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Publication Date

2003-08-20

**Merger Simulation with Brand-Level Margin Data:
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August 20, 2003

ABSTRACT

We present a method to calibrate empirically the demand parameters in a merger simulation model by using brand-level profit margin data. While the approach can be generalized, we develop these ideas within a particular framework — the PCAIDS (proportionality-calibrated AIDS) model We show that the brand-level margins effectively define product "nests" (products that are especially close substitutes) and substantially increase the flexibility of PCAIDS for modeling critical own- and cross-price elasticities. The model is particularly valuable for transactions at the wholesale level (where scanner data do not exist) and for geographic markets that span national borders (where comparable data may not be available), since other methods to derive elasticities, particularly those based on econometric estimation, may not be possible or may not be reliable.

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I.   INTRODUCTION

Merger simulation has developed rapidly within the field of industrial organization as an important tool to evaluate unilateral price effects of mergers involving differentiated goods.[1] Simulation typically calculates these effects as percentage changes in equilibrium prices between the pre and post merger markets, assuming the absence of overt collusion among competitors. A virtually unknown area a few years ago, the FTC has recently termed simulation among the past decade's "remarkable developments in the quantitative analysis of horizontal mergers."[2] The appeal of simulation is that it provides an economically coherent framework to quantify potential unilateral price increases, taking into account market shares, efficiencies, and other key features of a transaction.

In practice, simulation has to confront the significant practical constraints of the merger review process, including often limited amounts of data, the need to control costs, and regulations that often permit only a short amount of time for the evaluation of competitive effects. Much research relating to merger simulation has focused on other complications involving the appropriate specification of demand systems, the empirical estimation of parameters, the assumption of static versus dynamic pricing behavior, and other methodological issues.[3] Given these factors, there is increasing interest in methods

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[1] For a recent review of some of the relevant literature, including references to the work of Werden and Froeb, Berry, Levinsohn, and Pakes, and Hausman, Leonard, and Zona, see Baker and Rubinfeld (1999).

[2] See *Issues in Econometric Analysis of Scanner Data*, available at www.ftc.gov.

[3] See Pinske and Slade (2002) for a more detailed discussion.

to obtain values for the inputs to the analysis that can make simulation a feasible and persuasive option in a broader range of situations.[4]

In this article we describe a modeling strategy to achieve these goals that is fundamentally different from existing approaches that are based on structural econometric estimation.[5] Specifically, we integrate the PCAIDS (Proportionality-Calibrated Almost Ideal Demand System) merger simulation methodology (Epstein and Rubinfeld (2002)) with brand-level profit margin data, data that should be available (at least for the parties) in an actual merger review. Our approach uses the margin data to estimate the PCAIDS parameters that define "nests" (i.e., groups of products that are particularly close substitutes). The nests provide a flexible and relatively parsimonious structure for estimating pre-merger demand elasticities that are consistent with the observed margins and that exactly satisfy the Bertrand first-order conditions for profit maximization conventionally used in merger simulation.

Briefly, PCAIDS is an approximation to the Almost Ideal Demand System (AIDS) that is widely used in applied microeconomics.[7] PCAIDS relies on a generalized principle of proportionality to reduce greatly the number of free parameters in the demand model: a price increase for a single brand results in diversion of lost sales to the other brands in proportion to their current market shares. In its most basic form (i.e., with a single nest), PCAIDS can be fully specified with two parameters: the margin for a single brand and the price elasticity for the market as a whole. If the market is well characterized by proportionality, this specification will yield a close approximation to the elasticities from the unrestricted AIDS. When the actual pattern of demand deviates from "strict" proportionality, then the quality of the approximation can be improved by adding additional nests to generalize the analysis.[9] As the amount of a priori margin (or elasticity) information increases, PCAIDS is able to calibrate more and more nests

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[4] Werden and Froeb (2002).
[5] See, e.g., Berry, Levinsohn, and Pakes (1995) in addition to Pinske and Slade (2002).
[6] Nests are clusters of brands that are particularly close substitutes.
[7] Deaton and Muellbauer (1980).
[8] In Epstein and Rubinfeld (2002) PCAIDS in this setting is equivalently parameterized using the price elasticity for a single brand and the market elasticity.
[9] Nests in our analysis are analogous to the different levels of a multi-stage budgeting model, an approach that is sometimes used to make econometric estimation tractable.

empirically. The resulting elasticities are less constrained than those implied by strict proportionality and can result in a much closer approximation to a full AIDS model.

The key to our analysis is the linkage between the "nesting parameters" in PCAIDS and the accounting data on profit margins.[10] We will show that in some circumstances the margin data are sufficient to identify the nesting parameters exactly, while in other cases the nesting parameters are not fully identified. Lack of identification is not necessarily a serious issue because the margin data nonetheless yield bounds on the possible values for the nesting parameters. In still other cases (corresponding roughly to a situation of many margins and relatively few nests) the nesting parameters will be overidentified, necessitating a further discussion of calibration strategies.

Properly measured accounting margins hold out the prospect of more accurate model calibration compared to conventional econometric modeling of pre-merger own and cross-price elasticities. The new approach may be particularly appealing for transactions at the wholesale level (where scanner data do not exist) and for geographic markets that span national borders (where comparable data may not be available). The econometric approach is not well suited for many transactions because of a lack of adequate data even when strong assumptions are made about market structure (such as a multi-stage budgeting process) to reduce the typically large number of parameters to be estimated. Even with relatively large datasets, the empirical results can be problematic, with wrong signs, implausible magnitudes, and low statistical reliability for the estimated coefficients.

The balance of this article is organized as follows. In Section II we set the stage by briefly reviewing the structure of PCAIDS with nests. In Section III we explain how the accounting profit margin data can be used to infer nesting parameters empirically. Section IV briefly discusses some of the relevant considerations when using accounting data in this context. Section V presents several examples of the analysis. Section VI contains a brief conclusion.

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[10] The term "nesting parameter" will be used in place of "odds ratio factor" and "scaling factor" used in Epstein and Rubinfeld (2002), pp. 896, 897.

II. MERGER SIMULATION WITH PCAIDS

Merger simulation models for differentiated products typically assume that prices in the market can be analyzed using Bertrand assumptions. According to this theory, the first-order conditions ("FOCs") for profit maximization by each firm can be specified in terms of market shares, incremental profit margins, and price elasticities. The market is assumed to be in Bertrand equilibrium both pre- and post-merger.

