MATHEMATICAL MODEL OF AN INVERSION ANKLE SPRAIN

By

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ABSTRACT

Inversion ankle sprains, or IAS, occur very often in everyday life, mostly during sports and exercise. Detecting and evaluating the severity of an IAS are very important, but we need to understand how to differentiate a normal ankle from an injured one. The goals of this Capstone project are to find out what internal properties are impacted by inversion ankle sprains and how they change the functions of the ankle ligament complex and to quantitatively differentiate between a normal ankle and one affected by an IAS. In order to complete these goals, we developed a mathematical model and simulated the stress relaxation behavior of the normal and sprained ankles on MATLAB. We conducted a literature review on the possible differential equations we could possibly use that calculate stress over time, we calibrated the possible options for equations using experimental data from literature and then chose the best fitting equation as our mathematical model, and then we tested our chosen equation on virtual patient data to reveal the differences in the model parameters between the normal and injured ankles. This would hopefully increase our understanding of how we can assess ankle sprains, and doctors and physicians can make more accurate diagnoses and create the best treatment necessary to heal these types of injuries.

ACKNOWLEDGMENTS

I especially like to thank my faculty mentor, Prof. Heyrim Cho of the Department of Mathematics, for guiding me through my Capstone project. I also like to thank Dr. Richard Cardullo of University Honors for organizing the Capstone project. I have enjoyed working with these very intelligent and hardworking people over the course of my undergraduate career, and hopefully the skills that I’ve gained from this Capstone project will help me succeed in my future academic and professional endeavors.

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INTRODUCTION

Ankle injuries happen a lot in everyday life. Most people experience some type of ankle injury sometime in their life, like a high ankle sprain, a twisted ankle, or even ankle fatigue and soreness. Ankle injuries are one of the most common sports-related traumas, with ankle sprains being the most common in this category of injuries. Ankle sprains occur very frequently in sports and exercise, but it could also happen in everyday activity as well, like walking or tripping on a curb. Depending on how severe the damage is, ankle sprains can cause bruising, swelling, difficulty walking, and ankle instability for the patient. There are two types of ankle sprains, eversion ankle sprains (EAS) and inversion ankle sprains (IAS). Eversion ankle sprains occur when the foot rolls outwards and away from the body, while inversion ankle sprains occur when the foot rolls inwards and towards the body. While EAS is more severe than IAS, IAS happens a lot more often than EAS.

For this project, we are going to only focus on inversion ankle sprains. According to Daniel Tik-Pui Fong and his team from BMC Sports Science, Medicine and Rehabilitation, inversion (or lateral) ankle sprains are the most common injuries in sports and exercise, accounting for approximately 14% of all sports-related injuries (Fong et al., 2009). IAS damages and tears the ligaments on the lateral side of the talocrural joint, or more commonly known as the ankle joint. The ligaments that can be affected by IAS include the anterior talofibular ligament (ATFL), the calcaneofibular ligament (CFL), and the posterior talofibular ligament (PTFL). The ATFL is affected the most in this type of injury due to the fact that this ligament is the weakest out of the three, having the lowest maximum load at 138.9 N. Compared to the other ligaments, the CFL's ultimate load is almost three times the amount that the ATFL can hold at 345.7 N, while the PTFL's ultimate load is approximately double the amount at 261.2 N (Fong et al., 2009).

Inversion ankle sprains are mainly caused by the combination of excessive internal rotation and ankle plantar flexion (Purevsuren et al., 2017), explosive inversion velocity (Chu et al., 2010), and force, or load, exceeding the injury threshold on the ankle (Kristianslund et al., 2011). IAS can lead to partial or complete tears to the lateral ligaments of the ankle (Fong et al., 2009). Additionally, changes in viscoelastic behaviors, such as stress relaxation and creep (Lin et al., 2015), and the loss of end-range stiffness in the ankle ligament complex (Kovaleski et al., 2014) may also occur. The severity of damage to the ankle mainly depends on how explosive the injury mechanisms were to the affected area, meaning how much high stress did the ligaments endure and how fast was the inversion of the ankle. The assessment of the damage can be determined by medical staff through x-rays and MRI scans to examine how much time the ankle needs to fully heal and to discover the best treatment for each patient.

The viscoelasticity of the ATFL, CFL, and PTFL of the ankle ligament complex is an important quantitative measure to detect inversion ankle sprains. Viscoelasticity involves both viscous and elastic behavior, so any viscoelastic material gains the ability to dissipate or lose some of the energy that is mainly used for deformation of the material (University of Michigan). Other characteristics associated with viscoelasticity are stress relaxation, which refers to stress decreasing over time under constant strain, and creep, which refers to strain increasing over time under constant stress (University of Michigan). Viscoelastic models can be derived from simple mechanical models that have combinations of both spring and dashpot properties. One simple model is the Maxwell Model, where a spring and a dashpot are put together in series (in sequence) and the total displacement of the system is the sum of the spring displacement and the dashpot displacement. In addition, the total stress of the system is the same as the stress applied in the spring or the dashpot. Another model is known as the Voigt Model, where a spring and a dashpot are put together in parallel and the total displacement is the same as the displacement of the spring or the dashpot. Also, the total stress of the system is the sum of the stress applied in the spring and the stress applied in the dashpot (University of Michigan). Each of these mechanical models has its own constitutive equations to calculate the stress, strain, and total force of the ligaments. Understanding these simple mechanical models would not only give us a better understanding of what our own mathematical model would be, but it also gives us a better understanding of the behavior of the viscoelastic material in the ankle ligament complex. It is important to note that the concept of viscoelasticity doesn't just apply to the ankle ligament complex. It can be applied to assess any type of injury to the soft connective tissues of the body, including ligaments, tendons, and cartilage.

