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Calculations of Single Particle Polarization
using a Realistic Two-Center Shell Model

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Abstract.

In "slow" nucleus-nucleus collisions the single particle states will adjust continuously to an instantaneous two-center potential. Rearrangement reactions will occur between polarized rather than between asymptotic states.

A previous investigation of polarized single particle states is extended to a realistic two-center model including Coulomb forces. The dependence of polarization upon physical parameters is discussed. Polarization of states which participate in transfer reactions initiated by $^{40}\text{Ca} + ^{16}\text{O}$, and in the reaction $^{208}\text{Pb}(^{12}\text{C}, ^{11}\text{B})^{209}\text{Bi}$, is analyzed in detail. In the former system a strong resonance polarization is found for some neutron states. A two-state approximation is employed to estimate the evolution of polarization in time. We predict that in many cases typical direct interaction times are sufficient for strong polarization to develop. Observable consequences of this are discussed in a qualitative way. We also make suggestions for further theoretical and experimental work.

1. Introduction.

It has been proposed recently that, in "slow" nucleus-nucleus collisions, the single particle states which participate in quasielastic reactions will be modified or polarized due to the proximity of the approaching nucleus and its force field.¹⁾⁻³⁾ In the adiabatic limit the polarized states at a distance R between the nuclei can be computed as the eigenstates of an instantaneous two-center potential which is formed during the collision. This conception is analogous to that of molecular states or perturbed stationary states (pss) in atomic collisions.^{4),5)}

Model calculations performed recently predict that the polarization effect admixes components near the polarizing nucleus into single particle states, thereby increasing the probability for transfer at large distance.³⁾ In a semiclassical picture this will result in a shift of angular distributions forward of what calculations based on unpolarized states predict, and in an enhancement of absolute cross section. The effects are generally stronger for more highly excited states. They will diminish and eventually disappear with increasing collision energy, i.e. decreasing adiabaticity.

The hypothesis of single particle polarization was based on the observation of systematic discrepancies between experimental data on few-nucleon transfer and DWBA-predictions.^{1),6)} As discussed in ref. 6) the nature of the discrepancies is in accord with the qualitative trends which are expected from a semiclassical picture of polarization effects. These trends have been confirmed in a calculation of the transfer reaction $^{40}\text{Ca}(^{16}\text{O}, ^{15}\text{N})^{41}\text{Sc}$, where it

was found that inclusion of polarization effects gave rise to a 18° forward shift in the angular distribution, close to what experimental data require.^{6),7)}

Inclusion of polarization strongly enhanced the absolute cross section. It was larger than the unpolarized prediction by a factor of 240, and exceeded the experimentally observed cross section by a factor of 43. Thus, while the calculation of ref. 6) gave additional support to the hypothesis of single particle polarization effects, it did overestimate them considerably. There are two possible reasons for this. (i) The calculation was based on a very schematic two-center model, namely, the modified two-center oscillator of Maruhn and Greiner.⁸⁾ The two essential features determining polarization effects - relative level positions in the colliding nuclei and the shape of the potential barrier between them - are not realistic in this model. Also, Coulomb forces are neglected. (ii) Even at the relatively low bombarding energy of 48 MeV at which the experiment of ref. 7) was performed the polarization may have been less than predicted from a static two-center model, which applies only in the adiabatic limit.

In this paper we remedy point (i), above, by investigating polarization effects using a realistic two-center Woods-Saxon model including Coulomb forces. Our analysis is based on the method of "asymptotic expansion", as developed in ref. 3). We also give simple estimates concerning the time- (or energy-) dependence of the polarization, which stands behind point (ii), above. The dynamic evolution of polarization during a collision will be

analyzed in more detail from a consideration of two-step processes in a forthcoming publication.⁹⁾

Section 2 describes the two-center model and our method of calculation. Useful insight into the polarization effect and its evolution in time can be drawn from a simple two-state approximation, as discussed in section 3.

Results of calculations for neutron and proton states in collisions of $^{40}\text{Ca} + ^{16}\text{O}$ and for proton states in $^{208}\text{Pb} + ^{12}\text{C}$ are presented and discussed in section 4. Our predictions for observable effects of the polarization, i.e. for discrepancies between experimental data and (unpolarized) DWBA - predictions, are only qualitative at present because we have not yet completed reaction calculations with the polarized states discussed in this paper (work in progress). A summary of our findings and suggestions for further work are given in section 5.

2. The Model.

2.1 Two-Center Model.

Generally speaking, the instantaneous two-center potential formed during a nucleus-nucleus collision will depend upon the nucleus-nucleus distance and the integrals of motion, notably the collision energy, in a complicated way.¹⁾ We are presently interested in quasielastic reactions between spherical nuclei at moderate collision energies. We expect that for these ^{reactions} the two-center potential will be close to a simple superposition of the asymptotic one-center potentials. Introducing an internal coordinate system with origin

at the center of nucleus A and 3-axis pointing to nucleus B at a distance R we take the two-center Hamiltonian as¹⁰⁾

$$h^R(\underline{r}) = \underbrace{T + V_A(\underline{r})}_{h_A} + V_B(\underline{r} - R\hat{e}_3) \quad (1)$$

The ansatz eq. (1) does not allow for a self-consistent adjustment of the approaching shell model potentials. Also, residual interactions are neglected in the present work. For the individual shell model potentials V_A , V_B the usual Woods-Saxon parametrization is used

$$V(\underline{r}) = U \cdot f(\underline{r}) + W \underline{\ell} \cdot \underline{s} \frac{df_s(\underline{r})}{r d r} + \left(\frac{1}{2} + t_3\right) \phi(\underline{r}) \quad (2)$$

where

$$f(\underline{r}) = \left[1 + \exp\left(\frac{r - r_0 A^{1/3}}{a}\right) \right]^{-1} \quad (3)$$

and an analogous definition for the radial dependence $f_s(r)$ of the $\underline{\ell} \cdot \underline{s}$ -potential. The Coulomb potential is taken to be that of a uniformly charged sphere with radius R_{ch}

$$\phi(\underline{r}) = \begin{cases} \frac{ze^2}{2R_{ch}} \left(3 - \frac{r^2}{R_{ch}^2} \right) & r \leq R_{ch} \\ \frac{ze^2}{r} & r > R_{ch} \end{cases} \quad (4)$$

Our method to obtain the eigenstates of the Hamiltonian eq. (1) is similar to that of Zint and Mosel¹¹⁾, and Vautherin¹²⁾. Namely, we numerically diagonalize h^R on the basis of the eigenstates of the two-center oscillator, which are expressed in terms of cylindrical coordinates z, ρ, φ .^{13), 8)} Matrix elements are calculated numerically using a fast two-dimensional Gaussian integration. The ρ -integration is performed with the standard Gauss-Laguerre method.^{14), 12)} The z -integration is explained in the appendix.

For spherically symmetric nuclei the two-center potential has axial symmetry. The only integrals of motion are energy $\mathcal{E}$ and projection Ω of the angular momentum on the 3-axis (intrinsic symmetry axis). The two-center states may be either specified with the quantum numbers $\mathcal{E}$ and Ω , or with the (asymptotic) quantum numbers $nlj\Omega$ of eigenstates of the spherically symmetric one-center shell models, which they approach as the two-center separation parameter $R \rightarrow \infty$. Thus we have

$$\begin{aligned}
 |\mathcal{E}\Omega\rangle &\equiv |Rnlj\Omega\rangle \\
 &= \sum_{n_z n_s} \left(a_{n_z n_s}^{nlj\Omega\uparrow}(R) |n_z n_s \Omega - \frac{1}{2}\rangle \chi_{\frac{1}{2}} + a_{n_z n_s}^{nlj\Omega\downarrow}(R) |n_z n_s \Omega + \frac{1}{2}\rangle \chi_{-\frac{1}{2}} \right)
 \end{aligned}
 \tag{5}$$

where we have introduced an obvious notation for spin-up and spin-down components. The $|n_z n_s m\rangle$ are eigenstates of the two-center oscillator. The phase conventions used here and in what follows are those of ref. 3).

2.2 The Polarization.

We are interested in the "polarization" present in the single particle states eq. (5), i.e. in their distortion away from their asymptotic form which is caused by the proximity of "the other" shell model potential. To analyze this polarization, and to eventually use two-center states in the calculation of transfer reactions, an expansion in terms of asymptotic states with good angular momentum is required. Using the method of ref. 3) we bring eq. (5) into the form

$$|Rn\ell j \Omega\rangle = \sum_{\nu' \ell' j'} A_{\nu' \ell' j'}^{n\ell j \Omega}(R) |\nu' \ell' j' \Omega\rangle \quad (6)$$

Here the $|\nu' \ell' j' \Omega\rangle$ are eigenstates of a spherical harmonic oscillator with $\ell \cdot s$ -force, centered at either the polarized or the polarizing nucleus. Because Ω is an integral of motion all "polarization amplitudes" A may be chosen real. Furthermore, as the two-center Hamiltonian eq. (1) is time-reversal invariant, A^{Ω} and $A^{-\Omega}$ differ only by a phase factor[#]

$$A_{\nu' \ell' j'}^{n\ell j -\Omega} = (-)^{j-\ell-(j'-\ell')} A_{\nu' \ell' j'}^{n\ell j \Omega} \quad (7)$$

A difference to the treatment given in ref. 3) for the two-center model of Maruhn and Greiner arises from the fact that eigenstates of the two-center Woods-Saxon potential are a superposition of basis states even asymptotically.

[#] Due to a misprint the phase factor was missing in eq. (8c) of ref. 3).

$$|Rn\ell j \Omega\rangle \underset{R \rightarrow \infty}{\sim} |n\ell j \Omega\rangle = \sum_{v'} A_{v'\ell j}^{n\ell j \Omega}(\infty) |v'\ell j \Omega\rangle \quad (8)$$

We distinguish Woods-Saxon states from harmonic oscillator states by denoting the radial quantum number with n and v , respectively. The coefficients $A(\infty)$ for the asymptotic one-center Woods-Saxon states are calculated with the same method as the $A(R)$ in eq. (6). The diagonalization of the Woods-Saxon potential however, by which the coefficients $a(R)$ in eq. (5) are determined, is made with harmonic oscillator instead of two-center oscillator states $|n_z\rangle$ for $R = \infty$.

