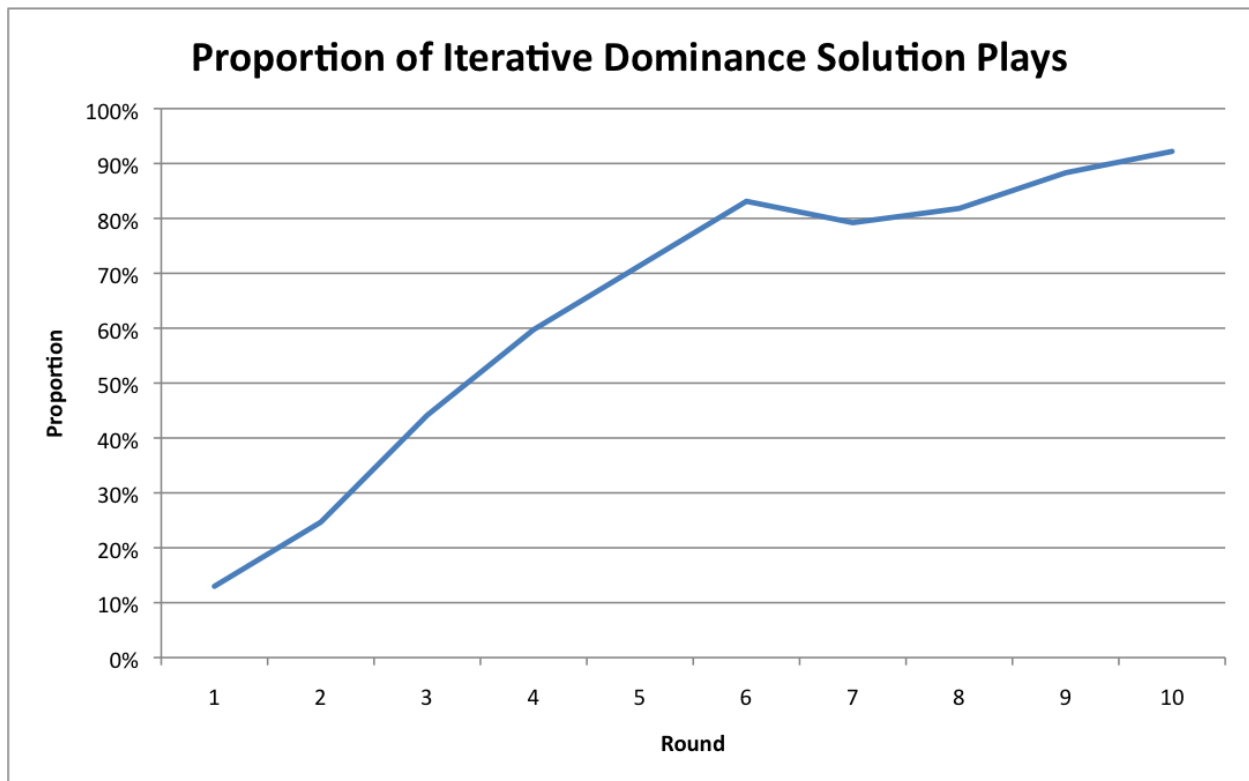


Figure 2.13: Proportion of Iterative Dominance Solution Plays

Table 2.2: Parameter Estimates and Maximized Log-likelihood for  $DL_r(t)$  Model and its Special Cases: Generalized Price Matching Game

Parameters/Models	$L_1(t)$	$L_r$	$L_r(t)$	$DL_r$	$DL_r(t)$
Initial Belief (Beta $(\alpha, \beta)$ )					
$\hat{\alpha}$	1	0.16 (1.15)	1.03 (0.18)	0.38 (0.11)	0.73 (0.16)
$\hat{\beta}$	$\infty$	0.02 (0.10)	0.72 (0.15)	0.24 (0.09)	0.60 (0.16)
Level-0's Choice at $t = 1$					
$\hat{\omega}(0)$	142.13 (4.11)	149.43 (11.61)	199.94 (0.04)	200.00 (0.92)	199.94 (0.03)
Adaptive Learning					
$\hat{\rho}_0$	0.35 (0.30)	$\infty$	3.70 (2.00)	$\infty$	3.73 (1.78)
$\hat{\phi}_0$	0.91 (0.20)	1	0.77 (0.07)	1	0.90 (0.10)
Sophisticated Learning					
$\hat{\rho}_d$	$\infty$	$\infty$	$\infty$	6.33 (1.56)	59.55 (19.48)
$\hat{\phi}_d$	1	1	1	0.68 (0.07)	0.98 (0.01)
Std. Dev. of Error					
$\hat{\sigma}$	25.87 (2.83)	28.98 (1.93)	18.96 (2.29)	20.39 (2.41)	18.90 (2.27)
$LL$	-789.13	-855.08	-646.02	-680.35	-641.98
$\chi^2$	294.31	426.20	8.08	76.76	-
( $p$ -value, dof)	(0.000, 4)	(0.000, 4)	(0.018, 2)	(0.000, 2)	-
AIC	1586.26	1718.06	1304.03	1372.71	1299.95*

same number of parameters, but the former attains a higher log-likelihood (the  $LL$  fit of  $L_r(t)$  is -646.02, while the  $LL$  fit of  $DL_r$  is -680.35). The AIC also selects the  $L_r(t)$  model over the  $DL_r$  model. Recall that this comparison was reversed for the  $p$ -beauty contest game. Hence, we find that adaptive learning is more pronounced in the generalized price matching game but sophisticated learning is more pronounced in the  $p$ -beauty contest game. These results point to the importance of  $DL_r(t)$  model, which allows for both adaptive and sophisticated learning. A model that allows for either type of learning alone will not be able to capture subjects' learning behavior in both games well.

When both adaptive and sophisticated learnings occur, a more restrictive model that considers only one type of learning may be misspecified. In the generalized price matching game, we find that the  $DL_r$  model is misspecified because it ignores adaptive learning. As a consequence, it overestimates the degree of sophisticated learning. Specifically, when adaptive learning is suppressed, the initial belief weight  $\rho_d$  has to be much lower in order to capture the dynamics in the data (i.e.,  $\hat{\rho}_d$  is 6.33

in  $DL_r$  whereas it is 59.55 in  $DL_r(t)$ ). However, we find that the adaptive learning parameters are consistently estimated in both  $L_r(t)$  and  $DL_r(t)$  models, since adaptive learning is more pronounced in the generalized price matching game.

We separately estimate the  $L_k$  model, which has a discrete rule hierarchy with a Poisson initial belief distribution with mean  $\lambda$ . The parameter estimates are  $\hat{\lambda} = 13.11$ ,  $\hat{\omega}(0) = 180.00$ , and  $\hat{\sigma} = 29.02$ , and the log-likelihood is  $-862.49$ . In contrast, the  $L_r$  model yielded a log-likelihood score of  $-855.08$ . These results show that the  $L_r$  model fits better than the  $L_k$  model, confirming again that a continuous rule hierarchy is useful in explaining the data.

Next, we examine the initial belief distributions obtained from the fitted  $L_r$ ,  $L_r(t)$ ,  $DL_r$ , and  $DL_r(t)$  models. The distributions are shown in Figure 2.14. We make three observations about these distributions.