There are n firms pre-merger. The ith firm produces n_i brands and there is a total of N brands in the market. A general expression for all of the FOCs for profit maximization is given by the matrix equation:

$$s + \text{diag}(E_1, E_2, \ldots, E_n)S\mu = 0. \tag{1}$$

In this expression, $s = (s_1, s_2, \ldots, s_N)'$ is the vector of market shares (in terms of revenue) and $S = \text{diag}(s)$. The corresponding vector of brand margins is $\mu = (\mu_1, \mu_2, \ldots, \mu_N)'$. For the ith firm, E_i is an n_i by n_i matrix of transposed price elasticities with element (k, j) equal to ε_{jk}. In the pre-merger equilibrium, the brands margins μ are given by

$$\mu = -S^{-1}\text{diag}(E_1, E_2, \ldots, E_n)^{-1}s. \tag{2}$$

Assume that the merger involves firms $n–1$ and n. The merged firm is characterized by an augmented elasticity matrix E^*_{n-1} for the n_{n-1} plus n_n brands it is now producing. The FOCs for the post-merger market are

$$s + \text{diag}(E_1, E_2, \ldots, E^*_{n-1})S\mu = 0, \tag{3}$$

where all variables are understood to be taken at their post-transaction values. Merger simulation consists of finding the post-merger prices that yield margins, shares, and elasticities that solve (3).

The solution to equation (3) depends on the functional form of the underlying demand model and a supply model that determines how total cost responds to incremental changes in post-merger output. The demand side can be modeled using a variety of specifications; the literature includes examples of linear, constant elasticity, and variants of logit and AIDS systems. The supply side is generally treated as a step-function for which incremental cost does not vary with output. The step allows for possible merger-specific efficiencies, which are analyzed by changing the level of post-

merger incremental costs (keeping the assumption that the new level of incremental cost does not vary with output).

In PCAIDS each s_i is a linear function of the natural logarithms of the vector of prices p of all of the brands in the relevant market. Letting p be the vector of prices of the brands, the model can be written as

$$s = a + B\ln(p)$$

where a is a vector of constants and B is a matrix of coefficients (that are assumed to be invariant to price changes). Unlike AIDS, PCAIDS suppresses the aggregate expenditure terms, i.e., the model imposes homotheticity, so that a change in total industry expenditure has no effects on share.[11] To proceed, differentiate each share equation totally to obtain:

$$ds = B(dp/p) \tag{4}$$

Equation (4) describes a simple relationship between the change in each brand's market share (ds) and the unilateral effects (dp/p). The elements b_{ij} of B act as weights to determine the amount of share lost or gained due to unilateral effects. Moreover, as is apparent from (4), knowledge of the a_i terms is unnecessary. The post-merger shares for use in equation (3) are given by $s^{post} = s^{pre} + ds$. PCAIDS therefore is a particularly convenient demand model for merger simulation.

The post-merger own and cross-price elasticities for each brand in the market in general will also depend on the vector dp/p of unilateral effects. It can be shown[12] that in PCAIDS:

Own-price elasticity for the ith brand: $\varepsilon_{ii} = -1 + \dfrac{b_{ii}}{s_i} + s_i(\varepsilon + 1)$ $\tag{5}$

Cross-price elasticity of the ith brand with respect to the price of the jth brand:

$$\varepsilon_{ij} = \frac{b_{ij}}{s_i} + s_j(\varepsilon + 1). \tag{6}$$

Here ε is the price elasticity of demand for the market as a whole, which is typically assumed to remain unchanged post-merger. Using the s^{post} vector in equations (5) and (6) yields the post-merger elasticities.

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[11] To our knowledge, the empirical AIDS models in the literature seldom indicate an economically important role for the expenditure term.
[12] Epstein and Rubinfeld (2002), Appendix.

Finally, the solution to (3) requires updated brand profit margins. Algebraically, it can be seen that for each brand, $\mu_i^{post} = 1 - (1 - \mu_i^{pre})/\exp(dp_i/p_i)$. This relationship is independent of the demand model. This structure is sufficient to solve the post-merger FOCs entirely in terms of the predicted unilateral effects dp/p.

A. Calibrating PCAIDS Under Strict Proportionality

The problem remains of finding appropriate values for the b_{ij}. PCAIDS assumes that the share lost as a result of a price increase is allocated to the other firms in the relevant market in proportion to their respective shares. In effect, the market shares define probabilities of making incremental sales for each of the competitors. We also impose homogeneity on the demand model in equation (4) as an appealing theoretical property, i.e., $\sum b_{ik} = 0 \; \forall k$. (Since shares must sum to 100%, the model also satisfies an adding-up constraint $\sum b_{ki} = 0 \; \forall k$ by definition). Homogeneity with the proportionality assumption implies symmetry of B, thereby satisfying Slutsky symmetry, as will be proved below.

A three-brand example will illustrate the basic proportionality assumption. Consider a price increase dp_1/p_1 with all other prices unchanged. With proportionality, sales are diverted to brands 2 and 3 in proportion to their market shares. That is, $ds_2/ds_3 = s_2/s_3$. Moreover, the sum of the *changes* in shares across all brands must equal zero (because shares must always sum to 100%). It follows that b_{21} equals $-s_2/(s_2+s_3)b_{11}$ and b_{31} equals $-s_3/(s_2+s_3)b_{11}$ (the minus sign is necessary to satisfy $\sum ds_i = 0$). The other coefficients in B can be calibrated similarly, e.g., a change dp_2/p_2 implies b_{12} equals $-s_1/(s_1+s_3)b_{22}$ and b_{32} equals $-s_3/(s_1+s_3)b_{22}$. B can accordingly be expressed in this example as:

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[13] See Horizontal Merger Guidelines at ¶2.211.
[14] Our discussion of PCAIDS focuses on implementation with aggregate market share information. However, the method is also applicable as a set of restrictions that could be imposed when estimating AIDS with scanner data.

$$
B = \begin{pmatrix}
b_{11} & \dfrac{s_1}{(s_1 + s_3)}b_{22} & \dfrac{s_1}{(s_1 + s_2)}b_{33} \\[2ex]
\dfrac{s_2}{(s_2 + s_3)}b_{11} & b_{22} & \dfrac{s_2}{(s_1 + s_2)}b_{33} \\[2ex]
\dfrac{s_3}{(s_2 + s_3)}b_{11} & \dfrac{s_3}{(s_1 + s_3)}b_{22} & b_{33}
\end{pmatrix}.
$$

The matrix is completely determined by the three unknown diagonal elements. Proportionality has dramatically reduced the calibration problem from order N^2 to order N.

