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Journal
Physical Review D, 45(5)

ISSN
2470-0010

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Publication Date
1992-03-01

DOI
10.1103/physrevd.45.1800

Z-mediated B-$\bar{B}$ mixing and B-meson CP-violating asymmetries in the light of new flavor-changing neutral-current bounds

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(Received 24 June 1991)

We impose new limits on flavor-changing neutral currents (FCNC's) applied to the 4×4 mixing matrix of the next heaviest down quark in a model with extra $SU(2)_L$ singlet down quarks, such as in E_6. We find the new bounds still allow tree-level FCNC Z exchange to dominate B_d^0-$\bar{B}_d^0$ mixing, but not B_s^0- $\bar{B}_s^0$ mixing. A stronger $b\to d$ bound by a factor of 1/10 could rule out the possibility of Z mediation with FCNC accounting for B_d^0-$\bar{B}_d^0$ mixing. The unitarity "d-b triangle" may still be a quadrangle with a detectable fourth side. The CP asymmetries from B_d^0 decay should show a new CP-violating phase if the Z-mediated process dominates B_d^0-$\bar{B}_d^0$ mixing.

PACS number(s): 14.40.Jz, 11.30.Er, 12.15.Mm, 13.20.Jf

I. INTRODUCTION

The new limits on $b\to s$ and $b\to d$ flavor-changing neutral currents (FCNC's) from UA1 and on $s\to d$ from new limits on $K_L\to\mu\mu$ are used to explore consequences for mixing models with new down quarks, which are $SU(2)_L$ singlets. These models have been explored before [1] and include three E_6 generations, each of which contain such a down quark. This mixing is truncated to the standard down quarks mixing only with the least massive new down quark in a 4×4 mixing matrix. The resulting bounds on FCNC's are compared with the Z-mediated rates for B_d^0- $\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ mixing to show that such tree-level mixing can still dominate B_d^0-$\bar{B}_d^0$ mixing over the standard-model box diagrams, but the Z-mediated process is bounded at less than half the amplitude for B_s^0- $\bar{B}_s^0$ mixing. The FCNC limits are then applied to the matrix elements of the 4×4 mixing matrix to find the limits on the mixing angles. If the Z-mediated FCNC dominates B_d^0-$\bar{B}_d^0$ mixing, we can show that the CP-violating asymmetries in B_d^0 meson decays must be due to new mixing angles and phases of the 4×4 mixing matrix. We also reevaluate the effects these FCNC and 4×4 unitarity can have in unitarity quadrangles that replace the 3×3 unitarity triangles. Throughout we take all data and bounds at 90% confidence limits (or 1.64σ), unless it is an absolute bound. This is a follow-up to the work done on this subject with Nir [2, 3].

II. FLAVOR-CHANGING NEUTRAL CURRENTS FROM MIXING

Mixing in a fourth down quark which is an $SU(2)_L$ singlet, using the 4×4 matrix V which diagonalizes the down quarks to the mass eigenstates, and using a basis where the three up-quark states are the mass eigenstates, the charged current interactions are [2]

$$\mathcal{L}=\frac{g}{\sqrt{2}}(W_\mu^- J^{\mu +}+W_\mu^+ J^{\mu -}), \tag{1}$$

$$J^{\mu -}=V_{ij}\bar{u}_{iL}\gamma^\mu d_{jL}. \tag{2}$$

Here the 3×4 submatrix of V couples the three up quarks to the four down quarks. The 3×3 submatrix of V for $i,j=1,2,3$ is the standard Cabibbo-Kobayashi-Maskawa (CKM) matrix.