It is important to understand how to differentiate between a normal ankle and an injured ankle (one affected by IAS). One study by Che-Yu Lin, Yio-Wha Shau, and their colleagues utilized a practical anterior drawer test to quantitatively assess the viscoelastic material in an IAS injury using a standard linear solid model and to examine the stress relaxation behavior of the lateral ankle ligaments using the stretched exponential function, $f = p_1 e^{-(\frac{t}{k_1})^{k_2}}$ (Lin et al., 2015). They discovered that the relaxation behavior of the sprained ankle had a much steeper decaying curve than that of the uninjured ankle, at least near the beginning of the relaxation process (*Figure 1*). Another study conducted by Tserenchimed Purevsuren and his team consists of an assessment of three accidental injury cases to reveal the causes of IAS. They concluded that excessive ankle inversion and slight internal rotation are the primary causes of IAS (Purevsuren et al., 2012). Finally, a third study from Vikki Wing-Shan Chu and her team explored various common sports motions, such as running, jumping, and walking, and simulated ankle sprain motions within them, specifically looking at ankle inversion velocity to identify any increased risk factors for ankle sprains. They found out that a threshold of ankle inversion velocity of 300°/s is an indication for an IAS (Chu et al., 2010).

This capstone project focuses on two important objectives. The first objective is to find out what internal properties are impacted by inversion ankle sprains and how they change the functions of the lateral ankle ligament complex, and the second objective is to quantitatively differentiate between a normal ankle and one affected by IAS.

DESIGN + APPROACH

In this Capstone project, my faculty mentor and I developed a mathematical model and simulated the relaxation behavior of the normal vs. injured ankle on a computational program on MATLAB. There were three benchmarks that were taken during the process of this Capstone project. First, we conducted a literature review on the differential equations that can calculate stress over time. Secondly, we calibrated the equations using experimental data from our literature, and then we chose the best fitting equation. Lastly, we tested our chosen equation on virtual patient data to reveal the differences between normal and injured ankles.

For our mathematical model, there were three options or equations we could use. The first option was the stretched exponential function (which was covered in the introduction), $f = p_1\, exp(-\, (\frac{t}{k_1})^{k_2})$. In this equation, f is stress, t is time, p_1 is the initial maximum stress, and k_1 and k_2 are the parameters that will be determined by curve-fitting. For every function that is nominated for our mathematical model, there will be two model equations derived from the original function with a different number of parameters in each of them. For example, for Model

1, which is $f = p_1 \, exp(-\frac{t}{k_1})$, we only include the parameter k_1, and for Model 2, which is $f = p_1 \, exp(-(\frac{t}{k_1})^{k_2})$, we include both parameters k_1 and k_2. To validate the functions, we calibrated them by curve-fitting (line of best fit), using stress relaxation data of injured and uninjured ankles from Lin, Shau, and their team (Lin et al., 2015) (*Figure 1*). As a result, Model 2 labeled in red had a much better fit than Model 1 in blue (*Figure 2*). The method we used for curve-fitting was the least squares method. This method continually searches for a curve where the sum of the offsets of data points is minimized. This method for curve-fitting will be used a lot during the calibration and virtual patient testing phases of this project.

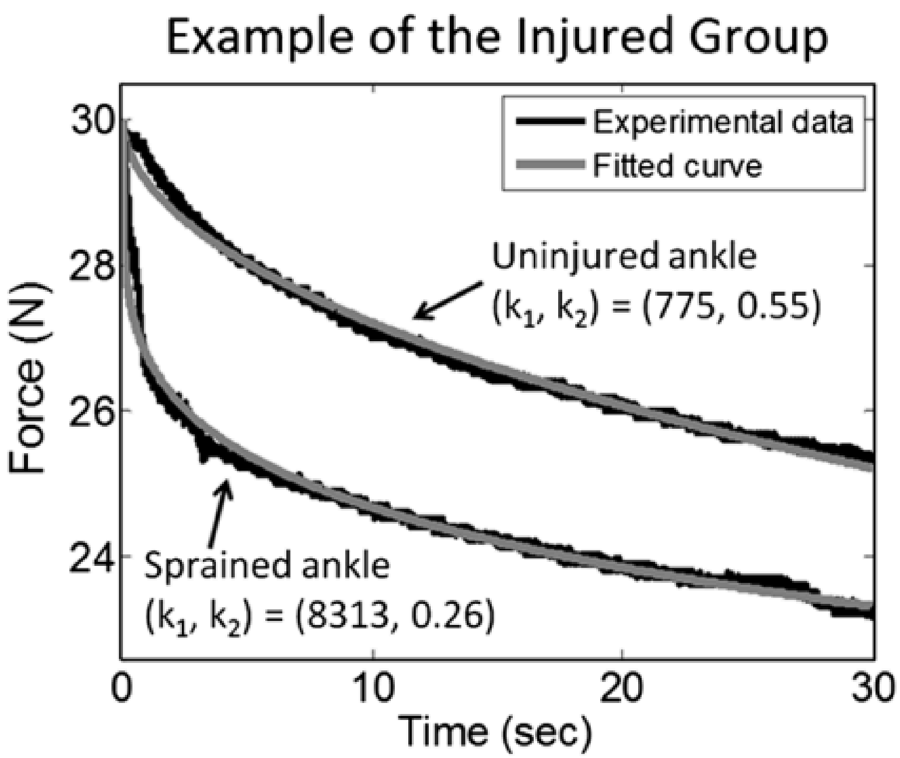


Figure 1: Relaxation behavior of the uninjured and sprained ankles, which was a result of the anterior drawer test performed by Che-Yu Lin and his colleagues (Lin et al., 2015).