Homogeneity and adding-up simplify the problem even further. Express b_{33} as $-b_{31}-b_{32}$ and substitute in $b_{11} = -b_{12} - b_{13}$ to find that $b_{22}=(s_2/s_1)(1-s_2)/(1-s_1)b_{11}$. Similarly, $b_{33}=(s_3/s_1)(1-s_3)/(1-s_1)b_{11}$. That is, the entire demand model under proportionality can be calibrated in terms of a single parameter. (We prove below that this result holds regardless of the number of brands in the market.) Assuming the own-price elasticity ε_{11} is known for the first brand and that the market elasticity of demand ε is also known, invert equation (5) to find

$$b_{11} = s_1(\varepsilon_{11} + 1 - s_1(\varepsilon + 1)).$$

The B matrix is then determined by appropriate scaling of b_{11} with the market shares.

These ideas can be illustrated as follows. Assume that the shares for the 3 brands (each sold by a different firm) are 20%, 30%, and 50%, respectively. Now, assume that there is a proposed merger between firms 1 and 2, the industry elasticity is –1, and the own-price elasticity for the first brand is –3. Table 1 shows the resulting B matrix and elasticities.

Table 1

PCAIDS Coefficients and Elasticities

	PCAIDS Coefficient with Respect to:				Elasticity with Respect to:		
Brand	p_1	p_2	p_3	Brand	p_1	p_2	p_3
1	–0.400	0.150	0.250	1	–3.00	0.75	1.25
2	0.150	–0.525	0.375	2	0.50	–2.75	1.25
3	0.250	0.375	–0.625	3	0.50	0.75	–2.25

The PCAIDS coefficients satisfy adding-up and homogeneity and are symmetric, as required.

Assume that the first two firms merge. PCAIDS simulation with these parameters predicts a unilateral post-merger price increase (absent efficiencies) of 13.8% for Brand 1 and 10.8% for Brand 2.

Table 1 illustrates an important feature of strict proportionality: it constrains the cross-price elasticities corresponding to a given price change to be equal, although they may still vary substantially with respect to price increases across brands. The ability to derive a large number of elasticities from a single parameter (e.g., b_{11}) therefore comes at the expense of some flexibility in the model. This constraint is the main difference between PCAIDS and the full, unrestricted AIDS. It is generally of most concern when products are highly differentiated, since proportionality may not accurately describe the diversion of sales in those circumstances.[15] The purpose of nests is to relax this constraint and allow a closer approximation to the unrestricted AIDS.

B. Nests and Deviations from Strict Proportionality

We allow a more general analysis of elasticity by grouping brands in "nests."[16] Proportionality governs diversion within a nest, where brands are relatively close substitutes. Brands are poorer substitutes across nests than indicated by proportionality, implying variation in the cross-price elasticities. While $\varepsilon_{ik} = \varepsilon_{jk}$ for brands in the same nest, the cross-price elasticities are (relatively) lower across nests, i.e., $\varepsilon_{mk} < \varepsilon_{ik}$.for brands m and i in different nests.

To illustrate, return to the three-brand example discussed in the previous section. In that example, brand 2's market share of 30% and brand 3's share of 50% implied that

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[15] Cf. *Horizontal Merger Guidelines* at ¶2.211: "The market shares of the merging firms' products may understate the competitive effect of concern, when, for example, the products of the merging firms are relatively more similar in their various attributes to one another than to other products in the relevant market. On the other hand, the market shares alone may overstate the competitive effects of concern when, for example, the relevant products are less similar in their attributes to one another than to other products in the relevant market."

[16] See Werden and Froeb (1994) for a discussion of nests in the context of a logit demand model.

37.5% (30/80) of the share lost by brand 1 when its price increased would be diverted to brand 2 and 62.5% (50/80) would be diverted to brand 3. This effect can be characterized using an *odds ratio*. Here, the odds ratio between brand 2 and brand 3 is 0.6 (0.375/0.625). That is, under proportionality, brand 2 is only 60% as likely to be chosen by consumers leaving brand 1 as brand 3. Now suppose instead that brand 2 is relatively "farther" from brand 1 in the sense that that fewer consumers would choose brand 2 in response to an increase in p_1 than would be predicted by proportionality. For example, brand 2 may only be "half as desirable" a substitute as brand 3 and the appropriate odds ratio really only 0.3. It is straightforward to calculate in this case that the share diversion to brand 2 becomes 23.1% and the diversion to brand 3 increases to 76.9% (an odds ratio of 0.3=.231/.769). As expected, fewer consumers leaving brand 1 would choose brand 2.

We use "nesting parameters" to generate the scaling factors that adjust diversion away from proportionality   Share diverted to a brand in a different nest is adjusted in the following sense: the odds ratio is equal to the odds ratio under proportionality, multiplied by a nesting parameter on the interval (0,1]. For brands within a nest, the nesting parameter effectively equals 1.   The result is that brands within a nest are closer substitutes than brands outside the nest. Proportionality for all brands can be thought of as a model with a single nest. PCAIDS with multiple nests allows a more flexible pattern of cross elasticities, as the model is no longer fully constrained by the proportionality assumption.[17]

To characterize the nest structure in above example we place brand 2 in a separate nest with a nesting parameter of 0.5. Table 2 reports the calculated elasticities for both the nested model and the original model.

Table 2

PCAIDS Elasticities with Nests

	Non-Nested Demand			Separate Brand 2 Nest, (Odds Ratio Factor = 0.5)		
	Elasticity with Respect to:				Elasticity with Respect to:	
Brand	P_1	p_2	p_3	Brand	p_1	p_2	p_3

---

[17] Given the fundamental role of nests in PCAIDS, discussions that ignore this feature of the model, e.g., Wu (2003), model are necessarily incomplete.

1	−3.00	0.75	1.25	1	−3.00	0.46	1.54
2	0.50	−2.75	1.25	2	0.31	−2.08	0.77
3	0.50	0.75	−2.25	3	0.62	0.46	−2.08

The cross-price elasticities ε_{2i} in the right-hand panel are scaled down by 50% relative to the other brands.[18] (The cross elasticities measuring the responses of brand 1 and brand 3 to the price of brand 2 remain equal, but at lower values, because brands 1 and 3 are in the same nest.) In general, the nesting treats brand 2 as a poorer substitute for brands 1 and 3, while brands 1 and 3 become better substitutes for each other

Simulation of a merger of brand 1 and brand 2 using this nested PCAIDS model predicts a unilateral price increase (without efficiencies) of 10.1% for both brand 1 and brand 2, compared to the original increases of 13.8% and 10.8% without nests. The unilateral effects are smaller because the merging brands are less close substitutes for each other.

