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Authors

Costinot, Arnaud Komunjer, Ivana

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UNIVERSITY OF CALIFORNIA, SAN DIEGO

DEPARTMENT OF ECONOMICS

What Goods Do Countries Trade? New Ricardian Predictions

By

Arnaud Costinot Ivana Komunjer Department of Economics University of California, San Diego

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WHAT GOODS DO COUNTRIES TRADE? NEW RICARDIAN PREDICTIONS

ARNAUD COSTINOT AND IVANA KOMUNJER

ABSTRACT. Though one of the pillars of the theory of international trade, the extreme predictions of the Ricardian model have made it unsuitable for empirical purposes. A seminal contribution of Eaton and Kortum (2002) is to demonstrate that random productivity shocks are sufficient to make the Ricardian model empirically relevant. While successful at explaining trade volumes, their model remains silent with regards to one important question: What goods do countries trade? Our main contribution is to generalize their approach and provide an empirically meaningful answer to this question.

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Affiliations and Contact information. Costinot: Department of Economics, University of California at San Diego, costinot@ucsd.edu; Komunjer: Department of Economics, University of California at San Diego, komunjer@ucsd.edu.

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1. INTRODUCTION

Though one of the pillars of the theory of international trade, the extreme predictions of the Ricardian model have made it unsuitable for empirical purposes. As Learner and Levinsohn (1995) point out: "The Ricardian link between trade patterns and relative labor costs is much too sharp to be found in any real data set."

A seminal contribution of Eaton and Kortum (2002) is to demonstrate that random productivity shocks are sufficient to make the Ricardian model empirically relevant. When drawn from an extreme value distribution, these shocks imply a gravity-like equation in a Ricardian framework with a continuum of goods, transport costs, and more than two countries. While successful at explaining trade volumes, their model remains silent with regards to one important question: What goods do countries trade? Our main contribution is to generalize their approach and provide an empirically meaningful answer to this question.

Section 2 describes the model. We consider an economy with one factor of production, labor, and multiple products, each available in many varieties. There are constant returns to scale in the production of each variety. The key assumption of our model is that labor productivity may be separated into: a deterministic component, which is country and industry specific; and a stochastic component, randomly drawn across countries, industries, and varieties. The former captures factors such as climate, infrastructure, and institutions that affect the productivity of all producers in a given country and industry,¹ whereas the latter reflects idiosyncratic differences in technological know-how across varieties.

Section 3 derives our predictions on the pattern of trade. Because of random productivity shocks, we can no longer predict trade flows in each variety. Yet, by assuming that each product comes in a large number of varieties, we generate sharp predictions at the industry level. In particular, we show that for any pair of exporters, the (first-order stochastic dominance) ranking of their relative labor productivity fully determines their relative export performance across industries. Compared to the standard Ricardian model—see e.g. Dornbusch, Fischer, and Samuelson (1977)—our predictions hold under fairly general assumptions on transport

¹Acemoglu, Antras, and Helpman (2006), Costinot (2005), Cuñat and Melitz (2006), Levchenko (2004), Matsuyama (2005), Nunn (2005), and Vogel (2004) explicitly model the impact of various institutional features—e.g. labor market flexibility, the quality of contract enforcement, or credit market imperfections on labor productivity across countries and industries.

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costs, the number of industries, and the number of countries.² Moreover, they do not imply the full specialization of countries in a given set of industries.

Section 4 investigates how well our model squares with the empirical evidence. We consider linear regressions tightly connected to our theoretical framework. Using OECD trade and labor productivity data from 1988 to 2003, we find strong support for our new Ricardian predictions: countries do tend to export relatively more (towards any importing country) in sectors where they are relatively more productive.

Our paper contributes to the previous trade literature in two ways. First, it contributes to the theory of comparative advantage. Our model generates clear predictions on the pattern of trade in environments—with both multiple countries and industries—where the standard Ricardian model loses most of its intuitive content; see e.g. Jones (1961) and Wilson (1980). Our approach mirrors Deardorff (1980) who shows how the law of comparative advantage may remain valid, under standard assumptions, when stated in terms of correlations between vectors of trade and autarky prices. In this paper, we weaken the standard Ricardian assumptions—the "chain of comparative advantage" only holds in terms of first-order stochastic dominance—and derive a deterministic relationship between exports and labor productivity across industries.

Second, our paper contributes to the empirical literature on international specialization, including the previous "tests" of the Ricardian model; see e.g. MacDougall (1951), Stern (1962), Balassa (1963), and more recently Golub and Hsieh (2000). While empirically successful, these tests have long been criticized for their lack of theoretical foundations; see Bhagwati (1964). Our model provides such foundations. Since it does not predict full international specialization, we do not have to focus on ad-hoc measures of export performance. Instead, we may use the theory to pin down explicitly what the dependent variable in crossindustry regressions ought to be.

As we discuss in our concluding remarks, our model may also provide an alternative theoretical underpinning of cross-industry regressions when labor is not the only factor of production. The validity of these regressions usually depends on strong assumptions on either demand—see e.g. Petri (1980) and the voluminous gravity literature based on Armington's

 $^{^{2}}$ Deardorff (2005) reviews the failures of simple models of comparative advantage at predicting the pattern of trade in economies with more than two goods and two countries.

preferences—or the structure of transport costs—see e.g. Harrigan (1997) and Romalis (2004). Our paper suggests that many of these assumptions may be relaxed, as long as there are stochastic productivity differences within each industry.

2. The Model

We consider a world economy comprising i = 1, ..., I countries and one factor of production labor. There are j = 1, ..., J products and constant returns to scale in the production of each product. Labor is perfectly mobile across industries and immobile across countries. The wage of workers in country i is denoted w_i . Up to this point, this is a standard Ricardian model. We generalize this model by introducing random productivity shocks. Following Eaton and Kortum (2002), we assume that each product j may come in N_j varieties $\omega = 1, ..., N_j$, and denote $a_{ij}(\omega)$ the constant unit labor requirements for the production of the ω th variety of product j in country i. Our first assumption is that:

A1. For all countries i, products j, and their varieties ω

(1)
$$\ln a_{ij}(\omega) = \ln a_{ij} + u_{ij}(\omega),$$

where $a_{ij} > 0$ and $u_{ij}(\omega)$ is a random variable drawn independently for each triplet (i, j, ω) from a continuous distribution $F(\cdot)$ such that: $E[u_{ij}(\omega)] = 0$.

We interpret a_{ij} as a measure of the fundamental productivity of country *i* in sector *j* and $u_{ij}(\omega)$ as a random productivity shock. The former, which can be estimated using aggregate data, captures cross-country and cross-industry heterogeneity. It reflects factors such as climate, infrastructure, and institutions that affect the productivity of all producers in a given country and industry. Random productivity shocks, on the other hand, capture intra-industry heterogeneity. They reflect idiosyncratic differences in technological knowhow across varieties, which are assumed to be drawn independently from a unique distribution $F(\cdot)$. In our setup, cross-country and cross-industry variations in the distribution of productivity levels derive from variations in a single parameter: a_{ij} .

We assume that trade barriers take the form of "iceberg" transport costs:

A2. For every unit of commodity *j* shipped from country *i* to country *n*, only $1/d_{ij}^n$ units arrive, where:

(2)
$$\begin{cases} d_{ij}^n = d_i^n \cdot d_j^n \ge 1, & \text{if } i \neq n, \\ d_{ij}^n = 1, & \text{otherwise.} \end{cases}$$

The indices i and n refer to the exporting and importing countries, respectively. The first parameter d_i^n measures the trade barriers which are specific to countries i and n. It includes factors such as: physical distance, existence of colonial ties, use of a common language, or participation in a monetary union. The second parameter d_j^n measures the policy barriers imposed by country n on product j, such as import tariffs and standards. In line with "the most-favored-nation" clause of the World Trade Organization, these impediments may not vary by country of origin.

We assume that markets are perfectly competitive.³ Together with constant returns to scale in production, perfect competition implies:

A3. In any country n, the price $p_j^n(\omega)$ paid by buyers of variety ω of product j is

(3)
$$p_j^n(\omega) = \min_{1 \le i \le I} \left[c_{ij}^n(\omega) \right],$$

where $c_{ij}^n(\omega) = d_{ij}^n \cdot w_i \cdot a_{ij}(\omega)$ is the cost of producing and delivering one unit of this variety from country *i* to country *n*.

For each variety ω of product j, buyers in country n are "shopping around the world" for the best price available. Here, random productivity shocks lead to random costs of production $c_{ij}^n(\omega)$ and in turn, to random prices $p_j^n(\omega)$. In what follows, we let $c_{ij}^n = d_{ij}^n$. $w_i \cdot a_{ij} > 0$.

On the demand side, we assume that consumers have a two-level utility function with CES preferences across varieties. This implies:

A4(i). In any country n, the total spending on variety ω of product j is

(4)
$$x_j^n(\omega) = \left[p_j^n(\omega)/p_j^n\right]^{1-\sigma} k_j^n,$$

where $k_j^n > 0$, $\sigma > 1$ and $p_j^n = [\sum_{\omega'=1}^{N_j} p_j^n(\omega')^{1-\sigma}]^{1/(1-\sigma)}$.

³The case of Bertrand competition is discussed in details in Appendix B.

