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CANONICAL-DISSIPATIVE FORMULATION OF RELATIVISTIC PLASMA
KINETIC THEORY WITH SELF-CONSISTENT MAXWELL FIELD

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Abstract

Following earlier discussions of the application of canonical Lie-Poisson and dissipative brackets in relativistic plasma physics, we propose a unified description which encompasses both selfconsistent, reversible Maxwell-Vlasov and irreversible Landau-like kinetic equations. Possible generalization of that theory to allow the study of plasmadynamic fluctuations is also discussed.

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The systematic use of the Lie-Poisson bracket technique [1] has led to considerable progress in various branches of nonlinear physics [2]. Application of that powerful mathematical method allows for a *canonical* formulation of reversible relativistic fluid and plasma dynamics [3-7]. On the other hand, it has been recently argued that one can describe the irreversible processes, for example those represented by Boltzmann-like equations, using the concept of the dissipative bracket [8-10]. Similar ideas have been advocated for a non-relativistic viscous fluid [11], and more recently for Heisenberg ferromagnets with Gilbert damping [12].

In this note, we present a mixed canonical-dissipative description of relativistic plasma, in which both the selfconsistent Maxwell field as well as direct particle collisions are included. We shall also discuss a possible generalization of the theory which would allow us to study the plasmadynamic fluctuations by means of the generalized Fokker-Planck equation. Non-relativistic plasmadynamic fluctuations have been studied previously by means of the linearized Vlasov-Landau-Langevin equation [13].

Consider a many-charged-particle system, its state being described by the electric $E(r,t)$ and magnetic $B(r,t)$ fields and a smooth one-particle distribution function $f(r,p,t)$. Here p stands for the relativistic particle *kinetic* momentum [3]. Clearly that description is a coarse-grained one, with considerable information lost in the process of, for example, averaging the exact Klimontovich equation [3]. We shall see later that one can restore part of that lost information by a suitable use of Langevin "forces".

According to the Lie-Poisson bracket approach, we begin by defining the fundamental Poisson brackets between components of the state vector $\psi = [f(r,p,t), E(r,t), B(r,t)]$. Following [3] we write:

$$\begin{aligned} \{f(r,p,t), f(r',p',t)\} &= [f(r,p',t) - f(r',p,t)] \nabla \cdot \partial \delta(z-z') \\ &+ eB(r,t) \cdot [\partial f(r,p,t) \times \partial] \delta(z-z') \end{aligned} \quad (1a)$$

$$\{f(r,p,t), B(r',t)\} = 0 \quad (1b)$$

$$\{f(r,p,t), E(r',t)\} = -e \partial f(r,p,t) \delta(z-z') \quad (1c)$$

$$\{E^a(r,t), B^b(r',t)\} = -e^{abc} \nabla_c \delta(r-r') \quad (1d)$$

where e is particle charge, $z = (r,p)$ and $\nabla = \partial/\partial r$, $\partial = \partial/\partial p$. We shall also use a shorthand notation for the formulae (1a-d), namely $\{\psi_A(z), \psi_B(z')\} = L_{AB}(z,z')$ with $A = f, E, B$. In order to study the system dynamics, we have to define its *free energy* $F(\psi)$. Assuming that we are dealing with neutral homogeneous plasma without external fields, we write $F(\psi)$ as

$$F(\psi) = U(\psi) - \theta S(\psi), \quad (2)$$

where

$$U(\psi) = \int dz (p^2 + m^2)^{\frac{1}{2}} f(z) + \frac{1}{2} \int dr (E^2 + B^2) \quad (3)$$

is the internal energy functional, and

$$S(\psi) = - \int dz f(z) [\ln f(z) - 1] \quad (4)$$

is the entropy functional. The constant θ can be identified with the temperature, by the standard requirement that the (absolute) minimum of $F(\psi)$ in ψ -space corresponds to the Maxwell-Boltzmann distribution function and zero fields. (No macroscopic internal fields E, B can be present at equilibrium).

It is now straightforward to check that the equations of motion for the state vector ψ are

$$\partial_t \psi_A(z) = \{\psi_A(z), F(\psi)\} = \int dz' L_{AB}(z,z') \delta F / \delta \psi_B(z') \quad (5)$$

and that they are equivalent to the relativistic Vlasov-Maxwell equations [3].

