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TPC/Two-Gamma Collaboration

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Measurement of the Kaon Content of Three-Prong Tau Decays

TPC/Two-Gamma Collaboration

August 30, 1993

Abstract

We present branching fraction measurements of the three-prong decays of the $\tau^$ lepton based on data from the TPC/Two-Gamma detector at PEP. The decays are classified according to the charged tracks to give simultaneous measurements of $B_{\pi^-\pi^+\pi^-}$, $B_{K^-\pi^+\pi^-}$ and $B_{K^-K^+\pi^-}$, and upper limits for $B_{\pi^-K^+\pi^-}$, $B_{K^-\pi^+K^-}$, and $B_{K^-K^+K^-}$, where additional neutrals may be present in each case. We find the mass distributions in the $K^-\pi^+\pi^-$ decay are consistent with K_1^- dominance, and obtain branching fractions for $\tau^- \to \nu_{\tau} K_1^-(1270)$ and $\tau^- \to \nu_{\tau} K_1^-(1400)$.

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Decays of τ lepton pairs produced at e^+e^- colliders provide an exceptionally clean environment for studies of the weak charged current and of the strong interaction below the τ mass. The well-measured hadronic decays of the τ^- lepton are observed to proceed through coupling of the W^- boson to $\bar{u}d$ or $\bar{u}s$ resonant states. (Charge conjugate processes are implicit.) The $(\pi\pi)^-$ and $(K\pi)^-$ decays are dominated by the ρ^- and K^{*-} vector channels [1], while the $(\pi\pi\pi)^-$ decays proceed through the a_1^- axial-vector channel [2]. However, the resonance structure of the $(K\pi\pi)^-$ decays is still an open question due to their small branching fractions and the difficulty of identifying or reconstructing the strange mesons. The obvious candidates for $(K\pi\pi)^-$ resonances are the $K_1^-(1270)$ and $K_1^-(1400)$ [3], which are mixtures of the $\bar{u}s$ analogs of the a_1^- and b_1^- .

We describe a set of measurements of the branching fractions of the τ^- lepton in which all three-prong decays, those with three charged particles among the decay products, are classified according to the identities of the charged tracks. We also examine the resonance structure of the $(K\pi\pi)^-$ decay through the threecharged mode, $K^-\pi^+\pi^-$, and report branching fractions for $\tau^- \rightarrow \nu_{\tau} K_1^-(1270)$ and $\tau^- \rightarrow \nu_{\tau} K_1^-(1400)$, assuming that resonances dominate the strange axialvector decays of the τ^- . Using predictions of the ratio of weak couplings [4], we determine a preferred mixing angle for the K_1 states.

The data were recorded with the TPC/Two-Gamma detector facility [5] at the PEP e^+e^- collider at SLAC during the 1982-1983 and 1984-1986 runs. The total sample has an integrated luminosity of 140 pb⁻¹ at an e^+e^- center of mass energy of 29 GeV. Central tracking was performed by a time projection chamber, TPC, which identifies charged particles through simultaneous measurements of momentum and ionization energy loss, dE/dx. At PEP energies the decay products of the $\tau^+\tau^$ events are well-separated into two hemispheres, giving a distinct signature for the selection of τ_{1+3} events, where the subscript gives the number of charged particles for each of the τ decays. The event selection is based entirely on kinematic and particle identification information from the charged tracks observed in the TPC. In addition to our basic set of τ_{1+3} selection criteria [6], we apply the following requirements in order to minimize $q\bar{q}$ and radiative Bhabha backgrounds and to ensure efficient π^{\pm}/K^{\pm} separation. There must be no reconstructed tracks in addition to the one-prong and three-prong candidates. The dE/dx of each of the three-prong tracks must satisfy $\chi_{\pi}^2 < 9.0$ or $\chi_K^2 < 9.0$, where χ_i^2 is the goodness of fit of the measured dE/dx and momentum to the expected dE/dx vs momentum curve for particle species i. Finally, each of the three-prong tracks with momentum greater than 2 GeV must have at least 80 wire dE/dx samples. The resulting $\tau_{1+3/2}$ sample consists of 518 events, with an estimated purity of 99.1%. Background estimates, based on Monte Carlo simulation, are 3.5 $q\bar{q}$ and 1.3 τ_{1+1} events with negligible contributions from Bhabha, $\mu^+\mu^-$, and two-photon events.

