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Minimization of Expected Distortion with Layer-Selective Relaying of Two-Layer Superposition Coding

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***Abstract*—This paper considers a relay system using two-layer superposition coding to minimize the expected distortion of a Gaussian source at the destination node. For the system, we propose two types of layer-selective relaying based on the local decoding result at the relay and the decoding result at the destination node fed back to the relay. One type of the proposed scheme uses decode-and-forward (DF) in the design of the relay signals, while the other type uses both DF and amplify-and-forward (AF). For the proposed scheme, we analyze the outage probabilities and evaluate the expected distortion according to the relay location. The results reveal that the proposed scheme improves the finite SNR performance, in particular when the relay node is closer to the source node than it is to the destination node.**

***Index Terms*—Superposition coding, Relay, Expected distortion, Feedback, Outage Probability**

I. Introduction

Cooperation and relaying among communicating nodes have attracted much attention from wireless researchers due to its capability of improving the reliability and coverage in information transfer. The frequently used strategies are amplify-and-forward (AF) and decode-and-forward (DF); the AF has a lower complexity than the DF but the latter outperforms the former at some relay locations [1]. With the pros and cons of the AF and DF, the researchers have developed various relaying schemes and analyzed their performances. However, the performances have been investigated mainly at the physical layer in terms of outage probabilities, symbol error rates, and average transmission rates [2].

Recently, the performances experienced at the higher layers are being incorporated in the design of physical layers for more efficient utilization of wireless resources. One of such efforts is to minimize the expected distortion between a transmitted source data and its reconstructed version at the receiver by taking into account source coding and channel transmission together [3]–[9]. In [4]–[6], various channel transmission schemes were studied in conjunction with successive refinement source coding when the instantaneous channel state information (CSI) is not available at the transmitter. The results showed that the superposition coding (SC) of the successively refinable source coding layers is among the most attractive solutions in reducing the expected distortion.

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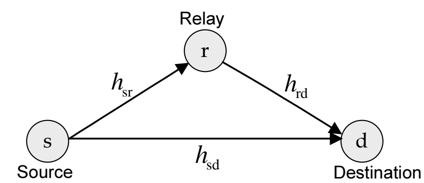

Fig. 1. A relay system with a non-vanishing direct channel.

In this paper, we also consider the problem of reducing expected distortion using SC in a relay network. The system model is similar to that in [3], [4], where the direct link between the source and destination nodes is available, and the number of layers in SC is limited to two, as in [3], [8], [9]. Unlike [3], [4] where the relay either DFs or AFs the received SC signal, the relay of the proposed scheme decodes the received signal and forwards only the layers required at the destination node by utilizing the decoding result at the destination node in the first slot. The decoding result of the destination node can be made available at the relay through the 2-bit feedback. In addition, the proposed scheme constructs the relay signals either by using only the DF signal for simplicity, or by using both the DF and AF signals to enhance the performance. For these schemes, we derive the outage probability of each layer when successive decoding is performed, and find the minimum expected distortion for various relay locations in the finite signal-to-noise power ratio (SNR) regime.

The rest of the paper is organized as follows. We describe the system model in Section II, and provide the proposed relaying scheme in Section III. The outage probabilities are analyzed in Section IV, with which the optimum expected distortion is computed in Section V. Finally, we provide concluding remarks in Section VI.

II. System Model

Consider a relay system described in Fig. 1, where a source node (s) wishes to send a memoryless, zero mean, unit variance, complex Gaussian source to a destination node (d) with the help of a relay node (r). The distortion-rate function of the Gaussian source is given by $D(R_S) = 2^{-R_S}$ at the source code rate R_S bits-per-source-sample under the squared-error distortion measure [10]. In the system, each block of K

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source samples is transmitted over a frame of N channel uses, which leads to a source-channel mismatch factor (also called bandwidth expansion ratio) [4]

$$b = \frac{N}{K} \text{ channel uses per source sample.} \quad (1)$$

For a channel code rate R_C bits-per-channel-use and a mismatch factor b, the source code rate is given by $R_S = bR_C$ bits-per-source-sample. Here, we assume that K and N are large enough to approach the rate-distortion bound and the channel capacity.

The channels between the nodes are assumed to be Rayleigh fading, independent of one another, and quasi-static over N channel uses. Let h_{uv} denote the complex channel gain, which is assumed constant over N channel uses between node u and node v at distance ς_{uv} for $uv \in \{\mathrm{sd}, \mathrm{sr}, \mathrm{rd}\}$ in Fig. 1: $h_{uv} \sim \mathcal{CN}(0, \varsigma_{uv}^{-\nu})$, where $\sim$ stands for 'distributed as', $\mathcal{CN}(m, \sigma^2)$ denotes the circularly symmetric complex Gaussian distribution with mean m and variance σ^2, and ν is the path loss exponent. In the system, N channel uses are divided into two time slots of length $N/2$, with the first time slot being used for the source node transmission and the second time slot being used for the relay node transmission.