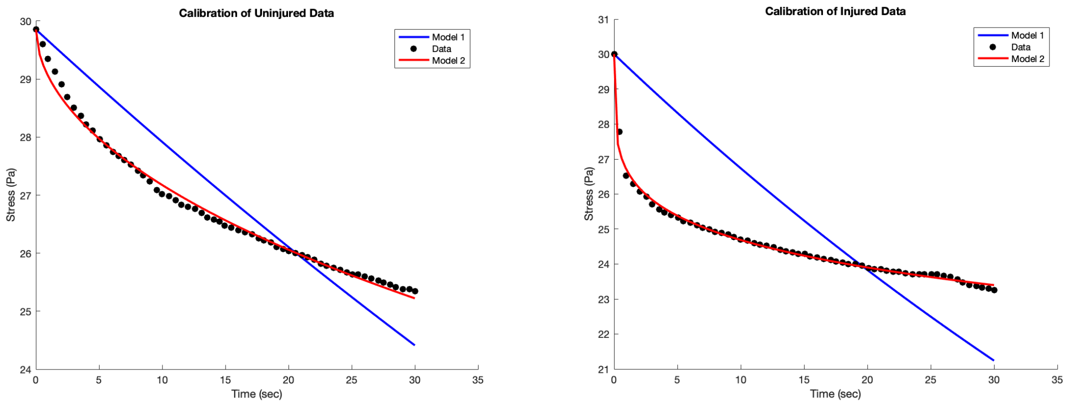


Figure 2: Calibration of Models 1 (blue) and 2 (red) of the Stretched Exponential Function through curve-fitting with the stress relaxation behavior of the uninjured vs. injured ankles from Lin et al.

The second option we had was the general decaying exponential function, $G(t) = ae^{-bt} + ce^{-dt} + ge^{-ht}$. This was used as a reduced relaxation function, which was part of the quasilinear viscoelastic theory used by Stephanie Toms, Greg Dakin, and their colleagues to quantify the stress-strain behavior of the human periodontal ligament (PDL) (Toms et al., 2002). In this equation, $G(t)$ is stress, t is time, and a, b, c, d, g, and h are the parameters that will be determined by curve-fitting. This equation is pretty straight forward since Models 1 and 2 just have a different number of parameters and exponential terms. For Model 1, $G(t) = ae^{-bt}$, we only include parameters a and b with one exponential term, while for Model 2, $G(t) = ae^{-bt} + ce^{-dt}$, we only include parameters a, b, c, and d with two exponential terms. Back to the calibration stage, Model 2 proved to be the more accurate fit for the data (*Figure 3*), and now we begin to see a common theme with the model with more parameters having a better fit of the stress relaxation data than the model with less parameters.

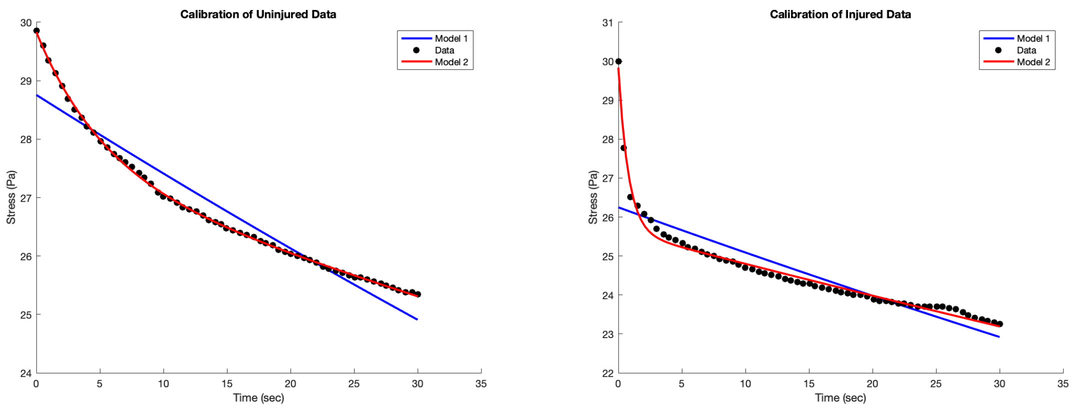


Figure 3: Calibration of Models 1 (blue) and 2 (red) of the General Decaying Exponential Function through curve-fitting with the stress relaxation behavior of the uninjured vs. injured ankles from Lin et al.

