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PHASE TRANSITIONS IN A SATURATING CHIRAL
THEORY OF NUCLEAR MATTER

N.K. Glendenning

April 1984

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Phase Transitions in a Saturating Chiral Theory
of Nuclear Matter

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Abstract

The formalism of a relativistically covariant saturating chiral field theory of nuclear matter is developed in the mean field approximation, and some of the properties are studied. The theory possesses the normal saturated state of nuclear matter and is compatible with the data on neutron star masses. The finite temperature properties are examined. A normal gas-liquid phase equilibrium region exists below $T \approx 23$ MeV. At all temperatures but low densities an abnormal phase exists which at zero temperature is an isolated point at zero baryon density. At finite temperature the abnormal phase, for which the baryon effective mass is vanishingly small, and in which there is an abundance of pairs, extends to finite density. The normal and abnormal phases are disconnected except at a single point where phase equilibrium occurs. Above a critical temperature the normal branch at low density disappears and the abnormal one is the only existing branch. The abnormal characteristic of small effective mass however is confined to low density. Chiral symmetry is restored in this sense as the temperature is increased, but only in matter of low baryon density.

I. Introduction

Chiral symmetry is a good hadron symmetry, ranking only below isotopic spin symmetry in accuracy.¹ For this reason it is highly desirable that any theory of dense matter should possess it. It is equally desirable that such a theory should be capable of describing the bulk properties of nuclear matter, most importantly the binding energy, saturation density, compressibility, and symmetry energy. Until recently a theory satisfying both conditions was not available. The chiral σ -model introduced by Gell-Mann and Levy² which embodies the results of current algebra, the partial conservation of the axial current, and low-energy pion phenomenology, has been widely used to discuss pion condensation in nuclear and neutron matter³ but it does not have a saturated state of nuclear matter.⁴ On the other hand, the Walecka⁵ model and its extensions by Boguta and Bodmer⁶ and by Banerjee, Glendenning, and Gyulassy⁷ possesses the desired bulk nuclear properties and describes many single-particle properties of finite nuclei as well, but does not possess chiral symmetry.

In the context of neutron star matter it was realized by Källman⁸ that a theory based on the coupling of nucleons and massless vector bosons, as suggested by Kirtznits and Linde,⁹ leads to an equation of state for dense matter that goes asymptotically to $p = 1/3\epsilon$. This contrasts with the causality limit $p = \epsilon$ emphasized by Zel'dovich,¹⁰ for massive vector bosons.

Recently Boguta¹¹ has made the very important observation that the chiral σ -model can be made to saturate by the same device of introducing a massless vector gauge field. In his work he simplified the problem by neglecting the pions and studied only the normal state of nuclear matter in its ground state. In view of the success in finally reconciling chiral

symmetry and low-energy pion physics and the bulk properties of nuclear matter in a single relativistically covariant theory, it is very appealing to develop the formalism in greater generality and to investigate some of the predictions of the theory for matter under extreme conditions of temperature and compression.

In this paper we derive the equations of motion for the mean fields in the general case for the chiral σ -model supplemented by a massless vector meson in interaction with the other hadrons, including baryon resonances.

We examine the compatibility of the theory with normal nuclear matter saturation and with neutron star masses. The finite temperature behavior is then studied. There exists an abnormal phase but it is very different in its temperature and density behavior than that of the original chiral σ -model. Chiral symmetry is restored only at low baryon density.

II. Lagrangian, Field Equations, Vacuum Masses

The gauge field ω_μ of a massless vector meson is introduced into the chiral σ -model through the covariant derivatives

$$D_\mu = \partial_\mu + ig_\omega \omega_\mu \quad (1)$$

Thus, using the conventions of Bjorken and Drell,¹² the Lagrangian is,

$$\mathcal{L}_\sigma = \frac{1}{2} [D_\mu^\star \sigma D^\mu \sigma + D_{\mu\pi}^\star \cdot D^\mu \pi] + \bar{\psi}_N i \gamma_\mu D^\mu \psi_N - g \bar{\psi}_N (\sigma + i \gamma_5 \underline{\tau} \cdot \underline{\pi}) \psi_N - V(\sigma, \underline{\pi}) - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} \quad (2)$$

$$\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \quad (3a)$$

$$V(\sigma, \underline{\pi}) = \frac{1}{4} \lambda (\sigma^2 + \underline{\pi}^2 - v^2)^2 - \eta \sigma \quad (3b)$$

The last term in (2) is the Lagrangian for the massless ω meson. The linear term, $\eta\sigma$, in (3) is the symmetry breaking term by which the pion acquires a finite mass. In addition to the ω -meson, which contributes a repulsive energy and carries spin and isospin $(1^+, 0)$, the theory has the scalar meson $\sigma(0^+, 0)$ and the pseudoscalar pion $(0^-, 1)$. The nucleon field is denoted by

$$\psi_N = \begin{pmatrix} P \\ N \end{pmatrix} \quad (4)$$

which is an eight component spinor. The Euler-Lagrange equations are

$$[i \not{\partial} - g_\omega \not{\omega} - g(\sigma + i \gamma_5 \underline{\tau} \cdot \underline{\pi})] \psi_N = 0 \quad (5a)$$

$$-\frac{\partial V}{\partial \sigma} - g \bar{\psi}_N \psi_N + g_\omega^2 \omega_\mu \omega^\mu \sigma = 0 \quad (5b)$$

$$-\frac{\partial V}{\partial \pi} - ig \bar{\psi}_N \gamma_5 \tau \psi_N + g_\omega^2 \omega_\mu \omega^\mu \pi = 0 \quad (5c)$$

$$g_\omega^2 (\sigma^2 + \pi^2) \omega^\mu - g_\omega \bar{\psi}_N \gamma^\mu \psi_N = 0 \omega^\mu - \partial^\mu \partial_\nu \omega^\nu \quad (5d)$$

The fields should be expanded around the stable point defined by the potential.¹³ This leads to the vacuum masses defined in terms of the potential parameters:

$$m_\sigma^2 = \lambda(3\sigma_0^2 - v^2) \quad (6a)$$

$$m_\pi^2 = \eta/\sigma_0 = \lambda(\sigma_0^2 - v^2) \quad (6b)$$

$$m = g\sigma_0 \quad (6c)$$

$$m_\omega = g_\omega \sigma_0 \quad (6d)$$

where σ_0 is the solution of

$$\left(\frac{dV}{d\sigma}\right)_{\pi=0} = 0 = \lambda \sigma_0 (\sigma_0^2 - v^2) - \eta \quad (7)$$

The canonical form of the PCAC theorem is obtained by choosing the symmetry breaking term $\eta\sigma$ of $\mathcal{L}_\sigma$ in the form $\eta = f_\pi m_\pi^2$.¹³ Hence from the above, $\sigma_0 = f_\pi$ where f_π is the pion decay coupling constant. Then the vacuum nucleon mass relates the coupling constant, g , appearing in $\mathcal{L}_\sigma$ to f_π through,

$$m = g f_{\pi} \quad , \quad (8)$$

which is the σ -model version of the Goldberger-Treiman relation.