The number of nesting parameters required in the model obviously depends on the number of nests. More specifically, the number of parameters equals the number of pairs of nests, because each parameter modifies the share diversion between the two associated nests. With 2 nests there is one nesting parameter; a 3-nest specification requires three parameters; and a 4-nest specification requires six parameters. Because the number of nesting factors increases exponentially with the number of nests, a tractable simulation model probably should not have more than 3 or 4 nests.

Assume that there are w nests, $2 \leq w \leq N$, with each brand assigned to a nest. The number of nesting parameters is $w(w-1)/2$. To summarize the nesting parameters, it is helpful to arrange them in a matrix. In the case of three nests the matrix takes the form:

$$\Omega = \begin{pmatrix} & f_1 & f_2 & f_3 \\ f_1 & 100\% & \omega_1 & \omega_2 \\ f_2 & \omega_1 & 100\% & \omega_3 \\ f_3 & \omega_2 & \omega_3 & 100\% \end{pmatrix} \tag{7}$$

---

[18] The calculations continue to assume an own-price elasticity of −3 for Brand 1 and an industry elasticity of −1. It would be incorrect to scale the non-nested elasticities in the left-hand panel directly. Nests affect the impact of adding-up, homogeneity, and symmetry and the appropriate calculation takes account of these constraints to generate economically consistent elasticities.

The matrix is symmetric. Given a price increase for a brand in nest f_i, the diversion of share to a brand nest f_j deviates from proportionality by the nesting parameter $0 < \Omega(\Im(i), \Im(j)) \le 1$. $\Im(k)$ is an indicator function that returns the nest containing brand k. Proportionality is the special case where $\Omega(\Im(i), \Im(j)) = 100\%$.

We illustrate a general calculation of b_{ij} with nesting parameters with the example used for Table 2. There are two nests, so Ω contains a single sub-diagonal element ω_1. We assume that $\Omega(\Im(1), \Im(2)) = \omega_1$ and that $\Omega(\Im(1), \Im(3)) = 100\%$. That is, brands 1 and 3 are in a common nest and brand 2 is in a separate nest. The share diversion for the price change for brand 1 can be expressed (imposing the adding-up condition) as

$$-ds_1 + \lambda_2 s_2/(1-s_1)ds_1 + \lambda_3 s_3/(1-s_1)ds_1 = 0 \tag{8}$$

or, rewriting,

$$\lambda_2 s_2/(1-s_1) + \lambda_3 s_3/(1-s_1) = 1, \tag{9}$$

where the λ's are share-diversion weights to be determined. In the case of strict proportionality, $\lambda_i = 100\%$. The adjustment due to the deviation from proportionality in this example satisfies

$$\lambda_2 / \lambda_3 = \Omega(\Im(1), \Im(2)) / \Omega(\Im(1), \Im(3)) = \omega_1.$$

It follows from our prior assumption that $\lambda_2 = \omega_1 \lambda_3$, and by substitution in equation (9), $\lambda_3 = (1-s_1)/(\omega_1 s_2 + s_3)$. Finally, equation (8) can be written as

$$-ds_1 + \omega_1 s_2/(\omega_1 s_2 + s_3)ds_1 + s_3/(\omega_1 s_2 + s_3)ds_1 = 0$$

That is, $b_{21} = \omega_1 s_2/(\omega_1 s_2 + s_3)b_{11}$ and $b_{31} = s_3/(\omega_1 s_2 + s_3)b_{11}$. To evaluate these expressions substitute the shares (30%, 50%), $b_{11} = -0.400$ (unchanged from Table 1), and $\omega_1 = 0.5$. The results are 0.231 and 0.769, agreeing with the share diversion percentages that we calculated at the beginning of this section.

C. General Calibration of PCAIDS with Nests

We generalize the determination of B with nests as follows. Each element of B can be written as $b_{ik} = \theta_{ik} b_{kk}$, where the θ's are known, but the diagonal elements b_{kk} are unknown. Impose adding-up and homogeneity. The constraints imply a system of $N-1$ independent equations in the N unknown diagonal elements. Without loss of generality,

11

normalize with respect to the first brand and define a vector β with $N–1$ elements equal to $b_{jj}/b_{11} = \beta_j$, $j > 1$. The equation system is then non-singular and can be written as

$$
\begin{pmatrix}
\theta_{12} & \theta_{13} & \cdots & \theta_{1N} \\
1 & \theta_{23} & \cdots & \theta_{2N} \\
\vdots & & & \\
\theta_{N-1,2} & \cdots & 1 & \theta_{N-1,N}
\end{pmatrix}
\begin{pmatrix}
\beta_2 \\
\vdots \\
\beta_N
\end{pmatrix}
=
\begin{pmatrix}
-1 \\
-\theta_{21} \\
\vdots \\
-\theta_{N-1,1}
\end{pmatrix}
\tag{10}
$$

Equation (10) can be inverted to solve for the β vector. B can therefore be specified entirely in terms of b_{11} and the θ's. With known ε_{11} and ε, calibration is completed by using (5) to solve for the value for b_{11}.

The θ's are known functions of the market shares and the nesting parameters. In general it can be shown that:

$$
\theta_{ik} = -s_i \frac{\Omega(\Im(k), \Im(i))}{\sum_{m \neq k} s_m \Omega(\Im(k), \Im(m))} \quad , i \neq k. \tag{11}
$$

With strict proportionality (i.e., a single nest that contains all of the brands), the nesting parameter equals 100% and equation (11) reduces to $\theta_{ik} = -s_i/(1 - s_k)$.

We now show that the matrix B of PCAIDS coefficients is symmetric both under strict proportionality and with nests. Since equation (10) has a unique solution, any feasible solution is also unique. We try the symmetric solution for B and assume that $b_{j1} = \theta_{j1} b_{11}$ and $b_{1j} = \theta_{1j} b_{jj}$, symmetry implies that $\beta_j = \theta_{j1} / \theta_{1j}$. From equation (11) it follows that

$$
\beta_j = \frac{s_j}{s_1} \frac{\sum_{m \neq j} s_m \Omega(\Im(j), \Im(m))}{\sum_{m \neq 1} s_m \Omega(\Im(1), \Im(m))} \tag{12}
$$

and from before, $b_{jj} = \beta_j b_{11}$.