The above expenditure function is a standard feature of the "new trade" literature; see e.g. Helpman and Krugman (1985). k_j^n is an endogenous variable that represents total spending on product j in country n. It depends on the upper tier utility function in this country and the equilibrium prices. p_j^n is the CES price index, and σ is the elasticity of substitution between varieties. It is worth emphasizing that while the elasticity of substitution σ is assumed to be constant, total spending, and hence demand conditions, may vary across countries and industries: k_j^n is a function of n and j.

Finally, we assume that:

A4(ii). In any country n, the elasticity of substitution σ between two varieties of product j is such that $E\left[p_{i}^{n}(\omega)^{1-\sigma}\right] < \infty$.

Assumption A4(ii) is a technical assumption that guarantees the existence of a well defined price index. Whether or not A4(ii) is satisfied ultimately depends on the shape of the distribution $F(\cdot)$.⁴

In the rest of the paper, we let $x_{ij}^n = \sum_{\omega=1}^{N_j} x_{ij}^n(\omega)$ denote the value of exports from country *i* to country *n* in sector *j*, where total spending on each variety $x_{ij}^n(\omega)$ is given by:

(5)
$$\begin{cases} x_{ij}^n(\omega) = x_j^n(\omega), & \text{if } c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega), \\ x_{ij}^n(\omega) = 0, & \text{otherwise.} \end{cases}$$

⁴Suppose, for example, that $u_{ij}(\omega)$'s are drawn from a (negative) exponential distribution with mean zero: $F(u) = \exp[\theta u - 1]$ for $-\infty < u \leq 1/\theta$ and $\theta > 0$. This corresponds to the case where labor productivity $z_{ij}(\omega) \equiv 1/a_{ij}(\omega)$ is drawn from a Pareto distribution: $G_{ij}(z) = 1 - (b_{ij}/z)^{\theta}$ for $0 < b_{ij} \leq z$ and $b_{ij} \equiv a_{ij}^{-1} \exp(-\theta^{-1})$, as assumed in various applications and extensions of Melitz's (2003) model; see e.g. Helpman, Melitz, and Yeaple (2004), Antras and Helpman (2004), Ghironi and Melitz (2005) and Bernard, Redding, and Schott (2006). Then, our assumption A4(ii) holds if the elasticity of substitution $\sigma < 1 + \theta$. Alternatively, suppose that $u_{ij}(\omega)$'s are distributed as a (negative) Gumbel random variable with mean zero: $F(u) = 1 - \exp[-\exp(\theta u - \mathbf{e})]$ for $u \in \mathbb{R}$ and $\theta > 0$, where \mathbf{e} is Euler's constant $\mathbf{e} \simeq 0.577$. This corresponds to the case where labor productivity $z_{ij}(\omega)$ is drawn from a Fréchet distribution: $G_{ij}(z) = \exp(-b_{ij}z^{-\theta})$ for $z \geq 0$ and $b_{ij} \equiv a_{ij}^{-\theta} \exp(-\mathbf{e})$, as assumed, for example, in Eaton and Kortum (2002) and Bernard, Eaton, Jensen, and Kortum (2003). Then, like in the Pareto case, A4(ii) holds if $\sigma < 1 + \theta$.

Similarly, we denote $\pi_{ij}^n(\omega)$ the probability that country *i* exports a variety ω of product *j* to country *n*:

(6)
$$\pi_{ij}^n(\omega) = \Pr\left\{c_{ij}^n(\omega) = \min_{1 \le i' \le I} [c_{i'j}^n(\omega)]\right\}.$$

By Assumption A1, the probabilities $\pi_{ij}^n(\omega)$ remain the same across all varieties ω of product j, so we can let $\pi_{ij}^n(\omega) = \pi_{ij}^n$ in Equation (6).

3. The Pattern of Trade

We now describe the restrictions that Assumptions A1–A4 impose on the pattern of trade; and how they relate to those of the standard Ricardian model.

3.1. **Predictions.** In order to make predictions on the pattern of trade, we follow a twostep approach. First, we relate total exports x_{ij}^n to the expected value of exports coming from country *i*, using the law of large numbers. Second, we derive a log-linear relationship between this expected value and the fundamental productivity level a_{ij} , using a first-order Taylor series development around a symmetric situation where costs are identical across exporters, $(c_{1j}^n = ... = c_{Ij}^n)$. While our approximation admittedly lacks the elegance of Eaton and Kortum's (2002) closed form solution, it presents one important advantage: it remains valid irrespectively of the distribution of random productivity shocks $F(\cdot)$.

Our main result can be stated as follows.

Theorem 1. Suppose that Assumptions A1-A4 hold. In addition, assume that the number of varieties N_j of any product j is large, and that technological differences across exporters are small: $c_{1j}^n \simeq ... \simeq c_{Ij}^n$. Then, for any exporter i, any importer $n \neq i$, and any product j,

(7)
$$\ln x_{ij}^n \simeq \alpha_i^n + \beta_j^n + \gamma \ln a_{ij}.$$

where $\gamma < 0$.

The proof can be found in Appendix A. The first term α_i^n is importer and exporter specific; it reflects wages w_i in the exporting country and trade barriers d_i^n between countries *i* and *n*. The second term β_j^n is importer and industry specific; it reflects the policy barriers d_j^n imposed by country *n* on product *j* and demand differences k_j^n across countries and industries. The main insight of Theorem 1 comes from the third term $\gamma \ln a_{ij}$, in which the parameter

 γ is constant across countries and industries. Since $\gamma < 0$, Theorem 1 predicts that $\ln x_{ij}^n$ should be decreasing in $\ln a_{ij}$: ceteris paribus, countries should export less in sectors where their firms are, on average, less efficient.

It is worth emphasizing that Theorem 1 *cannot* be used for comparative static analysis. If the fundamental productivity level goes up in a given country and industry, this will affect wages and, in turn, exports in other countries and industries through general equilibrium effects. In other words, changes in a_{ij} also lead to changes in the country and industry fixed effects, α_i^n and β_j^n . By contrast, Theorem 1 *can* be used to analyze the cross-sectional variations of bilateral exports, as we shall further explore in Section 4.

Though the assumptions of Theorem 1 may seem unreasonably strong—in particular, technological differences across *all* exporters are unlikely to be small—its predictions hold more generally. Suppose that, for each product and each importing country, exporters can be separated into two groups: small exporters, whose costs are very large (formally, close to infinity), and large exporters, whose costs of production are small and of similar magnitude. Then, small exporters export with probability close to zero and the results of Theorem 1 still apply to the group of large exporters.⁵

If we impose more structure on the distribution of random productivity shocks, we can further weaken the assumptions of Theorem 1. Suppose that the distribution $F(\cdot)$ of $u_{ij}(\omega)$ is Gumbel as in Eaton and Kortum (2002). Then, it can be shown that the property in Equation (7) holds *exactly* for any $(c_{1j}^n, ..., c_{Ij}^n)$. In other words, if Eaton and Kortum's (2002) distributional assumption is satisfied, then our local results become global; they extend to environments where technological differences across all countries are large; see Appendix C for details.⁶

⁵In other words, our theory does not require Gambia and Japan to have similar costs of producing and delivering cars in the United States. It simply requires that Japan and Germany do.

⁶If $F(\cdot)$ is Gumbel, one can further show that α_i^n , β_j^n , and γ do not depend on the elasticity of substitution σ . In this case, the predictions of Theorem 1 still hold if we relax Assumption A4(i), so that the elasticity of substitution may vary across countries and industries: $\sigma \equiv \sigma_j^n$. This derives from a key property of the Gumbel distribution: conditional on exporting a given variety to country n, the expected value of exports has to be identical across countries. Hence, transport costs, wages and fundamental productivity levels only affect the extensive margin—how many varieties are being exported—not the intensive margin—how much of each variety is being exported. Unfortunately, this property does not easily generalize to other distributions; see Appendix C.

In order to prepare the comparison between our results and those of the standard Ricardian model, we conclude this section by offering a Corollary to Theorem 1. Consider an arbitrary pair of exporters i_1 and i_2 , an importer $n \neq i_1, i_2$ and an arbitrary pair of goods j_1 and j_2 . Taking the differences-in-differences in Equation (7) we get that $(\ln x_{i_1j_1}^n - \ln x_{i_1j_2}^n) - (\ln x_{i_2j_1}^n - \ln x_{i_2j_2}^n) \simeq \gamma [(\ln a_{i_1j_1} - \ln a_{i_1j_2}) - (\ln a_{i_2j_1} - \ln a_{i_2j_2})]$, for N_{j_1} and N_{j_2} large enough. Since $\gamma < 0$, we then obtain that

(8)
$$\frac{a_{i_1j_1}}{a_{i_2j_1}} > \frac{a_{i_1j_2}}{a_{i_2j_2}} \Rightarrow \frac{x_{i_1j_1}^n}{x_{i_2j_1}^n} < \frac{x_{i_1j_2}^n}{x_{i_2j_2}^n}$$

Still considering the pair of exporters i_1 and i_2 and generalizing the above reasoning to all J products, we derive the following Corollary:

Corollary 2. Suppose that the assumptions of Theorem 2 hold. Then, the ranking of relative unit labor requirements determines the ranking of relative exports:

$$\left\{\frac{a_{i_11}}{a_{i_21}} > \dots > \frac{a_{i_1j}}{a_{i_2j}} > \dots > \frac{a_{i_1J}}{a_{i_2J}}\right\} \Rightarrow \left\{\frac{x_{i_11}^n}{x_{i_21}^n} < \dots < \frac{x_{i_1j}^n}{x_{i_2j}^n} < \dots < \frac{x_{i_1J}^n}{x_{i_2J}^n}\right\}.$$

3.2. Relation to the standard Ricardian model. Note that we can always index the J products so that:

(9)
$$\frac{a_{i_11}}{a_{i_21}} > \dots > \frac{a_{i_1j}}{a_{i_2j}} > \dots > \frac{a_{i_1J}}{a_{i_2J}}$$

Ranking (9) is at the heart of the standard Ricardian model; see e.g. Dornbusch, Fischer, and Samuelson (1977). When there are no random productivity shocks, Ranking (9) merely states that country i_1 has a comparative advantage in (all varieties of) the high j products. If there only are two countries, the pattern of trade follows: i_1 produces and exports the high j products, while i_2 produces and exports the low j products. If there are more than two countries, however, the pattern of pairwise comparative advantage no longer determines the pattern of trade. In this case, the standard Ricardian model loses most of its intuitive content; see e.g. Jones (1961) and Wilson (1980).

