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**Z-mediated B̄-B mixing and B-meson CP-violating asymmetries in the light of new flavor-changing neutral-current bounds**

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We impose new limits on flavor-changing neutral currents (FCNC’s) applied to the 4 × 4 mixing matrix of the next heaviest down quark in a model with extra SU(2)_L singlet down quarks, such as in Eq. 1. We find the new bounds still allow tree-level FCNC Z exchange to dominate B^0_d-B^0_s^+ mixing, but not B^0_u-B^0_d^+ mixing. A stronger b → d bound by a factor of 1/10 could rule out the possibility of Z mediation with FCNC accounting for B^0_d-B^0_u mixing. The unitarity “d-b triangle” may still be a quadrangle with a detectable fourth side. The CP asymmetries from B^0_d decay should show a new CP-violating phase if the Z-mediated process dominates B^0_d-B^0_u mixing.

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**I. INTRODUCTION**

The new limits on b → s and b → d flavor-changing neutral currents (FCNC’s) from UA1 and on s → d from new limits on K^- → μν are used to explore consequences for mixing models with new down quarks, which are SU(2)_L singlets. These models have been explored before [1] and include three E6 generations, each of which contain such a down quark. This mixing is truncated to the standard down quarks mixing only with the least massive new down quark in a 4 × 4 mixing matrix. The resulting bounds on FCNC’s are compared with the Z-mediated rates for B^0_d-B^0_s^+ and B^0_u-B^0_d^+ mixing to show that such tree-level mixing can still dominate B^0_d-B^0_s^+ mixing over the standard-model box diagrams, but the Z-mediated process is bounded at less than half the amplitude for B^0_u-B^0_d^+ mixing. The FCNC limits are then applied to the matrix elements of the 4 × 4 mixing matrix to find the limits on the mixing angles. If the Z-mediated FCNC dominates B^0_d-B^0_s^+ mixing, we can show that the CP-violating asymmetries in B^0_d meson decays must be due to new mixing angles and phases of the 4 × 4 mixing matrix. We also reevaluate the effects these FCNC and 4 × 4 unitarity can have in unitarity quadrangles that replace the 3 × 3 unitarity triangles. Throughout we take all data and bounds at 90% confidence limits (or 1.64σ), unless it is an absolute bound. This is a follow-up to the work done on this subject with Nir [2, 3].

**II. FLAVOR-CHANGING NEUTRAL CURRENTS FROM MIXING**

Mixing in a fourth down quark which is an SU(2)_L singlet, using the 4 × 4 matrix V which diagonalizes the down quarks to the mass eigenstates, and using a basis where the three up-quark states are the mass eigenstates, the charged current interactions are [2]

\[
L = \frac{g}{\sqrt{2}} (W^\mu J^{\mu+} + W^+ J^{\mu-}),
\]

\[
J^{\mu-} = V_{ij} u_i \gamma^\mu d_j.
\]

Here the 3 × 4 submatrix of V couples the three up quarks to the four down quarks. The 3 × 3 submatrix of V for i, j = 1, 2, 3 is the standard Cabibbo-Kobayashi-Maskawa (CKM) matrix.

The flavor-changing neutral currents are given by the mixings to the fourth down quark by [2]

\[
-U_{ij} \equiv V_{ij}^a = \text{for } i \neq j.
\]

The FCNC couplings of the down quarks to the Z are then given by [2]

\[
L_{FCNC}^Z = \frac{e}{2\sin \theta_W \cos \theta_W} U_{ij} V_{ij} \gamma^\mu d_j Z^\mu.
\]

**III. THE BOUND FROM B → μμX**

The Z-mediated process forms μ pairs from the FCNC b → d and b → s with amplitudes \(U_{db}\) and \(U_{sb}\), respectively. The branching ratio of this to the \(W\)-mediated semileptonic decay is

\[
\frac{B(B → \mu\mu X)}{B(B → \mu\nu X)} = \left[\left(\frac{1}{2} - \sin^2 \theta_W \right)^2 + (\sin^2 \theta_W)^2\right]\times \frac{|U_{db}|^2 + |U_{sb}|^2}{|V_{ub}|^2 + F_{ps}|V_{cb}|^2},
\]

where \(F_{ps} \approx 0.5\).