Inverting the above relations, the potential parameters of the theory can be defined by the experimental masses and coupling constants

$$\eta = f_{\pi} m_{\pi}^2 \quad (9a)$$

$$\sigma_0 = f_{\pi} \quad (9b)$$

$$\lambda = (m_{\sigma}^2 - m_{\pi}^2)/(2f_{\pi}^2) \quad (9c)$$

$$v^2 = \frac{m_{\sigma}^2 - 3m_{\pi}^2}{m_{\sigma}^2 - m_{\pi}^2} f_{\pi}^2 \approx f_{\pi}^2 \quad (9d)$$

In the normal state ($\pi = 0$) of the nucleon vacuum, the energy density is, from (3b) and (9),

$$\epsilon_{\text{vac}} = \frac{1}{4\lambda} \left(\frac{\eta}{f_{\pi}} \right)^2 - \eta f_{\pi} = \frac{1}{2} \frac{3m_{\pi}^2 - 2m_{\sigma}^2}{m_{\sigma}^2 - m_{\pi}^2} \left(\frac{m_{\pi}}{f_{\pi}} \right)^2 \quad (10)$$

which is negative if $m_{\sigma} > \sqrt{3/2} m_{\pi}$. In consequence of this vacuum energy density, the physical energy density and pressure, ϵ' and p' , should be measured against the nucleon vacuum,

$$\epsilon' = \epsilon - \epsilon_{\text{vac}} \quad p' = p + \epsilon_{\text{vac}} \quad (11)$$

(The result for p' follows from $p' = \rho^2 \frac{\partial}{\partial \rho} \left(\frac{\epsilon'}{\rho} \right)$.)

III. Charged Pion Condensate

In this section the theory will be developed, in the mean field approximation, for a charged pion condensate defined by

$$\pi_{\pm} = \frac{1}{\sqrt{2}} \langle \pi_1 \pm i\pi_2 \rangle = \frac{r}{\sqrt{2}} \sin\theta e^{\pm ikx}, \quad (kx = k_0 x_0 - k \cdot x) \quad (12a)$$

$$\pi_0 = 0 \quad (12b)$$

$$\sigma = r \cos\theta \quad (12c)$$

where the mean fields are parametrized by a chiral radius and angle, r, θ . We will derive the Dirac equation for quasi-particle fields in the presence of this condensate, the dispersion relation for the eigenvalues, and the propagator. Then the self-consistency conditions for the meson field strengths, which are non-linear and coupled through various nucleon currents, will be derived. The energy density and pressure are obtained from the canonical form of the stress-energy tensor, and the conditions for the lowest energy pion mode are derived. Then we will show how to calculate the various ground state expectation values of the nucleon currents that appear in the theory.

a) Dirac Equation, Dispersion Relation, Propagator

The Dirac equation (5a) for the nucleon contains the space-time varying pion field. The space-time variation can be eliminated by a transformation to quasi-particles as is usually done. To see what transformation will achieve this end, note that the interaction piece of the Dirac equation can be written

$$\sigma + i\gamma_5 \tau_1 \cdot \pi = r e^{-\frac{i}{2} kx\tau_3} [\cos\theta + i\gamma_5 \tau_1 \sin\theta] e^{\frac{i}{2} kx\tau_3} . \quad (13)$$

The Dirac equation then appears as

$$[(i\not{\partial} - g\omega) e^{-i\beta} - g r e^{-i\beta} e^{i\gamma_5 \tau_1 \theta}] e^{i\beta} \psi_N = 0 \quad (14)$$

where for brevity, we write

$$\beta = \frac{1}{2} kx \tau_3 . \quad (15)$$

Since

$$\not{\partial} e^{-\frac{i}{2} \gamma_5 \tau_1 \theta} = e^{\frac{i}{2} \gamma_5 \tau_1 \theta} \not{\partial} \quad (16a)$$

$$\not{\partial} \tau_3 e^{-\frac{i}{2} \gamma_5 \tau_1 \theta} = e^{-\frac{i}{2} \gamma_5 \tau_1 \theta} \not{\partial} \tau_3 \quad (16b)$$

the Dirac equation can be manipulated into the form

$$(\not{\partial} - g_\omega \omega - M + \frac{1}{2} \not{\partial} \tau_3 \cos\theta + \frac{1}{2} \not{\partial} \gamma_5 \tau_2 \sin\theta) \psi = 0 \quad (17)$$

where ψ is a quasi-fermion field defined by

$$\psi = e^{\frac{i}{2} \gamma_5 \tau_1 \theta} e^{\frac{i}{2} kx\tau_3} \psi_N \quad (18)$$

and the nucleon effective mass in the medium is

$$M = gr \quad . \quad (19)$$

The Dirac equation (17) for the new field, ψ , is now space-time independent. Clearly, the spinor ψ describes a combination of neutron and proton which we refer to as a quasi-particle.

It will be useful to recognize the Dirac Hamiltonian of the quasi-particles,

$$H_D = \gamma_0 [\gamma \cdot p + g_\omega \not{\omega} + M - \frac{1}{2} \not{\epsilon} \tau_3 \cos\theta - \frac{1}{2} \not{\epsilon} \gamma_5 \tau_2 \sin\theta] \quad . \quad (20)$$

The dispersion relation for the eigenvalues can be found by rationalizing the Dirac operator in (17). The result is

$$D(p_0, \underline{p}) \equiv [P_\mu P^\mu - \epsilon_0^2]^2 - (P_\mu k^\mu)^2 + M^2 k_\mu k^\mu \sin^2\theta \quad (21)$$

with

$$\epsilon_0^2 = M^2 - \frac{1}{4} k_\mu k^\mu \quad (22a)$$

$$P_\mu = p_\mu - g_\omega \omega_\mu \quad (22b)$$

The first term of (21) corresponds to the Walecka model, or if the meson fields are zero, to the free Dirac equation. The quasiparticle eigenvalues are the solutions of

$$D(p_0, \underline{p}) = 0 \quad . \quad (23)$$

This fourth order equation for p_0 is easily solved if $k_0 = 0$ (which we shall see corresponds to symmetric nuclear matter). In this case the two particle and two antiparticle eigenvalues, denoted by $\epsilon_{\pm}$ and $\bar{\epsilon}_{\pm}$, are

$$\epsilon_{\pm} = g_{\omega} \omega_0 + E_{\pm}, \quad \bar{\epsilon}_{\pm} = -g_{\omega} \omega_0 + E_{\pm} \quad (24a)$$

$$E_{\pm} \equiv p_0 = \left\{ p^2 + \epsilon_0^2 \pm \sqrt{(p \cdot k)^2 + M^2 k \cdot k \sin^2 \theta} \right\}^{1/2} \quad (24b)$$

, if $(k_0 \equiv 0)$

$$\epsilon_0 = M^2 + \frac{1}{4} k \cdot k \quad (24c)$$

Each momentum state, $\epsilon_{\pm}$, can be occupied by two quasiparticles. In the general case, $(k_0 \neq 0)$, the eigenvalues do not have simple analytic expressions. Nevertheless, such problems can be solved.^{7b-c} The schematic form of the eigenvalue spectrum is shown in Fig. 1.

