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- Fluctuations in dipole generation increase during polarity transition
- Axial dipole may flip several times during a reversal

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- Supporting Information S1

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Dipole fluctuations and the duration of geomagnetic polarity transitions

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Abstract Geomagnetic polarity transitions are often accompanied by a decrease in magnetic intensity. The time required to reestablish the magnetic intensity after a transition is usually longer than the duration based on magnetic direction. Analysis of the paleomagnetic axial dipole moment from the PADM2M model shows a return to the time-averaged intensity after 42 kyr. A shorter time is required to recover a fraction of the time-averaged moment, but the correspondence between recovery time and fraction of recovery is nonlinear. Predictions of a stochastic model reproduce the general trends in recovery time and suggest that fluctuations in dipole generation increase substantially during polarity transitions relative to times of stable polarity. These large fluctuations could reflect larger convective velocities in the core or represent a change in the efficiency of dipole generation. In either case, large fluctuations during polarity transitions can flip the sign of the axial dipole field several times before a polarity transition is completed.

1. Introduction

Paleomagnetic observations reveal a complex chronology of geomagnetic polarity transitions. Long intervals of stable polarity are interrupted by brief transition states. The duration of a transition state depends on how the transition is defined [*Merrill and McFadden*, 1999]. Direction measurements suggest an average duration of 7 kyr [*Clement*, 2004], although substantial variations are attributed to the influence of the nondipole field [*Brown et al.*, 2007], even for records from the same transition. Direction transitions are often preceded by a decrease in magnetic intensity [*Kent and Schneider*, 1995; *Hartl and Tauxe*, 1996], so the duration of a transition based on intensity is longer than estimates inferred from directions measurements alone.

There is little consensus on the definition of a transition state based on magnetic intensity. A 50% reduction in the dipole field is sufficient to permit large angular deviations [*Quidelleur et al.*, 1999], but the physical connection to the onset of a reversal is not obvious. An alternative strategy is to consider a range of intensity thresholds for the onset (or termination) of a reversal. Different choices for the threshold yield systematic changes in duration. Trends in the duration from available observations offer valuable insights into the underlying processes.

Individual transitions can vary significantly between reversals, so the use of time averages is necessary to extract meaningful quantities. A similar reasoning is applied to define a mean reversal rate, even though the lengths of individual chrons vary widely [*Cox*, 1968]. A second motivation for using time averages is that we can relate the mean transition duration to the predictions of stochastic models. This comparison yields quantitative information about the nature of magnetic field generation in the Earth's core, particularly during times when the dipole field is weak.

An observational constraint on the transition duration comes from models of the paleomagnetic axial dipole moment for the past two million years [*Valet et al.*, 2005; *Ziegler et al.*, 2011]. Figure 1 shows signed values of the axial dipole moment from the PADM2M model of *Ziegler et al.* [2011] using the known geomagnetic polarity timescale [*Cande and Kent*, 1992]. A zero crossing in the signed dipole moment serves as a useful reference point for defining a duration. The subsequent time interval required to reestablish a specified intensity (say 50% of the time-averaged intensity) gives a quantitative measure of the transition duration. This particular definition is well suited for comparisons with the stochastic models. Indeed, we find that the predictions of the stochastic models are most sensitive to the amplitude of the so-called noise term during polarity transitions. Because the noise term can be related to the strength and structure of convective fluctuations in the core [*Buffett et al.*, 2014], we gain new insights into changes in convection during reversals from paleomagnetic

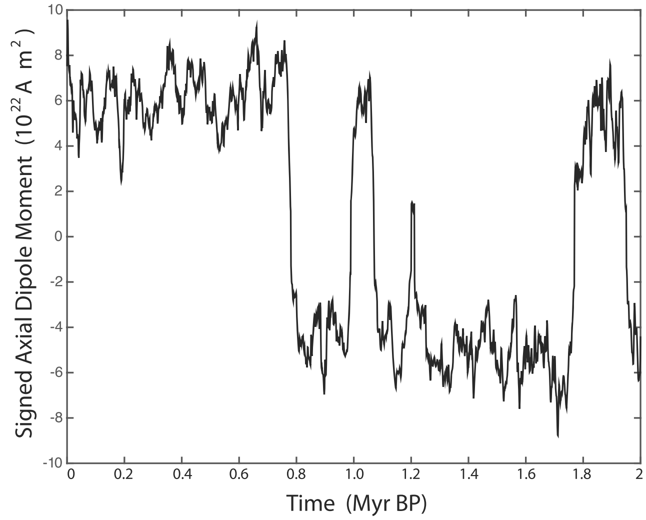

Figure 1. Signed estimate of the paleomagnetic axial dipole moment from the PADM2M model of *Ziegler et al.* [2011].