Finally, the third option was the general quasilinear viscoelastic function, $T(t) = A\gamma(e^{B\lambda} - 1)\frac{adhe^{-b(t-t_0)} + cbhe^{-d(t-t_0)} + gbde^{-h(t-t_0)} - adhe^{-bt} - cbhe^{-dt} - gbde^{-ht}}{bdh}$. This was utilized in an “BME 332: Introduction to Biosolid Mechanics” lecture in the University of Michigan, where the instructors derived it from the QLV (quasilinear viscoelastic) theory that Toms and her team used in their experiments. The general quasilinear viscoelastic function was an extended version of the general decaying exponential function but now with an instantaneous nonlinear elastic response. The instantaneous nonlinear elastic response is labeled by this term, $A\gamma(e^{B\lambda} - 1)$, where λ is the principal stretch ratio, γ is the strain rate up from 0 to t_0, and A and B are constants determined by experiment (University of Michigan). Since this function was overly complex, we had to make some assumptions for simplification. We assumed that the parameters A, B, γ, λ, and t_0 are all constant values, which means that, in turn, the instantaneous nonlinear elastic response can be represented as a constant. According to the lecture from the University of Michigan, $A = 8.92 \times 10^{-3}$, $B = 8.79$, $\lambda = 1.3$, $\gamma = 4$, and $t_0 = 0.0862$ (University of Michigan). In addition to these applied constants, $T(t)$ is stress, t is time, and a, b, c, d, g, and h are the parameters that will be determined by curve-fitting. For Model 1,

$T(t) = A\gamma(e^{B\lambda} - 1)\frac{ae^{-b(t-t_0)} - ae^{-bt}}{b}$, we only include parameters a and b, while for Model 2,

$T(t) = A\gamma(e^{B\lambda} - 1)\frac{ade^{-b(t-t_0)} + cbe^{-d(t-t_0)} - ade^{-bt} - cbe^{-dt}}{bd}$, we only include parameters a, b, c, and d.

Back to the calibration stage for the third time, Model 2 was the more accurate fit of the data than Model 1 once again (*Figure 4*).

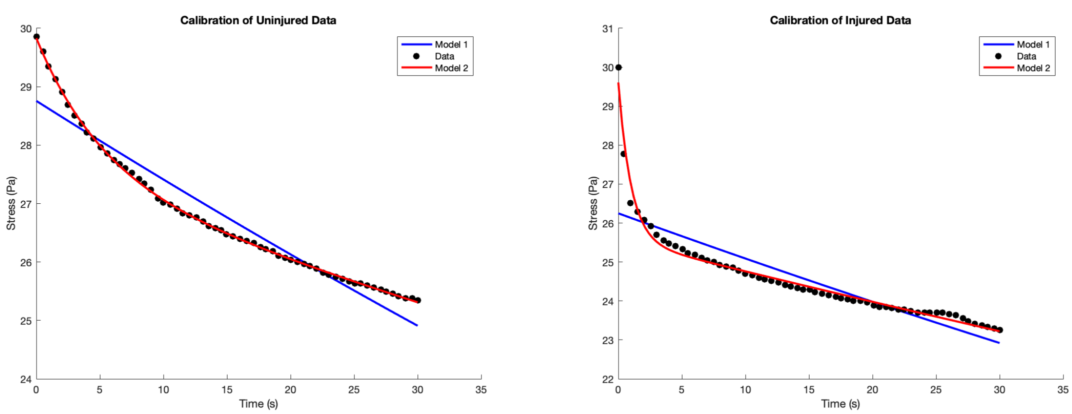


Figure 4: Calibration of Models 1 (blue) and 2 (red) of the General Quasilinear Viscoelastic Function through curve-fitting with the stress relaxation behavior of the uninjured vs. injured ankles from Lin et al.

After considering all of the three options, my faculty mentor and I decided that option #3, which was the general quasilinear viscoelastic function, was the best option as our mathematical model for the project.

$$T(t) = A\gamma(e^{B\lambda} - 1)\frac{adhe^{-b(t-t_0)}+cbhe^{-d(t-t_0)}+gbde^{-h(t-t_0)}-adhe^{-bt}-cbhe^{-dt}-gbde^{-ht}}{bdh}$$

Even though this was a very complex function, with the assumptions that we made to simplify terms, it was the right amount of complexity that was able to most accurately curve-fit the stress relaxation data out of the three nominated functions during calibration.

Now we move on to phase three of this Capstone project, which is virtual patient testing. (Just a quick note, no human subjects were tested and monitored for this Capstone project.) My faculty mentor and I created 15 virtual patients with a normal, uninjured ankle on one side and an injured ankle on the other side. These patients have been chosen for a stress relaxation test of both their normal and injured ankles, which will be performed over a 30-second duration. The purpose of this test is to identify the parameters in our mathematical model that are mostly affected by an ankle sprain, so we could quantitatively differentiate between a normal ankle and a sprained ankle. Once the stress vs. time data has been collected from the relaxation test of both the injured and uninjured ankles of all 15 patients, with the use of MATLAB programming, Models 1 and 2 of the general quasilinear viscoelastic function will be curve-fitted (line of best fit) to the uninjured and injured ankle stress relaxation data of each virtual patient. As a result, the parameter values for Model 1 (a and b) and the parameter values for Model 2 (a, b, c, and d) would be found as well as their respective error values. With these results, we will be able to observe any changes in each parameter value between the normal ankle and the injured ankle and any trends in the relaxation behavior (normal vs. injured) in this pool of patients.