The new bound from UA1 is [4]

\[
B(B → \mu\mu X) < 5 \times 10^{-5},
\]

which when scaled by the branching ratio for the \(W\)-mediated decay to \(\mu\nu\) bounds the ratios [3]

\[
\left|\frac{U_{db}}{V_{cb}}\right| < 0.041, \quad \left|\frac{U_{sb}}{V_{cb}}\right| < 0.041.
\]

This is an improvement by a factor of 5 on the \(|U_{qb}|\) from the previous bounds from this process. With the values \(|V_{cb}| = 0.044 \pm 0.014\), the range of the bounds on \(U_{db}\) and \(U_{sb}\) are

\[
|U_{db}| \leq 0.0018 \pm 0.0006, \quad |U_{sb}| \leq 0.0018 \pm 0.0006.
\]
IV. B–B MIXING

Mixing may occur by the b–d quarks in a B annihilating to a virtual Z through a FCNC with amplitude \( U_{db} \), and the Z then creates b–d quarks through another FCNC, and this then becomes a B meson. The mixing for this is given by

\[
(x_d)_{Z} = \frac{\sqrt{2}}{6} G_{F} B_{B} f_{B}^{2} M_{B} \eta_{b} |U_{db}|^{2}.
\]

(9)

The experimental and calculated parameters are (extended to 90% C.L.)

\[
x_d = 0.66 \pm 0.18,
\]

(10)

\[
\tau_{b} |V_{cb}|^{2} = (3.5 \pm 1.0) \times 10^{9} \text{ GeV}^{-1},
\]

(11)

\[
\sqrt{B_{B} f_{B}} = 0.15 \pm 0.08 \text{ GeV}.
\]

(12)

Requiring that this account for the observed value of \( B_{d}^{0} \)

\( B_{s}^{0} \) mixing would result in

\[
0.013 < \frac{|U_{ds}|}{|V_{ub}|} < 0.045.
\]

(13)

Thus the \( B_{d}^{0}, B_{s}^{0} \) mixing by Z-mediated FCNC’s is still allowed by the new bound on \( B \rightarrow \mu \mu X \), Eq. (7). The upper limit on \( U_{ds} \) obtained from \( B_{d}^{0}, B_{s}^{0} \) mixing [1,3] is of the same order as the new bound.

V. COMPARISON WITH BOX DIAGRAMS FOR MIXING

The standard-model box diagrams for meson mixing have net flavor-changing processes with two charged currents dominated by couplings to the top quark because of its large mass. For the \( Z \)-mediated FCNC to dominate over the standard model amplitude for \( B \rightarrow B \) mixing requires [3]

\[
|U_{db}| / (|V_{td}^{c}| V_{tb}) \geq 0.07 \text{ to } 0.14,
\]

(14)

\[
|U_{bs}| / (|V_{ts}^{c}| V_{tb}) \geq 0.07 \text{ to } 0.14,
\]

(15)

where the lower value for the limit is for \( m_{t} = 90 \text{ GeV} \) and the upper value is for \( m_{t} = 200 \text{ GeV} \).

For \( B_{d}^{0}, B_{s}^{0} \) mixing, Z-mediated dominance requires

\[
|U_{db}| \geq 0.07 |V_{td}^{c}|
\]

(16)

and with \( |V_{td}^{c}| \) between 0.003 and 0.02 requires \( |U_{db}| \) greater than 0.00021 to 0.0014. With the UA1 bound of Eq. (8) on \( U_{ds} \) this range is allowed.

For \( B_{d}^{0}, B_{s}^{0} \) mixing to be dominantly \( Z \)-mediated requires

\[
|U_{sb}| / (|V_{ts}^{c}| V_{tb}) \geq 0.07.
\]

(17)

With \( V_{tb} \equiv 1 \) and from the s–b unitarity triangle relation

\[
V = \begin{pmatrix}
1 & s_{12} & s_{13} e^{-i \theta_{13}} & s_{14} e^{-i \theta_{14}} \\
s_{12} & 1 & s_{23} & s_{24} e^{-i \theta_{24}} \\
(s_{12} s_{23} - s_{13} e^{i \theta_{13}}) & -s_{23} & 1 & s_{34}
\end{pmatrix}.
\]

(28)

The complex conjugates of the couplings to the fourth down quark are

\[
\left| \frac{U_{ub}}{V_{tb}} \right| \geq 0.07.
\]

(18)

This violates the UA1 bound of 0.041 on this ratio, Eq. (7), by a factor of 2. However, Z-mediated mixing could still interfere with the box diagrams and significantly affect the phase.

