Field Determination of the Hydrologic Properties and Parameters that Control the Vertical Component of Groundwater Movement

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FIELD DETERMINATION OF THE HYDROLOGIC PROPERTIES AND PARAMETERS THAT CONTROL THE VERTICAL COMPONENT OF GROUNDWATER MOVEMENT

(A Topical Report)

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EXECUTIVE SUMMARY

The vertical movement of groundwater through geologic formations generally constitutes the shortest pathway for transport of contaminated fluid from polluted aquifers to the accessible environment. If such movement is encouraged by unfavorable conditions such as an upward hydraulic gradient or relatively permeable zones, it could jeopardize the safety of an underground nuclear waste repository. In this report, a description has been attempted of some of the conventional techniques for measurement of the properties and parameters which control the vertical movement and travel time of groundwater.

The most important properties and parameters which control the vertical movement and travel time of groundwater are the efective porosity, hydraulic conductivity, and storativity of the geologic media, and the distribution of hydraulic head within the formations around the repository. Given a candidate repository site, a thorough knowledge of the magnitude and variation of these parameters within the adjacent geologic formations is essential in evaluating the safety of the site.

Determination of these parameters in aquifers has been a routine task for hydrologists for several decades. In tight formations or scarcely fractured media which have little or no significance from a water supply point of view, conventional methods are not reliable. During recent years when the need for further information has increased, some new techniques have been developed. However, these methods are far from complete and in some cases there is much room for improvement.

For those methods which are designed to indirectly measure a certain parameter through application of a particular theory, the theory together with the assumptions and constraints under which it has been developed are briefly discussed in this report. These assumptions and constraints should always be compared with actual field conditions when attempting to analyze test results. The application of each procedure is described step by step, and the basic equations and type curves used to analyze the field data are presented. Uncertainties and limitations associated with each method have also been brought out.

Two types of field tests have been described for measurement of porosity: logging techniques and tracer applications. Logging techniques are well developed but can only respond to the porosity of a small part of the medium around the borehole. Therefore, a large number of wells are required to give a clear picture of the porosity variation within the medium of interest. Tracer methods have been used for determining the porosity of permeable formations. However, for materials with permeability of the order of 10^{-7} m/s or less, the travel time of a tracer from one well to the other spaced a meaningful distance apart may be too long to be practical. This suggests the need for a large number of small scale tests.

Two kinds of field tests have been discussed for determination of the vertical component of hydraulic conductivity: single well tests and large scale pumping tests. As with logging techniques, the hydraulic conductivity measured by a single-well test is only representative of a small zone around the testing interval. Since a large number of these tests are required to give an overall distribution of the vertical hydraulic conductivity, application of this method may be limited to relatively shallow formations to be cost effective.

Large scale pumping tests have the capability of determining the average hydraulic conductivity of the formations being drained. However, none of the available methods of interpretation can independently give the vertical hydraulic conductivity of the less permeable layers (called aquitards or aquicludes). Each method requires an independent measurement or estimate of the value of storativity of these layers before the vertical hydraulic conductivity can be calculated. Further improvement in the design and interpretation of this type of test is needed.

The storativity of permeable layers can be easily calculated by interpretation of the pump test data. Storativity of a low permeability layer such as aquitard and aquiclude, however, cannot normally be directly calculated from pump tests. In such cases storativity must be estimated by rule of thumb. However, since the specific storage reported for different formations varies by several orders of magnitude, one could easily choose a value which is an order of magnitude from the correct value. More study is therefore needed for determining the storativity of low permeable materials.

Measurement of the hydraulic head within permeable materials is a simple, routine task. For low permeability materials a careful test design and accurate instrumentation are essential.

1.0 INTRODUCTION

1.1 Purpose

The purpose of this report is to review the conventional field techniques used for determination of hydrological properties and parameters that control the vertical component of groundwater movement within less permeable geological formations.

1.2 Background

The United States Department of Energy (DOE) has the responsibility for identifying sites and constructing repositories for the geological disposal of high level nuclear waste. These facilities will be licensed by the United States Nuclear Regulatory Commission (NRC) under the rules and procedures defined in the Code of Federal Regulations, 10 CFR Part 60.

In 1954 the United States Atomic Energy Commission (AEC) approached the National Research Council and the National Academy of Science (NAS) for a possible solution to the radioactive waste disposal problem (Hess, 1957). In 1955 the NAS and National Research Council appointed a steering committee for Radioactive Waste Disposal which sponsored a conference to consider methods and areas suitable for land disposal. The conference concluded that the most promising place for disposal was in rock-salt formations. This conclusion has led to an extensive investigation of different salt deposits. Bedded salt and dome salt have been extensively studied by different agencies sponsored by DOE. Currently, however, three additional types of rock, namely basalt, granite and tuff, as well as bedded salt, are being investigated as potential repository media.

When the DOE elects to submit an application for construction of a repository at a particular site, a Site Characterization Report will be submitted to the NRC prior to the application for a license. The Site Characterization Report will include a description of the candidate site based on the available data and a description of the site characterization program proposed to address the ability of the site to host a safe repository for radioactive waste. The NRC will review the Site Characterization Report and may make specific objections or recommendations regarding the proposed program. This report considers one of the important characterization issues that will have to be addressed at any site in bedded media. This will include sites in basalt and tuff, as well as salt. Several conventional field methods for resolving this issue are presented. The description of each method generally includes the purpose and procedure of the test, the theory and assumptions upon which the method is based, method for analysis of field data, and limitations and uncertainties associated with the technique. An overall evaluatin of the methods will then be presented.

1.3 Importance of Problem

Groundwater is the most likely means of transport of radionuclides from repository level to the accessible environment. Should migrating radionuclides enter the aquifers adjacent to the repository site, the specific flowpaths to be followed by the waste will depend on the natural or man made hydraulic head distribution in the region of study. Within the undisturbed strata the natural pathways of groundwater would generally be parallel to the layers, away from the source of pollution and towards the discharge area of that particular aquifer. Although resistance to flow along this path is commonly minimum, because of the very low hydraulic gradients and long distances, it would usually take a relatively long time for the waste to reach the point of discharge. Another path through which groundwater may transport hazardous waste to the upper fresh-water aquifers or accessible environment is upward in a direction perpendicular to the strata. This path is usually several orders of magnitude shorter than the lateral path. However, because of a generally very low permeability of the confining beds, the actual transport time is often not less than the first path. The important point is that one cannot always be sure that the confining beds are sufficiently impermeable. The occurrence of faults, joints and other similar features in stratified material could considerably increase the velocity of the vertical groundwater movement and thus provide the fastest way for transfer of waste materials to the accessible environment.

Evaluation of the hydrological properties of relatively low permeability geological formations confining aquifers which are potentially subject to invasion by hazardous materials is clearly important to the assessment of suitability for nuclear waste disposal and estimating groundwater travel time to the accessible environment. The need for resolution of this issue is reflected in several sections of 10 CFR Part 60:

a) Section 60.2(c) requires that the Safety Analysis Report include:

   (1) a description and analysis of the hydrological aspects of the site that bear significantly on the suitability of the geologic repository for disposal of radioactive waste.

   (2) an analysis and evaluation of the effectiveness of natural barriers, including barriers that may not be themselves a part of the geologic repository operations area, against the release of radioactive material to the environment.

b) Section 60.31(a) indicates that the commission shall consider whether the DOE has adequately described the hydrologic characteristics of the proposed site prior to construction authorization.

c) Section 60.111(b) requires that the geological setting shall be selected and the subsurface facility designed so as to assure that releases of radioactive materials from the geologic repository following permanent closure conform to such generally applicable environmental radiation protection standards as may have been established by the Environmental Protection Agency.

d) Section 60.112(c) requires that the geologic repository shall be located so that pre-waste emplacement groundwater travel times through the far field to the accessible environment are at least 1,000 years.

e) Section 60.123(b) indicates that potential for creating new pathways for radionuclide migration due to presence of faults or fracture zone in the disturbed zone irrespective of the age of last movement may compromise site suitability and will required careful analysis.

1.4 Properties and Parameters Concerned

Should the repository leak and the radioactive waste find its way into the aquifers surrounding the site, these aquifers could behave as natural barriers provided they can delay the waste long enough such that by the time the waste reaches the accessible environment its level of radiation is lower than the required limit. This condition may be fulfilled if (1) the aquifer is hydraulically isolated from the upper permeable formations, and (2) the transport time to the accessible environment is at least 1,000 years (10 CFR, Section 60.112(c)).

The first condition depends on the thickness and tightness of the confining layer. Undisturbed layers of shales and stiff clay with significant thickness may be able to separate effectively the host aquifer from the fresh water aquifers located above them. As noted earlier, disturbed zones containing faults, fractures, crushed zones, and other pertinent features, however, may easily provide a short cut path for transport of nuclear waste to the upper fresh water aquifer.

Investigation of the capability of the sedimentary strata around a repository site to serve as a natural barrier requires a thorough study of the hydrological properties and parameters which may control the vertical groundwater movement in the disturbed zone. The best way to recognize these properties is through the generalized Darcy's law.

$$\vec{\upsilon} = \frac{\vec{V}}{\alpha} = -\frac{[K]}{\alpha}\nabla h = -\frac{\rho g[k]}{\mu\alpha}\nabla h \qquad (1\text{-}1)$$

where

$\vec{\upsilon}$ = vector of seepage velocity having three compoents in x, y and z direction

$\vec{V}$ = vector of apparent velocity or Darcy's velocity

α = effective porosity

$[K]$ = 3x3 matrix of hydraulic conductivity

h = hydraulic head

∇ = gradient operator

ρ = density of fluid at the point of interest

μ = dynamic viscosity

$[k]$ = 3x3 matrix of permeability

g = acceleration of gravity.

Thus the hydrologic properties and parameters of our interest are effective porosity, permeability tensor, hydraulic head and fluid properties. In the case of a homogeneous fluid, where ρ and μ remain constant, the hydraulic conductivity tensor $[K]$ can be measured directly. When dealing with transient fluid flow, the storativity (S) becomes a major property of the system.

In the following sections, each of these hydrologic properties or parameters is defined. Then some of the conventional methods for its measurement in the field will be presented. The limitations and uncertainty associated with each method will also be discussed.

2.0 EFFECTIVE POROSITY

The porosity of a material is defined as the ratio of void space to total bulk volume. Sometimes some of the voids are isolated and do not play a role in transmitting fluid. This is the reason for introducing effective porosity, which is defined as the ratio of the volume of connected pores to the bulk volume of the material. Porosity is a scalar property of the rock, which means it is independent of direction.

2.1 Methods of Measurement

There are several methods which are commonly used in the laboratory to measure the porosity of a rock sample. These techniques, including the direct method, mercury injection, gas expansion, and imbibition have been fully discussed in an American Petroleum Institute report (1960). Because laboratory techniques are out of the scope of this report we shall not discuss these methods further.

In the field, porosity may be obtained by several methods including well logging and tracer tests. Here we shall discuss the techniques of Sonic Logs, Formation Density Logs, and Neutron Logs, as well as a two-well tracer method.

2.1.1 Sonic Log Method

A more detailed description of this method and additional references are given in Schlumberger (1972). Generally, for a given rock, when porosity increases the sonic velocity decreases. The Sonic Log is a recording of interval transit time (Δt) versus depth. The interval transit time is the time required for a compressional sound wave to traverse one foot of formation. The interval transit time for a given formation is a function of its lithology and porosity. The sonic Log is therefore a useful means of obtaining the porosity, provided the lithology is known.

Procedure

- Consider an uncased well filled with drilling mud or other fluid.
- A Sonic tool consisting of two transmitters and two pairs of receivers is lowered into the well, (see Fig. 2-1).
- A pulse is generated by each of the two transmitters and the difference between the arrival times of the first wave at the corresponding pair of receivers is measured.
- The Δt values from the two sets of receivers are averaged and recorded as a function of depth.

Theory

The wave generated by the transmitter will travel through different available media. However, since the speed of the wave in the formation is generally larger than that in the drilling fluid or the sonde itself, the wave which will first arrive at the receivers is the one which has traveled through the formation very close to the wall of the hole. As we measure the difference in travel time to the two receivers, the portion of time corresponding to travel through the drilling fluid is cancelled out. As a result, knowing the constant of the instrument, the measured Δt can be adjusted to show the reciprocal of the velocity in the formation. Δt is generally recorded in microsecond/foot (μsec/ft) and it varies between about 44 μsec/ft (for zero porosity dense dolomite) to about 190μsec/ft for pure water.

Wyllie et al. (1956, 1958) have proposed the following empirical formula for determination of the porosity ϕ of a consolidated

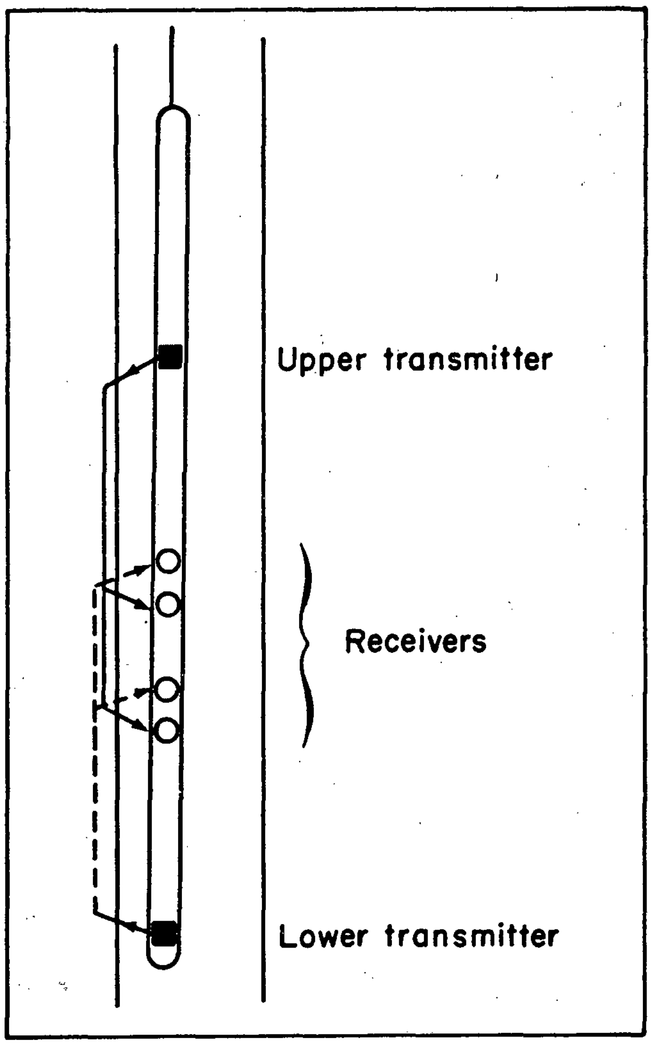

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Fig. 2-1 Sketch of a sonic tool, showing ray paths for transmitter-receiver sets (modified from Kokesh et al., 1965).

formation with uniformly distributed pores:

$$\phi = \frac{\Delta t_{log} - \Delta t_{ma}}{\Delta t_f - \Delta t_{ma}} \tag{2-1}$$

where

Δt_{log} = reading on the Sonic Log in μsec/ft

Δt_{ma} = transit time for the matrix material (values for different rocks are given in Table 2-1)

Δt_f = the inverse of the velocity of a Sonic Wave in the pore fluid (about 189 μsec/ft).

<u>How to Calculate Porosity</u>

- At each depth, identify the type of rock from the core and determine the value of Δt_{ma} from Table 2-1.
- Measure the magnitude of Δt_{log} from the Sonic Log for that particular depth.
- Calculate $\Delta t_f = \frac{1}{V_f}$, where V_f is the sonic wave velocity of the fluid filling the pores.
- Calculate porosity ϕ from equation (2-1).

<u>Uncertainties</u>

- The depth of penetration of the recorded wave is only a few inches from the wall of the hole. Thus the value of porosity obtained by this method is limited to a very small zone around the well.
- According to Wyllie et al. (1956, 1958), the velocity of sound in vuggy materials depends mostly on the primary porosity. Therefore,

Table 2-1. Values of transit time for common rocks and casing (modified from Schlumberger, 1972).

ROCK	Δt_{ma} (μsec/ft)
Sandstones	51.0 - 55.5
Limestones	47.5
Dolomites	43.5
Anhydrite	50.0
Salt	67.0
Casing (iron)	57.0

the sonic method tends to ignore secondary porosity such as fractures. The sonic logs in comparison with the density logs and neutron logs could, however, give a measure of secondary porosity.

- The method is not suitable for finding effective porosity if a significant volume of isolated pore space is available.

2.1.2 Density Log Method

A radioactive source, in contact with the wall of a hole, emits medium-energy gamma rays into a formation. After colliding with electrons in the formation, the scattered gamma rays are counted by a detector placed at a fixed distance from the source. The response of such a test is determined essentially by the electron density of the formation. Electron density is a function of the true bulk density ρ_b. Therefore the porosity of the formation may be calculated if the density of the rock matrix and the pore fluid density are known.

Procedure

- Consider an uncased hole filled with drilling mud or other fluid.
- A Formation Density Logging Device consisting of a source and one or two detectors attached to a skid is lowered into the well. The device is designed such that the source and detectors come in contact with the wall of the well.
- Record the vairation of bulk density against depth. Note that the tools are usually calibrated to directly record the apparent bulk density.

Theory

The electron density index ρ_e, which is proportional to electron density, is defined as:

$$\rho_e = \rho_b \left(2 \frac{\Sigma Z}{Mol.Wt.}\right) \qquad (2-2)$$

where

ΣZ = the sum of the atomic numbers of atoms making up the molecules (equal to the number of electrons per molecule).

Mol.Wt. = the molecular weight.

ρ_b = bulk density.

The density logging tool is calibrated such that the measured apparent bulk density ρ_a is related to ρ_e with the following formula:

$$\rho_a = 1.0704\ \rho_e - 0.1883 \qquad (2-3)$$

For liquid-filled sandstones, limestones, and dolomites, the apparent density ρ_a read by the tool is practically identical to the actual bulk density ρ_b. For a few other rocks such as rock salt, gypsum, and anhydrite a small correction is required. Figure 2-2 provides a means for such a correction.

How to Calculate Porosity

- At each given depth read apparent bulk density ρ_a from the log.
- Look at the core log at that depth. If the formation material is sandstone, limestone or dolomite and if it is in the zone

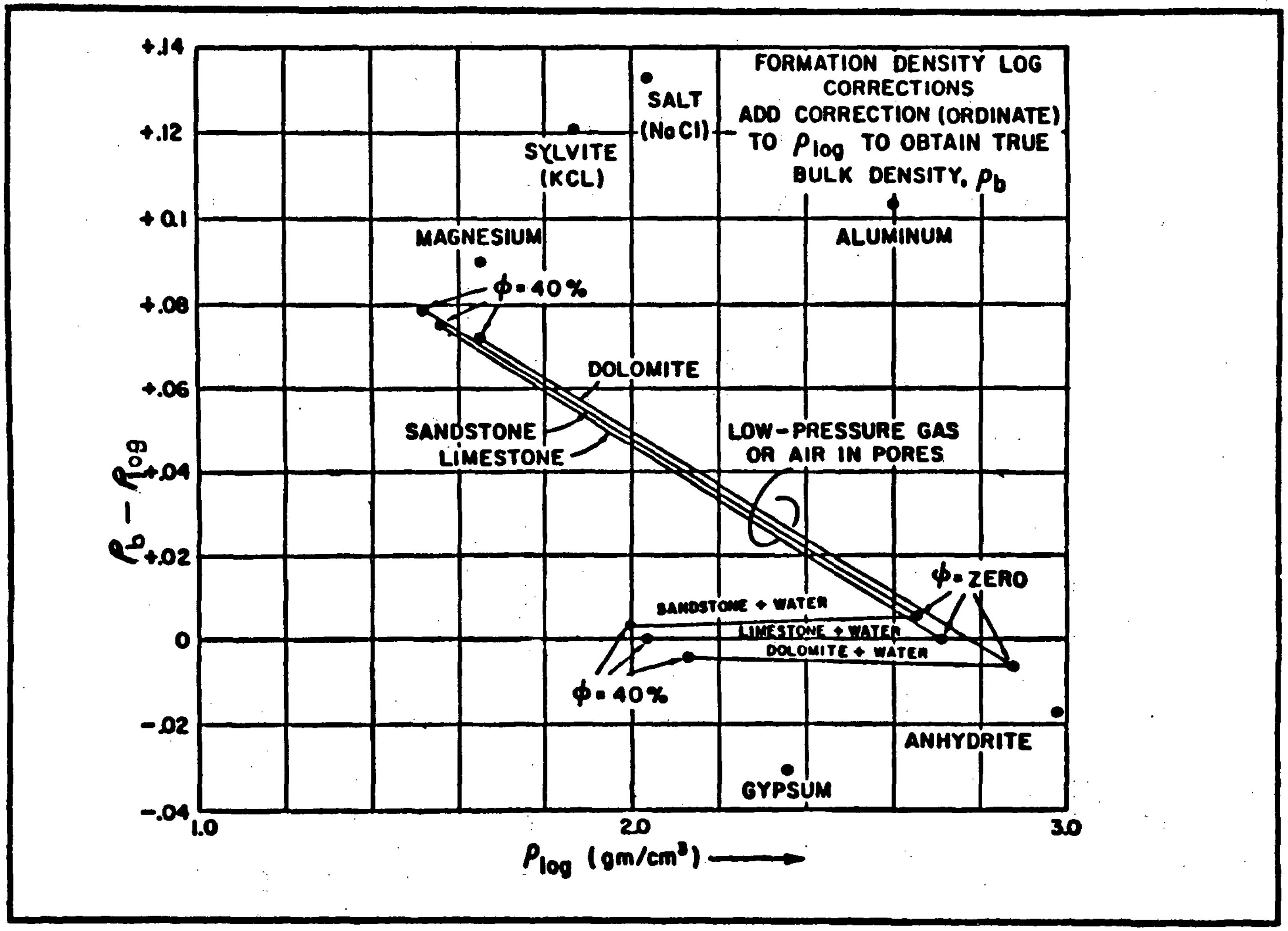

Fig. 2-2 Correction needed to get true bulk density from log density, (modified from Tittman and Wahl, 1965).

of saturation, consider ρ_a to be equal to the bulk density ρ_b. If the rock is salt, anhydrite or gypsum, find ρ_b from Fig. 2-2.