RESULTS

Model 1:	**Uninjured**		**Injured**	
Patient #	**a**	**b**	**a**	**b**
1	0.1057	0.0053	0.1025	0.0046
2	0.1051	0.0053	0.0968	0.0063
3	0.1051	0.0028	0.1007	0.0048
4	0.1014	0.0044	0.1018	0.0061
5	0.1018	0.0039	0.1012	0.0065
6	0.1024	0.0048	0.1011	0.0042
7	0.1017	0.0052	0.0949	0.0061
8	0.0999	0.0042	0.1007	0.0056
9	0.0986	0.0041	0.0991	0.0075
10	0.1024	0.0042	0.0950	0.0045
11	0.1036	0.0031	0.1010	0.0031
12	0.1033	0.0022	0.0987	0.0064
13	0.0984	0.0038	0.0985	0.0064
14	0.1008	0.0043	0.0989	0.0036
15	0.1003	0.0037	0.0972	0.0038

Table 1: Parameter values of Model 1 determined by curve-fitting with the virtual patient data, showing the changes in stress relaxation behavior from the uninjured ankle to the injured ankle.

Model 2:	**Uninjured**				**Injured**			
Patient #	**a**	**b**	**c**	**d**	**a**	**b**	**c**	**d**
1	0.0012	0.1460	0.1049	0.0050	0.0057	0.2121	0.0995	0.0033
2	0.1029	0.0045	0.0032	0.1255	0.0108	0.4474	0.0934	0.0046

3	0.0024	0.1362	0.1035	0.0022	0.0977	0.0033	0.0072	0.2974
4	0.0058	0.3154	0.0991	0.0033	0.0987	0.0046	0.0064	0.2355
5	0.1000	0.0031	0.0056	0.4181	0.0072	0.3096	0.0982	0.0050
6	0.0996	0.0035	0.0060	0.2438	0.0063	0.3856	0.0990	0.0031
7	0.0063	0.3225	0.0992	0.0040	0.0118	0.6127	0.0920	0.0045
8	0.0978	0.0032	0.0070	0.4608	0.0982	0.0044	0.0071	0.3650
9	0.0966	0.0031	0.0081	0.6242	0.0948	0.0054	0.0096	0.2745
10	0.1004	0.0032	0.0050	0.3100	0.0924	0.0031	0.0114	0.6703
11	0.0036	0.2561	0.1020	0.0024	0.0053	0.5482	0.0996	0.0024
12	0.0034	0.3417	0.1021	0.0016	0.0088	0.3859	0.0956	0.0049
13	0.0084	0.5930	0.0963	0.0028	0.0092	0.3252	0.0948	0.0045
14	0.0062	0.3783	0.0986	0.0033	0.0975	0.0028	0.0077	0.8627
15	0.0068	0.5152	0.0984	0.0028	0.0953	0.0028	0.0094	0.8237

Table 2: Parameter values of Model 2 determined by curve-fitting with the virtual data, showing the changes in stress relaxation behavior from the uninjured ankle to the injured ankle.

Each model was accurately curve-fitted with the virtual patient data, limiting the amount of error value in the parameters as much as possible. After curve-fitting both Models 1 and 2 with the uninjured and injured ankle stress relaxation data for all 15 patients, I observed some common themes among the different plots. First, and most obvious, Model 2 (red line) proved to be the more accurate fit of the data than Model 1 (blue line) in all plots of the patients (*Figure 5*). Secondly, when comparing the uninjured vs. injured stress relaxation behavior, for at least the majority of the patients, the exponentially decaying curve was much steeper in the injured plot than that of the uninjured plot. It is most revealed by the comparison of the initial drop in stress towards the beginning of the plots (*Figure 5*).