Equations (11) and (12) imply that

$$
b_{ij} = -\frac{s_i s_j}{s_1} \frac{\Omega(\Im(i), \Im(j))}{\sum_{k=2}^{N} s_k \Omega(\Im(k), \Im(1))} b_{11}. \tag{13}
$$

for $i \neq j$. Symmetry of B follows directly. It can be shown that the b_{ij} from equation (13) satisfy adding-up and homogeneity. They therefore comprise the unique solution to equation (10).

What remains is the problem of finding appropriate values for the nesting parameters. A coarse grid on the parameters (e.g., 0.75, 0.50, and 0.25) is generally sufficient to assess the sensitivity of the simulation of different nests. This is easy to carry out in the case of two nests but rapidly becomes unwieldy as the number of nests increases.

III. MARGINS AND NESTS IN PCAIDS

In this section we show how margin data can be used to estimate nesting parameters. Begin with the ("FOCs") in the pre-transaction market:

$$s + \mathrm{diag}(E_1, E_2, \ldots, E_n)S\mu = 0 \qquad\qquad (14)$$

Each firm may sell brands in different nests and every brand must be assigned to a nest. As a theoretical matter, since proportionality is unlikely to hold exactly, each brand might be put in a separate nest. However, as previously discussed, this strategy rapidly generates an intractable number of required parameters. *A priori* information must be available to group brands more broadly. To simplify the analysis, we aggregate the brands in the same nest for a given firm into a composite brand whose share is equal to the sum of the shares of the underlying brands. That is, the number of different nests that a firm sells into equals the number of (possibly composite) brands that it produces.

Assume that firms 1 and 2 are the merger partners and that the margins are known for each of the brands they produce. Each nest has either one or two margins associated with it; one margin if only one firm sells a brand in the nest, and two otherwise. These data are likely to be available to the merger parties and to the appropriate enforcement agency. To further simplify, assume also that no other margins are known and that the industry elasticity equals –1 (these assumptions are not essential to the results that follow). From equations (5) and (12), it can be seen that the elasticity matrices E_i in the FOCs are functions of the margins, market shares (and industry elasticity), and the unknown nesting factors.

The problem of determining nesting parameters amounts to finding values for them that generate E matrices that satisfy the FOCs with the pre-determined elements of the margin vector μ. Because margins are only known for firms 1 and 2, the nesting

parameters depend only on the FOCs for the two merger partners (more generally, the parameters will depend on the FOC equations for the brands with the known margins). The structure of PCAIDS leads to a simple solution.

To simplify the notation, let the subscript $\dagger$ denote the rows and columns in equation (14) that refer to the brands sold by the merging firms. Then $E_\dagger$ is the n_1+n_2 transposed block-diagonal matrix of elasticities for firms 1 and 2, and $B_\dagger$ is the corresponding n_1+n_2 block-diagonal matrix of coefficients from the B matrix of PCAIDS coefficients. It is straightforward to show that $E_\dagger = B_\dagger S_\dagger^{-1} - I$, where I is the identity matrix with rank n_1+n_2. Rewrite (14) as

$$S_\dagger \mu_\dagger = -E_\dagger^{-1} s_\dagger$$

where the expression just includes the shares for firms 1 and 2. By substitution,

$$S_\dagger \mu_\dagger = -(B_\dagger S_\dagger^{-1} - I_\dagger)^{-1} s_\dagger$$

implying

$$(B_\dagger S_\dagger^{-1} - I_\dagger) S_\dagger \mu_\dagger = -s_\dagger.$$

Rewriting,

$$B_\dagger \mu_\dagger = S_\dagger (\mu_\dagger - \mathbf{1}) \tag{15}$$

where $\mathbf{1}$ is a column vector of 1's. That is, the FOCs can be rewritten as a system of linear equations in the PCAIDS coefficients.

The next step is to express $B_\dagger$ in terms of the nesting factors. Using equations (12) and (13), it follows that

$$B_\Uparrow = \left[-ss' \otimes (\omega - I) + diag(s_i \sum_{m \neq i} s_m \Omega(\Im(j), \Im(m))) \right] \times \tau$$

where

$$\tau = \frac{b_{11}}{s_1 \sum_{m>1} s_m \Omega(\Im(j), \Im(m))}.$$

Finally, rewrite (15) as

$$\left[-ss' \otimes (\omega - I) + diag(s_i \sum_{m \neq i} s_m \Omega(\Im(j), \Im(m))) \right] \mu = S(\mu - 1) \times \frac{s_1 \sum_{m>1} s_m \Omega(\Im(j), \Im(m))}{b_{11}}$$

$$(16)$$

This shows that the FOC's are also a linear function of the nest factors, conditional on knowledge of b_{11}.

The solution to (16) depends on the number of nests. We begin with the cases that are likely to be most common and most tractable - either 2 or 3 nests. With two nests (and two margins) there is an exact solution. In this case, equation (16) reduces to a system of 2 equations in 2 unknowns, the single nesting parameter and the b_{11} coefficient. With three nests, (16) becomes a system of 3 equations in 4 unknowns, three nesting parameters, and the b_{11} coefficient. The most convenient solution in this case is to solve (16) by setting an exogenous value for one of the nesting parameters. A range of solutions is possible, depending on the selection of the exogenous parameter.

This 3-nest system is underidentified. However, (16) still places bounds on the solution because the nesting parameters must lie on the (0,1] interval. Depending on the particular values of the key parameters, these bounds can be fairly tight, resulting in a set of simulations that are highly informative.

Now consider the possibility of overidentification. When there are multiple margins for the same nest, estimates of nesting parameters are no longer unique. The 2-nest system will illustrate. Suppose initially that firm 1 produces a single brand A_{11} and firm 2 produces a single brand B_{21}. A_{11} is in nest f_1 and B_{21} is in nest f_2. This system has an exact solution. Now suppose that firm 1 also produces a second brand A_{12} in nest f_2. The system still has two nests, but f_2 now has two brands available for the calibration. We believe it appropriate to use the additional margin information to evaluate the sensitivity of the merger simulation analysis to the specification of the demand model.

To test the specification of the model, we propose that one calibrate (16) using only the A_{11} and B_{21} margins, while treating A_{12} as belonging to f_2, but with an unknown margin. The results would then be used to solve equation (14) for the implied remaining margins in the model. This would yield a predicted value for the A_{12} margin. A predicted margin that was close to the actual A_{12} margin would provide evidence that it was reasonable to place A_{12} in the same nest as B_{21} and the test would stop.

