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NUCLEAR DEMOCRACY AND BOOTSTRAP DYNAMICS

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ABSTRACT

The assumption of the dynamical equivalence of all nuclear particles is discussed and contrasted with conventional electrodynamics and weak interaction theory. The relation with the notion that all strong forces are Yukawa forces is brought out, and an attempt is made to explain the difference in properties between baryons and mesons on one hand and ordinary nuclei on the other. The bootstrap mechanism is explained, the crucial role of crossing matrices and of the Froissart limit being emphasized. It is shown that the combined requirements of unitarity and analyticity of the S matrix can place both lower and upper limits on the particle population in a democracy.

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Introduction

What is meant by "nuclear democracy," a term which I believe was first used by Gell-Mann? In this survey I shall mean the idea that all strongly interacting nuclear particles from the least massive, the pion, up to excited states of transuranic nuclei stand on a dynamically equivalent basis. More precisely, each may be regarded as a stable or metastable bound state of those channels with which it communicates, the mass, spin and partial widths (coupling constants) of each particle being calculable in terms of the Yukawa forces that act in the communicating channels.

What do I mean by a "Yukawa force"? I mean a force arising through the crossing principle from states that communicate with crossed channels. The Yukawa force may be said to originate in the "exchange" of such states, the most important of which appear to be single particles. The characteristics of a Yukawa force are uniquely determined by the properties of the particle exchanged, the range and form by the mass, spin and parity and the strength by the partial reduced widths coupling to the channels in question. The force may be either attractive or repulsive, depending on the internal quantum numbers of the particle exchanged and the channels in which the force acts.

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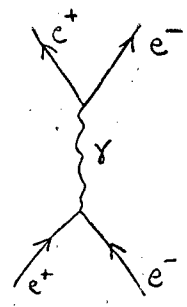
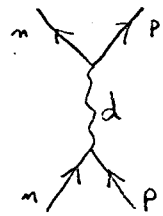
The best known example of a Yukawa force is that between two nucleons, where the long-range parts have been successfully explained in terms of single-particle exchanges of the known zero-strangeness mesons having mass less than 1 GeV, plus two-pion exchange. The long-range parts of the force between a pion and a nucleon have similarly been explained, single baryon exchange here playing the dominant role. It is, in fact, now generally accepted that all long-range forces between any pair of particles are controlled by the Yukawa mechanism. Doubt remains only about what we shall here call "short-range" forces, a euphemism for interactions in states having angular momentum 0, 1/2 and 1 but no higher. In conventional electrodynamics and weak interaction theory there appear to be non-Yukawa forces acting in certain of these low-J states, forces that correspond to an aristocratic rather than a democratic particle society. The assumption of nuclear democracy excludes non-Yukawa forces from strong interactions. It goes without saying that experimental evidence for strong non-Yukawa forces has never been seen.

To avoid misunderstanding on this point let me give as a typical example of a non-Yukawa interaction the contribution to positron-electron scattering that arises from "direct" transitions to the intermediate photon. This interaction, affecting J = 1 only, cannot be described as a force due to particle exchange, but in a conventional calculation one nevertheless inserts such a term at the beginning, employing



the mass and coupling constant of the photon as given parameters. The photon, in other words, is not expected to "emerge" as a bound state of

the electron-positron system, with properties determined by the dynamics. By contrast, in a calculation of n-p scattering it is felt unnecessary to insert directly the transition to the intermediate deuteron. The existence and properties of the deuteron are supposed to be consequences of the Yukawa forces due to meson exchange.



In conventional electrodynamics and weak interaction theory, the photon with spin-1 and the leptons with spin-1/2 are aristocrats in the sense that their properties are assumed not to be dynamically calculable, while other particles such as positronium can be predicted as bound states. In a nuclear democracy all strongly interacting particles are predictable, including the nucleon and the spin-1 mesons. A substantial amount of theoretical attention is being devoted to schemes where certain spin-1 mesons are given a status parallel to that of the photon. Needless to say, such a philosophy is fundamentally undemocratic.