observations. The PADM2M model suggests the presence of anomalously large fluctuations in dipole generation during transition states, possibly due to a change in columnar convection in the core [*Olson et al.*, 2011]. One prediction that follows from large fluctuations in dipole generation is the possibility of multiple changes in the sign of the dipole before the transition state is completed.

2. Stochastic Description of Dipole Fluctuations

Our definition of the transition duration is motivated by a stochastic description of dipole fluctuations. We represent the time variations in the axial dipole moment, $x(t)$, by a standard Langevin model [e.g., *Van Kampen*, 2007]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v(x) + \sqrt{D(x)}\,\Gamma(t) \tag{1}$$

where the drift term, $v(x)$, describes the deterministic evolution of the dipole moment and the noise term, $D(x)$, defines the amplitude of random variations associated with convective fluctuations in the core. The time dependence of the random process, $\Gamma(t)$, is assumed to be Gaussian with time average

$$< \Gamma(t) > = 0 \tag{2}$$

and autocovariance function

$$< \Gamma(t_1)\Gamma(t_2) > = 2\delta(t_1 - t_2)\,. \tag{3}$$

Together these conditions characterize zero mean and uncorrelated noise.

Estimates for $v(x)$ and $D(x)$ can be recovered from a realization of the stochastic process. The drift term is given by

$$v(x) = \frac{< x(t+\Delta t) - x(t) >}{\Delta t} \tag{4}$$

and the noise term is approximated by

$$D(x) = \frac{< [x(t+\Delta t) - x(t)]^2 >}{2\Delta t} \tag{5}$$

where the time averages are taken for each value of $x(t)$. In practice, the full range of values for $x(t)$ are

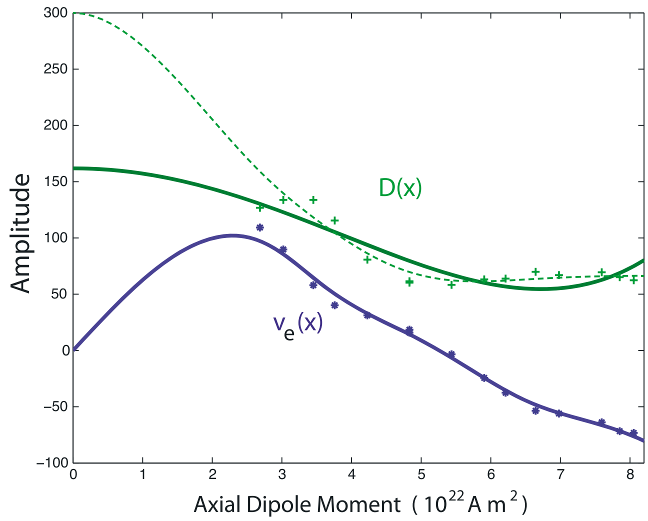

Figure 2. Discrete estimates of the drift $v_e(x)$ in units 10^{22} A m^2 Myr^{-1} and noise $D(x)$ in units 10^{44} A^2 m^4 Myr^{-1} from the PADM2M model. Smooth (solid) lines are fit through the discrete estimates assuming that $v_e(x)$ and $D(x)$ are odd and even functions of x, respectively. A smooth spline (dashed line) is fit to $D(x)$, allowing the unconstrained value at $x = 0$ to be nearly a factor of 2 higher.

discretized into bins and a time average is taken for each bin. The time increment, Δt, is chosen to be long enough to ensure that $\Gamma(t)$ and $\Gamma(t + \Delta t)$ are uncorrelated, consistent with the assumed form of the autocovariance function in (3).