1. The initial belief distributions of the  $DL_r(t)$  and  $L_r(t)$  models are quite similar. The  $DL_r(t)$  model, compared to the  $L_r(t)$  model, estimates that more players are of lower rule levels. This is because the  $DL_r(t)$  model allows players to update their beliefs dynamically and thus those players with lower rule levels can also choose the iterative dominance solution play over time by updating their beliefs.<sup>32</sup>
2. The initial belief distribution of  $DL_r$  model, compared to that of  $L_r(t)$  model, has a sharper increase at rule levels both near 0 and 12, meaning that more players have lower or higher level beliefs. This is because the  $DL_r$  model allows players to change their beliefs over time. In contrast, in the  $L_r(t)$  model which captures only adaptive learning behavior, players' beliefs remain static and thus are estimated to be more evenly distributed between 0 and 12.
3. In the  $L_r$  model, the predicted choice of level-0 players in the first round is estimated to be 149.43 and thus any initial belief higher than 6 (i.e., that opponents play rule level 6 or above) will induce players to choose the iterative dominance solution (which coincides with rule level 7 or above in this case). Since neither type of learning is allowed in  $L_r$ , about 90% of players are estimated to choose the iterative dominance solution right from the onset of the game in order to explain smaller choices and the fast convergence in the data over time.

<sup>32</sup>To get a better sense of how initial belief parameters  $\alpha$  and  $\beta$  are jointly estimated, we compared the proportion of level-0 players and the average rule level resulting from each set of estimates from 120 bootstrap runs with those resulting from the full-sample estimates. The standard deviations over those 120 estimates are listed in the parentheses. In  $DL_r(t)$ , the average proportion of level-0 players and the average rule level are 2.93% (1.67%) and 6.69 (0.52) respectively, while those based on full-sample estimates are 1.98% and 6.60; in  $DL_r$ , the average proportion of level-0 players and the average rule level are 8.62% (4.72%) and 7.15 (0.56) respectively, while those based on full-sample estimates are 7.20% and 7.31; in  $L_r(t)$ , the average proportion of level-0 players and the average rule level are 1.97% (1.30%) and 6.73 (0.56) respectively, while those based on full-sample estimates are 0.52% and 7.05; in  $L_r$ , the average proportion of level-0 players and the average rule level are 2.97% (2.58%) and 11.14 (0.41) respectively, while those based on full-sample estimates are 4.23% and 10.94. Overall, the pattern of initial belief distribution from bootstrapping appears to be quite similar on average to that based on full-sample estimates.

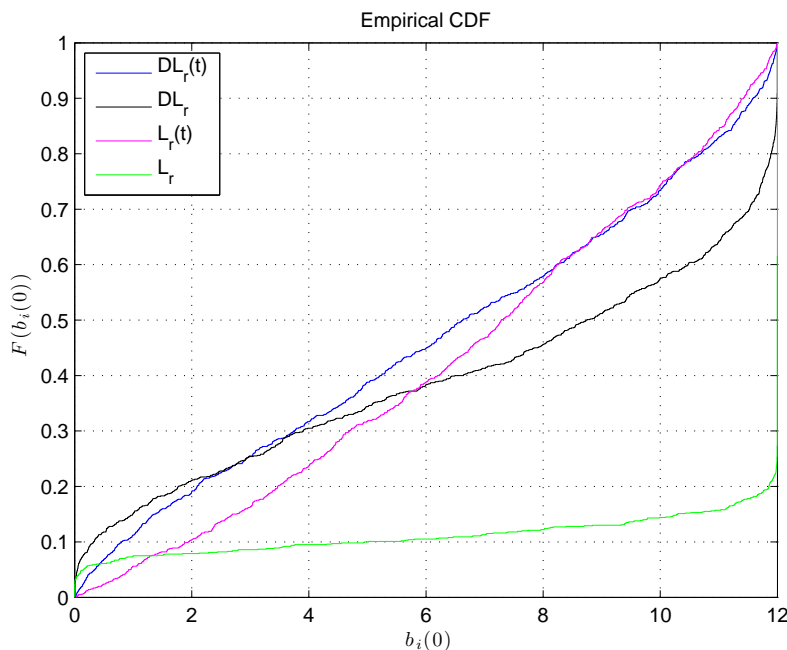


Figure 2.14: Distribution of Initial Beliefs

Finally, Figures 2.15 through 2.20 show the three-dimensional bar graphs of choice frequencies in the observed data as well as the model predictions of the  $DL_r(t)$ ,  $DL_r$ ,  $L_r(t)$ ,  $L_r$  and  $L_1(t)$  models, respectively. All plots show the combined results for both group sizes ( $n = 3$  and  $n = 7$ ), weighted by the number of observations in each condition.

Note the following:

1. The data for the generalized price matching game in Figure 2.15 shows rapid convergence to the iterative dominance solution. By round 4, more than 50% of subjects choose the iterative dominance solution. In contrast, for the  $p$ -beauty contest game (see Figure 2.5), the proportion of choices of the iterative dominance solution remains less than 40% through all 10 rounds.
2. Figure 2.16 and Figure 2.18 show the model predictions of the  $DL_r(t)$  model (best fit) and the  $L_r(t)$  model, respectively. These two figures look very similar, since dynamic behavior is driven mainly by adaptive learning, which is effectively captured by the  $L_r(t)$  model. Nevertheless, the  $DL_r(t)$  prediction is closer to the actual data and  $L_r(t)$  over-predicts the frequency of the iterative dominance solution by a greater extent in earlier rounds. Thus, as is the case for  $p$ -beauty contest game, the  $DL_r(t)$  model distinguishes itself from the second best fitting model in its power to better explain earlier rounds' data.
3. As shown in Figure 2.17, the  $DL_r$  model significantly over-predicts iterative dominance solution in the first round because it does not allow for adaptive learning dynamics and thus