The source node transmits its Gaussian source with two-layer SC, where layer 2 carries the refinement information of layer 1. The transmit symbol at the nth channel use is then expressed as

$$x(n) = \sum_{l=1}^{2} \sqrt{\mathcal{P}_\mathrm{s} \alpha_l} x_l(n), \ n = 1, 2, \cdots, N/2, \quad (2)$$

where $x_l(n)$ is the channel coding output of layer l, $\mathcal{P}_\mathrm{s}$ is the transmit power of the source node, and $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$ is the power allocation vector to the layers subject to $\alpha_1 \geq 0$, $\alpha_2 \geq 0$, and $\alpha_1 + \alpha_2 = 1$. Here, $x_l(n) \sim \mathcal{CN}(0, 1)$ by applying independent identically distributed (iid) complex Gaussian codebooks for each layer. The channel code rate of layer l is given by R_l in bits-per-channel-use, averaged over N channel uses. The source code rate of layer l is then given by bR_l bits-per-source-sample.

In the first slot, the received signal is given by

$$y_{v,1}(n) = h_{\mathrm{s}v} x(n) + w_{v,1}(n), \ n = 1, 2, \cdots, N/2, \quad (3)$$

where $v \in \{\mathrm{r}, \mathrm{d}\}$ denotes the receiving node and $w_{v,i}(n) \sim \mathcal{CN}(0, \sigma^2)$ is the additive noise at node v in the ith time slot. In the second slot, the relay node transmits a signal $z(n)$, which is generated from $y_{\mathrm{r},1}(n)$ under the relay power constraint $\mathcal{P}_\mathrm{r}$: The proposed signal $z(n)$ will be described in the following section. The received signal at the destination node in the second slot is then given by

$$y_{\mathrm{d},2}(n) = h_{\mathrm{rd}} z(n) + w_{\mathrm{d},2}(n), \ n = 1, 2, \cdots, N/2. \quad (4)$$

In our system, the receiving nodes perform successive decoding, which decodes layer 1 first and then decodes layer 2 only if layer 1 is decoded successfully and removed from the received signal. We assume this latter cancellation is done perfectly.

TABLE I
PROPOSED RELAYING STRATEGIES ACCORDING THE DECODING RESULTS $(Q_\mathrm{d}, Q_\mathrm{r})$ IN THE FIRST SLOT.

$(Q_\mathrm{d}, Q_\mathrm{r})$	PSR-D	PSR-M
$(0, 0)$	No tx	AF: all layers
$(0, 1)$	DF: first layer	DF: first layer
$(0, 2)$	DF: all layers	DF: all layers
$(1, 0)$	No tx	AF: all layers
$(1, 1)$	No tx	AF: second layer
$(1, 2)$	DF: second layer	DF: second layer
others	No tx	No tx

III. The Proposed Relaying Scheme

In the conventional SC-based relaying (CSR) scheme considered in [3], the relay chooses its signal according to its own decoding result $Q_\mathrm{r} \in \{0, 1, 2\}$, where 0 denotes the decoding failure of both layers, 1 denotes the decoding success of layer 1 only, and 2 denotes the decoding success of layers 1 and 2. Specifically, the relay transmits nothing if $Q_\mathrm{r} = 0$, transmits layer 1 with full power if $Q_\mathrm{r} = 1$, and transmits both layers with the power allocation used at the source node if $Q_\mathrm{r} = 2$. The proposed SC-based relaying (PSR) scheme extends the CSR by allowing feedback information from the destination node to the relay node on the decoding result $Q_\mathrm{d} \in \{0, 1, 2\}$ in the first slot. Thereby, the relay selects a relay signal based on $(Q_\mathrm{d}, Q_\mathrm{r})$ to avoid unnecessary relaying of the information already available at the destination node. In this paper, we consider two types of the PSR, PSR-D supporting DF signals only and PSR-M supporting both DF and AF signals, as summarized in Table I.

The relay signal of PSR-D is given by

$$z^D(n) = \begin{cases} G_{Q_\mathrm{d}, Q_\mathrm{r}} \sum\limits_{l=Q_\mathrm{d}+1}^{Q_\mathrm{r}} \sqrt{\alpha_l} x_l(n), & \text{if } Q_\mathrm{d} < Q_\mathrm{r} \\ 0, & \text{if } Q_\mathrm{d} \geq Q_\mathrm{r} \end{cases}, \quad (5)$$

while the relay signal of PSR-M is given by

$$z^M(n) = \begin{cases} z^D(n), & \text{if } Q_\mathrm{d} < Q_\mathrm{r} \\ G_{Q_\mathrm{d}, Q_\mathrm{r}} \left\{ y_{\mathrm{r},1}(n) - h_{\mathrm{sr}} \sum\limits_{l=1}^{Q_\mathrm{r}} \sqrt{\mathcal{P}_\mathrm{s} \alpha_l} x_l(n) \right\}, & \\ & \text{if } Q_\mathrm{d} \geq Q_\mathrm{r} \end{cases}, \quad (6)$$

where $G_{Q_\mathrm{d}, Q_\mathrm{r}} = \sqrt{\mathcal{P}_\mathrm{r} / \sum_{l=Q_\mathrm{d}}^{Q_\mathrm{r}} \alpha_l}$ if $Q_\mathrm{d} < Q_\mathrm{r}$, and $G_{Q_\mathrm{d}, Q_\mathrm{r}} = \sqrt{\mathcal{P}_\mathrm{r} / (\mathcal{P}_\mathrm{s} |h_{\mathrm{sr}}|^2 \sum_{l=Q_\mathrm{r}+1}^{Q_\mathrm{d}} \alpha_l + \sigma^2)}$ if $Q_\mathrm{d} \geq Q_\mathrm{r}$, to make the relay signal power $\mathcal{P}_\mathrm{r}$. The received signal (4) can then be rewritten as