Figure 2 shows discrete values for $v(x)$ and $D(x)$ computed from the PADM2M model of *Ziegler et al.* [2011]. Smooth curves are fit through these discrete values assuming that $v(x)$ and $D(x)$ are odd and even functions of x, respectively. These symmetries are based on the invariance of the magnetic induction equation to a change in the sign of the magnetic field (see *Buffett et al.* [2013] for details). Because the PADM2M model includes relatively few records of low field strengths, the corresponding values of $v(x)$ and $D(x)$ at low x are poorly constrained. However, both of these terms contribute to the prediction of transition durations. This means that we can use observations of transition durations to gain new information about $v(x)$ and $D(x)$ at low x. The key step in recovering this information is to provide a quantitative relationship between transition durations and the coefficients of the stochastic model.

Quantitative predictions for the mean transition duration are based on a probabilistic description of the stochastic process. The probability distribution for $x(t)$ is denoted by $P(x, t)$, and obeys the Fokker-Planck equation [e.g., *Risken*, 1989]

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}[v_e(x)P(x,t)] + \frac{\partial^2}{\partial x^2}[D(x)P(x,t)] \tag{6}$$

where

$$v_e(x) = v(x) + \frac{1}{2}\frac{\mathrm{d}D(x)}{\mathrm{d}x} \tag{7}$$

defines the effective drift for the physically relevant case of a continuously evolving noise source. In this specific case (sometimes called the Stratonovich convention) the autocovariance function in (3) approximates a process with a short correlation time relative to the sampling of $x(t)$. When the noise term varies with x, positive random fluctuations will differ systematically from negative fluctuations, depending on the sign of $\mathrm{d}D/\mathrm{d}x$. This systematic difference contributes to a noise-induced drift, as described by (7). However, the time average in (4) already includes this noise-induced drift, so the expression in (4) specifically represents the effective drift, $v_e(x)$, as required in the Fokker-Planck equation.

Solutions to (6) are subject to suitable initial and boundary conditions. When the initial value of the axial dipole moment is known (say $x(0) = x_0$), the corresponding probability distribution is $P(x, 0) = \delta(x - x_0)$. The relevant boundary conditions are defined by the problem of interest. The goal of determining the average transition

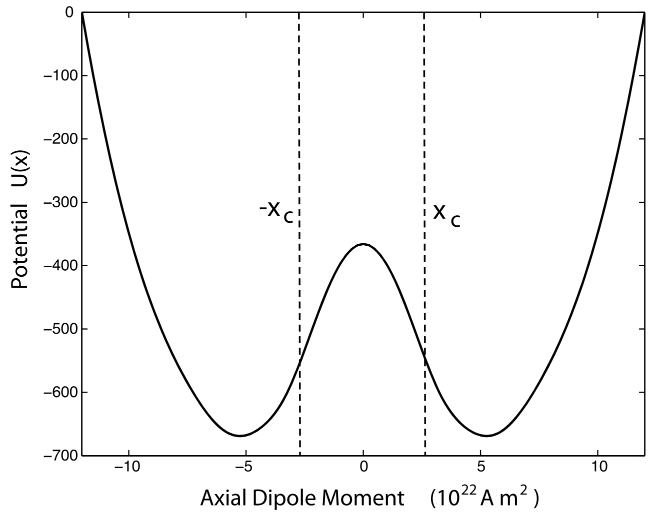

Figure 3. A potential $U(x)$ represents the drift term in the form $v_e(x) = -\nabla U(x)$. A realization started at $x = 0$ eventually descends into one of the two potential wells. The mean time taken for a realization to exceed the threshold at $\pm x_c$ defines a transition duration in this study.

duration reduces to the problem of computing a mean first-passage time. In other words we ask how long it takes on average for a realization $x(t)$ to leave a prescribed interval $x_- < x(t) < x_+$. Realizations that leave this interval are removed from consideration (and prevented from reentering the interval) by imposing the boundary conditions $P(x_-, t) = P(x_+, t) = 0$. For present purposes it suffices to note that numerical solutions of (6) are used to evaluate the mean first-passage time. Additional details can be found in the supporting information.

3. Predictions for the Duration of Polarity Transitions

Suppose that we start with the axial dipole moment at the point of reversing (i.e., $x(0) = 0$). The time required for $x(t)$ to rise above a threshold x_c is equivalent to the time required for a realization to leave the interval $-x_c < x(t) < x_c$. Thus, the mean time is obtained from a solution of the Fokker-Planck equation using initial condition $P(x, 0) = \delta(x)$ and boundary conditions $P(-x_c, t) = P(x_c, t) = 0$. The initial condition is approximated in the numerical solution of (6) using a very narrow Gaussian distribution.