- Calculate porosity of the formation ϕ from

$$\phi = \frac{\rho_{ma} - \rho_b}{\rho_{ma} - \rho_f} \tag{2-4}$$

where

ρ_{ma} = matrix density, 2.65 for sandstone and quartzite; 2.68 for limy sands and sandy limes; 2.71 for limestone and 2.87 for dolomite.

ρ_f = the density of fluid filling the pores very close to the well.

Uncertainties

- This method determines total porosity. It does not differentiate between connected and isolated pore spaces within the formation.
- The presence of shale or clay in the formation introduces some errors into the results.

2.1.3 Neutron Log Method

This method can determine the amount of liquid-filled porosity of a given material in situ. Principally, this technique is based on a measurement of the amount of available hydrogen in the formation under consideration. If the pore space of the rock is filled with fluid and no other source of hydrogen, such as the water in gypsum ($Caso^4 + 2H_{20}$), is present, then the response of this test is a measure of porosity.

Procedure

There are at least three different kinds of loggers which are currently used. GNT (Gamma Ray Neutron Tool), SNP (Sidewall Neuron Porosity), and CNL (Compensated Neutron Log) are three loggers which employ plutonium-beryllium or americium-beryllium as sources of neutrons with initial energies of several million electron volts (Schlumberger, 1972). Here we shall only address the SNP logger. Information about other tools and additional references on the cited tools may be obtained from Schlumberger (1972).

In the SNP a neutron source and a detector are mounted on a skid which is lowered in an uncased well, preferably without fluid and drilling mud. This tool is designed such that the logger comes in contact with the wall of the hole. The neutrons emitted by the source, after penetrating the formation and colliding with the nuclei of the formation material are received by the detector. The response, after correction on a panel, is recorded against depth. The surface panel automatically makes necessary corrections for salinity, temperature, and hole size variation and records the porosity directly. If the hole is filled with drilling mud, values of porosity should be corrected for the mud-cake thickness through available charts.

Theory

Neutrons are electrically neutral particles, each with the mass of a hydrogen atom (Tittman, 1956). The source on the logger continuously emits fast neutrons. These neutrons collide with nuclei of the formation materials and lose some of their energy. The amount of energy which a neutron loses in each collision depends on the relative mass of the nucleus with which the neutron collides. Collision with a hydrogen nucleus causes the maximum energy loss. Thus, the slowing-down of neutrons depends largely on the amount of hydrogen in the formation which in turn is related to the amount of water in the formation. The SNP method has the advantages that borehole effects are minimized and that most of the corrections required are performed automatically in the panel.

Uncertainties

- This method can measure effective porosity only if the isolated pores are free of liquid, otherwise the method does not differentiate between connected and isolated pores.
- The tool responds to all the hydrogen atoms in the formation including those chemically combined in formation materials, which do not correspond to porosity.
- In shaly formations the porosity derived from the neutron response will be greater than the effective porosity.
- The zone of influence of this method depends on the porosity of the formation, but generally it is limited to a short distance from the wall of the hole.

2.1.4 Tracer Techniques

There are several tracer methods for determination of aquifer parameters. The literature is replete with descriptions of experiments of this type. Many types of radioactive and nonradioactive tracers have been used. A list of some of the tracers which have been used in groundwater studies has been given by Thompson (1981). The most promising method used for determination of effective porosity seems to be the two-well injection-withdrawal test.

2.1.5 <u>Two Well Tracer Method</u>

In this test, water is pumped from a well, and, after being labled with an appropriate tracer, is injected in another well in the vicinity of the pumping well. In so doing, the maximum possible hydraulic gradient is developed between two wells and thus the time required to run the test is minimized. This test is commonly applied to measure effective porosity. Grove and Beetem (1971) have described a tracer technique for obtaining porosity and dispersivity. Their approach is a generalized form of the method proposed by Webster et al. (1970). Grove and Beetem (1971) and Claassen and Cordes (1975) employed this method using tritium as a tracer to determine the porosity and dispersivity of highly conductive fractured carbonate aquifers in New Mexico and Nevada, respectively. The following is a brief description of the method proposed by Grove and Beetem (1971).

<u>Procedure</u>

- Consider two wells which completely penetrate and are open to the total thickness of the formation to be investigated. The distance between these two wells depends on the hydraulic conductivity of the formation. Distances from 50 m to 120 m have been selected for very conductive aquifers. Smaller distances should be used in aquifers having smaller hydraulic conductivity.
- Water should be pumped from one of the wells and transferred to

be injected into the other well until a steady state condition is reached. The rate of pumping Q should be measured at the steady condition. Water samples are taken to measure the background concentration.

- A certain volume of tracer is mixed with the water to be injected over a finite period of time.
- Samples of water should be collected from the discharging well and tested for the concentration of the tracer C. This process should continue until the tracer concentration becomes almost constant.

Theory

Let us consider a pair of recharge and discharge wells such that the rate of discharge from one is equal to the rate of recharge from another. If we ignore the regional flow field, the pattern of streamlines developed by such a system, after a steady state condition has been reached, may be shown on Fig. 2-3. The length of each of the streamlines connecting the two wells may be given by

$$L = \frac{2\,a\,\Theta}{\sin\Theta} \tag{2-5}$$

where

a = half the distance between wells

$\Theta = \pi(1 + \frac{2\psi}{q})$ which varies between 0 to π

q = pumping rate per unit aquifer thickness

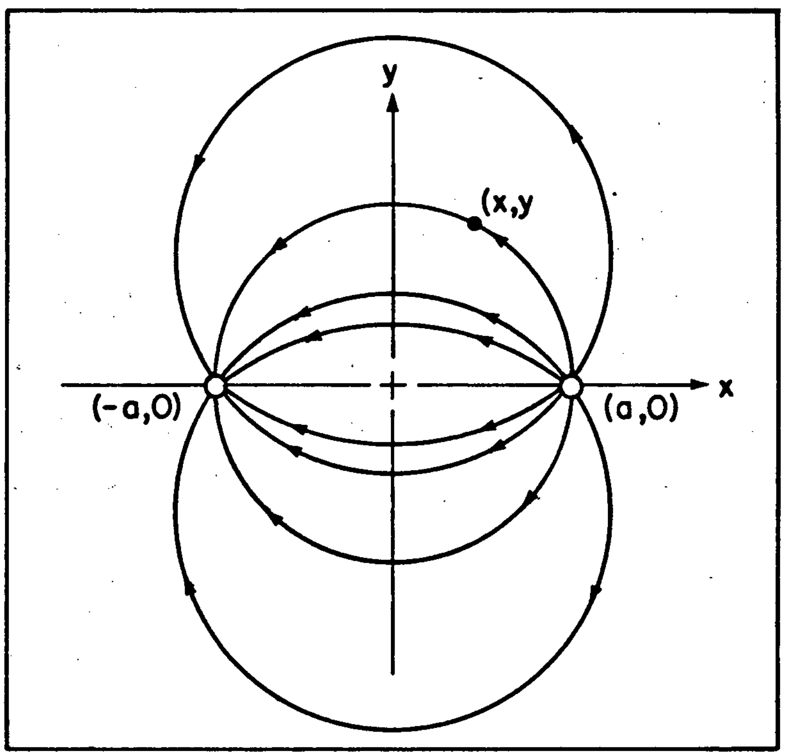

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Fig. 2-3 Pattern of streamlines formed by a recharging-discharging well pair.

ψ = stream function which is equal to $\frac{q}{2\pi} \tan^{-1} \frac{2ay}{a^2-x^2-y^2}$, where

(x,y) are the coordinates of the point through which the streamline passes.

The time T for a water particle to travel along a particular streamline between two wells may be given by

$$T = \frac{4\pi\alpha a^2}{q \sin^2\Theta} [\Theta \cot \Theta -1] \quad (2\text{-}6)$$

where

α = effective porosity.

If the tracer concentration at the recharge point of any of the flow channels shown on Fig. 2-3 is C_o, the value of the dimensionless concentration C/C_o as a function of time at the other end of the channel may be given by

$$\frac{C}{C_o} = 1-\exp[p(2-t_D)] \sum_{n=1}^{\infty} \frac{\lambda_n \sin(2\lambda_n)}{(\lambda_n^2 + p^2 + p)} \cdot \exp(-\frac{\lambda_n^2 t_D}{p}) \quad (2\text{-}7)$$

where

C = concentration of tracer at the discharge point of the flow channel

$t_D = \frac{t}{T}$ = dimensionless time; t is time since the injection started and T can be obtained from Equation (2-6).

$P = \frac{L}{4D_m}$ = Péclet number; L can be obtained from Equation (2-5)

D_m = dispersion constant or dispersivity

λ_n = the nth positive root of $\tan 2\lambda = 2\lambda P/(\lambda^2 - P^2)$

Grove and Beetem (1971) suggest that Equation (2-7) be used whenever P/t_D is less than one, and for P/t_D equal or greater than one the following equation is recommended:

$$\frac{C}{C_o} = \frac{1}{2} \operatorname{erfc}[(P/t_D)^{1/2} (1-t_D)] + (4Pt_D/\pi)^{1/2}[3 + 2P(1+t_D)]$$

$$\cdot \exp[-P(1-t_D)^2/t_D] - [1/2+2P(3+4t_D) + 4P^2(1+t_D)^2]$$

$$\cdot \exp(4P) \operatorname{erfc}[(P/t_D)^{1/2}(1+t_D)] \qquad (2\text{-}8)$$

where

erfc = complementary error function

Analysis of Field Data

- A set of type curves for different values of α and D_m should be prepared as per following instructions:
- Divide the well flow pattern to N different flow channels each represented by an arch connecting two wells.
- Given q and a, calculate L and T for each arc from Equations (2-5) and (2-6).
- Calculate the Péclet number for each arc.

- Using equation (2-7) or (2-8), calculate the values of C/C_o for different values of time since the injection started.
- For each given time t, add the values of C/C_o of all flow channels.
- A plot of C/C_o obtained from summation of all flow channels versus time would give a breakthrough curve for the assumed values of α and D_m and the given q and a of the test.
- Compare the plot of observed variation of C/C_o versus time with the breakthrough curves prepared for different values of α and D_m until a good match is obtained. The porosity and dispersivity of the formation being tested may now be given by those for which the type curve was prepared.

Uncertainties

- It is preassumed that the flow field between the two wells reaches steady state condition before the tracer is injected. This is a reasonable assumption when we are dealing with a highly conductive formation. However, when hydraulic conductivity is of the order of 10^{-8} cm/sec or less, achievement of a steady state condition in a reasonable length of time is impossible. In addition, the magnitude of the pumping rate, if pumping is even possible, is so small that the time required for the tracer to travel from one well to another well within a reasonable distance, is too long to be practical.
- The effect of the regional flow system is considered to be negligible. Depending on the magnitude of regional velocity, this assumption may or may not introduce an appreciable error.
- The whole development is based on two dimensional, homogenous aquifers.

3.0 HYDRAULIC CONDUCTIVITY

Hydraulic conductivity is the constant of proportionality in Darcy's law,

$$V = - K \frac{dh}{dx} \tag{3-1}$$

where

K = hydraulic conductivity

V = Darcy's velocity

$\frac{dh}{dx}$ = hydraulic gradient.

Hydraulic conductivity, which is sometimes called the coefficient of permeability, has been shown to be related to the fluid properties and the permeability of the porous medium by the following formula (Hubbert, 1940):

$$K = \frac{k\rho g}{\mu}$$

where

k = specific or intrinsic permeability of the porous medium

ρ = density of fluid

μ = dynamic viscosity of fluid

g = gravitational acceleration

Intrinsic permeability k which is a function of mean grain diameter, grain size distribution, sphericity, and roundness of the grains, is a measure of the ability of the medium to transfer fluids.

The hydraulic conductivity of geological materials varies from approximately 1 to 10^{-13} m/s. This is a very wide range of variation.

There are very few physical parameters that take on values over 13 orders of magnitude (Freeze and Cherry, 1979). Values of hydraulic conductivity of a geological formation can vary in space. This property of the medium is called heterogeneity. They can also show variations with the direction of measurement at any given point. This property is called anisotropy and is quite common in sedimentary rocks. In sedimentary rocks hydraulic conductivity along the layers is sometimes several orders of magnitude larger than across the layers. This property becomes especially important in layered formations where some thin layers of very low permeability appear within highly permeable sediments. Anisotropy is also quite common in fractured rocks where aperture and spacing of joints varies with direction.

As a result, in an anisotropic medium, hydraulic conductivity in its general form may be represented by a 3x3 symmetric matrix. The components of fluid velocity in an anisotropic medium may then be written by the following equations:

$$v_x = -K_{xx}\frac{\partial h}{\partial x} - K_{xy}\frac{\partial h}{\partial y} - K_{xz}\frac{\partial h}{\partial z} \qquad (3\text{-}2)$$

$$v_y = -K_{yx}\frac{\partial h}{\partial x} - K_{yy}\frac{\partial h}{\partial y} - K_{yz}\frac{\partial h}{\partial z} \qquad (3\text{-}3)$$

$$v_z = -K_{zx}\frac{\partial h}{\partial x} - K_{zy}\frac{\partial h}{\partial y} - K_{zz}\frac{\partial h}{\partial z} \qquad (3\text{-}4)$$

The values of K in the above equations are components of the hydraulic conductivity matrix. It has been shown that an appropriate selection of coordinate system enables one to diagonalize a symmetric matrix. The necessary and sufficient condition that allows such a transformation is that the principal directions of anisotropy coincide with the x,y, and z coordinate axes. If the system allows such a simplification, then the three components of flow velocity may be presented by the following equations

$$v_x = -K_x \frac{\partial h}{\partial x} \tag{3-5}$$

$$v_y = -K_y \frac{\partial h}{\partial y} \tag{3-6}$$

$$v_z = -K_z \frac{\partial h}{\partial z} \tag{3-7}$$

where K_x, K_y and K_z are principal values of hydraulic conductivity which are now in the direction of x,y, and z. Therefore, depending on the media, the vertical velocity of groundwater movement may be given by the one of the two equations, (3-4) or (3-7). Equation (3-7) indicates that the vertical component of groundwater motion is controlled by K_z alone. In cases where vertical velocity is given by equation (3-4), values of hydraulic conductivity in other directions are also required.

3.1 Methods of Measurement

In this section some of the conventional methods for determination of in situ hydraulic conductivity in geological materials will be discussed. Emphasis will be placed on the methods that lead to determination of vertical hydraulic conductivity. Some methods which have been recently developed for finding horizontal hydraulic conductivity in tight formations will also be examined.

In general these tests may be divided into two categories: those which are performed in a single well and those whose execution requires more than one well.

3.2 Single-Well Tests

3.2.1 Burns' Single-Well Test

Burns (1969) proposed a method of estimating vertical permeability of rocks. Following is a modification of that method.

Purpose

The purpose of this test is to find in situ vertical permeability of geological material in the vicinity of the test well. Horizontal permeability may also be estimated by this method.

Procedure

This test can be performed with several alternative arrangements of down-hole equipment. Two useful arrangements proposed by Burns are illustrated in Fig. 3-1. Here the procedure for the more simple test (Fig. 3-1A) is described. For further detail the reader is referred to Burns (1969).

- A well is drilled into the zone of interest. Assuming the well is cased, the annulus between the casing and the formation should be tightly cemented to prevent any sort of vertical flow. Arnold and Paap (1979) have presented a method for monitoring water flow behind a well casing. If the process of cementing fills up the voids in the vicinity of the well it may cause an artifical reduction of permeability. Then the casing and cement should be perforated at least at two different intervals separated from each other by a few feet.
- A packer is placed between these two perforated intervals to seal the hydraulic connection between them from inside the casing. Care should be taken that the change of pressure on one side does not

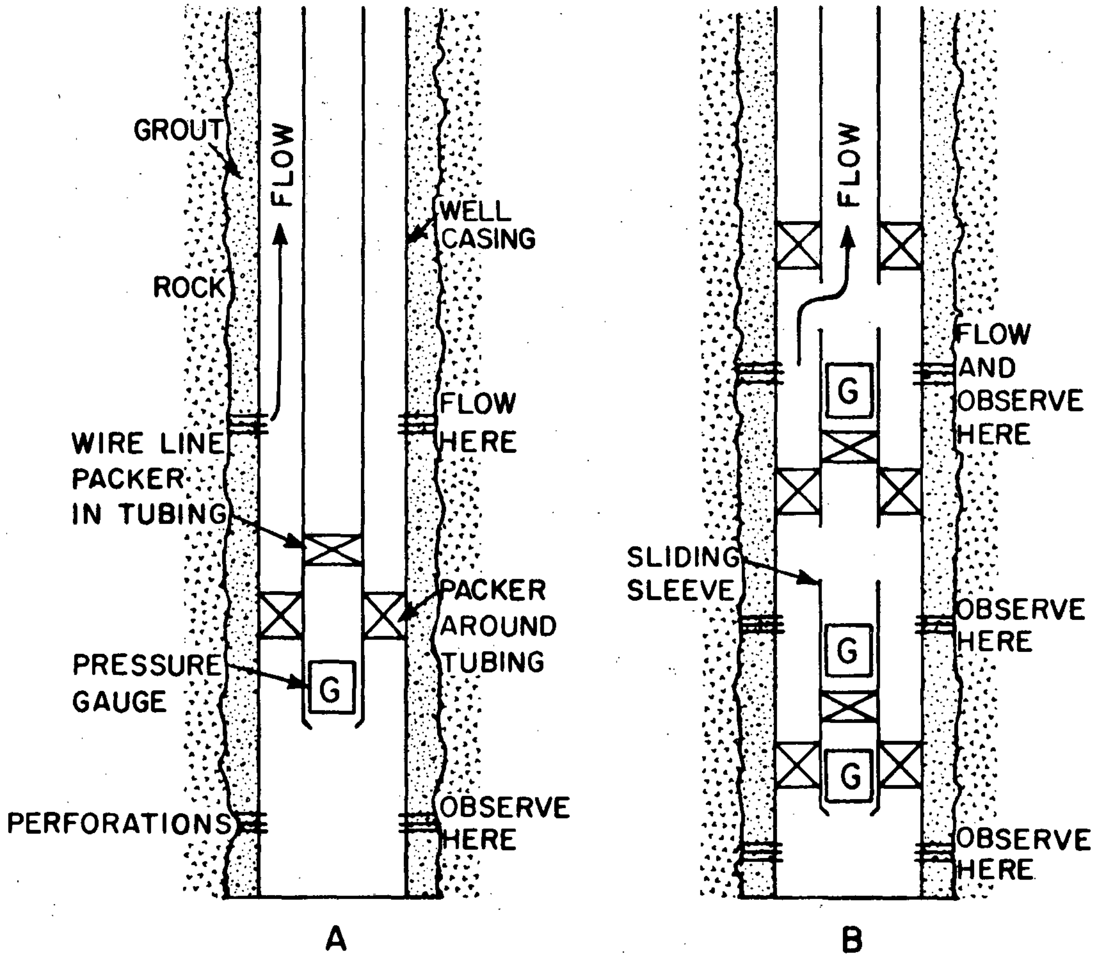

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Fig. 3-1 Downhole equipment arrangements for vertical well tests; A) single interval test, and B) multiple interval test with sliding sleeve (after Burns, 1969).

transmit through the packer. Installation of two packers with about half a foot distance may achieve this goal.

