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1. INTRODUCTION

A high-frequency description of the scattering from complex structures that exhibits surface discontinuities such as edges and vertices, is of importance in a wide variety of practical applications. To this end , the Geometrical Theory of Diffraction (GTD) and its uniform extension (UTD) provided very effective tools for most engineering purposes. Within this framework, an important canonical problem is that of a corner at the interconnection of two straight edges, joined by a plane angular sector.

It should be noted that for most practical purposes, the need for a corner diffraction coefficient in a UTD scheme mainly arises when the leading, edge diffracted field experiences a discontinuity, as it occurs when the diffraction point disappears from an edge or changes abruptly its location from one edge to the other.

The exact solution for this canonical problem was obtained in [1], but unfortunately is not well suited for pratical calculations. A heuristic corner diffraction coefficient was conjectured in [2]. An improvement was introduced in [3], where the radiation integral of the currents induced by a single diffraction mechanism from one edge, is extended only to the plane angular domain delimited by the other adjacent edge. Although their solution is cast in a nicely uniform form, no extimate has been introduced therein of the distortion of the currents due to the presence of the second edge. A spectral PTD approach has been developed by Ivrissimtzis and Marehfka [4]. There, a significant improvement has been obtained by introducing secondary non-uniform currents; also, their solution includes a uniform formulation. Recently, corner diffraction coefficients have been derived in the plane wave-far field regime by using the induction theorem [5]. These non-uniform coefficients, that account for second order interactions between the two edges, exhibit the expected singularities at the caustics of single and doubly diffracted rays.

In this paper, the above solution is used to weight the plane wave spectrum representation of a source, located at a finite distance from the vertex. Next, via a suitable, asymptotic evaluation of the spectral integrals, a high-frequency uniform solution is obtained. For the sake of simplicity, the scalar case is treated in this paper and the formulation for hard boundary conditions is given explicitly hereinafter. The same basic procedure can be extended without any significant difficulty, to treat the more general electromagnetic vector problem.

2. PLANE WAVE – FAR FIELD DIFFRACTION COEFFICIENT

The geometry at a plane angular sector interconnecting two edges is shown in Fig. 1. Let us denote by Ω the angle between the two edges. For the sake of simplicity in the discussion, let us suppose that hard boundary condition are imposed on the plane sector. At each edge (n=1,2) it is useful to define a local coordinate system (x_n, y_n, z_n) with its origin at the vertex; the z_n-axis is chosen along the edge, and the y_n-axis is perpendicular to surface; accordingly, a spherical coordinate system (r, β_n, ϕ_n) is also defined.

The far field scattered by the hard, plane angular sector, when it is illuminated by a plane wave propagating in a direction (β_n', ϕ_n'), may be represented as [5]

$$D= D_{21}(\cos\beta_1', \cos\beta_2') + D_{12}(\cos\beta_2', \cos\beta_1') \tag{1}$$

where

$$D_{mn}(\cos\beta_n', \cos\beta_m')= \frac{2 \sin\Omega \sin\frac{\phi_n'}{2} \sin\frac{\phi_m}{2} \sqrt{\sin\beta_n'\sin\beta_m}}{(\cos\beta_n'-\cos\beta_n)(\cos\beta_m'-\cos\beta_m)} \frac{\bar{\bar{d}}_{mn}}{\bar{d}_{mn}} \tag{2}$$

$$\bar{\bar{d}}_{mn}=\sqrt{\sin\left(\tfrac{1}{2}(-\beta_m+\beta_n'+\Omega)\right)}\ ;\ \bar{d}_{mn}= \sqrt{\sin\left(\tfrac{1}{2}(-\beta_n'+\beta_m+\Omega)\right)}$$

It is worth noting that both D_{21} and D_{12} exhibit the expected singularities at the first order diffraction caustics. Furthemore, a square-root type singularity occurs at the second order diffraction caustics ($\bar{d}_{mn}$).

3. SPECTRAL REPRESENTATION FOR A SOURCE AT FINITE DISTANCE FROM THE TIP.

Let us now consider the angular sector when it is illuminated by a point source at (x_1,y_1,z_1). First, the incident field is represented as a superposition of plane waves

$$\frac{e^{-jkR}}{4\pi R} = \frac{1}{8\pi^2 j} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{k_y}\, e^{-j[k_x(x_i-x_i')+k_z(z_i-z_i')+|y_i-y_i'|\, k_y]}\, dk_x dk_z \tag{3}$$

where $k_y=\sqrt{k^2-k_x^2-k_z^2}$ $(k_y=-j\sqrt{k_x^2+k_z^2-k^2})$ for $k^2 > (<)\ k_x^2+k_z^2$