The separation of the three-prong tracks into π^{\pm} and K^{\pm} candidates is based on their momentum and dE/dx values, shown in Figure 1. The dE/dx distributions for charged particles are known to be Gaussian out to three standard deviations,



Figure 1: dE/dx vs momentum for the three-prong tracks of the τ_{1+3} event sample.

with a well-understood resolution that is a function of the track angle and number of wire samples [5]. The average dE/dx resolution is 3.2% for the tracks in our sample with momentum above 2 GeV. Tracks below 2 GeV are considered to be π^{\pm} , since Monte Carlo simulations predict that fewer than 4% of the K^{\pm} from threeprong τ^{-} decays will have such low momenta. Protons from background sources pose a particular concern since they would likely be misidentified as K^{\pm} due to their low dE/dx values through the high momentum region. However, Monte Carlo simulations predict fewer than one proton in the sample from $q\bar{q}$ background, while protons from nuclear interactions tend to have much lower momenta and would be easily distinguishable. Moreover, the tracks in our sample with momentum above 2 GeV and dE/dx below the expected K^{\pm} value show no preference for positive charge.

To determine the decay populations in the sample we use both an extended maximum likelihood fit and an identification matrix inversion technique [7]. The statistical nature of these methods requires that we consider eight decay classes, since each of the randomly ordered tracks may be π or K. The population estimates for $K^-\pi^+\pi^-$ and $\pi^-\pi^+K^-$ are then added together, as are those for $K^-K^+\pi^-$ and $\pi^-K^+K^-$, to give event populations for the six physically distinct decay modes.

For the likelihood method [8], we derive the expression

$$\mathcal{L}_E = \exp(-\sum_j N_j) \prod_a \left(\sum_j N_j f_j(I_1^a, I_2^a, I_3^a) \right),$$

Decay	Population		Branching Fraction (%)	
$(\geq 0 \text{ neutrals})$	Matrix	Likelihood	$\mathrm{TPC}/2\gamma$	DELCO
$\frac{1}{\nu_{\tau} \pi^{-} \pi^{+} \pi^{-}}$	489.6 ± 22.6	$492.4 \stackrel{+22.7}{_{-22.1}}$	$13.34 \stackrel{+0.27}{_{-0.27}}$	
$ u_{ au}K^{-}\pi^{+}\pi^{-}$	19.7 ± 5.9	$19.9 \stackrel{+5.4}{-4.6}$	$0.54 \stackrel{+0.14}{_{-0.12}}$	$0.22 \stackrel{+0.16}{_{-0.13}}$
$ u_{ au} \pi^- K^+ \pi^-$	3.5 ± 3.4	$1.6{\pm}2.9$	< 0.22	
$ u_{ au}K^-K^+\pi^-$	$5.4{\pm}2.9$	$4.1 \stackrel{+2.5}{_{-1.8}}$	$0.14 \stackrel{+0.08}{-0.06}$	$0.22 \stackrel{+0.17}{_{-0.11}}$
$ u_{ au}K^{-}\pi^{+}K^{-}$	-0.1 ± 0.1	$0.0{\pm}0.5$	< 0.08	
$ u_{ au}K^{-}K^{+}K^{-}$	0.0 ± 0.0	$0.0{\pm}0.5$	< 0.19	

Table 1: Event populations with statistical errors for τ^- decay classes. The branching fractions are normalized to the world average three-prong topological branching fraction. The upper limits are at the 95% confidence level, including systematic uncertainties.

where N_j is the expected number of events in decay class j, the index a runs over the events, and the I_i^a are the measured dE/dx values for the three tracks in an event. The distribution f_j is taken to be a product of three Gaussian distributions, each centered on the expected dE/dx value given the measured momentum and the particle species that define class j; momentum uncertainties are unimportant. The exponential factor takes account of Poisson fluctuations in the sample. Using the package MINUIT [9], we determine the values of N_j that give a global maximum for the expression.