When there are stochastic productivity differences at the industry level, Assumption A1 and Ranking (9) further imply:

(10)
$$\frac{a_{i_11}(\omega)}{a_{i_21}(\omega)} \succ \dots \succ \frac{a_{i_1j}(\omega)}{a_{i_2j}(\omega)} \succ \dots \succ \frac{a_{i_1J}(\omega)}{a_{i_2J}(\omega)}$$

where \succ denotes the first-order stochastic dominance order among distributions.⁷ In other words, Ranking (10) is just a stochastic—hence weaker—version of the ordering of fundamental costs a_{ij} , which is at the heart of the Ricardian theory. Like its deterministic counterpart in (9), Ranking (10) captures the idea that country i_1 is relatively better at producing the high j products. But whatever j is, country i_2 may still have lower costs of production on some of its varieties.

According to Corollary 2, Ranking (10) does not imply that country i_1 should only produce and export the high j products, but instead that it should produce and export relatively more of these products. This is true irrespective of the number of countries in the economy. Unlike the standard Ricardian model, our stochastic theory of comparative advantage generates a clear and intuitive correspondence between labor productivity and exports. In our model, the pattern of comparative advantage for any pair of exporters fully determines their relative export performance across industries.

This may seem paradoxical. As we have just mentioned, Ranking (10) is a weaker version of the ordering at the heart of the standard theory. If so, how does our stochastic theory lead to finer predictions? The answer is simple: it does not. While the standard Ricardian model is concerned with trade flows in each variety of each product, we only are concerned with the total trade flows in each product. Unlike the standard model, we recognize that random shocks—whose origins remain outside the scope of our model—may affect the costs of production of any variety. Yet, by assuming that these shocks are identically distributed across a large number of varieties, we manage to generate sharp predictions at the industry level.

4. Empirical Evidence

We now investigate whether the predictions of Theorem 1 are consistent with the data.

⁷To see this, note that for any $A \in \mathbb{R}^+$ we have $\Pr\{a_{i_1j}(\omega)/a_{i_2j}(\omega) \leq A\} = \Pr\{u_{i_1j}(\omega) - u_{i_2j}(\omega) \leq \ln A - \ln a_{i_1j} + \ln a_{i_2j}\}$. Since for any j < j', $u_{i_1j}(\omega) - u_{i_2j}(\omega)$ and $u_{i_1j'}(\omega) - u_{i_2j'}(\omega)$ are drawn from the same distribution by A1, Ranking (9) implies:

$$\Pr\left\{\frac{a_{i_1j}(\omega)}{a_{i_2j}(\omega)} \le A\right\} < \Pr\left\{\frac{a_{i_1j'}(\omega)}{a_{i_2j'}(\omega)} \le A\right\} \Leftrightarrow \frac{a_{i_1j}(\omega)}{a_{i_2j}(\omega)} \succ \frac{a_{i_1j'}(\omega)}{a_{i_2j'}(\omega)}$$

4.1. **Data description.** We use yearly data from the OECD Structural Analysis (STAN) Databases from 1988 to 2003. Our sample includes 25 exporters, all OECD countries, and 49 importers, both OECD and non-OECD countries. It covers 21 manufacturing sectors aggregated (roughly) at the 2-digit ISIC rev 3 level. To the best of our knowledge, it corresponds to the largest data set available with *both* bilateral trade data *and* comparable labor productivity data. See Table 1 for details.

The value of exports x_{ij}^n by exporting country *i*, importing country *n*, and industry *j* is directly available (in thousands of US dollars, at current prices) in the STAN Bilateral Trade Database. The unit labor requirement a_{ij} in country *i* and industry *j* is measured as total employment divided by value added (in millions or billions of national currency, at current prices), which can both be found in the STAN Industry Database.⁸

4.2. Specification. The main testable implication derived in Theorem 1 is that:

(11)
$$\left(\partial \ln x_{ij}^n\right) / \left(\partial \ln a_{ij}\right) = \gamma < 0$$

In other words, the elasticity of exports with respect to the average unit labor requirement should be negative (and constant across importers, exporters, and industries). Accordingly, we shall consider a linear regression model of the form

(12)
$$\ln x_{ij}^n = \alpha_i^n + \beta_j^n + \gamma \ln a_{ij} + \varepsilon_{ij}^n$$

where α_i^n and β_j^n are treated as importer–exporter and importer–industry fixed effects, respectively, and ε_{ij}^n is an error term.

There are (at least) two possible interpretations of the error term ε_{ij}^n . First, we can think of ε_{ij}^n as a measurement error in trade flows. This is the standard approach in the gravity literature; see e.g. Anderson and Wincoop (2003). Alternatively, we can think of ε_{ij}^n as representing the impact of unobserved trade barriers, not accounted for in Assumption A2. Indeed, we can generalize A2 as $\ln d_{ij}^n = \ln d_i^n + \ln d_j^n + \tilde{\varepsilon}_{ij}^n$. Then, setting $\varepsilon_{ij}^n = \gamma \tilde{\varepsilon}_{ij}^n$ —and using the expressions of α_i^n and β_j^n provided in the proof of Theorem 1—immediately leads

⁸Any difference in units of account across countries shall be treated as an exporter fixed effect in our regression (12). Hence, we do not need to convert our measures of a_{ij} into a common currency. Similarly, we do not correct for the number of hours worked per person and per year, which only is available for a very small fraction of our sample. But it should be clear that cross-country differences in hours worked will also be captured by our exporter fixed effect.

Table 1: Data Set Description

Source: OECD Structural Analysis (STAN) Databases

Years 1988-2003

- *Exporters:* Twenty-five OECD countries (Australia, Austria, Belgium, Canada, Czech Republic, Danemark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Korea, Luxembourg, Netherlands, Norway, Poland, Portugal, Slovak Republic, Spain, Sweden, United Kingdom, United States)
- *Importers:* Exporters + Five OECD Countries (Iceland, Mexico, New Zealand, Switzerland, Turkey) + Others (Argentina, Brazil, Chile, China, Cyprus, Estonia, Hong Kong, India, Indonisia, Latvia, Lithuania, Malaysia, Malta, Phillipines, Russian Federation, Singapore, Slovenia, South Africa, Thailand)

Product Classification System: The industrial breakdown presented for the STAN indicators database is based upon the International Standard Industrial Classification (ISIC) Revision 3.

Industry:	ISIC Rev. 3
Food products, beverages and tobacco	15-16
l extiles, textile products, leather and footwear	17-19
Wood and products of wood and cork	20
Pulp, paper, paper products, printing and publishing	21-22
Coke, refined petroleum products and nuclear fuel	23
Pharmaceuticals	243
Rubber and plastics products	25
Other non-metallic mineral products	26
Iron and steel	271
Non-ferrous metals	272+2732
Fabricated metal products, except machinery and equipment	28
Machinery and equipment, n.e.c.	29
Office, accounting and computing machinery	30
Electrical machinery and apparatus, n.e.c.	31
Radio, television and communication equipment	32
Medical, precision and optical instruments, watches and clocks	33
Motor vehicles, trailers and semi-trailers	34
Building and repairing of ships and boats	351
Aircraft and spacecraft	353
Railroad equipment and transport equipment n.e.c.	352+359
Manufacturing n.e.c.	36-37

to Equation (12). This is the approach followed by Eaton and Kortum (2002) and Helpman, Melitz, and Rubinstein (2005). Under either interpretation, we shall assume that ε_{ij}^n is independent across countries *i* and *n* as well as across industries *j*; that ε_{ij}^n is heteroskedastic conditional on *i*, *n* and *j*; and that ε_{ij}^n is uncorrelated with $\ln a_{ij}$.