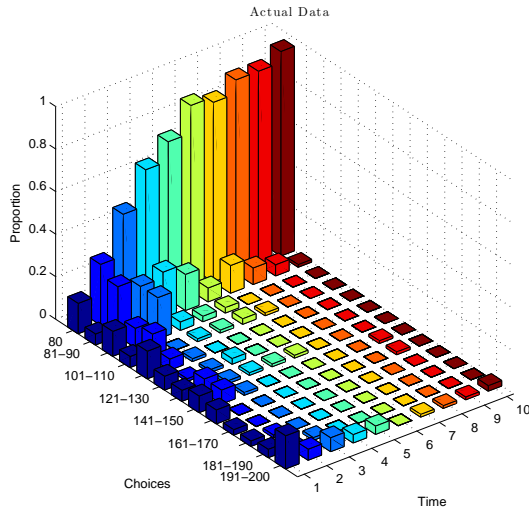


Figure 2.15: Actual Data

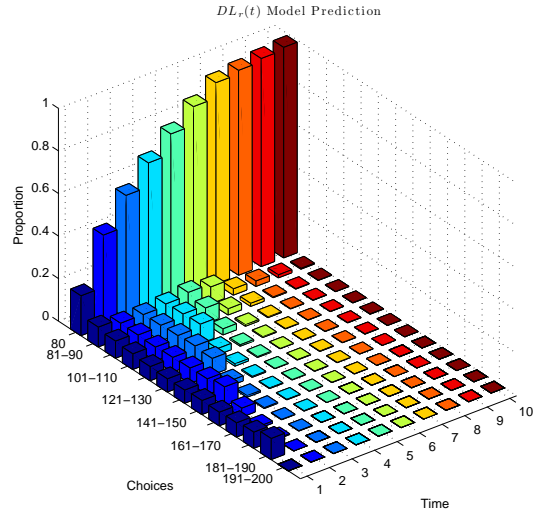


Figure 2.16:  $DL_r(t)$  Model Prediction

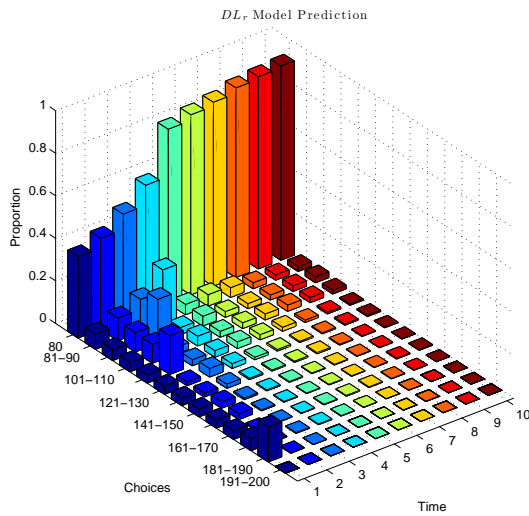


Figure 2.17:  $DL_r$  Model Prediction

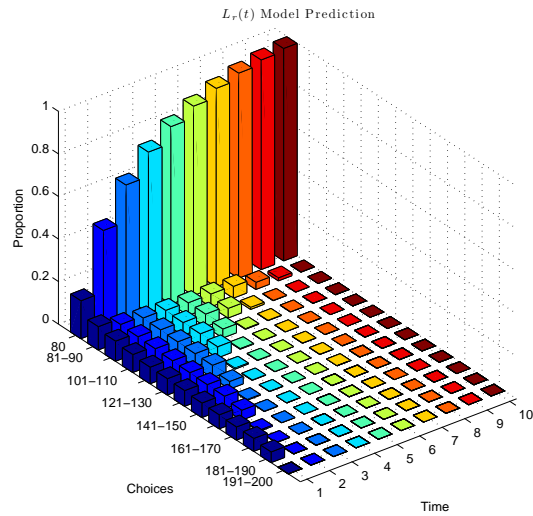


Figure 2.18:  $L_r(t)$  Model Prediction

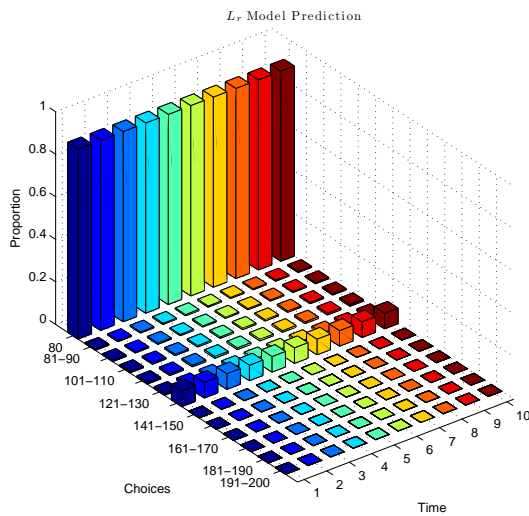


Figure 2.19:  $L_r$  Model Prediction

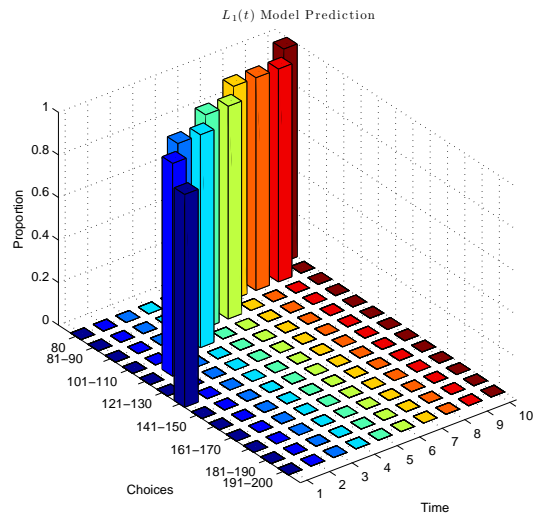


Figure 2.20:  $L_1(t)$  Model Prediction

more players are estimated to have higher level initial beliefs.

4. As shown in Figure 2.14, in the  $L_r$  model, about 90% of players are estimated to have high beliefs that induce the iterative dominance solution play. Hence, Figure 2.19 shows that  $L_r$  predicts most of choices to be 80 throughout the game and fails to capture the choice dynamics over time.<sup>33</sup>
5. Figure 2.20 shows that in  $L_1(t)$ , all players are weighted fictitious players and their choices gradually converge to 80 over time. It under-predicts the rate of convergence.

## 2.5 Conclusion

Dominance solvable games have a compelling theoretical prediction, but subjects who are motivated by substantial financial incentives often deviate from this unique iterative dominance solution. To explain their nonequilibrium behavior, the level- $k$  model is an attractive candidate because it is based on an iterative reasoning process (analogous to the iterated elimination of dominated strategies). Each iteration in the level- $k$  model is akin to a thinking step, in which players advance one level by best-responding to their counterparts who reason one step less. Lower level players do not perform enough thinking steps to reach the equilibrium initially. Nevertheless, empirical evidence shows that behavior converges to the equilibrium over time. To capture subjects' dynamic learning behavior, this paper proposes a model that generalizes the standard level- $k$  model.

Our model extends the level- $k$  model in 3 significant ways. First, we allow players to revise their rule levels over time. Specifically, players form beliefs over opponents' rule levels, update their beliefs upon observing opponents' choices, and choose the optimal rule level that best-responds to their beliefs. In this way, players may approach equilibrium play by choosing higher level rules, i.e., they become more sophisticated over time. Second, we specify the level-0 rule as the weighted average of ex post best responses in preceding rounds. As a result, as game play evolves, choices corresponding to the same rule level may move toward equilibrium. By adaptively responding to historical game play, players may approach the equilibrium without advancing in rule levels, i.e., they exhibit adaptive learning. Third, we generalize the standard level- $k$  approach, which uses a discrete rule hierarchy, to a level- $r$  framework, which accommodates a continuous rule hierarchy. This added flexibility decreases the reliance on the use of error structure to account for subjects' observed behavior.

Interestingly, the extended model provides a novel unification of two separate streams of research on non-equilibrium structural models of strategic behavior: 1) level- $k$  and 2) belief learning models. On one hand, level- $k$  and cognitive hierarchy models are "static" models that have been used to predict behaviors in one-shot games. On the other hand, belief learning models have been used to predict choice dynamics in repeated games. The two classes of models are treated separately

<sup>33</sup>Using parameter estimates of the  $L_k$  model, we also compare the model prediction of  $L_k$  with that of  $L_r$ . Like  $L_r$ , the  $L_k$  model also predicts that choices are highly concentrated around 80 throughout the game. However, having a more restrictive initial belief distribution,  $L_k$  model prediction is less spread out and does not show the cluster of choices in the 131-140 bin in the  $L_r$  model prediction.

in the literature. The extended model integrates these seemingly distinct streams of research and provides a sensible way to model behaviors in both one-shot and repeated games in a common and tractable model framework.

We apply the extended model to two classes of games:  $p$ -beauty contest game and generalized price matching game. Both types of games are dominance solvable and utilize a continuous strategy space. For the former, we fit our model to the data from Ho et al. (1998), and for the latter, we collect new data by running additional experiments. We find strong evidence that our generalized model explains the data better than its special cases, e.g., nested models that capture either adaptive or sophisticated learning but not both. Furthermore, our structural estimates allow us to identify whether adaptive learning or sophisticated learning is dominant in each class of game. We find that although subjects' dynamic behavior and convergence to the iterative dominance solution is readily observable in the experimental data for both games, the underlying learning dynamics that drives such convergence is quite different across these games: in  $p$ -beauty contest games, players' dynamic behavior is driven by sophisticated learning, whereas in generalized price matching games, it is driven by adaptive learning. Further, we show that a more restrictive model that allows only one type of learning may be misspecified, and it would not explain behavior well across both classes of games.

This paper incorporates both adaptive and sophisticated learning into the standard level- $k$  framework. Moreover, we prove that subjects' behavior in our model will converge to the iterative dominance equilibrium, provided that either adaptive or sophisticated learning is present. Therefore, our model can be viewed as a characterization of the equilibration process. This view bears the same spirit as Harsanyi's "tracing procedure" (Harsanyi and Selten, 1988), in which players' successive choices, as they react to new information in strategic environments, trace a path towards the eventual equilibrium outcome.