$$y_{\mathrm{d},2}(n) = h_{\mathrm{rd}} G_{Q_\mathrm{d}, Q_\mathrm{r}} \sum_{l=Q_\mathrm{d}+1}^{Q_\mathrm{r}} \sqrt{\alpha_l} x_l(n) + w_{\mathrm{d},2}(n) \quad (7)$$

for $Q_\mathrm{d} < Q_\mathrm{r}$, for both PSR-D and PSR-M. When $Q_\mathrm{d} \geq Q_\mathrm{r}$, $y_{\mathrm{d},2}(n) = w_{\mathrm{d},2}(n)$ for PSR-D and

$$y_{\mathrm{d},2}(n) = h_{\mathrm{rd}} G_{Q_\mathrm{d}, Q_\mathrm{r}} h_{\mathrm{sr}} \sum_{l=Q_\mathrm{r}+1}^{2} \sqrt{\mathcal{P}_\mathrm{s} \alpha_l} x_l(n) + \tilde{w}_{\mathrm{d},2}(n) \quad (8)$$

with $\tilde{w}_{\mathrm{d},2}(n) = G_{Q_\mathrm{d}, Q_\mathrm{r}} h_{\mathrm{rd}} w_{\mathrm{r},1}(n) + w_{\mathrm{d},2}(n)$ for PSR-M.

For the proposed scheme, the destination node needs to decode the layers in the first slot to feed back $Q_{\rm d}$. In the second slot, the destination node attempts to decode layer l only if $l > Q_{\rm d}$ (i.e., when layer l is is not decoded successfully in the first slot) and the relay signal contains the information on layer l (i.e., when combining the relay signal with the directly received signal improves the SNR). Therefore, the final decoding of layer l at the destination node can be performed in the first slot or in the second slot, according to $(Q_{\rm d}, Q_{\rm r})$. To identify the SNR improvement provided by the proposed scheme, let $\eta_{v,l}$ for $v \in \{{\rm r}, {\rm d}\}$ denote the effective SNR experienced by layer l when node v performs decoding of layer l in the first slot, and let $\eta_{{\rm f},l}$ denote the final effective SNR of layer l experienced by layer l when the destination node performs the final decoding of layer l; from successive decoding, $\eta_{x,1}$ is obtained by regarding layer 2 as interference while $\eta_{x,2}$ is obtained by assuming perfect cancelation of layer 1 for $x \in \{{\rm r}, {\rm d}, {\rm f}\}$.

Note that $\eta_{v,l}$, for $v \in \{{\rm r}, {\rm d}\}$, is given by

$$\eta_{v,1} = \frac{\alpha_1 \mathcal{P}_{\rm s} |h_{{\rm s}v}|^2}{\alpha_2 \mathcal{P}_{\rm s} |h_{{\rm s}v}|^2 + \sigma^2} = \Psi_1(\gamma_{{\rm s}v}), \quad (9)$$

$$\eta_{v,2} = \frac{\alpha_2 \mathcal{P}_{\rm s} |h_{{\rm s}v}|^2}{\sigma^2} = \Psi_2(\gamma_{{\rm s}v}) \quad (10)$$

from (3), where $\gamma_{{\rm s}v} = \frac{\mathcal{P}_{\rm s}|h_{{\rm s}v}|^2}{\sigma^2}$, $\Psi_1(x) = \frac{\alpha_1 x}{\alpha_2 x + 1}$, and $Psi_2(x) = \alpha_2 x$. When the final decoding is performed in the second slot, the effective SNR is obtained with optimal combining [11] of

$$\tilde{\boldsymbol{y}}_1(n) = [y_{{\rm d},1}(n) \ y_{{\rm d},2}(n)]^T = \boldsymbol{h}_1 x_1(n) + \tilde{\boldsymbol{y}}_2(n) \quad (11)$$

for layer 1, and

$$\tilde{\boldsymbol{y}}_2(n) = \boldsymbol{h}_2 x_2(n) + \tilde{\boldsymbol{y}}_3(n). \quad (12)$$

for layer 2. Here,

$$\boldsymbol{h}_l = [h_{\rm sr}\sqrt{\mathcal{P}_{\rm s}\alpha_l} \ G_{Q_{\rm d},Q_{\rm r}} h_{\rm rd}\sqrt{\alpha_l}]^T \quad (13)$$

$$\tilde{\boldsymbol{y}}_3(n) = [w_{{\rm d},1}(n) \ w_{{\rm d},2}(n)]^T \quad (14)$$

if $Q_{\rm d} < l \le Q_{\rm r}$ for both PSR-D and PSR-M, while

$$\boldsymbol{h}_l = [h_{\rm sr}\sqrt{\mathcal{P}_{\rm s}\alpha_l} \ \ G_{Q_{\rm d},Q_{\rm r}} h_{\rm rd} h_{\rm sr}\sqrt{\mathcal{P}_{\rm s}\alpha_l}]^T \quad (15)$$

$$\tilde{\boldsymbol{y}}_3(n) = [w_{{\rm d},1}(n) \ \ w_{{\rm d},2}(n) + G_{Q_{\rm d},Q_{\rm r}} h_{\rm rd} w_{{\rm r},1}(n)]^T \quad (16)$$

if $l > Q_{\rm d} \ge Q_{\rm r}$ for PSR-M.

