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Authors

Farrell, Joseph
Maskin, Eric

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UNIVERSITY OF CALIFORNIA, BERKELEY

Department of Economics

Berkeley, California 94720

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RENEGOTIATION IN REPEATED GAMES

Joseph Farrell and Eric Maskin

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Abstract

We analyze the possibility of renegotiation in supergame equilibrium. We show that it strictly reduces the set of equilibrium outcomes in most games, though not in the Prisoner's Dilemma. We give necessary and sufficient conditions for a payoff vector to be sustainable for large enough discount factors.

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Joseph Farrell, U.C. Berkeley
Eric Maskin, Harvard University

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1. Introduction

When players repeat a game infinitely often, many outcomes become Nash equilibria that would not be so without repetition. This is because players may refrain from using their one-shot best responses to others' actions in the face of threats (or promises) about how others will respond if they do so. In fact, the Folk Theorem tells us that any individually rational vector of feasible payoffs--those Pareto-dominating the minimax point--can arise as average payoffs in a Nash equilibrium of the repeated game (provided there is sufficiently little discounting of future payoffs).¹

The most famous example of the effect of repetition is the Prisoner's Dilemma. In the one-shot game (see Table 1),

	C	U
C	2,2	-1,3
U	3,-1	0,0

Table 1

the only equilibrium involves both players playing uncooperatively (i.e., choosing "U" in Table 1), giving payoffs (0,0). This is an unfortunate outcome because both players would be better off with the cooperative strategy pair (C,C). Happily, repetition makes mutual cooperation an equilibrium. The most familiar way to enforce

 1. See Aumann-Shapley [1976], Rubinstein [1979], and Fudenberg-Maskin [1986].

such cooperation is through "trigger" strategies: if either player ever fails to cooperate, then both players play uncooperatively forever after. With sufficiently little discounting, the trigger strategy pair forms a Nash equilibrium that leads to cooperation. Indeed, the equilibrium is not only Nash, but (subgame)-perfect.

But despite its perfection, this familiar equilibrium is not entirely convincing. If one of the players were to deviate from cooperation, so that the strategies tell players to play uncooperatively thereafter, they have an incentive to reconsider. They might well say to each other, "Why should we submit to this bleak prospect? Why not 'let bygones be bygones' and return to the original equilibrium path, which gives higher payoffs for us both?" Ex post, of course, this argument for forgiveness promotes efficiency, but it has potentially devastating effects on deterrence. If players knew in advance that they would "renegotiate", then a player would no longer be deterred from cheating, for he would predict that his transgression would be ignored.²

This criticism seems particularly apt if, as is common, we interpret equilibria as self-enforcing agreements. In this interpretation, players convene before the game to discuss (negotiate) which strategies they are going to play. Because they

2. The problem would arise even if deviations were punished by moving to uncooperative play for a finite number of periods and then returning to cooperative behavior. This is because, at the time a punishment is in fact about to start, both players would prefer the cooperating equilibrium.

have no external enforcement mechanism, they presumably must agree on equilibrium strategies, for only then will no player have an incentive to deviate once the game begins; an equilibrium is self-enforcing. Indeed, thinking of an equilibrium as a self-enforcing negotiated agreement in this way is especially attractive in repeated games, where there are so many equilibria that we may otherwise have trouble explaining how players know which one to play.

But once we open the door to negotiation, it is difficult to close the door to renegotiation. Unless players somehow cut the lines of communication, it seems plausible that they can renegotiate after the game begins, and that they will not follow a mutually unpleasant subgame-equilibrium path when there is a Pareto-dominating alternative available -- even if they had agreed to do so before the game began.

These considerations suggest what we might mean by renegotiation-proof equilibria. It is evident first that we have to talk about renegotiation-proof equilibrium sets rather than single equilibria. Every perfect equilibrium gives rise to a collection of many continuation equilibria. For renegotiation-proofness we require that all these continuation equilibria be plausible when players can renegotiate. More formally, we will define a weakly renegotiation-proof (WRP) set to be a collection S of perfect equilibria and all their continuation equilibria, such that at no point in the game tree of any equilibrium in S would players all prefer moving to another member of S . Such sets always exist. For

instance, one WRP set is the singleton where players use one-shot equilibrium strategies in each period regardless of the previous history. (In fact, this is the only kind of singleton WRP set). Clearly, however, we are more interested in bigger and better WRP sets.

In this paper we ask what payoffs are enforceable in a renegotiation-proof way. For simplicity, we confine attention to two-person games, although our results generalize readily to more players (see Section 9, however, for a discussion of the additional conceptual issues that games of three or more players create). In Section 2 we introduce our notation and formalize the concept of WRP described above. In Section 3 we show that, although WRP is unrestrictive for the Prisoner's Dilemma (that is, in that game, any individually rational payoffs can be sustained in a renegotiation-proof way for high enough discount factors), WRP does, in general, reduce the equilibrium set. Indeed, for general, infinitely repeated two-person games, we characterize the payoff vectors that can arise as a part of a WRP equilibrium set (Theorem 1). (Loosely, what is required is that it should be possible to punish each player in such a way that the other player actually benefits.) In Section 4, we use this result to show that Cournot duopolists can sustain a given collusive outcome if and only if it gives each duopolist a big enough share of the joint monopoly profit. We also demonstrate that equal sharing of monopoly profit is possible in Bertrand duopoly.

The spirit of our WRP analysis is that S represents a logically consistent "theory" that the players share about what outcomes are "plausible," given that there can be no external enforcement and that they can always renegotiate within S .³ But there are typically many different such sets S , and one could imagine an ambitious negotiator trying to persuade his fellow players to change theories in mid-game. It seems reasonable to argue that this should be harder than persuading them to go to another equilibrium within the theory, which is one reason why we believe that it is worthwhile to study WRP outcomes. But it would be presumptuous to claim that such ambitious renegotiation is impossible.

Clearly, such change-the-theory renegotiation is no threat if the WRP set consists entirely of Pareto-efficient equilibria, as can be arranged, for example, in the Prisoners' Dilemma. Accordingly, in Section 5, we give necessary and sufficient conditions under which, with sufficiently little discounting, there is such a set (Theorem 2). We demonstrate, moreover, that these conditions are satisfied by Cournot duopoly. But when the conditions do not hold, as in the case of Bertrand duopoly, we may have to worry about such renegotiation. Thus, in Section 6, we define a stronger concept of renegotiation-proofness: a weakly renegotiation-proof set S is strongly RP if all its elements lie on the Pareto frontier of the

3. Pearce [1987] proposes a different theory of renegotiation-proofness in which the focus is precisely on renegotiation across sets of equilibria, not (as in our theory) within them. See Section 9 for further elaboration.

union of all WRP sets (for a given discount factor). We show, by example, that such sets (unlike their weaker counterparts) need not exist. We therefore consider three weaker concepts. In particular, Theorems 3 and 4 establish that "relative" strongly RP sets exist and, when there is sufficiently little discounting, so do "epsilon" strongly RP sets.