- One pressure transducer is installed on each side of the packer. The ambient pressure trend is monitored by both of the transducers for some period before injection. To facilitate test data interpretation, the values of ambient pressure should either remain constant or change linearly during the trend monitoring period.
- Then start injecting into or pumping from the upper perforated zone. The rate of flow should remain constant during this period. Flow rate should be monitored very accurately.
- Production or injection may be stopped after the pressure change recorded in the lower part reaches at least 10 times the sensitivity of the gauge.
- Recording of pressure at both intervals should continue throughout the producing or injection period and afterwards for a period equal to at least 20 percent of the elapsed flow period.
- Caution: extra packers may be used to minimize the effect of well bore storage.

Theory

The theory behind this method rests on the derivation of pressure changes due to a finite-length vertical line source in a homogeneous, anisotropic infinite aquifer bounded between two impermeable confining layers. The solution of this problem has been given by Hantush (1957), and Nisle (1958). See also Hantush (1964).

According to Hantush the change of hydraulic head or drawdown $s(r,z,t)$ in a piezometer having a depth of penetration z and being at a distance r from a steadily discharging well (with infinitesimal diameter) that is screened between the penetration depths d and l in the anisotropic aquifer of Fig 3-2 s given by

$$s = (Q/4\pi\, K_r b)\ \{W(u_r) + f\} \qquad (3\text{-}8)$$

where

$$f = [2b/\pi(l-d)] \sum_{n=1}^{\infty} 1/n\ [\sin \frac{n\pi l}{b} - \sin \frac{n\pi d}{b}] \cdot \cos \frac{n\pi z}{b} \cdot$$

$$\cdot\ W\{u_r, \sqrt{\frac{K_z}{K_r} (\frac{n\pi r}{b})^2}\ \} \qquad 3\text{-}9)$$

$$\text{and } u_r = \frac{r^2 S}{4K_r bt}$$

Two functions of $W(u_r)$ and $W\{u_r, \sqrt{\frac{K_z}{K_r} (\frac{n\pi r}{b})^2}\ \}$ have been tabulated and given by Hantush (1964). The solution presented by Burns is based on the more complicated form which Nisle (1958) presented.

Introducing the following dimensionless parameters

$$s_D = \frac{4\pi K_r bs}{Q} \quad \text{and} \quad t_D = \frac{bK_z t}{S\, r_w^2} \quad , \qquad (3\text{-}10)$$

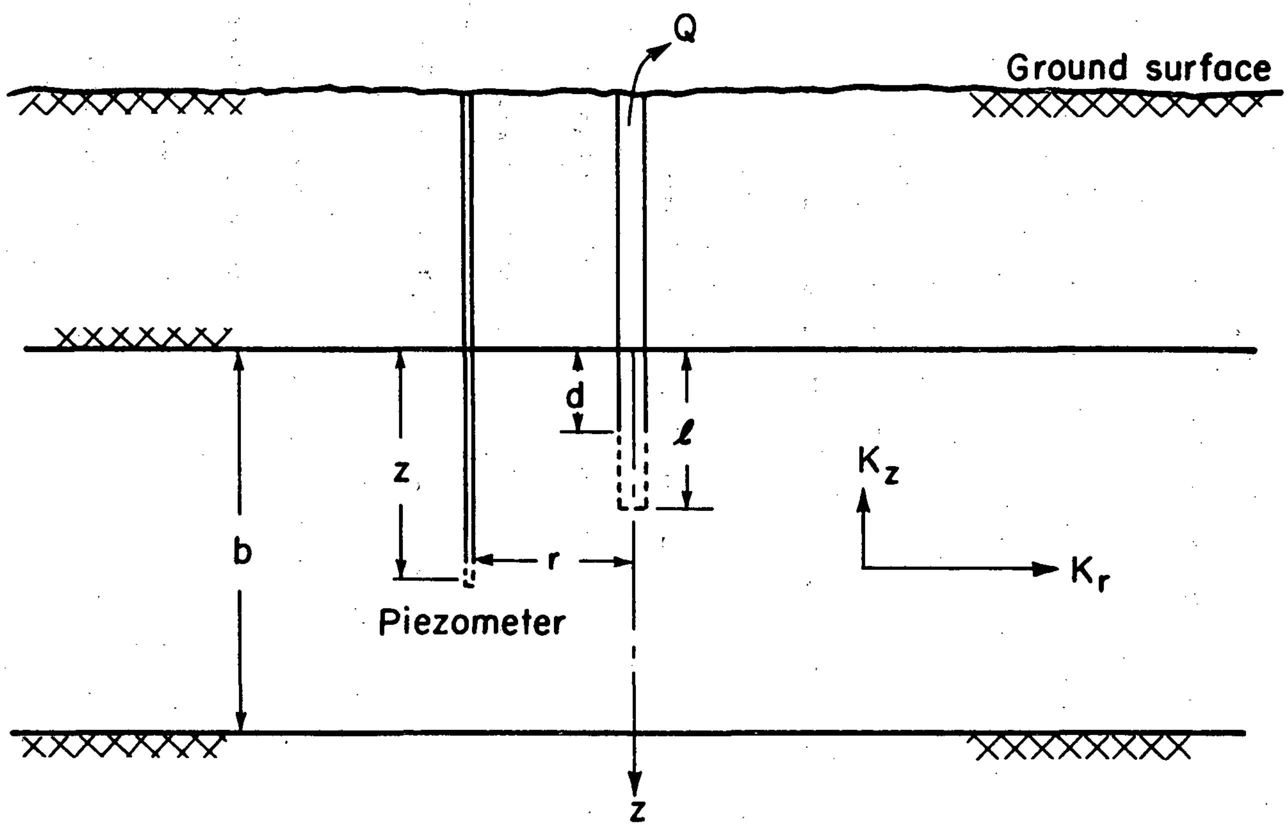

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Fig. 3-2 Aquifer with a partially penetrating well.

one can compute families of type curves of s_D versus t_D for different dimensionless parameters such as $\frac{z}{b}$, $\frac{l}{b}$, $\frac{d}{b}$, and $\frac{r}{b}$.

Analysis of Field Data

- Plot observed values of pressure versus time on rectangular coordinates.
- Draw the best straight line through the pressure response measurements during the trend-monitoring period, and extend it to the end of the flow period.
- The difference between the measured pressures and the original trend, $\Delta P = \gamma s$, is determined as a function of time since initiation of flow.
- Knowing dimensionless parameters such as $\frac{z}{b}$, $\frac{l}{b}$, $\frac{d}{b}$, and $\frac{r_w}{b}$, a family of type curves (log-log plot of s_D vs t_D) is prepared from equations (3-8) through (3-10) for different values of $\frac{K_z}{K_r}$.
- Variation of s versus time is plotted on another log-log paper with the same scale as the type curve plots.
- The observed plot is then compared with the type curves.
- Keeping the axes of two plots parallel, find the position that the observed plot matches best with one of the type curves.
- Read the value of $\frac{K_z}{K_r}$ and pick up a point on the top paper and identify the corresponding point right beneath that on the other plot. Read the coordinates of the two points i.e. s,t, s_D and t_D.

- Calculate the value of K_r from the definition of s_D, and K_z from the ratio of $\frac{K_z}{K_r}$.
- The value of S may now be computed from equation (3-10).

Multiple Tests in the Same Well

Several tests may be performed over different portions of a formation in the same well. In this case two or more packers may be used to isolate the testing portions of the well. Multiple tests can sometimes determine whether the response is characteristic of the formation or is a result of behind-casing leaks arising from poor cementing.

Uncertainties

- This test relies heavily on the assumption that the cementing behind the casing is not leaking. The existance of cement leaks behind the casing could result in an abnormally high vertical permeability measurements. Sufficiently large values of leakage behind the casing could cause almost equal response at the transducers in the flow and measurement zones.
- If the well has skin damage or if discontinuous shale barriers are locally present in the tested interval, then the calculated vertical permeability would be lower than the actual regional value.
- Within low permeable materials, if proper instrumentation is not utilized, the period of time required to reach a stabilized pressure before beginning the test might be long. In this case linear extrapolation of test pressure trends might lead to errors.

- The value of the hydraulic conductivity calculated by this method corresponds to a small volume of rock located in the vicinity of the testing zone.

3.2.2 Prats' Single-Well Test

Prats (1970) proposed a method for estimating in situ vertical permeability of geological materials which we shall describe here. This test requires injection or production at a constant rate from a short perforated interval and measurement of the pressure response at another perforated interval that is isolated from the first by a packer.

The purpose of this test is estimating the in situ vetical permeability of materials in the vicinity of a well. The test procedure is essentially the same as for the previously discussed Burns' test (1969), but probably less accurate.

Procedure

- Consider a single well with a casing cemented to the rock.
- Perforate two small intervals into the casing in the zone to be tested.
- Set a packer in the casing between the two perforations.
- Set one pressure transducer close to each perforation and monitor changes in pressure with time. As was discussed in the previous test, to avoid transfer of pressure through the packer, more than two packers may be used for separation of the flow and measurement zones.
- After pressure is almost stabilized, inject into the formation with a constant rate Q for some period of time until a reasonable amount of pressure response is picked up by the transducer at the

other perforation zone. In order to minimize the time required for pressure to stabilize, isolate the injection and observation zones from the rest of the well.

- Stop injection and continue to monitor the change of pressure at both transducers until the original ambient condition is almost reached.

Theory

The supporting theory behind this method is based on the pressure response of a confined homogeneous, anisotropic infinite aquifer due to a continuous point source. Thus, not only is the well considered to be of zero radius, but the perforation length of both injection zone and pressure measurement zone are also assumed to be vanishingly small. Based on these assumptions, the pressure change at a point z due to the release of a constant rate of flow Q at the point z', both located on the axis of the well, may be given by

$$\Delta p = \frac{Q\gamma}{4\pi K_r b} \sum_{n=-\infty}^{+\infty} \left[\frac{\text{erfc}\, \frac{|Z_D - Z'_D - 2n|}{2\sqrt{\tau}}}{|Z_D - Z'_D - 2n|} + \frac{\text{erfc}\, \frac{|Z_D + Z'_D - 2n|}{2\sqrt{\tau}}}{|Z_D + Z'_D - 2n|} \right] \qquad (3\text{-}11)$$

where

erfc = complementary error function

$$Z_D = \frac{z}{b}$$

$$Z'_D = \frac{z'}{b}$$

b = aquifer thickness

$\tau = \dfrac{K_z t}{S\ b}$

γ = unit weight of fluid

z = vertical distance of the point of measurement from the base of the aquifer

z' = vertical distance of the point of injection from the base of the aquifer

For large times, equation (3-11) may be simplified to

$$\Delta p = \frac{Q\gamma}{4\pi K_r b} \left\{ \ln \frac{K_z t}{S\ b} + G(Z_D, Z'_D) + \frac{b}{|z-z'|} \right\} \tag{3-12}$$

where $G(Z_D, Z'_D)$ may be obtained from Table 3-1.

Analysis of Field Data

- Calculate pressure changes at the measuring interval with the same procedure mentioned in the previous test.
- Plot pressure changes at the measuring interval versus time on a semilogarithmic paper.
- If the test was run long enough, the above curve should become a straight line at large values of time. Measure the slope m of that portion as ΔP/cycle.

TABLE 3-1 VALUES OF $G(Z_D, Z_D')$

Z_D' \ Z_D	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	4.188	2.511	1.685	1.210	0.919	0.743	0.648	0.617	0.644	0.729
0.2	2.542	1.701	1.210	0.904	0.712	0.600	0.550	0.555	0.613	0.729
0.3	1.742	1.237	0.916	0.709	0.582	0.517	0.505	0.544	0.638	0.796
0.4	1.289	0.953	0.732	0.591	0.512	0.485	0.509	0.586	0.725	0.944
0.5	1.017	0.781	0.625	0.532	0.492	0.502	0.565	0.689	0.891	1.207
0.6	0.857	0.685	0.577	0.523	0.520	0.569	0.680	0.868	1.169	1.653
0.7	0.777	0.651	0.581	0.563	0.599	0.696	0.872	1.159	1.629	2.446
0.8	0.759	0.669	0.634	0.655	0.738	0.900	1.174	1.631	2.435	4.086
0.9	0.797	0.740	0.741	0.807	0.954	1.214	1.657	2.488	4.087	9.072

- Calculate the radial hydraulic conductivity.

  from $K_r = \frac{2.3Q\gamma}{4\pi bm}$.

- Extrapolate the straight-line portion of the plot to a value of $t = 1$ hr, and read the pressure change at that time. This pressure change will be denoted as $\Delta P(1)$.
- Read the value of $G(Z_D, Z_D')$ from Table 3-1.
- Determine K_z from the following formula:

$$K_z = \frac{S\ b}{3600} \ \exp\left[\frac{4\pi K_r b}{Q\gamma}\Delta P(1) - G(Z_D - Z_D') - \frac{b}{|z-z'|}\right]. \qquad (3\text{-}13)$$

  All dimensions in equation (3-13) are in SI units. Note that K_2 can be calculated only when S is known.

Advantages and Limitations

The advantage of this method over the Burns' method is its simplicity in application. No type curve is necessary and analysis may be carried out with a small calculator.

Major limitations are as follow:

- The injection and measuring intervals must be short compared with the distance between them, probably 10 percent or less.
- If the distance between the injection (production) interval and the measuring interval is relatively long and the net vertical permeability is low, the pressure response may not be measured even in weeks. If this distance is relatively short, then the assumptions of point recharge (discharge) and point measurement become questionable.

- The thickness of the aquifer and the coefficient of storage are assumed to be known from other sources of information.
- The method will probably produce representative results in sands containing shaly streaks of limited extent, say not more than a few feet in radius. But its application is subject to question in the case of a reservoir with rather extended lenses of shale which could have significant local but not regional effects on vertical permeability.
- The method is rather sensitive to variations in the mass rate of fluid injection (production). The rate of flow is supposed to be constant.
- The method can only give the horizontal and vertical hydraulic conductivity of the materials immediately adjacent to the well being tested.

3.2.3 Hirasaki's Single-Well Pulse Test

Hirasaki (1974) has proposed a pulse test technique for estimating in situ vertical permeability. The test consists of pumping or injecting a small interval of a well for a short time, shutting in, and then measuring the time for the maximum pressure response to occur at another small interval of the well. This method has been used to estimate the vertical permeability of a low-permeability zone in the Fahud field, Oman (Rijnders, 1973).

Purpose

The purpose of this technique is also to provide of a simple means of estimating in situ vertical permeability of an aquifer in the vicinity of the testing location.

Procedure

- Perforate the casing of the well over a short interval at the top of the aquifer just beneath the confining layer.
- Perforate another short interval at a distance z below the first interval.
- Isolate these two intervals with a packer.
- Pump water from or inject into the upper perforated interval for a short time and measure pressure changes at the lower interval.
- Stop pumping or injection and continue measuring pressure change at the lower interval until the major part of the pulse test curve is obtained. Figure 3-3 shows a typical curve which may be obtained from such a pulse test. Note that the pumping or injection period should be short compared with the time required to reach the maximum pressure response (e.g., less than 10 percent) in the lower interval.

Theory

The theory of this technique rests on an approximation of the recovery equation for a continuous point source in a homogeneous anisotropic medium. Consider a continuous point source at $z = 0$ on the axis of the well, operating for a period $t = t_1$, as shown in Fig. 3-4. The pressure response of a semi-infinite medium (b is so large that the lower boundary is not touched) to the source at the point z and at any time $t > t_1$ may be given by

$$P_D = \frac{2}{Z_D} \left[\operatorname{erfc} \left(\frac{Z_D}{2\sqrt{\tau}} \right) - \operatorname{erfc} \left(\frac{Z_D}{2\sqrt{\tau - \tau_1}} \right) \right] \tag{3-14}$$

where

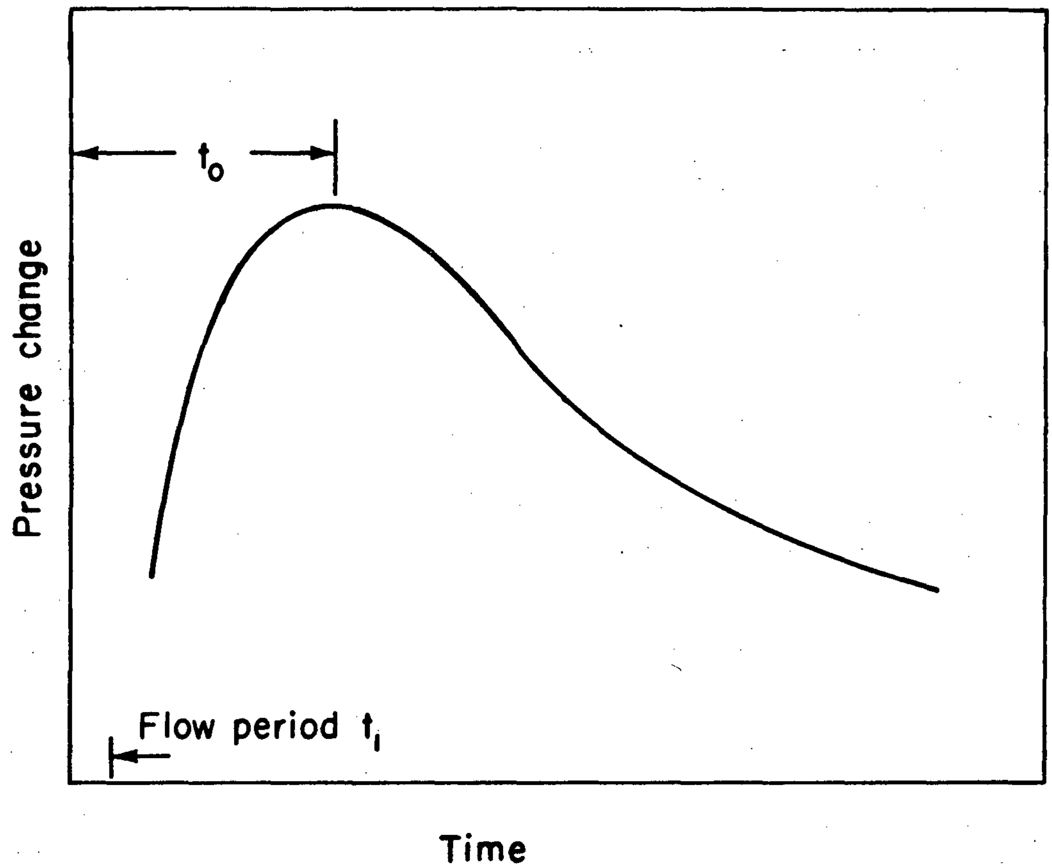

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Fig. 3-3 A typical pulse test response in the lower perforated interval, (modified from Hirasaki, 1974).