The flavor-changing neutral currents are given by the mixings to the fourth down quark by [2]

$$-U_{ij}\equiv V_{4i}^* V_{4j}\quad \text{for } i\neq j. \tag{3}$$

The FCNC couplings of the down quarks to the Z are then given by [2]

$$\mathcal{L}^Z_{\text{FCNC}}=-\frac{e}{2\sin\theta_W\cos\theta_W}U_{ij}\bar{d}_{iL}\gamma^\mu d_{jL}Z_\mu. \tag{4}$$

III. THE BOUND FROM $B\to\mu\mu X$

The Z-mediated process forms μ pairs from the FCNC $b\to d$ and $b\to s$ with amplitudes U_{db} and U_{sb}, respectively. The branching ratio of this to the W-mediated semileptonic decay is

$$\frac{B(B\to\mu\mu X)}{B(B\to\mu\nu X)}=\left[(\tfrac{1}{2}-\sin^2\theta_W)^2+(\sin^2\theta_W)^2\right]\times\frac{|U_{db}|^2+|U_{sb}|^2}{|V_{ub}|^2+F_{ps}|V_{cb}|^2}, \tag{5}$$

where $F_{ps}\simeq 0.5$.

The new bound from UA1 is [4]

$$B(B\to\mu\mu X)<5\times 10^{-5}, \tag{6}$$

which when scaled by the branching ratio for the W-mediated decay to $\mu\nu$ bounds the ratios [3]

$$\left|\frac{U_{db}}{V_{cb}}\right|<0.041,\quad \left|\frac{U_{sb}}{V_{cb}}\right|<0.041. \tag{7}$$

This is an improvement by a factor of 5 on the $|U_{qb}|$ from the previous bounds from this process. With the values $|V_{cb}|=0.044\pm 0.014$, the range of the bounds on U_{db} and U_{sb} are

$$|U_{db}|\leq 0.0018\pm 0.0006,\quad |U_{sb}|\leq 0.0018\pm 0.0006. \tag{8}$$

IV. B-$\bar{B}$ MIXING

Mixing may occur by the b-$\bar{d}$ quarks in a $\bar{B}$ annihilating to a virtual Z through a FCNC with amplitude U_{db}, and the Z then creates $\bar{b}$-d quarks through another FCNC, and this then becomes a B meson. The mixing for this is given by

$$(x_d)_Z = \frac{\sqrt{2}}{6} G_F B_B f_B^2 M_B \eta \tau_b |U_{db}|^2. \tag{9}$$

The experimental and calculated parameters are (extended to 90% C.L.)

$$x_d = 0.66 \pm 0.18, \tag{10}$$

$$\tau_b |V_{cb}|^2 = (3.5 \pm 1.0) \times 10^9 \text{ GeV}^{-1}, \tag{11}$$

$$\sqrt{B_B} f_B = 0.15 \pm 0.08 \text{ GeV}. \tag{12}$$

Requiring that this account for the observed value of B_d^0-$\bar{B}_d^0$ mixing would result in

$$0.013 < \left| \frac{U_{db}}{V_{cb}} \right| < 0.045. \tag{13}$$

Thus the B_d^0-$\bar{B}_d^0$ mixing by Z-mediated FCNC's is still allowed by the new bound on $B \to \mu\mu X$, Eq. (7). The upper limit on U_{db} obtained from B_d^0-$\bar{B}_d^0$ mixing [1, 3] is of the same order as the new bound.

V. COMPARISON WITH BOX DIAGRAMS FOR MIXING

The standard-model box diagrams for meson mixing have net flavor-changing processes with two charged currents dominated by couplings to the top quark because of its large mass. For the Z mediated FCNC to dominate over the standard model amplitude for B-$\bar{B}$ mixing requires [3]

$$|U_{db}/(V_{td}^* V_{tb})| \geq 0.07 \text{ to } 0.14, \tag{14}$$

$$|U_{sb}/(V_{ts}^* V_{tb})| \geq 0.07 \text{ to } 0.14, \tag{15}$$

where the lower value for the limit is for $m_t = 90$ GeV and the upper value is for $m_t = 200$ GeV.

For B_d^0-$\bar{B}_d^0$ mixing, Z-mediated dominance requires

$$|U_{db}| \geq 0.07 |V_{td}^*|, \tag{16}$$

and with $|V_{td}|$ between 0.003 and 0.02 requires $|U_{db}|$ greater than 0.00021 to 0.0014. With the UA1 bound of Eq. (8) on U_{db} this range is allowed.