In the mean field approximation the nucleon ground state is constructed as a degenerate Hartree state out of the solutions of the Dirac equation with all meson fields replaced by their mean values. The nucleon currents that appear in the mean field equations are replaced by the expectation values in the ground state so defined. The resulting coupled equations define the self-consistent values of the mean meson fields.

It is useful to have the propagator expressed so that the operators are in the numerator. It can be used to evaluate ground state expectation values. The result is

$$S \equiv (\not{p} - M + \frac{1}{2} \not{k} \tau_3 \cos \theta + \frac{1}{2} \not{k} \gamma_5 \tau_2 \sin \theta)^{-1} = N/D \quad (25a)$$

where D is given by (21), and

$$N(p_0, p) = \{P_\mu P^\mu - \epsilon_0^2 - P_\mu k^\mu \tau_3 \cos\theta - \sin\theta [P_\mu k^\mu + M \not{k}] \gamma_5 \tau_2\} \\ \times \{\not{P} + M + \frac{1}{2} \not{k} [\tau_3 \cos\theta - \gamma_5 \tau_2 \sin\theta]\} \quad (25b)$$

b) Self-Consistency Conditions for Mean Meson Fields

The nucleon currents appearing in the field equations need to be re-expressed in terms of the quasiparticle fields whose eigenvalue spectrum was discussed above. These are found to be

$$\bar{\psi}_N \gamma_\mu \psi_N = \bar{\psi} \gamma_\mu \psi \quad (26a)$$

$$\bar{\psi}_N \psi_N = \bar{\psi} \psi \cos\theta - i \bar{\psi} \gamma_5 \tau_1 \psi \sin\theta \quad (26b)$$

$$\bar{\psi}_N \gamma_5 \tau_\pm \psi_N = \frac{1}{2} e^{\pm i k x} [\bar{\psi} \gamma_5 \tau_1 \psi \cos\theta - i \bar{\psi} \psi \sin\theta \pm i \bar{\psi} \gamma_5 \tau_2 \psi] \quad (26c)$$

It is convenient to define the four densities

$$\rho_S = \langle \bar{\psi} \psi \rangle \quad (27a)$$

$$\rho = \langle \bar{\psi} \gamma_0 \psi \rangle = \langle \psi^\dagger \psi \rangle \quad (27b)$$

$$\rho_{51} = \langle \bar{\psi} \gamma_5 \tau_1 \psi \rangle \quad (27c)$$

$$\rho_{52} = \langle \bar{\psi} \gamma_5 \tau_2 \psi \rangle \quad (27d)$$

The bracket denotes the expectation value in the ground state that was defined

earlier. The first two are the scalar and the nucleon densities or more precisely in the second case the net baryon charge. When the above results are used in the field equations after the ground state expectation value is taken, the results can be arranged as follows

$$\rho_{52} \equiv 0 \quad (28a)$$

$$[k_\mu k^\mu \sin^2 \theta - \lambda(r^2 - v^2) + g_{\omega_0}^2]r + n \cos \theta - g_s \rho = 0 \quad (28b)$$

$$k_\mu k^\mu r \sin \theta \cos \theta - ig \rho_{51} - n \sin \theta = 0 \quad (28c)$$

$$g_{\omega_0} r^2 = \rho \quad (28d)$$

$$g_{\omega} r^2 = \langle \bar{\psi} \gamma \psi \rangle = 0 \quad (28e)$$

Notice that the dynamic ω mass in the baryonic medium is

$$M_\omega = g_\omega r \quad (29)$$

which can be inferred upon comparing (28d) with the mean field equation for a massive vector boson, $g_{\omega_0} = (g_\omega/M_\omega)^2 \rho$.⁵ The vanishing of the current in (28e) in the stationary ground state will be proven below. The consequence is $\omega \equiv 0$, while (28d) specifies ω_0 in terms of r and of the nucleon density. Notice that the sign of g_{ω_0} is determined by whether the matter has a preponderance of particles ($\rho > 0$) or antiparticles ($\rho < 0$). Consequently, in view of (24) the vector field is repulsive for particles and

attractive for antiparticles in matter, and vice versa in anti-matter. The two equations (28b,c) are the self-consistency conditions for the chiral radius and angle, r, θ of the σ - π field. They are coupled through the nucleon source currents ρ_S and ρ_{51} , which are to be calculated from the solutions of the Dirac equation (17). Before calculating these sources, we will derive additional results of the theory, which also require ground state expectation values of various operators.

c) Conserved Currents

The conserved four-currents can be constructed through Noether's theorem and correspond to the invariance of the lagrangian under various infinitesimal transformations. The charge, baryon, and isospin currents are

$$j^\mu = \langle \bar{\psi}_N \gamma^\mu \frac{1+\tau_3}{2} \psi_N \rangle + \langle \pi \times \frac{\partial \mathcal{L}}{\partial (\partial_\mu \pi)} \rangle_3 \quad (30a)$$

$$j_B^\mu = \langle \bar{\psi}_N \gamma^\mu \psi_N \rangle \quad (30b)$$

$$\tilde{j}^\mu = \langle \bar{\psi}_N \gamma^\mu \frac{1}{2} \tau \psi_N \rangle + \langle \pi \times \frac{\partial \mathcal{L}}{\partial (\partial_\mu \pi)} \rangle \quad (30c)$$

There are three isospin currents denoted by $\tilde{j}^\mu$, corresponding to the three isospin axis. The expected relation,

$$j^\mu = J_3^\mu + \frac{1}{2} j_B^\mu \quad (31)$$

follows. The time-like component reads,

$$q = J_3^0 + \frac{1}{2} \rho \quad (32)$$

giving the isospin-density in terms of the charge and nucleon density. After some algebra we find in particular,

$$j^\mu = \langle \bar{\psi} \gamma^\mu \frac{1+\tau_3}{2} \psi \rangle + k^\mu r^2 \sin^2 \theta \quad (33a)$$

$$j_B^\mu = \langle \bar{\psi} \gamma^\mu \psi \rangle \quad (33b)$$

$$j_3^\mu = \frac{1}{2} \langle \bar{\psi} \gamma^\mu e^{i\gamma_5 \tau_1 \theta} \tau_3 \psi \rangle + k^\mu r^2 \sin^2 \theta \quad (33c)$$

where the currents are now expressed in terms of the quasiparticles. The other two isospin four-currents can be shown to vanish identically (c.f. ref. 7b).

d) Stress-Energy Tensor

The canonical stress-energy tensor is

$$\mathcal{T}_{\mu\nu} = -g_{\mu\nu} \mathcal{L} + \sum \frac{\partial \mathcal{L}}{\partial (\partial^\mu \phi)} \partial_\nu \phi \quad (34)$$

where the sum is over the fields ψ , ω_μ , σ , π . Thus

$$\mathcal{T}_{\mu\nu} = -g_{\mu\nu} \mathcal{L} + \partial_\mu \sigma \partial_\nu \sigma + \partial_\mu \pi \partial_\nu \pi + \bar{\psi}_N i \gamma_\mu \partial_\nu \psi_N \quad (35)$$