If $Q_{\rm d} < l \le Q_{\rm r}$, optimal combining is performed for both PSR-D and PSR-M, when $(Q_{\rm d}, Q_{\rm r}) = (0,1), (0,2)$ if $l = 1$ and $(Q_{\rm d}, Q_{\rm r}) = (0,1), (0,2), (1,2)$ if $l = 2$, which contains the information on layer l in the relay signal. If $l > Q_{\rm d} \ge Q_{\rm r}$, optimal combining is performed for PSR-M only, when $(Q_{\rm d}, Q_{\rm r}) = (0,0)$ if $l = 1$ and $(Q_{\rm d}, Q_{\rm r}) = (0,0), (1,0), (1,1)$ if $l = 2$. For the other cases of $(Q_{\rm d}, Q_{\rm r})$, $\eta_{{\rm f},l} = \eta_{{\rm d},l}$, since there is no decoding in the second slot. The resulting final effective SNRs, $\eta_{{\rm f},l}^D$ and $\eta_{{\rm f},l}^M$, for PSR-D and PSR-M, respectively, are summarized in Table II, where $\gamma_{\rm rd} = \frac{\mathcal{P}_{\rm r}|h_{\rm rd}|^2}{\sigma^2}$, $\gamma_{\rm eq} = \gamma_{\rm sd} + \frac{\gamma_{\rm sr}\gamma_{\rm rd}}{\gamma_{\rm sr} + \gamma_{\rm rd} + 1}$, $\gamma_{\rm eq2} = \gamma_{\rm sd} + \frac{\gamma_{\rm sr}\gamma_{\rm rd}}{\alpha_2\gamma_{\rm sr} + \gamma_{\rm rd} + 1}$.

TABLE II
FINAL EFFECTIVE SNRS.

$(Q_{\rm d}, Q_{\rm r})$	PSR-D		PSR-M	
	$\eta_{{\rm f},1}^D$	$\eta_{{\rm f},2}^D$	$\eta_{{\rm f},1}^M$	$\eta_{{\rm f},2}^M$
$(0,0)$	$\Psi_1(\gamma_{\rm sd})$	$\Psi_2(\gamma_{\rm sd})$	$\Psi_1(\gamma_{\rm eq})$	$\Psi_2(\gamma_{\rm eq})$
$(0,1)$	$\Psi_1(\gamma_{\rm sd}) + \gamma_{\rm rd}$	$\Psi_2(\gamma_{\rm sd})$	$\eta_{{\rm f},1}^D$	$\eta_{{\rm f},2}^D$
$(0,2)$	$\Psi_1(\gamma_{\rm sd} + \gamma_{\rm rd})$	$\Psi_2(\gamma_{\rm sd} + \gamma_{\rm rd})$	$\eta_{{\rm f},1}^D$	$\eta_{{\rm f},2}^D$
$(1,0)$	$\Psi_1(\gamma_{\rm sd})$	$\Psi_2(\gamma_{\rm sd})$	$\eta_{{\rm f},1}^D$	$\Psi_2(\gamma_{\rm eq})$
$(1,1)$	$\Psi_1(\gamma_{\rm sd})$	$\Psi_2(\gamma_{\rm sd})$	$\eta_{{\rm f},1}^D$	$\Psi_2(\gamma_{\rm eq2})$
$(1,2)$	$\Psi_1(\gamma_{\rm sd})$	$\Psi_2(\gamma_{\rm sd}) + \gamma_{\rm rd}$	$\eta_{{\rm f},1}^D$	$\eta_{{\rm f},2}^D$
others	$\Psi_1(\gamma_{\rm sd})$	$\Psi_2(\gamma_{\rm sd})$	$\eta_{{\rm f},1}^D$	$\eta_{{\rm f},2}^D$

IV. OUTAGE PROBABILITY

In the following, we derive the outage probability $P_{{\rm out},l}$ of each layer of the PSR for given rate and power allocation $(\boldsymbol{R}, \boldsymbol{\alpha})$. The superscript X (= D or M) is used to denote the related PSR type in the sequel.

Consider the decoding result Q_v for $v \in \{{\rm r}, {\rm d}\}$, given by

$$Q_v = \begin{cases} 0, & \text{if } \frac{1}{2}C(\eta_{v,1}) \le R_1 \\ 1, & \text{if } \{\frac{1}{2}C(\eta_{v,1}) > R_1, \ \frac{1}{2}C(\eta_{v,2}) \le R_2\} \\ 2, & \text{if } \frac{1}{2}C(\eta_{v,2}) > R_2 \end{cases}, \quad (17)$$

where $C(x) = \log_2(1+x)$. The region $\mathcal{A}_{v,i}$ of $\gamma_{{\rm s}v}$ leading to $\{Q_v = i\}$ is then given by

$$\mathcal{A}_{v,i} = \{\Gamma_i \le \gamma_{{\rm s}v} < \Gamma_{i+1}\}, \ i = 0, 1, 2, \quad (18)$$

with $\Gamma_0 = 0$, $\Gamma_1 = \Psi_1^{-1}(2^{2R_1} - 1)$, $\Gamma_2 = \Psi_2^{-1}(2^{2R_2} - 1)$, and $\Gamma_3 = \infty$, where $\Psi_1^{-1}(x) = \frac{x}{\alpha_1 - \alpha_2 x}$ and $\Psi_2^{-1}(x) = \frac{x}{\alpha_2}$ from (9) and (10). On the other hand, the final decoding result, $Q_{\rm f}$, is determined by the region of $\boldsymbol{\gamma} = (\gamma_{\rm sd}, \gamma_{\rm sr}, \gamma_{\rm rd})$, since $\eta_{{\rm f},1}$ and $\eta_{{\rm f},2}$ are functions of $\boldsymbol{\gamma}$ and $(Q_{\rm d}, Q_{\rm r})$.