All dynamical calculations to predict nuclear particles are based on two related assumptions: (a) That communicating channels with thresholds sufficiently far above the mass of the particle in question may be neglected. (b) That exchanged systems of sufficiently high mass make negligible contributions to the overall Yukawa force. It is not completely clear why such assumptions should be valid, although plausibility arguments have been given both in the special framework of the Schrödinger equation and, more generally, in terms of the unitarity and analyticity of the S matrix. Experience shows, in any event, that the two assumptions actually work. The only question is a quantitative one: To how high an energy in the communicating

channels and in the exchanged systems must one go in order to achieve the required accuracy? The answer varies from one case to another, and theoretical efforts naturally tend to concentrate on predicting those particles where a small number of communicating channels and exchanged systems promise to constitute a reasonable approximation.

In this connection the relative positions of "classical nuclear physics" and "high energy nuclear physics" deserve some appreciation. Any particle with $A \geqslant 2$, whether in its ground state or an excited state, is supposed to be a citizen of our democracy with status equivalent to that of any meson $(A = 0)$ or baryon $(A = 1)$. We happen, however, to achieve a good dynamical approximation to particles with $A \geqslant 2$, $S = 0$, by ignoring channels that contain any $A = 0$ particles or $A = 1$ particles other than nucleons. The channels considered have thresholds separated only by energies of the order of MeV from the mass of the nucleus being studied. The neglected channels do not begin until several hundred MeV higher. The existence of the higher-threshold channels affects the properties of the nucleus, but the effect is small and nuclear physics has in consequence been broken by its students into two separate subjects, one dealing with $A = 0, 1$ and one dealing with $A \geqslant 2$. If one tries to understand <u>why</u> there is such a large gap between the two sets of thresholds, the explanation appears to involve the "unusual" smallness of the pion mass. Almost no current theories, whether democratic or not, give the pion a special status these days, but the fact remains that it <u>is</u> the least massive nuclear particle by a substantial margin and correspondingly it generates "unusually" long range forces for those channels where it can be exchanged. The pion <u>is</u> exchanged between two nucleons, and

the short range force here happens to be repulsive. As a consequence we have acting between nucleons an average force of long range that corresponds to a potential energy of only $\sim m_\pi^2/2M$ or 10-20 MeV. This turns out to be barely strong enough to make bound states at all, so that nuclear masses for $A \geqslant 2$ occur close to the lowest communicating channel threshold.

In contrast, because of conservation laws, a pion generally cannot be exchanged between two particles if at least one of the two has $A = 0$, and the shorter range forces that do act here turn out to be sometimes attractive. The average potential energy in such channels is higher than for two baryons by $\sim m^2/m_\pi^2 \gtrsim 30$ so that the characteristic energy of the dynamics turns out to be hundreds of MeV or even BeV. It is by such arguments that one tries to explain how, even in a democracy with equal treatment under the law, the population can become divided into two groups, possessing what seem to be qualitatively different characteristics.

It has been clear for some time, thanks to the circumstances just described, that all particles with $A \geqslant 2$ can be predicted as dynamical bound states, once one knows the properties of particles with $A = 0$ and $A = 1$. Current interest revolves about the predictability of the latter, and it is here that the bootstrap idea becomes essential. In other words, the same particles that one is attempting to predict may be involved in generating the important Yukawa forces. I emphasize "important" because particles with $A \geqslant 2$ also generate forces through their exchange, but they are so massive that the effect is small. The least massive particles are the most important force-producers and these all have $A = 0$ or 1.

For $A \geq 2$ the dynamical calculation has generally proceeded via a potential and the Schrödinger equation, nonrelativistic concepts that can be justified when the average force range is substantially larger than the Compton wavelength of the particles which are to be bound together. Such a condition is not satisfied, however, when the force-producing particles are as massive as those feeling the forces. Bootstrap dynamics for $A = 0, 1$ consequently must be carried out in a fully relativistic framework. There is no Schrödinger equation available, so what does one do? A first thought might be to use the Bethe-Salpeter equation, but, apart from any questions of principle, there are so many variables in this approach that no serious bootstrap calculations have ever been carried out. On the other hand, if one stays on the mass shell, working with the analytically continued S matrix, the number of variables is much fewer and realistic calculations seem feasible. The equations here are the Cauchy formulae, or dispersion relations, that represent an analytic function in terms of its singularities.