Through Section 6 our concern is with infinitely repeated games. We turn briefly, in Section 7, to games that are repeated only finitely many times. In Section 8, we discuss other game-theoretic work on renegotiation. Finally, Section 9 offers a few concluding remarks about games with incomplete information or imperfect observability and about games of more than two players.

2. Fundamentals

We consider a two-person finite game

$$g: A_1 \times A_2 \rightarrow R^2,$$

where A_i is player i 's action space and $g_i(a_1, a_2)$ is player i 's payoff. For convenience we shall assume that A_i contains not only player i 's actions, but all randomizations over these actions (i.e., mixed strategies) as well. This assumption has force because below we will suppose that repeated game strategies can be contingent on all past actions. Thus, if past actions can be random, we are supposing that players can observe each other's private randomizations ex post.^{4,5} The set of payoffs is

4. This would be so if, for example, players could inspect the

$$V = \{(v_1, v_2) \mid \text{there exists } (a_1, a_2) \text{ with } (v_1, v_2) = g(a_1, a_2)\},$$

where $g(a_1, a_2)$ must be interpreted as an expected payoff if players use mixed strategies. Player 1's minimax payoff $\underline{v}_1$ is defined as

$$\underline{v}_1 = \min_{a_2} \max_{a_1} g(a_1, a_2).$$

Player 2's minimax payoff is defined analogously. Let us assume henceforth that payoffs are normalized so that $(\underline{v}_1, \underline{v}_2) = (0, 0)$. Then, the set of feasible, (strictly) individually rational payoffs consists of

$$V^* = \{(v_1, v_2) \in V \mid v_i > 0 \text{ } i=1, 2\}.$$

For later reference, let $\bar{v}_i$ ($i=1, 2$) be player i 's maximax payoff:

$$\bar{v}_i = \max_{a_1} \max_{a_2} g_i(a_1, a_2).$$

In the infinitely repeated version of g , players maximize the (expected) discounted sum of their one-shot payoffs. Thus, player i maximizes the expectation of

$$(1) \quad \sum_{t=1}^{\infty} \delta^{t-1} g_i(a_1(t), a_2(t)),$$

where δ is the discount factor and $(a_1(t), a_2(t))$ are the actions chosen in period t . The average or "per period" payoff corresponding to (1) is

realizations of each other's randomizing devices after the fact.

5. This assumption can, however, be dispensed with (see Section 6 of Fudenberg-Maskin [1986]).

$$(1-\delta) \sum_{t=1}^{\infty} \delta^{t-1} g_i(a_1(t), a_2(t)).$$

The history in period t is a list of all the actions taken through period $t-1$: $(a_1(1), a_2(1)), \dots, (a_1(t-1), a_2(t-1))$. The actions chosen in period t may depend on this history. Thus, a repeated game strategy for player i is a function σ_i that, for each period t and each possible previous history, prescribes a period t action (or a lottery over actions). The strategy pair (σ_1, σ_2) forms a (subgame-)perfect equilibrium if it constitutes a Nash equilibrium in every subgame. Thus, a perfect equilibrium induces an infinity of continuation equilibria, the equilibria beginning in the subgames.

Our first concept of renegotiation-proofness is the following: a weakly renegotiation-proof (WRP) set S is a set of pairs of payoffs such that

- (i) for every $(v_1, v_2) \in S$, there exists a perfect equilibrium (σ_1, σ_2) such that (v_1, v_2) are the average payoffs corresponding to (σ_1, σ_2) , and all continuation payoffs for (σ_1, σ_2) belong to S ;

and

- (ii) no point in S strictly Pareto-dominates any other point in S .⁶

6. We adopt the criterion of strict Pareto-domination (both players must be strictly better off for one point to dominate another) largely for technical reasons. In particular, it enables us to conclude that the closure of a WRP set is itself WRP. (See

As we mentioned above, one can think of the set S as the players' theory of which outcomes (average payoffs) of the repeated game can be agreed to. Presumably, it is thought-out or negotiated in advance. Notice that the set of "agreeable" payoffs does not vary with time or with the previous history; we are imposing stationarity. This reflects the fact that the repeated game itself is stationary: the possibilities for the future do not depend on the past. The requirement that the points in S be perfect equilibrium payoffs embodies the idea that agreements between the players be self-enforcing, even after histories that would not occur if players stuck to the agreement. Finally, the stipulation that there be no Pareto-domination within S is really a perfection requirement applied to the two players jointly. It supposes that, at any point in the game tree, the players are collectively rational (i.e., that they never settle for a path that they both find inferior to another that they believe is available), just as ordinary perfection requires individual maximization at all points in the game tree.

The weakness in the concept of WRP, as mentioned before, is that players may be better renegotiators than it allows for. If (v_1, v_2) lies in S , although no other payoffs in S dominate it, there may be payoffs (v_1^*, v_2^*) in another WRP set S^* that do. Thus perhaps (v_1, v_2) would be abandoned in favor of (v_1^*, v_2^*) .

the Lemma below).

Of course, the counterargument can be advanced that the players may decide in advance to limit their future attention and negotiation to the set S . But commitment to such a decision may be difficult and is perhaps at odds with our collective perfection requirement. For this reason, we will turn to stronger RP requirements in Section 6. Even if WRP is rejected as being too weak, however, it still deserves study since it provides the basis for the stronger concepts; it is at least a necessary condition.

Implicit in our emphasis on Pareto-domination is the idea that agreements are broken (equilibrium paths are abandoned) only if it is in the interest of both players to do so. This restriction would certainly make sense if players could actually sign binding contracts. Given that our agreements are only self-enforcing, however, the idea may not be altogether convincing. Player 1, say, might bully player 2 into moving to an equilibrium that improves the former's but reduces the latter's welfare, and Player 2 might have no recourse against such tactics.

To take an even more extreme view, one could ask why the ex-ante agreement should have any force at all if renegotiation is so easy. Since the past is "sunk", one could argue that the same thing should happen at every node no matter what the history.

Of course, such a view completely rules out any cooperation due to repetition: we would inevitably have one-shot Nash payoffs every period. This view -- that the ex-ante agreement has no force -- is the polar opposite of the standard view that the ex-ante agreement is followed provided only that it is an equilibrium.