The polarization effect is characterized by a large number (≈ 200) of "significant" amplitudes A in eq. (6). An overall measure of the strength of the polarization which ignores the details of its structure is provided by the total probability of the non-asymptotic components in the state given by eq. (6). Because of eq. (8) this quantity can not be expressed in the simple form eq. (13) of ref. 3). To generalize, we introduce an operator to project out the non-asymptotic components for every polarized state $|Rn\ell j \Omega\rangle$

$$\hat{p}^{n\ell j \Omega}(R) = 1 - |n\ell j \Omega\rangle \langle n\ell j \Omega| \quad (9)$$

where the asymptotic Woods-Saxon state $|n\ell j \Omega\rangle$ is centered at the polarized nucleus. The polarization strength, or simply polarization, is defined as

the expectation value of this operator in a polarized state. Using eq. (6)

we have

$$\begin{aligned}
 p^{nlj\Omega}(R) &\equiv \langle Rnlj\Omega | \hat{P}^{nlj\Omega}(R) | Rnlj\Omega \rangle \\
 &= 1 - \left[\sum_{v'} A_{v'lj}^{nlj\Omega}(\infty) A_{v'lj}^{nlj\Omega}(R) \right]^2
 \end{aligned} \quad (10)$$

The non-asymptotic part of the state eq. (6) can be obtained by projecting

$$\begin{aligned}
 |Rnlj\Omega\rangle &\equiv \hat{P}^{nlj\Omega}(R) |Rnlj\Omega\rangle \\
 &= \sum_{v'l'j'} \left[A_{v'l'j'}^{nlj\Omega}(R) - \sqrt{1 - p^{nlj\Omega}(R)} \delta_{ll'} \delta_{jj'} A_{v'l'j'}^{nlj\Omega}(\infty) \right] |v'l'j'\Omega\rangle
 \end{aligned} \quad (11)$$

The normalization of this state equals $p^{nlj\Omega}(R)$, of course.

2.3 Accuracy of Calculation and Choice of Basis.

The diagonalization of the two-center Hamiltonian on the basis of two-center oscillator states is more difficult for the two-center Woods-Saxon model (TCWS) than it was for the two-center model of Maruhn and Greiner.⁸⁾ The reason for this is that the z-components of the basis functions were very close to the actual eigenstates in the Maruhn-Greiner model, the correspondence becoming more perfect for increasing R. This is not the case for the TCWS-model. Any failure to very accurately determine the TCWS-states within one

nucleus, due to limited basis size and improper choice of basis functions, would reduce the overlap of the TCWS-state with the corresponding unpolarized one-center Woods-Saxon state, thereby increasing the polarization, eq. (10), beyond what actually is present. The polarized states, eqs. (6), (11), are superpositions of many components and hence are expected to be more accurately determined than individual amplitudes A and the polarization, eq. (10).

The accuracy requirements become more severe for increasing R , because the TCWS-state then deviates less and less from the corresponding asymptotic unpolarized Woods-Saxon state, and it is this small deviation which represents the polarization effects. For example, if the squared overlap between a TCWS-state and the corresponding asymptotic state is $(1 - \delta)$ with an accuracy of $\pm \eta$, the polarization according to eq. (10) is $1 - [(1 - \delta) \pm \eta] = \delta \pm \eta$. Thus for a polarization of e.g. 10^{-3} to be accurate to within 1 % the squared overlap is required with an accuracy of 10^{-3} %. The situation is alleviated somewhat from the fact that the more tightly bound and hence less strongly polarized states automatically are determined with a higher absolute accuracy by the diagonalization procedure.

For the nuclei with $A \leq 40$ studied in this work we have chosen the $\hbar\omega_{A,B}$ of the basis oscillator to reproduce the measured rms-radii. For $A > 40$ we take the standard value $\hbar\omega = 41/A^{1/3}$ (see figs. 1 and 2). The accuracy of the calculation was checked by comparing results obtained with different - within reasonable limits - basis parameters and basis sizes. In order to achieve an accuracy of the order of 1 % for the polarization, in both the systems $^{40}\text{Ca} + ^{16}\text{O}$

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and $^{208}\text{Pb} + ^{12}\text{C}$, 25 (one-dimensional) two-center oscillator states were required ($1 \leq n_z \leq 25$), corresponding to respectively $14 + 11$ and $17 + 8$ oscillator shells asymptotically (separated nuclei). In our method the frequency ω_g of the g -basis-oscillator (plane perpendicular to the symmetry axis) is conveniently chosen equal to either ω_A or ω_B , depending upon whether we want the asymptotic expansion, eq. (6), to be made on the basis of asymptotic states of nucleus A or B. Thus the ω_g of the basis is far from optimal for one of the two nuclei, particularly so in a very asymmetric case like $\text{Pb} + \text{C}$. To get the required accuracy we needed the lowest 11 g -oscillator states, corresponding to states with up to 20 phonons above the ground state. As all amplitudes $a_{n_z n_g}$ in eq. (5) generally decrease for high n_z we diminish the number of g -states by 1 for every increase in n_z by 2, up to a minimum of four g -states per n_z for $n_z \geq 14$. With this the dimension of the two-center oscillator basis for the diagonalization of the TCWS - Hamiltonian, eq. (1), was 278.

The time required to calculate the eigenstates of the Hamiltonian eq. (1) was one minute for a given R, Ω on a CDC 7600 computer. This time includes calculation of two-center oscillator basis states, calculation of matrix elements on a 60×25 grid, and diagonalization of the Hamiltonian matrix. The steps and times involved in the calculation of the polarization amplitudes, eq. (6), are as discussed in ref. (3).

In the grazing region, results for the polarization obtained with substantially different basis parameters (see figs. 1 and 2) differed by 1 % on the

average for $^{40}\text{Ca} + ^{16}\text{O}$, and by 2 % for $^{208}\text{Pb} + ^{12}\text{C}$, with little difference between strongly and weakly bound states due to the absolutely much stronger polarization of the latter. Single particle energies then were accurate to within 0.1 keV (!) for the most deeply bound states and to within 1 keV for the highest states in both systems.

The sensitivity of the calculation may also be appreciated by comparing results obtained with a smaller set of basis functions. For $^{40}\text{Ca} + ^{16}\text{O}$ at $R = 12$ fm we decreased the basis size to 16 n_z - and 8 n_g - states. This resulted in moderate errors for single particle energies, ranging from 1 keV $\cong 10^{-2}$ % for the lowest to 20 keV $\cong 1$ % for the highest states. The polarizations in contrast turned out too large by 50 - 200 % for the basis parameters marked "b" in fig. 1, and too large by up to a factor of 10 for the basis "a".

3. Two-State Approximation.

Certain systematics of the polarization effect as well as polarization resonances can be conveniently discussed within the framework of a two-state approximation.¹⁵⁾ Also, simple estimates as to the evolution of polarization in time can be made.

3.1 Static Case.

Suppose the isolated nuclei A and B have two eigenstates ϕ_1 and ϕ_2 ,

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respectively, which are close in energy

$$|\Delta E| = |\varepsilon_1 - \varepsilon_2| \ll |\varepsilon_1 - \varepsilon_n| \quad (n \neq 1, 2) \quad (12)$$

It is well known that in this case the combined system, characterized in shell model approximation by the Hamiltonian eq. (1), has two eigenstates which, at a distance R between the nuclei, are approximately

$$\phi_{u,\ell}^R = N_{u,\ell} \left([\sin \alpha_{u,\ell}(R)] \phi_1 + [\cos \alpha_{u,\ell}(R)] \phi_2 \right) \quad (13)$$

Here and in what follows u, ℓ refers to upper and lower sign, respectively.

$N_{u,\ell}$ is the normalization

$$N_{u,\ell} = \left(1 + [\sin 2\alpha_{u,\ell}] \cdot \sigma_{12} \right)^{-1/2} \quad (14)$$

and

$$\sigma_{12} = \langle \phi_1 | \phi_2 \rangle_R \quad (15)$$

is the overlap between the asymptotic states at distance R . The admixture amplitudes are determined from

$$\cot \alpha_{u,\ell}(R) = \pm [\operatorname{sgn} h_{12}(R)] \sqrt{1 + q(R)^2} - q(R) \quad (16)$$

where h_{12} is the matrix element of the two-center Hamiltonian, eq. (1)

$$h_{12}(R) = \langle \phi_1 | h^R | \phi_2 \rangle = \langle \phi_1 | V_B^R | \phi_2 \rangle_R + \epsilon_1 \sigma_{12} \quad (17)$$

$V_B^R(\mathbf{r}) = V_B(\mathbf{r} - R \mathbf{e}_3)$ is the shell model potential of nucleus B, centered at a distance R from A. The important parameter is q, which relates the level spacing to the perturbing interaction:

$$q(R) = \frac{\Delta \epsilon + \Delta V^R}{2 h_{12}} \quad (18)$$

where

$$\Delta V^R = \langle \phi_1 | V_B^R | \phi_1 \rangle_R - \langle \phi_2 | V_A | \phi_2 \rangle_R \quad (19)$$

The eigenenergies are approximately

$$E_{u,l}(R) = \frac{1}{2} [\epsilon_1 + \epsilon_2 + \langle \phi_2 | V_A | \phi_2 \rangle + \langle \phi_1 | V_B^R | \phi_1 \rangle \pm \Delta(R)] \quad (20)$$

where

$$\Delta(R) = 2 |h_{12}| \sqrt{1+q^2} \quad (21)$$

Note that the equations and definitions above hold also if ϕ_1 and ϕ_2 belong to the same nucleus. For $R \rightarrow \infty$ $\phi_{u,l} \rightarrow \phi_{1,2}$ or vice versa. Assume without loss of generality that it is ϕ_u which tends to ϕ_1 asymptotically. The polarization of this state is according to eqs. (9), (10)

$$P(R) \equiv 1 - |\langle \phi_1 | \phi_u^R \rangle|^2 = \frac{1 - \sigma_{12}^2}{1 + \tan^2 \alpha_u + 2 \sigma_{12} \tan \alpha_u} \quad (22)$$

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In grazing collisions $|\sigma_{12}| \ll 1$ and we have approximately

$$P(R) = \frac{1}{2} \left(1 - \sqrt{1 - \frac{1}{1+q^2}} \right) \sim \begin{cases} \frac{h_{12}^2}{(\Delta E + \Delta V^R)^2} & \text{if } q^2 \gg 1 \\ \frac{1}{2} & \text{if } q^2 \ll 1 \end{cases} \quad (23)$$

Also,

$$|h_{12}(R)| = |\Delta(R)| \cdot \sqrt{P(1-P)} \quad (24)$$

and

$$|\Delta E + \Delta V^R| = 2 \cdot |\Delta(R)| \cdot \sqrt{1/4 - P(1-P)} \quad (25)$$

The following features are now apparent.