If, however, the predicted and actual A_{12} margins were substantially different, we would reevaluate the original nesting assumptions. One possibility is that it is more appropriate to place A_{12} in f_1. This could be tested by putting A_{12} in a new nest f_3 with the constraints (using the notation in equation (7)) that $\omega_2=100\%$ and $\omega_3=\omega_2$. In this case, equation (14) would be recalibrated using the new nest structure and the two

additional, pre-determined nesting parameters. Solving (14) again for the new predicted A_{12} margin would now test whether the brand belongs in f_1. If the predicted and actual margins still diverged, this would be evidence that A_{12} should be placed in its own nest with independent nesting parameters.

Analogous considerations apply to the 3-nest system.

IV. MEASURING INCREMENTAL PROFIT MARGINS USING ACCOUNTING DATA

The relevant profit margin to calibrate the model in principle should be based on the profit associated with an incremental increase in output, i.e., the difference between the incremental revenue and the incremental cost associated with the additional output. While seemingly straightforward, this information generally is not shown explicitly in the firm's financial statements, implying the need for further accounting and economic analysis.   For example, external reporting under Generally Accepted Accounting Principles (GAAP) mandates expense categories that typically commingle fixed costs with variable costs. In addition, certain costs of capacity should be treated as fixed in some circumstances but variable in others. Finally, adjustments for common costs and joint products may be required for measuring net incremental costs for a single product of a multi-product firm.

In particular, the gross margin (revenue minus cost of goods sold, or COGS) reported on a firm's income statement prepared under GAAP usually is not the relevant measure of the incremental profit margin.   The main issue is that COGS includes allocated fixed costs of production, such as rent and depreciation, which may not vary with output. It also excludes variable sales, marketing, and administrative expenses (such as commissions and warranties). If the firm's current production is at or near capacity (which can be limited by physical plant and equipment, personnel, and intangible assets such as quotas under licenses), then additional capacity would be required for the incremental production and the associated costs should be included in the profit margin calculation.

Empirically, two approaches are typically used to determine incremental costs. The first approach relies on a regression analysis in which the dependent variable is total

operating costs and the independent variable is either quantity produced or sales revenue. The regression coefficient on the quantity variable provides an estimate of incremental cost. The regression coefficient on the revenue variable is an estimate of 1 minus μ, where μ is the incremental profit margin percent. Multiple regression analysis is also used, where the independent variables are various "cost drivers" (e.g., machine hours) that are correlated with output.[19] While potentially highly informative, regression analysis is subject to a variety of statistical pitfalls that can lead to unreliable estimation results. It can also be misleading to extrapolate the results of a regression when cost effects due to capacity limits or multi-product production are relevant.

The second accounting analysis approach involves identifying each line item in a firm's detailed, internal cost reporting system as either "fixed" or "variable." This can be especially useful when the adjustments to COGS are reasonably straightforward, e.g., subtracting out depreciation or adding in sales commissions. However, account analysis can entail subjective assumptions when the individual line items still combine fixed and variable components. Account analysis can also become arbitrary in the case of multi-product firms, where a variable cost item may have to be allocated across different product lines.

Costs associated with the need for additional capacity are also likely to require separate analysis. When capacity can be rented, market prices should be available to include in incremental cost. If the firm must undertake additional investment, economic and financial analysis is required to determine the optimal scale of the investment and the associated capital charge. That is, the capacity cost in this situation is incremental and needs to be amortized over the economic useful life of the investment.

V. EXAMPLES

To illustrate the ideas developed to this point we now discuss several different examples. We begin with model calibration in the 3-firm market example described previously. We then turn to a well-known analysis of the beer market published by

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[19] Horngren, Dukar, and Foster (2003).

Hausman, Leonard, and Zona ("HLZ"). Our last example focuses on identification issues.

A. Calibration in a 3-firm market

Recall that each of the 3 firms produces a single brand; the shares for the firms are 20%, 30%, and 50%, respectively. The industry elasticity is –1, and the merger partners are firms 1 and 2, with firm 1 having an incremental profit margin of 33.3%.

Suppose first that all brands are assumed to belong in a single nest and that the simulation model is calibrated with the firm 1 margin. PCAIDS generates an elasticity of –2.75 for firm 2 (see Table 1), implying a corresponding margin of 36.4% (the negative reciprocal of the elasticity). If the actual incremental profit margin were close to 36.4%, we would take this as support for the strict proportionality assumption and use of a single nest in the model. Conversely, if the model were calibrated with the firm 2 margin, the resulting elasticity and margin for firm 1 would be –3 and 33.3%, respectively, which would also support the proportionality assumption.

Suppose, however, that firm 2 had a profit margin of 48.1%. This would suggest that firm 2 faces less competition than implied by proportionality. We would infer that the model requires two nests, with firms 1 and 3 in the same nest and firm 2 in a separate nest. The model is exactly identified because the two margins map into the two unknown parameters in equation (16). The solution yields a nesting parameter equal to 0.5, since (see Table 2) this value for the nesting parameter results in an elasticity for firm 2 of – 2.08, which corresponds to the observed 48.1% margin.

The decision as to how to group brands into nests is an important one. Thus, firm 2 could also have faced reduced competition if it were in the same nest as firm 3 and firm 1 was in a separate nest. Obviously, the nesting parameter, and the resulting elasticities in the model, can be sensitive to this choice. Beyond the intuition just given, there is one additional helpful guide. A nesting parameter must lie in the interval (0,1] to be economically meaningful. In this case, the alternative nest structure can only satisfy the observed margins if the nesting parameter had the value 2.16, an extraneous solution that in our view rules out this nest from further consideration. We conclude that the most appropriate model should use the nests in Table 2.

We believe it reasonable to utilize a principle of "maximum proportionality" as an additional model selection criterion that is relevant to the grouping problem. When different nest structures are consistent with the margin data and have valid nesting parameters, we advise selecting the structure for which the length of a vector of 1's minus the vector of feasible nesting parameters is a minimum. This metric yields a solution with a minimum deviation from strict proportionality. In the case of a single parameter, for example, we would use the nests for which the parameter is closest to 1.0. We view this as an application of Occam's razor, since proportionality has the virtue of simplicity.[20]

Grouping and maximum proportionality in this example are further illustrated in Table 3. For each of the three possible configuration of nests, the table shows the ranges of feasible elasticities for firms 1 and 2. More specifically, for each configuration we first hold the elasticity for firm 1 at –3 (the observed 33.3% margin) and solve for the minimum and maximum possible elasticities (and associated margins) for firm 2 obtained by varying the nesting parameter over (0,1]. We then hold the firm 2 elasticity at –2.75 (implied by the firm 1 margin and strict proportionality) and similarly solve for the possible elasticity range for firm 1. That is, each configuration "starts" with a nesting parameter of 1.0 and then maps the feasible deviations from strict proportionality that are consistent with the observed margins. One of the main implications of Table 3 is that the various configurations of nests are able to accommodate an extremely wide range of margins.