Our view is intermediate: we suppose that the outcome is affected by the ex-ante agreement, but not to the extent of enforcing joint irrationality. That is, in the contractual spirit, we suppose that an ex-ante agreement can determine which efficient (within the available set) continuation the players follow ex-post, but is not strong enough to make them do something inefficient ex-post.

In the next section, we show that renegotiation does not preclude cooperation in the repeated Prisoner's Dilemma, and then characterize the set of WRP payoff vectors in a general game. Following the successful lead of the Folk Theorem literature, we refrain from trying to characterize sustainable payoffs for a given discount factor δ , and instead give conditions under which a payoff vector is sustainable for large enough δ .

3. Weakly Renegotiation-Proof Sets

As we noted in the introduction, sustaining cooperation in the Prisoner's Dilemma by the threat of reverting to uncooperative behavior forever is not renegotiation-proof. Such a threat is simply not credible; the players will renegotiate if ever faced with the uncooperative prospect.

Nevertheless, cooperation is not inconsistent with renegotiation-proofness. To see this, refer again to Table 1 and consider the following candidate equilibria:

(2) e^* is defined so that players

- begin by playing (C,C); thereafter,
- if last period (C,C) was played, they play (C,C) again;
- if last period player 1 played U, they play e^1 (which punishes player 1);
- if last period (C,U) was played, they play e^2 (which punishes player 2).

(3) e^1 is defined so that players

- begin by playing (C,U) for T periods;
- if during these T periods player 1 ever plays U, the players immediately restart e^1
- if player 1 conforms to C for T periods, then the players restart e^* .

(4) e^2 is defined symmetrically with e^1 .

The length of the "punishment" phase T is chosen so that

$2 > 3(1-\delta) + (\delta - \delta^{T+1})(-1) + 2\delta^{T+1} > 0$. One can readily confirm that, as long as δ is near 1, such a T can be found. For such δ , e^* , e^1 , and e^2 are all perfect equilibria. Moreover, e^1 is worse for player 1 and better for player 2 than e^* , and vice versa for e^2 . Hence, for δ near 1, the the payoff pairs corresponding to e^* , e^1 , e^2 together with their continuation payoffs, constitute a WRP set that includes cooperative behavior (see Figure 1).⁷

7. These punishments have the flavor of making the defector

In fact, for the Prisoner's Dilemma, any feasible, individually rational payoff pair (i.e., any pair in V^*) is part of a WRP set, for δ near enough 1, as is shown by van Damme [1986].⁸ Since the "perfect Folk Theorem" for two-player games asserts that precisely the payoffs in V^* can arise as ordinary perfect equilibria,⁹ we conclude that the added requirement of WRP does not restrict the set of possible equilibrium payoffs for this game.

This is not true of all games. To see that, in general, WRP is restrictive, refer to the game in Table 2.

	L	R
T	2,2	0,4
B	-2,0	-2,0

Table 2

Because the minimax point is (0,0), the perfect Folk Theorem tells us that any feasible point with positive payoffs can arise as the average payoffs of a perfect equilibrium when δ is near 1 (see

atone for his sins, as compared with the standard "withdrawal of cooperation" or "do all we can to hurt him" punishments. They are reminiscent of TIT-FOR-TAT strategies: after you defect, you must cooperate while I defect. But actually they are more forgiving than TIT-FOR-TAT: the latter implies eternal alternation between (C,U) and (U,C) after a defection (the vendetta principle).

8. Actually, as van Damme shows, even for given δ the WRP requirement is not binding in this game: every outcome sustainable in perfect equilibrium is also WRP.

9. A "nondegeneracy" condition must be imposed before the same can be concluded for games of more than two players (see Fudenberg-Maskin [1986]).

Figure 2). However, only those points lying above the line segment connecting $(-2,0)$ and $(1,3)$ (the shaded area of Figure 2) remain when we impose WRP as well. We can understand this as follows. Let S be a WRP set and assume that (v_1, v_2) is that pair in S where player 2's payoff is minimal.¹⁰ We can express (v_1, v_2) as

$$(v_1, v_2) = (1-\delta)(v_1^1, v_2^1) + \delta(v_1^c, v_2^c),$$

where (v_1^1, v_2^1) are the expected first-period payoffs in the perfect equilibrium corresponding to (v_1, v_2) , and (v_1^c, v_2^c) are the expected continuation payoffs. Now, because v_2 is the minimal payoff for player 2 in S , we have

$$(5) \quad v_2^c \geq v_2 \geq v_2^1,$$

and, hence, from footnote 10 and to avoid Pareto-domination,

$$(6) \quad v_1^c \leq v_1 \leq v_1^1.$$

Now, (v_1^1, v_2^1) is a convex combination of the elements of Table 2:

$$(7) \quad (v_1^1, v_2^1) = \lambda^T \lambda^L (2, 2) + (1-\lambda^T) (-2, 0) + \lambda^T (1-\lambda^L) (0, 4),$$

10. If there is more than one such pair, take the one in which player 1's payoff is maximal.

where λ^T is the probability that player 1 plays T and λ^L is the probability that player 2 plays L. Because v_2 is minimal, it cannot be less than the payoff that player 2 obtains from playing action R with probability 1 in the first period (since after playing R his worst possible continuation payoff would be v_2):

$$(8) \quad v_2 \geq 4\lambda^T.$$

From the second inequality of (6)

$$(9) \quad v_1 \leq 2[\lambda^T \lambda^L - (1 - \lambda^T)].$$

But $(2[\lambda^T \lambda^L - 1 + \lambda^T], 4\lambda^T)$ lies above the line segment connecting $(-2, 0)$ and $(1, 3)$. Thus, from (8) and (9), so does (v_1, v_2) . We conclude that all members of S must lie above this line segment as well.

We have shown that, regardless of the discount factor, WRP equilibrium payoffs are confined to the shaded region in Figure 2. Our first theorem--which for general games characterizes those payoffs that belong to WRP sets--implies that such payoffs coincide exactly with the shaded region (except perhaps on the boundaries).