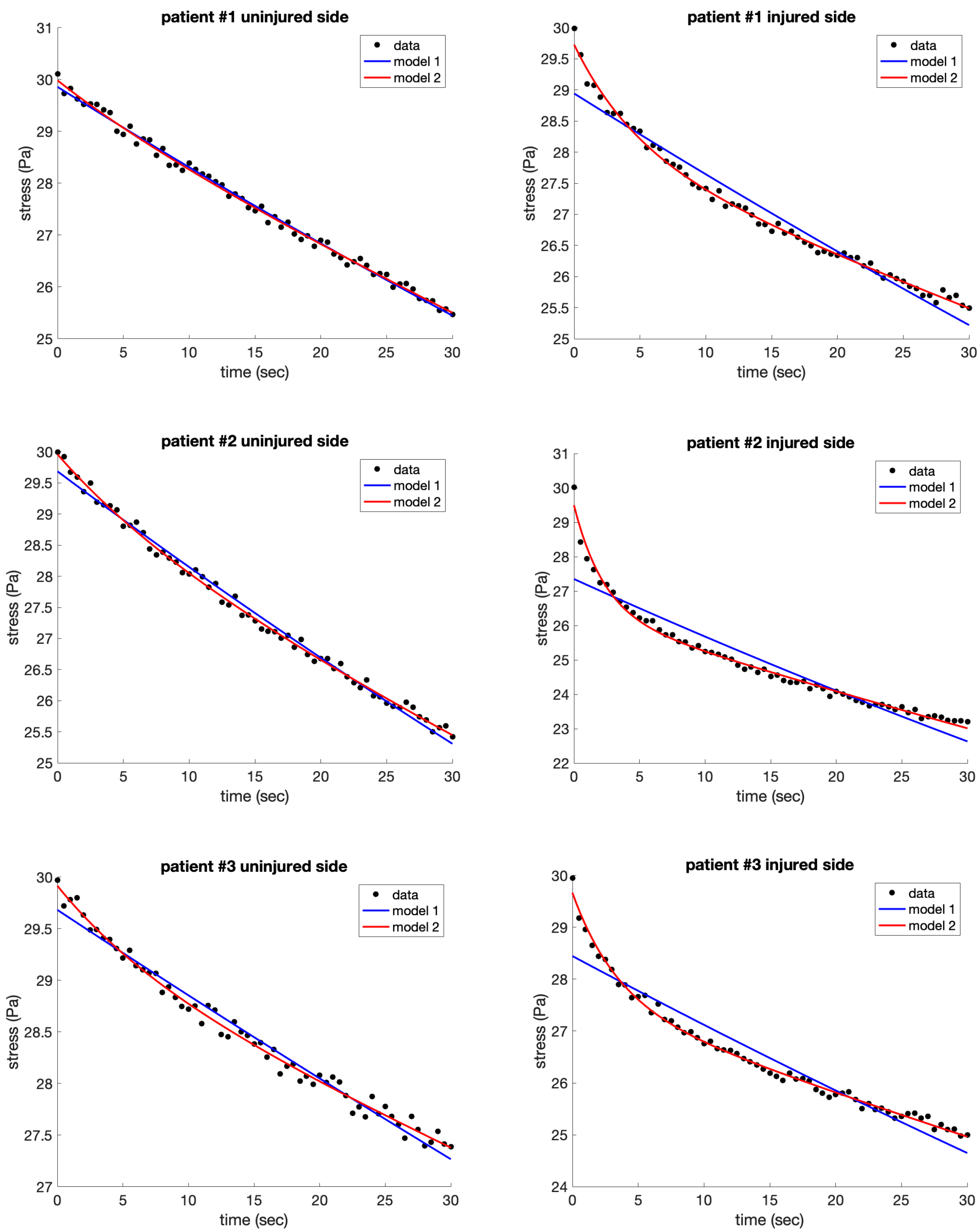
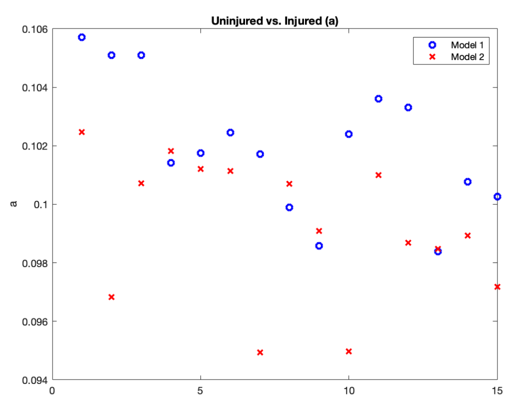


Figure 5: Stress relaxation plots of the uninjured vs. injured ankles for Patients #1-3. (NOTE: The reason I didn't include the other patients was because all of the plots, including the ones not shown, convey the same message that Model 2 is a much better fit of the data than Model 1, and

that the stress relaxation behavior has a steeper downward curve for the injured ankle than that of the uninjured ankle).

For Model 1, the values for parameters a and b were consistent for the most part surrounding all of the patients. For the uninjured ankle, the average value of parameter a was 0.1020 with a standard deviation of 0.0023, and the average value of parameter b was 0.0041 with a standard deviation of 9.0034 x 10^{-4}. On the other hand, for the injured (sprained) ankle, the average value of parameter a was 0.0992 with a standard deviation of 0.0024, and the average value of parameter b was 0.0053 with a standard deviation of 0.0013. This means that the difference between the uninjured and injured ankle average values of parameter a is -0.0028, and the difference between the average values of parameter b is 0.0012. These difference values show that from the uninjured ankle to the injured ankle, there is a sufficient drop in parameter a and a slight rise in parameter b. Even though these numbers aren't all that high, the plots comparing the changes in parameter values from uninjured ankle stress data to injured ankle stress data were most revealing (*Figure 6*). So, in summary, parameter a was more affected by the injured ankle than parameter b.



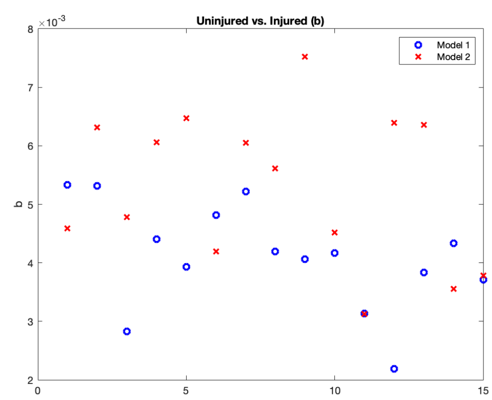


Figure 6: Plots showing the changes in parameters a and b from the stress relaxation behavior of the uninjured ankle to that of the injured ankle.

For Model 2, the values for each parameter (a, b, c, and d) were mostly all over the place considering all of the patients. For the uninjured ankle, the average value of parameter a was 0.0428 with a standard deviation of 0.0480, the average value of parameter b was 0.2017 with a standard deviation of 0.2028, the average value of parameter c was 0.0626 with a standard deviation of 0.0480, and the average value of parameter d was 0.1473 with a standard deviation of 0.2109. On the contrary, for the injured ankle, the average value of parameter a was 0.0493 with a standard deviation of 0.0456, the average value of parameter b was 0.2169 with a standard deviation of 0.2260, the average value of parameter c was 0.0554 with a standard deviation of 0.0456, and the average value of parameter d was 0.2374 with a standard deviation of 0.3159. This means that the difference between the uninjured and injured ankle average values of parameter a was 0.0065, the difference between the average values of parameter b was 0.0152, the difference between the average values of parameter c was -0.0072, and the difference between the average values of parameter d was 0.0901. These difference values show that from the uninjured ankle to the injured ankle, there is a slight rise in parameter a, a slight drop in parameter c, and huge increases in parameter b and d. Again, even though these numbers aren't so high in value, the plots comparing the parameter values for uninjured vs. injured ankles prove that these changes in parameters are significant to mention (*Figure 7*). So, in short, parameters b and d were most affected by the injured ankle than parameters a and c.