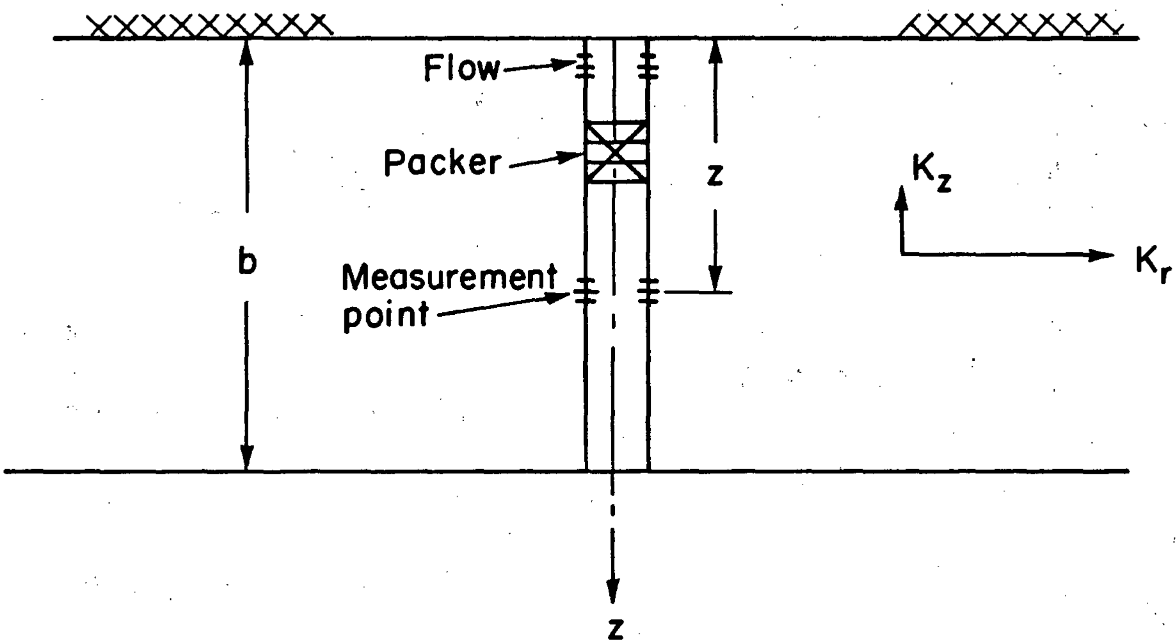

Fig. 3-4 Sketch of the Hirasaki's test configuration.

$$Z_D = \frac{z}{b}$$

$$\tau = \frac{K_z t}{S\ b}$$

$$\tau_1 = \frac{K_z t_1}{S\ b}$$

$$P_D = \frac{4\pi K_r b\ \Delta P}{Q\gamma}$$

If t_1 is much less than t, then equation (3-14) may be approximated by

$$P_D = \frac{\tau_1}{\sqrt{\pi}}\ \frac{e^{-\frac{Z_D^2}{4\tau}}}{\tau^{3/2}} \qquad (3\text{-}15)$$

Equation (3-15) represents the pluse-pressure curve. The arrival time of the peak of this curve may easily be obtained by setting its derivative equal to zero, which would give

$$\tau_o = \frac{Z_D^{\ 2}}{6} \qquad (3\text{-}16)$$

Substituting for τ_o gives

$$K_z = \frac{S\ bZ_D^2}{6t_o} = \frac{S_s\ z^2}{6t_o} \qquad (3\text{-}17)$$

As was mentioned above equations (3-14) through (3-17) apply to a a semi-infinite medium. Two other cases have also been considered by this author. In one case the aquifer is considered to be finite in thickness, which is treated by the introduction of a no flow condition at the lower boundary. In the other case the lower boundary is assumed to remain at constant head. A family of curves has been presented in Fig. 3-5, which gives the variation of τ_o versus Z_D for all three cases. It is interesting to note that the equation (3-17) holds for all three cases as long as $Z_D \leq 0.6$.

Analysis of Field Data

- Plot the variation of ΔP versus time as measured at the lower interval. The same precautions for measuring ΔP apply here as were discussed in previous methods.
- If this curve shows a peak like that on Fig. 3-3, then measure the time t_o corresponding to the maximum pressure response.
- Modify the time t_o by subtracting half of the flow period t_1

$$t_o' = t_o - \frac{1}{2} t_1 \qquad (3\text{-}18)$$

- If the distance, z between the upper and lower intervals is relatively short with respect to the thickness of the aquifer (z<0.6b) then calculate K_z using equation (3-17) by employing t_o' instead of t_o.
- If the distance z is larger than 0.6b, then calculate Z_D = z/b and determine the type of lower boundary which best approximates field condtions.
- Determine the value of τ_o from the appropriate curve of Fig. 3-5.
- Calculate vertical hydraulic conductivity K_z from following equation:

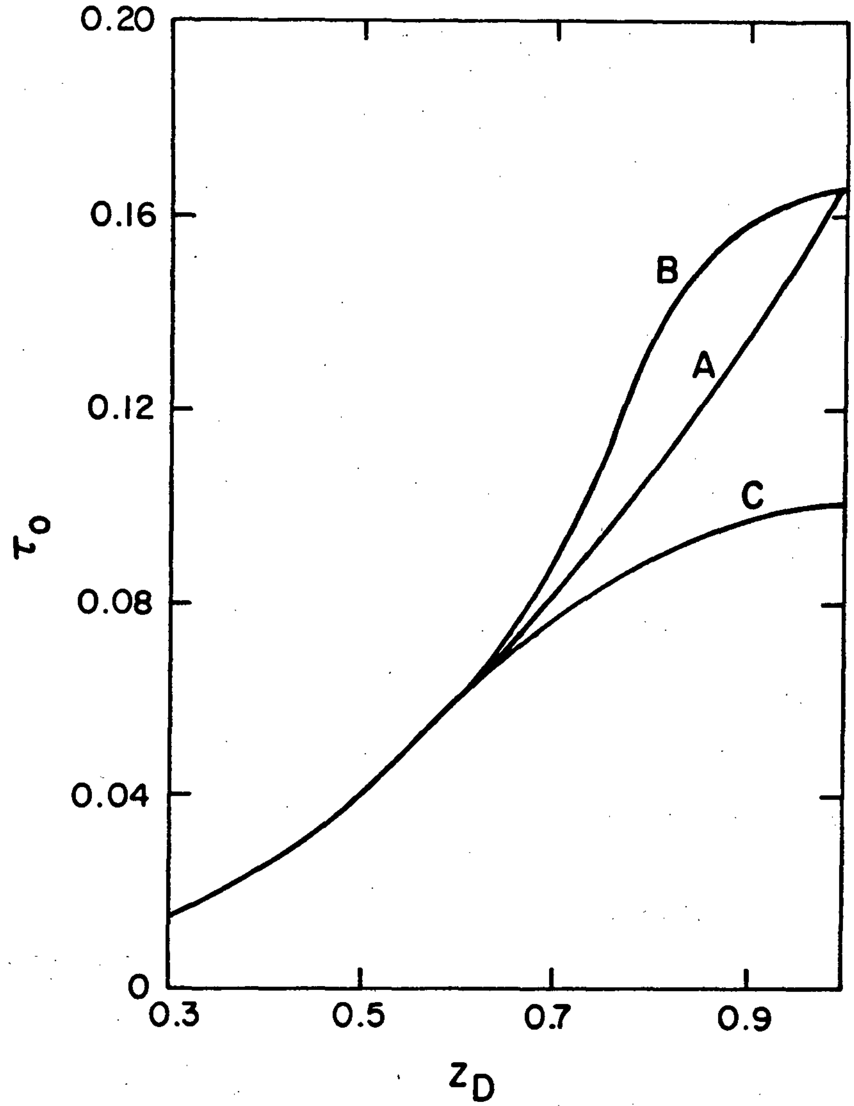

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Fig. 3-5 Dimensionless response time for pulse test; A for semi-infinite case, B for a finite thickness layer with an impermeable lower boundary, and C for a finite thickness layer and constant head at the lower boundary, (modified from Hirasaki, 1974).

$$K_z = \frac{S_s b^2 \tau_o}{t_o'} \tag{3-19}$$

Here again it is assumed that the specific storage S_s is known from other information.

Uncertainties

- This test is based on the assumption that the period of injection or pumping is almost negligible in comparison with the time to reach the maximum pressure response.
- Possible leaks behind the casing lead to erroneously high values of vertical hydraulic conductivity.
- The hydraulic conductivity measured by this method is representative of materials very close to the well.

3.2.4 Bredehoeft-Papadopulos Single-Well Test

Bredehoeft and Papadopulos (1980) have proposed a method of measuring permeability which is a modification of the conventional slug test. Although their method is designed for measuring horizontal rather than vertical hydraulic conductivity, we shall discuss it here because (1) the conventional methods for measuring vertical hydraulic conductivity in tight formations are associated with uncertainties, and the value of horizontal hydraulic conductivity could give an upper limit for the vertical component provided that major vertical fractures are absent, and (2) as we saw before, in some cases in addition to the vertical value one also needs horizontal hydraulic conductivity to evaluate the vertical component of fluid flow.

Purpose

The purpose of this test is to measure in situ horizontal hydraulic conductivity of so called 'tight formations', such as tightly compacted clays, rock units in which fractures, if they exist, are essentially closed or filled, or matrix rock between fractures.

Procedure

Figure 3-6 depicts setups for the test in (a) an unconsolidated formation and (b) a consolidated formation. Depending on the time elapsed since the well has been drilled, the water level in the hole may or may not have stabilized to the ambient hydraulic head at the interval to be tested. To start the test, the test system is filled with water and, after a period of observing the water level for ambient conditions, the test interval is suddenly pressurized by injecting an additional amount of water with a high pressure pump. The test interval is then shut-in, and the head change H_o caused by the pressurization is allowed to decay. As water slowly penetrates into the formation H_o will drop. The variation of H_o with time is recorded.

Theory

Apart from the conventional initial-boundary value formulation which is generally adopted for simple radial flow in a confined and infinitly long aquifer, one specific constraint used in this development is as follows.

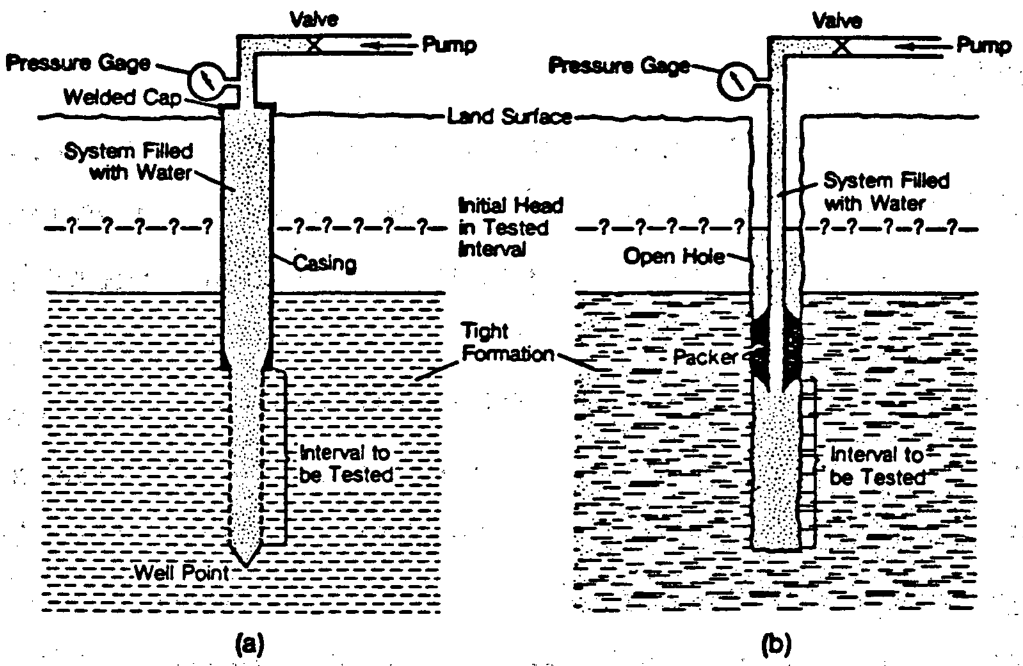

Fig. 3-6 Possible arrangements for conducting pressurized test (a) in unconsolidated formations and (b) in consolidated formations, (after Bredehoeft and Papadopulos, 1980).

The driving force governing the movement of water from the well into the formation is the expansion that the water stored within the pressurized system undergoes as the head, or the pressure within the system, declines. Thus, the rate at which water flows from the well is equal to the rate of expansion. In a conventional slug test the water flow into the formation comes directly from the volume of stored water in the system under normal hydrostatic pressure. The solution for the modified slug test has been presented in the form

$$\frac{H}{H_o} = F(\alpha,\beta) \tag{3-20}$$

where H_o and H are values of head measurement in the hole at the time of shut-in and following that with respect to the background head, respectively. α and β are given by

$$\alpha = \frac{\pi r_s^2 S}{V_w C_w \rho_w g} \tag{3-21}$$

$$\beta = \frac{\pi T t}{V_w C_w \rho_w g} \tag{3-22}$$

where

r_s = radius of well in the tested interval

t = time

S = storage coefficient of the tested interval

V_w = volume of water within the pressurized section of the system

C_w = Compressibility of water

ρ_w = density of water,

T = transmissivity of the tested interval

g = gravitational acceleration

Tables of the function $F(\alpha, \beta)$ for a large range of variation of α and β are given by the above authors as well as Cooper et al. [1967] and Papadopulos et al. [1973].

Major assumptions applied in development of this method are as follow:

- Flow in the tested interval is radial, which will also imply that the flow at any distance from the well is limited to the radial zone defined by the tested interval.
- Hydraulic properties of the formation remain constant throughout the test.
- The casing and the formation on the side of the borehole containing the water are rigid and do not expand or contract during the test.
- Before the system is pressurized water level in the well has come to a near equilibrium condition with the aquifer.

Analysis of Field Data

Bredehoeft and Papadopulos have proposed two different techniques, one for $\alpha \leq 0.1$ and the other for $\alpha > 0.1$. If $\alpha \leq 0.1$ the following steps should be taken.

- Prepare a family of type curves, one for each α, of $F(\alpha, \beta)$ against β on a semilogarithmic paper. A table giving the value of $F(\alpha, \beta)$ as a formation of α and β is presented by Bredehoeft and Papadopulos (1980).
- Plot observed values of H/H_o versus time t on another semilog paper of the same scale as the type curves.

- Match the observed curve with one of the type curves keeping the β and t axes coincident and moving the plots horizontally.
- Note the value of α of the matched type curve, and the values of β and t from the match point.
- Calculate values of S and T from the definitions of α and β given by equations (3-21) and (3-22).

The above method is not suitable for $\alpha > 0.1$. In this range of α, this method can only give the product of transmissivity and storage coefficient, TS. This product maybe calculated by matching the field curve of H/H_o versus time t with a type curve family of $F(\alpha,\beta)$ versus the product $\alpha\beta$ (Fig. 3-7).

Merits of the Method

As the authors have shown in one example, a conventional slug test in a formation with hydraulic conductivity of $K = 10^{-12}$ m/s may last more than one year whereas the modified slug test method as discussed here may take only a few hours.

Uncertainties

- The major assumption employed in this method is that "volumetric changes due to expansion and contraction of other components of the system are negligible." In other words, expansion of the pipes and contraction of the rock in the test zone is negligible relative to that of water. This assumption may introduce large errors into the calculation of hydraulic conductivity. Neuzil (1982) has referred to a test in which the compressibility in the

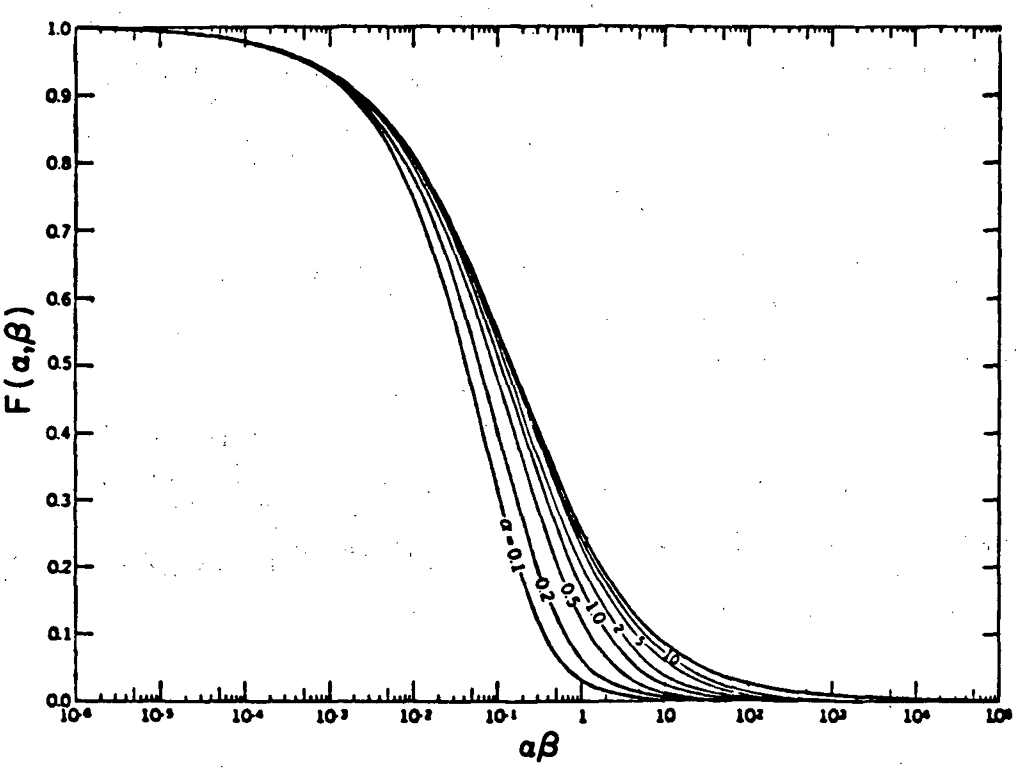

Fig. 3-7 Type curves of the function $F(\alpha,\beta)$ against the product parameter $\alpha\beta$, (after, Bredehoeft and Papadopulos, 1980).

shut-in well was approximately six times larger than the compressibility of water.

- The other major assumption which was employed in this method was that before the system was pressurized either the water level in the well had come to near equilibrium condition with the aquifer or that the observed trend could be extended throughout the test. Neuzil (1982) has pointed out that this assumption may also lead to erroneous results. He argues that the pressure changes due to nonequilibrium conditions before shut-in become much more rapid after the well is pressurized. Neuzil (1982) has proposed the following modifications in the setup and procedure for performing the test.
- Modify the test equipment to that shown on Fig. 3-8.
- Fill the borehole with water and set two packers near each other.
- Set up two pressure transducers as shown in the figure.
- Close the valve, shutting in the test section, and monitor the pressures in both sections until they are changing very slowly.
- Open the valve, pressurize the test section by pumping in a known volume of water, and reclose the valve.
- Measure the net pressure decay (slug) by subtracting the decline due to transient flow prior to the test from the measured total pressure.
- Analyze data using the technique prepared by Bredehoeft and Papadopulos (1980) as was mentioned before, except that the term for the compressibility of water c_w is replaced by the ratio c, defined as

$$c = \frac{\frac{\Delta V}{V}}{\Delta P} \qquad (3\text{-}23)$$

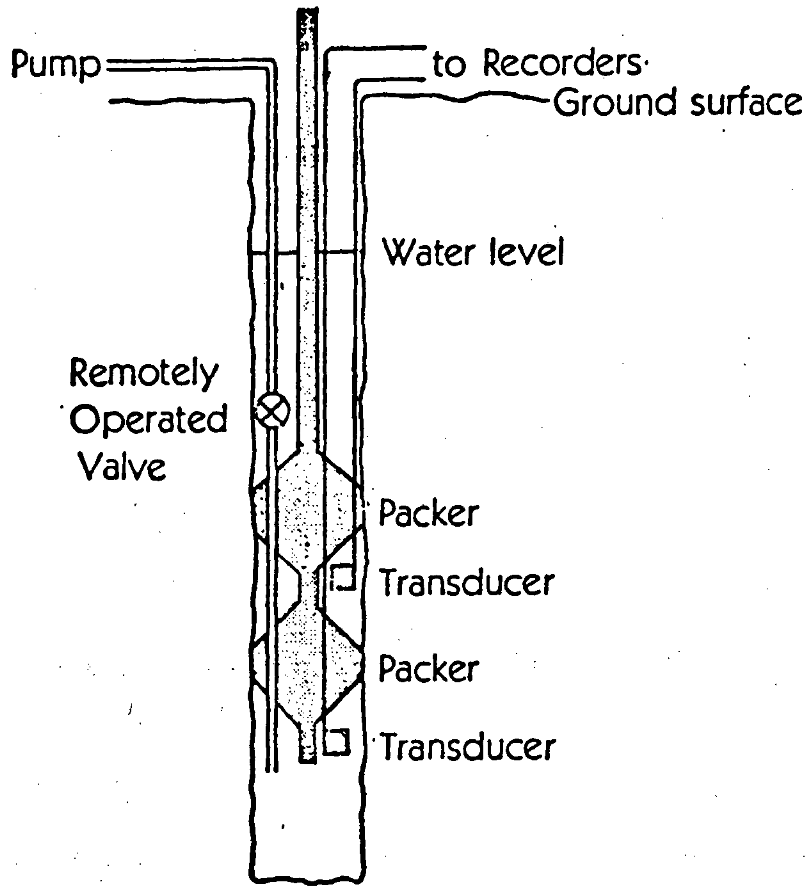

Fig. 3-8 Arrangement of the borehole instrumentation as suggested by Neuzil (1982).

where v is the volume of the shut-in section, and ΔV is the volume of water added to generate a pressure change of ΔP. Neuzil (1982) indicates that a rise in pressure measured by the transducer between the two packers may indicate leakage upward from the test section. However, two other phenomena may cause some rise of pressure in the middle section. One is increase of pressure inside the formation adjacent to the test section, which may or may not be significant. The other reason is the possibility of transfer of pressure by the packer itself, from the test section to the middle section.

3.2.5. General Comments About Single-Well Tests

The following problems are inherent in all single well tests.