Crossing Matrices

Bootstrap dynamics rest on the crossing principle, that is, on the assumption that the physical regions of different nuclear reactions are connected by an analytic continuation of the S matrix. The reactions so connected differ by having ingoing particles in one replaced by outgoing antiparticles in another, the energy variable for one reaction being a momentum transfer for a crossed reaction. In the energy variable for a particular reaction there are poles corresponding to all the particles that communicate with the initial and final channels of the reaction. The position of a pole corresponds to the particle mass and lifetime and

the residue to the product of the partial reduced widths coupling the particle to the two channels. According to the crossing principle all these particle-poles appear in momentum-transfer variables of the reactions reached by crossing and may there be interpreted as what we have called "Yukawa forces."

The quantum numbers of a pole with respect to its communicating reaction are "pure", but under crossing a given pole appears with a mixture of the quantum numbers belonging to the crossed reaction. The matrix of coefficients that tells how a particular pure set of quantum numbers for one reaction becomes distributed among the quantum numbers of a crossed reaction is called the "crossing matrix." We shall hear a great deal about crossing matrices from Dr. Cutkosky in the following paper. I simply want to remark here that it is the elements of the crossing matrix that determine by their sign and magnitude whether the exchange of a particular particle constitutes an attractive or repulsive, strong or weak, force for some particular set of communicating channels. Obviously the structure of these matrices plays a crucial role in bootstrap dynamics.

A striking general feature of crossing matrices has recently been established by Neville, following up on a conjecture made two years ago by Frautschi, Mandelstam and me and recent observations of Pignotti. In the particle-antiparticle (two-body) channels coupled to the quantum numbers of the vacuum, <u>all</u> the individual force components are attractive and the sum necessarily larger than the attractive force for any other channels. In bootstrap dynamics, therefore, whatever particles and symmetries exist the strongest attraction inevitably occurs for the quantum numbers of the vacuum. I shall return to this point at the end of my paper.

Angular Momentum Interpolation and Regge Poles

Let us now consider the role of angular momentum analyticity in bootstrap dynamics. It has been shown by Froissart and Gribov that one may define a unique analytic interpolation between the large physical J values for 4-line connected parts, and in a democracy this same interpolation must connect all physical J values, including $J = 0$, $1/2$, and 1. It follows that the positions and residues of all energy poles are analytic functions of angular momentum and vice-versa. That is, there are poles in the angular momentum complex plane whose positions and residues are analytic functions of the energy; these are, of course, the Regge poles.

Mandelstam has shown that the singularities in complex angular momentum include branch points as well as poles, but at zero total energy no J singularities occur to the right of the rightmost pole, whose position determines the dominant asymptotic behavior in momentum transfer. By crossing, therefore, one finds that the high energy behavior of any two-particle reaction amplitude at zero momentum transfer is determined by $S^{\alpha_c(0)}$, where S is the square of the total energy in the barycentric system and $\alpha_c(0)$ is the position in the crossed-reaction angular-momentum plane of the rightmost Regge pole. Now Froissart has proved from a combination of unitarity and analyticity considerations that two-particle reaction amplitudes cannot increase faster than the first power of S. One arrives then at the requirement that no Regge poles may occur to the right of $J = 1$ at zero total energy.

This constraint of Froissart plays an enormous role in bootstrap dynamics. It means, first of all, that particles of spin greater than 1 and low mass are not to be expected. Furthermore, the exchange of high-spin particles does not produce the strongly singular forces that might

be expected, because the trajectories on which they lie must retreat below $J = 1$ before one reaches the physical region of the channels in which the force is exerted. The net result is that low-spin particles play a dominant role in producing forces for the bootstrap. Put more accurately, if we understand the low-spin citizens of our democracy we can calculate the forces without worrying in detail about high-spin citizens. The practical importance of such a circumstance is obvious. I shall defer to the end of this talk what seems to me an even more important aspect of the Froissart limit than the foregoing.

Just how does one go about a bootstrap calculation in the framework of the analytically continued S matrix? Let me sketch the first stages, although we shall come nowhere near a realistic situation.