VI. FCNC BOUNDS FROM K-MESON PHYSICS

The new BNL result [5] for \( K_{L} \rightarrow \mu \mu \) is (with 90% C.L. limits)

\[
B(K_{L} \rightarrow \mu \mu ) = (7.0 \pm 0.82) \times 10^{-9}.
\]

(19)

The result from the 2γ intermediate state is [6] \( (6.83 \pm 0.46) \times 10^{-9} \). The 2γ absorptive contribution is purely imaginary, and the FCNC Z-mediated decays sum to contribute only to the real part. Bounding the difference of the rates as being due to the real part gives

\[
|A_{K}|^{2} = (0.17 \pm 0.94) \times 10^{-9}.
\]

(20)

The relation to the FCNC amplitude is [3]

\[
|A_{K}|^{2} = 13.7 |\text{Re}(U_{ds})|^{2}.
\]

(21)

Using the difference plus error as the maximum bound on \( |A_{K}|^{2} \leq 1.11 \times 10^{-9} \) gives

\[
\text{Re}(U_{ds}) \leq 0.90 \times 10^{-5}.
\]

(22)

The Z-mediated FCNC also contribute to the neutral-K-meson mass difference. Requiring \( (\Delta M_{K})_{Z} \leq \Delta M_{K} \), the bound from this is [3]

\[
\text{Re}(U_{ds})^{2} \leq 4.1 \times 10^{-7}.
\]

(23)

The Z-mediated diagram also contribute to \( \epsilon \) given the bound [3]

\[
\text{Im}(U_{ds})^{2} \leq 2.6 \times 10^{-9}.
\]

(24)

Putting together the above two bounds we find, for the \( U_{ds} \) parts,

\[
\text{Re}(U_{ds}) \leq 0.90 \times 10^{-5},
\]

(25)

\[
\text{Im}(U_{ds}) \leq 0.64 \times 10^{-3},
\]

(26)

\[
|U_{ds}| \leq 0.64 \times 10^{-3}.
\]

(27)

VII. BOUNDS ON MIXING ANGLES

All cosine terms in the \( 4 \times 4 \) mixing matrix are near 1. The leading terms in the \( 3 \times 4 \) part of \( V \), in terms of the sines of the mixing angles are [7]
Using the bound on |Re(U_{4d})| above, for the four-quark mixing form -U_{ij} = V_{4i}^* V_{4j}, we get

\[ |\text{Re}(V_{4d} V_{4s})| \leq 0.90 \times 10^{-5}. \]  

(32)

We now insert the forms for the V_{4d} and V_{4s} in the above and keep the quadratic terms in each of the new mixing angles s_{34}, s_{24} e^{-i\delta_4}, and s_{14} e^{-i\delta_4} and use the above bound to bound each mixing angle separately. For s_{24} e^{-i\delta_4} and s_{14} e^{-i\delta_4} these give

\[ |s_{24} e^{-i\delta_4}| \leq 0.0064. \]  

(33)

\[ |s_{14} e^{-i\delta_4}| \leq 0.0064. \]  

(34)

For s_{34} we note that its coefficient in V_{4d} is -V_{4d} and that its coefficient in V_{4s} is mainly s_{23} = V_{cb}. The bound becomes

\[ s_{34}^2 \leq 0.90 \times 10^{-5} |V_{cb} \text{Re}(V_{4d})|. \]  

(35)

Using the Particle Data Table 90% C.L. lower bound for |V_{td}| of 0.003 and the lower bound |V_{cb}| = 0.030 gives

\[ |s_{34}| \leq 0.32. \]  

(36)

The bounds on b -> d and b -> s do not give better bounds on s_{24} e^{-i\delta_4} and s_{14} e^{-i\delta_4}, but the b -> s bound, Eq. (7), gives

\[ |s_{34}| \leq 0.20, \]  

(37)

independent of the value of V_{cb} = s_{23}, which cancels out in the ratio.