The drift term vanishes at $x = 0$ but the noise term perturbs the realization to a nonzero value. A useful way to visualize the process is to relate the drift term to a potential $U(x)$, defined by $v_e(x) = -\nabla U(x)$. Figure 3 shows the potential obtained by integrating the estimate of $v_e(x)$ from Figure 2. An arbitrary constant in the definition of the potential is fixed by setting $U(x) = 0$ at large x. The two minima in the potential $U(x)$ define the nominal location of the axial dipole moment during times of stable polarity. A sequence of large fluctuations due to the noise term occasionally send the axial dipole moment over the barrier at $x = 0$, causing a polarity transition. A realization started at $x = 0$ will eventually descend into one of the two potential wells. The actual path toward either a positive or a negative polarity does not matter because the mean first-passage time depends on $x(t)$ passing over $-x_c$ or $+x_c$.

Figure 4 show the numerical prediction of the mean transition duration as a function of the threshold x_c. We also show a small-amplitude approximation, which is obtained using the simplification that $v_e(x)$ is small near $x = 0$ and $D(x)$ is nearly constant. The mean transition duration for small x_c is approximated by

$$\tau \approx \frac{4x_c^2}{D(0)\pi^2} \tag{8}$$

where the amplitude of the noise term is evaluated at $x = 0$. This approximate solution agrees well with the numerical solution when $x_c < 2.5 \times 10^{22}$ A m^2, which means that transition durations for small thresholds depend mainly on $D(0)$. Thus, observations of τ provide information about $D(0)$. At larger x_c the approximation in (8) tends to overestimate τ. Conceptually, the descent of $x(t)$ into one of the potential wells shortens the time required to pass the threshold x_c, relative to predictions driven solely by noise.

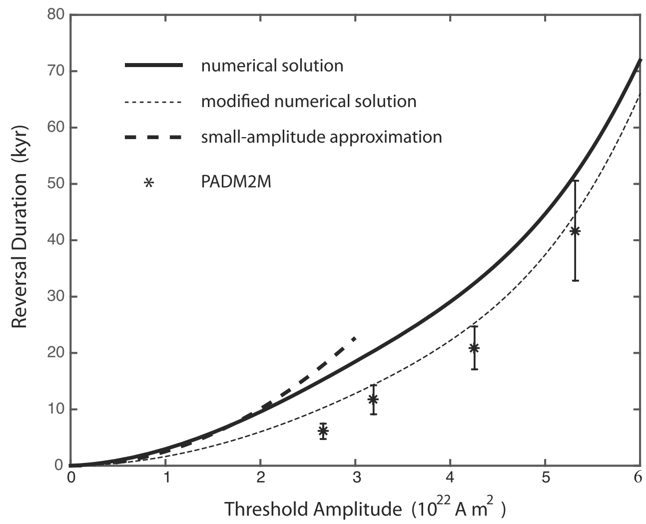

Figure 4. Transition duration as a function of threshold x_c. A numerical solution of the stochastic model (solid line) is compared with a time average from five distinct reversals in the PADM2M model, excluding the Cobb Mountain subchron. The error bars reflect the standard deviation of the sample mean. The small-amplitude approximation (dashed line) is given by equation (8). A modified numerical solution (dotted line) is obtained with a larger value for $D(x)$ at $x = 0$, improving agreement with the observed durations.

Observed estimates for τ from the PADM2M model are also shown in Figure 4. Each reversal over the past two million years yields a distinct time to exceed the threshold x_c, but the time average gives a trend that parallels the prediction. A total of five reversals contribute to the time average after excluding the Cobb Mountain subchron because the axial dipole moment does not fully recover to the average value in the PADM2M model. The error bars represent the standard deviation of the sample mean.

A comparison of the predictions and the observations suggests that the stochastic model overestimates the transition duration, particularly at low values for the threshold x_c. The simplest interpretation is that our extrapolation of $D(x)$ to low x underestimates $D(0)$. According to (8), a low value for $D(0)$ gives long transition durations. As an experiment we use a smooth spline fit through the discrete values of $D(x)$ from the PADM2M model but allow the unconstrained value at $x = 0$ to increase by nearly a factor of 2 (shown as a dashed line in Figure 2). The corresponding prediction for the reversal duration is shown in Figure 4 as the modified numerical solution. This new prediction is in much better agreement with the observed reversal durations from PADM2M. At the same time the smooth spline fit for $D(x)$ in Figure 2 preserves good agreement with the discrete estimates.