Suppose we are in the situation where firm 1 has a 33.3% margin and firm 2 has a 48.1% margin. Table 3 shows that the only feasible nest structure to attain the required elasticities of –3 and –2.08 is Configuration A (with implied nesting parameter equal to 0.5). The other two nest structures are not consistent with the margin data. Suppose instead that firm 2 had an elasticity of –2.75 (36.4% margin), but that firm 1 had an elasticity of –1.5 (66.7% margin). Then C is the only feasible solution (with a nesting

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[20] The assumption of "maximum proportionality" appears consistent with the views of other economists who specialize in merger analysis. See Werden and Froeb (2002) p. 14 who state, "Absent contrary evidence, substitution in proportion is often viewed as the most natural default assumption. We share that view…" and their cites to Willig (1991) and the Horizontal Merger Guidelines.

parameter equal to 0.19). Finally, suppose the elasticities for firms 1 and 2 were –3.50 and –2.75, respectively. Configurations A and B both provide solutions, with nesting parameters 0.71 and 0.07, respectively. By maximum proportionality, A is preferred. A parameter of 0.07 suggests that firm 3 virtually does not compete with firms 1 or 2. Ideally, other information external to the model would also indicate that firm 3's brand was a reasonably good substitute for firms 1 and 2 to support an inference that 0.71 is reasonable and 0.07 is implausibly small.

Table 3

**Nest Configurations and Feasible Elasticities
for the Merging Firms**

| | Firm 1 Margin = 33.3% | | Firm 2 Margin = 36.4% | |
| | Implied Firm 2 Elasticity: | | Implied Firm 1 Elasticity: | |
Nest Configuration	Minimum	Maximum	Minimum	Maximum
A: (1,3) —(2)	–1.02	–2.75	–3.00	–10.00
B: (1,2)—(3)	–2.34	–2.75	–2.75	–3.60
C: (2,3)—(1)	–2.75	–10.00	–1.02	–3.00

Note: Only elasticities in the interval [–10, –1) are included.

B. The Light Beer Market Revisited

We first illustrate PCAIDS with nests by analyzing a model of retail demand for beer published by Hausman, Leonard, and Zona ("HLZ").[21] We focus on the light beer segment that was estimated as part of a multi-stage budgeting model of a broader beer market. HLZ used a panel of weekly store-level data to estimate demand for five different brands (Genesee Light, Coors Light, Old Milwaukee Light, Miller Lite, and Molson Light) using an AIDS model. Their AIDS specification imposed symmetry and homogeneity, but was otherwise unrestricted. Although HLZ did not report market

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[21] Hausman, Leonard, and Zona (1994).

shares for the brands, the estimation results contain sufficient information for us to infer reasonable values. For purpose of this example we have assumed the shares shown in Table 4.[22]

Table 4
Estimated Light Beer Market Shares

Brand	Share (%)
Genesee Lite	37.1
Coors Light	25.7
Old Milwaukee Light	11.4
Miller Lite	15.9
Molson Lite	9.9
Total	100.0%

We calibrate PCAIDS using HLZ's estimates of an unconditional price elasticity for Genesee of –3.763 and a light beer segment elasticity of –2.424. These values imply $b_{11} = -0.813$. The resulting PCAIDS coefficients with no nests (using equations (12) and (13) are:

$$
\begin{array}{ccccc}
-0.813 & 0.332 & 0.148 & 0.206 & 0.127 \\
0.332 & -0.665 & 0.102 & 0.143 & 0.088 \\
0.148 & 0.102 & -0.353 & 0.063 & 0.039 \\
0.206 & 0.143 & 0.063 & -0.467 & 0.055 \\
0.127 & 0.088 & 0.039 & 0.055 & -0.309
\end{array}
$$

with implied elasticities:

Table 5
PCAIDS Elasticities (No Nests)

Genesee Light	-3.72	0.53	0.24	0.33	0.20
Coors Light	0.76	-3.95	0.24	0.33	0.20
Old Milwaukee Light	0.76	0.53	-4.25	0.33	0.20
Miller Lite	0.76	0.53	0.24	-4.16	0.20
Molson Light	0.76	0.53	0.24	0.33	-4.28

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[22] It appears that their data were for stores in upstate New York, which (at least several years ago) would account for the brands and relative shares.

As expected under strict proportionality, the cross-price elasticities in each column are identical.

Compare the PCAIDS elasticities to those estimated by HLZ:

Table 6
HLZ Light Beer Elasticities

Genesee Light	-3.76	0.46	0.40	0.25	0.20
Coors Light	0.57	-4.60	0.41	0.45	0.48
Old Milwaukee Light	1.23	0.96	-6.10	0.84	0.57
Miller Lite	0.51	0.74	0.59	-5.04	0.58
Molson Light	0.68	1.21	0.61	0.89	-5.84
Ave. HLZ cross price elasticity	0.75	0.84	0.50	0.61	0.46

The variation in the HLZ cross-price elasticities suggests that strict proportionality is not satisfied within a segment defined as these five brands. (In contrast, HLZ's results support proportionality much more strongly in their premium beer segment.)

Deviating from proportionality requires assumptions about nesting. While beer lovers can debate what brands are the closest substitutes, HLZ suggest that Coors Light and Miller Lite are particularly close. We put these brands in nest f_1, along with Molson Light as another high quality, heavily advertised national brand. We find it plausible that Old Milwaukee Light occupies a middle ground between those three brands and Genesee Light, a regional label that is not without its charms. Accordingly, we put them in separate nests f_2 and f_3, respectively. Finally, we hypothesize the following nesting parameters:

$$\Omega = \begin{array}{c|ccc} & f_1 & f_2 & f_3 \\ \hline f_1 & 100\% & 75\% & 50\% \\ f_2 & 75\% & 100\% & 75\% \\ f_3 & 50\% & 75\% & 100\% \end{array}$$

These parameters imply that f_2 is "equally far" from f_1 and f_3, and that f_3 is relatively farther from f_1.