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# Appendix A

## Appendix to Chapter 1

### A.1 Proof of Theorems

*Proof of Theorem 1.1.* Recall that

$$\begin{aligned} u_{kj}^m &= -\beta_k \cdot p_j^m + \mathbf{x}_j^m \cdot \gamma_k + \xi_j^m + \epsilon_{kj}^m \\ &= v_{kj}^m + \epsilon_{kj}^m \end{aligned}$$

Let  $u_k^m(i)$  be the  $i$ -th order statistic among  $u_{kj}^m$  that result from realized  $\{\epsilon_{k0}^m, \dots, \epsilon_{kJ}^m\}$ . Thus,  $A_{kj}^m$  can be interpreted as the region where  $u_k^m(1) = u_{kj}^m$ . Then, the total consumer surplus can be expressed as:

$$\begin{aligned} \Phi^m &= \int_D \int_v \left( \sum_{j \in \mathcal{J}^m} \left( \int_{A_{kj}^m} \frac{u_{kj}^m - \epsilon_{k0}^m}{\beta_k} dF_\epsilon(\epsilon) \right) \right) dF_v(v) dF_D(D) \\ &= \int_D \int_v \left( \int \frac{u_k^m(1) - u_{k0}^m}{\beta_k} dF_\epsilon(\epsilon) \right) dF_v(v) dF_D(D) \\ &= \int_D \int_v \left( \int \frac{u_k^m(1)}{\beta_k} dF_\epsilon(\epsilon) \right) dF_v(v) dF_D(D) - \int_D \int_v \left( \int \frac{u_{k0}^m}{\beta_k} dF_\epsilon(\epsilon) \right) dF_v(v) dF_D(D) \end{aligned}$$

Since the second term is irrespective of  $p_j^m$ ,

$$\begin{aligned} \frac{\partial \Phi^m}{\partial p_j^m} &= \frac{\partial}{\partial p_j^m} \left( \int_D \int_v \left( \int \frac{u_k^m(1)}{\beta_k} dF_\epsilon(\epsilon) \right) dF_v(v) dF_D(D) \right) \\ &= \int_D \int_v \frac{1}{\beta_k} \cdot \left( \frac{\partial}{\partial p_j^m} u_k^m(1) dF_\epsilon(\epsilon) \right) dF_v(v) dF_D(D) \end{aligned}$$

Thus, it suffices to show that  $\frac{\partial}{\partial p_j^m} \int u_k^m(1) dF_\epsilon(\epsilon) < 0$ . Let  $G(u_k^m(1) | p_j^m, p_{-j}^m)$  be the distribution of order statistic  $u_k^m(1)$ , given the price of product  $j$ ,  $p_j^m$ , and the price vector of all other products,  $p_{-j}^m$ . We complete the proof by showing below that  $G(u_k^m(1) | p_j^m, p_{-j}^m)$  first-order stochastically



dominates  $G(u_k^m(1) | p', p_{-j}^m)$  for any  $p' < p_j^m$ .

For any  $p' < p_j^m$ ,

$$\begin{aligned}
\Pr[u_k^m(1) \leq z | p_j^m, p_{-j}^m] &= \prod_{j' \in \mathcal{J}^m} \Pr[u_{kj'}^m \leq z | p_{j'}^m] \\
&= \Pr[u_{kj}^m \leq z | p_j^m] \cdot \prod_{j' \neq j, j' \in \mathcal{J}^m} \Pr[u_{kj'}^m \leq z | p_{j'}^m] \\
&= \Pr[\epsilon_{kj}^m \leq z + \beta_k \cdot p_j^m - (\mathbf{x}_j^m \cdot \gamma_k + \xi_j^m)] \cdot \prod_{j' \neq j, j' \in \mathcal{J}^m} \Pr[u_{kj'}^m \leq z | p_{j'}^m] \\
&< \Pr[\epsilon_{kj}^m \leq z + \beta_k \cdot p' - (\mathbf{x}_j^m \cdot \gamma_k + \xi_j^m)] \cdot \prod_{j' \neq j, j' \in \mathcal{J}^m} \Pr[u_{kj'}^m \leq z | p_{j'}^m] \\
&= \Pr[u_{kj}^m \leq z | p'] \cdot \prod_{j' \neq j, j' \in \mathcal{J}^m} \Pr[u_{kj'}^m \leq z | p_{j'}^m] \\
&= \Pr[u_k^m(1) \leq z | p', p_{-j}^m]
\end{aligned}$$

Hence,  $G(u_k^m(1) | p_j^m, p_{-j}^m)$  first-order stochastically dominates  $G(u_k^m(1) | p', p_{-j}^m)$  for any  $p' < p_j^m$  and the theorem is proved.  $\square$

*Proof of Theorem 1.2.* The equilibrium price  $p_j^m(\alpha_i)$  of product  $j$  in market  $m$  satisfies its first order condition:

$$(1 - \alpha_i) \cdot \frac{\partial \pi_i^m}{\partial p_j^m(\alpha_i)} + \alpha_i \cdot \frac{\partial \Phi^m}{\partial p_j^m(\alpha_i)} = 0$$

Let  $g(\alpha_i, p_j^m(\alpha_i))$  denote the lefthand side of the above equation so that  $\alpha_i$  and corresponding equilibrium price  $p_j^m(\alpha_i)$  satisfies

$$g(\alpha_i, p_j^m(\alpha_i)) = 0$$

Rearranging  $g(\alpha_i, p_j^m(\alpha_i))$ , we obtain

$$\frac{\partial \pi_i^m}{\partial p_j^m(\alpha_i)} + \alpha_i \cdot \left( \frac{\partial \Phi^m}{\partial p_j^m(\alpha_i)} - \frac{\partial \pi_i^m}{\partial p_j^m(\alpha_i)} \right) = 0$$

Since  $\frac{\partial \Phi^m}{\partial p_j^m(\alpha_i)} < 0$  by theorem 1.1, it must be the case that

$$\frac{\partial \pi_i^m}{\partial p_j^m(\alpha_i)} > 0 \tag{A.1}$$

so that  $g(\alpha_i, p_j^m(\alpha_i)) = 0$ . Hence,

$$\left( \frac{\partial \Phi^m}{\partial p_j^m(\alpha_i)} - \frac{\partial \pi_i^m}{\partial p_j^m(\alpha_i)} \right) < 0 \tag{A.2}$$

Besides, if  $p_j^m(\alpha_i)$  is the equilibrium price of product  $j$  of firm  $i$ , then firm  $i$ 's weighted objective function has to be concave in  $p_j^m$  at  $p_j^m(\alpha_i)$  since  $p_j^m(\alpha_i)$  is the objective function maximizer. Specifically,

$$\left. \frac{\partial g(\alpha_i, p_j^m)}{\partial p_j^m} \right|_{p_j^m(\alpha_i)} = (1 - \alpha_i) \cdot \left. \frac{\partial^2 \pi_i^m}{(\partial p_j^m)^2} \right|_{p_j^m(\alpha_i)} + \alpha_i \cdot \left. \frac{\partial^2 \Phi^m}{(\partial p_j^m)^2} \right|_{p_j^m(\alpha_i)} < 0 \quad (\text{A.3})$$

We shall use the implicit function theorem to prove the theorem. By the implicit function theorem, it suffices to show that

$$\frac{\partial p_j^m(\alpha_i)}{\partial \alpha_i} = - \frac{\frac{\partial g(\alpha, p_j^m)}{\partial \alpha_i}}{\frac{\partial g(\alpha, p_j^m)}{\partial p_j^m}} < 0$$

