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Unconventionally sharp dynamic resonances from a disordered Wigner crystal

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Abstract. Motivated by puzzling results of recent experiments, we re-examine the response of a weakly pinned two-dimensional Wigner crystal to a uniform AC electric field. We confirm that at some disorder and magnetic field dependent frequency ω_p , an inhomogeneously broadened absorption line emerges. Although the line is conventionally broad in zero magnetic field, in strong fields it appears as a sharp resonance whose width is related to the density of states in the low-frequency tail of the zero-field phonon spectrum. This behavior originates due to the long-range Coulomb interactions.

1. MOTIVATION

The problem of pinning of an elastic manifold by a random potential has been a long-standing problem in condensed matter physics, first addressed by Larkin [1] in 1970. Physical systems where this problem arises include charge-density-waves [2] and vortex lattices in superconductors [3]. About one decade ago another interesting system became available. This is a two-dimensional (2D) Wigner crystal (WC), which forms in quantum wells of semiconductor heterostructures at sufficiently strong magnetic fields and for low enough density of charge carriers. Recently, experimentalists began to study very low disorder samples where the pinning is presumably collective. In this regime individual defects of the substrate are too weak to significantly deform the crystalline lattice, so the WC is well ordered and unpinned at small length scales. However, the cumulative effect of the disorder dominates the elasticity at a pinning length that is much larger than the lattice constant.

The pinning is also known to modify the dynamical properties of the system, such as the linear response conductivity. In this work we focus on the real part of the dissipative conductivity $\sigma_{xx}(q = 0, \omega)$ that determines the power absorption. If the frequency ω is not too small, the absorption is dominated by the collective excitations of the WC, the phonons. In a magnetic field there are two types of such excitations: magnetophonons and magnetoplasmons. Without the pinning potential, the frequency and therefore the oscillator strength of the magnetophonon vanish at $q = 0$. The absorption is possible only by exciting the magnetoplasmon mode, at the frequency ω_c (the cyclotron frequency) set by the magnetic field B . The pinning shifts the original $q = 0$ magnetophonon mode to a nonzero frequency ω_p and also broadens it. The resulting structure is called a pinning mode. It gives rise to an additional absorption near ω_p , as was originally discussed by Fukuyama and Lee [4]. In that work as well as in later studies [5] the linewidth $\Delta\omega_p$ of the pinning mode was predicted to be of the order of ω_p itself. It came as a surprise when recent experiments [6,7] revealed that $\Delta\omega_p$ can be a factor of 30 smaller than ω_p . The explanation of this puzzling discrepancy is the subject of this paper.

2. QUALITATIVE DISCUSSION

The long-range Coulomb interactions play an important role in reducing the inhomogeneous broadening of the pinning mode, as was recently emphasized by Fertig [8]. In his analysis, which involves several simplifications (effectively, a commensurate pinning potential and a projection on the lowest Landau level) the linewidth is exponentially small [9] for weak pinning. We studied a more realistic model where the pinning is provided by a Gaussian random potential of a general type, and we do not

use the projection on the lowest Landau level. We confirm the crucial role of the Coulomb interaction but find that (a) two factors are needed to make the line narrow: the long-range interaction and the strong magnetic field, and (b) $\Delta\omega_p/\omega_p$ is not an exponential but instead a power-law function of the disorder strength,

$$\Delta\omega_p/\omega_p \sim (\omega_p/\omega_c)^s, \quad (1)$$

where s is close to 1.

The explanation of the surprisingly sharp dynamical response of a rather disordered system, such as a pinned WC, is based on the careful and novel analysis of the mutual interactions among the pinning potential and the two dynamical degrees of freedom of the WC: the longitudinal (l-) and the transverse (t-) phonons in the $B = 0$ case or, similarly, the l- and t-components of the magnetophonon in the large B case. At zero temperature and in the Ohmic regime the broadening of the absorption line occurs due to the scattering and localization of the low-frequency phonon modes by disorder. For a fixed disorder, the amount of broadening decreases with the group velocity $d\omega/dq$ of the phonons. Thus, in $B = 0$ case the slower t-phonons are scattered much stronger and become localized at the pinning length scale. In contrast, the direct scattering of the l-phonons by disorder, i.e., $l \rightarrow l$ channel, is considerably less effective. Dispersion relation $\omega \propto \sqrt{q}$ of the l-phonons caused by the long-range Coulomb interactions renders the "ordinary" Gaussian type of disorder marginal in 2D and the scattering rate in the $l \rightarrow l$ channel exponentially small. Instead, the main scattering channel is through an intermediate t-state: $l \rightarrow t \rightarrow l$ where the necessary l-t mixing is provided by disorder and (if $B \neq 0$) by the Lorentz force. The advantage of such a process is the possibility of the resonant enhancement if the frequencies of the incoming l-phonon and the virtual t-phonons are closely matched. This leads to an interesting picture of certain localized t-phonon modes playing the role of strong resonant scatterers for the l-phonons, which propagate on this sort of a dynamical background. This picture indicates that the dynamics of a weakly pinned WC is in some sense similar to that of the WC pinned by strong dilute impurities.