To generalize this reversible description to the irreversible,

information-destroying processes, we follow Ref. 8 and define the *dissipative* bracket of the state vector components as:

$$\langle \psi_A(z), \psi_B(z') \rangle = D'_{AB}(z, z') \quad (6)$$

where the symmetric matrix $D'_{AB}(z, z')$ has only one non-vanishing matrix element D'_{ff} . Indeed we expect that the total loss of information about the state of the system is due to the direct particle-particle "collisions" only. Explicitly we write

$$\langle f(z), f(z') \rangle = (1/2) \int dz_1 dz_2 f(z_1) \Delta(z, z'; z_1, z_2) f(z_2), \quad (7)$$

with

$$\begin{aligned} \Delta(z, z'; z_1, z_2) = \int dk \alpha_k(z_1, z_2) \delta(k \cdot (v(p) - v(p'))) \cdot \\ k \cdot [\partial_1 \delta(z - z_1) - \partial_2 \delta(z - z_2)] k \cdot [\partial_1 \delta(z' - z_1) - \partial_2 \delta(z' - z_2)] \end{aligned} \quad (8)$$

where $v(p) = p/(p^2 + m^2)^{1/2}$ is the relativistic particle velocity, and $\alpha_k(z_1, z_2)$ describes the precise nature of the information-destroying processes.

The full set of the canonical-dissipative equations for the Maxwell field and the one-particle distribution function can now be written as:

$$\partial_t \psi_A(z) = \int dz' D_{AB}(z, z') \delta F / \delta \psi_B(z') \quad (9)$$

with the kernel matrix $D_{AB}(z, z') \equiv L_{AB}(z, z') - D'_{AB}(z, z')$.

Eq. (9) has precisely the same form as used in other applications of the mixed canonical-dissipative formulation. Working out all the matrix elements of Eq. (9), one finds that they represent the Maxwell equations with sources given in terms of particle density and particle current, and the Landau- or Balescu-Lenard kinetic equation, depending on what form of $\alpha_k(z, z')$ we use. We conclude therefore that the kinetic theory of the

collisional relativistic plasma with self-consistent Maxwell field can be cast into the form of the mixed-canonical dissipative formulation.

Having written our fundamental equation (9), we can now speculate about its use in possible generalizations of the theory. Indeed, a description of our system only in terms of the fields E, B and the one-particle distribution f is inadequate. Information was lost in the process of averaging the exact one-particle Klimontovich function $f^K(z, t) = \sum_i \delta(z - z_i)$, $\langle f^K(z, t) \rangle = f(z, t)$, where as usual $\langle \dots \rangle$ denotes initial ensemble averaging. One can attempt to restore at least part of that lost information by amending the RHS of Eq. (9) with a suitable Langevin "force". We write then:

$$\partial_t \psi_A(z, t) = \int dz' D_{AB}(z, z') \delta F / \delta \psi_B(z') + \Lambda_A(z, t) \quad (10)$$

with

$$\langle \langle \Lambda_A(z, t) \Lambda_B(z', t') \rangle \rangle = C_{AB}(z, z') \delta(t - t') \quad (11)$$

Here $\langle \langle \dots \rangle \rangle$ denotes an average over the realizations of the forces Λ_A :

Similarly as in [11], we can now write the generalized Fokker-Planck equation for the probability-distribution functional in ψ space $\rho(\psi; t)$. We have

$$\partial_t \rho(\psi; t) = P(\delta / \delta \psi) \rho(\psi; t) \quad (12)$$

with the Fokker-Planck operator P in the form

$$P = \int dz_1 dz_2 \delta / \delta \psi_A(z_1) D_{AB}(z_1, z_2) \delta F / \delta \psi_B(z_2) + \int dz_1 dz_2 \delta / \delta \psi_A(z_1) C_{AB}(z_1, z_2) \delta / \delta \psi_B(z_2) \quad (13)$$

Eq. (13) is now the starting point for the analysis of plasmadynamic fluctuations. We emphasize that our theory, although not explicitly

covariant, is nevertheless relativistically invariant. Thus Eq. (13) is also a relativistic Fokker-Planck equation for the state vector ψ . The precise nature of the fluctuations is determined by the correlation function C_{AB} . As in the nonrelativistic theory, that should be determined from the equilibrium statistical mechanics of the system and then should obey known potentiality conditions, which have been discussed in detail in [9]. We hope to report on extensions of our work in this direction in a separate publication.

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