By equation (A.2),

$$\frac{\partial g(\alpha, p_j^m)}{\partial \alpha_i} = \left( \frac{\partial \Phi^m}{\partial p_j^m(\alpha_i)} - \frac{\partial \pi_i^m}{\partial p_j^m(\alpha_i)} \right) < 0$$

and by equation (A.3),

$$\frac{\partial g(\alpha, p_j^m)}{\partial p_j^m} < 0$$

Hence,

$$\frac{\partial p_j^m(\alpha_i)}{\partial \alpha_i} < 0$$

as desired. □

*Proof of Theorem 1.3.* By chain rule,

$$\frac{\partial \pi_i^m}{\partial \alpha_i} = \sum_{j \in \mathcal{J}^m} \frac{\partial p_j^m}{\partial \alpha_i} \cdot \frac{\partial \pi_i^m(\mathbf{p}_i^m, \mathbf{p}_{-i}^m)}{\partial p_j^m}$$

Since the other firms' prices are assumed to remain unchanged, we have

$$\frac{\partial p_j^m}{\partial \alpha_i} = 0, \quad \forall j \notin \mathcal{J}_i^m$$

Thus,

$$\frac{\partial \pi_i^m}{\partial \alpha_i} = \sum_{j \in \mathcal{J}_i^m} \frac{\partial p_j^m}{\partial \alpha_i} \cdot \frac{\partial \pi_i^m(\mathbf{p}_i^m, \mathbf{p}_{-i}^m)}{\partial p_j^m}$$

We showed that  $\frac{\partial p_j^m}{\partial \alpha_i} < 0, \forall j \in \mathcal{J}_i^m$  in theorem 1.2 and  $\frac{\partial \pi_i^m(\mathbf{p}_i^m, \mathbf{p}_{-i}^m)}{\partial p_j^m} > 0, \forall j \in \mathcal{J}_i^m$  in equation (A.1).

Hence,  $\frac{\partial \pi_i^m}{\partial \alpha_i} < 0$  as desired. □

## A.2 Definition of Variables

- *INCOME* : *INCOME* is monthly income input as one of the 11 income brackets of [\$0, \$1000], [[\$1000, \$1500], [\$1500, \$2000], [\$2000, \$2500], [\$2500, \$3000], [\$3000, \$3500], [\$3500, \$4000], [\$4000, \$6000], [\$6000, \$8000], [\$8000, \$10000], and above \$10000.
- *HHOLDSIZE* : *HHOLDSIZE* is the size of the household.
- *AGE* : *AGE* is the age of the primary grocery buyer.
- *DWELLING* : *DWELLING* is a dummy variable that is equal to one if the household lives in a subsidized public housing and zero if in a private housing.
- *WORK* : *WORK* is a dummy variable that is equal to one if the primary grocery buyer works.
- *MAID* : *MAID* is a dummy variable that is equal to one if the household has a maid.
- *CHILD04* : *CHILD04* is a dummy variable that is equal to one if the household has a child aged 4 or below.
- *CHILD514* : *CHILD514* is a dummy variable that is equal to one if the household has a child aged between 5 and 14.
- *FAMILY* : *FAMILY* is a dummy variable that is equal to one if the household is of singles/couples type and zero if it is of family type.
- *FEMALE* : *FEMALE* is a dummy variable that is equal to one if the household has a female at age of 30 years or older.

### A.3 Key Position Holders of *NTUC*

Name	Position at <i>NTUC</i>	Term	Political Career
Devan Nair	Secretary General	'61-'65	President of Singapore ('81-'85)
	Secretary General	'70-'79	
	President	'79-'81	
ST Nagayan	Secretary General	'65-'66	Member of Parliament
Ho See Beng	President	'62-'64	Member of Parliament
	Secretary General	'66-'67	
	Chairman	'66-'67	
Seah Mui Kok	Secretary General	'67-'70	Member of Parliament
Lim Chee Ong	Secretary General	'79-'83	Member of Parliament
Ong Teng Cheong	Secretary General	'83-'93	Cabinet Minister
			Deputy Prime Minister
			President of Singapore ('93-'99)
Lim Boon Heng	Secretary General	'93-'06	Cabinet Minister Chairman of PAP <sup>a</sup>
Lim Swee Say	Secretary General	'07-Present	Member of Parliament

<sup>a</sup>People's Action Party is the single most dominant political party in Singapore historically occupying 93% to 100% seats of Singapore parliament.

# Appendix B

## Appendix to Chapter 2

### B.1 Proof of Theorems

*Proof of Theorem 2.4 and Theorem 2.5.* First, we show that repetition unraveling property holds if  $\rho_d < \infty$ . Let  $w_n(t) = \frac{(n-1)}{\rho_d \cdot \phi_d^t + \phi_d^{t-1} \cdot (n-1) + \dots + (n-1)}$ . We have

$$\begin{aligned} b_i(t) &= \frac{\rho_d \cdot \phi_d^t \cdot b_i(0) + \phi_d^{t-1} \cdot (n-1) \cdot r(z_{-i}(1)) + \dots + (n-1) \cdot r(z_{-i}(t))}{\rho_d \cdot \phi_d^t + \phi_d^{t-1} \cdot (n-1) + \dots + (n-1)} \\ &= (1 - w_n(t)) \cdot b_i(t-1) + w_n(t) \cdot r(z_{-i}(t)) \end{aligned} \quad (\text{B.1})$$

Let  $r_{max}$  be the rule level corresponding to the iterative dominance solution, i.e., the highest rule level that a player can play. Naturally,  $b_i(t) \in [0, r_{max}]$ . Note that  $r_{max}$  may be  $\infty$ , which is the case in the  $p$ -beauty contest game with strategy space  $[0, U]$ . Then, the below lemma proves Theorem 2.4 and Theorem 2.5 for  $\rho_d < \infty$ .

**Lemma B.1.** *If  $\rho_d < \infty$ , then  $\lim_{t \rightarrow \infty} b_i(t) = r_{max}$ .*

*Proof of Lemma B.1.* It suffices to show that the lower corner of range of  $b_i(t)$  converges to  $r_{max}$ . The lower corner of the range of  $b_i(t)$  is realized when all players' initial belief is lowest possible, i.e.,  $b_i(0) = 0, \forall i$ . Since all players start with the same initial belief, their observations are the same in each subsequent round, and thus they update the same after each round. As a consequence, all players choose the same number at every time  $t$ . Hence,  $b_i(t) = b_j(t) \forall j \neq i, \forall t$  and

$$r(z_{-i}(t)) = \min\{b_i(t-1) + 1, r_{max}\} \quad \forall i \text{ and } \forall t. \quad (\text{B.2})$$

First, if  $r_{max} = \infty$ , we have the lower corner of range of 1st round belief as

$$\begin{aligned} b_i(1) &= (1 - w_n(1)) \cdot b_i(0) + w_n(1) \cdot r(z_{-i}(1)) && \text{by (B.1)} \\ &= (1 - w_n(1)) \cdot b_i(0) + w_n(1) \cdot (b_i(0) + 1) && \text{by (B.2)} \\ &= b_i(0) + w_n(1) \\ &= w_n(1) \end{aligned}$$

and iteratively, we have the lower corner of range of  $t$ -th round belief as

$$b_i(t) = \sum_{s=1}^t w_n(s).$$

If  $\rho_d < \infty$  and  $\phi_d \in [0, 1)$ , then  $\lim_{t \rightarrow \infty} w_n(t) = 1 - \phi_d > 0$ . Thus,  $\lim_{t \rightarrow \infty} b_i(t) = r_{max} = \infty$ . If  $\rho_d < \infty$  and  $\phi_d = 1$ , let  $\rho_d < k \cdot (n - 1)$  for some positive integer  $k$ . Then,  $\lim_{t \rightarrow \infty} b_i(t) = \lim_{t \rightarrow \infty} \sum_{s=1}^t w_n(s) > \lim_{t \rightarrow \infty} \sum_{s=1}^t \frac{(n-1)}{(k+s) \cdot (n-1)} = \lim_{t \rightarrow \infty} \sum_{s=1}^t \frac{1}{k+s} = \infty$ . Thus, regardless of level-0 player's choice, players' choices converge to the iterative dominance solution as  $t \rightarrow \infty$ .