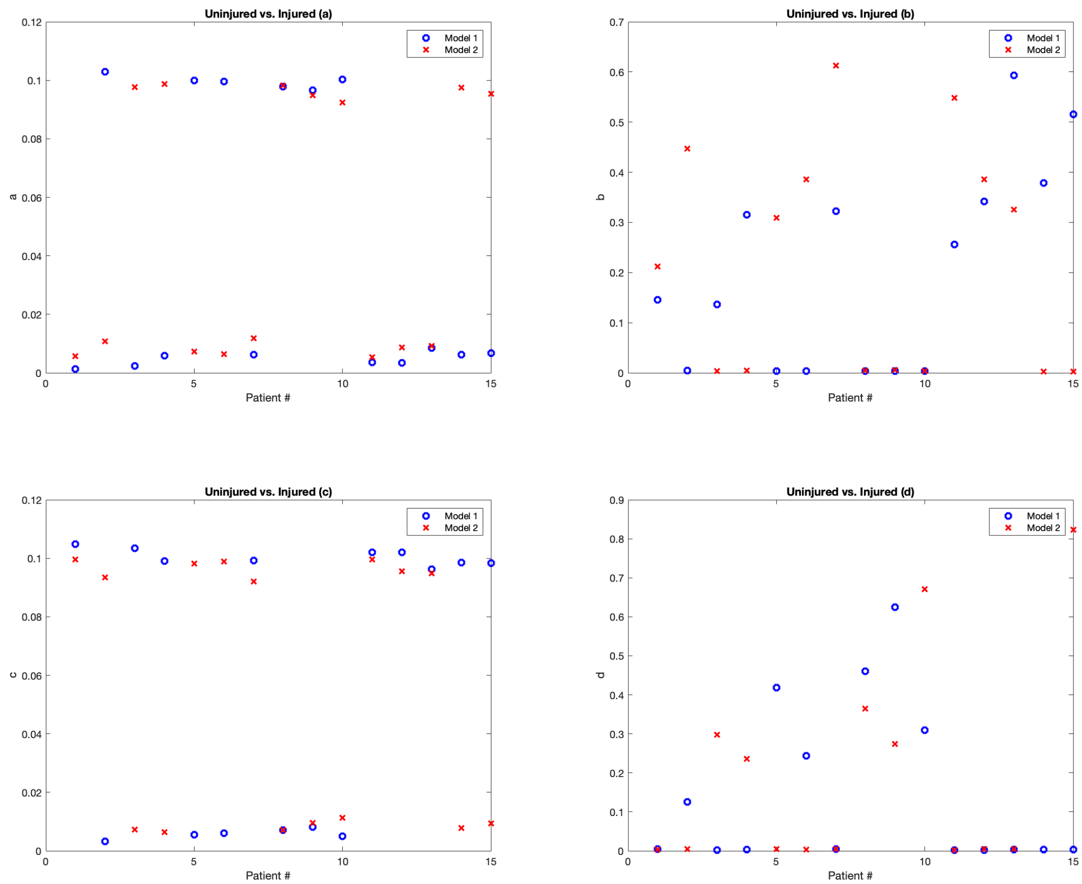


Figure 7: Plots showing the changes in parameters a, b, c, and d from the stress relaxation behavior of the uninjured ankle to that of the injured ankle.

DISCUSSION

Overall, the virtual patient testing was successful. In all of the ankle stress relaxation plots for the patients (uninjured vs. injured), Model 2 had a more accurate fit of the virtual patient data than Model 1 by a long shot. Model 2 always looked like an exponential decaying curve, while Model 1 usually looked more like a linear curve. The probable reason for this consistency in curve-fitting results is that the equation for Model 2 has more parameters than the equation for Model 1, which add to the complexity of the equation and can make a better fit with the experimental data. In addition, since the stress relaxation behavior looks like an exponential decaying curve, adding more exponential terms can also make the shape of the curve a better fit of the data. Since the instantaneous nonlinear elastic response is constant, we were able to reveal the linear downward slope towards the beginning of the stress relaxation plots (*Figure 7*). This, in turn, made the plots of Model 2 look very similar to the plots made by Lin and his team when they performed the anterior drawer test.

Regarding Model 1, the values for parameters a and b were very similar in comparison with each patient, regardless of whether it was the injured or uninjured ankle. This made it easier for us to predict the average value of these parameters. In terms of changes in parameter values, the coefficients before the exponential terms of this function, a, had a larger change from the uninjured ankle to the injured ankle than the parameter located where the exponents of the exponential terms are, b. This makes sense because the coefficients of a function usually affect the slope and y-intercept of the corresponding line or curve. So the relaxation behavior plot of the model was mostly affected by the change in parameter a.

