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Some simple analytics of peak-load pricing

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and

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Consider a public utility that offers its service at two different times. We study the effects of a change from uniform pricing throughout the day to peak-load pricing. We show that for a utility constrained to operate with a fixed rate of return on capital, the introduction of peak-load pricing can plausibly reduce the price of the service both in peak and off-peak times. We also find that peak-load pricing can lead to either greater or smaller capacity than uniform pricing. We find a simple criterion for determining whether a particular individual gains or loses from peak-load pricing.

1. Introduction

■ Consider a public utility that offers its product at two different times, morning and afternoon. Capacity in place can be used in both periods, but the amount consumed in either period must not exceed capacity. If price is the same in both periods, afternoon demand will exceed morning demand. Accordingly, we refer to afternoon as the *peak* and morning as the *off-peak* demand period.

Suppose that the utility, which has been constrained to charge the same price at both times of day, is allowed to use peak-load pricing. Will prices in the peak period necessarily rise? Will equilibrium capacity increase or decrease? Which consumers will gain and which will lose? The comparative statics of peak-load pricing work out in a neat and decisive way, and we find rather surprising answers to these questions. For example, under plausible demand conditions, peak-load pricing will reduce prices in *both* the peak and off-peak periods. Also, introducing time-of-day pricing may lead to an *increase* in capacity.

A related article is by Bailey and White (1974), who analyze peak-load pricing under alternative assumptions about market structure and regulation.[1] Unlike Bailey and White, who assume that price in either period has no effect on demand in the other period, we allow substitutability or complementarity between peak and off-peak consumption. Several extensions and generalizations of the results found here appear in a (much longer) previous version of this article, Bergstrom and MacKie-Mason (1989).

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[1] Additional literature is surveyed by Brown and Sibley (1986).

2. The basic model

■ Technology and costs.

Assumption T. A firm with capacity K, morning (off-peak) output x_M, and afternoon (peak) output x_A will have total costs $rK + u_A x_A + u_M x_M$. Output in each period is constrained by capacity, so that max $(x_M, x_A) \leq K$. Morning and afternoon "user costs" u_A and u_M may differ, but it is assumed that $u_A + r > u_M$.

□ **Preferences and demand.** Let p_M and p_A denote the prices charged in the morning and the afternoon and let x_M and x_A denote total consumption in the morning and in the afternoon. We assume the following about preferences:

Assumption P1. Preferences are weakly separable between utility services and other goods, with a homothetic aggregator for preferences over utility services. The utility function of consumer, i, is of the form $U_i(y^i, f(x_M^i, x_A^i))$, where y^i is i's consumption of "other goods," and $f(\cdot, \cdot)$ is homothetic, twice differentiable, and strictly quasi-concave.

The assumption of homothetic separability with identical aggregators simplifies analysis because it makes the ratio of aggregate demand for afternoon consumption to aggregate demand for morning consumption a function of the ratio of afternoon price to morning price. This assumption is common in the empirical literature on peak-load pricing.[2]

Let "other goods" be the *numeraire* and let aggregate demands for morning and afternoon use of the utility be functions, $x_A(p_A, p_M)$ and $x_M(p_A, p_M)$. At an interior maximum, an optimizing consumer will choose a consumption bundle such that her marginal rate of substitution between afternoon and morning consumption equals the price ratio p_A/p_M. We define the price ratio as $\rho = p_A/p_M$. Since the functions f are homothetic and strictly quasi-concave, it must be that person i's marginal rate of substitution between afternoon and morning consumption is determined by the ratio x_A^i/x_M^i, and is a strictly monotone decreasing function of this ratio. Then for all i,

$$MRS(x_M^i/x_A^i) = \frac{f_2(x_M^i, x_A^i)}{f_1(x_M^i, x_A^i)} = \frac{p_A}{p_M}. \tag{1}$$

Strict quasi-concavity of f implies that as the ratio x_M^i/x_A^i ranges from zero to infinity, $MRS(x_M^i/x_A^i)$ decreases monotonically over a real interval, R_ρ. Since the function $MRS(x_M^i/x_A^i)$ is monotonic, it has an inverse. That is, for any price ratio, $p_A/p_M \in R_\rho$, there is a unique ratio x_M^i/x_A^i such that $MRS(x_M^i/x_A^i) = p_A/p_M$. Indeed, since all individuals have the same aggregator function, f, and all face the same price ratio, p_A/p_M, it must be that x_A^i/x_M^i is the same for all i. Therefore the ratio $x_A(p_A, p_M)/x_M(p_A, p_M)$ is determined by the price ratio p_A/p_M. These facts allow us to make the following definitions.