For the *identification matrix inversion method*, each of the three-prong tracks is initially classified as a π^{\pm} or K^{\pm} . Each track with momentum greater than 2 GeV and with dE/dx that satisfies $\chi_K^2 < (\chi_\pi^2 - 4.0)$ is classified as K^{\pm} , while all other tracks are classified as π^{\pm} . From the assumption of Gaussian dE/dx distributions for charged particle species, an identification matrix

$$\boldsymbol{Q}^{a} = \left(\begin{array}{cc} Q^{a}_{\pi \leftarrow \pi} & Q^{a}_{\pi \leftarrow K} \\ Q^{a}_{K \leftarrow \pi} & Q^{a}_{K \leftarrow K} \end{array}\right),$$

is generated for each of the tracks, where the element $Q_{K \leftarrow \pi}$, for example, is the probability of π^{\pm} being misidentified as K^{\pm} . These elements are functions of the momentum and dE/dx resolution of the track, with an average value for $Q_{\pi \leftarrow \pi}$ of 0.99 and for $Q_{K \leftarrow K}$ of 0.86. The outer product of the three *track* identification matrices gives an *event* identification matrix

$$\boldsymbol{P}^{\boldsymbol{a}}(p_1,p_2,p_3) \equiv \boldsymbol{Q}^{\boldsymbol{a}}(p_1) \otimes \boldsymbol{Q}^{\boldsymbol{a}}(p_2) \otimes \boldsymbol{Q}^{\boldsymbol{a}}(p_3),$$

where a is an event label and p_i are the measured parameters for track *i* of the event. For event a, P^a relates the true decay class, given by vector N^a , to the assigned decay class, given by vector M^a , according to

$$M^a = P^a N^a.$$



Figure 2: Invariant mass distributions for the 23 $K^-\pi^+\pi^-$ candidates in the τ_{1+3} sample. The dashed curve shows the estimated background from misidentified $\pi^-\pi^+\pi^-$ events, while the solid curve shows the background plus the best fit combination of $K_1^-(1270)$ and $K_1^-(1400)$.

The element of M^a that corresponds to the observed decay class is set to one, while all other elements are set to zero. We then solve for N^a to obtain a vector of weights for the possible decay modes. The summation of these weights over all of the events,

$$N_j = \sum_a N_j^a,$$

provides another estimate of the decay populations N_i within the sample.

The decay class populations for the two methods, given in Table 1, agree very well. Note that these are inclusive decay classes defined by the identity of their charged particles and may include decays with different numbers of neutrals. To estimate the event selection efficiency for the $\pi^-\pi^+\pi^-$ decay class we use Monte Carlo simulations of $\tau^- \rightarrow \nu_\tau \pi^- \pi^+ \pi^-$ and $\tau^- \rightarrow \nu_\tau \pi^- \pi^+ \pi^- \pi^0$, weighted according to current values for their branching fractions. For the decay classes with K^{\pm} we use the decay mode with no additional π^0 to determine the efficiency. To calculate the branching fractions, we correct the likelihood populations for variations in acceptance among the different decay classes and normalize to the world average three-prong topological branching fraction of $14.06 \pm 0.25\%$ [10]. We estimate a systematic error of 20% for the decay classes with K^{\pm} , primarily due to uncertainties in dE/dx parameterization and in event selection efficiencies. Compared with previous results from DELCO [11], our values for $B_{K^-\pi^+\pi^-}$ and $B_{K^-K^+\pi^-}$ are 1.6σ higher and 0.6σ lower, respectively. Table 1 also shows our upper limits for $B_{\pi^-K^+\pi^-}$, $B_{K^-\pi^+K^-}$, and $B_{K^-K^+K^-}$.