For B_s^0-$\bar{B}_s^0$ mixing to be dominantly Z mediated requires

$$\left| \frac{U_{sb}}{V_{ts}^* V_{tb}} \right| \geq 0.07. \tag{17}$$

With $V_{tb} \cong 1$ and from the s-b unitarity triangle relation $V_{ts}^* \cong -V_{cb}$ the above requirement becomes

$$\left| \frac{U_{sb}}{V_{cb}} \right| \geq 0.07. \tag{18}$$

This violates the UA1 upper bound of 0.041 on this ratio, Eq. (7), by a factor of 2. However, Z-mediated mixing could still interfere with the box diagrams and significantly affect the phase.

VI. FCNC BOUNDS FROM K-MESON PHYSICS

The new BNL result [5] for $K_L \to \mu\mu$ is (with 90% C.L. limits)

$$B(K_L \to \mu\mu) = (7.0 \pm 0.82) \times 10^{-9}. \tag{19}$$

The result from the 2γ intermediate state is [6] $(6.83 \pm 0.46) \times 10^{-9}$. The 2γ absorptive contribution is purely imaginary, and the FCNC Z mediated decays sum to contribute only to the real part. Bounding the difference of the rates as being due to the real part gives

$$|A_R|^2 = (0.17 \pm 0.94) \times 10^{-9}. \tag{20}$$

The relation to the FCNC amplitude is [3]

$$|A_R|^2 = 13.7 [\text{Re}(U_{ds})]^2. \tag{21}$$

Using the difference plus error as the maximum bound on $|A_R|^2 \leq 1.11 \times 10^{-9}$ gives

$$|\text{Re}(U_{ds})| \leq 0.90 \times 10^{-5}. \tag{22}$$

The Z-mediated FCNC also contribute to the neutral-K-meson mass difference. Requiring $(\Delta M_K)_Z \leq \Delta M_K$, the bound from this is [3]

$$|\text{Re}[(U_{ds})^2]| \leq 4.1 \times 10^{-7}. \tag{23}$$

The Z-mediated diagram also contribute to ϵ giving the bound [3]

$$|\text{Im}[(U_{ds})^2]| \leq 2.6 \times 10^{-9}. \tag{24}$$

Putting together the above two bounds we find, for the U_{ds} parts,

$$|\text{Re}(U_{ds})| \leq 0.90 \times 10^{-5}, \tag{25}$$

$$|\text{Im}(U_{ds})| \leq 0.64 \times 10^{-3}, \tag{26}$$

$$|U_{ds}| \leq 0.64 \times 10^{-3}. \tag{27}$$

VII. BOUNDS ON MIXING ANGLES

All cosine terms in the 4×4 mixing matrix are near 1. The leading terms in the 3×4 part of V, in terms of the sines of the mixing angles are [7]

$$V = \begin{pmatrix} 1 & s_{12} & s_{13} e^{-i\delta_{13}} & s_{14} e^{-i\delta_{14}} \\ -s_{12} & 1 & s_{23} & s_{24} e^{-i\delta_{24}} \\ (s_{12} s_{23} - s_{13} e^{i\delta_{13}}) & -s_{23} & 1 & s_{34} \end{pmatrix}. \tag{28}$$

The complex conjugates of the couplings to the fourth down quark are

$$V_{4d}^* = -s_{14}e^{-i\delta_{14}} + s_{24}e^{-i\delta_{24}}s_{12} - s_{34}(s_{12}s_{23} - s_{13}e^{-i\delta_{13}}), \tag{29}$$
$$V_{4s}^* = -s_{14}e^{-i\delta_{14}}s_{12} - s_{24}e^{-i\delta_{24}} + s_{34}(s_{23} + s_{12}s_{13}e^{-i\delta_{13}}), \tag{30}$$
$$V_{4b}^* = -s_{34} - s_{24}e^{-i\delta_{24}}s_{23} - s_{14}e^{-i\delta_{14}}s_{13}e^{i\delta_{13}}. \tag{31}$$