Note that our orthogonality condition rules out situations where country n tends to discriminate relatively more against a given country i in sectors where i is relatively more productive. Were these situations prevalent in practice, due to endogenous trade protection,

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					/			
Variable	2003	2002	2001	2000	1999	1998	1997	1996
ln a	-0.78	-0.67	-0.91	-0.80	-0.71	-0.89	-0.95	-0.73
	(-10.31)***	(-12.74)***	(-16.47)***	(-15.91)***	(-12.87)***	(-13.72)***	(-14.16)***	(-10.69)***
Exporter-Importer FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry-Importer FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	8778	15051	18167	18597	18805	18187	18233	17780
R^2	0.805	0.794	0.793	0.787	0.792	0.792	0.789	0.785
Variable	1995	1994	1993	1992	1991	1990	1989	1988
Variable In a	1995 -0.79	1994 -0.75	1993 -0.53	1992 -0.50	1991 -0.42	1990 -0.37	1989 -0.39	1988 -0.08
Variable In a	1995 -0.79 (-12.22)***	1994 -0.75 (-9.79)***	1993 -0.53 (-7.74)***	1992 -0.50 (-7.03)***	1991 -0.42 (-5.60)***	1990 -0.37 (-5.05)***	1989 -0.39 (-5.77)***	1988 -0.08 (-1.06)
Variable In a Exporter-Importer FE	1995 -0.79 (-12.22)*** Yes	1994 -0.75 (-9.79)*** Yes	1993 -0.53 (-7.74)*** Yes	1992 -0.50 (-7.03)*** Yes	1991 -0.42 (-5.60)*** Yes	1990 -0.37 (-5.05)*** Yes	1989 -0.39 (-5.77)*** Yes	1988 -0.08 (-1.06) Yes
Variable In a Exporter-Importer FE Industry-Importer FE	1995 -0.79 (-12.22)*** Yes Yes	1994 -0.75 (-9.79)*** Yes Yes	1993 -0.53 (-7.74)*** Yes Yes	1992 -0.50 (-7.03)*** Yes Yes	1991 -0.42 (-5.60)*** Yes Yes	1990 -0.37 (-5.05)*** Yes Yes	1989 -0.39 (-5.77)*** Yes Yes	1988 -0.08 (-1.06) Yes Yes
Variable In a Exporter-Importer FE Industry-Importer FE Observations	1995 -0.79 (-12.22)*** Yes Yes 17423	1994 -0.75 (-9.79)*** Yes Yes 14245	1993 -0.53 (-7.74)*** Yes Yes 13489	1992 -0.50 (-7.03)*** Yes Yes 12755	1991 -0.42 (-5.60)*** Yes Yes 11827	1990 -0.37 (-5.05)*** Yes Yes 11655	1989 -0.39 (-5.77)*** Yes Yes 11606	1988 -0.08 (-1.06) Yes Yes 10672

Table 2: Year-by-Year OLS Regressions

Note: Absolute value of t-statistics in parentheses, calculated from heteroskedasticity-consistent (White) standard errors

* Significant at 10% confidence level

** Significant at 5% confidence level

*** Significant at 1% confidence level

our OLS estimates of γ would be biased (upward) towards zero.⁹ Similarly, our orthogonality condition rules out any potential errors in the measurement of labor productivity at the industry level, which obviously is a very strong assumption. The presence of measurement errors in the data should further bias our OLS estimates of γ (upward) towards zero.

4.3. Estimation Results. Table 2 reports the OLS estimates of the regression parameter γ obtained independently for each year 1988-2003. In line with our theory—and in spite of the potential upward biases discussed above—we find that the regression parameter γ is

⁹Formally, suppose that trade barriers, d_{ij}^n , and exports, x_{ij}^n , are simultaneously determined according to

$$\begin{cases} \ln d_{ij}^n = \ln d_i^n + \ln d_j^n + \mu \ln x_{ij}^n \\ \ln x_{ij}^n = \hat{\alpha}_i^n + \hat{\beta}_j^n + \gamma \ln a_{ij} + \gamma \ln d_{ij}^n \end{cases}$$

where $\mu > 0$ captures the fact that higher levels of import penetration lead to higher levels of protection. The previous system can be rearranged as

$$\begin{cases} \ln d_{ij}^n = (1 - \mu\gamma)^{-1} [\ln d_i^n + \ln d_j^n + \mu \hat{\alpha}_i^n + \mu \hat{\beta}_j^n + \mu\gamma \ln a_{ij}] \\ \ln x_{ij}^n = \alpha_i^n + \beta_j^n + \gamma \ln a_{ij} + \varepsilon_{ij}^n \end{cases}$$

where $\alpha_i^n = (1 - \mu\gamma)^{-1} [\hat{\alpha}_i^n + \gamma \ln d_i^n], \ \beta_j^n = (1 - \mu\gamma)^{-1} [\hat{\beta}_j^n + \gamma \ln d_j^n], \ \text{and} \ \varepsilon_{ij}^n = \mu\gamma^2 (1 - \mu\gamma)^{-1} \ln a_{ij}.$ This directly implies $E[\ln a_{ij}\varepsilon_{ij}^n] = \mu\gamma^2 (1 - \mu\gamma)^{-1}E[(\ln a_{ij})^2] > 0$, and in turn, the upward bias in the OLS estimate of γ .

negative for every year in the sample. Further, it is significant at the 1% level for 15 out of 16 years, the only exception being 1988 (which also is the only year for which we do not have US data).¹⁰ Overall, we view these results as strongly supportive of our new Ricardian predictions.

Is the impact of labor productivity on the pattern of international specialization economically significant as well? As mentioned in Section 3, we cannot use our estimate of γ to predict the changes in *levels* of exports associated with a given change in labor productivity. However, we can follow a difference-in-difference approach to predict *relative* changes in exports across countries and industries. Consider, for example, two exporters, i_1 and i_2 , and two industries, j_1 and j_2 , in 2003. If $a_{i_1j_1}$ decreases by 10%, then our prediction is that:

$$\left(\Delta \ln x_{i_1 j_1}^n - \Delta \ln x_{i_1 j_2}^n\right) - \left(\Delta \ln x_{i_2 j_1}^n - \Delta \ln x_{i_2 j_2}^n\right) = -\widehat{\gamma}_{2003} \Delta \ln a_{i_1 j_1} \simeq 7.8\%$$

This is consistent with a scenario where country i_1 's exports of good j_1 (towards any importer) go up by 5% and those of j_2 go down by 2.8%, while they remain unchanged in both sectors in country i_2 .

5. Concluding Remarks

The Ricardian model has long been perceived has a useful pedagogical tool with, ultimately, little empirical content. Building on the seminal work of Eaton and Kortum (2002), we introduce random productivity shocks in a standard Ricardian model with multiple countries and industries. The predictions that we derive are both intuitive and empirically meaningful: countries should export relatively more (towards any importing country) in sectors where they are relatively more productive. Using OECD trade and labor productivity data from 1988 to 2003, we find strong support for our new Ricardian predictions.

We believe that the tight connection between the theory and the empirical analysis that our paper offers is a significant step beyond the existing literature. First, we do not have to rely on ad-hoc measures of export performance. The theory tells us exactly what the dependent variable in the cross-industry regressions ought to be: ln(exports), disaggregated by exporting and importing countries. This allows us to move away from the country-pair

¹⁰This may suggest that the good performance of our theory is entirely driven by US data. In Appendix D, we show that this is not the case; running our regressions without the United States leads to similar results.

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comparisons inspired by the two-country model, and in turn, to take advantage of a much richer data set. Second, our clear theoretical foundations make it possible to discuss the economic origins of the error terms—measurement errors in trade flows or unobserved trade barriers—and as a result, the plausibility of our orthogonality conditions.

Another attractive feature of our theoretical approach is that it relies on fairly general assumptions on preferences, transport costs, and the number of industries and countries. Hence, we believe that it may be fruitfully applied to more general environments, where labor is not the only factor of production. The basic idea, already suggested by Bhagwati (1964), is to reinterpret differences in a_{ij} as differences in total factor productivity. With multiple factors of production, the probability of being an exporter, and in turn the volume of exports, would be a function of both technological differences, captured by a_{ij} , and differences in relative factor prices. The rest of our analysis would remain unchanged.

References

- ACEMOGLU, D., P. ANTRAS, AND E. HELPMAN (2006): "Contracts and Technology Adoption," *mimeo Harvard University*.
- ANDERSON, J. A., AND E. V. WINCOOP (2003): "Gravity with Gravitas: A Solution to the Border Puzzle," *The American Economic Review*, pp. 170–192.
- ANTRAS, P., AND E. HELPMAN (2004): "Global Sourcing," *Journal of Political Economy*, pp. 552–580.
- BALASSA, B. (1963): "An Empirical Demonstration of Comparative Cost Theory," *Review* of *Economics and Statistics*, pp. 231–238.
- BERNARD, A. B., J. EATON, J. B. JENSEN, AND S. KORTUM (2003): "Plants and Productivity in International Trade," *The American Economic Review*, pp. 1268–1290.
- BERNARD, A. B., S. J. REDDING, AND P. K. SCHOTT (2006): "Comparative Advantage and Heterogeneous Firms," (forthcoming) The Review of Economic Studies.
- BHAGWATI, J. (1964): "The Pure Theory of International Trade: A Survey," The Economic Journal, pp. 1–84.
- BILLINGSLEY, P. (1995): Probability and Measure. John Wiley & Sons, Inc.
- COSTINOT, A. (2005): "Contract Enforcement, Division of Labor, and the Pattern of Trade," *mimeo University of California, San Diego*.