(i) If ϕ_1 and ϕ_2 belong to the same nucleus the perturbing potential will act between two exponential tails. Hence h_{12} will generally be larger if ϕ_1 and ϕ_2 belong to different nuclei. Thus the polarization effect will preferably admix components near the center of the polarizing nucleus into single particle states.

(ii) The polarization obtained for large q^2 , eq. (23), resembles the first order perturbation expression as given in ref. 3) (see eq. (34)); except that for the latter all states had to be referred to the same nucleus.

(iii) When the perturbation h_{12} is of the order of or larger than the level spacing $\Delta\epsilon + \Delta V^R$, i.e. $q^2 \lesssim 1$, we encounter a strong resonance polarization. For neutron states ΔV^R will be small in the grazing region (typically < 100 keV, see table 5), due to the short range of the shell model potentials. Hence a resonance will occur only if the nuclei A and B accidentally have close lying single particle states. Note that the effect depends upon the actual energies $\epsilon_{1,2}$ of the states, which differ from the experimentally accessible separation energies $\epsilon_{1,2}^s$ by a rearrangement energy $\epsilon_{1,2}^r = \epsilon_{1,2} - \epsilon_{1,2}^s$.¹⁶⁾ Thus it is conceivable that further investigation of the resonance polarization effect may lead to a determination of rearrangement energies.

The situation is entirely different for protons in an asymmetric nucleus-nucleus collision. The long range Coulomb potential shifts single particle levels in the smaller nucleus up relative to those in the larger nucleus.

We have approximately

$$\Delta V^R = \frac{(z_B - z_A)e^2}{R} \quad (26)$$

In very asymmetric systems such as $^{208}\text{Pb} + ^{12}\text{C}$ this shift amounts to nearly 10 MeV in the grazing region (see subsect. 4.5). Thus resonance conditions can be met, at different R , for many levels. If $\epsilon_1 > \epsilon_2$ (we suppose that ϕ_1 belongs to the larger nucleus, A say) we have for the separation distance at which resonance occurs ($q(R_{\text{res}}) = 0$)

$$R_{\text{res}} = \frac{(z_A - z_B)e^2}{\epsilon_1 - \epsilon_2} \quad (27)$$

Resonance conditions are confined to a neighborhood $|\Delta R| \approx 2|h_{12}|R_{\text{res}}/(\epsilon_1 - \epsilon_2)$ around R_{res} . The resonance region becomes narrower for more asymmetric systems. In strongly asymmetric cases it is traversed in a time too short for resonance polarization to occur, see subsect. 4.5.

3.2 Time Dependence.

In order to analyze the evolution of polarization in time it is tempting to employ a semiclassical picture in which the vector $\underline{R}$ connecting the colliding nuclei is considered a function of time.^{17), 10)} Polarization would then be described quantum mechanically as internal excitation caused by the time-dependent perturbation from the force field of the approaching nucleus. Such a model meets with the principal difficulty, however, that integrals of motion for the system as a whole, notably energy and angular momentum, can only be conserved on the average. As noted by Bertsch and Schaeffer this approximation breaks down completely for weak internal excitations.¹⁰⁾ The polarization effect essentially corresponds to a coherent superposition of many weak transitions into highly excited states with large angular momentum. In fact, for many of the components $|v'l'j'\Omega\rangle$ contained in a polarized state the excitation $\mathcal{E}^* = (2v' + l')\hbar\omega - \epsilon_{nlj}$ is so high ($15\hbar\omega$ and more, see figs. 5 and 6) that

at moderate collision energies the kinetic energy of relative motion in those channels will be negative. Hence they must disappear entirely outside the interaction region. In a semiclassical approach, however, these states would diminish the kinetic energy of relative motion only by $|A_{v' \ell' j'}^{n \ell j - \Omega}|^2 \epsilon^*$, and hence can have non-zero amplitudes even for $t \rightarrow \infty$. A similar argument can be made for the angular momentum transfer $j' - j$, which often is of the order of 10 - 20 units, but semiclassically would diminish the angular momentum of relative motion only by $|A|^2 (j' - j)$.

We conclude that the polarization effect can not be described within the approach sketched above. A full quantum mechanical treatment is required to investigate the dependence of polarization on time or, equivalently, on collision energy. Work on this is in progress and will be reported elsewhere. We shall here quote a simple result obtained in the framework of the two-state approximation, which in some cases can provide a rough estimate.

We envisage the colliding nuclei being held fixed at a distance R and study the dependence of polarization on time. The wave function describing the time evolution of the system can be approximated as a linear combination of the stationary states $\phi_{u, \ell}$, eq. (13), complemented with their time dependence $\exp(-i \epsilon_{u, \ell} \cdot t / \hbar)$.¹⁸⁾ Requiring the polarization to vanish initially, $\psi(t = 0) = \phi_1$, we get from eqs. (13) and (20)

$$\Psi(t) = \left\{ \cos \frac{\Delta}{2\hbar} t \cdot \phi_1 - i \frac{q}{2q\hbar_{12} \sqrt{1+q^2}} \sin \frac{\Delta}{2\hbar} t \cdot \phi_1 - i \frac{1}{2q\hbar_{12} \sqrt{1+q^2}} \sin \frac{\Delta}{2\hbar} t \cdot \phi_2 \right\} \exp(-i\epsilon t/\hbar) \quad (28)$$

Here $\epsilon = (\epsilon_u + \epsilon_\ell)/2$ is the average of the energies of the two states (cf. eq. (20)) and $\Delta = \epsilon_u - \epsilon_\ell$ is their difference. The polarization of the state eq. (28) is approximately

$$P(t) = \frac{1}{1+q^2} \sin^2 \frac{\Delta}{2\hbar} t \quad (29)$$

with a maximum $1/(1+q^2)$, which is close to four times the static limit for large q^2 , and approaches twice the static limit for a resonance $q^2 \rightarrow 0$ (cf. eq. (23)). For $q^2 \gg 1$ the time required to reach the static limit is

$$t_{\text{stat}} = \frac{\pi\hbar}{3\Delta} \approx \frac{7 \times 10^{-22}}{\Delta} \quad (\text{MeV, sec}) \quad (30)$$

For a resonance ($q^2 \rightarrow 0$) t_{stat} is 50 % larger than given by eq. (30).

Initially, the polarization increases quadratically with time.

$$P(t) = \left| \frac{\hbar_{12}}{\hbar} \right|^2 t^2 \quad \text{for } t \ll t_{\text{stat}} \quad (31)$$

Thus, the rate at which polarization starts to build up depends only upon the matrix element h_{12} , and not upon the energies of the states involved. It is obvious that a resonance will not be felt if $t \ll t_{\text{stat}}$. From eq. (31) one may calculate the time which is required to reach a certain "appreciable" polarization, 1 % say. Comparing this time with typical collision times will then give an indication as to whether such a polarization may or may not be expected to occur under the conditions of a particular experiment.

4. Results.

4.1 Choice of Parameters.

The results of the two-state approximation show that, in the adiabatic limit, the polarization of a specific single particle state at a given distance R depends upon (i) the single particle energies of other states of the system, notably of close lying states in the polarizing nucleus, and (ii) upon the strength of the matrix element h_{12} of the polarizing interaction. In a realistic calculation (for the adiabatic limit) shell model parameters therefore have to be adjusted such as to (i) yield a good fit to the single particle spectra of the individual nuclei, and (ii) give a realistic potential barrier between the nuclei, as it is this barrier upon which h_{12} sensitively depends. The latter requirement calls for realistic values of nuclear radius and diffuseness parameters.

It is not always possible to fit single particle spectra and rms-radii simultaneously within the framework of a local state-independent Woods-Saxon potential. We find the discrepancies to be largest for lighter nuclei. For these nuclei radii are usually underestimated if parameters are adjusted to fit single particle levels. Adjusting the nuclear radii to values as required from electron scattering data generally yields too high a level density near the Fermi energy.

In this work we investigate single particle polarization effects in the systems $^{208}\text{Pb} + ^{12}\text{C}$ and $^{40}\text{Ca} + ^{16}\text{O}$. In the former system single particle energies of proton states in ^{12}C are less relevant than the overall size of the ^{12}C - nucleus, because no ^{12}C - level is close to one of the levels which can be populated in proton transfer to ^{208}Pb .[#] Thus moderate inaccuracies in ^{12}C - levels do not lead to wrong resonance situations. In the system $^{40}\text{Ca} + ^{16}\text{O}$, however, relative positions are critical for both proton and neutron levels, because some of them are very close (see table 1). It is not known at present whether sufficient adiabaticity can be achieved in experiment to actually reach the strong resonance polarization which we obtain for some states (see subsect. 4.3). Thus it may seem more relevant to get more accuracy for the non-resonant polarization via a realistic choice of nuclear sizes, while sacrificing on single particle energies. Inaccurate relative positions of single particle levels, however, can give rise to erroneous resonance situations. The best strategy to reliably predict the polarization in the

[#] Note that in the grazing region ^{12}C - proton - levels are shifted up relative to those of ^{208}Pb by more than 8 MeV due to Coulomb forces.

adiabatic limit for a certain single particle state would be to adjust shell model parameters to give proper binding for the state in question as well as for neighboring levels in the polarizing nucleus; and this under the constraint of reproducing the rms-radii. Then only binding energies of more distant levels would be inaccurate, which is of minor significance.

Adjusting shell model parameters to best describe the polarization of individual states is outside the scope of the present work, however, which is of an exploratory nature. Where we have been unable to fit both single particle levels and nuclear radii simultaneously we have determined alternative parameter sets to fit one or the other.

In what follows we shall discuss shell model parameters for the nuclei investigated in this work (see tables 1 through 4). In ^{208}Pb we have investigated proton states only. The potential parameters were taken from the work of Batty and Greenlees;¹⁹⁾ they give a good fit to both single particle energies and electron scattering data (table 3). The parameters for ^{40}Ca as determined by Batty and Greenlees yield good results for electron scattering. Their calculated level densities are substantially too high near the Fermi energy, however, and the mean error of single particle energies is a large 1.5 MeV for protons and 1.2 MeV for neutrons. With the Batty/Greenlees -parameters as starting values we have performed a parameter search which resulted in a very good fit to single particle energies (parameter set I, tables 1 and 2). The rms-radius of the proton distribution is underestimated by 15 %, however.

We have then varied potential parameters to bring the radius in better agreement with the experimental value, while at the same time maintaining as good a fit as possible to occupied proton states. The resultant parameter set II (tables 1 and 2) gives 3 MeV too strong a binding for the (unfilled) $1f_{7/2}$ -proton level. This level is of considerable interest here because it can be populated in proton transfer to ^{40}Ca and, because of its weak binding, is a candidate for strong polarization effects. The transfer reaction $^{40}\text{Ca}(^{16}\text{O}, ^{15}\text{N})^{41}\text{Sc}$ populating this level could not be described with ordinary DWBA.⁷⁾ For neutrons potential parameters can be found which fit the single particle levels very well and yield an rms-radius close to what has been measured for protons (table 1).