Punishments that hurt everyone are not credible. So we must use punishments that the innocent player actually wants to enforce. Since that is typically not true of "minimax the offender" (though it is in the Prisoner's Dilemma!), we can expect that punishments will require the cooperation of the player being punished. We must

therefore give him an incentive to cooperate in his own punishment. This goal is aided if we make the punishment finite in length, and follow it with a return to the cooperative outcome that we were originally trying to sustain. Then, if someone deviates during his punishment, we simply start the punishment again. Of course, a player being punished should not be tempted simply to play his best response and to accept the fact that they will never return to cooperation. These requirements are formalized in Theorem 1:

Theorem 1: A necessary condition for $(v_1, v_2) \in V^*$ to belong to some WRP set is that there exist pairs of actions (a_1^1, a_2^1) and (a_1^2, a_2^2) such that, if we define

$$(10) \quad \hat{v}_1^1 = \max_{a_1} g_1(a_1, a_2^1), \quad (v_1^1, v_2^1) = g(a_1^1, a_2^1)$$

and

$$(11) \quad (v_1^2, v_2^2) = g(a_1^2, a_2^2), \quad \hat{v}_2^2 = \max_{a_2} g_2(a_1^2, a_2),$$

then we have (see Figure 3)

$$(12) \quad \hat{v}_1^1 \leq v_1 \leq v_1^2 \quad \text{and} \quad \hat{v}_2^2 \leq v_2 \leq v_2^1.$$

Conversely, if there exist actions satisfying (10)-(12), where the inequalities of (12) hold strictly, then there exists $\underline{\delta} < 1$ such that, for all $\delta > \underline{\delta}$, (v_1, v_2) belongs to some WRP set.

Proof: We first establish sufficiency. Suppose that for $(v_1, v_2) \in V^*$ there exist actions satisfying (10)-(12), where (12) holds strictly. By definition, $v_i^i \leq \hat{v}_i^i$. Choose δ near enough 1 so that

$$(13) \quad (1-\delta)\bar{v}_i + \delta \hat{v}_i^i < v_i$$

and choose integer t_i so that (13) continues to hold when $\hat{v}_i^i$ is replaced by

$$(14) \quad \hat{v}_i^i = (1-\delta^{t_i})v_i^i + \delta^{t_i}v_i,$$

where $\hat{v}_i^i \geq \hat{v}_i^i$. Such a t_i (possibly infinite) exists when

δ is near 1 because, (i) for $t_i=1$, the right side of (14) is bigger than $\hat{v}_i^i$; (ii) for $t_i=\infty$, the right side is no greater than $\hat{v}_i^i$, and (iii) for δ near 1, the right side changes almost continuously as integer t_i varies.

Take

$$(15) \quad \hat{v}_j^i = (1-\delta^{t_i})v_j^i + \delta^{t_i}v_j, \quad j \neq i.$$

We now construct a WRP set containing (v_1, v_2) (see Figure 3); our construction parallels the one we devised for the Prisoner's Dilemma above. We first define an equilibrium, e^* , giving average payoffs (v_1, v_2) . We then construct two others, e^1 and e^2 , intended as punishments against players 1 and 2, respectively, for deviations from e^* .

To construct e^* , choose (a_1, a_2) such that $(v_1, v_2) = g(a_1, a_2)$. The

equilibrium begins in the "normal" phase. In this phase, each player i plays a_i . The normal phase continues indefinitely unless some player i deviates from a_i , in which case play switches to equilibrium e^i (if both players deviate simultaneously from (a_1, a_2) , the deviation is ignored). In equilibrium e^i , players play (a_1^i, a_2^i) for t_i periods (the "punishment" phase). After this punishment phase, play reverts to the equilibrium e^* . If player i deviates from the prescribed play during equilibrium e^i , then e^i begins again. If player j ($j \neq i$) deviates during e^i , play moves to equilibrium e^j .

Notice that, as desired, average payoffs from equilibrium e^* are (v_1, v_2) . Formulas (14) and (15) imply that at the beginning of equilibrium e^i the average payoffs are $(\hat{v}_1^i, \hat{v}_2^i)$, whereas continuation payoffs in e^i are convex combinations of $(\hat{v}_1^i, \hat{v}_2^i)$ and (v_1, v_2) .

To see that e^* , e^1 and e^2 are perfect equilibria, notice that formula (13) implies that neither player will deviate in the normal phase. Because $v_j < \hat{v}_j^i$ for $j \neq i$ (from (12) and (15)), (13) also implies that player j will not deviate during player i 's punishment phase. If player i deviates during his punishment phase, he can get at most $\hat{v}_i^i$ that period (from (10) and (11)). Since the punishment phase then begins again, moreover, his average payoff starting the following period is also $\hat{v}_i^i$, so his overall average payoff is no greater than $\hat{v}_i^i$. But if

instead he conforms to the prescribed strategy, his average payoff starting at any point in the punishment phase is bounded below by $\hat{v}_i^i$, since the "low" payoffs v_i^i of the punishment phase are followed by the "high" payoffs v_i . Hence, player i has no incentive to deviate when he is being punished. We conclude that S consisting of the average payoffs for e^* , e^1 , and e^2 , together with their continuation equilibrium payoffs, constitutes a WRP set. This establishes the sufficiency of (10)-(12) when the the inequalities are strict and δ is near 1.

To see that (10)-(12) are necessary we require a lemma that is proved in the Appendix.

Lemma: If S is a WRP set, then so is its closure.

Consider $(v_1, v_2) \in V^*$ such that

$$(16) \quad \text{for all } (\hat{a}_1, \hat{a}_2) \text{ with } g_1(\hat{a}_1, \hat{a}_2) \geq v_1, \max_{a_2} g_2(\hat{a}_1, a_2) > v_2.$$

If (v_1, v_2) is part of some WRP set S , then let (v_1^0, v_2^0) be that point in the closure of S such that

$$(17) \quad v_2^0 = \min \{ \tilde{v}_2 \mid (\tilde{v}_1, \tilde{v}_2) \text{ is in the closure of } S \}$$

and

$$(18) \quad v_1^0 = \max \{ \tilde{v}_1 \mid (\tilde{v}_1, v_2^0) \text{ is in the closure of } S \}.$$

From the Lemma, there exists a perfect equilibrium (σ_1^0, σ_2^0) , with corresponding average payoffs (v_1^0, v_2^0) , such that all continuation payoffs lie in the closure of S . Suppose that player 2 can deviate in the first period of (σ_1^0, σ_2^0) and obtain a payoff higher than v_2^0 . Then, from (17), his average payoff in subsequent periods is no less than v_2^0 , and so his overall average payoff is greater than v_2^0 , a contradiction. Hence, if (a_1^0, a_2^0) are the first period actions for

(σ_1^0, σ_2^0) , $\max_{a_2} g_2(a_1^0, a_2) \leq v_2^0 \leq v_2$. Thus (17) implies that

$g_1(a_1^0, a_2^0) < v_1 \leq v_1^0$. But then player 1's continuation payoff v_1^{oo} beginning in period 2 must exceed v_1^0 , i.e., $v_1^{oo} > v_1^0$. If v_2^{oo} is player 2's continuation payoff beginning in period 2, then, because (v_1^{oo}, v_2^{oo}) cannot Pareto-dominate (v_1^0, v_2^0) , $v_2^{oo} \leq v_2^0$. But this, together with $v_1^{oo} > v_1^0$, contradicts (17) and (18). Hence (v_1, v_2) cannot be part of a WRP set.