The challenge in estimating $v(x)$ and $D(x)$ from (4) and (5) is that there are insufficient records of the field at low x to obtain a reliable time average. By recovering estimates for the reversal duration from PADM2M, we are able to make indirect inferences about $v(x)$ and $D(x)$ through a solution of the Fokker-Planck equation for the mean first-passage time. The observed and predicted durations depend on the choice of the threshold, x_c, so it is reasonable to question whether low values for x_c can be reliably detected in the observations. Fortunately, the restrictions on x_c do not limit the utility of the approach. Even large thresholds ($x_c > 4 \times 10^{22}$ A m^2) are distinctly affected by the value of the noise term at low x. In order to explain both the discrete estimates of $D(x)$ in Figure 2 and the reversal durations in PADM2M at large x_c, we require a substantial increase in the noise term at low x. Matching the reversal duration at lower x_c would argue for even larger increases in the noise term.

4. Implications for the Temporal Complexity of Reversals

A higher noise term during polarity transitions has several consequences. First, it implies larger fluctuations in dipole generation. We denote the dipole generation by $S(t)$, so the random fluctuations are defined by

$$\Delta S(t) \equiv S(t) - < S > \tag{9}$$

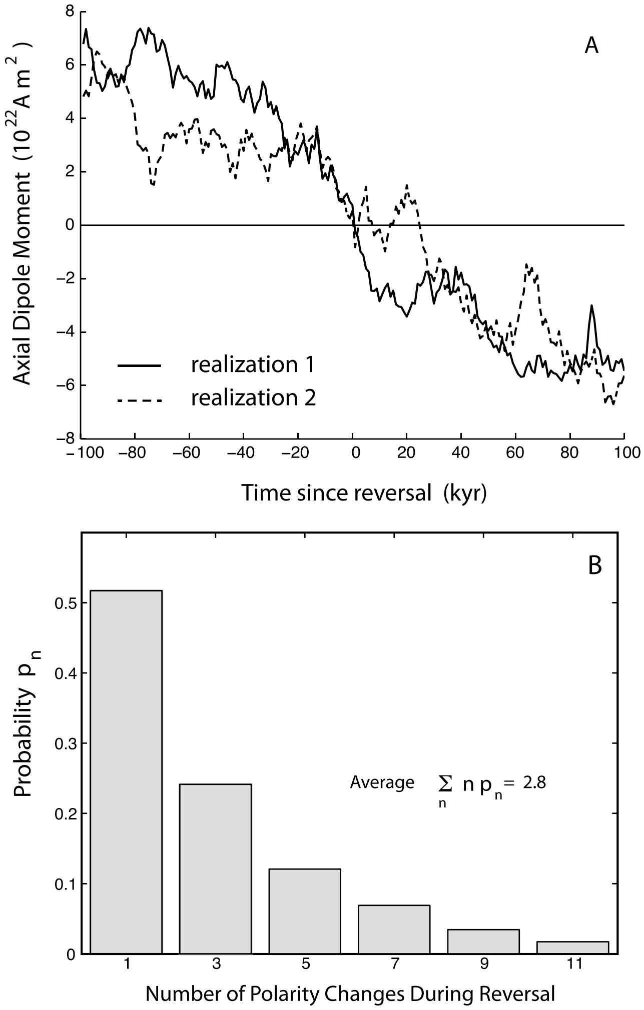

Figure 5. (a) Numerical realizations of the stochastic model. Realization 1 (solid) yields a single polarity change, whereas realization 2 (dashed) has multiple polarity changes. (b) A histogram of sign changes from a long (60 Myr) realization indicates that a single sign change occurs about half of the polarity transitions. The average number of sign changes is 2.8, so the additional number of polarity changes after the initial transition is 1.8.

where $< S >$ is the time-averaged generation. The root-mean-squared fluctuations in $S(t)$ are related to $D(x)$ by [*Buffett et al.*, 2014]

$$< \Delta S^2 >^{1/2} \approx \sqrt{\frac{2D(x)}{\Delta t}} \tag{10}$$

where Δt is the time difference required to ensure that $\Gamma(t)$ is uncorrelated. Consequently, a higher noise term means larger fluctuations in $S(t)$ during polarity transitions.