Definition 1. Define the function $\chi(\rho)$ implicitly by the equation $MRS(\chi(\rho)) = \rho$. That is, $\chi(\rho)$ is the ratio of demand for afternoon consumption to demand for morning consumption, when their price ratio is ρ.

Definition 2. The elasticity of substitution between afternoon and morning consumption is $\sigma(\rho) = -d \ln \chi(\rho)/d \ln \rho$.

Definition 3. The expenditure shares of afternoon and morning consumption are respectively

$$\theta_A = \frac{p_A x_A}{p_M x_M + p_A x_A} \quad \text{and} \quad \theta_M = \frac{p_M x_M}{p_M x_M + p_A x_A}.$$

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[2] See, e.g., Parks and Weitzel (1984); Howrey and Varian (1984); Caves, Christensen, and Herriges (1984), and Hausman, Kinnucan, and McFadden (1979).

Notice that since $MRS(\rho)$ is a monotone decreasing function and $X(\rho)$ is its inverse function, it must be that $X(\rho)$ is monotone decreasing. Therefore $\sigma(\rho) > 0$ for all ρ.

We assume that if prices are the same in both periods, demand in the afternoon will exceed demand in the morning. This assumption is expressed in our notation as $X(1) > 1$. The assumption that the afternoon is the peak-load period at *uniform* prices does not exclude the possibility that at some prices morning demand might be higher.

□ **Pareto-efficient pricing.** Where marginal costs are well defined, a necessary condition for Pareto efficiency is that consumers' marginal rates of substitution between morning and afternoon consumption equal the ratio of marginal costs. So long as demand in the afternoon exceeds demand in the morning, an additional bit of service can be provided in the morning without changing capacity, while an extra bit provided in the afternoon requires a corresponding increase in capacity. Therefore, if afternoon demand exceeds morning demand when the price ratio is $\rho^* = \dfrac{r + u_A}{u_M}$, ρ^* is the Pareto-efficient price ratio. If morning demand exceeds afternoon demand at the price ratio ρ^*, then Pareto efficiency requires that demands be equalized with a price ratio ρ^{**} satisfying $1 < \rho^{**} < \rho^*$.

□ **Constrained rate of return and equilibrium.** We assume that the public utility is constrained to operate at a fixed rate of return on capital. This constraint might be enforced by a regulatory agency, or it might be an equilibrium rate of return that is enforced by potential competition. The capital base on which the utility is allowed to earn this rate of return is proportional to its capacity. Let c_K denote the return per unit of capacity that will yield the allowable rate of return on capital. If the regulatory agency seeks a Pareto-optimal outcome, it will set $c_K = r$. But it might allow a rate of return $c_K > r$, as in the model of Averch and Johnson (1962).

Assumption R. The firm is constrained to operate at a fixed rate of return on capital, so that $p_A x_A + p_M x_M - u_A x_A - u_M x_M = c_K K$.

We assume that the utility produces to meet demand in each period. This means that the prices and quantities chosen must satisfy equation (1). We also assume that the utility uses its full capacity at least some time during the day, so that when $x_A > x_M$, it must be that $x_A = K$. These assumptions restrict the set of possible equilibria to a "one-dimensional continuum" determined by the parameter ρ. This fact is expressed by the following lemma.

Lemma 1. For any ρ such that $X(\rho) \geq 1$, there is exactly one set of equilibrium prices and quantities, p_A, p_M, x_A, and x_M, that satisfies Assumption R and equates capacity to peak-load demand.

Proof. See the Appendix.

In consequence of Lemma 1, we can study the comparative statics effects of moving from uniform pricing to peak-load pricing by studying the derivatives of equilibrium prices and quantities with respect to the variable ρ. This allows one to use the same tools to analyze an Averch-Johnson regulator—who allows the firm to maximize profits subject to a maximum rate of return; a Ramsey regulator—who requires a declining-cost firm to maximize welfare subject to a minimum rate of return; or a market in which free entry maintains the rate of return on capital at the market rate of interest.

3. Does peak-load pricing make peak prices rise or fall?

■ With time-of-day pricing, it is possible to allocate capacity more efficiently between morning and afternoon use. Since the rate of return on capacity is fixed, this gain in efficiency may lead to a decline in the equilibrium prices in both periods.

□ **Two easy cases.** To see how this works, it helps to think about two special cases that are easy enough to "solve in one's head." In each of these special cases, we assume that user costs $u_M = u_A = 0$ and the allowable rate of return on capacity is c_K.