- CUÑAT, A., AND M. MELITZ (2006): "Labor Market Flexibility and Comparative Advantage," *mimeo Harvard University*.
- DAVIDSON, J. (1994): Stochastic Limit Theory. Oxford University Press, Oxford.
- DEARDORFF, A. V. (1980): "The General Validity of the Law of Comparative Advantage," Journal of Political Economy, pp. 941–957.
- (2005): "How Robust is Comparative Advantage," mimeo University of Michigan.
- DORNBUSCH, R., S. FISCHER, AND P. SAMUELSON (1977): "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review*, 67, 823–839.
- EATON, J., AND S. KORTUM (2002): "Technology, Geography, and Trade," *Econometrica*, 70, 1741–1779.
- GHIRONI, F., AND M. MELITZ (2005): "International Trade and Macroeconomic Dynamics with Heterogeneous Firms," *Quarterly Journal of Economics*, pp. 865–915.
- GOLUB, S. S., AND C.-T. HSIEH (2000): "Classical Ricardian Theory of Comparative Advantage Revisited," *Review of International Economics*, pp. 221–234.
- HARRIGAN, J. (1997): "Technology, Factor Supplies, and International Specialization: Estimating the Neo-Classical Model," *The American Economic Review*, pp. 475–494.
- HELPMAN, E., AND P. R. KRUGMAN (1985): Market Structure and Foreign Trade. MIT Press.
- HELPMAN, E., M. MELITZ, AND Y. RUBINSTEIN (2005): "Trading Partners and Trading Volumes," *mimeo Harvard University*.
- HELPMAN, E., M. MELITZ, AND S. YEAPLE (2004): "Export versus FDI with Heterogeneous Firms," *The American Economic Review*, pp. 300–316.
- JONES, R. W. (1961): "Comparative Advantage and the Theory of Tariffs: A Multi-Country, Multi-Commodity Model," *The Review of Economic Studies*, 28(3), 161–175.
- LEAMER, E., AND J. LEVINSOHN (1995): "International Trade Theory: The Evidence," in *Handbook of International Economics*, ed. by G. Grossman, and K. Rogoff, vol. 3, pp. 1139–1194. Elsevier Science, New York.
- LEVCHENKO, A. (2004): "Institutional Quality and International Trade," *IMF working paper*.

- MACDOUGALL, G. (1951): "British and American Exports: A Study Suggested by the Theory of Comparative Costs, Part," *Economic Journal*, pp. 697–724.
- MATSUYAMA, K. (2005): "Credit Market Imperfections and Patterns of International Trade and Capital Flows," *Journal of the European Economic Association*, pp. 714–723.
- MELITZ, M. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Inudstry Productivity," *Econometrica*, pp. 1695–1725.
- NUNN, N. (2005): "Relationship Specificity, Incomplete Contracts and the Pattern of Trade," *mimeo University of Toronto*.
- PETRI, P. A. (1980): "A Ricardian Model of Market Sharing," Journal of International Economics, pp. 201–211.
- ROMALIS, J. (2004): "Factor Proportions and the Structure of Commodity Trade," *The American Economic Review*, pp. 67–97.
- STERN, R. M. (1962): "British and American Productivity and Comparative Costs in International Trade," Oxford Economic Papers, pp. 275–303.
- VOGEL, J. (2004): "Institutions and Moral Hazard in Open Economies," *mimeo Princeton University*.
- WILSON, C. A. (1980): "On the General Structure of Ricardian Models with a Continuum of Goods: Applications to Growth, Tariff Theory, and Technical Change," *Econometrica*, pp. 1675–1702.

APPENDIX A: PROOF OF THEOREM 1

Proof of Theorem 1. Fix $i \neq n$; by the definition of total exports x_{ij}^n and Assumption A4(i), we have

$$\begin{aligned} x_{ij}^{n} &= \sum_{\omega=1}^{N_{j}} x_{j}^{n}(\omega) \cdot \mathbb{I}\left\{c_{ij}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\} \\ &= \frac{k_{j}^{n}}{\left(p_{j}^{n}\right)^{1-\sigma}} \sum_{\omega=1}^{N_{j}} p_{j}^{n}(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{c_{ij}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\} \\ &= k_{j}^{n} \left[\frac{1}{N_{j}} \sum_{\omega'=1}^{N_{j}} p_{j}^{n}(\omega')^{1-\sigma}\right]^{-1} \left[\frac{1}{N_{j}} \sum_{\omega=1}^{N_{j}} p_{j}^{n}(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{c_{ij}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\}\right], \end{aligned}$$

where the function $\mathbb{I}\{\cdot\}$ is the standard indicator function, i.e. for any event A, we have $\mathbb{I}\{A\} = 1$ if A true, and $\mathbb{I}\{A\} = 0$ otherwise. By Assumption A1, $u_{ij}(\omega)$ is independent and identically distributed (i.i.d.) across varieties so same holds for $c_{ij}^n(\omega)$. In addition, $u_{ij}(\omega)$ is i.i.d. across countries so $\mathbb{I}\{c_{ij}^n(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^n(\omega)\}$ is i.i.d. across varieties as well. This implies that $p_j^n(\omega)^{1-\sigma}$ and $p_j^n(\omega)^{1-\sigma} \cdot \mathbb{I}\{c_{ij}^n(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^n(\omega)\}$ are i.i.d. across varieties. Moreover, by Assumption A4(ii), $E\left[p_j^n(\omega)^{1-\sigma}\right] < \infty$ so we can use the strong law of large numbers for i.i.d. random variables (e.g. Theorem 22.1 in Billingsley (1995)) to show that

(13)
$$\frac{1}{N_j} \sum_{\omega'=1}^{N_j} \left[p_j^n(\omega') \right]^{1-\sigma} \stackrel{a.s.}{\to} E\left[p_j^n(\omega)^{1-\sigma} \right],$$

as $N_j \to \infty$. Note that $a_{ij} > 0$, $d_{ij}^n \ge 1$ ensure that $c_{ij}^n > 0$ whenever $w_i > 0$; hence $E\left[p_j^n(\omega)^{1-\sigma}\right] > 0$. Similarly, Assumption A4(ii) implies that

$$E\left[p_j^n(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega)\right\}\right] < \infty,$$

so we can again use the strong law of large numbers for i.i.d. random variables (e.g. Theorem 22.1 in Billingsley (1995)) to show that

(14)
$$\frac{1}{N_j} \sum_{\omega=1}^{N_j} p_j^n(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega)\right\}$$
$$\stackrel{a.s.}{\to} E\left[p_j^n(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega)\right\}\right]$$

as $N_j \to \infty$. Combining Equations (14) and (13) together with the continuity of the inverse function $x \mapsto x^{-1}$ away from 0, yields by continuous mapping theorem (e.g. Theorem 18.10 (i) in Davidson (1994))

(15)
$$\left[\frac{1}{N_j} \sum_{\omega'=1}^{N_j} p_j^n(\omega')^{1-\sigma} \right]^{-1} \left[\frac{1}{N_j} \sum_{\omega=1}^{N_j} p_j^n(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{ c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega) \right\} \right]$$
$$\stackrel{a.s.}{\to} \left\{ E\left[p_j^n(\omega)^{1-\sigma} \right] \right\}^{-1} \left\{ E\left[p_j^n(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{ c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega) \right\} \right] \right\},$$

as $N_j \to \infty$. Note that the quantities in Equation (15) are positive; hence, applying again the continuous mapping theorem (e.g. Theorem 18.10 (i) in Davidson (1994)) to their logarithm we get, with probability one,

(16)
$$\ln x_{ij}^n \to \ln k_j^n + \ln E\left[p_j^n(\omega)^{1-\sigma} \cdot \mathbb{1}\left\{c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega)\right\}\right] - \ln E\left[p_j^n(\omega)^{1-\sigma}\right],$$

as $N_j \to \infty$.

Consider $H_i(c_{1j}^n, ..., c_{Ij}^n) \equiv E\left[p_j^n(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega)\right\}\right]$. Assumptions A1, A3 and straightforward computations yield

(17)
$$H_i(c_{1j}^n, ..., c_{Ij}^n) = (c_{ij}^n)^{1-\sigma} \int_{-\infty}^{+\infty} \exp\left[(1-\sigma)u\right] f(u) \prod_{k \neq i} \left[1 - F(\ln c_{ij}^n - \ln c_{kj}^n + u)\right] du.$$

where we let $f(u) \equiv F'(u)$.

We now approximate $\ln \tilde{H}_i(c_{1j}^n, ..., c_{Ij}^n) \equiv \ln H_i(c_{1j}^n, ..., c_{Ij}^n) - (1 - \sigma) \ln c_{ij}^n$ obtained from Equation (17) by its first order Taylor series around the symmetric case $\ln c_{1j}^n = ... = \ln c_{Ij}^n = \ln c$. Without loss of generality, we choose units of account in each sector j such that $\ln c = 0$. We have

(18)
$$\tilde{H}_{i}(c_{1j}^{n},...,c_{Ij}^{n})\Big|_{(0,...,0)} = \int_{-\infty}^{+\infty} \exp\left[(1-\sigma)u\right]f(u)\left[1-F(u)\right]^{I-1}du,$$

(19)
$$\frac{\partial \tilde{H}_i(c_{1j}^n, ..., c_{Ij}^n)}{\partial \ln c_{ij}^n} \bigg|_{(0,...,0)} = -(I-1) \int_{-\infty}^{+\infty} \exp\left[(1-\sigma)u\right] f^2(u) \left[1-F(u)\right]^{I-2} du,$$

and, for $i' \neq i$,

(20)
$$\frac{\partial \tilde{H}_i(c_{1j}^n, ..., c_{Ij}^n)}{\partial \ln c_{i'j}^n} \bigg|_{(0,...,0)} = \int_{-\infty}^{+\infty} \exp\left[((1-\sigma)u\right] f^2(u) \left[1-F(u)\right]^{I-2} du.$$