For both ^{16}O and ^{12}C we have performed parameter searches, starting from standard values of $r_0 = r_s = 1.2$ fm and $a = a_s = 0.55$ fm. For ^{16}O we have obtained an excellent fit to single particle energies for both protons and neutrons (parameter set I, tables 1 and 2), at the expense of underestimating the rms-radius by as much as 20 %. Varying potential parameters to fit the radius while maintaining a fit to occupied levels yields much too strong binding for the sd - shell (parameter set II). In ^{12}C we required to fit the experimental position of the lp - proton levels and varied potential parameters to obtain as good a fit as possible to the sd - levels. We were unable to find parameters which would bind the $2s_{1/2}$ - state more strongly than the $1d_{5/2}$ - state, as required by experiment. Parameter set I was the

best as far as avoiding a strong overbinding of the $1d_{5/2}$ - level is concerned (table 3 and 4). The radius comes out 12 % below the experimental value. Varying parameters to increase the radius invariably led to a strong overbinding of the $1d_{5/2}$ - state. A best fit to both $1p$ - states and rms-radius would be intermediate between parameter sets II and III. Their differences are small and serve to illustrate the sensitivity of polarization effects on rather small changes in radius and diffuseness parameters (see fig. 13). Charge radii have been computed after Myers.²⁰⁾

4.2 General Results.

The general features of our results for single particle polarization agree with the calculations of ref. 3), which were based on the two-center model of Maruhn and Greiner.⁸⁾ The main points are as follows.

- (i) The most dramatic effect of the polarization consists in admixing components into single particle wave functions which are concentrated near the center of the polarizing nucleus (figs. 3 and 4).
- (ii) As a consequence of the nature of the admixtures, the polarization amplitudes generally vary smoothly and slowly as a function of ν', ℓ' (figs. 5 and 6). The exception are amplitudes with small ν', ℓ' , corresponding to a small rms-radius $\langle r^2 \rangle = (2\nu' + \ell' + 3/2)\hbar/m\omega$, which represent a distortion within

the polarized nucleus (this is the main effect of Coulomb forces, see subsect. 4.5).

(iii) "Persistent shape": As R decreases the general shape of the admixtures near the center of the polarizing nucleus persists, while they increase in amplitude. Hence the polarization amplitudes maintain their general pattern as R decreases, with all structures moving towards smaller ν', ℓ' , while increasing in amplitude.

(iv) For neutrons the polarization decreases very nearly exponentially with increasing R . For protons the dependence is exponentially at smaller R and then turns into $\sim R^{-4}$ approximately, due to the long range Coulomb force (see figs. 7, 8, 10).

(v) The polarization generally is stronger for more highly excited states. If two levels in the colliding nuclei are close, however, a strong resonance polarization occurs. Due to Coulomb forces, which had been neglected in previous work³⁾, resonances are commonplace for protons (figs. 10, 13).

(vi) The polarization is largest for $|\Omega| = 1/2$ and decreases fast with increasing $|\Omega|$ (see fig. 7). The reason for this is that states with smaller $|\Omega|$ have a larger amplitude near the 3-axis (symmetry axis) and hence "feel" the polarizing potential at larger R . In fact, as $|Y_{\ell}^m| \sim (1 - \cos^2 \vartheta)^{m/2}$ for small ϑ , states with smaller m have a larger overlap with the polarizing potential. For the same geometrical reason the $j_> = \ell + 1/2$ - states of a spin-orbit doublet are often more strongly polarized than their $j_< = \ell - 1/2$

counterparts, although the latter are more weakly bound. Namely, the ratio of $m_{<} = |\Omega| - 1/2$ to $m_{>} = |\Omega| + 1/2$ components in a state

$$|\ell s j \Omega\rangle = \sum_{m_{\mu}} C_{m_{\mu} \Omega}^{\ell s j} Y_{\ell}^m \chi_s^{\mu} \quad (32)$$

is $\sqrt{(\ell + |\Omega| + 1/2)/(\ell - |\Omega| + 1/2)} > 1$ for the $j_{>}$ and the reciprocal of this value for the $j_{<}$ -state.

Because polarization effects in peripheral collisions are ^{most} significant for states with $|\Omega| = 1/2$ the discussion in subsect. 4.3 - 4.5 will always refer to these if not stated otherwise.

(vii) Energy shifts due to polarization, i.e. deviations from the asymptotic single particle energies, generally are small (of the order of 100 keV in the grazing region); except of course for the large shifts induced by the monopole part of the Coulomb potential.

In what follows we shall discuss more specifically the polarization of states which can participate in one-nucleon transfer reactions induced by $^{40}\text{Ca} + ^{16}\text{O}$, or in the one-proton transfer reaction $^{208}\text{Pb}(^{12}\text{C}, ^{11}\text{B})^{209}\text{Bi}$, with a view on inferring observable effects. Full reaction calculations based on the polarized states discussed in this work are in progress and will be reported separately upon completion.

4.3 Neutron States in $^{40}\text{Ca} + ^{16}\text{O}$.

We have calculated polarized neutron states in ^{40}Ca in the range $8 \leq R \leq 13$ fm in steps of 1 fm. Polarized neutron states in ^{16}O were calculated at $R = 10$ fm only. Results are displayed in figs. 3, 5, 8, and 9.

At grazing distance $R = 10$ fm⁷⁾ results obtained with basis parameters (a) and (b) (cf. fig. 1) agreed to within 1 % for two-center parameters I (cf. tables 1 and 2), and to within 0.1 % for two-center parameters II.

From eq. (23) and point (i) of the subsequent discussion it follows that the polarization of neutron states should behave approximately as

$$P(R) \sim e^{-2\alpha R} / R^2 \quad (33)$$

at large R , where $\alpha = \sqrt{2m|\mathcal{E}|}/\hbar$ with $\mathcal{E} < 0$ the binding energy of the polarized state.[#] Fig. 8 shows that, with the exception of the resonating $2p_{1/2}$ - and $1d_{3/2}$ - states, the polarization indeed increases very nearly exponentially with decreasing R , the slight curvature reflecting the $1/R^2$ factor.

The parameter α (eq. (33)) as determined from the calculated polarization turns out to be somewhat larger than expected from the binding energy, and corresponds to an effective binding of $\approx \mathcal{E} - 2 \frac{\text{MeV}}{}$ on the average. This effect goes beyond the perturbation expression eq. (33) and reflects the enhance-

[#]In a self-consistent treatment the polarization of neutron states is expected, due to proton-neutron forces, to have an R -dependence very similar to that of proton states (see subsect. 4.4).

ment of the tail of the polarized state in the region between the nuclei due to the tail of the polarizing potential. The "saturation" of the polarization for the higher lying states at smaller R , which is evident from fig. 8, reflects higher order effects which go beyond the realm of validity of eqs. (23) and (33).

The following discussion will focus on the states which participate in neutron transfer reactions between ^{40}Ca and $^{16,17}\text{O}$, i.e. the $1p_{1/2}^-$, $1d_{5/2}^-$ and $2s_{1/2}^-$ states in oxygen and the fp - shell in calcium. In a particle-core picture, ignoring rearrangement effects, our calculation should apply to single particle states in nuclei differing from ^{40}Ca and ^{16}O respectively by not more than one neutron.

Experience from a previous calculation for the proton transfer reaction $^{40}\text{Ca}(^{16}\text{O}, ^{15}\text{N})^{41}\text{Sc}$ indicates that a polarization of only 1 % at grazing distance can result in a cross section for transfer which exceeds that of the 99 % unpolarized components.⁶⁾ The results as given in figs. 8 and 9 then indicate that in a "slow" collision polarization effects would be expected to occur for all available states in ^{40}Ca (fp - shell). While they would be rather weak for the $1f_{7/2}$ - ground state of ^{41}Ca they would be stronger for the first excited ($2p_{3/2}$) and very strong for the two highest lying states ($2p_{1/2}$ and $1f_{5/2}$). In an actual scattering experiment the polarization will be weaker than predicted for the adiabatic limit. This will be particularly so for the two highest states. Their very strong polarization in the adiabatic limit is due to an accidental near-resonance with the $1d_{5/2}^-$ and $2s_{1/2}^-$ states

in ^{17}O , which are equally strongly polarized. From eq. (30) it can be seen that the polarization time, i.e. the time required to reach the adiabatic limit of polarization, is particularly long for a resonance situation with small energy difference Δ . Another resonance occurs between the $lp_{1/2} - 0$ and the $ld_{3/2} - \text{Ca}$ levels.

We employ the two-state approximation as discussed in sect. 3 to investigate the chance that a very strong polarization may actually occur in experiment. The relevant quantities at grazing distance $R = 10$ fm are given in table 5.

For each state we list the asymptotic experimental and calculated single particle energies ϵ_{exp} and ϵ_{calc} , respectively, and the respective energy differences $\Delta\epsilon_{\text{exp}}$ and $\Delta\epsilon_{\text{calc}}$. $\Delta = \epsilon_u - \epsilon_l$ is the energy difference at grazing distance $R = 10$ fm as calculated in the two-center shell model (compare eq. (20)), and P is the static polarization computed from eq. (10) for the same distance.

The first step in the analysis is to establish the approximate validity of the two-state approximation for the indicated resonances. For every pair of resonating states we have calculated the squared overlap of the polarized state with the resonating asymptotic state in the polarizing nucleus (P_{res} in table 5). This was done from the expansions eqs. (6) and (8). Neglecting interference with other states and effects of non-orthogonality of states belonging to different nuclei this quantity may be identified as the "resonant part" of the polarization. The validity of this approximation can be seen (cf. table 5) from the fact that the squared overlaps are approximately symmetric between the resonating states. E.g., the admixture of the $lp_{1/2} - 0$ state into the polarized $ld_{3/2} - \text{Ca}$ state is nearly the same as the admixture of the $ld_{3/2} - \text{Ca}$ state into the polarized $lp_{1/2} - 0$ state. The same holds true for the other pairs. Using the calculated values for energy differences and squared overlaps we calculate the matrix element h_{12} from eq. (24). We want

to correct our calculated value for the resonance polarization for the (small) inaccuracy in single particle energies. This can be done with the help of eqs. (18) and (23) if we substitute for $\Delta\mathcal{E}$ the experimental value. We also require ΔV^R , which can be determined from eq. (25) once the sign of $\Delta\mathcal{E} + \Delta V^R$ is known. If at $R = 10$ fm $\Delta\mathcal{E} + \Delta V^R$ would differ in sign from its asymptotic value, which simply is $\Delta\mathcal{E}$, then it would have gone through zero for a certain $R_0 > 10$ fm. From eqs. (18) and (23) it follows that $P(R_0) = 50\%$. Inspection of fig. 8 shows that this does not occur for any of the resonating states, from which it follows that $\text{sgn}(\Delta\mathcal{E} + \Delta V^R(R=10)) = \text{sgn}\Delta\mathcal{E}$ in all cases.