Introducing the mean meson fields and the quasiparticle fields this becomes

$$\begin{aligned} \bar{\mathcal{T}}_{\mu\nu} &\equiv \langle \mathcal{T}_{\mu\nu} \rangle \\ &= -g_{\mu\nu} \bar{\mathcal{L}} + k_\mu k_\nu r^2 \sin^2 \theta + \langle \bar{\psi} \gamma_\mu (p_\nu + \frac{1}{2} e^{i\gamma_5 \tau_1 \theta} k_\nu \tau_3) \psi \rangle \\ &= -g_{\mu\nu} \bar{\mathcal{L}} + \langle \bar{\psi} \gamma_\mu p_\nu \psi \rangle + k_\mu J_{3\nu} \end{aligned} \quad (36)$$

and

$$\bar{\mathcal{L}} \equiv \langle \mathcal{L}_\sigma \rangle = \frac{1}{2} (k_\mu k^\mu \sin^2 \theta + g_{\omega\mu}^2 \omega^\mu) r^2 - \frac{1}{4} \lambda (r^2 - v^2)^2 + n r \cos \theta \quad (37)$$

The energy density is therefore

$$\epsilon = \mathcal{F}_{00} = -\bar{\mathcal{L}} + \langle \bar{\psi} \gamma_0 p_0 \psi \rangle + k_0 J_3^0 \quad (38)$$

where the last term is recognized as the time-like component of the isospin four-current, which is related to the charge and nucleon density by (32). It is very useful to recognize this fact because variations of the energy density at fixed charge and nucleon density leave J_3^0 constant. In particular, we wish to find the lowest energy mode in which the pions can condense. This will be given by the conditions

$$\frac{\delta \epsilon}{\delta k_0} = \frac{\delta \epsilon}{\delta \tilde{k}} = 0 \quad (39)$$

where the variation δ means that charge and nucleon density are held fixed.

We have

$$\frac{\delta \epsilon}{\delta \tilde{k}} = \frac{\partial \epsilon}{\partial \tilde{k}} + \frac{\partial \epsilon}{\partial \sigma} \frac{\delta \sigma}{\delta \tilde{k}} + \frac{\partial \epsilon}{\partial \pi} \frac{\delta \pi}{\delta \tilde{k}} + \dots \quad (40)$$

The partial derivatives $\partial \epsilon / \partial \sigma$, etc., all vanish by virtue of the field equations. Hence

$$\frac{\delta \epsilon}{\delta \tilde{k}} = -\frac{\partial \bar{\mathcal{L}}}{\partial \tilde{k}} + \frac{\partial}{\partial \tilde{k}} \langle \bar{\psi} \gamma_0 p_0 \psi \rangle = -\frac{\partial \bar{\mathcal{L}}}{\partial \tilde{k}} + \frac{\partial}{\partial \tilde{k}} \langle \psi^\dagger H_D \psi \rangle \quad (41)$$

where in the last step the Dirac equation was used to replace p_0 by H_D .
The derivative of its expectation value is,

$$\frac{\partial}{\partial k} \langle \psi^\dagger H_D \psi \rangle = \langle \psi^\dagger \frac{\partial H_D}{\partial k} \psi \rangle + \epsilon_D \frac{\partial}{\partial k} \langle \psi^\dagger \psi \rangle \quad (42)$$

where

$$\epsilon_D = \langle \psi^\dagger H_D \psi \rangle \quad (43)$$

is the sum of the occupied quasiparticle eigenvalues. The last term in (42) vanishes at constant nucleon density. We find therefore, taking account of (20, 37) that

$$-\frac{\delta \epsilon}{\delta k} = J_3 \quad (44)$$

where the right side is the 3-current of isospin (33c). Therefore the condition that the pions condense in the lowest mode places a condition on their wave vector k , which is to be chosen so that the isospin 3-current vanishes. Likewise the first condition of (39) places a constraint on k_0 . Together they read,

$$J_3^0 = \frac{1}{2} \langle \bar{\psi} \gamma^0 e^{i\gamma_5 \tau_1 \theta} \tau_3 \psi \rangle + k^0 r^2 \sin^2 \theta = q - \frac{1}{2} \rho \quad (45a)$$

$$J_3 = \frac{1}{2} \langle \bar{\psi} \gamma e^{i\gamma_5 \tau_1 \theta} \tau_3 \psi \rangle + k r^2 \sin^2 \theta = 0 \quad (45b)$$

The vanishing of the three-current corresponding to the conserved isospin density, which was derived above, can also be inferred by Baym's theorem,¹⁴

which states that the mean value of three-currents corresponding to conserved charges vanish in the ground state.

In view of (45), the stress-energy tensor simplifies,

$$\mathcal{T}_{ij} = \bar{\mathcal{L}} + \langle \bar{\psi} \gamma_i p_j \psi \rangle \quad (\text{unsummed}) \quad (46)$$

the three diagonal space components are equal as can be seen from the methods used in ref. 7b. In particular, let p, κ characterize a single quasi-particle state [where in the case $k_0 = 0, \kappa = \pm$ characterizes the two eigenvalues (24)]. For such a single particle state

$$(\psi^\dagger H_D \psi)_{p, \kappa} = p_0(p, \kappa) = E_\kappa(p) \pm g_\omega \omega_0 \quad \begin{cases} \text{particle} \\ \text{anti-particle} \end{cases} \quad (47)$$

Recall from (28d) that ω_0 is fixed by the nucleon density. Hence, recalling (20) we obtain, by taking a derivative with respect to p_i

$$(\bar{\psi} \gamma_i \psi)_{p, \kappa} = \frac{\partial}{\partial p_i} E_\kappa(p) \quad (48)$$

With this, the ground state expectation value $\langle \bar{\psi} \gamma_i p_i \psi \rangle$ can be evaluated

$$\langle \bar{\psi} \gamma_i p_i \psi \rangle = \sum_\kappa \int \frac{d^3 p}{(2\pi)^3} \theta[E_F - E_\kappa(p)] p_i \frac{\partial}{\partial p_i} E_\kappa(p) \quad (49)$$

where E_F denotes the Fermi energy and θ is the usual step function that vanishes for negative argument and is otherwise unity. The integral can be evaluated easily as in Ref. 7b to yield

$$\langle \bar{\psi} \gamma_i p_i \psi \rangle = E_F \rho - \sum_{\kappa} \int \frac{d^3 p}{(2\pi)^3} \theta_{\kappa} E_{\kappa}(p) \quad , \quad (i \text{ unsummed}) \quad . \quad (50)$$

Since this is independent of the space index i , all three such expectation values are equal, and we can write

$$p = \bar{\mathcal{L}} + \frac{1}{3} \langle \bar{\psi} \gamma \cdot p \psi \rangle \quad . \quad (51)$$

Notice in connection with (49) that the eigenstates E_{κ} are doubly degenerate if both neutrons and protons are present and singly degenerate otherwise. The sum on κ is understood to be summed over this degeneracy. (Otherwise insert a factor 2 or 1 as the case may be, and sum only over distinct energies.)