Using the bound on $\mathrm{Re}(U_{ds})$ above, for the four-quark mixing form $-U_{ij} = V_{4i}^* V_{4j}$, we get

$$|\mathrm{Re}(V_{4d}^* V_{4s})| \leq 0.90 \times 10^{-5}. \tag{32}$$

We now insert the forms for the V_{4d}^* and V_{4s} in the above and keep the quadratic terms in each of the new mixing angles $s_{34}, s_{24}e^{-i\delta_{24}}$, and $s_{14}e^{-i\delta_{14}}$ and use the above bound to bound each mixing angle separately. For $s_{24}e^{-i\delta_{24}}$ and $s_{14}e^{-i\delta_{14}}$ these give

$$|s_{24}e^{-i\delta_{24}}| \leq 0.0064, \tag{33}$$
$$|s_{14}e^{-i\delta_{14}}| \leq 0.0064. \tag{34}$$

For s_{34} we note that its coefficient in V_{4d} is $-V_{td}$ and that its coefficient in V_{4s} is mainly $s_{23} = V_{cb}$. The bound becomes

$$s_{34}^2 \leq \frac{0.90 \times 10^{-5}}{|V_{cb}\mathrm{Re}(V_{td})|}. \tag{35}$$

Using the Particle Data Table 90% C.L. lower bound for $|V_{td}|$ of 0.003 and the lower bound $|V_{cb}| = 0.030$ gives

$$|s_{34}| \leq 0.32. \tag{36}$$

The bounds on $b \to d$ and $b \to s$ do not give better bounds on $s_{24}e^{-i\delta_{24}}$ and $s_{14}e^{-i\delta_{14}}$, but the $b \to s$ bound, Eq. (7), gives

$$|s_{34}| \leq 0.20, \tag{37}$$

independent of the value of $V_{cb} = s_{23}$, which cancels out in the ratio.

We can now form separate limits on the mixing elements V_{4q} based on the above limits on the mixing angles. Adding possible contributions by assuming no cancellations gives

$$|V_{4d}| \leq 0.012, \tag{38}$$
$$|V_{4s}| \leq 0.019, \tag{39}$$
$$|V_{4b}| \leq 0.20. \tag{40}$$

Substituting these back into $-U_{ij} = V_{4i}^* V_{4j}$ we can look for improvements. We find

$$|U_{ds}| \leq 0.23 \times 10^{-3}, \tag{41}$$

which is better than Eqs. (27) and (26) on the magnitude and imaginary part, but does not improve Eq. (25) on the real part. On U_{sb} the product $|V_{4s}||V_{4b}|$ gives the bound 0.0038, which is not as good as the direct bound in Eq. (8). On U_{db} the product $|V_{4d}||V_{4b}|$ gives the bound 0.0024 which is the same as in Eq. (8).

VIII. CORRECTIONS TO CKM UNITARITY TRIANGLES

The unitarity of the 4×4 mixing matrix requires orthogonality of the different rows. Instead of the three terms in the CKM orthogonality relation, which give a triangle in the complex plane, we now have four terms which give a quadrangle. The fourth terms are the $U_{ij} = -V_{4i}^* V_{4j}$ and the orthogonality relations are

$$U_{ds} = V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts}, \tag{42}$$
$$U_{sb} = V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb}, \tag{43}$$
$$U_{db} = V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb}. \tag{44}$$

In the U_{ds} equation, the first two terms on the right are of magnitude 0.22 while the third term is ≤ 0.0012. The magnitude of U_{ds} is $\leq 2 \times 10^{-4}$, so to one part in 200 the first two terms simply cancel each other.

In the U_{sb} equation, the new bound of $0.04V_{cb}$ is to be compared with the middle term of V_{cb}. Thus the correction to the cancellation of the second and third terms can only be as large as 0.04 of either of them. In the complex plane, this is now an almost closed triangle.

In the U_{db} equation, however, the U_{db} bounded by $0.04V_{cb}$ can be compared to the middle or "base" term of $0.22V_{cb}$. Thus the quadrangle is not yet ruled out here, with the fourth side possibly as large as 20% of the base.