To examine the resonance structure of the decay $\tau^- \rightarrow \nu_{\tau} K^- \pi^+ \pi^-$, we select 23 candidate events where exactly one of the charged tracks satisfies the K^- criteria. The $K^-\pi^+\pi^-$, $K^-\pi^+$, and $\pi^+\pi^-$ invariant mass plots for these events are shown

Decay	Population	Efficiency	Branching Fraction (%)
	Likelihood	$\epsilon_{K_1}/\epsilon_3$	$\mathrm{TPC}/2\gamma$
$\overline{\nu_{ au}K_{1}^{-}(1270)}$	5.4 + 5.4 - 4.6	0.36 ± 0.04	$0.41 \stackrel{+0.41}{-0.35}$
$\nu_{\tau} K_1^-(1400)$	$11.0 \begin{array}{c} +5.8 \\ -4.8 \end{array}$	0.39 ± 0.04	$0.76 \stackrel{+0.40}{_{-0.33}}$
$ u_{ au}K_1^-$	$16.4 \stackrel{+5.7}{-5.1}$	0.38 ± 0.04	$1.17 \stackrel{+0.41}{_{-0.37}}$

Table 2: Branching fractions for decays of the τ^- to $\nu_{\tau} K_1^-(1270)$, $\nu_{\tau} K_1^-(1400)$, and total $\nu_{\tau} K_1^-$, assuming that the process $\tau^- \rightarrow \nu_{\tau} K^- \pi^+ \pi^-$ proceeds entirely through the K_1^- channels. The efficiency definition is described in the text.

in Figure 2. Using Monte Carlo methods, we obtain analytical expressions for the expected distributions of $K_1^-(1270)$ and $K_1^-(1400)$ events in the three-dimensional invariant mass space defined by $m_{K^-\pi^+\pi^-}^2$, $m_{K^-\pi^+}^2$, and $m_{\pi^+\pi^-}^2$. Events are generated with the τ^- generator KORALB [12], to which we have added decays to the K_1^- resonances. These events are then passed through a detector simulation in order to model acceptance and resolution effects. For each of the decay modes of the $K_1^-(1270)$ and $K_1^-(1400)$ [10], we fit the analytical expression

$$N \cdot g(K\pi\pi) \cdot g(K\pi) \cdot g(\pi\pi) \cdot \left(rac{m_ au^2}{m_{K\pi\pi}^2} - 1
ight),$$

to the invariant mass distribution of the Monte Carlo events. Here N is the normalization, and g is a Breit-Wigner function for resonant charged particle combinations and a constant otherwise. The fit determines the best values for the effective mass and width parameters. Finally, these expressions for the individual decay modes are combined, according to their relative selection efficiencies and branching fractions, to give predicted net invariant mass distributions for the $K_1^-(1270)$ and the $K_1^-(1400)$.

Using the $\pi^-\pi^+\pi^-$ candidates of the τ_{1+3} data sample, we obtain a similar analytical expression for the expected invariant mass distribution for the background due to particle misidentification. For each π^- with momentum greater than 2 GeV, we replace the π^- mass with that of the K^- and calculate $m_{K^-\pi^+\pi^-}^2$, $m_{K^-\pi^+}^2$, and $m_{\pi^+\pi^-}^2$ for the event. The expression above is then fit to these events, weighted by their misidentification probabilities $Q_{K\leftarrow\pi}$, which range from 0.4% to 2.3%. The values of $Q_{K\leftarrow\pi}$ are also used to obtain a background estimate of 6.3 ± 2.7 events, including systematic uncertainties.

Assuming that the decay $\tau^- \rightarrow \nu_\tau K^- \pi^+ \pi^-$ is dominated by the two K_1^- channels, we construct an extended maximum likelihood function,

$$\mathcal{L}_E = \exp(-\sum_j N_j) \prod_a \left(\sum_j N_j h_j(m^a_{K\pi\pi}, m^a_{K\pi}, m^a_{\pi\pi})
ight),$$

where a runs over the $K^-\pi^+\pi^-$ candidate events. The N_j and h_j are the expected populations and predicted invariant mass distributions for $K_1^-(1270)$, $K_1^-(1400)$,