Notice that (48) can be used to infer that

$$\langle \bar{\psi} \gamma \psi \rangle = 0 \quad (52)$$

as used in (28e).

e) Ground State Expectation Values

Here explicit form will be given to the various ground state expectation values that appear in the equations above. They can in general be evaluated by a trace theorem (c.f. ref. 7a)

$$\langle \bar{\psi} \Gamma \psi \rangle = \sum_{\pm} \int \frac{d^3 p}{(2\pi)^3} \theta_{\pm} \left[\frac{P_0 - E_{\pm}(p)}{D(p_0, p)} \text{Tr}(\Gamma N(p_0, p)) \right]_{P_0 = E_{\pm}(p)} \quad . \quad (53)$$

The trace is over spinor and isospin indices so we have altered the notation from κ of (48) to $\pm$.

The ground state expectation values for which we need explicit forms are given below, with γ_S denoting the isospin degeneracy

$$\rho = \langle \psi^\dagger \psi \rangle = \gamma_S \sum_{\pm} \int \frac{d^3 p}{(2\pi)^3} \theta_{\pm} \quad (54)$$

$$\rho_S = \langle \bar{\psi} \psi \rangle = M \gamma_S \sum_{\pm} \int \frac{d^3 p}{(2\pi)^3} \theta_{\pm} \left[\frac{2(p_\mu p^\mu - \epsilon_0^2) - k_\mu k^\mu \sin^2 \theta}{2(p_\mu p^\mu - \epsilon_0^2) p_0 - p_\mu k^\mu k_0} \right]_{p_0=E_{\pm}} \quad (55)$$

$$\begin{aligned} \rho_{51} &= \langle \bar{\psi} \gamma_5 \tau_1 \psi \rangle \\ &= 2 i M k_\mu k^\mu \sin \theta \cos \theta \gamma_S \sum_{\pm} \int \frac{d^3 p}{(2\pi)^3} \theta_{\pm} \left[\frac{p_0 - E_{\pm}}{D} \right]_{p_0=E_{\pm}} \end{aligned} \quad (56)$$

$$\begin{aligned} \langle \bar{\psi} \gamma_\mu e^{i\gamma_5 \tau_1 \theta} \tau_3 \psi \rangle &= \\ 2\gamma_S \sum_{\pm} \int \frac{d^3 p}{(2\pi)^3} \theta_{\pm} &\frac{(p_\nu p^\nu - \epsilon_0^2) k_\mu + 2M^2 k_\mu \sin^2 \theta - 2(p_\nu k^\nu) p_\mu}{4(p_\nu p^\nu - \epsilon_0^2) p_0 - 2(p_\nu k^\nu) k_0} \end{aligned} \quad (57)$$

$$\langle \bar{\psi} \not{k} \gamma_5 \tau_2 e^{-i\gamma_5 \tau_1 \theta} \psi \rangle = -2i M \rho_{51} \quad (58)$$

$$\langle \bar{\psi} \gamma_5 \gamma_\mu \tau_1 \psi \rangle = \langle \bar{\psi} \gamma_\mu \gamma_5 \tau_3 \psi \rangle = \langle \bar{\psi} \gamma_5 \tau_2 \psi \rangle = 0 \quad (59)$$

f) Axial Vector Current

The Lagrangian, $\mathcal{L}_\sigma$, exclusive of the linear term, $n\sigma$, is invariant to the axial transformation

$$\pi^i \rightarrow \pi^i - \alpha^i \sigma, \quad \sigma \rightarrow \sigma + \alpha \cdot \vec{\pi},$$

$$\psi_N \rightarrow \psi_N + \frac{i}{2} \vec{\tau} \cdot \vec{\alpha} \gamma_5 \psi_N \quad (60)$$

The axial current can be calculated from Noether's theorem as

$$A_j^\mu = \frac{1}{2} \bar{\psi}_N \gamma^\mu \gamma_5 \tau_j \psi_N + \sigma \partial^\mu \pi_j - \pi_j \partial^\mu \sigma \quad . \quad (61)$$

We wish to express this in terms of the quasi-particle fields ψ . The result, after some algebra, is

$$\begin{aligned} A_\pm^\mu &\equiv \frac{1}{\sqrt{2}} (A_1 \pm i A_2) \\ &= \pm \frac{i}{2\sqrt{2}} e^{\pm i k x} \langle \bar{\psi} \gamma^\mu \gamma_5 e^{i \gamma_5 \tau_1 \theta} \tau_2 \psi \rangle \pm i k^\mu \sigma \pi_\pm \end{aligned} \quad (62)$$

$$A_3^\mu = \frac{1}{2} \langle \bar{\psi} \gamma^\mu \gamma_5 e^{i \gamma_5 \tau_1 \theta} \tau_3 \psi \rangle = 0 \quad . \quad (63)$$

From this we can evaluate

$$\begin{aligned} \partial_\mu A_\pm^\mu &= -i M \rho_{51} \frac{1}{\sqrt{2}} e^{\pm i k x} + k_\mu k^\mu \sigma \pi_\pm \\ &= + f_\pi m_\pi^2 \pi_\pm \quad . \end{aligned} \quad (64)$$

where in the last step the field equation was used. This confirms the expected form of the PCAC.

g) Specialization to Symmetric Nuclear Matter

When $k_0 = 0$, we find from (45a, 57) that

$$0 = J_3^0 = q - \frac{1}{2} \rho \quad (65)$$

which is the condition for symmetric nuclear matter. In this case the Fermion eigenvalues can be obtained explicitly as

$$\epsilon_{\pm} = g_{\omega} \omega_0 + E_{\pm} \quad , \quad \bar{\epsilon}_{\pm} = -g_{\omega} \omega_0 + E_{\pm} \quad (66a)$$

$$E_{\pm} = \{ \tilde{p}^2 + \epsilon_0^2 \pm [(\tilde{p} \cdot \tilde{k})^2 + M^2 \tilde{k}^2 \sin^2 \theta]^{1/2} \} \quad (66b)$$

$$\epsilon_0^2 = M^2 + \frac{1}{4} \tilde{k}^2 \quad . \quad (66c)$$

The field equations take the form

$$\rho = r^2 g_{\omega} \omega_0 \quad , \quad \tilde{\omega} = 0 \quad (67a)$$

$$g_{\rho_S} = [\rho^2 / r^4 - \tilde{k}^2 \sin^2 \theta - \lambda(r^2 - v^2)]r + \eta \cos \theta \quad (67b)$$

$$-ig_{\rho_{51}} = \tilde{k}^2 r \cos \theta \sin \theta + \eta \sin \theta \quad . \quad (67c)$$

The condition for the equilibrium value of $k = |\tilde{k}|$ takes the form

$$-\gamma_S \sum_{\pm} \int \frac{d^3 p}{(2\pi)^3} \frac{\theta_{\pm}}{2E_{\pm}} \{ \frac{1}{2} \tilde{k}^2 \pm [(\tilde{p} \cdot \tilde{k})^2 + M^2 \tilde{k}^2 \sin^2 \theta]^{1/2} \} = \tilde{k}^2 r^2 \sin^2 \theta \quad (67d)$$