Q.E.D.

As we mentioned, the construction in Theorem 1 is a generalization of the argument showing that cooperation in the repeated Prisoner's Dilemma is consistent with renegotiation-proofness. If (10)-(12) hold strictly, then, for δ near 1, (v_1, v_2) can be sustained in a WRP way by an equilibrium in which (a) players play actions (a_1, a_2) (where $(v_1, v_2) = g(a_1, a_2)$), and, (b) if player i should deviate, he is "punished" for t_i periods in a way that gives player 2 a payoff higher than v_2 (inequality (12) guarantees that

this is possible), after which (c) players return to (a_1, a_2) . Conversely, if (10)-(12) fail to hold, then one of the players, say player 1, cannot be punished without also driving player 2's payoff below v_2 (i.e., we cannot have $\hat{v}_1^1 < v_1$ without also $v_2^1 < v_2$). Hence, any plan to "punish" player 1 for deviating will be renegotiated.

One can apply Theorem 1 to the Prisoner's Dilemma to show that, for that game, any point in V^* belongs to some WRP set when there is sufficiently little discounting (see also van Damme's [1986] direct demonstration). We can also use it to understand why the shaded region in the game of Figure 1 corresponds to the WRP payoffs in the game of Table 2. We have already argued that every WRP payoff vector is in the shaded region. To see the converse, choose (v_1, v_2) in this region. Let the actions used to punish player 1, (a_1^1, a_2^1) , be (T,R) (see Table 2). Then $(v_1^1, v_2^1) = (0, 4)$ and $\hat{v}_1^1 = 0$, so that $\hat{v}_1^1 < v_1$ and $v_2 < v_2^1$. Similarly, let the actions (a_1^2, a_2^2) used to punish player 2 be as follows: $a_2^2 = L$ and a_1^2 is a randomization between T, with probability $(v_2/4) - \epsilon$, and B, where $\epsilon > 0$. Then,

$$v_1^2 = 2\left(\frac{v_2}{4} - \epsilon\right) - 2\left(1 - \frac{v_2}{4} + \epsilon\right) = v_2 - 4\epsilon - 2.$$

But, for (v_1, v_2) in the shaded region, $v_2 - 2 > v_1$. Hence, for ϵ small, $v_1^2 > v_1$. Furthermore,

$$\hat{v}_2^2 = 4\left(\frac{v_2}{4} - \epsilon\right) = v_2 - 4\epsilon < v_2,$$

as (12) requires.

4. Duopoly

We can use Theorem 1 to investigate when collusive outcomes in duopolistic markets can be attained in a renegotiation-proof way. Let us start with Cournot duopoly.

Let demand be given by $D(p)=2-p$, and suppose that output is costless to produce. If firms choose quantities as strategies, then the feasible and individually rational set V^* is given by

$$V^* = \{(v_1, v_2) \mid v_1 > 0, v_2 > 0, v_1 + v_2 \leq 1\}.$$

If we did not require renegotiation-proofness, then, with sufficiently little discounting, any point on the Pareto frontier $\{(v_1, v_2) \in V^* \mid v_1 + v_2 = 1\}$ could be sustained as a perfect equilibrium of the repeated game. Let us investigate which points on the frontier are renegotiation-proof. If the point $(\pi, 1-\pi)$ is part of a WRP set, then Theorem 1 tells us that there must exist actions (a_1^1, a_2^1) (corresponding to a punishment against firm 1) such that

$$(19) \quad a_2^1(2 - a_1^1 - a_2^1) \geq 1 - \pi,$$

i.e., firm 2's payoff from punishing 1 must be greater than $1-\pi$, and

$$(20) \quad \max_{a_1} a_1(2 - a_1 - a_2^1) = \frac{1}{4}(2 - a_2^1)^2 \leq \pi,$$

i.e., firm 1's best response to a_2^1 yields a payoff lower than π .

From (20), $a_2^1 \geq 2 - 2\sqrt{\pi}$. Substituting this value into (19)¹¹ and

11. It should be clear that (19) could not be easier to solve if

solving for a_1^1 , we obtain

$$(21) \quad 2J\pi - \frac{(1-\pi)}{2-2J\pi} \geq a_1^1.$$

But (21) can be satisfied if and only if the left hand side is positive, i.e.

$$(22) \quad \frac{1}{9} \leq \pi \leq 1.$$

From (22) and the symmetric argument for punishments against player 2, Theorem 1 implies that the Pareto optimal point $(\pi, 1-\pi)$ is part of a WRP set for δ near 1 if and only if¹²

$$(23) \quad \frac{1}{9} \leq \pi \leq \frac{8}{9}.$$

Turning next to Bertrand duopoly, let us continue to assume that the demand function is $D(p)=2-p$ and that production is costless. If prices are strategies, then the firm setting the lower price captures the entire market. If their prices are the same, the firms split demand equally. As before the Pareto frontier is the set $\{(v_1, v_2) | v_1 + v_2 = 1, v_i > 0, i=1,2\}$. Rather than characterizing all points that belong to WRP sets, we shall show simply that $(\frac{1}{2}, \frac{1}{2})$ belongs to such a set for δ near 1. Now, $(\frac{1}{2}, \frac{1}{2})$ arises when both firms set a price of 1. It is clear that if firm 1 is to be punished for deviating from this price, firm 2 cannot use a pure strategy: any price that earns firm 2 a profit more than $\frac{1}{2}$ can be slightly undercut by firm 1, implying that firm 1 is not held below $\frac{1}{2}$. Suppose, however, that firm 1 is punished by firm 2's

we took a value of a_2^1 that satisfied (20) strictly.

12. For sufficiency, however, the inequalities should be strict.

randomizing 50-50 between $\frac{1}{4}$ and $\frac{3}{4}$. If, at the same time, firm 1 sets a price above $\frac{3}{4}$, then firm 2's expected profit is $11/16$, which exceeds $\frac{1}{2}$, as required by Theorem 1. Moreover, firm 1's best response to the randomization is to set a price just below $\frac{3}{4}$, resulting in an expected profit of $15/32$, which is less than $\frac{1}{2}$. Because the firms are symmetric, Theorem 1 thus implies that, when δ is big enough, $(\frac{1}{2}, \frac{1}{2})$ belongs to a WRP in which randomizing punishments are used.

5. Pareto Optimality

Presumably the players in a repeated game are interested in sustaining Pareto optimal outcomes, if possible. Although Theorem 1 characterizes those outcomes that are sustainable in a renegotiation-proof way for low discount rates, it does not make clear when Pareto optimality is consistent with WRP. We do not know whether there is always a WRP set containing a Pareto optimum for δ close to 1 (although we strongly conjecture that there is). However, our next result develops necessary and sufficient conditions under which there exists a WRP set consisting entirely of Pareto optima when there is not too much discounting.