Let

$$\kappa \equiv \int_{-\infty}^{+\infty} \exp\left[(1-\sigma)u\right] f(u) \left[1-F(u)\right]^{I-1} du$$

and

$$\delta \equiv \kappa^{-1} \left[\int_{-\infty}^{+\infty} \exp\left[(1-\sigma)u \right] f^2(u) \left[1 - F(u) \right]^{I-2} du \right]$$

Combining Equations (18), (19), and (20), we then get

$$\ln H_{i}(c_{1j}^{n},...,c_{Ij}^{n}) = \ln \kappa + (1-\sigma)\ln c_{ij}^{n} - (I-1)\,\delta \ln c_{ij}^{n} + \delta \sum_{i'\neq i} \ln c_{i'j}^{n} + o\left(\left\|\ln c_{j}^{n}\right\|\right)$$

$$(21) = \ln \kappa - (\delta I + \sigma - 1)\ln c_{ij}^{n} + \delta \sum_{i'=1}^{I} \ln c_{i'j}^{n} + o\left(\left\|\ln c_{j}^{n}\right\|\right),$$

where $\|\ln c_j^n\|^2 = \sum_{i'=1}^{I} [\ln c_{i'j}^n]^2$ denotes the usual L_2 -norm, and $\delta > 0$ only depends on $f(\cdot), F(\cdot), \sigma$ and I. Combining Equation (21) with the definition of $c_{ij}^n = d_{ij}^n \cdot \mathbf{w}_i \cdot a_{ij}$ and Assumption A2, then gives

(22)
$$\ln H_i(c_{1j}^n, ..., c_{Ij}^n) \simeq \alpha_i^n + b_j^n + \gamma \ln a_{ij},$$

where

$$\begin{aligned} \alpha_i^n &\equiv \ln \kappa - (\delta I + \sigma - 1) \ln(d_i^n \cdot \mathbf{w}_i) \\ b_j^n &\equiv -(\delta I + \sigma - 1) \ln d_j^n + \delta \sum_{i'=1}^{I} \ln c_{i'j}^n \\ \gamma &\equiv -(\delta I + \sigma - 1). \end{aligned}$$

Note that α_i^n does not depend on the product index j, b_j^n does not depend on the country index i and $\gamma < 0$ is a negative constant which only depends on $f(\cdot)$, $F(\cdot)$, σ and I. Combining Equations (16) and (22) then yields

$$\ln x_{ij}^n \simeq \alpha_i^n + \beta_j^n + \gamma \ln a_{ij},$$

for N_j large, where we have let $\beta_j^n \equiv \ln k_j^n + b_j^n - \ln E\left[p_j^n(\omega)^{1-\sigma}\right]$. This completes the proof of Theorem 1.

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Appendix B: Bertrand Competition

Instead of Assumption A3, we now consider:

A3'. In any country n, the price $p_j^n(\omega)$ paid by buyers of variety ω of product j is

$$p_j^n(\omega) = \min\left\{\min_{i' \neq i^*} \left[c_{i'j}^n(\omega)\right], \overline{m}c_{i^*j}^n(\omega)\right\},\,$$

where $c_{i^*j}^n(\omega) = \min_{1 \le i' \le I} c_{ij}^n(\omega)$ and $\overline{m} = \sigma/(\sigma - 1)$ is the monopoly markup.

This is in the spirit of Bernard, Eaton, Jensen, and Kortum (2003): the producer with the minimum cost may either charge the cost of its closest competitor or the monopoly price. We then have the following result:

Theorem 3. Suppose that Assumptions A1, A2, A3', and A4 hold. In addition, assume that the number of varieties N_j of any product j is large, and that technological differences across exporters are small: $c_{1j}^n \simeq ... \simeq c_{Ij}^n$. Then, for any exporter i, any importer $n \neq i$, and any product j,

(23)
$$\ln x_{ij}^n \simeq \widetilde{\alpha}_i^n + \widetilde{\beta}_j^n + \widetilde{\gamma} \ln a_{ij}.$$

where $\widetilde{\gamma} < (\sigma - 1)/(I - 1)$.

Under Bertrand competition, the qualitative insights of Theorem 1 remain valid, albeit in a weaker form. We obtain new importer–exporter and importer–industry fixed effects, $\tilde{\alpha}_i^n$ and $\tilde{\beta}_j^n$, and a new parameter $\tilde{\gamma}$ constant across countries and industries. However, the restriction $\tilde{\gamma} < (\sigma - 1)/(I - 1)$ is less stringent than in the case of perfect competition. When $\sigma \to 1$, that is when varieties become perfect substitutes, or when $I \to +\infty$, that is when the number of exporters is very large, this collapses to: $\tilde{\gamma} \leq 0$.

Proof of Theorem 3. Compared to the proof of Theorem 1, the only difference comes from the expression of $H_i(c_{1j}^n, ..., c_{Ij}^n) = E\left[p_j^n(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega)\right\}\right]$. Assumptions

A1, A3' and straightforward computations now yield

(24)
$$H_{i}(c_{1j}^{n},...,c_{Ij}^{n}) = (c_{ij}^{n})^{1-\sigma} \int_{-\infty}^{+\infty} f(u_{1}) du_{1} \int_{u_{1}}^{+\infty} [\min\left(\exp u_{2},\overline{m}\exp u_{1}\right)]^{1-\sigma} \cdot \sum_{i'\neq i} \left\{ \prod_{i''\neq i,i'} \left[1 - F(\ln c_{ij}^{n} - \ln c_{i''j}^{n} + u_{2}) \right] f(\ln c_{ij}^{n} - \ln c_{i'j}^{n} + u_{2}) \right\} du_{2}$$

where we let $f(u) \equiv F'(u)$.

As previously, we approximate $\ln \tilde{H}_i(c_{1j}^n, ..., c_{Ij}^n) \equiv \ln H_i(c_{1j}^n, ..., c_{Ij}^n) - (1-\sigma) \ln c_{ij}^n$, obtained from Equation (24), by its first order Taylor series around the symmetric case $\ln c_{1j}^n = ... = \ln c_{Ij}^n = 0$. We have

(25)
$$\tilde{H}_{i}(c_{1j}^{n},...,c_{Ij}^{n})\Big|_{(0,...,0)} = \int_{-\infty}^{+\infty} f(u_{1}) du_{1} \int_{u_{1}}^{+\infty} [\min\left(\exp u_{2},\overline{m}\exp u_{1}\right)]^{1-\sigma} \cdot (I-1) \left[1-F(u_{2})\right]^{I-2} f(u_{2}) du_{2},$$

$$\frac{\partial \tilde{H}_{i}(c_{1j}^{n},...,c_{Ij}^{n})}{\partial \ln c_{ij}^{n}}\bigg|_{(0,...,0)} = -(I-1)\int_{-\infty}^{+\infty} f(u_{1}) du_{1} \int_{u_{1}}^{+\infty} [\min\left(\exp u_{2},\overline{m}\exp u_{1}\right)]^{1-\sigma} \cdot \left\{-f'(u_{2})\left[1-F(u_{2})\right]^{I-2} + (I-2) f^{2}(u_{2})\left[1-F(u_{2})\right]^{I-3}\right\} du_{2},$$
(26)

and, for $i' \neq i$,

$$\frac{\partial H_i(c_{1j}^n, ..., c_{Ij}^n)}{\partial \ln c_{i'j}^n} \bigg|_{(0,...,0)} = \int_{-\infty}^{+\infty} f(u_1) \, du_1 \int_{u_1}^{+\infty} \left[\min\left(\exp u_2, \overline{m} \exp u_1\right)\right]^{1-\sigma} \cdot \left\{-f'(u_2) \left[1 - F(u_2)\right]^{I-2} + (I-2) \, f^2(u_2) \left[1 - F(u_2)\right]^{I-3}\right\} \, du_2.$$
(27)

Let then

(28)
$$\kappa \equiv (I-1) \int_{-\infty}^{+\infty} f(u_1) \, du_1 \int_{u_1}^{+\infty} \left[\min\left(\exp u_2, \overline{m} \exp u_1\right) \right]^{1-\sigma} \left[1 - F(u_2) \right]^{I-2} f(u_2) \, du_2,$$

and

(29)
$$\delta \equiv \kappa^{-1} \int_{-\infty}^{+\infty} f(u_1) \, du_1 \int_{u_1}^{+\infty} \left[\min\left(\exp u_2, \overline{m} \exp u_1\right) \right]^{1-\sigma} \cdot \left\{ -f'(u_2) \left[1 - F(u_2) \right]^{I-2} + (I-2) \, f^2(u_2) \left[1 - F(u_2) \right]^{I-3} \right\} du_2.$$

Combining Equations (25), (26), and (27), we get

$$\ln H_{i}(c_{1j}^{n},...,c_{Ij}^{n}) = \ln \kappa + (1-\sigma)\ln c_{ij}^{n} - (I-1)\,\delta\ln c_{ij}^{n} + \delta \sum_{i'\neq i}\ln c_{i'j}^{n} + o\left(\left\|\ln c_{j}^{n}\right\|\right)$$
$$= \ln \kappa - (\delta I + \sigma - 1)\ln c_{ij}^{n} + \delta \sum_{i'=1}^{I}\ln c_{i'j}^{n} + o\left(\left\|\ln c_{j}^{n}\right\|\right),$$

where $\|\ln c_j^n\|^2 = \sum_{i'=1}^{I} \left[\ln c_{i'j}^n\right]^2$ as previously, and δ only depends on $f(\cdot)$, $F(\cdot)$, σ and I. Let

$$\widetilde{\gamma} \equiv -(\delta I + \sigma - 1).$$

It remains to be shown that $\widetilde{\gamma} < (\sigma - 1)/(I - 1)$.