The simplest conceivable nuclear democracy would consist of one particle, or more precisely, one Regge trajectory reaching the right-half J plane. If Neville's results are as general as they seem this trajectory would have to belong to the quantum numbers of the vacuum. The energy at which such a trajectory cuts $J = 0$ is the mass of the scalar meson that constitutes our lonely citizen. This scalar meson would be a bound state of the communicating channels consisting of two mesons, three mesons, etc. In particular, in the S matrix we have a pole at $E = m$ and $J = 0$ in the amplitude for the reaction $a + a \rightarrow a + a$. By crossing, this means three poles, because three reactions of this type are connected by analytic continuation. The reaction amplitude is an analytic function of energy and momentum transfer with branch points that depend, through the Landau conditions, on the pole positions. There are an infinite number of poles on the multi-sheeted Reimann surface, but we have assumed that only three are

close to the physical regions. Correspondingly the Landau branch points close to the low energy physical regions are controlled by the same three poles and have a simple structure. The discontinuities across the cuts running from the branch points are given by formulas associated with the name of Cutkosky.

I have said that, basic to bootstrap dynamics, is the assumption that channels with high thresholds and exchanged particles with high masses may be neglected. In the S-matrix framework this principle is equivalent to the dominance of "nearby" poles and branch points when one expresses an analytic function at a particular point through the Cauchy formula in terms of the residues of its poles and the discontinuities across its cuts. We would therefore, in a first approximation, attempt to express our reaction amplitude in terms of its three poles and those branch points that correspond to the two-particle channel communicating with each of the three reactions. This was the approach of Mandelstam and it leads to the so-called "strip approximation."

My time here is far too brief for a proper explanation, but when the requirement of analytic interpolation in angular momentum is adjoined to the strip approximation, one finds a set of integral equations analogous to those for nonrelativistic scattering by a Yukawa potential. One solves these equations and tries to require that the mass and reduced width of the bound state match the range and strength of the potential. Because there is only a single mass in the problem considered, however, the mass merely serves to set the energy scale, and the residue of the pole (or reduced width) has the impossible task of satisfying two different conditions by itself. In the strip approximation, at least, the nuclear world cannot bootstrap itself with a single particle. The real world must be more complicated.

Dr. Cutkosky will speak of some of the theoretical possibilities when more particles are allowed, and Dr. Zachariasen will tell of attempts to verify the bootstrap character of the actual nuclear world that we see, which of course has a substantial number of low mass-low spin particles. In this latter connection I will just mention a current program of strip-approximation calculations at Berkeley by Jones, Teplitz and me, the distinctive feature of which is to add to the force from each exchanged particle a correlated contribution from the communicating two-body channels so as to guarantee Regge asymptotic behavior. In other words we are deriving our forces from the exchange of composite particles--not point particles. We are hoping to get quantitatively important improvement on previous calculations, but no results can be reported yet.

I conclude with the remark that it is already possible to discern from the bootstrap mechanism not only a lower limit to the total number of low mass-low spin particles in a self-sustaining nuclear democracy but also an upper limit. The argument is as follows: The stronger the net attractive force acting for a given set of quantum numbers the higher will be the angular momentum that can be bound at a given energy. Neville has shown that in the strip approximation, no matter how many and what kind of particles exist, all the particles supply attractive forces to the vacuum singlet, while the forces acting for other quantum numbers are a mixture of attraction and repulsion so that the net attraction is always less. We understand immediately, therefore, why the so-called Pomeranchuk trajectory has the top-ranking position in a plot of angular momentum versus energy. Less immediate but strongly suggested by Neville's result is that the spacing in angular

momentum at a given energy between the Pomeranchuk trajectory and the next highest trajectory increases indefinitely as the total number of particles increases. Now, if other particles exist at all there certainly must be trajectories close to $J = 0$ at zero total energy, if not above zero. Therefore an indefinite increase in the total number of particles would sooner or later push the Pomeranchuk trajectory above the Froissart limit at $J = 1$.

It is possible to make an immediate and practical application of this mechanism to rule out two decuplets of vector mesons which Neville observed might otherwise be able to bootstrap themselves. Were they to exist, the total attractive force acting on the vacuum singlet in an SU_3 strip approximation would be <u>seven</u> times as great as on the vector octet. In the absence of the decuplets and with only the octet, the force ratio is two, which estimates have indicated is about right to explain the observed spacing of the Pomeranchuk and vector trajectories.

The general principle is, of course, much more interesting than this particular application. We have here a hint of how the combination of unitarity and analyticity can determine the population of a nuclear democracy.