Obviously, a necessary condition is that there should be an efficient point $v=(v_1, v_2)$ that can be sustained by punishment actions, $a^i=(a_1^i, a_2^i)$, $i=1,2$, that are efficient. (Although the continuation payoff from punishment is not the same as the payoff from the a 's, if the latter is inefficient then so is the former.) One might conjecture that this condition would also be sufficient,

but it is not. The reason is that the continuation payoffs during the punishment phase are not the payoffs from the a 's, but are convex combinations of those payoffs and the v 's. To be sure that S contains only efficient payoffs, we need the entire line segment between $g(a^i)$ and v to be on the frontier (see Figure 4). If the frontier "bows out" between $g(a^i)$ and v , then the construction fails. Theorem 2 formalizes this idea:

Theorem 2: Suppose that there exist two Pareto optima, (a_1^1, a_2^1) and (a_1^2, a_2^2) , such that (10)-(11) are satisfied,

$$(24) \quad \hat{v}_1^1 < v_1^2 \text{ and } \hat{v}_2^2 < v_2^1,$$

and there exist Pareto optimal payoffs (v_1^{oi}, v_2^{oi}) , $i=1,2$, such that for some $\hat{v}_j^i$, $(\hat{v}_1^i, \hat{v}_2^i)$ is Pareto optimal and a strict convex combination of (v_1^i, v_2^i) and (v_1^{oi}, v_2^{oi}) (see Figure 4). Then for δ greater than some $\underline{\delta} < 1$, there exists a WRP set consisting entirely of Pareto optima. Furthermore, the above conditions--where (24) holds weakly and the convex combinations need not be strict--are necessary for there to exist such a set.

Proof: The argument follows the proof of Theorem 1. To establish sufficiency, suppose that the hypotheses are satisfied. Because $(\hat{v}_1^i, \hat{v}_2^i)$ is a strict convex combination of (v_1^i, v_2^i) and (v_1^{oi}, v_2^{oi}) , we can, for δ near enough 1, choose t_i so that

$$(25) \quad (\hat{v}_1^i, \hat{v}_2^i) = (1 - \delta^i)(v_1^i, v_2^i) + \delta^i(v_1^{oi}, v_2^{oi}),$$

where $\hat{v}_2^i \leq \hat{v}_1^i$. Then equilibrium e^i consists of players choosing (a_1^i, a_2^i) for t_i periods, followed by actions leading to (v_1^{oi}, v_2^{oi}) thereafter. If player i deviates from this path, e^i starts again. If player j ($j \neq i$) deviates, play moves to equilibrium e^j . As the WRP set, we take the equilibrium payoffs for e^i ($i=1,2$) together with all continuation payoffs.

To establish the converse, consider a WRP set S consisting of Pareto optima. From our Lemma, its closure, $\bar{S}$, is also WRP. Let $(\hat{v}_1^i, \hat{v}_2^i)$ be the element of $\bar{S}$ that minimizes player i 's payoff (if there are multiple such elements, choose the one that maximizes player j 's payoff). In the equilibrium leading to payoffs $(\hat{v}_1^i, \hat{v}_2^i)$, let (a_1^i, a_2^i) be the actions chosen in the first period and take $(v_1^i, v_2^i) = g(a_1^i, a_2^i)$. Let (v_1^{oi}, v_2^{oi}) be the continuation equilibrium payoffs beginning in the second period of this equilibrium. It is then straightforward to verify that $(\hat{v}_1^i, \hat{v}_2^i)$, (a_1^i, a_2^i) , and (v_1^{oi}, v_2^{oi}) satisfy the hypotheses of the theorem.

Q.E.D.

Rubinstein [1980] defines a concept of strong perfect equilibrium (SPE). In a two-player game, a perfect equilibrium e^* is SPE if in no subgame does there exist a perfect equilibrium that Pareto-dominates the continuation equilibrium of e^* . Rubinstein provides an example of a repeated game in which no SPE exists. It

is clear, however, that when the hypotheses of Theorem 2 are satisfied, an SPE exists for δ near 1.

The hypotheses of Theorem 2 are satisfied by the Prisoner's Dilemma. Indeed, the WRP set we constructed in Section 3 consists entirely of Pareto optima. Let us next apply Theorem 2 to the Cournot model of Section 4. We saw before that, for the point $(\pi, 1-\pi)$ to belong to a WRP set, π must satisfy (23). If, in addition, the punishment against player 1 is to be Pareto efficient, then, without loss of generality, we can take a_2^1 in (19) and (20) to be 1 (and $a_1^1=0$). But then (20) reduces to $\frac{1}{4} \leq \pi$. Hence, a necessary (and, if the inequalities are strict, sufficient) condition for $(\pi, 1-\pi)$ to belong to a Pareto optimal WRP set is that

$$(26) \quad \frac{1}{4} \leq \pi \leq \frac{3}{4}.$$

Notice that (26) is more stringent than (22). This is because Pareto efficient punishments involve the punisher producing less output than do some punishments that sustain collusion close to the limits of (22). Since the punisher is producing less, the firm being punished can do better if it decides not to cooperate in its punishment. This in turn limits the severity of punishment, and hence limits the set of sustainable collusive arrangements.

We saw above that, in the Bertrand model, punishments for deviating from the joint monopoly price must involve the punisher randomizing if they are to be renegotiation-proof. This implies that the profit pair $(\frac{1}{2}, \frac{1}{2})$ cannot be part of a Pareto optimal WRP

set, nor, for the same reason, can any other pair $(\pi, 1-\pi)$. That is, Bertrand duopoly violates the hypotheses of Theorem 2.

6. Strongly Renegotiation Proof Sets

We acknowledged at the outset that the concept of a WRP set might be too weak to capture the idea of renegotiation proofness fully, since players could in principle renegotiate to points outside a given WRP set. Of course, for games satisfying the hypotheses of Theorem 2, this problem does not arise; if a WRP set consists entirely of Pareto optima, there are no opportunities for mutually advantageous renegotiation either inside or outside the set.

But for games like Bertrand duopoly, which violate the hypotheses, we need a concept that is stronger than weak renegotiation-proofness but not so strong as to require Pareto optimality. A natural criterion is to require optimality within the union of WRP sets. Formally, for any discount factor δ , let

$$\Sigma(\delta) = \{(v_1, v_2) \mid \text{there exists a WRP set } S \text{ with } (v_1, v_2) \in S \text{ when the discount factor is } \delta\}.$$

We shall call a set S strongly renegotiation proof (SRP) for discount factor δ if

- (i) it is a WRP set

and

- (ii) for all $(v_1, v_2) \in S$, there is no $(\hat{v}_1, \hat{v}_2) \in \Sigma(\delta)$ that strictly

Pareto dominates (v_1, v_2) .