First consider the extreme case of "perfect substitution" between morning and afternoon consumption where a unit of afternoon consumption is exactly as good as two units of morning consumption. The aggregator function for utility services is

$$f(x_A, x_M) = 2x_A + x_M.$$

If there is a uniform price p, the only demand for utility services will be in the afternoon. The zero-profit constraint requires that the entire cost of capacity be repaid by afternoon usage, so that $p = c_K$. On the other hand, peak-load pricing would equalize morning and afternoon demands. This happens when $p_A = 2p_M$. At these prices, consumers are indifferent between using the service in the morning and afternoon, and consumption in both periods can be set equal to capacity. In effect, peak-load pricing allows the firm to sell its entire capacity twice, once in the morning and once in the afternoon. The profit constraint is satisfied when $p_A + p_M = c_K$. Since $p_A = 2p_M$, it must be that with peak-load pricing, $p_A = 2c_K/3$, and $p_M = c_K/3$. Moving from uniform pricing to peak-load pricing results in lower prices in both periods.

Now consider the case of "perfect complements," where at any price, consumers always want to consume exactly twice as much in the afternoon as in the morning. Let

$$f(x_A, x_M) = \min\{x_A, 2x_M\}.$$

At any price, consumers choose $x_A/x_M = 2$. No matter what prices it chooses, the utility can sell all of its capacity in the afternoon and only half of its capacity in the morning. Therefore the profit constraint is satisfied for any pair of prices, p_A and p_M, on the locus $p_A + p_M/2 = c_K$.[3] In this example, increasing the price ratio from uniform pricing requires an increase in the peak price and a decrease in the off-peak price to remain on the profit constraint locus.

□ **The case of zero user costs.** More generally, assume that $u_A = u_M = 0$ and that full capacity is used in the afternoon. The constraint on the rate of return simplifies to $p_A x_A + p_M x_M = c_K x_A$. Multiply both sides of this equation by θ_A/x_A to obtain $p_A = \theta_A c_K$. It follows that an increase in the price ratio ρ will make afternoon consumption go up or down depending on whether the expenditure share θ_A is an increasing or decreasing function of ρ. A familiar result from production theory is that θ_A is an increasing (decreasing) function of ρ if and only if the elasticity of substitution σ is less than (greater than) one. Therefore as ρ is increased, the price of afternoon consumption will rise if $\sigma < 1$, fall if $\sigma > 1$, and stay constant if $\sigma = 1$.

□ **A general answer.** Define the ratio of net return on morning sales to price of morning consumption as[4]

$$L_M = (p_M - u_M)/p_M = 1 - u_M/p_M.$$

Lemma 2 supplies explicit formulae for the change in price in each period as the price ratio, ρ, is changed.

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[3] There is not a unique solution as in Lemma 1 because the utility function does not satisfy Assumption P1.

[4] In the discussion below, we implicitly assume that $L_M \geq 0$. But our equations apply whether L_M is positive or negative. The results apply even when regulators require the utility to set the morning price below variable user cost. Wenders (1976) shows that a profit-maximizing utility with a regulated rate of return on installed capital may set the off-peak price below marginal cost in order to encourage the expansion of capital-intensive base-load capacity.

Lemma 2. Assume the technology, preferences and profit constraint given in Assumptions T, P1, and R. Then for all ρ such that $\chi(\rho) > 1$,

$$\frac{d \ln p_A}{d \ln \rho} = \theta_M(1 - L_M\sigma(\rho)) \tag{2}$$

and

$$\frac{d \ln p_M}{d \ln \rho} = \frac{d \ln p_A}{d \ln \rho} - 1 = -(\theta_A + L_M\sigma(\rho)\theta_M). \tag{3}$$

Proof. See the Appendix.

From equation (2), it is apparent that the sign of $dp_A/d\rho$ is the same as that of $1 - L_M\sigma$. From equation (3), we see that $dp_M/d\rho$ is always negative. This permits us to claim

Theorem 1. Under Assumptions T, P1, and R, moving toward peak-load pricing results in (a) lower prices in both peak and off-peak times if the elasticity of substitution between peak and off-peak consumption is greater than $p_M/(p_M - u_M)$ and (b) higher prices in peak times and lower prices in off-peak times if the elasticity of substitution is less than $p_M/(p_M - u_M)$.