Regarding Model 2, the values for parameters a, b, c, and d were very scattered between all of the patients, regardless of whether it was the normal or injured ankle. This made it very difficult for us to predict the average value of these parameters. On the contrary from Model 1, the parameters, b and d, had a much larger change from the uninjured ankle to the injured ankle than the parameters a and c. This is because the parameters, b and d, are located in multiple areas of the equation. More specifically, the parameters b and d can be found in the coefficients before the exponential terms, as part of the exponents of the exponential terms, and in the denominator of the function. Since these parameters are found in multiple places, a change in these variables will cause a bigger effect on the stress relaxation curve. A common theme can be found between Models 1 and 2, which is the fact that the parameters representing the coefficients of the functions (more specifically, the coefficients used before the exponential terms of the functions) are mostly affected in the stress relaxation behavior when transitioning from an uninjured ankle to an injured one.

CONCLUSION

In summary, inversion ankle sprains ultimately cause changes in coefficient parameters in our mathematical model. Such changes affect the stress relaxation behavior of the ankle by creating a steeper decaying curve. By curve-fitting multiple sub-models with experimental data, we were able to discover the differences in shape, fit, and parameter values for these functions.

Our mathematical model successfully depicted the quantitative differences between a normal ankle and an injured (sprained) one. Keep in mind that this mathematical model doesn't have to apply to just quantitatively assessing inversion ankle sprains. This model could be used to assess eversion ankle sprains and its affected ligaments. It could also be used to assess other ligaments, like the UCL (ulnar collateral ligament) or the ACL (anterior cruciate ligament). It could even be used to assess injuries in other soft connective tissues of the body, such as the Achilles tendon and articular cartilage. So our mathematical model can adapt to any quantitative assessment of soft connective tissue.

In the future, we could even extend the virtual patient testing by performing stress relaxation tests on patients some time later where the once injured ankle is now fully healed. This is so we could evaluate and compare the stress relaxation behavior of a normal ankle vs. a healed ankle of a patient. Since the ankle ligaments that are healed from the inversion ankle sprain would look similar to the normal ankle, we could infer to see similar stress relaxation curves between these two ankles. We could also see new changes in different parameters in the sub-models of our mathematical model. There could be a possibility that different parameters are mostly affected by a healed ankle, and not necessarily the same parameters that are mostly affected by an injured ankle.

As another extension to the virtual patient testing, we could install different treatments for the injured ankles of the patients, such as pain relievers, the RICE method, or a combination of both. Then, after some time, we could perform the stress relaxation test on the patients' treated ankles to see how their stress relaxation behavior compares to that of their once injured ankles. Not only could we see differences in parameter values of the relaxation curves for our mathematical model, we could also reveal the best treatment to aid the injured ankle for the patient.

Inversion ankle sprains are being detected, evaluated, and treated almost every day, and these practices and methods are currently improving and/or being innovated. The use of mathematical modeling and computer programming is an interesting, yet innovative concept that could be applied to not only the field of biomechanics but also to the field of medicine and health. It could give us a new perspective on how the parts of our body function on a cellular, tissue, or organ level, and how they can be affected by trauma or disease. This Capstone project provided a practical application for injury evaluations by quantitatively assessing inversion ankle sprains.

REFERENCES

Chu, Vikki Wing-Shan, et al. “Differentiation of Ankle Sprain Motion and Common Sporting Motion by Ankle Inversion Velocity.” *Journal of Biomechanics*, vol. 43, no. 10, 20 July 2010, pp. 2035–2038., doi:10.1016/j.jbiomech.2010.03.029.

Fong, Daniel Tik-Pui, et al. “Understanding Acute Ankle Ligamentous Sprain Injury in Sports.” *BMC Sports Science, Medicine and Rehabilitation*, vol. 1, no. 1, 30 July 2009, doi:10.1186/1758-2555-1-14.

Kovaleski, John E., et al. “Joint Stability Characteristics of the Ankle Complex after Lateral Ligamentous Injury, Part I: A Laboratory Comparison Using Arthrometric Measurement.” *Journal of Athletic Training*, vol. 49, no. 2, 2014, pp. 192–197., doi:10.4085/1062-6050-49.2.07.

Kristianslund, Eirik, et al. “Kinematics and Kinetics of an Accidental Lateral Ankle Sprain.” *Journal of Biomechanics*, vol. 44, no. 14, 23 Sept. 2011, pp. 2576–2578., doi:10.1016/j.jbiomech.2011.07.014.

Lin, Che-Yu, et al. “Modeling and Analysis of the Viscoelastic Response of the Ankle Ligament Complex in Inversion Ankle Sprain.” *Ann Biomed Eng*, vol. 43, no. 9, 7 Jan. 2015, pp. 2047–2055., doi:10.1007/s10439-014-1240-0.

Purevsuren, Tserenchimed, et al. “Investigation of Ligament Strains in Lateral Ankle Sprain Using Computational Simulation of Accidental Injury Cases.” *Journal of Mechanical Science and Technology*, vol. 31, no. 7, 1 Aug. 2017, pp. 3627–3632., doi:10.1007/s12206-017-0650-y.

Toms, Stephanie R, et al. “Quasi-Linear Viscoelastic Behavior of the Human Periodontal Ligament.” *Journal of Biomechanics*, vol. 35, no. 10, Oct. 2002, pp. 1411–1415, doi:10.1016/s0021-9290(02)00166-5.

University of Michigan. “Section 7: Constitutive Equations: Viscoelasticity.” *BME 332: Introduction to Biosolid Mechanics*, websites.umich.edu/~bme332/ch7consteqviscoelasticity/bme332consteqviscoelasticity.htm