The last three equations are to be solved, at given baryon density ρ , for r , θ , and $|\tilde{k}|$. The current densities ρ_S and ρ_{51} take the forms

$$\rho_S = \gamma_S \sum_{\pm} \int \frac{d^3 p}{(2\pi)^3} \theta_{\pm} \frac{M}{E_{\pm}} \left\{ 1 \pm \frac{\tilde{k}^2 \sin^2 \theta}{2[(\tilde{p} \cdot \tilde{k})^2 + M^2 \tilde{k}^2 \sin^2 \theta]^{1/2}} \right\} \quad (68)$$

$$\rho_{51} = \frac{1}{2} iM \tilde{k}^2 \sin\theta \cos\theta \gamma_S \int \frac{d^3p}{(2\pi)^3} \frac{1}{[(\tilde{p} \cdot \tilde{k})^2 + M^2 \tilde{k}^2 \sin^2\theta]^{1/2}} \left(\frac{\theta_-}{E_-} - \frac{\theta_+}{E_+} \right) \quad (69)$$

(Recall that $M = \text{gr.}$) The Fermi energy, E_F , appearing in the step function

$$\theta_{\pm} \equiv \theta[E_F - E_{\pm}(\tilde{p})] \quad , \quad (70)$$

is defined in terms of the nucleon density by

$$\rho = \gamma_S \sum_{\pm} \int \frac{d^3p}{(2\pi)^3} \theta_{\pm} \quad . \quad (71)$$

Equations (67) determine ω_0 , r , θ and $|\tilde{k}|$. the last three quantities, the chiral radius and angle, and the wave number of the pion condensate occur in a highly non-linear way in (67).

Once the solutions have been found, the energy density and pressure can be calculated from

$$\begin{aligned} \epsilon = & \frac{1}{2} \tilde{k}^2 r^2 \sin^2\theta + \frac{1}{2} \left(\frac{\rho}{r} \right)^2 + \frac{1}{4} \lambda (r^2 - v^2)^2 - \eta r \cos\theta \\ & + \gamma_S \sum_{\pm} \int \frac{d^3p}{(2\pi)^3} \theta_{\pm} E_{\pm} \end{aligned} \quad (72)$$

$$\begin{aligned} p = & -\frac{1}{2} \tilde{k}^2 r^2 \sin^2\theta + \frac{1}{2} \left(\frac{\rho}{r} \right)^2 - \frac{1}{4} \lambda (r^2 - v^2)^2 + \eta r \cos\theta \\ & + \frac{1}{3} \gamma_S \sum_{\pm} \int \frac{d^3p}{(2\pi)^3} \frac{\theta_{\pm}}{2E_{\pm}} \left\{ 2\tilde{p}^2 \pm \frac{(\tilde{p} \cdot \tilde{k})^2}{\sqrt{(\tilde{p} \cdot \tilde{k})^2 + M^2 \tilde{k}^2 \sin^2\theta}} \right\} . \end{aligned} \quad (73)$$

It is interesting to consider the high density limit for the normal state

($k = \theta = 0$). In this limit we find that $r \rightarrow k_F$ and

$$p \rightarrow \frac{1}{3} \epsilon, \quad \text{as} \quad \rho \rightarrow \infty, \quad (74)$$

where k_F is the Fermi momentum (c.f. 70). This limit, which was noticed by Kallman⁸ for a massless vector field is to be contrasted with the limit, $p \rightarrow \epsilon$, corresponding to the exchange of a massive vector boson as noted by Zel'dovich.¹⁰

IV. Coupling of Baryon Resonances

The theory can be easily extended to incorporate N^* resonances as Dirac fields. Such an extension becomes relevant in the high temperature or density regime because the effective baryon masses may become small. The effective mass, according to (19), is $M = g r$ and the vacuum mass according to (8) is $m = g f_\pi$. Accordingly the baryons should be coupled so that their vacuum masses are

$$m_B = g_B f_\pi, \quad (75)$$

where B denotes the spin-isospin multiplet of mass m_B and isospin degeneracy γ_B .

The interaction term in the Lagrangian is replaced as

$$g \bar{\psi} (\sigma + i \gamma_5 \underline{\tau} \cdot \underline{\pi}) \psi \rightarrow \sum_B g_B \bar{B} (\sigma + i \gamma_5 \underline{\tau} \cdot \underline{\pi}) B \quad (76)$$

where, in context, B denotes the spinor for baryon of mass m_B . The structure of all the densities computed previously remains unchanged except that they are now summed over the various nucleon masses and their spin degeneracies, with effective mass, $M = g r$, replaced by

$$M_B = g_B r. \quad (77)$$

V. Finite Temperature

The theory is easily extended to finite temperature in the usual way but the details are somewhat complicated by the fact that the Fermi momentum boundaries are not in general spherical (c.f. 24). The thermodynamic potential and logarithm of the partition function, Ω and $\ln Z$ for a volume V are

$$-\Omega = T \ln Z$$

$$= V\bar{\mathcal{L}} + VT \gamma_S \sum_{\pm} \int \frac{d^3 p}{(2\pi)^3} \left\{ \ln \left(1 + e^{-(\epsilon_{\pm} - \mu)/T} \right) + \ln \left(1 + e^{-(\bar{\epsilon}_{\pm} + \mu)/T} \right) \right\} \quad (78)$$

where $\bar{\mathcal{L}}$ is given by (37) and $\epsilon_{\pm}$, $\bar{\epsilon}_{\pm}$ denote the baryon and antibaryon eigenvalues with isospin degeneracy γ_S .

The finite temperature generalizations of the three field equations (28b-d) and the two conditions (39) are contained in the equilibrium expressions

$$\frac{\delta \Omega}{\delta r} = \frac{\delta \Omega}{\delta \sin \theta} = \frac{\delta \Omega}{\delta \omega_0} = \frac{\delta \Omega}{\delta k_0} = \frac{\delta \Omega}{\delta |\tilde{k}|} = 0 \quad (79)$$

where the variation, δ , is at constant volume, temperature and chemical potential.

The particle and antiparticle Fermi-Dirac distribution functions for momentum $\tilde{p}$, corresponding to the two Fermi seas, $\pm$, are,

$$n_{\tilde{p}}^{\pm} = T \frac{\delta \ln Z}{\delta \mu} = \left[1 + e^{(\epsilon_{\pm} - \mu)/T} \right]^{-1} \quad (80a)$$

$$\bar{n}_{\tilde{p}}^{\pm} = \left[1 + e^{(\bar{\epsilon}_{\pm} + \mu)/T} \right]^{-1} \quad (80b)$$

so that the baryon density is

$$\rho = \rho_B - \rho_{\bar{B}} = \gamma_S \sum_{\pm} \int \frac{d^3 p}{(2\pi)^3} (n_{\underline{p}}^{\pm} - \bar{n}_{\underline{p}}^{\pm}) \quad (81)$$

The entropy is defined by

$$S = \frac{\partial}{\partial T} (T \ln Z)_{V, \mu} \quad (82)$$

which leads to the expected relationship

$$sT = p + \epsilon - \mu\rho \quad (83)$$

where $s = S/V$ is the entropy density and ϵ is the energy density,

$$\epsilon = -\bar{\mathcal{L}} + g_{\omega\omega_0}\rho + \gamma_S \sum_{\pm} \int \frac{d^3 p}{(2\pi)^3} E_{\pm} (n_{\underline{p}}^{\pm} + \bar{n}_{\underline{p}}^{\pm}) \quad (84)$$

This is the obvious temperature generalization of the earlier result. The pressure as usual is

$$pV = T \ln Z \quad (85)$$

and the last of (79) ensures, as at $T = 0$, that the pressure is isotropic. To summarize, the field equations and the expression for p and ϵ hold, as before, with the replacement in the integrals,

$$\theta_{\pm} \rightarrow n_{\underline{p}}^{\pm} + \bar{n}_{\underline{p}}^{\pm} \quad (86)$$

except for the baryon density, (81) in which the difference appears. (That the ω -field is coupled to the difference (81) can be traced to (47).)