Second, if  $r_{max} < \infty$ , there exists smallest  $0 < t_0 < \infty$  such that  $r(z_{-i}(t)) = r_{max}$  for all  $t \geq t_0$ . Then for all  $t > t_0$ , we have the lower corner of range of  $b_i(t)$  as

$$b_i(t) = \prod_{s=t_0}^t (1 - w_n(s)) \cdot b_i(t_0 - 1) + \left( 1 - \prod_{s=t_0}^t (1 - w_n(s)) \right) \cdot r_{max}.$$

where  $b_i(t_0 - 1) < r_{max}$ . It is easy to show that if  $\rho_d < \infty$ , then  $\lim_{t \rightarrow \infty} \prod_{s=t_0}^t (1 - w_n(s)) = 0$  for any  $\phi_d \in [0, 1]$ . Hence,  $\lim_{t \rightarrow \infty} b_i(t) = r_{max}$  as desired.  $\square$

Next, we show that repetition unraveling property holds if  $\rho_0 < \infty$ . We assume the iterative dominance solution is  $L = 0$  in the  $p$ -beauty contest game. Proof for  $L > 0$  is analogous. In both games, it suffices to show that  $x_0(t)$  converges to the iterative dominance solution as  $t \rightarrow \infty$ . All players have at least level-0 belief and thus are at least of rule level 1. Hence, we have

$$\omega(1) \leq \frac{p \cdot (n-1)}{n-p} \cdot x_0(1) < x_0(1). \quad (\text{B.3})$$

Since level-0 rule's choice in round 2 equals  $x_0(2) = \frac{\rho_0 \cdot \phi_0 \cdot x_0(1) + \omega(1)}{\rho_0 \cdot \phi_0 + 1}$ ,  $\rho_0 < \infty$ , and  $\phi_0 \in [0, 1]$ , the weight given to  $\omega(1)$  is bigger than 0, i.e.,  $\frac{1}{\rho_0 \cdot \phi_0 + 1} > 0$ . Thus using (B.3),

$$x_0(2) \leq \frac{\rho_0 \cdot \phi_0 + u}{\rho_0 \cdot \phi_0 + 1} \cdot x_0(1),$$

where  $u = \frac{p \cdot (n-1)}{n-p}$ . Through similar iterative processes over time, we obtain

$$x_0(t) \leq \frac{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + u}{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + 1} \cdot x_0(t-1).$$

Letting  $q(t) = \rho_0 \cdot \phi_0^t + \phi_0^{t-1} + \dots + \phi_0 + 1$ , we have

$$x_0(t) \leq \prod_{s=1}^{t-1} \left( 1 - \frac{1-u}{q(s)} \right) \cdot x_0(1).$$

Thus, it suffices to show that

$$\lim_{t \rightarrow \infty} \prod_{s=1}^{t-1} \left(1 - \frac{1-u}{q(s)}\right) = 0.$$

Note that if  $\rho_0 < \infty$  and  $\phi_0 < 1$ , then  $\lim_{s \rightarrow \infty} q(s) = \frac{1}{1-\phi_0}$  and  $\lim_{s \rightarrow \infty} \left(1 - \frac{1-u}{q(s)}\right) = 1 - (1-u) \cdot (1-\phi_0)$ . Thus  $\lim_{s \rightarrow \infty} \left(1 - \frac{1-u}{q(s)}\right) < 1$  and it is readily shown that  $\lim_{t \rightarrow \infty} \prod_{s=1}^{t-1} \left(1 - \frac{1-u}{q(s)}\right) = 0$ . If  $\phi_0 = 1$ , it boils down to showing that

$$\lim_{t \rightarrow \infty} \sum_{s=1}^{t-1} \log \left(1 - \frac{1-u}{q(s)}\right) = -\infty.$$

To show the above, we have

$$\begin{aligned} \lim_{t \rightarrow \infty} \sum_{s=1}^{t-1} \log \left(1 - \frac{1-u}{q(s)}\right) &< \lim_{t \rightarrow \infty} \int_1^t \log \left(1 - \frac{1-u}{q(s)}\right) ds \\ &= \lim_{t \rightarrow \infty} \int_1^t \log \left(1 - \frac{1-u}{\rho_0 + s}\right) ds \\ &= \lim_{t \rightarrow \infty} \int_1^t - \sum_{k=1}^{\infty} \left( \frac{\left(\frac{1-u}{\rho_0+s}\right)^k}{k} \right) ds \\ &< \lim_{t \rightarrow \infty} \int_1^t - \frac{1-u}{\rho_0 + s} ds \\ &= \lim_{t \rightarrow \infty} -(1-u) \cdot \ln(\rho_0 + s) \Big|_1^t = -\infty. \end{aligned}$$

Note that we use a Taylor expansion so that  $\log \left(1 - \frac{1-u}{\rho_0+s}\right) = - \sum_{k=1}^{\infty} \left( \frac{\left(\frac{1-u}{\rho_0+s}\right)^k}{k} \right)$  since  $\left| \frac{1-u}{\rho_0+s} \right| < 1$ . Hence,  $x_0(t) \rightarrow 0$  and players' choices converge to iterative dominance solution over time.

In the generalized price matching game, let  $L$  be the lower corner of strategy space and the iterative dominance solution. Then,

$$\begin{aligned} x_0(t) &= \frac{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0}{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + 1} \cdot x_0(t-1) + \frac{1}{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + 1} \cdot \omega(t-1) \\ &\leq \frac{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0}{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + 1} \cdot x_0(t-1) \\ &\quad + \frac{1}{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + 1} \cdot \max\{L, x_0(t-1) - 2 \cdot s\}, \end{aligned}$$

since the minimum choice in round  $t - 1$  is at most  $x_0(t - 1) - s$  (because all players have at least level-0 belief and thus are of at least level-1) and the ex post best response  $\omega(t - 1)$  is  $s$  less than the former. Suppose  $\max\{L, x_0(t - 1) - 2 \cdot s\} = x_0(t - 1) - 2 \cdot s, \forall t$ . Then,  $x_0(t) \leq x_0(t - 1) - \frac{1}{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + 1} \cdot 2 \cdot s, \forall t$ . Since  $\sum_{t=1}^{\infty} \frac{1}{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + 1} = \infty, \forall \phi_0 \in [0, 1]$ , this implies that  $\lim_{t \rightarrow \infty} x_0(t) = -\infty$  which contradicts that  $x_0(t) \in [L, U]$ . Hence,  $x_0(t - 1) - 2 \cdot s$  eventually becomes smaller than  $L$  for sufficiently large  $t$ . We let  $t_0$  be the threshold period where  $\max\{L, x_0(t - 1) - 2 \cdot s\} = L$  for  $\forall t \geq t_0$ . Then,  $\forall t > t_0$ ,

$$x_0(t) \leq \frac{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0^{t-t_0}}{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + 1} \cdot x_0(t_0) + \frac{\sum_{s=0}^{t-t_0-1} \phi_0^s}{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + 1} \cdot L$$

If  $\rho_0 < \infty, \lim_{t \rightarrow \infty} \frac{\sum_{s=0}^{t-t_0-1} \phi_0^s}{\rho_0 \cdot \phi_0^{t-1} + \phi_0^{t-2} + \dots + \phi_0 + 1} = 1, \forall \phi_0 \in [0, 1]$ . Hence,  $x_0(t) \rightarrow L$  as desired.  $\square$

## B.2 Instructions for Generalized Price Matching Game

Below is the instruction for the experimental generalized price matching game for the group of size 3. That for the group of size 7 is exactly same but only with different numerical examples.

### Instructions

This is an experiment about economic decision-making. The instructions are simple; and if you follow them carefully and make good decisions, you may earn a considerable amount of money which will be paid to you in cash at the end of the experiment. It is important that you do not look at the decisions of others, and that you do not talk, laugh, or make noises during the experiment. You will be warned if you violate this rule the first time. If you violate this rule twice, you will be asked to leave the room immediately and your cash earnings will be \$0.