Eq. (30) yields the time required to actually reach the adiabatic limit of resonance polarization.[#] Finally, from eq. (31) we calculate the time required to reach 1 % and 0.1 % polarization respectively at the grazing distance $R = 10$ fm.

The time for full resonance polarization is larger than typical interaction times, which will be $\lesssim 10^{-21}$ sec, especially when considering that only a fraction of the total interaction time do the nuclei spend near their grazing distance, where polarizing forces are stronger than at larger distance.*

[#] Observe the remark following eq. (30).

* The polarization would not necessarily build up if orbiting near grazing distance would occur, because a rotating nucleus-nucleus system may spend only a short time in an $|\Omega| = 1/2$ configuration in which (adiabatic) polarization is strong.

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Due to the weakness of the matrix element h_{12} the polarization of both the $1d_{3/2}$ - Ca and the $1p_{1/2}$ - O state will most likely not be observable experimentally. For the $2p_{1/2}$ - and $1f_{5/2}$ - Ca and the $1d_{5/2}$ - and $2s_{1/2}$ - O states, however, 1 % polarization is reached after only a few 10^{-22} seconds, so that strong observable effects are expected in slow collisions.

It should be noted that all these estimates are conservative, because they are based on too small potential wells, which we had to use to fit experimental single particle levels with the high accuracy required for a description of resonance situations (see tables 1 and 2). Because the radius of the oxygen nucleus is about 0.5 fm too small in our calculation the polarization $P(R)$ should rather apply to $\approx R + 0.5$ fm.

In summary, we expect strong polarization effects in the following reactions

We have not^{yet} calculated cross sections from the polarized states discussed here and therefore we can only give qualitative predictions at present for the expected effects. At "sufficiently low" collision energies, a few MeV above the Coulomb barrier, experimental cross sections will be larger than predicted by DWBA perhaps by a factor of 10, and will peak at a smaller angle than predicted by DWBA. These effects will diminish with increasing collision

energy, i.e. decreasing adiabaticity.

In this connection the results of Vigdor et al. for the neutron transfer reaction $^{48}\text{Ca}(^{16}_0, ^{17}_0^*)^{47}\text{Ca}$ are very interesting.²¹⁾ The angular distribution for the ground state transition at 56 MeV bombarding energy is well described by DWBA, as one would expect for a not too badly Q-mismatched neutron transfer between low lying states of doubly magic nuclei. The angular distribution for the transition to the $2s_{1/2}$ - state in $^{17}_0$ at 0.87 MeV excitation, however, is shifted forward by more than 10° with respect to a DWBA - prediction. This may be due to a resonance polarization from the (unfilled) $2p_{1/2}$ - state in ^{48}Ca , which is only 0.16 MeV away from the $2s_{1/2}$ - state in $^{17}_0$. If it is, the effect would be expected to be even more dramatic for a bombarding energy closer to the Coulomb barrier, which is near 40 MeV (lab - energy).

4.4 Proton States in $^{40}\text{Ca} + ^{16}_0$.

Polarized proton states were calculated over the same range of nucleus-nucleus distances as neutron states. The results are given in figs. 7, 10, and 11.

Specific Coulomb effects on the R-dependence of polarization are evident from figs. 7 and 10. Namely, the polarization of proton states at large distance is much stronger than that of neutron states, compare fig. 8, and it also decreases less sharply with distance. These effects will be analyzed in detail in subsect. 4.5. Here we shall discuss the polarization of those states which

are involved in proton pick up or stripping reactions in a $^{40}\text{Ca} + ^{16}\text{O}$ collision. These are the $1d_{3/2}$ - and $1f_{7/2}$ - states in ^{40}Ca and the $1p_{1/2}$ -, $1d_{5/2}$ -, and $2s_{1/2}$ - states in ^{16}O .

Polarization of the $1d_{3/2}$ - state in ^{40}Ca is rather weak. The $1f_{7/2}$ - state which is populated in the reaction $^{40}\text{Ca}(^{16}\text{O}, ^{15}\text{N})^{41}\text{Sc}$ has been investigated previously, because a systematic discrepancy between experimental data and DWBA calculations indicated that it may be appreciably polarized.^{6), 7)} On the basis of the two-center model of Maruhn and Greiner we had calculated the polarization of this state to be 20.1 % at grazing distance $R = 10$ fm.³⁾ In that calculation shell model parameters had been adjusted to the rms-radii, which resulted in too high a density of ^{16}O - states near the $1f_{7/2}$ - Ca state, hence an overestimate of the polarization. The transfer cross section calculated for 20 % polarization was 43 times larger than the measured value.⁶⁾ It is interesting to note that with the parameter set II, which also is adjusted to rms-radii and overpredicts the density of ^{16}O - states, we now get $P^{1f_{7/2}}(R = 10) = 21.5$ %. With parameter set I, however, which fits single particle levels very well, we obtain a polarization of only 0.4 %. This is definitely an underestimate. Note that due to the underestimate of radii for parameters I the surfaces of the nuclei are 1.0 fm further apart at a given distance R than they should be. Hence realistically all curves in fig. 10 should be shifted to the right by approximately 1 fm,[#] and the polarization

Strictly speaking this is true only in the region where the nuclear part of the polarization dominates over the Coulomb part, as the latter is properly described.

of the $1f_{7/2}$ - state at grazing distance should be close to 2 %. Apart from effects of structural details the transfer cross section due to polarized components is approximately proportional to the polarization. A value of 2 % for the latter then will essentially resolve the previous overestimate of absolute cross section.

The $1d_{5/2}$ - and $2s_{1/2}$ - states in ^{16}O , which can be populated in proton pick up reactions, are very strongly polarized because of their weak binding. Thus strong effects are expected in the reaction $^{40}\text{Ca}(^{16}\text{O}, ^{17}\text{F}_{1d_{5/2}, 2s_{1/2}})^{39}\text{K}$ at moderate collision energies. The absolute cross section will be strongly enhanced over the small value which would be expected because of the large Q - mismatch.

Polarization of the $1p_{1/2}$ - state in ^{16}O is mostly due to a near-resonance with the neighbouring $2s_{1/2}$ - state in calcium. Asymptotically, the $1p_{1/2}$ - 0 state is 1.2 MeV more strongly bound, but because of the Coulomb shift on approach the levels cross at $R = 13.9$ fm. We use the two-state approximation in an analogous way as in subsect. 4.3 to analyze the time required for a certain (small) polarization to be reached. At a grazing distance $R = 10$ fm (which because of the underestimate of radii corresponds to $R \approx 9$ fm in fig. 10) we find $t_{1\%} = 6 \times 10^{-22}$ sec and $t_{0.1\%} = 2 \times 10^{-22}$ sec. Thus a certain albeit small polarization for this state can not be ruled out.

The level crossing at $R = 13.9$ fm (see eq. (27)), however, will be passed unnoticed. At this large distance the polarizing matrix element h_{12} is so weak that 8×10^{-21} sec would be required to reach a polarization of only 0.1 %.

4.5 Proton States in $^{208}\text{Pb} + ^{12}\text{C}$.

For the system $^{208}\text{Pb} + ^{12}\text{C}$ we have evaluated the polarization of proton states only. The investigation was motivated by the observation of systematic discrepancies between experimental data and DWBA - predictions for the reaction $^{208}\text{Pb}(^{12}\text{C}, ^{11}\text{B})^{209}\text{Pb}$.^{22) 23)} The nature of the discrepancies and their dependence upon physical parameters suggest that they may reflect polarization effects. Namely, measured angular distributions peak forward of what DWBA predicts by up to 10° , with discrepancies increasing with increasing final state excitation and decreasing with increasing bombarding energy.

Results of our calculations are presented in figs. 4, 6, 12, and 13, and in table 6. It is seen that the polarization generally is not very strong. At the energies at which the experiments have been performed ($78 \text{ MeV} \leq E_{\text{lab}} \leq 116 \text{ MeV}$) the grazing distance in the system $^{208}\text{Pb} + ^{12}\text{C}$ is close to $R_{\text{gr}} = 12.5 \text{ fm}$.²⁴⁾ At this distance the polarization of states which are populated in the $^{208}\text{Pb}(^{12}\text{C}, ^{11}\text{B})^{209}\text{Bi}^*$ - reaction is close to 1 %. Figs. 4 and 6 show that the structure of the polarization is little affected by shell model parameters, with the strength increasing with increasing ^{12}C - size ($P_{\text{I}} < P_{\text{II}} < P_{\text{III}}$). Note that the rather moderate increase in radius and diffuseness parameters from set II to set III (table 4) more than doubles the polarization.

In what follows we shall first discuss specific Coulomb effects, and then determine an approximation to the R - dependence of polarization which is useful in reaction calculations.

Coulomb polarization can easily be distinguished from nuclear polarization. Consider a perturbation expression for the polarization amplitudes as given in ref. 3)

$$A_{\nu' \ell' j'}^{\nu \ell j \Omega}(R) \simeq \frac{\langle \nu' \ell' j' \Omega | V_{\text{pol}}^R | \nu \ell j \Omega \rangle}{\epsilon_{\nu \ell j} - \epsilon_{\nu' \ell' j'}} \quad (34)$$

The nuclear part of the polarizing potential V_{pol}^R is the shell model potential V_B^R of the approaching nucleus B. As V_B^R has a short range of the order of the nuclear radius r_B , contributions to the matrix element in eq. (34) are essentially confined to a sphere with radius r_B at a distance R from the polarized nucleus. Significant contributions appear only if (a) the tail of the asymptotic state $|\nu \ell j \Omega\rangle^\#$ reaches V_B^R and if (b) the state $|\nu' \ell' j' \Omega\rangle$ has a large amplitude at a distance R from the center of the polarized nucleus, i.e. if its rms-radius $\sqrt{\langle r^2 \rangle} = \sqrt{(2\nu' + \ell' + 3/2)\hbar/m\omega}$ is comparable to R . From condition (a) it follows that the nuclear part of the polarization is short ranged with an approximately exponential R -dependence, and that it decreases fast with increasing $|\Omega|$; whereas from (b) it follows that mainly states with a large phonon number $N' = 2\nu' + \ell'$ will be admixed.