VI. Properties of Symmetric Matter

Here we compare the properties of normal matter in this theory with the empirical properties. Recall equations (6 and 9), which relate the parameters of the theory to physical quantities, the nucleon and meson masses, and the π -decay constant f_π . The vacuum masses of nucleon and pion are given their normal values of course. The σ -mass is taken as a parameter. The vector meson coupling, g_ω , appears in the combination $g_\omega \omega_0$ in infinite matter and can be eliminated through the field equation (67a). The coupling constant g is related through the vacuum nucleon mass to the pion decay constant f_π through

$$m = g f_\pi \quad (87)$$

The normal Goldhaber-Treeman relation, however, reads $g_A m = g f_\pi$ with $g_A = 1.25$, and $f_\pi = 93$ MeV.

In principle the theory has only one parameter, m_σ . This turns out to be insufficient to reproduce the three properties, saturation density, binding, and compressibility. This is not surprising, of course. So we take f_π as an additional parameter. With $m_\sigma = 642.5$ MeV and $f_\pi = 116$ MeV the saturation density and binding energy per nucleon in normal symmetric matter have the acceptable values

$$\rho_0 = 0.153 \text{ fm}^{-3}, \quad B/A = 15.95 \text{ MeV} \quad (88)$$

The compressibility, however, is large, $K = 650$ MeV, as compared to 240 MeV. The solution found here is close to that of Boguta,¹¹ as should be the case for the normal state in which the pion field vanishes. The only difference lies in the relationship (6) since he drops the pion completely from consideration in his calculations.

VII. Neutron Star Masses

As an additional test of the theory we calculate the mass of a neutron star approximating the matter as pure neutron matter described by this theory. One condition for stability of the star is that

$$dM/d\rho_c \geq 0 \tag{89}$$

where ρ_c is the central density. The equality yields the maximum mass that can be sustained against gravitational collapse. A second condition is that the adiabatic index of the equation of state $\gamma = d \ln p / d \ln \epsilon$ should be greater than or equal to 4/3. Values of γ that are less than 4/3 can lead to oscillations that may destroy the star. In Fig.2 the mass as a function of central density is shown. The central density corresponding to the Oppenheimer-Volkoff limit is $\rho_{0.V.} = 4.6\rho_0$ for which the star mass is $2.4 M_\odot$. Stars with central densities in the range $\rho_0 < \rho_c \leq \rho_{0.V.}$ are gravitationally stable. Almost all of these stars are also stable against oscillations in the core. Stars with central densities $\rho_c < 4.3 \rho_0$ possess this stability. The maximum star mass of $M_\odot$, which is stable in all respects, exceeds the observed masses of $1.4 M_\odot$ for the crab and Her-1. Thus the theory is acceptable as far as data on neutron star masses is concerned.

VIII. Low-Temperature Behavior: Gas-Liquid Transition

For the normal state ($\theta = 0$) of symmetric nuclear matter ($k_0 = 0$), we show in Fig.3 the pressure, as function of the net baryon density ρ in the low temperature domain. The saturation density of symmetric nuclear matter corresponds to the density at which $p = 0$ on the $T = 0$ isotherm. The pressure exhibits the typical Van de Waals behavior for which the Maxwell construction for phase equilibrium can be made as in Fig.4, by plotting the pressure as a function of chemical potential at constant temperature. The critical temperature above which the matter is homogeneous with respect to gas and liquid in the normal state of matter is $T_c \approx 23$ MeV. As we shall see in the next section, at considerably higher temperature new phase transitions are possible.

IX. Chiral Restoration in Low-Density High-Temperature Matter

The original chiral σ -model of Gell-Mann and Levy,² insofar as its nuclear matter properties are concerned, is remarkable for the absence of a normal saturated state, as was emphasized first by Kerman and Miller.⁴ It does possess both a pion condensed state³ and as discussed by Lee and Wick¹⁶ an abnormal state in which the nucleon mass becomes very small, above a critical baryon density or temperature.¹⁷

In contrast the present version of the chiral model, as emphasized by Boguta,¹¹ has a normal saturated state. The properties connected with abnormal state, as discussed below are also quite different.

In the following calculations, all well established $T = 1/2$ baryon resonances, strange and non-strange with masses up to 1.8 GeV are included, the strange ones only in baryon, anti-baryon pairs. Very similar results hold

if the resonances are neglected, but the critical temperatures discussed below are increased. For the present exploration, we confine ourselves to the $\theta = 0$ case (no condensed pions).

For vanishing baryon density, the field equation (67b) has a solution, which corresponds to very small effective nucleon mass, $M = \text{gr}$. This solution would be precisely $M = 0$ if chiral symmetry were exact ($n = f_{\pi} m_{\pi}^2 = 0$). But this is an isolated solution confined to $\rho = 0$ at $T = 0$. For finite temperature, the situation is rather more interesting. The reason, as has been stated elsewhere,¹⁸ is that the scalar field is coupled to the sum of baryon and antibaryon densities, as a consequence of which an instability can set in at finite temperature since the departure of the scalar field from its vacuum value reduces the baryon masses, thus making pair creation more energetically favorable, which further drives the scalar field reducing baryon masses even further. Therefore the new phase appears at an extremely low temperature compared to the vacuum mass of a pair because of the near vanishing of the effective mass. The vector field does not inhibit the instability because it is coupled to the net baryon density (c.f. 67a, 81 or alternately 66a). The present model possesses such an abnormal phase, corresponding to small baryon effective mass and abundant pairs, but it is confined to the low-density regime, again in contrast to the usual σ -model. This is due to the fact that here, the vector field is introduced as a gauge boson that is coupled to the scalar field. It derives its effective mass from this. The first term on the right side of (67b) originates from this coupling as the derivation shows, and this term prevents small values of r except for small ρ .

The above assertions are illustrated in Fig.5. Over some range of temperature and density there are three solutions to the field equations. Two of them correspond to minima in the free energy per nucleon regarded as a function, at fixed temperature and density, of the chiral radius r , or equivalently the effective mass $M = gr$, and one corresponds to the maximum lying at intermediate effective mass. The isolated solution lying at $M = \rho = 0$ at zero temperature develops into a pair of solutions at finite temperature, extending to finite baryon density. On one of the stable branches, the effective mass is near the vacuum value, and we call it the normal branch. On the other, the effective mass is very small, and we refer to it as abnormal. On the normal branch, the effective mass decreases as a function of density to a minimum and thereafter increases monotonically. It is the particular gauge coupling of the vector field as remarked above that gives rise to this behavior. As the temperature is raised the abnormal branch extends to higher and higher density until it intercepts the normal branch. Beyond the critical temperature, T_K , at which this occurs, the normal branch no longer extends continuously from low to high density. It becomes a remnant of its former self. In contrast, the abnormal branch extends continuously from low to high density. However as the density increases, its nature evolves, and it assumes the character of a normal state at high density. Beyond another critical temperature, T_C , it is the only solution and the remnant of the normal state ceases to exist.