The experiment consists of 10 decision rounds. In each decision round, you will be randomly assigned into groups of 3 participants. That is, you are randomly matched with other participants in each round, and your paired members may be different round by round.

### Experimental Procedure

In each decision round, all members in each group are asked to choose a number simultaneously between 80 and 200 (inclusive of 80 and 200, and up to 1 decimal point). Specifically, each member can choose any number among 80.0, 80.1, 80.2, ..., 199.8, 199.9, and 200.0. Note that each member must make a choice without knowing what his or her paired members will choose. Once all members in a group make their choices, the computer will determine each members point earning based on everyones chosen number. Ones point earning is determined by ones chosen number and



the minimum choice of ones paired members excluding oneself. Details of how point earning is determined will be given below. After each decision round, each participant will be told (a) his/her choice in that round, (b) the minimum choice of his/her paired members excluding him/herself in that round, and (c) his/her point earning in that round. Again, every participant will undertake this task 10 times, and will be randomly matched with other participants in each round.

### Determination of Point Earnings

Your total point earning in each round is the sum of 2 components: baseline earning and supplementary earning. Specifically, we have:

$$\text{TOTAL POINT EARNING} = \text{BASELINE EARNING} + \text{SUPPLEMENTARY EARNING.}$$

1. BASELINE EARNING: In each round, the baseline earning in a group is given by the minimum of all members choices in that group. That is, we will examine all choices in each group and use the minimum of the choices in the group to determine its baseline earning. Note that all members in a group receive the same baseline earning. Different groups, however, may receive different baseline earnings depending on the choices of their respective group members.
2. SUPPLEMENTARY EARNING: Depending on the relative magnitude between your own choice and the minimum of your paired members choices, your supplementary earning can be either positive, negative, or zero. Each case is described in full detail below.
  - a) Positive Supplementary Earning: You will receive a positive supplementary earning (on top of your baseline earning) if your own choice is smaller than the minimum choice of your paired members excluding yourself. This means that your choice is the only smallest number in the group. That is, you have chosen the lowest number, while all your paired members have chosen bigger numbers than you did. Since you receive a positive supplementary earning, your total point earning will be higher than your baseline earning as a consequence.

Precisely, your positive supplementary earning is 1.5 TIMES the difference between your choice and the minimum of your paired members choices. However, there is a cap on the maximum supplementary earning that you can earn. In this experiment, you can earn up to 15 supplementary points. Again, your total earning is the sum of your baseline earning and supplementary earning.

- b) Negative Supplementary Earning: You will receive a negative supplementary earning (i.e., a deduction from your baseline earning) if your choice is higher than the minimum choice of your paired members excluding yourself. This means that someone else in your group has chosen a smaller number than you did, and your choice is NOT the lowest number (or the minimum). Since you receive a negative supplementary earning, your total point earning will be lower than your baseline earning as a consequence.

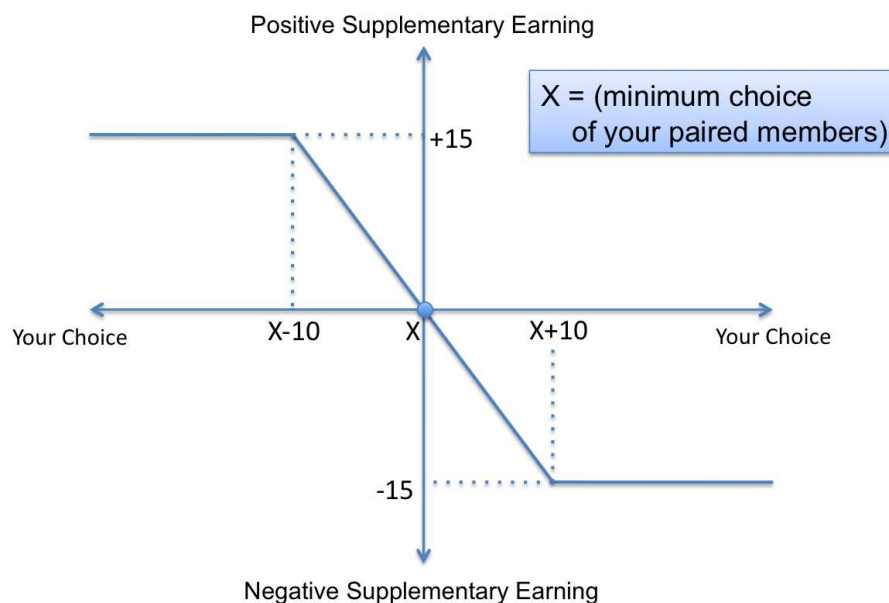


Figure 1: Supplementary Earning

Precisely, the magnitude of your negative supplementary earning is 1.5 TIMES the difference between your choice and the minimum of your paired members choices. However, there is a cap on the maximum supplementary points that will be deducted from your baseline earning. In this experiment, up to 15 supplementary points can be deducted from your baseline earning. Again, your total earning is the sum of your baseline earning and supplementary earning.

- c) No Supplementary Earning: You will receive no supplementary earning if your choice is the smallest in your group, but there are also other paired members who have chosen the same number as well. As a consequence, your total point earning will simply be your baseline earning.

Figure 1 summarizes the 3 possible cases of supplementary earning described above.

If your choice is smaller than the minimum of your paired members choices (on the left hand side of Figure 1), your positive supplementary earning increases as your choice gets even smaller and farther away from your paired members minimum choice (indicated as  $X$  at the origin). Your positive supplementary earning is exactly 1.5 times the difference between your choice and your paired members minimum choice. However, once the positive supplementary earning hits 15, your positive supplementary earning stops increasing and remains at 15. Note that as you choose a smaller number on the left hand side of Figure 1, the baseline earning for everyone in the group becomes smaller too (because the smaller number that you choose is also the minimum of all members choices).

If your choice is higher than the minimum of your paired members choices (on the right hand side of Figure 1), you will receive a negative supplementary earning. The magnitude of your negative supplementary earning increases as your choice gets even higher and farther away from the minimum of your paired members choices (indicated as X at the origin). The magnitude of your negative supplementary earning is exactly 1.5 times the difference between your choice and your paired members minimum choice. Likewise, once the negative supplementary earning hits  $-15$ , it stops getting larger and remains at  $-15$ . Note that as you choose a higher number on the right hand side of Figure 1, the baseline earning for everyone in the group remains the same at the minimum of your paired members choices (i.e., X) because X remains the minimum of all members choices.

Finally, if your choice is the same as the minimum of your paired members choices (when your choice is at the origin of Figure 1), your supplementary earning is zero, and your total earning is simply the baseline earning.

### Illustrative Examples

The numbers used in following examples are chosen purely for illustrative purposes.

**EXAMPLE 1:** Assume your paired members chose 160.0 and 167.2, respectively. Note that the minimum of your paired members choices is 160.0 (the smaller of 160.0 and 167.2). We shall use Figure 2 below (which we created by relabeling Figure 1) to determine your supplementary earning depending on possible choices you can make. Note that the origin is now set to 160.