There are two ways to interpret this result. One possibility is that $S(t)$ increases when the dipole is weak. This change might be accomplished by increasing the convective velocities, although numerical geodynamo models indicate modest changes in kinetic energy during polarity transitions [*Takahashi et al.*, 2007]. An alternative interpretation is that $< S >$ becomes small during a transition. A collapse of columnar convection during a reversal [*Olson et al.*, 2011] could reduce the efficiency of dipole generation and eliminate a persistent or time-averaged contribution to $S(t)$. When $< S >$ becomes small, $\Delta S(t)$ increases to become nearly equal $S(t)$. From the perspective of the stochastic model, we do not expect dipole generation at $x = 0$ to favor one polarity or the other, so it would be reasonable to assume that $< S >$ vanishes during a reversal.

A second consequence of the large noise term is that reversals are unlikely to be simple monotonic transitions between polarities. Instead, we can anticipate considerable temporal complexity [*Coe and Glen*, 2004; *Valet et al.*, 2012]. The stochastic model suggests that the evolution of dipole moment is dominated by the noise term at small x because the drift term is small. The resulting stochastic process can be viewed as a continuous version of a random walk in one dimension (sometimes called a Wiener process). A process started at $x = 0$ can potentially cross zero several times before the dipole moment settles into one of the two stable polarities.

A rough estimate of the average number of zero crossings during a transition can be obtained by treating the process as a discrete random walk. The duration of each discrete step or event in the random walk is set by the value $\Delta t = 5$ kyr, which is the time needed to ensure that $\Gamma(t)$ is uncorrelated. We limit the applicability of the Wiener process (or random walk) to $|x| < 2.5 \times 10^{22}$ A m^2, which is justified by the validity of the small-amplitude approximation in (8). As the drift term becomes more important at larger $|x|$, the evolution of the dipole moment is more likely to descend into a potential well than return to $x = 0$. The mean time to leave the interval $|x| < 2.5 \times 10^{22}$ A m^2 is roughly 15 kyr, assuming $D(0) \approx 160 \times 10^{44}$ A^2 m^4 Myr^{-1}, or about three steps of a random walk. The average number of the zero crossings for Gaussian noise is 0.44 [*DasGupta and Rubin*, 1998], which means that an additional zero crossing occurs roughly 40% of the time after the axial dipole moment reaches $x = 0$.

A higher value for $D(0)$ reduces the likelihood of multiple reversals during a transition because the dipole spends less time in the transition state. On the other hand, a time interval of $\Delta t = 5$ kyr might reflect the acquisition of magnetization in sediments, rather than the correlation time of convective fluctuations. If we adopt a correlation time of 1 kyr for convective fluctuations [*Buffett and Matsui*, 2014] and reduce the transition duration to about 10 kyr, consistent with the estimate from the modified stochastic model, then we might expect 10 random steps during a transition. The expected number of zero crossings increases to roughly 1.3 [*DasGupta and Rubin*, 1998], so a single additional reversal becomes likely after the axial dipole moment initially reaches $x = 0$ during a transition. Of course a single additional zero crossing would restore the original polarity, so a second zero crossing would be needed to reverse the field after the transition state is completed.

A more direct estimate of the transition behavior comes from realizations of the process. Numerically integrating (1) with the modified noise term produces realizations with many polarity transitions. Figure 5a shows two realizations around the time of a polarity transition. One realization crosses $x = 0$ once, whereas the other realization crosses zero 5 times (i.e., four extra sign changes). Figure 5b shows a normalized histogram (probability) of the number of zero crossings from a long (60 Myr) realization. Roughly half of the polarity transitions cross zero once, whereas the other half exhibit three or more sign changes. The average number of zero crossings is 2.8, so the number of sign changes after the initial transition is 1.8 (on average). The random walk analogy (above) gives a reasonable estimate of the average behavior. Whether this complexity is actually observed may depend on the contribution of the nondipole field [e.g., *Brown et al.*, 2007]. Nevertheless, it is plausible on the basis of the stochastic model that complex, multiple reversals can occur during a single transition interval [e.g., *Sagnotti et al.*, 2014].

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