- The hydraulic conductivity measured by these tests is only representative of a small zone around the testing interval. A thin lens of very small permeability located between injection and measuring zones could lead to an erroneously low vertical hydraulic conductivity, even if it is only locally present. This problem may be overcome by conducting several tests within the total thickness of a given formation. However, the lateral variation of vertical hydraulic conductivity could be another problem which requires either other types of testing or performance of a number of single-well tests.
- Because the horizontal permeability of sedimentary materials is usually much larger than the vertical permeability, flow lines generated by either injection or pumping in these tests are predominantly horizontal. Therefore, a long time may be

required to have significant pressure disturbances in measuring intervals located vertically above or below the flow zone. A small pressure change together with the possibility of leakage behind the casing due to poor cementing will result in an increased degree of uncertainty in the credibility of these tests in tight formations.

- Measurement of change of pressure due to pumping or injection in single-well tests is another source of uncertainty. This is because the test may often start before the pressure at the measuring interval has stabilized. One way to handle this problem is to minimize the volume of the measurement cavity in the well with the help of extra packers. This will shorten the time required for pressure stabilization.
- In a single-well test, injection is preferred over pumping unless the well will flow without artificial lift (Earlougher, 1980). In a tight formation, indeed, injection is the only feasible way to test.
- The injection or pumping zone should be packed off to minimize well bore storage.

3.3 Tests With Two Or More Wells

Tests involving two or more wells measure the response of a much larger volume of rocks than tests from a single well. Therefore, the value of hydraulic conductivity obtained from multiple well tests is usually more representative of the large scale behavior of the formation. The only problem with these tests is that they cannot be directly used within the formation of interest, once the permeability of that formation becomes very low. Wells completed in very low permeablility materials are unable to produce fluid for the required test period. Fluid could be injected in these wells; however, it may take years before any useful response can be measured in observation wells at a distance of 5 to 10 m.

In the following discussions readers are assumed to be familiar with general pump test design and operation. For more information on this subject readers are referred to Stallman (1971).

3.3.1 Weeks' Method

Weeks (1964) proposed a method of calculating vertical hydraulic conductivity of higher conductivity aquifers. A brief description of his method is given here.

Purpose

The purpose of this method is to determine in situ vertical and horizontal hydraulic conductivity of anisotropic aquifers.

Procedure

- Consider a pumping well which is only partially penetrating an anisotropic aquifer. The well is open to the aquifer over a length of (l-d), (see Fig. 3-2).
- Also consider one or more piezometers at distances r_i, from the axis of the pumping well, such that each r_i is smaller than half of the thickness of the aquifer.
- Pump the well with a constant rate of discharge Q, for a period of time.
- Measure water level variations in the piezometers and record these variations against the time of measurement.

Theory

The solution for the drawdown around a partially penetrating well in an anisotropic aquifer has been given by Hantush (1957, 1964).

$$s = \frac{Q}{4\pi K_r b} \left\{W(u) + f\right\} \tag{3-8}$$

where

$$f = \frac{2b}{\pi(l-d)} \sum_{n=1}^{\infty} \frac{1}{n} \left(\sin\frac{n\pi l}{b} - \sin\frac{n\pi d}{b}\right) \cos\frac{n\pi z}{b} \cdot W\left[u_r, \sqrt{\frac{K_z}{K_r}\left(\frac{n\pi r}{b}\right)^2}\right] \tag{3-9}$$

This equation was presented in a slightly different context in Sec. 3.2.1.

Hantush (1961) has given another form for f which is valid at large values of time. Weeks (1964) has modified this solution for anisotropic aquifers.

$$f = \frac{4b}{\pi(l-d)} \sum_{n=1}^{\infty} \frac{1}{n} \left(\sin\frac{n\pi l}{b} - \sin\frac{n\pi d}{b}\right) \cos\frac{n\pi z}{b} \cdot K_o \left[\frac{n\pi r}{b} \left(\frac{K_z}{K_r}\right)^{1/2}\right] \tag{3-24}$$

where K_o is modified Bessel function of the second kind and zero order. Equation (3-24) is valid for large values of time when

$$u_r < \left(\frac{\pi r}{b}\right)^2 \frac{K_z}{20K_r} \; . \tag{3-25}$$

$$\text{or } t > \frac{b\,S}{2K_z} \tag{3-26}$$

Let us introduce the following dimensionless terms:

$$s_D = \frac{4\pi K_r bs}{Q} \tag{3-27}$$

$$t_D = \frac{t\,T}{r^2 S} \qquad (3\text{-}28)$$

where T and S are transmissivity and storage coefficient of the aquifer, respectively.

Given the geometry of the system, one can calculate r/b, z/b, l/b, and d/b. Assuming different values for K_z/K_r, a family of type curves showing the variation of s_D against t_D can be prepared for the above known dimensionless parameters.

One may have noticed that the methods proposed by Weeks and Burns are both based on the same theory. Burns' method applies the theory to a single well, and Week's method applies it to multiple wells. Saad (1967) and Weeks (1969) have proposed other methods for calculating the ratio of horizontal to vertical permeability in aquifers. Both of those methods are also based on the theory of the Partially Penetrating Wells which was discussed above.

Analysis of Field Data

- Plot s_D versus t_D calculated from equations (3-8), (3-9), (3-27), and (3-28) for the dimensionless parameters of the system and for different values of K_z/K_r on log-log paper. Note that equation (3-24) is independent of time. Therefore, it is much simpler to use equation (3-24) in place of equation (3-9) for those times when $t > \frac{bS}{2K_z}$. This means that the order of magnitude of S and K_z should be estimated in advance. An extensive table evaluating equation (3-8) for the simple case of d=0 and K_z/K_r=1 is given by Witherspoon et al (1967), which could be easily modified for the case of $d \neq 0$ and an anisotropic medium.

- Plot values of drawdown versus time as measured by each piezometer on another log-log paper with the same scale as the type curves.
- Using the superposition technique, find the best match between the observed data and one of the type curves.
- When the best match is achieved read the K_z/K_r corresponding to the type curve and the cooridnates of a match point on both graphs.
- Calculate the radial hydraulic conductivity and the storage coefficient of the aquifer from the following equations.

$$K_r = \frac{s_D Q}{4 \pi b s} \tag{3-29}$$

$$S = \frac{t\, b\, K_r}{t_D r^2} \tag{3-30}$$

where s, t, t_D and s_D are coordinates of the match point.

- Calculate the vertical hydraulic conductivity of the aquifer from

$$K_z = \left(\frac{K_z}{K_r}\right) K_r \tag{3-31}$$

3.3.2 Tests Based on the Theory of Leaky Aquifers

The term leaky aquifer generally refers to a system in which an aquifer is overlain and/or underlain by much less permeable layers. Once the pressure in the aquifer drops while being pumped, water from saturated less permeable layers lying above or below leaks into the aquifer. Sometimes the amount of leakage is so great that its effect can be detected in the aquifer being pumped. In this case the confining beds are called 'aquitards' and the aquifer is referred to as being 'leaky'. When the amount of leakage is so little that its effect cannot be easily detected in the aquifer, then the confining beds are called 'aquicludes' and the aquifer is termed 'slightly leaky' (Neuman and Witherspoon, 1968).

Much work has been done on the theory of leaky aquifers. The first group of papers appeared before 1960 (Jacob, 1946, Hantush and Jacob, 1955, Hantush, 1956) and were based on the assumption that the storage capacity of the aquitard was negligible. Later, Hantush (1960) introduced a new solution for leaky aquifers in which he had considered the effect of storage capacity of the confining bed. Neuman and Witherspoon (1969, 1972) evaluted the significance of the assumptions applied in the earlier work and provided more generalized solutions. A brief description of these methods will be given in the following sections.

One may ask what the relation is between leaky aquifers and the subject of field determination of vertical hydraulic conductivity. Why should we study the leaky aquifer pump test techniques? As we shall see later, all of the leaky aquifer solutions which are discussed here are based on the assumption that the flow in the less permeable layer, above or below an aquifer, is essentially vertical. Therefore, application of these methods should give an overall vertical hydraulic conductivity for the confining layer.

3.3.2.1 Hantush and Jacob Solution

Jacob (1946) developed a partial differential equation for a leaky aquifer and solved it for a bounded reservoir. Hantush and Jacob (1955) solved the same problem for a radially infinite aquifer. Because of its simplicity, in spite of the fact that in some cases it leads to erroneous results, these solutions have been widely used by groundwater hydrologists.

Purpose

The purpose of this section is to evaluate the possibility of determining the vertical hydraulic conductivity of the confining layer and discuss the assumptions and limitations encompassing the method of approach.

Procedure

The procedure for conducting the test is similar to that for a standard pump test within a simple aquifer. From such a test one obtains a table of observed drawdown in an observation well or a piezometer against the time elapsed from the start of pumping.

Theory

Figure 3-9 depicts the arrangment of the system to be studied. A semi-permeable layer (aquitard) with a constant thickness of b' is overlying an aquifer with much higher hydraulic conductivity. The aquitard is overlain by another highly permeable extensive aquifer. The lower aquifer is being pumped with a constant rate of discharge Q. Hantush and Jacob (1955) obtained an expression which gives the drawdown distribution in the pumped aquifer as a function of time. Derivation of this solution was based on the following major assumptions: (1) flow is essentially horizontal in the aquifer and vertical in the aquitard, (2) no drawdown is permitted in the upper aquifer

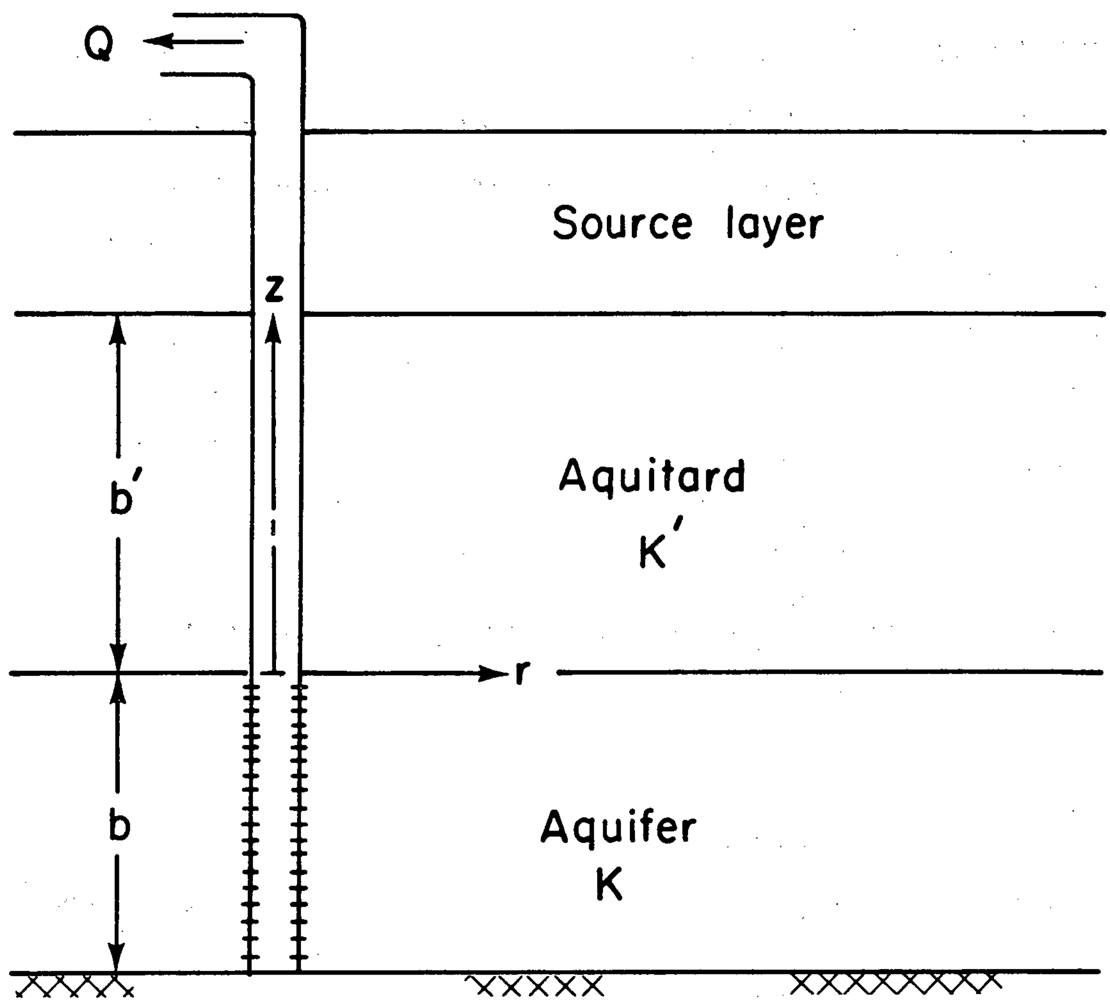

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Fig. 3-9 Leaky aquifer with a constant head boundary at the top of the aquitard.

because of pumping in the lower aquifer, (3) leakage into the pumped aquifer is proportional to the potential drop across the aquitard this last assumption is equivalent to assuming that the storage capacity of the confining bed is negligible and all the water leaking into the pumped aquifer comes directly from the upper aquifer, thus the aquitard behaves only as a conduit between the two aquifers. The solution to this problem as given by Hantush and Jacob (1955), sometimes referred to as the (r/B solution) is

$$s = \frac{Q}{4\pi Kb} W(u, r/B) \tag{3-32}$$

where

$$u = \frac{r^2 S_s}{4tK}$$

$B = \sqrt{\frac{Kbb'}{K'}}$ called the leakage factor

K, K' = hydraulic conductivity of the aquifer and aquitard, respectively

S_s = specific storage

s = drawdown in the aquifer

b, b' = thickness of the aquifer and the aquitard, respectively

$$W(u, r/B) = \int_u^\infty \exp\left(-y - \frac{r^2}{4yB^2}\right) \frac{dy}{y}$$

This last term is called the well function of leaky aquifers. This function has been extensively tabulated (Hantush, 1956).

Analysis of Field Test Data

Several methods based on the r/B solution are conventionally used for interpretation of leaky aquifer pump test data. Here we shall discuss two of these methods.

A. Walton's Type-Curve Method (1960)

- Prepare a family of type curves by plotting on a log-log paper the values of function W(u,r/B) versus 1/u with r/B as the running parameter of the curves. Note that the curve with r/B = 0 is the Theis curve.
- Plot the drawdowns versus time as were recorded within an observation well (after appropriate adjustments) on another log-log paper with the same scale as that used for the type curves.
- Follow the regular procedure for curve matching* and read the appropriate value of r/B by interpolating the position of the data curve among the type curves. Also read the dual coordinates of the matching point, s,t, 1/u, and W(u,r/B).
- Calculate the hydraulic conductivity of the pumped aquifer from

$$K = \frac{Q}{4\pi bs} W(u,\ r/B) \tag{3-33}$$

- Calculate the specific storage of the pumped aquifer from

$$S_s = \frac{4tK}{r^2(1/u)} \tag{3-34}$$

- Finally, calculate the vertical conductivity of the aquitard from

$$K' = \frac{Kbb'}{r^2} \left(\frac{r}{B}\right)^2 . \tag{3-35}$$

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*A unique fitting position is difficult to obtain unless sufficient data is available from the period when the leakage effect is insignificant (Hantush, 1964).

B. U.S.B.R. Method

U.S. Bureau of Reclamation (1977) has published a groundwater manual as a guide for field personnel in groundwater investigation. Following is the method which that manual suggests for interpretation of pump test data of a leaky aquifer. Fig. 3-10 shows a family of type curves prepared from Jacob's leaky aquifer solution (1946). As was discussed before, Jacob's solution was developed for a radially bounded aquifer. However, in developing Fig. 3-10 the outer boundary was located at a sufficient distance that the effect of pumping never reached it (Glover, Moody, and Tapp, 1960). This approach permits the curves to be used for infinite aquifers. The steps to be used in applying the USBR method are as follows:

- Drawdown versus time from two or more observation wells (after appropriate corrections) located at different radial distances r from the pumped well should be plotted on a log-log paper with the same scale as Fig. 3-10.
- Superimpose the field curve with those of Fig. 3-10.
- After obtaining the best match read the dual coordinates of a match point (s,t, u and η), and the x value of the best fitting type curve. Interpolation may be required to find the x value.
- Calculate the hydraulic conductivity of the aquifer from

$$K = \frac{Qu}{2\pi Ms} \qquad (3\text{-}36)$$

- Calculate the hydraulic conductivity of the aquitard from

$$K' = KMM' \left(\frac{x}{r}\right)^2 \qquad (3\text{-}37)$$

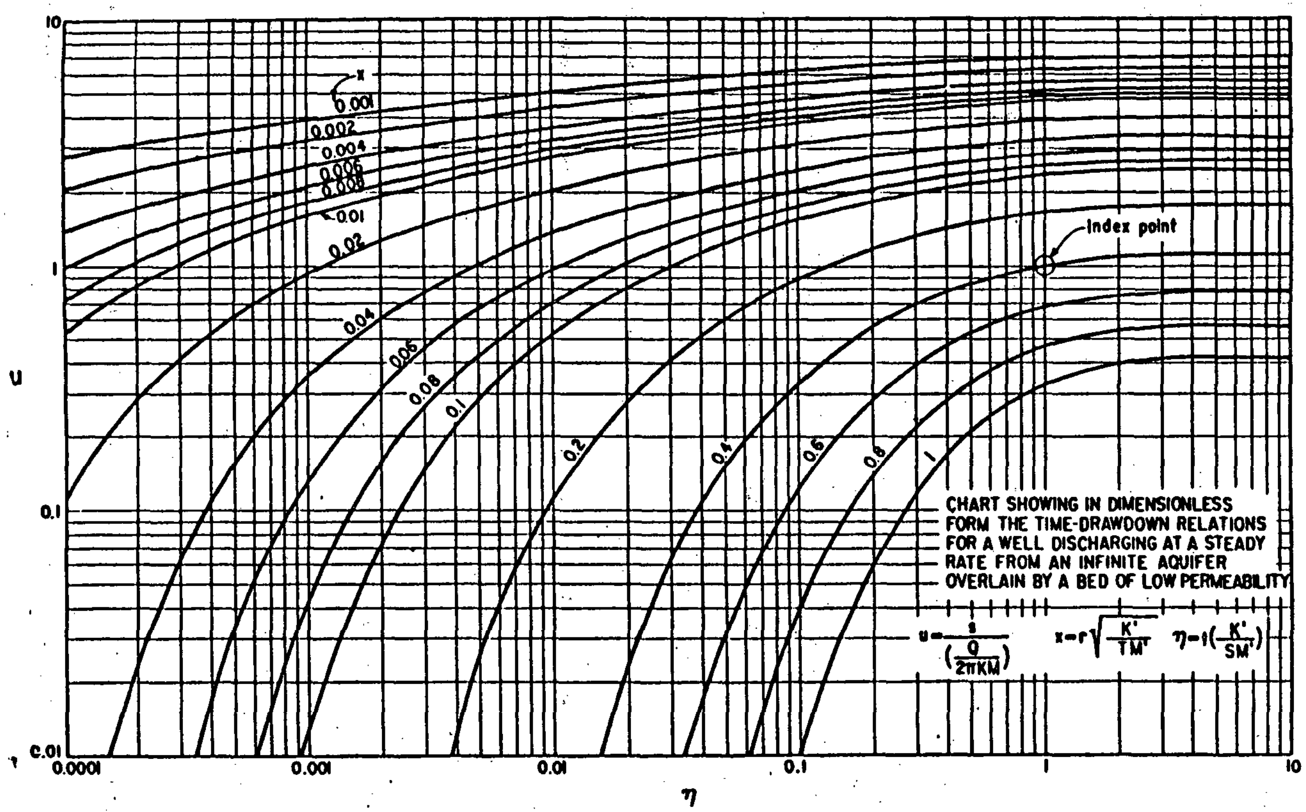

Fig. 3-10 Leaky aquifer type curves based on r/B approach (USBR, 1977).

- Finally, calculate the storage coefficient of the aquifer from

$$S = \frac{K't}{\eta M'} \tag{3-38}$$

In the above equations M and M' indicate the thickness of the aquifer and the aquitard, respectively. The ratio r/x is the leakage factor B used in the development of the theory. The definitions of other terms are given in Fig. 3-10.

The following is a quotation from the U.S.B.R. staff on the interpretation of leaky aquifer pump test data from the Missouri river basin project (Glover, Moody, and Tapp, 1960, p. 175).

> "When drawdown data from well tests are compared with drawdown curves computed for idealized conditions a lack of perfect agreement is generally evident".

Other methods of analysis of field data based on r/B solution have been suggested by Hantush (1964, p. 416-417), and Narasimhan (1968).