4. Does peak-load pricing increase or decrease industry capacity?

■   The conventional view seems to be that peak-load pricing will reduce utilities' demand for capacity. (See Berlin, Cicchetti, and Gillen (1974), Nemetz and Hankey (1984) and Caves, Christensen, and Herriges (1984).) But it isn't necessarily so. There are two forces at work here. Peak-load pricing allows more efficient use of capacity, since less capacity is idle off-peak. This means that less capacity is required to generate a given amount of the composite commodity, *utility services.* On the other hand, utility services become cheaper, which tends to increase the demand for utility services. Which effect is dominant turns out to depend in a simple way on the elasticity of demand for utility services.

The weakly separable functional form for utility allows us to define a composite commodity, x, such that the quantity of x is $f(x_M, x_A)$ and the "price" p of the composite commodity is just equal to the value of the expenditure function

$$p = e(p_M, p_A) = \min_{f(x_M, x_A)=1} p_M x_M + p_A x_A. \tag{4}$$

Since the equilibrium conditions determine p_A and p_M as functions of ρ, we can define

$$p(\rho) = e(p_M(\rho), p_A(\rho)). \tag{5}$$

Let $D_x(p)$ be the total demand for the composite commodity with price p, and let η denote the price elasticity of demand. We use duality theory to prove:

Lemma 3. Under Assumption T, P1, and R, the following hold:

$$\frac{d \ln p(\rho)}{d \ln \rho} = -L_M\sigma(1 - \theta_A).$$

$$\frac{d \ln K}{d \ln \rho} = -\theta_M\sigma(1 + L_M\eta).$$

Proof. See the Appendix.

The first result in Lemma 3 states that if demand in the afternoon exceeds demand in the morning at the price ratio $\rho = p_A/p_M$, then moving toward peak-load pricing must lower

the price of the composite good. This result is not surprising, since an increase in ρ tends to equalize demand in the morning and afternoon, thus allowing more "efficient" production of the composite commodity. But whether a reduction in its price results in an increase or a decrease in expenditure on the commodity depends on the magnitude of the elasticity of demand, as seen in the second result of the lemma. From these results we deduce

Theorem 2. Under Assumptions T, P1, and R, moving toward peak-load pricing will lower the price of the composite good, utility services, and will increase or decrease the equilibrium capacity depending on whether the absolute value of the price elasticity of demand for the composite good is greater or smaller than $p_M/(p_M - u_M)$.

In case $u_M = u_A = 0$, it is easy to interpret this result. Since the composite price p falls when prices move toward peak-load pricing, total consumer expenditures must increase if the aggregate elasticity is greater than one. But the rate of return on capacity is constrained to stay constant, and since user costs are zero it must be that in equilibrium the extra consumer expenditures are spent on more capacity. If instead user costs are positive, then lowering the morning price increases off-peak utilization of the capacity, which increases the total off-peak user costs. Only when the demand elasticity is large enough relative to the user cost effect $(-\eta > 1/L_M)$ will an increase in total expenditure require a higher equilibrium capacity.

5. Who gains and who loses from peak-load pricing?

■ In the last section we showed that when preferences over time of use are homothetic and identical, moving from uniform pricing toward time-of-day pricing will reduce the cost of the composite good, "utility services," for every consumer. All consumers benefit from the change. Now suppose that preferences differ between individuals. If the prices of morning and afternoon consumption both fall as the system is moved toward peak-load pricing, then of course all consumers will benefit. But if the price of afternoon consumption rises and the price of morning consumption falls, then those for whom an especially large proportion of consumption is in the afternoon might be worse off.

To analyze these effects, we assume:

Assumption P2. Different consumers have different aggregator functions, $f_i(x_M, x_A)$, that are homogeneous of degree one. The ratio χ of *total* afternoon demand to total morning demand is determined by the ratio of ρ of the afternoon price to the morning price.[5]

Then, just as in the earlier sections, we can define the elasticity of substitution to be $\sigma(\rho) = -d \ln \chi(\rho)/d \ln \rho$.

It is easy to figure out whether a consumer is a net gainer from the price change. Since the aggregator functions f_i are assumed to be homothetic, all we need to do to find out whether i is a gainer or a loser is to see whether the unit cost to i of producing one unit of the aggregate $f_i(x_A, x_M)$ has gone up or down. Using duality theory we can prove

Theorem 3. Under Assumptions T, P2, and R, a move toward peak-load pricing will benefit consumer i if

$$\frac{\theta_M^i}{\theta_M} > 1 - \sigma(\rho)L_M \tag{6}$$

and will make consumer i worse off if the inequality is reversed.