Despite its strength, strong renegotiation-proofness is a less demanding concept than that of strong perfect equilibrium (defined in Section 4). Specifically, a candidate for SPE is compared with all other perfect equilibria for possible Pareto-domination; there is no requirement that potential "blocking" equilibria themselves be strongly perfect. By contrast, the points in an SRP set are compared only with payoffs in $\Sigma(\delta)$. The SRP concept, in effect, endows players with some far-sightedness: they will not switch to another equilibrium path that itself is subject to renegotiation.

An obvious first question about SRP sets is whether they exist. We noted earlier that a trivial WRP set always exists: the singleton consisting solely of the payoffs from a one-shot Nash equilibrium. This set is also SRP if δ is close enough to zero,¹³ but not, in general, for higher δ . Clearly, any WRP set consisting solely of Pareto optima is SRP regardless of the discount factor, but, as we have seen, there may be no such set. Of course, for any finite game and any discount factor δ , the Pareto boundary of $\Sigma(\delta)$ will always exist. Does that mean that SRP sets always exist too? Unfortunately, the answer is no, as the game in Table 3 illustrates.¹⁴

13. Provided that there is no Pareto-superior one-shot Nash equilibrium. Obviously, there is always a one-shot equilibrium satisfying this condition.

14. As we mentioned, Rubinstein [1980] gives an example of a repeated game that has no strong perfect equilibrium when δ is near 1. That game, however, does have an SRP set.

	y_1	y_2	y_3	y_4
x_1	-B, 6	-B, 6	14, 6	-B, 8
x_2	-B, 6	-B, 7	-B, 7	10, 2
x_3	-B, 6	4, 12	-B, 6	-B, 17
x_4	2, 10	-B, 6	-B, 12	-B, 6
x_5	0, -B	0, 6	0, -B	0, 0

Table 3

In Table 3, "B" represents a large positive number. In the Appendix we demonstrate that when $\delta = \frac{1}{2}$ we can choose B large enough so that no SRP set exists. The argument is fairly complicated but boils down to showing that any WRP set that contains points on the boundary of $\Sigma(\frac{1}{2})$ must be very close to either $\{(4,12), (7,7)\}$ or $\{(2,10), (8,8)\}$. The former, however, is not SRP because (7,7) is dominated by (8,8), whereas the latter is not SRP since (2,10) is dominated by (4,12). (The complication in the proof comes from showing that no WRP is much different from these two).

Nonexistence of SRP sets arises when, as in Table 3, outcomes on the boundary of $\Sigma(\delta)$ require threats in the interior of $\Sigma(\delta)$. We also note that such examples occur because of the need to provide both players with incentives to take efficient actions. In games--such as a multiperiod principal-agent relationship--in which at least one player's incentive constraint can be ignored, SRP sets necessarily exist, (see, for example, Nehring [1982]) i.e., there exist equilibria all of whose

continuation payoffs lie on the Pareto boundary of $\Sigma(\delta)$.¹⁵

The fact that a simple game such as that of Table 3 may not have an SRP is unsettling. There are, however, at least three plausible ways of relaxing the SRP requirement to restore existence.

First, following Bernheim-Ray [1987], we shall call a WRP set S Pareto Perfect (PP) if there is no alternative WRP set $\hat{S}$ such that some point in S is strictly Pareto-dominated by a point in $\hat{S}$ and no point in $\hat{S}$ is strictly Pareto-dominated by a point in S . Such PP sets may not exist, as Bernheim and Ray show, because it is possible for a set S_2 to dominate S_1 (in the sense we have discussed), for S_3 to dominate S_2 , and yet for S_1 to dominate S_3 . In such a cycle prevents existence, however, they propose calling each of the sets in a maximal cycle a PP set.

Given that there may be no WRP set consisting entirely of points from the Pareto frontier of $\Sigma(\delta)$, a second possibility is to work with subsets of $\Sigma(\delta)$ relative to which Pareto WRP sets do exist. Accordingly, define a WRP set S to be relative strongly RP (RSRP) if all its points lie on the Pareto frontier of a set $\hat{\Sigma}(\delta)$, where $\hat{\Sigma}(\delta)$ is a maximal subset of $\Sigma(\delta)$ relative to which there is a Pareto WRP set.

One advantage that RSRP has over SRP is that RSRP sets exist for

15. Roughly speaking, this is because if, say, player 1 does not face incentive constraints, then in any period, we can increase his component of any payoff vector in the interior of $\Sigma(\delta)$ until the boundary of $\Sigma(\delta)$ is reached. Because player 2's payoffs are not changed, his incentives in previous periods are not affected.

any δ , as the following result confirms.

Theorem 3: For any δ , an RSRP set exists.

Proof: The proof is an application of Zorn's Lemma. Fix δ . Consider a sequence of sets $\{\Sigma^n\}$ for which $\Sigma^1 \subset \Sigma^2 \subset \dots$ and such that, for each n , there exists a WRP set S^n consisting only of points from the Pareto-boundary of Σ^n . Without loss of generality, we can assume that S^n consists of the payoffs for some perfect equilibrium together with all its continuation payoffs. Let (σ_1^n, σ_2^n) be the corresponding equilibrium strategies. Arguing exactly as in the proof of the above Lemma, we can choose a "diagonal" subsequence $\{(\sigma_1^{n,n}, \sigma_2^{n,n})\}$ that converges to a perfect equilibrium (σ_1, σ_2) . Let S be the set consisting of the payoffs and continuation payoffs of (σ_1, σ_2) . Clearly, S is a WRP set. By construction, S lies on the Pareto frontier of $\bigcup_{n=1}^{\infty} \Sigma^n$. Hence the hypotheses of Zorn's Lemma are satisfied and an RSRP set exists.

Q.E.D.

Given its maximality property, an RSRP set approximates strong renegotiation-proofness as closely as possible while still being guaranteed to exist. (The concept of ϵ -SRP, defined below, also gets at the idea of approximating SRP, but in a different sense). In the game of Table 3, $\{(4,12), (7,7)\}$ and $\{(12,10), (8,8)\}$ are

examples of such sets when $\delta = \frac{1}{2}$. We do not yet have a general characterization of RSRP sets a la Theorem 1.

Finally, we can weaken the SRP concept by relaxing the Pareto-domination criterion. For any $\epsilon > 0$, a set of payoffs S is ϵ -SRP for discount factor δ if

- (i) it is WRP

and

- (ii) for any $(v_1, v_2) \in S$ there is no $(\hat{v}_1, \hat{v}_2) \in \Sigma(\delta)$ such that $\hat{v}_i - v_i > \epsilon$ for $i=1,2$.