Fig.6 shows the evolution of the solutions as a function of temperature at zero baryon density. To understand this figure, we note that in the absence of baryons to drive the meson fields to finite values, there ought to be a solution with effective nucleon mass equal to its vacuum mass, and this is

indeed so for temperatures up to ~ 160 MeV. This is the normal branch. At remarkably low temperature, the instability with respect to pair production sets in, signaling the appearance of the abnormal phase together with an unstable solution. This one bifurcates with the normal one at $T_c \approx 163$ MeV, which corresponds to the end point of the normal branch at zero baryon density.

The abundance of pairs¹⁸ which besides the low-effective mass is the other remarkable feature of the abnormal phase, is shown in Fig.7.

The normal phase is one of broken chiral symmetry. What we have found above is that at high temperature, the symmetry is recovered, but only at low density, indeed, strictly speaking, at vanishing density. This result is quite different from the original chiral σ -model, and as remarked earlier, is due to the particular gauge coupling of the vector meson to the scalar field.

X. Summary

We have developed the self-consistency equations that govern the normal and pion-condensed states in a chiral theory with an vector meson introduced as a gauge field. As Boguta first observed, such a theory saturates in the normal state unlike the original σ -model. We have shown in addition that the theory is compatible with data on neutron star masses. This finding is non-trivial inasmuch as the ideal neutron gas equation of state which has the same asymptotic behavior, $\rho = 1/3 \epsilon$, as the present theory has too small a limiting mass.

The theory was extended to incorporate N^* resonances as Dirac fields, as is appropriate for studying the high temperature or high density regimes.

We then examined the finite temperature properties. A normal phase transition between liquid and gas occurs below $T \sim 23$ MeV. There exists as well at all temperatures an isolated abnormal phase characterized by small baryon effective mass, and at non-zero temperature, abundant baryon-antibaryon pairs of low effective mass. Phase equilibrium between this and the normal phase occurs at only a single point, T_K . Above a critical temperature T_C there is only one phase, which at low density corresponds to the chirally restored phase. As a function of density, and above T_C , this phase evolves continuously into a normal phase as the density is increased. In this theory, chiral symmetry is restored at high temperature but only in matter of low baryon density.

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References

1. H. Pagels, Phys. Rev. 16 219 (1975)
2. M. Gell-Mann and M. Levy, Nuovo Cim. 16 53 (1960)
3. C.-K. Au and G. Baym, Nucl. Phys. A236 50 (1974)
D. Campbell, R. Dashen and J. Manassah, Phys. Rev. D12 110 (1975)
M. Chanowitz and P. J. Siemens, Phys. Lett. 70B 175 (1977)
H. J. Pirner, K. Yazaki, P. Bonche, and M. Rho, Nucl. Phys. A329 491 (1979)
4. A. Kerman and L. D. Miller, in Second High Energy Heavy Ion Summer Study, 1974 Lawrence Berkeley Laboratory report 3675 (unpublished)
5. J. D. Walecka, Ann. of Phys. 83 491 (1974)
6. J. Boguta and A. R. Bodmer, Nucl. Phys. A292 413 (1974)
7. a) B. Banerjee, N. K. Glendenning, and M. Gyulassy, Nucl. Phys. A361 326 (1981)
b) N. K. Glendenning, B. Banerjee, and M. Gyulassy, Ann. of Phys. 149 1 (1983)
c) N. K. Glendenning, P. Hecking and V. Ruck, Ann. Phys. 149 22 (1983)
8. C.-G. Kallman, Fundamentals of Cosmic Physics 4 167 (1979) (published by Gordon and Breach Science Pub.)
9. D. A. Kirzhnits and A. D. Linde, Ann. Phys. 101 195 (1976)
10. Y. B. Zel'dovich, Zh. Eksp. Teoret. 41 1609 (1961)
11. J. Boguta, Phys. Lett. 120B 34 (1983)
12. J. D. Bjorken and S. D. Drell, "Relativistic Quantum Fields," McGraw-Hill, New York, 1965

13. G. Baym and D. K. Campbell, in Mesons in Nuclei Vol. III, ed. by M. Rho and D. Wilkinson (North Holland Publishing Company, 1979)
14. G. Baym, Phys. Rev. Lett. 30 1340 (1973)
15. J. R. Oppenheimer and H. Snyder, Phys. Rev. 56 455 (1939)
16. T. D. Lee and G. C. Wick, Phys. Rev. D9 2291 (1974)
17. M. Wakamatsu and A. Hayashi, Prog. Theoret. Phys. (Tokyo) 63 1688 (1980)
18. N. K. Glendenning, LBL-17653 (1984) to be published in Phys. Lett.; S. I. A. Garpman, N. K. Glendenning and Y. Karant, Nucl. Phys. A322 382 (1979)

XBL 845-10567

Fig.1 Schematic showing eigenvalue spectrum of Fermions in the special case $k = \theta = 0$ and in the general case of $k \neq 0, \theta \neq 0$.

XBL837-1890

Fig.2 Neutron star mass in units of solar mass $M_{\odot}$ as a function of central density ρ_c .

XBL 845-10562

Fig.3 Pressure as a function of density in the low-temperature regime of the liquid-gas phase coexistence. The $T = 0$ isotherm passes through $p = 0$ at the equilibrium density $\rho_0 = 0.153 \text{ fm}^{-3}$ of symmetric nuclear matter in this theory. The binding energy at saturation is 16 MeV/nucleon.

XBL 845-10563

Fig.4 Pressure as a function of chemical potential at fixed temperature. The intersection corresponds to the Gibbs phase equilibrium between gas and liquid.

XBL 845-10577

Fig.5 For four temperatures, the ratio of effective baryon mass to vacuum mass $M/m = r/f_\pi$ shown as a function of total baryon density. (All baryons scale in the same way, c.f. equations (75,76)). Three solutions exist in the low density regime below a critical density $T_c \approx 163$ MeV, two stable and one unstable. The normal branch with $M \approx m$ is marked by n, the abnormal with $M \approx 0$ is marked by a, and the unstable branch by u. Note at a critical temperature $T_K \approx 135$ MeV the abnormal branch intercepts the normal one, and the latter becomes a remnant that disappears above T_c .

XBL 845-10572

Fig.6 At zero baryon density $\mu = \rho = 0$, the three solutions are shown as a function of temperature. Above $T_c \approx 163$ MeV, only the abnormal solution exists.

XBL 845-10575

Fig.7 Total number of baryons and antibaryons as a function of net baryon density. The abnormal branch is characterized by an abundance of baryon-antibaryon pairs. The normal branch has almost no antibaryons.

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