Proof. See the Appendix.

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[5] If, for example, utility takes the quasi-linear form $U_i(y^i, f^i(x_M, x_A)) = y^i + f^i(x_M, x_A)$, this assumption will be satisfied.

In particular, a move toward peak-load pricing will (a) yield a Pareto improvement if $\sigma L_M > 1$ and (b) benefit customer i for *any* substitution elasticity σ if i has a higher than average off-peak expenditure share ($\theta_M^i > \theta_M$) and if $L_M \geq 0$.

6. Price reversals

■ We have implicitly assumed that if a public utility is enabled to use time-of-day pricing, it will increase the ratio of peak to off-peak price. Bailey and White (1974) show that time-of-day pricing may lead to "price reversals" for an Averch-Johnson monopolist, who is constrained to a rate of return, c_K, greater than the market rate of return on capital. That is, the monopolist may actually charge a *lower* price in the peak than in the off-peak period. In their model, demand in each period depends only on the price in that period. The same effect can occur in our model, where morning and afternoon consumption may be complements or substitutes.

It is quite easy to see in our case why this happens. Since the Averch-Johnson monopolist is constrained to the rate of return c_K on his investment, and since c_K exceeds the market rate of return, the monopolist's objective is equivalent to maximizing his capacity subject to his rate of return on capacity being at least c_K.[6]

Lemma 4 tells us how capacity changes with ρ. Indeed,

$$\frac{d \ln K}{d \ln \rho} = -\theta_M \sigma (1 + L_M \eta),$$

where $L_M = 1 - u_M/p_M$ and η is the elasticity of demand for the composite commodity, utility services. Thus, for example, in the case where $u_M = 0$, capacity will be a decreasing function of ρ if demand for utility services is inelastic. If this is the case, the Averch-Johnson monopolist who starts out with equal prices in the morning and the afternoon will want to reduce the ratio, ρ, of peak to off-peak prices.

The underlying reason is simple. The monopolist is assumed not to be able to increase his rate base by adding useless capital. The only method available to him for increasing his rate base is to use capital "wastefully" by using a "perverse" time-of-day pricing structure. An inefficient time-of-day pricing structure makes the cost of the composite commodity higher for consumers, but this does not reduce the monopolist's revenue when demand is inelastic.

7. Remarks

■ For public utilities and regulators who are considering the introduction of time-of-use pricing, the questions we have posed and partially answered are of great interest.[7] The answers to these questions depend on the size of two key empirical magnitudes: the elasticity of substitution between peak and off-peak services and the demand elasticity for the composite utility services commodity. What does the empirical literature say about these magnitudes? In the time-of-day electricity literature, most estimates of the substitution elasticity are quite low; the range of 0.10 to 0.14 reported in Caves, Christensen, and Herriges (1984) is typical. It is important to notice, however, that these studies have measured only short-run response to price changes. In the long run, when users have time to adjust their appliance holdings or redesign their factories, the substitution elasticity may be much greater in absolute value.

Estimates of short-run demand elasticities for electricity and telephone services are also

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[6] Like Bailey and White (1974), we assume that the monopolist is not allowed to include "useless" capital in its rate base, but must actually put the investment to work in the form of operating, peak-load capacity.

[7] We have also studied (1989) an unconstrained monopolist and more general technologies, and obtained similar results.

rather low. For instance, MacKie-Mason and Lawson (1991) find price elasticities for local telephone calling demand ranging from −0.1 to −0.4 depending on the time of day. The presence of adjustment costs should again imply that long-run elasticities are higher. (Taylor (1977) surveys several older studies that report long-run demand elasticities greater than one in magnitude.) According to our results, if the long-run demand elasticity is greater than one, then equilibrium capacity under time-of-day pricing may be greater than under uniform pricing.

Appendix

■ Proofs of Lemmas 1, 2, and 3 and Theorems 3 and 4 follow.

Proof of Lemma 1. In Assumption R, substitute x_A for K, substitute ρp_M for p_A, divide both sides by x_M, and rearrange. The resulting expression is

$$p_M\left(\rho\,\frac{x_A}{x_M} + 1\right) = (c_K + u_A)\,\frac{x_A}{x_M} + u_M. \tag{A1}$$

But $x_A/x_M = \chi(\rho)$. It follows from equation (A1) that

$$p_M = \frac{(c_K + u_A)\chi(\rho) + u_M}{1 + \rho\chi(\rho)}.$$

Therefore we see that ρ uniquely determines p_M. Since $p_A = \rho p_M$, both prices are determined by their ratio. Then quantities are determined by $x_A = x(p_A, p_M)$ and $x_M = x_M(p_A, p_M)$.   Q.E.D.