1. If you choose 130, everyone's baseline earning is 130 (since 130 is the minimum of the choices of all group members including yourself). As indicated in Point A, your supplementary earning will be 15. This is because your choice is 30 below 160, and 1.5 times this difference exceeds 15, leaving your positive earning at its cap of 15. As a consequence, your total earning is  $130 + 15 = 145$ .
2. If you choose 155, everyone's baseline earning is 155. As indicated in Point B, your supplementary earning will be 7.5 (1.5 times the difference between your choice of 155 and the minimum choice of the paired members of 160). As a consequence, your total earning is  $155 + 7.5 = 162.5$ .
3. If you choose 160, everyone's baseline earning is 160. As indicated in Point C, your supplementary earning is zero because your choice is identical to the minimum of the paired members. As a consequence, your total earning is  $160 + 0 = 160$ .
4. If you choose 165, everyone's baseline earning is 160 (since 160 is the minimum of choices of all group members including yourself). As indicated in Point D, your supplementary earning will be  $-7.5$ . This is because your choice is 5 above 160, and 1.5 times this difference is 7.5. As a consequence, your total earning is  $160 - 7.5 = 152.5$ .
5. If you choose 190, everyone's baseline earning is 160. As indicated in Point E, your supplementary earning will be  $-15$ , because 1.5 times the difference between your choice and the

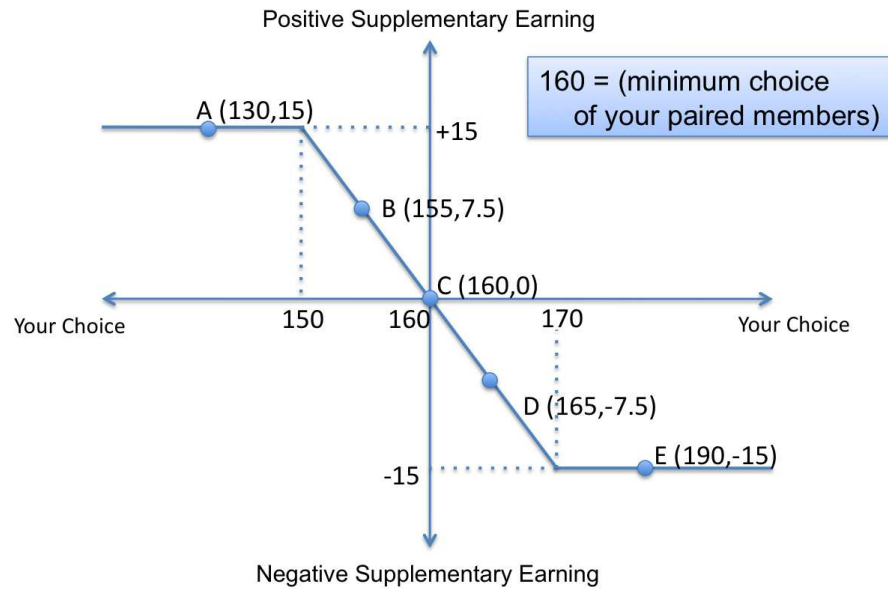


Figure 2: Choice Scenarios When the Paired Members Chose 160.0 and 167.2

Your Choice	Point Indicated in Figure 2	Your Paired Members' Minimum Choice	Baseline Earning	Supplementary Point Earning	Total Point Earning
130.0	A	160.0	130.0	+15.0	145.0
155.0	B	160.0	155.0	+7.5	162.5
160.0	C	160.0	160.0	0	160.0
165.0	D	160.0	160.0	-7.5	152.5
190.0	E	160.0	160.0	-15.0	145.0

Table 1: Summary of Earnings when the Paired Members Chose 160.0 and 167.2

minimum of the paired members choices is 45 which exceeds 15. As a consequence, your total earning is  $160 - 15 = 145$ .

Table 1 summarizes the baseline, supplementary and total point earnings for the 5 possible scenarios (when the paired members choose 160.0 and 167.2).

**EXAMPLE 2:** Assume your paired members chose 120.0 and 147.8, respectively. Note that the minimum of your paired members choices is 120.0 (the smaller of 120.0 and 147.8). We shall use Figure 3 below (which we created by relabeling Figure 1) to determine your supplementary earning depending on possible choices you can make. Note that the origin is now set to 120.

1. If you choose 90, everyone's baseline earning is 90 (since 90 is the minimum of the choices of all group members including yourself). As indicated in Point V, your supplementary earning

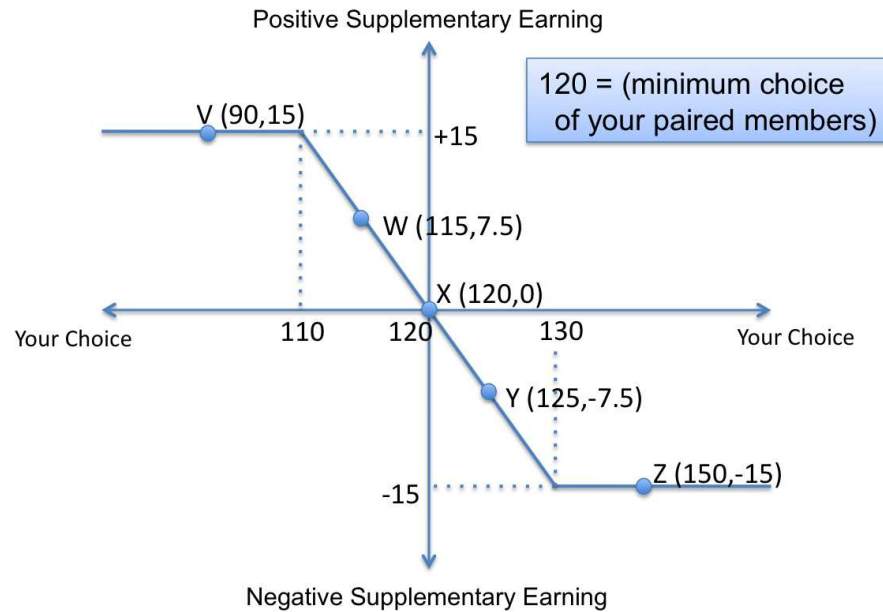


Figure 3: Choices Scenarios when the Paired Members Chose 120.0 and 147.8

- will be 15. This is because your choice is 30 below 120, and 1.5 times this difference exceeds 15, leaving your positive earning at its cap of 15. As a consequence, your total earning is  $90 + 15 = 105$ .
2. If you choose 115, everyone's baseline earning is 115. As indicated in Point W, your supplementary earning will be 7.5 (1.5 times the difference between your choice of 115 and the minimum choice of the paired members of 120). As a consequence, your total earning is  $115 + 7.5 = 122.5$ .
  3. If you choose 120, everyone's baseline earning is 120. As indicated in Point X, your supplementary earning is zero because your choice is identical to the minimum of the paired members' choices. As a consequence, your total earning is  $120 + 0 = 120$ .
  4. If you choose 125, everyone's baseline earning is 120 (since 120 is the minimum of choices of all group members including yourself). As indicated in Point Y, your supplementary earning will be  $-7.5$ . This is because your choice is 5 above 120, and 1.5 times this difference is 7.5. As a consequence, your total earning is  $120 - 7.5 = 112.5$ .
  5. If you choose 150, everyone's baseline earning is 120. As indicated in Point Z, your supplementary earning will be  $-15$  because 1.5 times the difference between your choice and the minimum of the paired members' choices is 45, which exceeds 15. As a consequence, your total earning is  $120 - 15 = 105$ .

Your Choice	Point Indicated in Figure 3	Your Paired Members' Minimum Choice	Baseline Earning	Supplementary Point Earning	Total Point Earning
90.0	V	120.0	90.0	+15.0	105.0
115.0	W	120.0	115.0	+7.5	122.5
120.0	X	120.0	120.0	0	120.0
125.0	Y	120.0	120.0	-7.5	112.5
150.0	Z	120.0	120.0	-15.0	105.0

Table 2: Summary of earnings when the paired members chose 120.0 and 147.8

Table 2 summarizes the baseline, supplementary and total point earnings for the 5 possible scenarios (when the paired members choose 120 and 147.8).

### Your Dollar Payoffs

At the end of the experiment, we will sum your point earning in each round to obtain your total point earning over 10 rounds. We will then multiply your total point earning by \$0.01 to obtain your dollar earning. In addition, we will add to this dollar earning \$5 show-up fee to determine your final dollar earning. The amount will be paid to you in cash before you leave the experiment today.