For this, let $I(u_1) \equiv \int_{u_1}^{+\infty} \left[\min\left(\exp u_2, \overline{m} \exp u_1\right)\right]^{1-\sigma} f'(u_2) \left[1 - F(u_2)\right]^{I-2} du_2$. We can rearrange $I(u_1)$ as

$$(30) I(u_1) = \int_{u_1}^{u_1+\ln\overline{m}} [\exp u_2]^{1-\sigma} f'(u_2) [1 - F(u_2)]^{I-2} du_2 + [\overline{m} \exp u_1]^{1-\sigma} \int_{u_1+\ln\overline{m}}^{+\infty} f'(u_2) [1 - F(u_2)]^{I-2} du_2 = - [\exp u_1]^{1-\sigma} f(u_1) [1 - F(u_1)]^{I-2} - (1 - \sigma) \int_{u_1}^{u_1+\ln\overline{m}} [\exp u_2]^{1-\sigma} f(u_2) [1 - F(u_2)]^{I-2} du_2 + (I - 2) \int_{u_1}^{+\infty} [\min (\exp u_2, \overline{m} \exp u_1)]^{1-\sigma} f^2(u_2) [1 - F(u_2)]^{I-3} du_2$$

where the second equality uses a simple integration by parts. Combining Equations (29) and (30), we then get

(31)
$$\delta = \kappa^{-1} \int_{-\infty}^{+\infty} f(u_1) \, du_1 \left\{ \left[\exp u_1 \right]^{1-\sigma} f(u_1) \left[1 - F(u_1) \right]^{I-2} - \left(\sigma - 1 \right) \int_{u_1}^{u_1 + \ln \overline{m}} \left[\exp u_2 \right]^{1-\sigma} f(u_2) \left[1 - F(u_2) \right]^{I-2} \, du_2 \right\}.$$

Using Equations (28) and (31), we then have

$$\begin{split} &(I-1)\delta + \sigma - 1 \\ &= (I-1)\kappa^{-1} \int_{-\infty}^{+\infty} [\exp u_1]^{1-\sigma} f^2(u_1) \left[1 - F(u_1)\right]^{I-2} du_1 \\ &- (I-1)(\sigma - 1)\kappa^{-1} \int_{-\infty}^{+\infty} f(u_1) du_1 \int_{u_1}^{u_1 + \ln \overline{m}} [\exp u_2]^{1-\sigma} f(u_2) \left[1 - F(u_2)\right]^{I-2} du_2 \\ &+ (I-1)(\sigma - 1)\kappa^{-1} \int_{-\infty}^{+\infty} f(u_1) du_1 \int_{u_1}^{+\infty} [\min \left(\exp u_2, \overline{m} \exp u_1\right)]^{1-\sigma} \left[1 - F(u_2)\right]^{I-2} f(u_2) du_2 \\ &= (I-1)\kappa^{-1} \int_{-\infty}^{+\infty} [\exp u_1]^{1-\sigma} f^2(u_1) \left[1 - F(u_1)\right]^{I-2} du_1 \\ &+ (I-1)(\sigma - 1)\kappa^{-1} \int_{-\infty}^{+\infty} f(u_1) du_1 \int_{u_1 + \ln \overline{m}}^{+\infty} [\overline{m} \exp u_1]^{1-\sigma} [1 - F(u_2)]^{I-2} f(u_2) du_2, \end{split}$$

which is positive by inspection. Hence, writing $\tilde{\gamma} = -I(I-1)^{-1}[(I-1)\delta + \sigma - 1] + (I-1)^{-1}(\sigma-1)$ and using $(I-1)\delta + \sigma - 1 > 0$ yields the desired result: $\tilde{\gamma} < (I-1)^{-1}(\sigma-1)$. \Box

APPENDIX C: THE WONDERFUL WORLD OF EATON AND KORTUM (2002)

We now impose more structure on the distribution of random productivity shocks:

A5. For all countries *i*, products *j*, and their varieties ω , $u_{ij}(\omega)$ is drawn from a (negative) Gumbel distribution with mean zero:

$$F(u) = 1 - \exp[-\exp(\theta u - \mathbf{e})]$$

where $u \in \mathbb{R}$, $\theta > \sigma - 1$, and **e** is Euler's constant **e** $\simeq 0.577$.

Assumption A5 corresponds to the case where labor productivity is drawn from a Fréchet distribution, as assumed in Eaton and Kortum (2002). We then have the following result:

Theorem 4. Suppose that Assumptions A1-A5 hold. In addition, assume that the number of varieties N_j of any product j is large. Then, for any exporter i, any importer $n \neq i$, any product j, and any vector of costs $(c_{1j}^n, ..., c_{Ij}^n)$

(32)
$$\ln x_{ij}^n \simeq \alpha_i^n + \beta_j^n - \theta \ln a_{ij}$$

As mentioned in the main text, Assumption A5 guarantees that the results of Theorem 1 hold globally. Assumption A5 also implies that the elasticity of exports with respect to the average unit labor requirement is equal to the shape parameter of the Gumbel θ . Hence, changes in the elasticity of substitution σ across countries and industries do not affect the predictions of Theorem 4.

Proof of Theorem 4. Since Assumption A1-A4 hold, the results of Theorem 1 apply. In particular, we know that, with probability one

$$\ln x_{ij}^n \to \ln k_j^n + \ln E\left[p_j^n(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega)\right\}\right] - \ln E\left[p_j^n(\omega)^{1-\sigma}\right],$$

as $N_j \to \infty$. Using Equation (17) together with the expressions for the (negative) Gumbel distribution and density, we then have

$$E\left[p_{j}^{n}(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{c_{ij}^{n}(\omega) = \min_{1 \le i' \le I} c_{i'j}^{n}(\omega)\right\}\right]$$

$$= (c_{ij}^{n})^{1-\sigma} \int_{-\infty}^{+\infty} \theta \exp\left\{(\theta + 1 - \sigma)u - \mathbf{e} - \left[1 + \sum_{k \ne i} (c_{ij}^{n}/c_{kj}^{n})^{\theta}\right] \exp\left(\theta u - \mathbf{e}\right)\right\} du$$

$$= (c_{ij}^{n})^{1-\sigma} \exp\left(-\mathbf{e}\frac{\sigma - 1}{\theta}\right) \Gamma\left(\frac{\theta + 1 - \sigma}{\theta}\right) \left[1 + \sum_{k \ne i} (c_{ij}^{n}/c_{kj}^{n})^{\theta}\right]^{-(\theta + 1 - \sigma)/\theta}$$

$$(33) = \exp\left(-\mathbf{e}\frac{\sigma - 1}{\theta}\right) \Gamma\left(\frac{\theta + 1 - \sigma}{\theta}\right) \frac{(c_{ij}^{n})^{-\theta}}{\left[\sum_{k=1}^{I} (c_{kj}^{n})^{-\theta}\right]^{(\theta + 1 - \sigma)/\theta}},$$

where the second equality uses the change of variable $v \equiv \left(1 + \sum_{k \neq i} (c_{ij}^n / c_{kj}^n)^{\theta}\right) \exp(\theta u - \mathbf{e})$, and where $\Gamma(\cdot)$ denotes the Gamma function, $\Gamma(t) = \int_0^{+\infty} v^{t-1} \exp(-v) dv$ for any t > 0. Note that

$$E\left[p_j^n(\omega)^{1-\sigma}\right] = \sum_{i=1}^{I} E\left[p_j^n(\omega)^{1-\sigma} \cdot \mathbb{I}\left\{c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega)\right\}\right],$$

so that by using Equation (33) we get

$$E\left[p_{j}^{n}(\omega)^{1-\sigma}\right] = \exp\left(-\mathbf{e}\frac{\sigma-1}{\theta}\right)\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)\frac{1}{\left[\sum_{k=1}^{I}(c_{kj}^{n})^{-\theta}\right]^{(1-\sigma)/\theta}}$$

and hence

$$\ln x_{ij}^n \simeq \ln k_j^n - \theta \ln c_{ij}^n - \ln \left(\sum_{k=1}^I (c_{ij}^n)^{-\theta} \right).$$

for N_j large. Combining the above with the definition of $c_{ij}^n = d_{ij}^n \cdot \mathbf{w}_i \cdot a_{ij}$ and Assumption A2, then gives

$$\ln x_{ij}^n \simeq \alpha_i^n + \beta_j^n - \theta \ln a_{ij},$$

where we have let $\alpha_i^n \equiv -\theta \ln(d_i^n \cdot w_i)$ and $\beta_j^n \equiv \ln k_j^n - \theta \ln d_j^n - \ln \left(\sum_{k=1}^I (c_{ij}^n)^{-\theta}\right)$. \Box

Theorem 4 crucially relies on the following property of the Gumbel distribution:

(34)
$$\Pr\left\{p_j^n(\omega) \le p\right\} = \Pr\left\{p_j^n(\omega) \le p \left| c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega) \right.\right\},$$

for any p > 0 and any $1 \le i \le I$. Property (34) states that the distribution of the price $p_j^n(\omega)$ of a given variety ω of product j in country n is independent of the country of origin i; see Eaton and Kortum (2002) p1748 for a detailed discussion. Unfortunately, this property does not easily generalize to other distributions, as we show in the following Theorem.