We emphasize that both the unscreened Coulomb interaction and a strong enough magnetic field are required to yield the sharp pinning resonance in 2D. In the absence of the magnetic field the strong resonant scatterers discussed above are abundant at the pinning frequency ω_0 and the line is broad. In fact, we derived the following estimate, $\Delta\omega_p/\omega_p \sim \nu(\omega_p^2)/\nu_0$, where $\nu(\epsilon)$ is the phonon density of states at "energy" $\epsilon = \omega^2$ and ν_0 is the density of states of the clean system [$\nu(\epsilon) = dN/d\epsilon$, dN being the number of phonon states whose frequencies satisfy $\epsilon < \omega^2 < \epsilon + d\epsilon$]. In the absence of the magnetic field $\nu(\omega_0^2) \sim \nu_0$, and so $\Delta\omega_p \sim \omega_p$. In contrast, a strong magnetic field shifts the central pinning frequency to a much lower value of $\omega_p = \omega_0^2/\omega_c$, which places it into the low-frequency tail of the zero-field density of states. The resonant scatterers of such a low frequency are therefore the zero-field "soft modes," which appear due to unusual, rare disorder configurations frustrating the ground state of the system (see, e.g., Ref. 10). For weak pinning and not too high a magnetic field, the marginality of the "ordinary" disorder allows such rare configurations to dominate the line broadening. It is expected that $\nu(\omega^2) \sim (\omega/\omega_0)^{2s}$ for $\omega \ll \omega_0$, which leads to Eq. (1).

Note that the exact value of s is controversial. In Ref. 11 $s \simeq 1$ was obtained for the one-dimensional case, while Aleiner and Ruzin [10] gave compelling arguments that $s = \frac{3}{2}$ in general. We leave the calculation of s in the *weak* pinning regime for future work.

Let us also mention another argument, which, although incomplete, still sheds the light on the physical mechanism of the narrowing of the absorption line. As we argued above, the application of a high magnetic field strongly decreases the scattering of the phonon modes. Correspondingly, the localization length of the phonons increases from its zero-field value L_l to a much larger value L_B . One can visualize each localized magneto-phonon as a coupled set of about $N = (L_B/L_l)^2$ localized phonons of the zero-field case. As a result, the $N \gg 1$ independent oscillators are in effect replaced by a single oscillator with the average frequency. The averaging reduces the inhomogeneous broadening and makes the line narrow.

Finally, to avoid a misunderstanding let us point out that $\Delta\omega_p/\omega_p$ does not decrease indefinitely

with B . Indeed, although the mean-free path $L_G \propto \exp(L_l/a)$ in the $l \rightarrow l$ channel is exponentially large (a being the WC lattice constant), L_B is increasing with B . At $\omega_c \propto L_G^{1/2s}$, L_B eventually reaches L_G . In even higher fields the resonant scatterers become so scarce that $l \rightarrow l$ channel dominates and $\Delta\omega_p/\omega_p$ saturates at the value of $\exp(-L_l/a)$.

3. COMPARISON WITH PREVIOUS WORK AND EXPERIMENTS

Now we compare our approach with other work on the subject. First of all, our interpretation of the line broadening in terms of the scattering of the coupled l - and t -phonon modes by disorder is completely novel. In some of the existing literature the physical picture is simply missing. Second, as far as the dynamical response is concerned, practically all other theories suggested so far, e.g., the variational replica method of Chitra *et al.* [12], are essentially equivalent to the Self-Consistent Born Approximation (SCBA). Let us briefly summarize the results thus obtained. Fukuyama and Lee [4] concluded that $\Delta\omega_p \sim \omega_p$ but did not present the details of the calculation. Fertig [9] obtained an exponentially small $\Delta\omega_p$. The approach of Chitra *et al.* [12], when analyzed further, would give $\Delta\omega_p/\omega_p \propto \omega_p^{1/4}/\rho_e^{3/8}$ where $\rho_e = 2/\sqrt{3}a^2$ is the average electron concentration. One reason for the variety of predictions is the uncertainty in the value of the dimensionless parameter $C \sim 1$, which enters the SCBA formalism. The results of Chitra *et al.* are recovered if $C = 1$; a slight reduction of C brings us to those of Fertig. In contrast, we found that in high magnetic fields $\Delta\omega_p$ is determined by the properties of the band tails (the very low frequency modes), in a way that is not captured by the SCBA.

Unfortunately, Eq. (1) with $s \sim 1$ strongly underestimates the measured linewidth [6,7]. There may be two reasons for such a discrepancy. On the one hand, other types of dilute strong scatterers such as residual impurities near the interface, which we do not include in our model, may be also contributing to $\Delta\omega_p$. On the other hand, the observed linewidths are significantly affected by temperature and (for p -type samples) by power broadening [13], so the intrinsic, zero-temperature, linear-response linewidth, which is what we have been discussing, may be much smaller than what is seen in the current experiments.

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