Coulomb polarization is caused by the interaction

$$V_{\text{Coul}}^R = \frac{z_B e^2}{|R - r|} = z_B e^2 \sqrt{4\pi} \sum_{\ell=0}^{\infty} \frac{r^\ell}{R^{\ell+1}} \frac{Y_\ell^0(\vartheta_\varphi)}{\sqrt{2\ell+1}} \quad (35)$$

Here the spherical oscillator state $|\nu \ell j \Omega\rangle$ is the main component of the Woods-Saxon state $|n \ell j \Omega\rangle$.

Here $\underline{r}$ is the position vector of the polarized nucleon, and the expansion has been written for the case $r < R$. In fact contributions of the Coulomb potential to the matrix element in eq. (34) are confined to the region where $|\nu \ell j \Omega\rangle$ has a large amplitude, i.e. inside the polarized nucleus. The rms-radius of admixed states differs little from that of the asymptotic state. E.g. the strongest component of the Coulomb field is the dipole ($\ell = 1$), which is proportional to $1/R^2$ and admixes the two states $|\nu \ell+1 j+1 \Omega\rangle$. The Coulomb polarization thus essentially causes only a distortion of the asymptotic state within the polarized nucleus. This is of much less significance for transfer reactions than the admixture of components near the polarizing nucleus, as induced by nuclear forces.

The above considerations are borne out by our calculations. More specifically we find:

(i) All polarized states have an appreciable admixture of components

$|\nu \ell+1 j+1 \Omega\rangle$. These two states typically account for a polarization of $\approx 0.1\%$ near grazing distance independent of the binding energy of the polarized state.

(ii) For the deeply bound states ($\epsilon < -10$ MeV) the polarization in the grazing region is almost entirely due to Coulomb forces. We have verified this directly by calculating the polarization with the nuclear field of the ^{12}C - nucleus omitted. The results agree to within a few % with those including nuclear forces. About two thirds of the total Coulomb polarization is

due to first order effects of the dipole field, i.e. to admixtures of the two states $|v \ell+1 j+1 \Omega\rangle$.

(iii) The R-dependence of polarization for the deeply bound states follows closely a $1/R^4$ - pattern, which would be expected from first order effects of the electric dipole field only.

(iv) At larger distance ($R \geq 14$ fm) the polarization with nuclear forces omitted ($U = W = 0$ for ^{12}C in eq. (2)) actually is stronger for most states than that with nuclear forces included. This reflects a destructive interference of nuclear and Coulomb forces. The tail of the polarizing nuclear field has a dipole moment about the center of the polarized nucleus which partly cancels the electric dipole field.

(v) Coulomb polarization has a long range and therefore also decreases much more slowly with increasing $|\Omega|$ than nuclear polarization. At grazing distance $R = 12.5$ fm the dipole part of the Coulomb polarization is about 10 % of the nuclear polarization for the $|\Omega| = 1/2$ - components of the ^{208}Pb - proton states above the Fermi level. For the $|\Omega| = 3/2$ - components of these levels the dipole Coulomb polarization is typically 50 % of the total polarization, and for the $|\Omega| = 5/2$ - components it is typically > 70 % (all at $R = 12.5$ fm). If the dipole part of the Coulomb polarization is omitted the polarization of $|\Omega| = 3/2$ - components is typically 10 % that of $|\Omega| = 1/2$ - components.

In order to calculate a transfer reaction with polarized single particle states in principle transition amplitudes have to be evaluated separately

for each component $|v'l'j'\Omega\rangle$ of the polarized state. Even if a cut-off is made e.g. at $|A_{v'l'j'}^{nlj\Omega}(R)| \leq 0.001$ this would be a prohibitively lengthy calculation, and further approximations are necessary. As discussed in ref. 3) the "persistent shape" feature of the polarization admixtures (see also subsect. 4.2) allows to approximately take the R - dependence of the polarization amplitudes $A_{v'l'j'}^{nlj\Omega}(R)$ to be independent of $v'l'j'$. The polarized part of the state $|Rnlj\Omega\rangle$ as given by eq. (11) is approximated by

$$|\overline{Rnlj\Omega}\rangle \approx f_{R_0}^{nlj\Omega}(R) |\overline{R_0nlj\Omega}\rangle \quad (36)$$

From normalization requirements the unpolarized part of $|Rnlj\Omega\rangle$ is then

$$\sqrt{1 - f_{R_0}^{nlj\Omega}(R) P^{nlj\Omega}(R_0)} |nlj\Omega\rangle \quad (37)$$

Eq. (36) allows to perform the v' - summation in eq. (11) before transition amplitudes are evaluated, which reduces the numerical work involved in a reaction calculation by a factor of 5 - 10.

R_0 is to be chosen close to grazing distance, and $f_{R_0}^{nlj\Omega}(R_0) = 1$. In ref. 3) we had neglected Coulomb forces, which made it possible to determine $f(R)$ directly from the polarization, requiring that

$$\langle \overline{Rnlj\Omega} | \overline{Rnlj\Omega} \rangle = P^{nlj\Omega}(R) = f_{R_0}^{nlj\Omega 2}(R) P^{nlj\Omega}(R_0) \quad (38)$$

Eq. (38) would give a bad approximation here, because the polarization from

Coulomb forces admixes different states than that coming from nuclear forces, and with a different R - dependence. The "important" part of the polarization which will strongly affect transfer form factors is due to nuclear forces. Thus $f(R)$ has to be determined such that eq. (36) gives as good an approximation as possible to the nuclear part of the polarization. The Coulomb part will then be overestimated for $R < R_0$ and underestimated for $R > R_0$, which will not lead to serious errors for reactions confined to a small annulus around R_0 . f can be determined from an analysis of extrema of the amplitudes $A_{\nu' \ell' j'}^{n \ell j \Omega}(R)$ as a function of $\nu' \ell'$ at different R , as was also discussed in ref. 3). We find that

$$f_{R_0}^{n \ell j 1/2}(R) = \exp(-\gamma^{n \ell j 1/2}(R-R_0)) \quad (39)$$

to a good approximation. Values for γ for the states which can be populated in proton transfer to ^{208}Pb are given in table 6.

Except for the $1i_{13/2}$ - state both range and strength of the polarization increase with decreasing binding, which supports our view that previously observed discrepancies between DWBA - predictions and experimental data may have resulted from a neglect of these polarization effects.

Finally we shall consider the likelihood that the initial proton state, $1p_{3/2}$ in ^{12}C , will be polarized from the approaching ^{208}Pb - nucleus. Of particular

interest in this connection is a level crossing with the $2d_{3/2}$ - state in Pb near $R = 14$ fm. We use the two-state approximation in a way as discussed in subsect. 4.3 to evaluate polarization times. We have calculated the overlap of the polarized $1p_{3/2}$ - ^{12}C state with the $2d_{3/2}$ - Pb state, from which with eq. (24) the matrix element h_{12} of the polarizing interaction can be determined. We find $h_{12}(R = 14 \text{ fm}) = 0.036 \text{ MeV}$ and $h_{12}(R = 12.5 \text{ fm}) = 0.167 \text{ MeV}$, corresponding to times for 1 % polarization of 18 and 4×10^{-22} sec, respectively. Thus, while the level crossing will be traversed in a time much too short to build up any significant polarization there, a polarization of the order of 1 % may be reached near grazing distance.

5. Conclusion.

In this work we have investigated the polarization of single particle states within a realistic two-center model. Systematic deficiencies in conventional DWBA-analysis and simple estimates of polarization times suggest that it will be between such polarized states that transfer reactions in "slow" nucleus-nucleus collisions proceed.

We have performed an extensive analysis of polarized single particle states for the system $^{40}\text{Ca} + ^{16}\text{O}$, and of proton states for the system $^{208}\text{Pb} + ^{12}\text{C}$. This confirmed the trends observed in a previous more schematic two-center calculation, while emphasizing important differences in detail due to a

sensitive dependence upon shell model parameters and single particle energies. A number of more weakly bound states are found to be strongly polarized. As a consequence cross sections for nucleon transfer to these states will be enhanced over "unpolarized" DWBA - predictions, and angular distributions will be shifted forward. Reaction calculations to quantitatively evaluate these effects are in progress.

In general single particle polarization consists of an admixture of a large number of excited (asymptotic) states, each of which contributes weakly and coherently. In the special case, however, when the energies of two single particle states in the colliding nuclei are close a very strong "resonance polarization" develops, which essentially mixes just these two states.

Resonance polarization, which so far is a hypothetical effect, is predicted e.g. in the system $^{40}\text{Ca} + ^{16}\text{O}$ for neutron transfer populating either the $2p_{1/2}$ - state in ^{40}Ca or the $1d_{5/2}$ - state in ^{16}O .

An important question concerning polarization effects is their dependence upon collision energy. Can sufficient adiabaticity be achieved in experiment so that polarization reaches the static nuclei limit, as implied by the two-center shell model? Our estimate of the time evolution of polarization within a simple two-state approximation indicates that the answer to this question is probably somewhat less than "yes". Adiabaticity requirements are particularly severe for resonance polarization.

Resonances occur frequently for proton states in asymmetric collisions, due to large level shifts introduced by the Coulomb force. Whether this will give rise to observable effects is uncertain at present, because resonance conditions are confined to a small region in space which may be traversed in too short a time to allow for resonance polarization to build up.

Further work is needed to clarify the dynamical interplay between relative motion and single particle polarization. In this connection it would be very interesting to experimentally search for effects of adiabatic polarization by performing transfer reaction experiments at energies close to the Coulomb barrier. Our analysis suggests a number of promising examples.

In a sufficiently slow collision not only the single particle states but also the shell model potentials will adjust to the relative motion. In such a situation polarized single particle states would have to be calculated from a self-consistent two-center potential, e.g. from the constrained Hartree-Fock method.

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Appendix: Numerical Computation of z - Matrix Elements.