Uncertainties

The problem of flow to a pumped well in a hydrologic system consisting of several aquifers separated by less pervious aquitards or aquicludes is in fact 3 dimensional. A rigorous approach to the solution of such a problem is analytically intractable. Therefore, it has been customary to simplify the problem by assuming that flow is essentially horizontal in the aquifers and vertical in the aquitards and aquicludes. The validity of this assumption which was used in the derivation of the r/B solution, was evaluated by Neuman and Witherspoon (1969). They noted that the errors introduced by this assumption are less than 5% provided that the conductivities of the aquifers are more than 3 orders of magnitude greater than that of the aquitards.

These errors increase with time and decrease with radial distance from the pumping well. One should note that the 5% error given by Neuman and Witherspoon (1969) is the percentage difference between drawdowns calculated by the analytic solution based on the above assumption and drawdowns obtained by a finite-element numerical analysis without that assumption. The magnitude of the error which may result in the calculation of the hydraulic properties of the confining layer is not known.

Another assumption used in the derivation of the r/B solution is that no water is released from storage in the aquitard. Neuman and Witherspoon (1969) have found that this assumption tends to result in overestimating the permeability of the aquifer and underestimating the permeability of the aquitard.

An important uncertainty about the r/B solution is that it does not provide a means of distinguishing whether the leaking bed lies above or below the aquifer being pumped. In case leakage occurs both from above and below the aquifer this method does not provide a means for determining conductivities of individual aquitards. This becomes particularly important when one is looking for the hydraulic conductivity of a certain confining bed rather than that of the aquifer itself.

When the hydraulic conductivity of the confining bed becomes so small that the ratio of K'/K tends to zero, the drawdown distribution in the aquifer becomes essentially the same as would be predicted by the Theis solution for an aquifer without leakage. As a result, techniques based on observation in the aquifer alone fail to give the properties of the confining bed.

3.3.2.2 Hantush Modified Solution

In 1960 Hantush published another paper in which he introduced a new treatment of leaky aquifers which overcome some of the difficulties of the r/B solution.

Purpose

The Hantush modified solution provides a more accurate approach to the evaluation of the vertical hydraulic conductivity of less permeable layers which confine permeable aquifers.

Procedure

The test procedure again follows the same steps as a regular pump test. The data needed for interpretation is a record of drawdown versus time in one or more observation wells around a pumping well.

Theory

In this development, in addition to assigning a storage capacity to the confining aquitard, Hantush (1960) solved the problem for two different cases: (1) an infinite horizontal aquifer overlain by an aquitard whose upper boundary does not experience any change in drawdown, and (2) the same situation but with an impermeable bed overlying the aquitard. Other assumptions applied in the development of the r/B solution, including vertical flow in the aquitard and horizontal flow in the aquifer, still hold. In this solution Hantush considered leakage into the aquifer from both above and below. He presented the solutions for two ranges of time t as indicated below.

Solutions for Small Values of Time

For t less than both $b'S'/10K'$ and $b''S''/10K''$, the solution for both cases is the same and is given by

$$s = \frac{Q}{4\pi Kb} H(u,\beta) \tag{3-39}$$

where

$$H(u,\beta) = \int_u^\infty \frac{e^{-y}}{y} \text{ erfc } \left(\beta \sqrt{u} / \sqrt{y(y-u)}\right) dy$$

$$\beta = (r\lambda)/4$$

$$\lambda = \sqrt{\frac{K'}{Kbb'} \frac{S'}{S}} + \sqrt{\frac{K''}{Kbb''} \frac{S''}{S}}$$

$$u = \frac{r^2 S}{4tbK}$$

s = drawdown in the aquifer

S'', S' = storage coefficient of the lower and upper aquitards, respectively

K'', K' = hydraulic conductivity of the lower and upper aquitards, respectively

r = radial distance of the observation well from the pumped well

b'', b' = thickness of aquitards below and above the aquifer, respectively.

$H(u,\beta)$ has been extensively tabulated (Hantush, 1960b). A short table of $H(u,\beta)$ is also available (Hantush, 1964).

Solution for Large Values of Time

Case 1.

In this case, t should be larger than both $5b'S'/K'$ and $5b''S''/K''$. The solution is then given by

$$s = \frac{Q}{4\pi bK} W(u\delta_1, \alpha) \qquad (3\text{-}40)$$

where

$$W(u\delta_1, \alpha) = \int_{u\delta_1}^{\infty} \frac{dy}{y} \exp(-y - \frac{\alpha^2}{4y})$$

is the well function for leaky aquifers which is tabulated by Hantush (1956);

$$\alpha = r \sqrt{\frac{K'}{bb'K} + \frac{K''}{bb''K}}$$

$$\delta_1 = 1 + \frac{S' + S''}{3\ S}$$

The other terms are the same as defined before.

Case 2.

For t greater than both 10b'S'/K' and 10b"S"/K" the expression for drawdown in the aquifer is

$$s = \frac{Q}{4\pi Kb} W(u\delta_2) \qquad (3\text{-}41)$$

where

$$W(u\delta_2) = \int_{u\delta_2}^{\infty} \frac{e^{-y}}{y} dy \quad \text{is the well function}$$

$$\delta_2 = 1 + \frac{S' + S''}{S}$$

At this point, before describing the method of interpretating the pump test data, the applicability of the different operations given above will be reviewed. For large values of time, equation (3-40) indicates that, even when one considers the storage capacity of the confining bed, the r/B solution could be safely used for evaluation of the aquifer and aquitard, provided that $t > \frac{5b'S'}{K'}$. This solution may qualify at relative small values of time when the aquitard is thin, when it has a relatively high hydraulic conductivity and incompressible (i.e. very small S'). For example, if b' = 5 m, K' = $2x10^{-7}$ m/s, and S' = $2x10^{-5}$, then the r/B solution is applicable after 625 seconds, or approximately 10.5 minutes after the start of the test. In applying the simpler r/B solution, note that u should be replaced by $u\left(1 + \frac{S'}{3S}\right)$. Also, the aquifer above the aquitard should not show any drawdown during the test. If the overlying aquifer does show some drawdown then the r/B solution tends to underestimate the hydraulic conductivity of the aquitard. On the other hand, if the confining bed is relatively thick and elastic with low hydraulic conductivity then the r/B solution is not applicable. For example, if b' = 50 m, K' = $5x10^{-9}$ m/s and S' = 10^{-3}, then the r/B solution is only applicable after $3.12x10^{8}$ seconds, or approximately 1 year after the test has started.

Equation (3-41) suggests that when the confining bed is thin, relatively permeable, and incompressible, and overlain by an impermeable layer which cannot supply water, the drawdown data in the aquifer will follow the Theis solution at relatively small values of time. In applying the Theis solution, note that u should be replaced by $u\left(1 + \frac{S'}{S}\right)$.

Equation (3-39) is the solution for small values of time. It can also be applied to relatively large values of time when the aquitard is thick, relatively impermeable and compressible. For example, if b' = 100 m, K' = 10^{-9} m/s, and S' = 10^{-3}, then equation (3-39) is applicable for 10^{7} seconds or the first 115 days of the test. Note that within this range of time the effect of pumping would not reach the upper boundary of the aquitard. Therefore, the assumption of a constant head boundary there does not introduce any error. The above discussion was made only with reference to the upper confining bed. In each case, however, both the upper and lower beds must meet the same criteria for these simplifications to apply.

Analysis of the Field Data

Figure 3-11 shows a family of type curves on a log-log plot of $H(u,\beta)$ versus $1/u$ which can be used for the analysis of the Hantush modified solution.

- Plot the variation of drawdown versus time on a log-log paper with the same scale as that of the type curves.
- Use the superposition method to find the best match between the observed plot and the appropriate type curve.
- Read the value of β from the type curve which matches the observed plot, and the dual coordinates $H(u,\beta)$ $1/u$, t, and s of the match point.
- Calculate the hydraulic conductivity of the aquifer from

$$K = \frac{Q}{4\pi bs} H(u,\beta) \tag{3-42}$$

- Calculate the storage coefficient of the aquifer from

$$S = \frac{4tbKu}{r^2} \tag{3-43}$$

- Calculate λ from

$$\lambda = \frac{4\beta}{r} \tag{3-44}$$

- If we assume that the lower layer is completely impermeable, then

$$K'S' = \lambda^2 Kbb'S$$

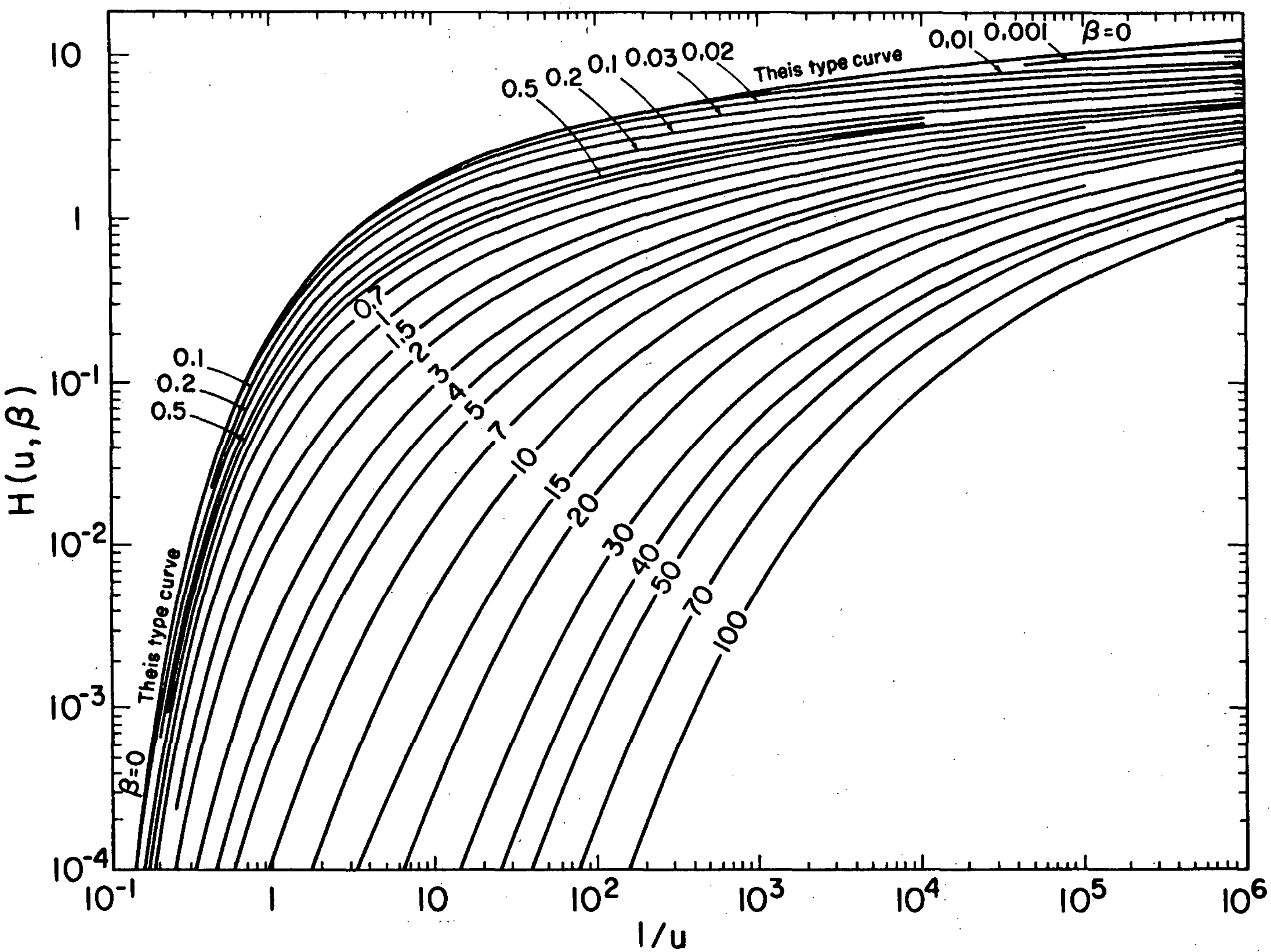

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Fig. 3-11 Type curves of the function H(U,β) against 1/U, for various values of β(after Lohman, 1972).

- If one can determine the magnitude of the storage coefficient of the aquitard S' from other methods, then the hydraulic conductivity of the aquitard may be obtained from

$$K' = \frac{\lambda^2 Kbb'S}{S'} .$$

Uncertainties

Except for very large values of β, the type curves have shapes that are not too different from the Theis curve. Thus, it is difficult to decide which of the type curves to use in matching against field data. When b is very small, one may easily choose a β which could be off by two orders of magnitude. Since $K'S' = (\beta^2)\,\frac{16Kbb'S}{r^2}$, an error in choosing β would lead to a much larger error in the calculation of (K'S'). Thus, two orders of magnitude error in estimating β would lead to four orders of magnitude error in (K'S').

In order to improve this problem, Weeks (1977) suggested that data from at least two observation wells at different distances from the pumping well should be used. A composit plot of the drawdown versus t/r^2 is made on a log-log paper with the same scale as that of the type curves. As a result, one should obtain two or more type curves each with different values of β proportional to the value of r. A unique match may then be obtained by adding the extra constraint that r values for observation wells must fall on curves having proportional β values (Weeks, 1977). This method could somewhat improve the results but, when $\beta<0.01$, type curves with different values for β are so close together that a unique match is still next to impossible.

Very often both layers above and below an aquifer constitute leakage to the aquifer. If this is the case, one may not be able to find the properties of either of the confining layers. All this method can give is the value of λ (equation 3-44), which is a parameter depending on the properties of both confining layers and the aquifer. This method provides no means for independently determining the properties of both confining layers.

Even when leakage comes only from one of the confining layers, this method gives the product of the hydraulic conductivity and the storage coefficient of the aquitard. The value of the storage coefficient for the aquitard should be found by some other means before one can finally obtain the vertical hydraulic conductivity.

3.3.2.3 Witherspoon and Neuman Ratio Method

When the ratio of K'/K decreases, both r/B and β, as defined in previous methods, decrease and equations (3-32) and (3-39) will eventually reduce to the Theis solution. Therefore, it is obvious that determining the hydraulic conductivity of a tight confining layer by observations in the aquifer alone, if at all possible, is associated with a great many uncertainties. Witherspoon et al. (1962) suggested a method of calculating the permeability of the caprock of gas storage reservoirs which was based on using observations of drawdown in both the aquifer and the overlying aquiclude. Later, Witherspoon and Neuman (1967) presented an improvement over the previous method. This work, together with their more recent works (Neuman and Witherspoon 1972), will be discussed here.

Purpose

The purpose of this section is to describe a method of determining the vertical diffusivity of a low permeable layer overlying an aquifer.

Procedure

- Complete a pumping well through the total thickness of the aquifer.
- Construct an observation well in the aquifer at a distance r from the axis of the pumping well.
- Establish at least three transducers at three different elevations within the confining bed as shown in Fig 3-12. It is required that the radial distance of all three transducers from the pumping well be the same as that of the observation well. To avoid the effect of possible inhomogenity of the media, it is preferred to have all the transducers in the same well close to the observation well.
- Start recording water level in the observation well and values of pressure measured by the transducers long before the start of the pumping test. It is very important that the values of pressure measured by the transducers come to an equilibrium condition before the beginning of the test.
- Start producing from the pumping well with a constant rate of Q. Pumping should continue until at least half a meter of drawdown is observed by the middle transducer in the aquiclude. Recently very accurate pressure-measurement instruments have been introduced to the market which are able to measure pressure changes equivalent to 1 cm or less of water. If such instruments are available for use, then 10 cm of drawdown would be sufficient. Recording of water level in the observation well and pressures measured by the transducers should continue at least a few days after pumping has stopped.

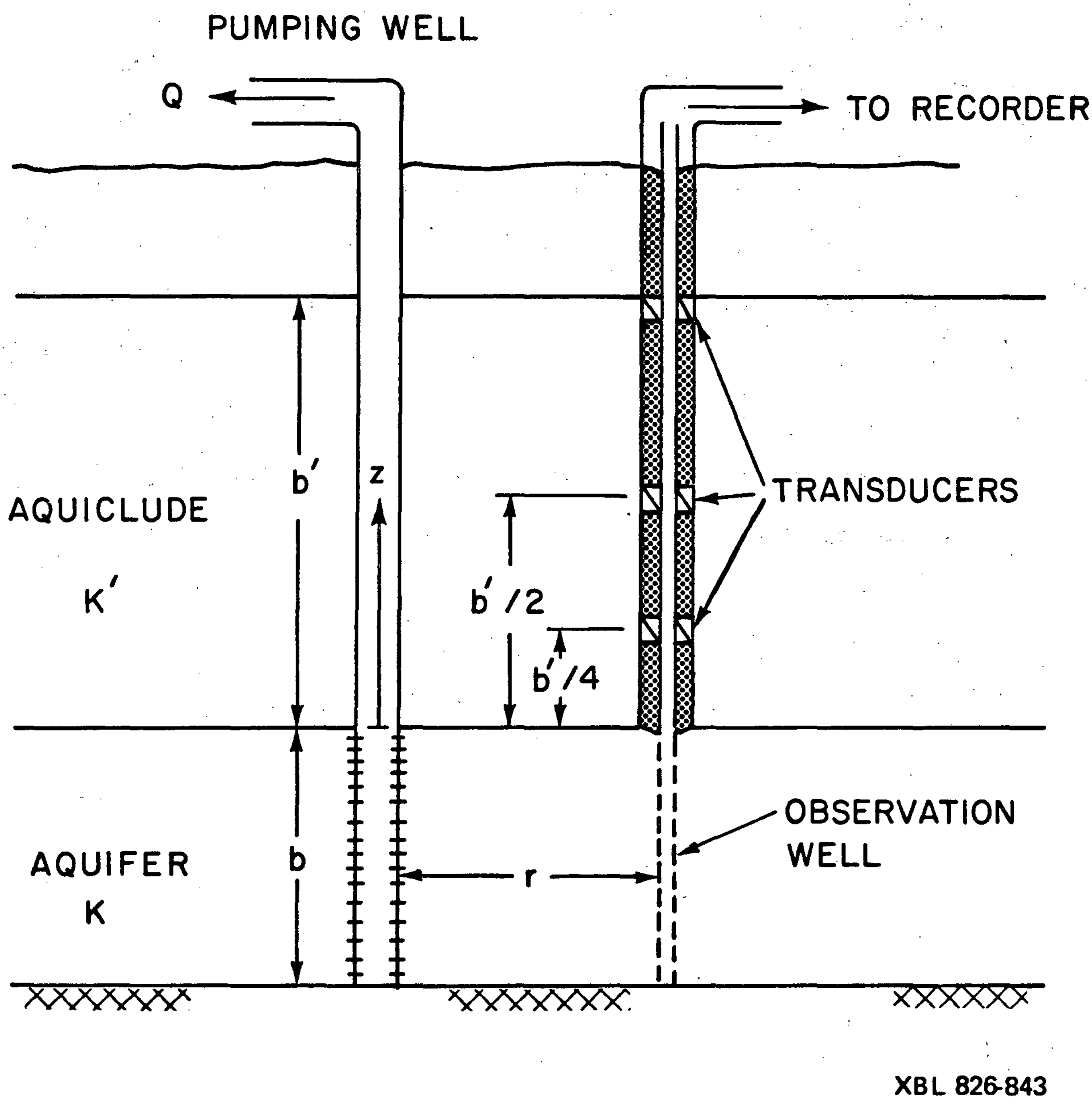

Fig. 3-12 A suggested arrangement for conducting a ratio-method test.

Theory

Let us first discuss the theory which was developed for evaluating a slightly permeable aquiclude. A review of more recent works from Neuman and Witherspoon will then follow.

Consider an aquifer of finite thickness overlain by a semi-infinite confining bed. When the ratio of K'/K is sufficiently small, then under the influence of pumping the aquifer, the flow in the confining bed is essentially vertical, and the drawdown in the aquifer can be closely approximated by the Theis solution. The term semi-infinite has been used to indicate that the aquiclude is so thick that the effect of pumping the aquifer does not reach the top of the aquiclude. With the above assumptions in mind, Witherspoon and Neuman (1967) derived the following expression which gives the drawdown in the aquiclude as a function of time t and elevation z above the top of the aquifer.

$$s' = \frac{Q}{2\pi^{3/2} Kb} \int_{\sqrt{1/4t'_D}}^{\infty} - Ei\left[- \frac{t'_D y^2}{t_D(4t'_D y^2 - 1)} \right] e^{-y^2} \, dy \qquad (3\text{-}45)$$

where

$$t'_D = \frac{K't}{S'_s \, z^2}$$

$$t_D = \frac{Kt}{S_s r^2}$$

$$-Ei(-x) = \int_x^{\infty} \frac{e^{-y}}{y} \, dy$$

z = vertical distance from the top of the aquifer

S_s, S'_s = specific storage of the aquifer and the aquiclude, respectively.