IX. PHASES IN B_d^0-$\bar{B}_d^0$ CP-VIOLATING DECAY ASYMMETRIES

The CP-violating decay asymmetries [8] rely on a relative phase between the B_d^0-$\bar{B}_d^0$ mixing and the b-quark decay amplitudes into final states of definite CP. Since we have found that B_d^0-$\bar{B}_d^0$ mixing may be dominated by Z-mediated FCNC processes, the phases of U_{db} could be the important ones. To leading order in s_{34}, $s_{24}e^{-i\delta_{24}}$, and $s_{14}e^{-i\delta_{14}}$ we have

$$U_{db} = -s_{34}(s_{34}V_{td}^* + s_{14}e^{-i\delta_{14}} - s_{24}e^{-i\delta_{24}}s_{12}). \tag{45}$$

The first term has the same phase as in the standard model where the box diagram with the t-quark dominates the mixing. This term is bounded by 0.0008. The term with $s_{24}e^{-i\delta_{24}}$ is bounded by 0.0003. The term with $s_{14}e^{-i\delta_{14}}$ has the largest bound of 0.0013. The sum gives the bounded value of 0.0024 to U_{db} . Thus if the FCNC dominates B_d^0-$\bar{B}_d^0$ mixing, it may do so by involving the new phase in $s_{14}e^{-i\delta_{14}}$, due to the 4×4 or larger nature of the down-quark mixing matrix.

Using the lowest bound for $m_t = 90$ GeV Eq. (14), we see U_{db} need only be as large as $0.07|V_{td}|$ to dominate over the box diagrams in B_d^0-$\bar{B}_d^0$ mixing. Since the term in $s_{34}^2 V_{td}^*$ in U_{db} could only be as large as $0.04|V_{td}|$ this term could not account for Z mediation dominating over the box diagrams in B_d^0-$\bar{B}_d^0$ mixing. If Z mediation does dominate, it must then be from $s_{14}e^{-i\delta_{14}}$. We thus expect a new phase to the CP asymmetries if Z-mediated diagrams dominate B_d^0-$\bar{B}_d^0$ mixing.

In Fig. 1 we show the relevant bounds for $|U_{db}|$ and the

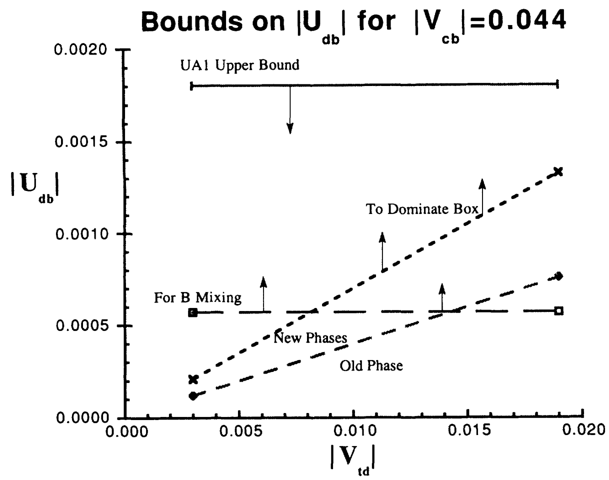

FIG. 1. Bounds on the flavor-changing amplitude $|U_{db}|$ versus the magnitude $|V_{td}|$ for the case $|V_{cb}| = 0.044$. The upper bound (solid) is from the new UA1 limit on $|U_{db}|$. The dashed horizontal line is the lower bound if the Z-mediated FCNC diagram is to be large enough to account for B_d^0-$\bar{B}_d^0$ mixing. The upper diagonal line is the lower bound for the Z-mediated FCNC to dominate the standard-model box diagram. The lower diagonal line is the upper bound on the first term in U_{db} in Eq. (45). This term has the same phase as the standard-model box diagram.