In this appendix we discuss the method used to numerically compute matrix elements in the two-center oscillator basis. As the form of these matrix elements has been given elsewhere we refer the interested reader to the literature for more details.^{12), 11)}

For the z -integration we require matrix elements of the form

$$M_{n'_2, n_2} = \int_{\pm} dz \exp\left\{-\frac{m\omega_z}{\hbar}(z-z_0)^2\right\} u_{n'_2}(z) M(z) u_{n_2}(z) \quad (A1)$$

where the integral runs over either the positive or negative semi-axis, as indicated by the $(\pm)$ -sign under the integral. The $u_{n_2}(z)$ are the modified parabolic cylinder functions defined by

$$\varphi_{n_2}(z) = \sqrt{\frac{m\omega_z}{\hbar}} \exp\left\{-\frac{m\omega_z}{2\hbar}(z-z_0)^2\right\} u_{n_2}(z) \quad (A2)$$

where φ_{n_2} is a normalized eigenstate of the one-dimensional two-center oscillator, and z_0 and ω_z are different for the positive and negative z -axis in the asymmetric case. For more details on the two-center oscillator eigenstates see refs. 8) and 3).

To evaluate the above integral we have employed Gauss' method by constructing an orthonormal set of polynomials $p_n(z, z_0)$ with respect to the weight function $w(z, z_0)$:

$$\int_z dz W(z, z_0) P_n(z, z_0) P_m(z, z_0) = \delta_{nm} \quad (A3)$$

$$(n, m = 0, 1, 2, \dots)$$

with

$$W(z, z_0) = \exp \left\{ -\frac{m\omega_z}{\hbar} (z - z_0)^2 \right\} \quad (A4)$$

For an N-point integration routine the integration points z_i are the zeros of P_N

$$P_N(z_i) = 0 ; \quad i = 1, \dots, N \quad (A5)$$

and the weights are given by³³⁾

$$w_i = \left(\frac{d^N P_N(z)}{dz^N} \right)_{z=0} \times \left[\left(\frac{d^{N-1} P_{N-1}(z)}{dz^{N-1}} \right)_{z=0} \left(\frac{d P_N(z)}{dz} \right)_{z=z_i} P_{N-1}(z_i) N \right]^{-1} \quad (A6)$$

The integral in eq. (A1) then becomes

$$M_{n_1 n_2} = \sum_{i=1}^N w_i M(z_i) u_{n_1}(z_i) u_{n_2}(z_i) \quad (A7)$$

Matrix elements involving derivatives may be handled in the same way. Here one needs to express the derivative of the two-center oscillator wave function in terms of modified parabolic cylinder functions:

$$\frac{d \varphi_{n_z}(z)}{dz} = \sqrt{\frac{m\omega_z}{\hbar}} \exp \left\{ -\frac{m\omega_z}{2\hbar} (z - z_0)^2 \right\} \tilde{u}_{n_z}(z) \quad (A8)$$

where

$$\tilde{u}_{n_z} = \frac{m\omega_z}{\hbar} \left[\sqrt{\frac{n_z}{2}} u_{n_z-1} - \sqrt{\frac{n_z+1}{2}} u_{n_z+1} \right] \quad (A9)$$

Note that in the asymmetric case separate sets of polynomials must be constructed for the positive and negative z -axis.

We remark that the modified parabolic cylinder functions are not in general polynomials but approach the modified Hermite polynomials as $z_0 \rightarrow \pm\infty$. The weights w_i and zeros z_i then approach those of a Gauss-Hermite integration. For finite separation z_0 the integration points are distributed about z_0 , being most dense and having the largest weights in the neighborhood of z_0 .

Table 2: Woods-Saxon potential parameters for ^{40}Ca and ^{16}O .

	^{40}Ca				^{16}O			
	protons		neutrons		protons		neutrons	
	I	II	I	II	I	II	I	II
U (MeV)	-70.8	-55.0	-52.7	-51.4	-79.1	-51.3	-74.9	-49.3
r_0 (fm)	1.05	1.27	1.26	1.28	0.95	1.36	0.97	1.38
a (fm)	0.67	0.67	0.76	0.76	0.585	0.60	0.60	0.60
W (MeV)	30.0	30.0	27.2	27.2	33.4	27.0	32.2	26.0
r_s (fm)	1.05	1.05	1.37	1.37	1.10	0.90	1.10	0.90
a_s (fm)	0.55	0.55	0.55	0.55	0.55	0.40	0.55	0.40
R_{ch} (fm)	3.83	3.83			2.77	2.77		

For a definition of the parameters see eqs. (2) - (4)

Table 3: Single particle energies and rms-radii for protons in ^{208}Pb and ^{12}C .

^{208}Pb			^{12}C				
state	exp	calc	state	exp	calc		
					I	II	III
2d 5/2	-9.70 ^a	-9.19	1s 1/2		-29.61	-26.37	-25.73
1h 11/2	-9.37 ^a	-9.12	1p 3/2	-15.9 ^c	-15.94	-15.75	-15.99
2d 3/2	-8.38 ^a	-7.89	1p 1/2	-1.94 ^d	-2.02	-1.92	-1.98
3s 1/2	-8.03 ^a	-7.08	2s 1/2	+0.43 ^c	+1.63	+0.97	+0.21
1h 9/2	-3.77 ^a	-3.54	1d 5/2	+1.2 ^e	-0.21	-4.19	-5.29
2f 7/2	-2.87 ^a	-2.75					
1i 13/2	-2.16 ^a	-2.30					
2f 5/2	-0.95 ^b	-0.77					
3p 3/2	-0.65 ^b	-0.08					
3p 1/2	-0.13 ^b	+0.61					
<hr/>			<hr/>				
ΔE		0.496			0.927	2.711	3.247
$\sqrt{\langle r^2 \rangle}$	5.42 ^f	5.56		2.42 ^g	2.13	2.39	2.45

(a) ref. 27); (b) ref. 23); (c) ref. 30); (d) ref. 31);

(e) estimated from 1d 5/2 - neutron level; (f) ref. 32);

(g) ref. 25).

The calculations are for shell model parameters as listed in table 4.

 ΔE is the mean deviation of single particle energies. (energies in MeV, radii in fm)

Table 4: Woods-Saxon potential parameters for protons in ^{208}Pb and ^{12}C .

	^{208}Pb	^{12}C		
		I	II	III
U (MeV)	-59.21	-65.5	-53.5	-53.5
r_0 (fm)	1.286	1.06	1.25	1.27
a (fm)	0.762	0.60	0.60	0.70
W (MeV)	23.88	44.0	70.0	74.0
r_s (fm)	0.88	0.80	1.10	1.10
a_s (fm)	0.60	0.20	0.55	0.55
R_{ch} (fm)	6.69	2.50	2.50	2.50

The parameters are defined in eqs. (2) - (4); the Pb - parameters were taken from ref. 19).

Table 5: Polarization resonances for neutrons in the system $^{40}\text{Ca} + ^{16}\text{O}$.

	<u>Ca</u> <u>O</u>		<u>Ca</u> <u>O</u>		<u>Ca</u> <u>O</u>	
states	1d 3/2	1p 1/2	2p 1/2	1d 5/2	1f 5/2	2s 1/2
ε_{exp}	-15.6	-15.7	-4.3	-4.14	-2.9	-3.27
$\varepsilon_{\text{calc}}$	-15.71	-15.74	-4.34	-4.14	-2.97	-3.31
$\Delta\varepsilon_{\text{exp}}$	0.1		-0.16		0.37	
$\Delta\varepsilon_{\text{calc}}$	0.03		-0.20		0.34	
Δ	0.126		0.730		0.556	
P	10.61	15.32	48.34	53.47	28.99	35.51
P_{res}	10.21	10.31	41.48	39.26	14.23	20.09
$ h_{12} $	0.0382	0.0383	0.360	0.356	0.194	0.223
ΔV	+0.07	-0.07	+0.08	-0.04	+0.06	+0.01
$P_{\text{res}}^{\text{corr}}$	4.39	4.41	44.48	41.69	12.88	18.60
t_{stat}	8.3×10^{-21}		1.4×10^{-21}		1.9×10^{-21}	
$t_{1\%}$	1.7×10^{-21}		1.8×10^{-22}		3.4×10^{-22}	
$t_{0.1\%}$	5.4×10^{-22}		0.6×10^{-22}		1.1×10^{-22}	

The table is explained in the text, subsect. 4.3. (energies in MeV, polarization in %, times in seconds)

Table 6: Parameters for exponential R-dependence of nuclear polarization
in $^{208}\text{Pb} + ^{12}\text{C}$.

state	γ	
	I	II
1h 9/2	1.12 ± 0.02	1.12 ± 0.02
2f 7/2	0.99 ± 0.04	0.95 ± 0.04
1i 13/2	0.98 ± 0.05	1.00 ± 0.02
2f 5/2	0.98 ± 0.02	0.98 ± 0.02
3p 3/2	0.88 ± 0.02	0.88 ± 0.02
3p 1/2	0.77 ± 0.02	0.82 ± 0.02

The table is explained in the text, subsect. 4.5. Note that the parameters γ are determined within narrow limits. The values are nearly equal for shell model parameters I and II (cf. table 4), and decrease systematically with decreasing binding of the polarized state (cf. table 3).

Figure Captions.Fig. 1: Two-center potentials for neutron states in $^{40}\text{Ca} + ^{16}\text{O}$.

The potentials are shown along the z-axis. The Ca-potential is on the left, and the O-potential is centered at grazing distance $R = 10$ fm to the right. The solid curve is a two-center Woods-Saxon potential (parameter set I, see table 2). The dashed and dotted curves are for parameter sets (a) and (b) of the two-center oscillator, respectively, which is used in the diagonalization. ((a): $\hbar\omega_{\text{Ca}} = 10.90$ MeV,²⁵⁾ $\hbar\omega_{\text{O}} = 13.00$ MeV;²⁶⁾ (b): $\hbar\omega_{\text{Ca}} = 13.30$ MeV, $\hbar\omega_{\text{O}} = 16.08$ MeV).

A number of (asymptotic) level positions referring to the different potentials are indicated. In the case of the oscillator potentials the levels are with inclusion of the usual $\ell\cdot\zeta$ -force $-2\kappa\hbar\omega\ell\cdot\zeta$, where $\kappa = 0.08$. The zero of the oscillator potentials has been adjusted such that the average position of the indicated ^{40}Ca - levels agrees with that in the Woods-Saxon potential.

Fig. 2: Two-center potentials for proton states in $^{208}\text{Pb} + ^{12}\text{C}$.

The potentials are shown along a line parallel to the z-axis, at a distance $g = 0.5832$ fm from the z-axis. The Pb-potential is on the left, and the C-potential is centered at a distance $R = 13$ fm (close to grazing) to the right. The solid curve is the superposition of a two-center Woods-Saxon potential and a Coulomb potential (parameter set II, table 4). The dashed and dotted curves are for parameter sets (a) and (b) of the basis oscillator, respectively

((a): $\hbar\omega_{\text{Pb}} = 6.92$ MeV, $\hbar\omega_{\text{C}} = 15.36$ MeV; ²⁵⁾ (b): $\hbar\omega_{\text{Pb}} = 9.28$ MeV, $\hbar\omega_{\text{C}} = 15.36$ MeV). The zero of the two-center oscillator potentials has been adjusted such that the average position of ^{208}Pb - levels near the Fermi energy agrees with that in the Woods-Saxon potential.