Equation (3-45) has been evaluated over a practical range for two parameters of t_D and t'_D . Calculated values of s' and s'/s for different t_D and t'_D have been tabulated in Appendix G of Witherspoon et al. (1967). Figure 3-13 shows a family of curves presenting variation of s'/s versus t'_D for different values of t_D.

A variation of the above problem, involving a finite thickness aquiclude, has also been solved by Neuman (1966). In this derivation the hydraulic head was assumed to be constant at the top of the aquiclude. This solution has been evaluated over a practical range of relevant dimensionless parameters and the results are tabulated in Appendix H of Witherspoon et al. (1967).

Later, Neuman and Witherspoon (1969a) developed a complete solution for the distribution of drawdown in a system consisting of an aquitard separated by two aquifers as shown on Fig. 3-14. In each aquifer the solution depends on five dimensionless parameters, and in the aquitard six dimensionless parameters are involved. Consequently, Neuman and Witherspoon (1972) stated that "This large number of dimensionless parameters make it practically impossible to construct a sufficient number of type curves to cover the entire range of values necessary for field application." Hantush (1960) apparently had noticed this problem before as he stated that "It should be remarked that rigorous solutions can be obtained for the actual nonsteady three-dimensional flow in layered aquifers, as well as solutions for flow systems in which the condition of vertical leakage is removed. These solutions, however, are very difficult to evaluate numerically and are therefore not presented here."

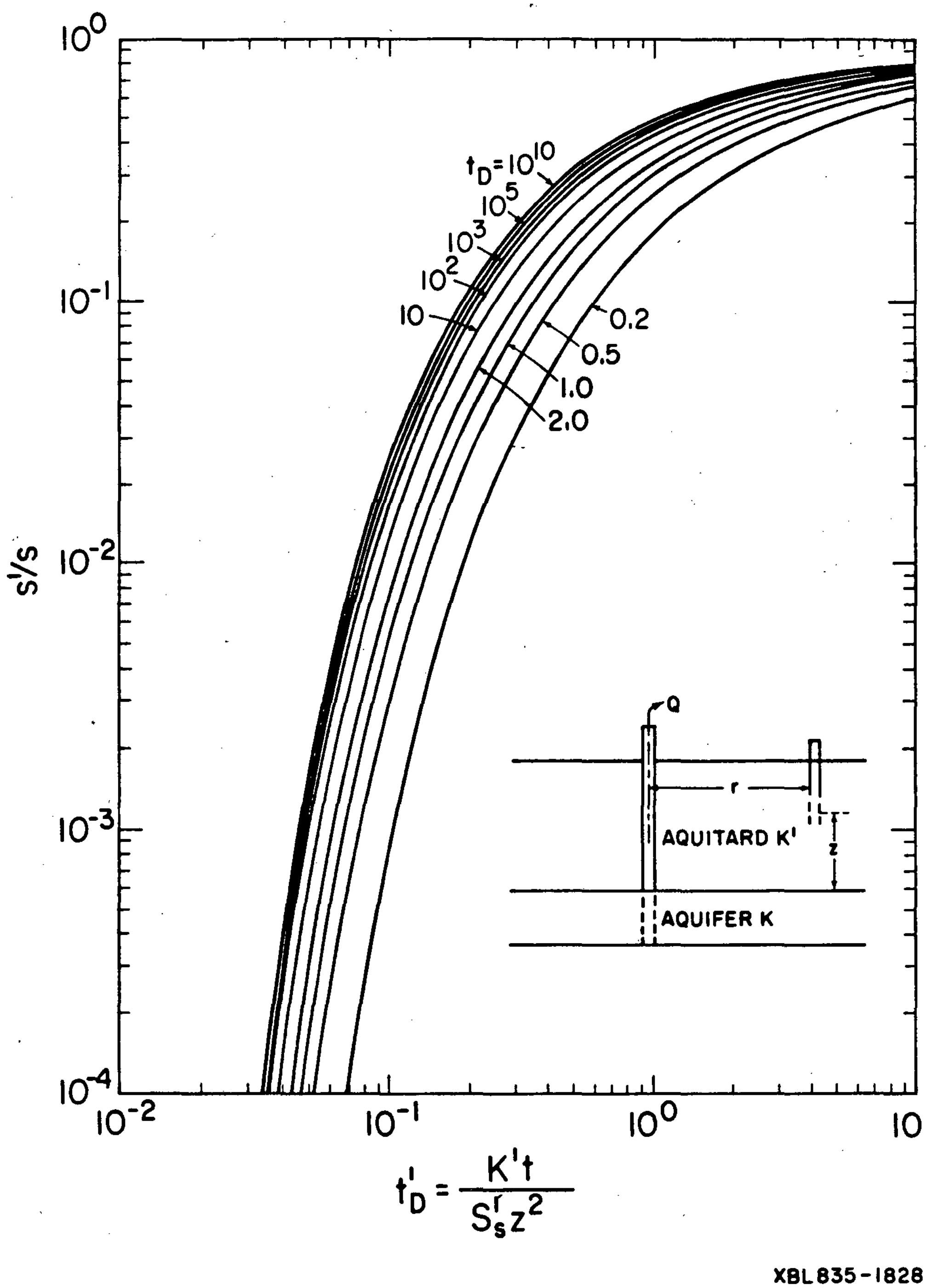

Fig. 3-13 The variation of s'/s with t_D' for a semi-infinite aquitard (modified from Witherspoon et al. 1967).

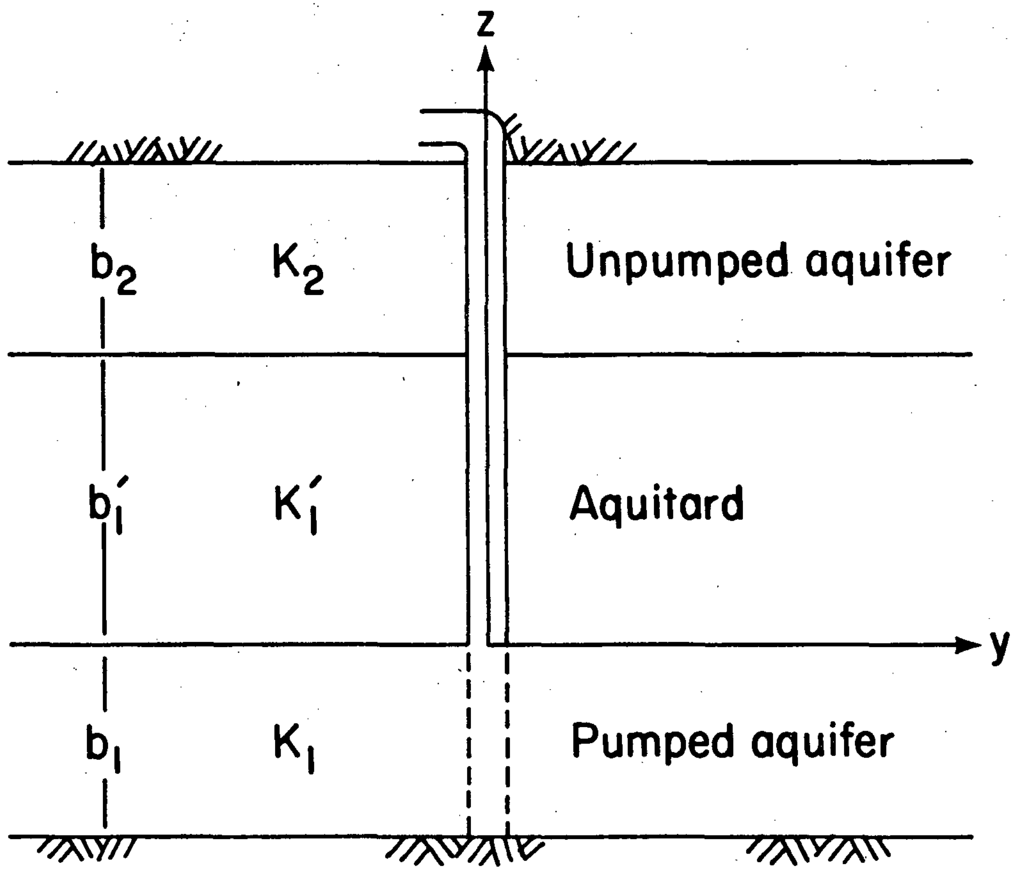

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Fig. 3-14 Schematic diagram of two aquifer system.

As a result, in spite of the development of more sophisticated theories, because of the difficulties which appear in the process of their application in the field, authorities seem to go back and recommend the simpler approaches. For example, all the methods of analysis of the leaky-aquifer pump tests described by Hantush (1964), appeared four years after he introduced the modified theory (1960), are based on the r/B solution. Neuman and Witherspoon (1972), too, stated that "We therefore decided to adopt the ratio method as a standard tool for evaluating the properties of aquitards." This happened 5 years after their original introduction of the ratio method (1967).

Analysis of the Field Data

- Observe the pressure record of the transducer at the top of the confining bed. If it shows any drawdown beyond the error limits of the system, note the time of such observation and ignore all records of drawdown measured after that time.
- Calculate the hydraulic conductivity K, and the specific storage S_s of the aquifer using Hantush's modified solution and the drawdown record from the observation well.
- Plot the values of drawdown, measured both in the aquifer and the aquiclude, on log-log paper and draw smooth curves through the data.
- Select several arbitrary values of time t. All values should be smaller than the time when drawdown was first noted at the top transducer.
- Calculate t_D for each selected value of t from the following equation

$$t_D = \frac{Kt}{S_s r^2} \tag{3-46}$$

- At each value of time select representative values of s and s' from the time drawdown plots.
- Using the appropriate curve corresponding to each value of t_D from Fig. 3-13, find t_D' for each ratio of s'/s.
- Calculate the vertical diffusivity of the confining bed for each value of t and z of a particular transducer from

$$\frac{K'}{S_s'} = \frac{z^2 t_D'}{t} \tag{3-47}$$

- For each value of z find the average value of K'/S_s' calculated for different selected times. The average value calculated for each z should represent the diffusivity of that part of the confining layer between the top of the aquifer and that particular elevation.

Advantages

As was noted before, if the aquifer received leakage from both above and below, then r/B and β methods, which relied on the measurement of drawdowns in the aquifer alone, failed to lend themselves to calculation of the hydraulic conductivities of the confining layers. The ratio method, on the other hand, can provide a means of calculating diffusivities of both upper and lower confining beds.

Uncertainties

- The ratio method can only lead to the calculation of the vertical diffusivity of the confining beds. If one can calculate the specific storage by other means, then the vertical hydraulic conductivity of those layers may be computed. Leahy (1976) has used the following approach to overcome the above difficulty. He used Hantush's (1960) β solution to find the product of K' and S_s', and the Witherspoon and Neuman (1967) ratio method to find the ratio of K'/S_s'. Then, he calculated the value of K' from

$$K' = \sqrt{\left(\frac{K'}{S_s'}\right) \cdot (K' \cdot S_s')} \qquad (3\text{-}48)$$

- The method is based on the assumption that the hydraulic head remains constant at the top of the confining bed. Depending on the thickness and the hydraulic properties of the aquitard, this may or may not cause errors in the result. If the aquitard is thin with a small storage coefficient, the transient effect may completely penetrate it at relatively early stages of the pump test.
- Wolff (1970) reported that piezometers completed in the aquitard exhibit reverse water-level fluctuations, in that water levels rise for some period of time after the start of pumping from the aquifer. He relates these changes to radial and vertical deformation of the aquifer and aquitard resulting from their compressibility. Because

the ratio method does not take such phenomena into account, Weeks (1977) warns the investigators against application of this method. This phenomenon has not been observed in other tests such as the ones reported by Leahy (1977), and Neuman and Witherspoon (1972).

4.0 STORAGE COEFFICIENT

The storage coefficient or the storativity S of a saturated confined geological bed of thickness b is defined as the volume of water that the bed releases from storage per unit surface area of the bed per unit decline in the component of hydraulic head normal to that surface. This term has been commonly defined for aquifers (Freeze and Cherry, 1979, Hantush, 1964). However, storativity has been generally used for aquitards and aquicludes as well. Some authors have used the term storativity for both confined and unconfined aquifers (USBR, 1977).

Note that in the above definition it is inherent that the hydraulic head is the same through the total thickness of the bed. This may not be a valid assumption for cases in which hydraulic head varies with elevation. Consequently, Hantush (1964) has used the term average hydraulic head in the definition of the storage coefficient in order to overcome the above problem. Therefore, a more accurate term to use is the specific storage. The specific storage S_s of a saturated confined geological bed is defined as the volume of water that a unit volume of that bed releases from storage under a unit decline in hydraulic head. For cases where the hydraulic head remains constant throughout the total thickness b of the bed, then the following relation holds

$$S = bS_s \qquad (3\text{-}49)$$

The storativity and the specific storage are scalar parameters. They could be space dependent, but they are independent of direction.

A decrease in hydraulic head leads to a decrease in fluid pressure and an increase in effective stress. Therefore, the volume of water that is released from storage due to decreasing the hydraulic head h is produced by two mechanisms: (1) the expansion of the water caused by decreasing the pore water pressure, and (2) the compaction of the skeleton of the medium caused by increasing the effective stress. The expansion of the water is controlled by its compressibility β and the compaction of the medium by the matrix compressibility α. Therefore, it can be shown that the specific storage S_s is given by

$$S_s = \rho g(\alpha + \phi\beta) \tag{3-50}$$

where

ρ = density of the water

g = acceleration of gravity

φ = porosity of the medium.

Equations (3-49) and (3-50) indicate that S_s has the dimension of $[L]^{-1}$ and S is dimensionless.

4.1 Methods of Measurement

In general, methods of in situ measurement of the storage coefficient fall into two categories: (1) methods which are based on well testing of aquifers, and (2) techniques which rely on the change of barometric pressure and earth tides. In addition, some indirect methods such as measurement of subsidence and consolidation have been used to obtain a rough estimate of the storativity of the shallower unconsolidated materials.

Storativity of an aquifer can be easily determined by the common pump test techniques. Some of these tests were discussed in previous sections. Unfortunately, the suitability of the current pump test techniques to determine the storativity of less pervious confining beds is questionable. As we discussed before, the original leaky aquifer theory (r/B method) simply ignores the storage capacity of the aquitard. The more recent theories such as the β solution of Hantush (1960) and the ratio method of Witherspoon and Neuman (1967) cannot single out the storativity of the aquitards. The Hantush solution could at best give the product of the specific storage and the hydraulic conductivity of the aquitard, and the ratio method yields the diffusivity of the aquitard. In fact, we noticed that calculation of hydraulic conductivity was only possible if one could obtain the storativity from other sources.

Application of the β solution of Hantush combined with the ratio method of Witherspoon and Neuman has been reported (Leahy, 1977) to yield a value for the storativity of the aquitard. However, despite the fact that the β method cannot differentiate properties of the two confining layers, above and below the aquifer, the procedure used by Leahy is suitable for cases where one is certain that leakage into the aquifer is from only one of the confining beds.

In regard to single well tests, the modified Burns' method as described in the previous section should give a value for the storativity of the layer being tested. Recalling the limitations of that test, the estimated value of storativity belongs to the materials very close to the well. Other single well tests are either unable to give S_s or, if they do, the reliability of the calculated value is questionable (Papadopolus et al., 1973).

Barometric efficiency of a well can also be used to find the storage coefficient of a confined aquifer (Jacob, 1940)

$$S = \frac{\phi \gamma b}{E_w B}$$

where

S = storage coefficient

ϕ = porosity

b = aquifer thickness

E_w = bulk modulus of elasticity of water

B = barometric efficienty

γ = specific weight of water

Fluctuation of the water level in a well due to the earth tide has occasionally been used to estimate the storativity of deep confined aquifers (Kanehiro and Narasimhan, 1980). However, because of uncertainties in estimating input data this method is not commonly applied in the field.

5.0 HYDRAULIC HEAD

As we saw in Section 1.4, an important parameter which controls the movement of groundwater is the hydraulic gradient. Distribution of hydraulic head within a given hydrologic system is generally controlled by the conditions at the boundaries of the system and the properties of the media.

The potential Φ of a given fluid at any point in space is generally defined as the mechanical energy per unit mass of the fluid, which has three components

$$\Phi = gz + \frac{v^2}{2} + \int_{P_o}^{P} \frac{dP}{\rho} \qquad (5\text{-}1)$$

where

g = gravitational acceleration

z = elevation of the point above datum

v = velocity of fluid

p = pore water pressure at the point

ρ = density of fluid

P_o = atmospheric pressure

The potential Φ is the amount of work required to bring a unit mass of fluid from an arbitrary standard state to the point under consideration. The standard state is usually considered at elevation $z=0$, velocity $v=0$, and pressure $P=P_o$ atmospheric pressure. The first term of the right hand side of Equation (5-1) represents the work necessary to bring a unit mass of fluid from the standard position to the elevation z. The second term is the work required to increase the dynamic energy of the unit mass from zero to $v^2/2$. Finally, the third term is the work required to bring the pressure of the fluid from P_o to P.

For the case of flow through porous media, where velocity is generally very small, the term $v^2/2$ may be ignored with respect to the other terms. In case of incompressible fluids, where ρ is not a function of pressure, the third term may also be simplified and equation (5-1) becomes

$$\Phi = gz + \frac{P-P_o}{\rho} \tag{5-2}$$

One may note that in some cases, such as fracture flow close to a well or a shaft where fluid velocity is relatively large, the term $v^2/2$ may not be so small as to be negligible.

If we refer to P as gauge pressure, then the atmospheric pressure may be set equal to zero and the expression for potential becomes

$$\Phi = gz + \frac{P}{\rho} \tag{5-3}$$

A term which is commonly used in groundwater hydrology is hydraulic head which is the enrgy per unit weight of the fluid. Therefore,

$$h = \frac{\Phi}{g} = z + \frac{P}{\rho g} \tag{5-4}$$

For a homogeneous fluid with constant ρ, Darcy's law shows that fluid flows from regions of higher heads toward regions of lower heads, and that the flow velocity is proportional to the gradient of the hydraulic head. However, if we have more than one type of fluid or ρ changes from one aquifer to the other, which could occur because of changes of temperature or salt concentration, then at each point in space one can define as many potentials as there are densities (Hubbert, 1940). For example, if we have three different densities such as ρ_1, ρ_2, and ρ_3, then at any point in the space with elevation z and pore pressure P, regardless of which fluid occupies that space, we can write

$$\Phi_1 = gz + \frac{P}{\rho_1} \tag{5-5}$$

$$\Phi_2 = gz + \frac{P}{\rho_2} \tag{5-6}$$

$$\Phi_3 = gz + \frac{P}{\rho_3} \tag{5-7}$$

In this case, according to Hubbert (1940), motion of fluid i with the density ρ_i should be solely studied by the distribution of its own potential Φ_i or $h_i = \Phi_i/g$. Let us emphasize that potential Φ_i based on the density ρ_i is defined everywhere in the space including the space occupied by fluids of other densities. This concept is very important when we are investigating flow between two aquifers of differing salinity separated by some semipermeable layer. To make this point clear let us consider the following example.

A look at Fig. 5-1 without attention to the quality of water of two aquifers makes one think that there is a drop of hydraulic head from the freshwater aquifer downward to the saline water, and thus flow is downward. However, the following calculations show that flow is actually occurring upward from the saline aquifer towards the fresh-water aquifer.