Let $\mathcal{B}_{ij,k}^X$ denote the region of $\boldsymbol{\gamma}$ leading to $(Q_{\rm d}, Q_{\rm r}) = (i,j)$ and $Q_{\rm f} = k$, when the PSR-X is employed. The outage probabilities can then be expressed $P_{{\rm out},1}^X = \sum_{i=0}^{2}\sum_{j=0}^{2} \Pr[\mathcal{B}_{ij,0}^X]$ and $P_{{\rm out},2}^X = 1 - \sum_{i=0}^{2}\sum_{j=0}^{2} \Pr[\mathcal{B}_{ij,2}^X]$. Since $\mathcal{B}_{ij,0}^X = \phi$ if $i \ge 1$ and $\sum_{j=0}^{2} \Pr[\mathcal{B}_{2j,2}^X] = \Pr[\mathcal{A}_{{\rm d},2}]$, we have

$$P_{{\rm out},1}^X = \sum_{i=0}^{2}\sum_{j=0}^{2} \Pr[\mathcal{B}_{ij,0}^X] = \sum_{j=0}^{2} \Pr[\mathcal{B}_{0j,0}^X] \quad (19)$$

$$P_{{\rm out},2}^X = 1 - \Pr[\mathcal{A}_{{\rm d},2}] - \sum_{i=0}^{1}\sum_{j=0}^{2} \Pr[\mathcal{B}_{ij,2}^X]. \quad (20)$$

With the region $\mathcal{B}_{ij,k}^X$ given, $\Pr[\mathcal{B}_{ij,k}^X]$ is then obtained with the joint probability density function (pdf) of $\boldsymbol{\gamma}$, given by

$$f_{\boldsymbol{\gamma}}(x,y,z) = \frac{1}{\Omega_{\rm sd}\Omega_{\rm sr}\Omega_{\rm rd}} e^{-\left(\frac{x}{\Omega_{\rm sd}} + \frac{y}{\Omega_{\rm sr}} + \frac{z}{\Omega_{\rm rd}}\right)}, \ x, y, z \ge 0, \quad (21)$$

when $\boldsymbol{\Omega} = E\{\boldsymbol{\gamma}\} = (\Omega_{\rm sd}, \Omega_{\rm sr}, \Omega_{\rm rd})$.

Let us now rewrite $\eta_{{\rm f},l}^X$ when $(Q_{\rm d}, Q_{\rm r}) = (i,j)$ as $\eta_{{\rm f},l}^X(i,j)$, with which the regions required for (19) and (20) can be expressed as

$$\mathcal{B}^X_{0j,0} = (\mathcal{A}_{\mathrm{d},0} \cap \mathcal{A}_{\mathrm{r},j}) \cap \{\eta^X_{\mathrm{f},1}(0,j) < \Psi_1(\Gamma_1)\} \quad (22)$$

$$\mathcal{B}^X_{ij,2} = (\mathcal{A}_{\mathrm{d},i} \cap \mathcal{A}_{\mathrm{r},j}) \cap \{\eta^X_{\mathrm{f},l}(i,j) \geq \Psi_l(\Gamma_l),\ l=1,2\}, \quad (23)$$

since $\Psi_l(\Gamma_l) = 2^{2R_l} - 1$ with Γ_l given below (18).

A. PSR-D

The regions $\{\mathcal{B}^D_{0j0}\}_{j=0}^2$ for PSR-D are obtained by inserting $\eta^D_{\mathrm{f},l}$ in Table II into (22) as

$$\mathcal{B}^D_{00,0} = \{0 \leq \gamma_{\mathrm{sd}} < \Gamma_1, 0 \leq \gamma_{\mathrm{sr}} < \Gamma_1\} \quad (24)$$

$$\mathcal{B}^D_{01,0} = \{0 \leq \gamma_{\mathrm{sd}} < \Gamma_1, \Gamma_1 \leq \gamma_{\mathrm{sr}} < \Gamma_2, \Psi_1(\gamma_{\mathrm{sd}}) + \gamma_{\mathrm{rd}} < \Psi_1(\Gamma_1)\} \quad (25)$$

$$\mathcal{B}^D_{02,0} = \{0 \leq \gamma_{\mathrm{sd}} < \Gamma_1, \gamma_{\mathrm{sr}} \geq \Gamma_2, \gamma_{\mathrm{sd}} + \gamma_{\mathrm{rd}} < \Gamma_1\}. \quad (26)$$

Similarly, by inserting $\{\eta^D_{\mathrm{f},l}\}_{l=1}^2$ into (23), we obtain

$$\mathcal{B}^D_{00,2} = \mathcal{B}^D_{01,2} = \mathcal{B}^D_{10,2} = \mathcal{B}^D_{11,2} = \phi \quad (27)$$

$$\mathcal{B}^D_{02,2} = \{0 < \gamma_{\mathrm{sd}} < \Gamma_1, \gamma_{\mathrm{sr}} \geq \Gamma_2, \gamma_{\mathrm{sd}} + \gamma_{\mathrm{rd}} \geq \Gamma_2\} \quad (28)$$

$$\mathcal{B}^D_{12,2} = \{\Gamma_1 \leq \gamma_{\mathrm{sd}} < \Gamma_2, \gamma_{\mathrm{sr}} \geq \Gamma_2, \alpha_2\gamma_{\mathrm{sd}} + \gamma_{\mathrm{rd}} \geq \alpha_2\Gamma_2\} \quad (29)$$

where (27) is straightforward, since no decoding for layer 2 is performed in the second slot.