Fig. 3: Probability density of resonating neutron states in $^{40}\text{Ca} + ^{16}\text{O}$.

Shown are contour maps of $|\psi|^2$ in the z - plane for the polarized neutron states $2p_{1/2\ 1/2}$ in Ca (left) and $1d_{5/2\ 1/2}$ in O (right), which have neighbouring energies and hence are mutually strongly polarized (cf. table 5). The states have been calculated for shell model parameters I (cf. table 2). The polarization is strong ($\approx 10\%$) already at the large distance $R = 13$ fm (upper figures), with the structure of the admixtures resembling that of the resonating state. Note that the polarized components are centered at the polarizing nucleus. At grazing distance $R = 10$ fm (lower figures) the states are smeared out completely over both nuclei (polarization $\approx 50\%$).

Fig. 4: Probability density of the $2f_{5/2\ 1/2}$ proton state in ^{208}Pb , polarized from ^{12}C .

The polarized $2f_{5/2\ 1/2}$ -state is shown with the polarizing ^{12}C - nucleus at grazing distance $R = 13$ fm for different shell model parameters (I - upper, II - middle, III - lower, cf. table 4). The admixtures are close to the center of the polarizing nucleus. With increasing size of the ^{12}C -potential ($\text{I} \rightarrow \text{II} \rightarrow \text{III}$) the polarized components are enhanced, with their structure little affected. A distortion of the state within the Pb-nucleus is evident for

parameters II and becomes more pronounced for parameters III.

Fig. 5: Polarization amplitudes for resonating neutron states in $^{40}\text{Ca} + ^{16}\text{O}$.

Polarization amplitudes (cf. eq. (6)) are shown for the neutron states $2p_{1/2\ 1/2}$ in Ca and $1d_{5/2\ 1/2}$ in O at grazing distance $R = 10$ fm. In order to display the $A_{\nu'\ell'j'}^{n\ell j\Omega}$ as contour maps we interpolate between neighbouring values of $\nu'\ell'$. Units along contours are 10^{-2} . The regular dependence of the A's upon $\nu'\ell'$ is striking. Similar structures are apparent in both maps, reflecting similar structures in spatial wave functions (fig. 3).

Fig. 6: Polarization amplitudes for the $2f_{5/2\ 1/2}$ - proton state in ^{208}Pb , polarized from ^{12}C .

Depicted is the same state as in fig. 4 (shell model parameters I - left, II - middle, III - right, cf. table 4). Units along contours are 10^{-2} . The structures are seen to be similar for the different ^{12}C - potential parameters, with their strength increasing with increasing size of the ^{12}C - potential well ($\text{I} \rightarrow \text{II} \rightarrow \text{III}$). The peculiar structure appearing for $\ell' = 3$, $j' = \ell' - 1/2$ reflects the expansion of the ^{208}Pb - Woods-Saxon state on a harmonic oscillator basis.

Fig. 7: Polarization of the $1f_{7/2}$ - states in ^{40}Ca due to ^{16}O .

For the $1f_{7/2}$ - states in ^{40}Ca (neutrons and protons) the polarization of the different $|\Omega|$ - components as computed from eq. (10) is shown as a function of the distance R of the polarizing ^{16}O - nucleus. The polarization is smaller for larger $|\Omega|$. For neutrons (left) the radial dependence is similar for the

different $|\Omega|$ - values, and approximately $P^{|\Omega|=1/2} \approx 4 \times P^{|\Omega|=3/2} \approx 70 \times P^{|\Omega|=5/2} \approx 10^4 \times P^{|\Omega|=7/2}$. The slow decrease of the polarization for the proton states at large R is due to the long range Coulomb field of the $^{16}_0$ - nucleus.

Fig. 8: Polarization of neutron states in ^{40}Ca due to $^{16}_0$.

The polarization as computed from eq. (10) for neutron states ($|\Omega| = 1/2$ - components) which asymptotically belong to ^{40}Ca is shown as a function of the distance of the polarizing $^{16}_0$ - potential (shell model parameters I, table 2). It is evident that for higher lying states the polarization generally is stronger and falls off less steeply with increasing R . The strong polarization of the deeply bound $1d_{3/2}$ - and $2s_{1/2}$ - states is due to a near-resonance with the $1p_{1/2}$ - state in $^{16}_0$. Observe the logarithmic scale, which makes the very strong resonance polarization of the $2p_{1/2}$ - state, and to a lesser extent the $1f_{5/2}$ - state, appear less dramatic. The levelling off of the polarization for the $1s$ - and $1p$ - states for $R > 11$ fm reflects limitations of numerical accuracy; the actual behaviour should follow the extrapolated curves (dashed lines). Where resonance effects are absent the polarization increases very nearly exponentially with decreasing R .

Fig. 9: Polarization of neutron states in $^{40}\text{Ca} + ^{16}_0$ at grazing distance $R = 10$ fm.

Polarization strengths as calculated from eq. (10) are shown as a function

of single particle energy for shell model parameters I (cf. table 2). The full line lengths show polarization of the $|\Omega| = 1/2$ - components, with polarization of higher $|\Omega|$ - components indicated by the end points of alternating solid and dashed lines. In all cases $P^{|\Omega|=1/2} > P^{|\Omega|=3/2} > P^{|\Omega|=5/2} > P^{|\Omega|=7/2}$. It is seen that the polarization generally increases with decreasing binding energy. Superimposed on this trend are resonance effects for a number of states (cf. table 5).

Fig. 10: Polarization of proton states in ^{40}Ca due to ^{16}O .

The same as fig. 8 except for protons (shell model parameters I, cf. table 2). Because of Coulomb effects the decrease of polarization with increasing R is much slower than for neutrons. The wiggles in the curves for the $1p$ - states come from an interference between Coulomb and nuclear polarization. The maximum in the polarization of the (weakly unbound) $2p_{3/2}$ - state near $R = 12$ fm is caused by a resonance (level crossing) with the $1d_{5/2}$ - state in ^{16}O . Another resonance between the $2s_{1/2}$ - state in Ca and the $1p_{1/2}$ - state in 0 occurs at $R = 14$ fm. The approach of this is visible near $R = 13$ fm, where the curve for the $2s_{1/2}$ - state bends slightly. Because of the vicinity of the $1p_{1/2} - 0$ level the $2s_{1/2}$ - state is more strongly polarized than the much more weakly bound $1f_{7/2}$ - state. Otherwise, polarization increases with decreasing binding.

Fig. 11: Polarization of proton states in $^{40}\text{Ca} + ^{16}\text{O}$ at grazing distance

$R = 10$ fm.

The same as fig. 9 except for protons, shell model parameters I. Because

of Coulomb effects the polarization decreases less rapidly with increasing binding and increasing $|\Omega|$ than it did for neutrons, cf. fig. 9 (observe the different scales!).

Fig. 12: Polarization of proton states in ^{208}Pb due to ^{12}C .

The polarization of (unfilled) proton states ($|\Omega| = 1/2$ - components) which asymptotically belong to ^{208}Pb is shown as a function of the distance of the polarizing ^{12}C - nucleus; shell model parameters II. For $R > 13$ fm the polarization is mostly Coulomb, for $R < 13$ fm mostly nuclear, with a destructive interference visible for the $1i_{13/2}$ - state. Near grazing distance, for $12 \text{ fm} \leq R \leq 13 \text{ fm}$, the falloff is nearly exponential, with non-perturbative effects coming in for $R < 12$ fm.

Fig. 13: Polarization of proton states in $^{208}\text{Pb} + ^{12}\text{C}$ near grazing distance at $R = 13$ fm.

Polarization of proton states ($|\Omega| = 1/2$ - components) as calculated from eq. (10) for shell model parameters II (cf. table 4) is shown as a function of single particle energy. For the more deeply bound states the polarization is almost entirely due to the Coulomb force. Mutually enhanced polarization occurs for the close lying $2p_{3/2} - 1g_{9/2}$ and $2d_{5/2} - 1h_{11/2}$ states in Pb. Resonance effects for the $2d_{3/2}$ - and $3s_{1/2}$ - states in Pb and the $1p_{3/2}$ - state in C are also evident.

For the states above the Fermi level for ^{208}Pb ($3s_{1/2}$) we have indicated the polarization obtained for shell model parameters I and III, too (III - full

length; II - end of dashed line; I - end of solid line). Note that the rather moderate increase in radius and diffuseness of the polarizing potential from II to III results in a 100 % increase of polarization, emphasizing the sensitivity of barrier effects.

XBL 772 - 376

Fig. 1: Two-center potentials for neutron states in $^{40}\text{Ca} + ^{16}\text{O}$.

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XBL774-883

Fig. 2: Two-center potentials for proton states in $^{208}\text{Pb} + ^{12}\text{C}$.

XBL 774-858

Fig. 3: Probability density of resonating neutron states in $^{40}\text{Ca} + ^{16}\text{O}$.

XBL774-856

Fig. 4: Probability density of the $2f\ 5/2\ 1/2$ proton state in ^{208}Pb , polarized from ^{12}C .

XBL774-857

Fig. 5: Polarization amplitudes for resonating neutron states in $^{40}\text{Ca} + ^{16}\text{O}$.

Fig. 6: Polarization amplitudes for the $2f\ 5/2\ 1/2$ proton state in ^{208}Pb ,
polarized from ^{12}C .

Fig. 7: Polarization of the $1f$ $7/2$ - states in ^{40}Ca due to ^{16}O .

XBL774-859

XBL772-374

Fig. 8: Polarization of neutron states in ^{40}Ca due to ^{16}O .

XBL 772 - 380

Fig. 9: Polarization of neutron states in $^{40}\text{Ca} + ^{16}\text{O}$ at grazing distance
 $R = 10$ fm.

XBL 772 - 373

Fig. 10: Polarization of proton states in ^{40}Ca due to ^{16}O .

XBL772 - 377

Fig. 11: Polarization of proton states in $^{40}\text{Ca} + ^{16}\text{O}$ at grazing distance $R = 10$ fm.

XBL 772 - 370

Fig. 12: Polarization of proton states in ^{208}Pb due to ^{12}C .

XBL772-381

Fig. 13: Polarization of proton states in $^{208}\text{Pb} + ^{12}\text{C}$ near grazing distance
at $R = 13$ fm.

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