Theorem 5. Suppose that Assumptions A1-A4 hold and that $f(u) \equiv F'(u) > 0$ for any u in \mathbb{R} . Then, for any p > 0 and any $1 \le i \le I$, we have:

$$\Pr\left\{p_j^n(\omega) \le p\right\} = \Pr\left\{p_j^n(\omega) \le p \left| c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega) \right.\right\} \Leftrightarrow F(\cdot) \text{ satisfies A5}$$

Put simply, the only distribution with full support satisfying Property (34) is the Gumbel.

Proof of Theorem 5. That Assumption A5 is sufficient for Equation (34) to hold is a matter of simple algebra. We now show that it is also necessary: if Equation (34) is satisfied, then $F(\cdot)$ is Gumbel. First, note that Equation (34) is equivalent to

$$\frac{\Pr\left\{p_j^n(\omega) \le p, c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega)\right\}}{\Pr\left\{c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega)\right\}} = \Pr\left\{p_j^n(\omega) \le p\right\},$$

for all p > 0 and any $1 \le i \le I$, which in turn is equivalent to having (35)

$$\frac{\Pr\left\{p_{j}^{n}(\omega) \leq p, c_{i_{1}j}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\}}{\Pr\left\{c_{i_{1}j}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\}} = \frac{\Pr\left\{p_{j}^{n}(\omega) \leq p, c_{i_{2}j}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\}}{\Pr\left\{c_{i_{2}j}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\}},$$

for all p > 0 and any $1 \le i_1, i_2 \le I$. Using Assumptions A1 and A3, we have

(36)
$$\Pr\left\{c_{i_{1}j}^{n}(\omega) = \min_{1 \le i' \le I} c_{i'j}^{n}(\omega)\right\} = \int_{-\infty}^{+\infty} f(u) \prod_{k \ne i_{1}} \left[1 - F(\ln c_{i_{1}j}^{n} - \ln c_{kj}^{n} + u)\right] du$$

and

(37)
$$\Pr\left\{p_{j}^{n}(\omega) \leq p, c_{i_{1}j}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\} = \int_{-\infty}^{\ln p - \ln c_{i_{1}j}^{n}} f(u) \prod_{k \neq i_{1}} \left[1 - F(\ln c_{i_{1}j}^{n} - \ln c_{kj}^{n} + u)\right] du,$$

with similar expressions for i_2 . So the condition in Equation (35) is equivalent to

$$\frac{\int_{-\infty}^{\ln p - \ln c_{i_{1j}}^{n}} f(u) \prod_{k \neq i_{1}} \left[1 - F(\ln c_{i_{1j}}^{n} - \ln c_{kj}^{n} + u) \right] du}{\int_{-\infty}^{+\infty} f(u) \prod_{k \neq i_{1}} \left[1 - F(\ln c_{i_{1j}}^{n} - \ln c_{kj}^{n} + u) \right] du} = \frac{\int_{-\infty}^{\ln p - \ln c_{i_{2j}}^{n}} f(u) \prod_{k \neq i_{2}} \left[1 - F(\ln c_{i_{2j}}^{n} - \ln c_{kj}^{n} + u) \right] du}{\int_{-\infty}^{+\infty} f(u) \prod_{k \neq i_{2}} \left[1 - F(\ln c_{i_{2j}}^{n} - \ln c_{kj}^{n} + u) \right] du},$$

for all p > 0 and any $1 \le i_1, i_2 \le I$. Differentiating the above equality with respect to $\ln p$ and using the fact that f(x) > 0 and hence F(x) < 1 for all $x \in \mathbb{R}$, this in turn implies

$$\frac{f(\ln p - \ln c_{i_{1j}}^{n})\left[1 - F(\ln p - \ln c_{i_{2j}}^{n})\right]}{f(\ln p - \ln c_{i_{2j}}^{n})\left[1 - F(\ln p - \ln c_{i_{1j}}^{n})\right]} = \frac{\int_{-\infty}^{+\infty} f(u) \prod_{k \neq i_{1}} \left[1 - F(\ln c_{i_{1j}}^{n} - \ln c_{kj}^{n} + u)\right] du}{\int_{-\infty}^{+\infty} f(u) \prod_{k \neq i_{2}} \left[1 - F(\ln c_{i_{2j}}^{n} - \ln c_{kj}^{n} + u)\right] du},$$

for all p > 0 and any $1 \le i_1, i_2 \le I$. Since the right-hand side of the above equality does not depend on p, we necessarily have that

(38)
$$\frac{h_F(p/c_{i_1j}^n)}{h_F(p/c_{i_2j}^n)} \text{ only depends on } c_{i_1j}^n, c_{i_2j}^n,$$

where $h_F(\cdot)$ is a modified hazard function of $F(\cdot)$, i.e. $h_F(x) \equiv [1 - F(\ln x)]^{-1} f(\ln x)$ for any x > 0. We now make use of the following Lemma:

Lemma 6. If for any positive constants c_1 and c_2 , $h_F(x/c_1)/h_F(x/c_2)$ only depends on c_1, c_2 , then necessarily $h_F(x)$ is of the form $h_F(x) = \mu x^{\theta}$ where $\mu > 0$ and θ real. Proof of Lemma 6. Let $U(t,x) \equiv h_F(tx)/h_F(x)$ for any x > 0 and any t > 0. Consider $t_1, t_2 > 0$: we have

(39)

$$U(t_{1}t_{2}, x) = \frac{h_{F}(t_{1}t_{2}x)}{h_{F}(x)}$$

$$= \frac{h_{F}(t_{1}t_{2}x)}{h_{F}(t_{1}x)} \cdot \frac{h_{F}(t_{1}x)}{h_{F}(x)}$$

$$= U(t_{2}, t_{1}x) \cdot U(t_{1}, x).$$

If the assumption of Lemma (6) holds then U(t, x) only depends on its first argument t and we can write it U(t). Hence the Equation (39) becomes

$$U(t_1t_2) = U(t_2) \cdot U(t_1).$$

So, $U(\cdot)$ solves the Hamel equation on \mathbb{R}^+_* and is of the form $U(t) = t^{\theta}$ for some real θ . This implies that

(40)
$$h_F(xt) = x^{\theta} h_F(t).$$

Consider t = 1 and let $\mu \equiv h_F(1) > 0$; Equation (40) then gives

$$h_X(x) = \mu x^{\theta},$$

which completes the proof of Lemma 6.

(Proof of Theorem 5 continued). The result of Lemma 6 allows us to characterize the class of distribution functions $F(\cdot)$ that satisfy Property (34). For any $u \in \mathbb{R}$, we have

(41)
$$\frac{f(u)}{1 - F(u)} = \mu \exp(\theta u)$$

Note that when $u \to -\infty$ we have $f(u), F(u) \to 0$ so that necessarily $\theta > 0$. We can now integrate Equation (41) to obtain for any $u \in \mathbb{R}$

(42)
$$F(u) = 1 - \exp\left[-\exp\left(\theta u + \ln\left(\frac{\mu}{\theta}\right)\right)\right] \text{ with } \mu > 0 \text{ and } \theta > 0,$$

which belongs to the (negative) Gumbel family. Noting the expected value of the (negative) Gumbel distribution in Equation (42) equals $-\theta^{-1}(\ln(\mu/\theta) + \mathbf{e})$, where \mathbf{e} is the Euler's constant, we necessarily have, by Assumptions A1 and A4(ii),

$$F(u) = 1 - \exp\left[-\exp\left(\theta u - \mathbf{e}\right)\right]$$
 with $\theta > \sigma - 1$ for any $u \in \mathbb{R}$,

which completes the proof of Theorem 5.

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APPENDIX D: OLS ESTIMATES WITHOUT US DATA

					/			
Variable	2003	2002	2001	2000	1999	1998	1997	1996
ln a	-0.67	-0.65	-0.88	-0.77	-0.69	-0.86	-0.93	-0.67
	(-8.60)***	(-12.23)***	(-15.78)***	(-15.12)***	(-12.37)***	(-13.06)***	(-13.63)***	(-9.76)***
Exporter-Importer FE	Yes	Yes						
Industry-Importer FE	Yes	Yes						
Observations	7749	14024	17319	17570	17777	17181	17225	16774
R ²	0.804	0.791	0.788	0.782	0.789	0.789	0.785	0.781
Variable	1995	1994	1993	1992	1991	1990	1989	1988
ln a	-0.76	-0.71	-0.47	-0.47	-0.37	-0.34	-0.37	-0.08
	(-11.45)***	(-9.13)***	(-6.82)***	(-6.52)***	(-4.90)***	(-4.64)***	(-5.36)***	(-1.06)
Exporter-Importer FE	Yes	Yes						
Industry-Importer FE	Yes	Yes						
Observations	16419	13244	12491	11779	10945	10775	10727	10672
R ²	0.7795	0.7823	0.7887	0.7903	0.780	0.778	0.776	0.776

Table 3: Year-by-Year OLS Regressions without US Data (Dependent Variable: Inx)

Note: Absolute value of t-statistics in parentheses, calculated from heteroskedasticity-consistent (White) standard errors

* Significant at 10% confidence level

** Significant at 5% confidence level

*** Significant at 1% confidence level