Figure 3: Statistical error contours for the $\tau^- \rightarrow \nu_{\tau} K_1^-(1270)$ and $\tau^- \rightarrow \nu_{\tau} K_1^-(1400)$ branching fractions. The shaded regions represent the ranges of the ratio of K_1^- branching fractions predicted for K_1^- mixing angles of 33° and 57° and SU(3) breaking of $|\delta| \leq 0.2$.

and background events. Using the background contribution given above, we determine the most probable values for the $K_1^-(1270)$, $K_1^-(1400)$, and total K_1^- populations, listed in Table 2. The branching fractions are calculated according to

$$B_{K_1} = B_3 \cdot (N_{K_1}/N_3)/(\epsilon_{K_1}/\epsilon_3),$$

where B_3 is the topological branching fraction to three-prongs, N_3 is the number of events in our τ_{1+3} sample, and $\epsilon_{K_1}/\epsilon_3$ is the selection efficiency for K_1^- events in the $K^-\pi^+\pi^-$ sample relative to that for three-prong events in the τ_{1+3} sample. We estimate a systematic error of 25% from uncertainties in dE/dx parameterization, event selection efficiencies, and branching fractions for the decays of the $K_1^-(1270)$ and $K_1^-(1400)$. The Monte Carlo invariant mass distributions for the best fit values of $K_1^-(1270)$ and $K_1^-(1400)$ are compared with those of the $K^-\pi^+\pi^-$ candidates in Figure 2, while the statistical error contours for the K_1^- branching fractions are shown in Figure 3.

From the relative magnitude of the CKM matrix elements $|V_{ud}|$ and $|V_{us}|$, the branching fraction for $\tau^- \rightarrow \nu_{\tau} K_1^-$ is expected to be roughly 5% of that for $\tau^- \rightarrow \nu_{\tau} a_1^-$, which is in the range of 16%-20%. This estimate is in good agreement with our measurement for the total branching fraction of $\tau^- \rightarrow \nu_{\tau} K_1^-$, listed in Table 2. Given the current branching fraction values for the decays $K_1(1270) \rightarrow K^*\pi$

and $K_1(1400) \to K^*\pi$ [10], we use our $\tau^- \to \nu_\tau K_1^-$ values to obtain

$$B(\tau^- \to \nu_\tau \bar{K}^{*0} \pi^-) = 0.51 \stackrel{+0.26}{_{-0.21}} \pm 0.13\%,$$

consistent with a measurement from CLEO [13] of $0.38 \pm 0.11 \pm 0.13\%$. The partial decay rates and masses of the $K_1(1270)$ and $K_1(1400)$ predict a mixing angle of $\theta_{K_1} \approx 33^\circ$ or 57° [4]. Our ratio of K_1^- branching fractions favors the mixing angle of $\theta_{K_1} \approx 33^\circ$ for reasonable estimates of SU(3) breaking, as shown in Figure 3.

In order to check for non- K_1^- contributions to the decay $\tau^- \rightarrow \nu_{\tau} K^- \pi^+ \pi^-$, we derive expressions for the expected invariant mass distributions of non-resonant $\bar{K}^{*0}\pi^-$, $K^-\rho^0$, and $K^-\pi^+\pi^-$. We perform likelihood fits where one of these nonresonant components is included along with the $K_1^-(1270)$, the $K_1^-(1400)$, and background. The most likely value for the branching fraction of $\bar{K}^{*0}\pi^-$, which has an invariant mass distribution very similar to the $K_1^-(1400)$, is $0.19 \stackrel{+0.24}{_{-0.20}}$, while branching fractions for the $K^-\rho^0$ and $K^-\pi^+\pi^-$ are $0.01 \stackrel{+0.11}{_{-0.07}}$ and $0.04 \stackrel{+0.11}{_{-0.11}}$. Although we are unable to rule out contributions from these non-resonant channels, our overall results are consistent with K_1^- dominance of the $(K\pi\pi)^-$ decays of the τ^- lepton.

In conclusion, we obtain simultaneous measurements of several three-prong τ^- decays including K^{\pm} . Our resonance analysis provides first evidence for the strange axial-vector decay of the τ^- lepton.

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