With (24)-(29), (19) and (20) for PSR-D are derived as

$$P^D_{\mathrm{out},1} = 1 - e^{-\frac{\Gamma_1}{\Omega_{\mathrm{sd}}}} - \left(e^{-\frac{\Gamma_1}{\Omega_{\mathrm{sr}}}} - e^{-\frac{\Gamma_2}{\Omega_{\mathrm{sr}}}}\right)\Lambda - e^{-\left(\frac{\Gamma_2}{\Omega_{\mathrm{sr}}} + \frac{\Gamma_1}{\Omega_{\mathrm{rd}}}\right)} \frac{1}{a_1}\left\{1 - e^{-\frac{a_1\Gamma_1}{\Omega_{\mathrm{sd}}}}\right\}, \quad (30)$$

$$P^D_{\mathrm{out},2} = 1 - e^{-\frac{\Gamma_2}{\Omega_{\mathrm{sd}}}} - \frac{1}{a_1} e^{-\left(\frac{\Gamma_2}{\Omega_{\mathrm{sr}}} + \frac{\Gamma_2}{\Omega_{\mathrm{rd}}}\right)}\left\{1 - e^{-\frac{a_1\Gamma_1}{\Omega_{\mathrm{sd}}}}\right\} - \frac{1}{a_2} e^{-\left(\frac{\Gamma_1}{\Omega_{\mathrm{sd}}} + \frac{\Gamma_2}{\Omega_{\mathrm{sr}}} + \frac{\alpha_2(\Gamma_2-\Gamma_1)}{\Omega_{\mathrm{rd}}}\right)}\left\{1 - e^{-\frac{a_2(\Gamma_2-\Gamma_1)}{\Omega_{\mathrm{sd}}}}\right\}, \quad (31)$$

where $\Lambda = \frac{1}{\Omega_{\mathrm{sd}}}\int_0^{\Gamma_1} e^{-\left(\frac{x}{\Omega_{\mathrm{sd}}} + \frac{\Psi_1(\Gamma_1)-\Psi_1(x)}{\Omega_{\mathrm{rd}}}\right)} dx$, $a_1 = 1 - \frac{\Omega_{\mathrm{sd}}}{\Omega_{\mathrm{rd}}}$, and $a_2 = 1 - \frac{\alpha_2\Omega_{\mathrm{sd}}}{\Omega_{\mathrm{rd}}}$. To avoid the numerical integration, we obtain an upper bound on Λ given by

$$\overline{\Lambda} = \frac{1}{c_1} e^{-\frac{\Psi_1(\tau_1)}{\Omega_{rd}}}\left\{1 - e^{-\frac{c_1 x_o}{\Omega_{sd}}}\right\} + \frac{1}{c_2} e^{-\frac{\tau_1 + c_2(x_o - \tau_1)}{\Omega_{sd}}}\left\{1 - e^{-\frac{c_2(\tau_1 - x_o)}{\Omega_{sd}}}\right\}, \quad (32)$$

where $c_1 = 1 - \frac{\alpha_1\Omega_{sd}}{\Omega_{rd}}$, $c_2 = 1 - \frac{\Psi_1'(\tau_1)\Omega_{sd}}{\Omega_{rd}}$, $\Psi_1'(x) = \frac{d\Psi_1(x)}{dx}$, and $x_o = \frac{\Gamma_1}{\alpha_2\Gamma_1 + 2}$. Here, we apply the concave property of $\Psi_1(x)$ such that $\Psi_1(x) \leq \Psi_1'(0)x$ for $0 \leq x < x_o$ and $\Psi_1(x) \leq \Psi_1'(\Gamma_1)(x - \Gamma_1) + \Psi_1(\Gamma_1)$ for $x_o \leq x < \Gamma_1$, with x_o being the intersection of $\Psi_1'(0)x$ and $\Psi_1'(\Gamma_1)(x - \Gamma_1) + \Psi_1(\Gamma_1)$.