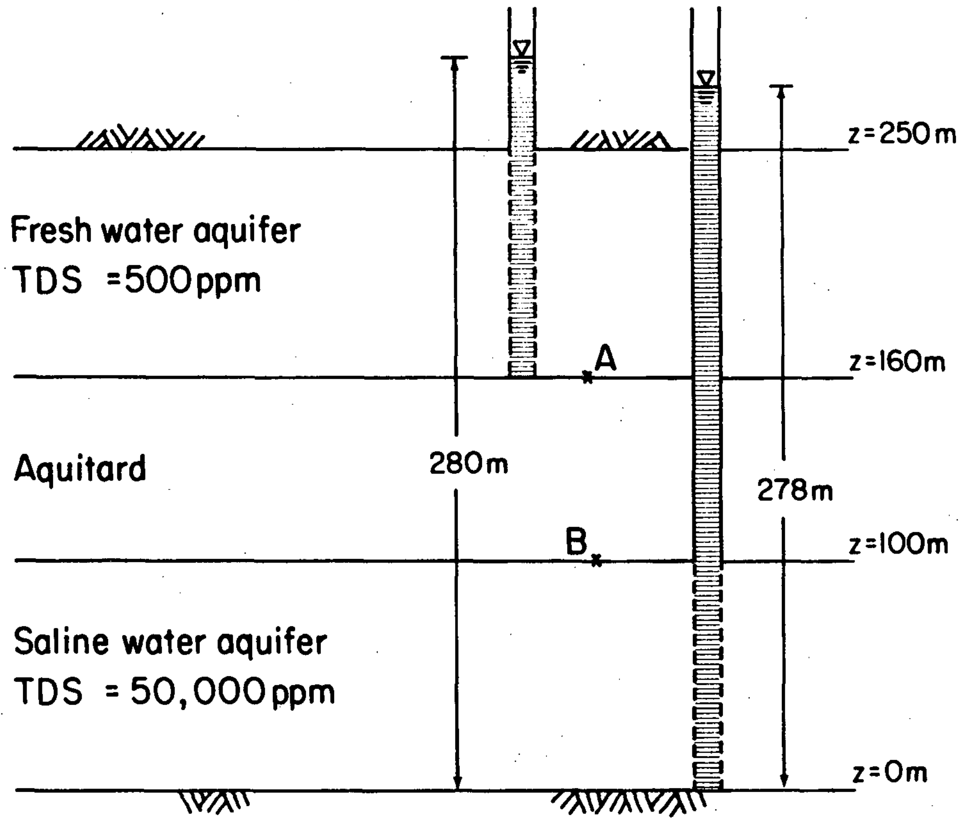

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Fig. 5-1 Schematic diagram showing two observation wells, one open in the top fresh-water aquifer and the other screened in the lower saline aquifer.

The values of potential of fresh water at points A and B, top and bottom of the aquitard, are

$$\Phi_{f_A} = 160\ g + \frac{120\,\rho_f g}{\rho_f} = 280\ g \tag{5-8}$$

$$\Phi_{f_B} = 100\ g + \frac{178\,\rho_s g}{\rho_f} = 284.5\ g \tag{5-9}$$

It is now apparent that the fresh-water potential at point B is larger than A, thus causing an upward flow from B to A. Assuming a linear variation of potential between A and B, salt-water potential gradient between B and A is

$$\frac{\Phi_{s_B} - \Phi_{s_A}}{60} = \frac{(278 - 275.77)}{60}\ g = 0.0371\ g$$

The vertical velocity component is upward with the magnitude of

$$v_z = -\frac{k_z}{\mu_s}\,\rho_s\,\frac{\partial\Phi}{\partial z} = -\frac{k_z}{\mu_s}\,(0.0385\ g\rho_f) \tag{5-10}$$

In the above calculations density of saline water ρ_s is 1.036 gr/cm^3.

For the study of groundwater movement in a porous medium containing fresh, diffused, and salt water, Lusczynski (1961) has introduced three different types of head at each point i within the medium: fresh-water head h_{if}, point-water head h_{ip}, and environmental-water head h_{in}, which are defined as follows.

Fresh-water head at point i, Fig. 5-2.B, is defined as the height of the water above the datum in a well filled with fresh-water from point i to a level high enough to balance the existing pressure at point i. Based on this definition, fresh-water head at point i may be written as

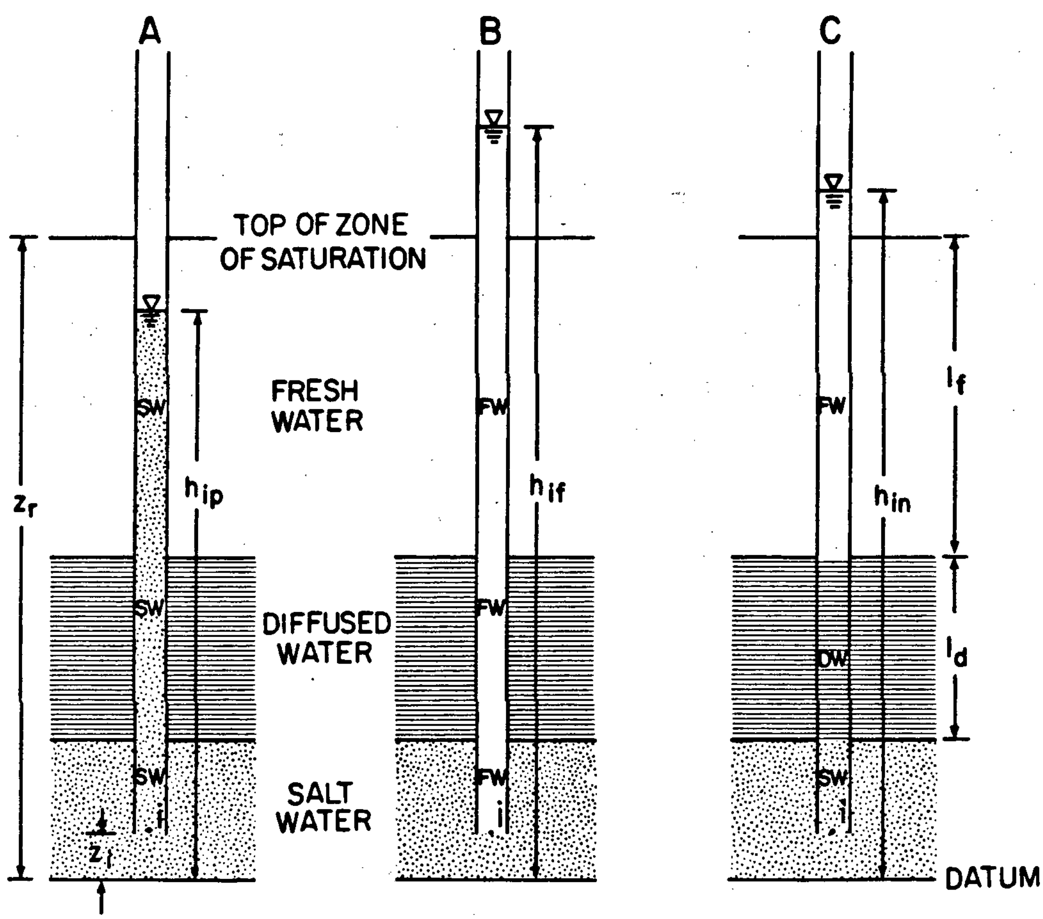

Fig. 5-2 Heads in groundwater of variable density, (A) point-water head, (B) fresh-water head, and (C) environmental head, (modified from Lusczynski, 1961).

$$h_{if} = z_i + \frac{P}{g\rho_f} \tag{5-11}$$

where

h_{if} = fresh-water head at the point i

z_i = elevation of point i

P = pressure at point i

ρ_f = density of fresh-water

As defined above, h_{if} is the energy per unit weight of fresh water at the point i, as was defined by Hubbert (1940).

Point-water head at point i, Fig. 5-2A, in groundwater of variable density, is defined as the water level above the datum in a well filled with water of the type found at point i to balance the existing pressure at point i. From this definition one can write

$$h_{ip} = z_i + \frac{P}{g\rho_i} \tag{5-12}$$

where

h_{ip} = point-water head at point i

ρ_i = density of water at i

Environmental-water head at a given point i, Fig. (5-2C), in groundwater of variable density is defined as the fresh-water head reduced by an amount corresponding to the difference of salt mass in fresh-water and the environmental water between point i and the top of the zone of saturation.

Environmental water between point i and the top of the zone of saturation is herein defined to be the water of constant or variable density occurring in the environment along a vertical line between point i and the top of the zone of saturation.

Based on the above definition, environmental-water head at point i may be written as

$$h_{in} = \frac{P}{\rho_f g} - (z_r - z_i) \frac{\rho_a}{\rho_f} + z_r \tag{5-13}$$

or in terms of fresh-water head one can write

$$h_{in} = h_{if} - (z_r - z_i) \left(\frac{\rho_a}{\rho_f} - 1\right) \tag{5-14}$$

where

h_{in} = environmental head at point i

z_r = vertical distance between datum and top of the zone of of saturation.

ρ_a = average density of water between point i and top of the zone of saturation

$$\rho_a = \frac{1}{z_r - z_i} \int_{z_i}^{z_r} \rho dz \tag{5-15}$$

Lusczynski (1961) states that "fresh-water heads define hydraulic gradients along a horizontal. However, along a vertical environmental-water heads should be used to define the hydraulic gradient". Although along a horizontal the Lusczynski and Hubbert theories match, in a vertical direction their theories lead to different values of gradients. Based on the Lusczynski approach, environmental head at point A of the example on Fig. 5-1 is the same as the fresh-water head which is 280 m. The value of environmental head at point B may be calculated from equations (5-9, 5-14, and 5-15). Assuming that the aquitard is occupied by saline water, the value of ρ_a may be calculated from equation (5-15) to be $1.0146\rho_f$. Substituting for ρ_a and h_{if} in equation (5-14), one obtains the value of h_{in} at B to be 282.31m. Therefore, Lusczynski's approach also gives the direction of flow from B to A. The magnitude of gradient is also the same as that obtained by the Hubbert approach for saline water. If the concentration of water in the aquitard is somewhere between fresh and saline water, then the environmental water head at point B would be larger than 282.31m leading to a gradient different from those obtained by the Hubbert approach.

Lusczynski (1961) has given the following formula for calculation of components of velocity in the horizontal and vertical directions at point i

$$v_h = -\frac{k_h g}{\mu_i}\left(\rho_f \frac{\partial h_{if}}{\partial x}\right) \qquad (5\text{-}16)$$

$$v_z = -\frac{k_z g}{\mu_i}\left(\rho_f \frac{\partial h_{in}}{\partial z}\right) \qquad (5\text{-}17)$$

Some authors believe that in dealing with problems in which density is a function of space, it is more convenient to formulate the groundwater flow equation in terms of pressure rather than head, because pressure head ($P/\rho g$) is dependent on fluid density which in turn is dependent on salt concentration (Anderson, 1979). In terms of pressure Darcy's law at a point i in a groundwater system may be written as (Scheideger, 1960):

$$\vec{V}_i = -\frac{[k]}{\mu_i}\left(\nabla P_i - \rho_i \vec{g}\right) \qquad (5\text{-}18)$$

where

$\vec{V}_i$ = Darcy's velocity vector

∇P_i = gradient of pressure at point i

μ_i = viscosity of fluid at point i

[k] = permeability matrix

ρ_i = density of fluid at point i

$\vec{g}$ = gravity vector

Let us now examine the example in Fig. (5-1) with the approach of equation (5-18). Values of pressure at points A and B are $120\rho_f$ g and $184.5\rho_f$ g, respectively. If we assume a linear variation of pressure between A and B, then the component of pressure gradient in the vertical direction becomes

$$\frac{\partial P}{\partial z} = \left(\frac{184.5-120}{60}\right) g\rho_f = 1.075 g\rho_f \qquad (5\text{-}19)$$

and

$$\frac{\partial P}{\partial z} - \rho_B g = (1.075-1.0365) g\rho_f = 0.0385\ g\rho_f \qquad (5\text{-}20)$$

and the vertical velocity component at the point B is

$$v_{z_B} = -\frac{k_z}{\mu_s}(0.0385\ g\rho_f) \qquad (5\text{-}21)$$

which is exactly the same magnitude as obtained from the Hubbert approach.

The above discussion was based on the gradient of hydraulic head alone. Other types of gradient such as chemical, electrical, and thermal are also effective in moving fluid, even in the absence of any hydraulic head (Philip and de Vries, 1957; Casagrande, 1952). In particular, for a problem such as the above example, where one is dealing with a big contrast of concentration, a certain amount of water moves from the higher concentration zone to the lower one. The law governing this type of motion is called Fick's first law which is

$$F = - D \frac{dC}{dx} \tag{5-22}$$

where

F = mass of solute passing from a unit area per unit time

D = diffusion coefficient

C = concentration of solute

Although the value of D is generally very small, over a long period of time this process could cause a considerable amount of contaminant transport. Note that in the above example the chemical gradient acts in the same direction as the hydraulic head gradient.

Clarification of one point seems to be in order here. A layer of compacted clay restricts the passage of ions while allowing relatively unrestricted passage of neutral species (Freeze and Cherry, 1979). Thus, saline water may not easily move across a compacted clay layer while fresh-water may, if, of course the hydraulic gradient allows.

5.1 Measurement of Hydraulic Head

Hydraulic head at a given point in a geological formation occupied by a fluid may be measured both directly and indirectly. Hydraulic head may be measured directly by a pipe with one end open at the point of interest and the other end open to the atmosphere. This pipe is generally referred to as a piezometer. The elevation of fluid in this pipe at the equilibrium is the hydraulic head at the point of interest where water is allowed to enter the pipe. The end of the pipe which allows water to enter is usually equipped with a small section of slotted pipe or a device called a well point. Hydraulic head may be obtained indirectly by measuring the pore water pressure at any point with the help of a transducer. Commercially available transducers generate a voltage proportional to pressure which can be converted to the actual pressure of the water at the point. The value of pressure and the elevation of the point of measurement may be substituted into equation (5-4) to give the hydraulic head at the point of interest.

Within a single-layer aquifer where flow is essentially horizontal and equipotential lines are vertical (hydraulic head remains constant with depth), water level in an observation well which is screened along all or part of the thickness of the aquifer would give the value of hydraulic head of the aquifer at the position of the well. If for some reason such as stratification of the aquifer, proximity to the zones of recharge or discharge, or change in water quality with depth, hydraulic head varies with depth, then the observation well can only give an average value of head of the aquifer for the screened interval. This head may not be accurate enough for a critical study of groundwater movement.

As we discussed above, an important parameter which one should always measure together with hydraulic head is the density of fluid at the point of measurement. Density varies with temperature and chemical properties. It is recommended that a water sample be taken from the point of interest for chemical analysis. If the medium is occupied by freshwater, i.e. total dissolved solid (TDS) less than 1000 mg/l, one can ignore the effect of density variation, and hydraulic head as defined by equation (5-4) is adequate for calculation of the velocity components of groundwater movement. If, however, TDS is very high and its magnitude changes with the space, then one must consider the density of water at each point where hydraulic head is measured. At 75°F and atmospheric pressure, the relation between NaCl water salinity and water density may be approximated by

$$\rho = 1 + .73\ C \tag{5-22}$$

where C is NaCl concentration in ppm x 10^{-6}. A chart showing variation of water density with temperature and pressure at different Na Cl concentrations is given in page 47 of Schlumberger (1972).

The above methods of hydraulic head measurement are only practical when the formation is reasonably permeable such that height of water within the pipe comes to equilibrium with the formation pressure at the point within a reasonably short period of time. Measurement of hydraulic head in less permeable formations is quite involved. This is because of the long period of time required for water pressure in the pipe to come to equilibrium with the formation pressure. To overcome this difficulty one should pack off the test interval from the rest of the hole to minimize the volume of water needed to be produced by the formation. For further information about installation of piezometers in fine-textured soils and application of inflatable straddle packers for hydrologic testing readers are referred to Johnson (1965) and Shuter and Pemberton (1978), respectively.

6.0 CONCLUSIONS

The evaluation of properties and parameters controlling the vertical component of groundwater movement through the geological materials around a radioactive repository site is an essential task of data base preparation for effective hydrological modeling. This in turn is an essential part of site evaluation. Essentially there are three properties of geologic materials, namely porosity, vertical hydraulic conductivity, and storativity which, together with the gradient of the hydraulic head, control vertical groundwater movement. Determination of these four items in a formation with a relatively high permeability is a routine job of hydrogeologists. In low permeability materials, however, determination of these items is a challenging task.

In this report some of the conventional methods of determining porosity, vertical hydraulic conductivity, storativity, and hydraulic head were described. The following conclusion may be drawn from these descriptions:

6.1 Porosity

It situ effective porosity of geological materials may be determined by either logging or tracer techniques. Logging methods tend to estimate the porosity of a small zone around the well being logged. Thus, unless the medium is homogeneous from a porosity point of view, a large number of wells is required to give a reasonable picture of porosity variation within the formation. On the other hand, the tracer method can determine the effective porosity of a more extensive zone. However, almost all tests of this type, so far have been performed in highly conductive formations. For rocks with a hydraulic conductivity of the order of 10^{-8}cm/sec or less this type of test is practically impossible.

6.2 Hydraulic Conductivity

Vertical hydraulic conductivity of permeable formations can be easily obtained by the analysis of appropriate aquifer pump tests. For less permeable formations, two general types of field tests are available which could estimate vertical hydraulic conductivity. The first includes methods based on single well tests in the low permeability formation itself, while the second includes large scale multiple well pumping tests designed and interpreted based on the various theories of leaky aquifer systems.

The problem inherent in the first type of tests is that the measured hydraulic conductivity is normally only representative of a small zone around the testing interval. Hence, again, a large number of testing wells is required to give a clear picture of the distribution of the vertical hydraulic conductivity in the area of interest.

The most commonly used method among the second type of test is based on an early leaky aquifer solution od Hantush (1956). This solution ignores the storativity of the confining bed. Neuman and Witherspoon (1969) have noted that the application of this method tends to overestimate the hydraulic conductivity of the aquifer and underestimate that of the confining bed. When the confining layer is thin and relatively permeable and incompressible, however, this method could give useful results.

Hantush's (1960) modified method and the "ratio method" of Neuman and Witherspoon (1972) are two other techniques of the second type of tests which under certain circumstances could be used for determination of (KS_s) and (K/S_s), respectively. Unfortunately, neither of these two methods can yield vertical hydraulic conductivity unless the specific storage of the low permeability layer is independently identified. Furthermore, Hantush's method is unable to separately distinguish the contribution of leakage from upper and lower confining beds, thus introducing further difficulties in calculation of the vertical hydraulic conductivity of the individual confining layers.

6.3 Storativity

Storativity of an aquifer can be easily determined by the common pump test techniques. However, the suitability of these pump tests in determining storativity of low permeability confining beds is questionable.

6.4 Hydraulic Head

Measurement of hydraulic head in permeable geological materials is routinely done through observation wells or by piezometers installed at the appropriate location in the well. In very low permeability media, however, measurement of head is quite a challenging job and requires either very long term measurements with piezometers or packing off the measurement zone and applying special transducers.

NOMENCLATURE

a	half the distance between recharge and discharge walls in the tracer test, (L).
B	$\sqrt{\frac{kbb'}{K'}}$, leakage factor, (L).
b,b'	thickness of an aquifer and aquitard, respectively, (L).
C	Concentration of the tracer, (ML^{-3}).
C_o	input concentration of the tracer, (ML^{-3}).
c_w	compressibility of water, (LT^2M^{-1}).
D_m	dispersion constant or dispersivity, (L).
$-Ei(-u)$	$\int_u^\infty \frac{e^{-y}}{y}\,dy = w(u)$, well function.
E_w	bulk modulus of elasticity of water, ($ML^{-1}T^{-2}$).
$erfc(x)$	$1-erf(x) = 1-\frac{2}{\sqrt{\pi}}\int_o^\infty e^{-y^2}dy$, complementary error function.
$\vec{g}$	acceleration of gravity vector, (LT^{-2}).
h	hydraulic head, (L).
K,K'	hydraulic conductivities of an aquifer and aquitard, respectively, (LT^{-1}).
K_r,K_z	Components of hydraulic conductivity in radial and vertical directions, respectively, (LT^{-1}).
k	intrinsic permeability, (L^2).
l	depth of penetration, (L).
M,M'	thickness of an aquifer and aquitard, respectively, (L).
P	Peclet number.

p_D	dimensionless pressure.
Q	rate of pumping, (L^3T^{-1}).
q	pumping rate perunit aquifer thickness, (L^2T^{-1}).
r	radial distance from a pumping well, (L).
r_s	radius of well in the tested interval, (L).
S,S'	Storage coefficient of an aquifer and aquitard, respectively.
S_s,S'_s	specific storage of an aquifer and aquitard respectively, (L^{-1}).
s,s'	drawdowns in an aquifer and aquitard, respectively (L).
s_D	dimensionless drawdown.
T	time for a water particle to travel along a particular streamline between two wells, (T).
t	time, (T).
t_D	dimensionless time.
u	$\frac{r^2S_s}{4tK}$.
$\vec{V}$	vector of apparent or Darcy's velocity, (LT^{-1}).
V_f	sonic wave velocity of fluid filling the pores, (LT^{-1}).
v_x,v_y,v_z	Components of apparent velocity vector, (LT^{-1}).
$\vec{v}$	vector of seepage velocity, (LT^{-1}).
$W(u,r/B)$	well function of leaky aquifers.
x,y,z	coordinate system.
α	effective porosity.
γ	specific weight of water, $(ML^{-2}T^{-2})$.
Φ	hydraulic potential, (L^2T^{-2}).
ϕ	porosity
ρ	density of fluid, (ML^{-3}).

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