We can now form separate limits on the mixing elements V_{4d} based on the above limits on the mixing angles. Adding possible contributions by assuming no cancellations gives

\[ |V_{4d}| \leq 0.012. \]  

(38)

\[ |V_{4s}| \leq 0.019. \]  

(39)

\[ |V_{4b}| \leq 0.20. \]  

(40)

Substituting these back into -U_{ij} = V_{4i}^* V_{4j} we can look for improvements. We find

\[ |V_{4d}| \leq 0.23 \times 10^{-3}. \]  

(41)

which is better than Eqs. (27) and (26) on the magnitude and imaginary part, but does not improve Eq. (25) on the real part. On U_{ab} the product |V_{4e}^* V_{4b}| gives the bound 0.0038, which is not as good as the direct bound in Eq. (8). On U_{ab} the product |V_{4d}^* V_{4b}| gives the bound 0.0024 which is the same as in Eq. (8).

VIII. CORRECTIONS TO CKM UNITARITY TRIANGLES

The unitarity of the 4x4 mixing matrix requires orthogonality of the different rows. Instead of the three terms in the CKM orthogonality relation, which give a triangle in the complex plane, we now have four terms which give a quadrangle. The fourth terms are the

\[ U_{ij} = \frac{V_{4i}^* V_{4j}}{V_{4d} \text{Re}(V_{4d})}. \]  

The first term has the same phase as in the standard model where the box diagram with the t-quark dominates the mixing. This term is bounded by 0.0008. The term with s_{24} e^{-i\delta_4} is bounded by 0.0004. The term with s_{14} e^{-i\delta_4} has the largest bound of 0.0013. The sum gives the bounded value of 0.0024 to U_{ab}. Thus if the FCNC dominates B_s^0 \to B_d^0 mixing, it may do so by involving the new phase in s_{14} e^{-i\delta_4}, due to the 4 x 4 or larger nature of the down-quark mixing matrix.

Using the lowest bound for m_t = 90 GeV Eq. (14), we see U_{ab} need only be as large as 0.07 |V_{td}| to dominate over the box diagrams in B_d^0 \to B_d^0 mixing. Since the term in s_{24} V_{4d}^* V_{4d} could only be as large as 0.04 |V_{td}| this term could not account for Z mediation dominantly over the box diagrams in B_d^0 \to B_d^0 mixing. If Z mediation dominates, it must then be from s_{14} e^{-i\delta_4}. We thus expect a new phase to the CP asymmetries if Z-mediated diagrams dominate B_d^0 \to B_d^0 mixing.

In Fig. 1 we show the relevant bounds for |U_{ab}| and the
XI. EFFECTS OF IMPROVEMENT OF $B \to D$ BOUND BY 1/10

An experiment at Fermilab may improve the bound on $B \to \mu \mu X$ by a factor of 1/10 that in Eq. (6), namely, to

$$B(B \to \mu \mu X) \leq 5 \times 10^{-6}.$$  \hspace{1cm} (46)

This bounds the FCNC amplitude at 90% C.L.:

$$\left| \frac{U_{ab}}{V_{cb}} \right| \leq 0.013.$$  \hspace{1cm} (47)

Comparing this to the requirement for $B_0^d \to B_0^d$ mixing to be accounted for by $Z$-mediated FCNC in Eq. (7), which is that the above ratio be greater than 0.013 at 90% C.L., we would have a clear discrepancy. By combining the two offsets at a 90% C.L. or 1.64$\sigma$ each, we would get a 2.32$\sigma$ discrepancy, and the $Z$ mediation would be ruled out at the 98% C.L. In Fig. 1 the upper bound line (solid) would be lowered to $|U_{ab}| < 0.006$, or coincident with the lower bound for $B$ mixing by FCNC. Although the figure is shown only for the central value of $|V_{cb}|$, the above conclusion would be valid independent of its value since both bounds are scaled by it.

The improved bound would also improve the bound on $\sin 2\beta$ in Eq. (37), giving

$$|\sin 2\beta| \leq 0.11,$$  \hspace{1cm} (48)

which would limit the standard model phase term in Eq. (45) to 0.01$V_{td}$.

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