B. PSR-M

The outage probabilities of the PSR-M are related to those of the PSR-D as follows:

$$P^D_{\mathrm{out},1} = P^D_{\mathrm{out},1} - \Pr[\mathcal{B}^D_{00,0}] + \Pr[\mathcal{B}^M_{00,0}] \quad (33)$$

$$P^M_{\mathrm{out},2} = P^D_{\mathrm{out},2} - \Pr[\mathcal{B}^M_{00,2}] - \Pr[\mathcal{B}^M_{10,2}] - \Pr[\mathcal{B}^M_{11,2}] \quad (34)$$

since $\eta^M_{\mathrm{f},l} = \eta^D_{\mathrm{f},l}$ when $(Q_{\mathrm{d}}, Q_{\mathrm{r}}) = (0,1), (0,2), (1,2)$ and $\Pr[\mathcal{B}^D_{00,2}] = \Pr[\mathcal{B}^D_{10,2}] = \Pr[\mathcal{B}^D_{11,2}] = 0$. By inserting $\{\eta^M_{\mathrm{f},l}\}$ in Table II into (22) and (23), we have

$$\mathcal{B}^M_{00,0} = (\mathcal{A}_{\mathrm{d},0} \cap \mathcal{A}_{\mathrm{r},0}) \cap \{\gamma_{\mathrm{sd}} + \frac{\gamma_{\mathrm{sr}}\gamma_{\mathrm{rd}}}{\gamma_{\mathrm{sr}} + \gamma_{\mathrm{rd}} + 1} \leq \Gamma_1\} \quad (35)$$

$$\mathcal{B}^M_{00,2} = (\mathcal{A}_{\mathrm{d},0} \cap \mathcal{A}_{\mathrm{r},0}) \cap \{\gamma_{\mathrm{sd}} + \frac{\gamma_{\mathrm{sr}}\gamma_{\mathrm{rd}}}{\gamma_{\mathrm{sr}} + \gamma_{\mathrm{rd}} + 1} \geq \Gamma_2\} \quad (36)$$

$$\mathcal{B}^M_{10,2} = (\mathcal{A}_{\mathrm{d},1} \cap \mathcal{A}_{\mathrm{r},0}) \cap \{\gamma_{\mathrm{sd}} + \frac{\gamma_{\mathrm{sr}}\gamma_{\mathrm{rd}}}{\gamma_{\mathrm{sr}} + \gamma_{\mathrm{rd}} + 1} \geq \Gamma_2\} \quad (37)$$

$$\mathcal{B}^M_{11,2} = (\mathcal{A}_{\mathrm{d},1} \cap \mathcal{A}_{\mathrm{r},1}) \cap \{\gamma_{\mathrm{sd}} + \frac{\gamma_{\mathrm{sr}}\gamma_{\mathrm{rd}}}{\alpha_2\gamma_{\mathrm{sr}} + \gamma_{\mathrm{rd}} + 1} \geq \Gamma_2\} \quad (38)$$

We then obtain the probabilities in (33) and (34), via numerical integration, as $\Pr[\mathcal{B}^M_{ij,k}] = \int_{\mathcal{B}^M_{ij,k}} f_{\boldsymbol{\gamma}}(x,y,z)dzdydx$.

V. Performance Evaluation

With the source model described in Section II, and $P_{\mathrm{out},l}$ derived in Section IV, the expected distortion for given $\boldsymbol{R}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\Omega}$ is expressed as [4], [7]

$$ED(\boldsymbol{R},\boldsymbol{\alpha},\boldsymbol{\Omega}) = \sum_{l=0}^{2}(P_{\mathrm{out},l+1} - P_{\mathrm{out},l})2^{-b\sum_{m=1}^{l} R_m} \quad (39)$$

where $P_{\mathrm{out},0} = 0$ and $P_{\mathrm{out},3} = 1$. For each $\boldsymbol{\Omega}$, we then obtain the optimum expected distortion as

$$\mathrm{ED}(\boldsymbol{\Omega}) = \min_{(\boldsymbol{R},\boldsymbol{\alpha})} ED(\boldsymbol{R},\boldsymbol{\alpha},\boldsymbol{\Omega}) \quad (40)$$

$$\text{s.t. } R_l \geq 0,\ \alpha_l \geq 0,\ \alpha_1 + \alpha_2 = 1. \quad (41)$$

Since the optimization problem (41) is not convex, the optimum value is searched over the feasible set of $(\boldsymbol{R},\boldsymbol{\alpha})$. To reduce the searching time caused by the numerical integration in evaluating (30), we can use the lower bound on $P_{\mathrm{out},1}$ obtained by replacing Λ with its upper bound (32) in (30).

In the following performance evaluation, we assume that all nodes are located on a straight line so that $d_{\mathrm{rd}} = d_{\mathrm{sd}} - d_{\mathrm{sr}}$, the path loss exponent is set to $\nu = 3$, and $\mathcal{P}_{\mathrm{s}} = \mathcal{P}_{\mathrm{r}}$, so that $\Omega_{\mathrm{sr}} = \Omega\left(\frac{d_{\mathrm{sr}}}{d_{\mathrm{sd}}}\right)^{-\nu}$ and $\Omega_{\mathrm{rd}} = \Omega\left(1 - \frac{d_{\mathrm{sr}}}{d_{\mathrm{sd}}}\right)^{-\nu}$ when $\Omega_{\mathrm{sd}} = \Omega$.

We first verify our analysis on $P_{\mathrm{out},l}$ by comparing the results from a Monte Carlo simulation in Fig. 2 as Ω varies when $\frac{d_{\mathrm{sr}}}{d_{\mathrm{sd}}} = 0.5$, $\boldsymbol{R} = (1/2, 1/2)$, and $\boldsymbol{\alpha} = (0.8, 0.2)$. The marks and lines denote the results from simulation and analysis, respectively. The figure shows that the results from analysis agree with those from simulation. With the parameters used herein, PSR-M performs better than the PSR-D.