Next, the convenient variable transformation is introduced

$$k_z= k\, u_n'; \quad k_x= \frac{k}{\sin\Omega}(-u_n'\cos\Omega + u_m') \quad (n,m=1,2\ ;\ n \neq m) \tag{4}$$

so that u_1' and u_2' may be interpreted as the director cosines af each spectral plane wave in (3) with respect to the axis z_1 and z_2, respectively. Then, by weigthing each plane wave by its relevant plane wave response of the vertex, one obtains the following spectral representation of the scattered field in far zone

$$\mathfrak{D}_v= \mathfrak{D}_{21} + \mathfrak{D}_{12} \tag{5}$$

$$\mathfrak{D}_{mn} =\frac{1}{8\pi^2 j} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J(u_n', u_m')\, D_{mn}(u_n', u_m')\, \exp^{-j\, kr'\, q(u_n', u_m')} \tag{6}$$

where r' is the distance of the source from the tip, J is the Jacobian of the variable transformation, and $kr'q(u_n', u_m')$ is the phase shift between the point source and the tip of each spectral plane wave. Taking into account (2), it is easily seen that the integrand in (6) contains two separeted poles in the two variables of integration, arising from the first order caustics of the corresponding plane-wave/far field representation. Furtermore, it contains a branch singularity in one of the two variables, which arises from double diffraction caustics.

4. UNIFORM ASYMPTOTIC EVALUATION.

The spectral integral representation (6) is now asymptotically evaluated. To this end the integrand is represented as the sum of two contributions; the first one contains only the pole singularities and the second one the branch singularity. Next the two contributions are approximated by two different asymptotic expressions. The first part wich contain the poles is evaluated as in [3]; the second part is evaluated by means of cylindric parabolic functions. The final results is

$$\mathfrak{D} = \mathfrak{D}_{21}' + \mathfrak{D}_{12}'' + \mathfrak{D}_{21}' + \mathfrak{D}_{21}'' + \mathrm{U}_1\mathfrak{D}_1^{utd} + \mathrm{U}_2\mathfrak{D}_2^{utd} + \mathfrak{D}^{go}\,\bar{\mathrm{U}}_1\bar{\mathrm{U}}_2 \tag{7}$$

where $\mathfrak{D}^{go}$ is the geometrical optics contribution, $\mathfrak{D}_i^{utd}$ is the usual UTD diffraction coefficient at the i^{th} edge; $\mathrm{U}_i = \mathrm{U}(\cos\beta_i - \cos\beta_i')$ and $\bar{\mathrm{U}}_i = \mathrm{U}(\pi - \phi_i - \phi_i')$ are the unit step functions that account for the occurrence of diffraction and reflection points within the plane angular sector. Furthermore in (7)

$$\mathfrak{D}_{mn}' = \mathrm{T}(\bar{\delta}_n, \delta_n, \bar{\delta}_m, \delta_m, \mathrm{kr}') \cdot \frac{\bar{\mathrm{c}}_{mn}\left(\mathrm{c}_{mn}' - \frac{1}{2}\bar{\mathrm{s}}_{mn}\right) + \bar{\bar{\mathrm{c}}}_{mn}\left(\mathrm{c}_{mn} - \frac{1}{2}\bar{\bar{\mathrm{s}}}_{mn}\right)}{(\cos\beta_n' - \cos\beta_n)\,(\cos\beta_m' - \cos\beta_m)} \tag{8}$$

$$\mathfrak{D}_{mn}'' = \mathrm{T}(\bar{\delta}_n, \delta_n, \bar{\delta}_m, \delta_m, \mathrm{kr}') \cdot \frac{\bar{\bar{\mathrm{d}}}_{mn}\,\mathrm{T}_W\left(2\sqrt{-\mathrm{jK}}\ \bar{\mathrm{d}}_{mn}^{\,2}\right)}{\left(\mathrm{c}_{mn}' + \bar{\mathrm{s}}_{mn}\right)\left(\mathrm{c}_{mn} + \bar{\bar{\mathrm{s}}}_{mn}\right)\bar{\mathrm{d}}_{mn}} \tag{9}$$

$$\delta_i = \sqrt{2}\sin\left(\frac{\beta_i' - \beta_i}{2}\right) \quad ; \quad \bar{\delta}_i = \pm\sqrt{2\sin\beta_i\sin\beta_i'}\cos\left(\frac{\phi_i' \pm \phi_i}{2}\right) \tag{10}$$