Recalibrating PCAIDS with these additional parameters results in the following matrix of elasticities:

Table 7

PCAIDS Elasticities (3 Nests)

Genesee Light	-3.76	0.48	0.40	0.30	0.18
Coors Light	0.69	-4.82	0.40	0.82	0.51
Old Milwaukee Light	1.30	0.90	-5.50	0.56	0.35
Miller Lite	0.69	1.32	0.40	-5.32	0.51
Molson Light	0.69	1.32	0.40	0.82	-5.63
Average PCAIDS cross-price elasticity with nests	0.85	1.01	0.40	0.62	0.39

The three nesting parameters bring the PCAIDS elasticities quite close to the unrestricted HLZ results. We might view them as analogous to a "sufficient statistic" for the demand system. That is, it appears that the four PCAIDS parameters (b_{11} and three nesting parameters) contain essentially the same information as the 25 coefficients estimated by HLZ.

C. Calibration in a 3-Nest System

Suppose there are five firms in the market, the merger partners (A, B) plus three competitors. Firm A sells two brands. The other firms sell one brand. There are three nests (Popular, Budget, and Prestige). Collectively, firms A and B sell into all three nests. In the absence of nests, we would calibrate the PCAIDS model if we knew one brand elasticity. Now, however, we have the case of 3 FOC equations (for the 3 brands produced by the merger partners) and 4 unknowns (b_{11} and the three nesting parameters). The shares and nests in the market are as follows:

Table 8
Shares in a 3 Nest Market

Firm-Brand	Share (%)	Nest
A-1	10.0	Popular
A-2	7.5	Prestige
B	12.5	Budget

C	15.0	Popular
D	25.0	Budget
E	30.0	Popular
Total	100.0	

We consider three different margin scenarios to create three different examples of nest calibration.

Scenario 1

Suppose the margins for brands A-1, A-2, and B are 40.0%, 55.0%, and 45.0%, respectively, and the overall market demand elasticity is –1.0. We use the solution method outlined above that sets one of the nesting parameters exogenously. First, we test whether the 3 nests can be reduced to 2 nests. This is done by solving equation (16) three times, each time fixing a different ω_i at 100%. We found that in no case was there a feasible solution. That is, each solution to equation (16) with $\omega_i = 100\%$ required at least one nesting parameter outside (0,1] to satisfy the FOCs. This is an important piece of information that supports the necessity of using a model specification with at least three nests.

We next find the range of feasible solutions for equation (16) by using a coarse grid to set ω_3 exogenously. We found that a value of approximately 0.75 was the highest value for ω_3 that permitted a feasible solution. We then solved equation (16) using the grid (.75, .50, .25, and .01). Each exogenous value for ω_3 implies corresponding unique values for ω_1 and ω_2. Table 9 reports the range of solutions.

Table 9
40%, 55%, and 45% Margins: Nest Factors

ω_1 Popular— Prestige	ω_2 Budget— Popular	ω_3 Budget— Prestige
0.03	0.24	0.75
0.24	0.30	0.50
0.46	0.36	0.25

0.66	0.41	0.01

The results show a fairly tight range for ω_2, the Budget—Popular nesting parameter, centered at approximately 0.32. The other two parameters move inversely. If there is evidence that both Popular and Budget are substitutable for Prestige, then the parameters close to 0 may be ruled out as solutions. That is, we could exclude the solutions where $\omega_3 = 0.75$ or $\omega_3 = 0.01$. On this basis, Table 8 suggests that a reasonable solution is to assume that *all* nest factors are in the range 0.25 to 0.50. This is likely to offer sufficient precision for the nest factors in many cases. Alternatively, the principle of maximum proportionality generates a solution vector of approximately (0.37, 0.34, 0.35), a similar result.

Scenario 2

Suppose the margins are 40.0%, 35.0%, and 35.0%. In this case, there is *no* feasible solution for any set of nesting factors in the (0,1] range. There are several possible explanations. First, the margins may not have been measured or reported accurately. It is possible, of course, that firms are not Bertrand pricers, or that markets are not in equilibrium. A second possibility – the one that we find the most intriguing -- is that the assignment of the brands to the nests was not reasonable. For example, there is a feasible solution if C is moved from Popular to Prestige. This suggests that using margin data with PCAIDS can offer a useful methodology for model specification purposes.

Scenario 3

Finally, suppose the margins are 40.0% for all three brands. There is a feasible solution in this case. Moreover, the range of feasible solutions is tight, as Table 10 illustrates.

Table 10

40% Margins for all Brands: Nest Factors

Popular— Prestige	Budget— Popular	Budget-- Prestige
0.76	0.71	1.00
1.00	0.80	0.80

The nest factors are not identified, but are nevertheless estimated with a high level of precision.

VI. CONCLUDING REMARKS

We believe that the direct use of brand-margin data to estimate the parameters of demand systems offers a fruitful empirical basis for merger simulation analysis. We have shown, within the PCAIDS framework, that the margins serve to calibrate empirically the nest structure (groups of products that are particularly close substitutes) and generalize the analysis of own- and cross-price elasticities. PCAIDS in its basic formulation relies on an assumption of "proportionality" which greatly reduces the number of free parameters but constrains all cross-price elasticities corresponding to a given price change to be equal.   By incorporating nests, however, PCAIDS can relax the proportionality constraint and more closely approximate an unconstrained AIDS model.

This modeling strategy exploits information that is likely to be available in the typical merger investigation (at least for the merging parties), but that has not been integrated into existing econometric models used for merger simulation. In some cases, the nesting parameters will be exactly identified.   In other cases, they will be underidentified, but simulated price effects can be still bounded. In still other cases, the nesting parameters will be overidentified, which permits an analysis of the robustness of the underlying demand specification.

PCAIDS may be especially appealing for mergers at the wholesale level (where scanner data do not exist) and for geographic markets that span national borders (where comparable data are often difficult to gather).   Available brand margin data, market

shares, and an estimate of the market demand elasticity should be sufficient to make PCAIDS feasible in such cases. In contrast, it may be difficult or even impossible to implement more data-intensive methodologies that rely on the econometric estimation of demand systems. Some of the areas for continued work include: how does PCAIDS compare in practice and in theory to a logit model with nests? to a multi-stage AIDS model? What are the most appropriate procedures for testing and evaluating the robustness of the PCAIDS results to the choice of nesting parameters? These and other questions are ripe for future research.

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