Fig. 3 shows the optimum expected distortion in dB relative to the maximum distortion of unity, when the relay is located at $\frac{d_{\mathrm{sr}}}{d_{\mathrm{sd}}} = 0.2$. In the figure, 'DT' denotes the direct transmission (DT) of SC without relaying [7] while 'CSR' and 'CAF' denote the conventional SC relaying schemes based on DF [3] and AF [4], respectively. When $\frac{d_{\mathrm{sr}}}{d_{\mathrm{sd}}} = 0.2$, PSR-D and PSR-M perform almost identically, and outperform the CAF and CSR; similar performance of PSR-D and PSR-M results from the fact that the relay closer to the source node is likely to decode two layers successfully, so that the relay signals of PSR-D and PSR-M tend to be the same. The proposed scheme performs better than the DT in the finite SNR regime even

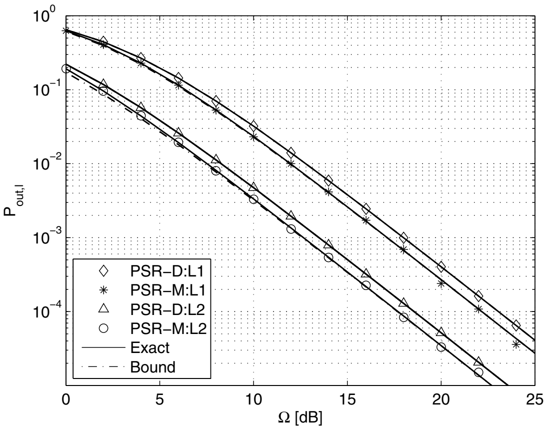

Fig. 2. Outage probabilities for the average SNR Ω of the direct channel when $\boldsymbol{R} = (0.5, 0.5)$ and $\boldsymbol{\alpha} = (0.8, 0.2)$.

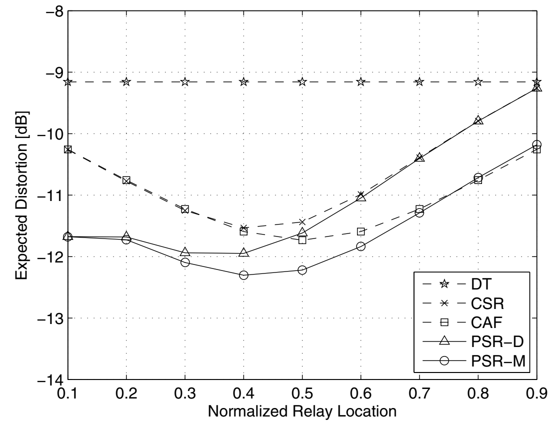

Fig. 4. Expected distortion for $d_{\rm sr}/d_{\rm sd}$ when $b = 2$ and $\Omega = 15$ dB.

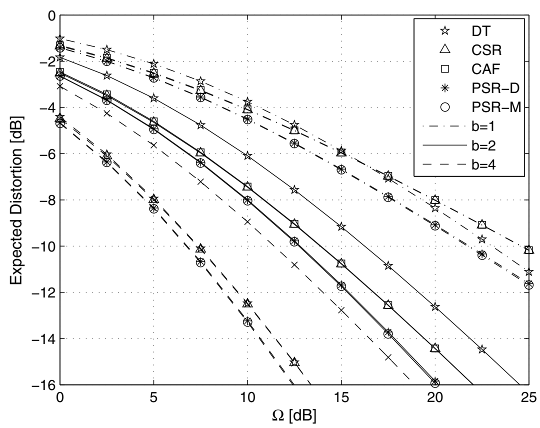

Fig. 3. Expected distortion for Ω when $d_{\rm sr}/d_{\rm sd} = 0.2$.

when $b = 1$, unlike the observation in [4] regarding the high SNR behavior. Furthermore, as b increases, the relay schemes tend to perform better than the DT, since the distortion due to the outages in channel transmission becomes more dominant than the distortion due to source coding, and the relaying schemes have lower outage probabilities than the DT.

We provide the expected distortion for the normalized relay location $\frac{d_{\rm sr}}{d_{\rm sd}}$ in Fig. 4 when $\Omega = 15$ dB and $b = 2$. It is observed that PSR-D can perform better than the CSR or CAF only when $\frac{d_{\rm sr}}{d_{\rm sd}} \lesssim 0.5$, while PSR-M performs best among the schemes when $\frac{d_{\rm sr}}{d_{\rm sd}} \lesssim 0.8$, at the cost of the feedback information. It is also observed that the gain of the proposed scheme over the CSR or CAF is larger as the relay is closer to the source node, since the proposed scheme can allocate a higher rate to the second layer by sending the first layer reliably over the direct channel in the first slot, while sending the second layer over the relay channel in the second slot.

VI. Conclusions

To reduce the expected distortion, we have considered a two-layer SC transmission over a three-node relay network and have proposed a relaying scheme using a simple feedback technique. The PSR constructs a relay signal according to the decoding results at the relay and destination nodes in the first slot. In constructing relay signals, we considered the PSR-D using DF signals only, and the PSR-M using both DF and AF signals. For both types of PSR, we evaluated the optimal expected distortion in the finite SNR regime. The results show that both PSR-D and PSR-M outperform the conventional DT and relaying schemes when the relay is located closer to the source node. In particular, PSR-M provides uniformly best performance over a wide range of relay locations using both AF and DF signals at the cost of feedback overhead of 2 bits.

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