(+, − signs apply to $\phi_i <,> \pi$, respectively)

$$\bar{\mathrm{c}}_{mn} = \sqrt{\cos\beta_m - \cos(\beta_n' + \Omega)} \quad ; \quad \bar{\bar{\mathrm{c}}}_{mn} = \sqrt{\cos(\beta_m - \Omega) - \cos\beta_n'} \tag{11}$$

$$\bar{\mathrm{s}}_{mn} = \sqrt{\cos(\beta_n' - \Omega) - \cos\beta_m} \quad ; \quad \bar{\bar{\mathrm{s}}}_{mn} = \sqrt{\cos\beta_n' - \cos(\beta_m + \Omega)} \tag{12}$$

$$\mathrm{c}_{mn}' = \sqrt{\cos(\beta_n' - \Omega) - \cos\beta_m'} \quad ; \quad \mathrm{c}_{mn} = \sqrt{\cos\beta_n - \cos(\beta_m + \Omega)} \tag{13}$$

$$\sqrt{-\mathrm{jK}} = \mathrm{e}^{-\mathrm{j}\frac{\pi}{4}}\sqrt{\mathrm{kr}'} \quad ; \quad \mathrm{T}_W(x) = \mathrm{e}^{-x^2/4}\frac{\sqrt{-x}}{\sqrt{2\pi}}\int \frac{1}{\sqrt{t}}\,\mathrm{e}^{-\left(\frac{t^2}{2} + xt\right)}\,\mathrm{d}t \tag{14}$$

In (8) and (9) the transition function

$$\mathrm{T}(\bar{x}, x, \bar{y}, y, \mathrm{K}) = \mathrm{jK}\frac{(\bar{x}^2 + x^2)xy}{(x\bar{y} + y\bar{x})}\left(\epsilon_{\bar{x}}\epsilon_x \mathrm{G}(\mathrm{K}\bar{x}^2, \mathrm{K}x^2) + \epsilon_{\bar{y}}\epsilon_y \mathrm{G}(\mathrm{K}\bar{y}^2, \mathrm{K}y^2)\right) \tag{15}$$

(in which $\epsilon_z = \mathrm{sgn}(z)$) is the same as that defined in [3] and involves the generalized Fresnel integral $\mathrm{G}(a, b)$. In (9) and (14) the transition function $\mathrm{T}_W(x)$ involves a parabolic cilindric function of order $-\frac{1}{2}$.

5. NUMERICAL RESULTS.

The same formulation as that adopted above apply to the case of plane wave incidence and observation point at finite distance. This latter case is discussed in the following numerical examples which are oriented to demonstrate the effectiveness of the transition functions adopted here. In all the plots, the dashed, dotted and continuous lines represent the UTD field, the tip contribution and the total scattered field, respectively. In Fig. 1 the amplitude of a field is plotted at distance $r=2\lambda$ from the tip of a $\Omega=90°$ plane angular sector. The scan plane and the direction of the incident plane wave are specified in the inset of the same figure. Here, the first order diffraction cone (DC) is represented for convenience. When the point of observation (P) passes through this cone, the UTD diffracted field disappears abruptly and the tip contribution provides the required continuity of the total scattered field. In Fig. 2, the same scan is adopted for an acute plane angular sector ($\Omega=45°$). Here the interaction between the two edges becomes stronger; the observation point passes through both first order DCs and a second order DC. In this case both the two transition functions T and T_W act when P approaches first and second order DCs, respectively.

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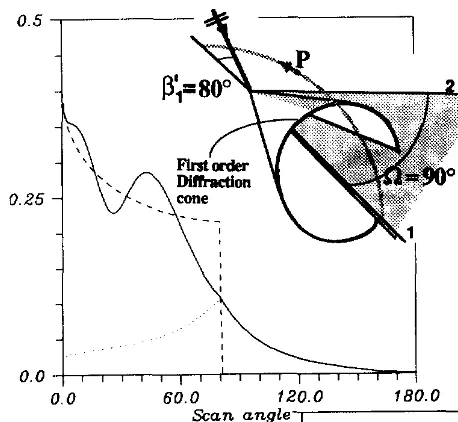

Fig.1

Amplitude of the field scattered by a plane angular sector. the distance of P from the tip is 2λ.
The plane cut intersect only one first order diffraction cone (DC).

- - - - ***First order UTD***
· · · · · ***Tip contribution***
______ ***Total field***

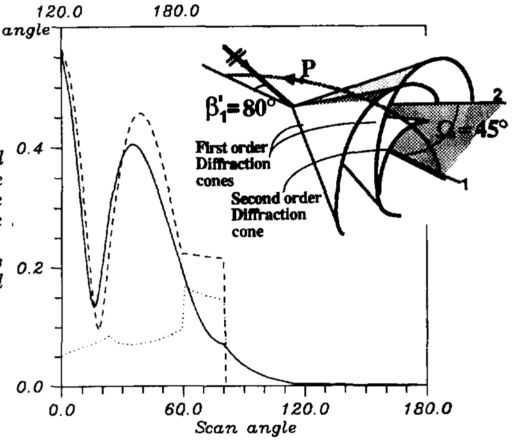

Fig. 2

Amplitude of the field scattered by a plane angular sector. the distance of P from the tip is 2λ.
The plane cut intersects two first order DCs and one second order DC.

- - - - ***First order UTD***
· · · · · ***Tip contribution***
______ ***Total field***