bound on the first term in $|U_{db}|$ in Eq. (45), which has the standard-model phase. The bounds are plotted for the case $|V_{cb}| = 0.044$, its central value. The top horizontal line is the upper bound for $|U_{db}|$ from the UA1 FCNC experiment in Eq. (8). The lower horizontal line (dashed) is the lower limit for $|U_{db}|$ if it is to be large enough to account for B_d^0-$\bar{B}_d^0$ mixing, from Eq. (13). The top diagonal line is the lower limit of U_{db} for Z-mediated FCNC to dominate the standard-model box diagram. Since the box process is proportional to V_{td}^2 and the Z-mediated one to U_{db}^2, the bound for Z mediation to dominate is a straight line. The lowest experimental limit for $m_t = 90$ GeV is used here. The limit would increase upward by a factor of 2 if $m_t = 200$ GeV. The lower diagonal line is the upper bound on the first term in U_{db} in Eq. (45) which contains the same 3×3 phase of V_{td}. If U_{db} is above this line it must be from contributions from the second and/or third terms in Eq. (45) which bring in one or both of the new phases from the 4×4 mixing matrix. For $|V_{cb}|$ equal to the upper or lower limits, 0.58 or 0.30, both horizontal curves would shift up or down, respectively, by 32%.

X. EFFECTS OF IMPROVEMENT OF $B \to D$ BOUND BY 1/10

An experiment at Fermilab may improve the bound on $B \to \mu\mu X$ by a factor of 1/10 that in Eq. (6), namely, to

$$B(B \to \mu\mu X) \le 5 \times 10^{-6}. \tag{46}$$

This bounds the FCNC amplitude at 90% C.L.:

$$\left|\frac{U_{db}}{V_{db}}\right| \le 0.013. \tag{47}$$

Comparing this to the requirement for B_d^0-$\bar{B}_d^0$ mixing to be accounted for by Z-mediated FCNC in Eq. (7), which is that the above ratio be greater than 0.013 at 90% C.L., we would have a clear discrepancy. By combining the two offsets at a 90% C.L. or 1.64σ each, we would get a 2.32σ discrepancy, and the Z mediation would be ruled out at the 98% C.L. In Fig. 1 the upper bound line (solid) would be lowered to $|U_{db}| \le 0.006$, or coincident with the lower bound for B mixing by FCNC. Although the figure is shown only for the central value of $|V_{cb}|$, the above conclusion would be valid independent of its value since both bounds are scaled by it.

The improved bound would also improve the bound on s_{34} in Eq. (37), giving

$$|s_{34}| \le 0.11, \tag{48}$$

which would limit the standard model phase term in Eq. (45) to $0.01V_{td}^*$.

XI. CONCLUSIONS

We have used the newest bounds on $b \to d$, $b \to s$, and $s \to d$ FCNC to restrict the values for possible Z-mediated contributions to B-$\bar{B}$ mixing. We found that these processes can now be shown not to dominate B_s^0-$\bar{B}_s^0$ mixing, but are still just allowed for B_d^0-$\bar{B}_d^0$ mixing. We have set limits on the mixing angles in a 4×4 down-quark mixing matrix. These show that new phases from the 4×4 matrix should contribute to the CP-violating B_d^0 meson decay asymmetries if the Z-mediated processes dominate B_d^0-$\bar{B}_d^0$ mixing.

ACKNOWLEDGMENTS

We wish to thank D. Cline, W. Molzon, K. Ankoviak, and A. Heinson for discussions about the experimental results.

[1] M. Shin, M. Bander, and D. Silverman, Phys. Lett. B **219**, 381 (1989)
[2] Y. Nir and D. Silverman, Nucl. Phys. **B345**, 301 (1990).
[3] Y. Nir and D. Silverman, Phys. Rev. D **42**, 1477 (1990).
[4] C. Albajar *et al.*, Phys. Lett. B **262**, 163 (1991); K. Ankoviak, *Rare and Exclusive B and K Decays and Novel Flavor Factories*, Los Angeles, edited by D. Cline (AIP, New York, in press).
[5] A. Heinson *et al.*, Phys. Rev. D **44**, 1 (1991).
[6] C. Q. Geng and J. N. Ng, Phys. Rev. D **41**, 2351 (1990).
[7] F. J. Botella and L. L. Chau, Phys. Lett. **168B**, 97 (1986); H. Harari and M. Leuler, Phys. Lett. B **181**, 123 (1986).
[8] A. B. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980); Phys. Rev. D **23**, 1567 (1981); I. I. Bigi and A. I. Sanda, Nucl. Phys. **B193**, 85 (1981); **B281**, 41 (1987).