Fact 1. Under Assumptions T, P1, and R, with $\sigma(\rho)$ the (negative of the) elasticity of substitution between x_M and x_A at the price ratio ρ,

$$\frac{d \ln \theta_A}{d \ln \rho} = \theta_M(1 - \sigma(\rho)).$$

Proof. See Bergstrom and MacKie-Mason (1989).

Proof of Lemma 2. If we multiply both sides of the equation in Assumption R by θ_A/x_A, we have

$$p_A = \left(c_K + u_A + u_M\,\frac{x_M}{x_A}\right)\theta_A. \tag{A2}$$

From the fact that $\theta_M = 1 - \theta_A$, it follows that $\dfrac{x_M}{x_A}\theta_A = \rho(1 - \theta_A)$. Therefore, equation (A2) is equivalent to

$$p_A = (c_K + u_A - u_M\rho)\theta_A + u_M\rho. \tag{A3}$$

Differentiating, multiplying both sides by ρ/p_A, and simplifying, we have

$$\frac{d \ln p_A}{d \ln \rho} = (c_K + u_A - u_M\rho)\,\frac{\theta_A}{p_A}\,\frac{d \ln \theta_A}{d \ln \rho} + \frac{u_M}{p_A}\,\rho(1 - \theta_A). \tag{A4}$$

From (A3) and the definition of L_M, we see that

$$(c_K + u_A - u_M\rho)\,\frac{\theta_A}{p_A} = 1 - \frac{u_M}{p_A}\,\rho = 1 - \frac{u_M}{p_M} = L_M. \tag{A5}$$

Therefore, using Fact 1 and (A5) we can simplify (A4) to

$$\frac{d \ln p_A}{d \ln \rho} = L_M\theta_M(1 - \sigma(\rho)) + \frac{u_M}{p_A}\,\rho(1 - \theta_A). \tag{A6}$$

Since $\dfrac{u_M}{p_A}\,\rho(1 - \theta_A) = (1 - L_M)\theta_M$, we can further simplify (A6) to

$$\frac{d \ln p_A}{d \ln \rho} = \theta_M(1 - L_M\sigma(\rho)).$$

This is the first equation claimed in Lemma 2. The second equation is a trivial consequence of the first.   Q.E.D.

Proof of Lemma 3. The first result follows from logarithmically differentiating text equation (5) and applying standard duality results. For the second result, note that total revenue from the sales of utility services, $p_A x_A + p_M x_M$, is equal to $p D_x(p)$. Therefore when $\chi(\rho) > 1$, so that $K = x_A$, we can write

$$\theta_A = \frac{p_A K}{p(\rho) D_x(p(\rho))} .$$ (A7)

Logarithmically differentiating both sides of (A7) and making substitutions from text equations (2) and (3) yields the result. *Q.E.D.*

Proof of Theorem 3. Subscript equation (5) by i to allow the expenditure function to vary across individuals. From standard duality results,

$$\frac{d \ln p_i(\rho)}{d \ln \rho} = \theta_A^i \frac{d \ln p_A}{d \ln \rho} + \theta_M^i \frac{d \ln p_M}{d \ln \rho} ,$$ (A8)

where θ_A^i and θ_M^i are the afternoon's and morning's shares of i's expenditures on utility services. Substituting from equations (2) and (3) into (A8) and rearranging terms, we find that

$$\frac{d \ln p^i(\rho)}{d \ln \rho} = \theta_M(1 - \sigma(\rho) L_M) - \theta_M^i.$$ (A9)

The theorem restates equation (A9) in terms of the conditions for an individual's composite price to fall. *Q.E.D.*

Proof of Theorem 4. Let the optimal uniform price be $\bar{p}$. Suppose that the uniform price is higher than both optimal time-of-day prices. Since demands are independent, charging instead a uniform price equal to the time-of-day peak-period price would maximize profits from that service. With a concave profit function for off-peak services, off-peak profits would also increase by moving the uniform price closer to the time-of-day optimum. Thus, the uniform price cannot be higher than the time-of-day peak-period price. Strict inequality is established by noting that marginal peak-period profit is zero at the optimal peak-period price, but marginal off-peak profits are negative at that price, so the uniform price will be set lower than the peak-period price. A similar argument works for a uniform price below p_M, so $p_M < \bar{p} < p_A$. *Q.E.D.*

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