Notice that an ϵ -SRP set S is almost SRP except that we allow points in S to be Pareto-dominated by a point in another WRP set as long as both players do not gain by more than ϵ . Happily, ϵ -SRP sets exist for δ near enough 1.

Theorem 4: For any $\epsilon > 0$ there exists $\underline{\delta} < 1$ such that for all $\delta > \underline{\delta}$ there exists an ϵ -SRP set for discount factor δ .

Proof: Fix ϵ . For each δ and $i=1,2$ choose the point $w^i(\delta) = (w_1^i(\delta), w_2^i(\delta))$ in $\Sigma(\delta)$ that maximizes player i 's payoff. These points exist because $\Sigma(\delta)$ is bounded and, from the argument proving the above Lemma, closed. If there is more than one such point, choose the one that minimizes player i 's payoff ($j \neq i$).

As $\delta \rightarrow 1$, $w^i(\delta)$ converges to some limiting value $w^i(1)$. Take $\underline{\delta}$ close enough to 1 so that $w^1(\underline{\delta})$ and $w^2(\underline{\delta})$ are within ϵ of $w^1(1)$ and $w^2(1)$, respectively.

Case I: $w^1(\underline{\delta}) = w^2(\underline{\delta}) = w(\underline{\delta})$.

Consider a WRP set (to which $w(\underline{\delta})$ belongs) and the corresponding equilibrium $e(\underline{\delta})$ giving rise to payoffs $w(\underline{\delta})$. Consider the actions $a(\underline{\delta}) = (a_1(\underline{\delta}), a_2(\underline{\delta}))$ that players use in the first period of $e(\underline{\delta})$. We claim that $a(\underline{\delta})$ is a Nash equilibrium of the one-shot game and that $g(a(\underline{\delta})) = w(\underline{\delta})$.

Now, $g_i(a(\underline{\delta})) \geq w_i(\underline{\delta})$ for $i=1,2$, otherwise player i 's continuation payoff $w_i^c(\underline{\delta})$ (beginning in period 2) exceeds $w_i(\underline{\delta}) (= w_i^i(\underline{\delta}))$, a contradiction of the definition of $w^i(\underline{\delta})$. If, say, $g_1(a(\underline{\delta})) > w_1(\underline{\delta})$, then,

$$(27) \quad w_1^c(\underline{\delta}) < w_1(\underline{\delta}),$$

and, thus, to avoid $w(\underline{\delta})$ Pareto-dominating $w^c(\underline{\delta})$, we must have

$$(28) \quad w_2^c(\underline{\delta}) = w_2(\underline{\delta}).$$

But, by assumption, $w(\underline{\delta}) (= w^2(\underline{\delta}))$ minimizes player 1's payoff among those vectors where 2's payoff is $w_2(\underline{\delta})$, a contradiction of (27) and (28). We conclude that $g(a(\underline{\delta})) = w(\underline{\delta})$.

If player 1 could deviate from $a(\underline{\delta})$ and obtain a higher payoff than $w_1(\underline{\delta})$, he would have to be punished by the players' moving to some equilibrium with payoffs $(\hat{v}_1, \hat{v}_2)$, where $\hat{v}_1 < w_1(\underline{\delta})$. But, by definition of $w(\underline{\delta})$, $\hat{v}_2 \leq w_2(\underline{\delta})$, and so to avoid Pareto-domination, $\hat{v}_2 = w_2(\underline{\delta})$. But then $(\hat{v}_1, \hat{v}_2)$ violates the assumption that $w(\underline{\delta})$

minimizes player 1's payoff among vectors where 2's payoff is $w_2(\underline{\delta})$.

Hence $a(\underline{\delta})$ is, after all, a Nash equilibrium, and the singleton $\{w(\underline{\delta})\}$ thus constitutes a WRP set. But $w(\underline{\delta})$ is on the boundary of $\Sigma(\underline{\delta})$, and so $\{w(\underline{\delta})\}$ is a SRP set.

Case II: $w^1(\underline{\delta}) \neq w^2(\underline{\delta})$.

Choose $(v_1, v_2) \in V^*$ such that

$$(29) \quad w_1^2(\underline{\delta}) < v_1 \leq w_1^1(\underline{\delta})$$

and

$$(30) \quad w_2^1(\underline{\delta}) < v_2 \leq w_2^2(\underline{\delta}).$$

Let (a_1^1, a_2^1) be the first period actions in the equilibrium corresponding to $w^2(\underline{\delta})$ (recall that $w^2(\underline{\delta})$ belongs to a WRP set, so that the equilibrium exists). By definition of $w_2^2(\underline{\delta})$,

$$(31) \quad g_2(a_1^1, a_2^1) \geq w_2^2(\underline{\delta}).$$

Moreover, to prevent Pareto-domination,

$$(32) \quad g_1(a_1^1, a_2^1) \leq w_1^2(\underline{\delta}).$$

Now, if player 1 could deviate from (a_1^1, a_2^1) and obtain a payoff higher than $w_1^2(\underline{\delta})$, he would have to be punished by the players'

moving to continuation payoffs $(\hat{v}_1, \hat{v}_2)$, where $\hat{v}_1 < w_1^2(\underline{\delta})$. But then, to prevent Pareto-domination, $\hat{v}_2 > w_2^2(\underline{\delta})$, a contradiction of the definition of $w_2^2(\underline{\delta})$. We conclude that

$$(33) \quad \max_{a_1} g_1(a_1, a_2^1) \leq w_1^1(\underline{\delta}).$$

Thus, (29) and (30)-(33) imply, via Theorem 1, that we can punish player 1 for deviating from the payoffs (v_1, v_2) in a way that is WRP. Similarly, we can punish player 2.

We conclude that $(v_1, v_2) \in \Sigma(\underline{\delta})$. Hence there exists a pair $v(\underline{\delta}) = (v_1(\underline{\delta}), v_2(\underline{\delta}))$ satisfying (29) and (30) on the boundary of $\Sigma(\underline{\delta})$. For $\underline{\delta}$ close enough to 1, $v(\underline{\delta})$ is within ϵ of the boundary of $\Sigma(\delta)$ for all $\delta > \underline{\delta}$. Following Theorem 1, we can construct a WRP set to which $v(\underline{\delta})$ belongs. In the equilibrium sustaining $v(\underline{\delta})$, player 1 is punished by players switching to (a_1^1, a_2^1) for a while and then returning to $v(\underline{\delta})$. If δ is close enough to 1 the equilibrium average payoffs $(\hat{v}_1^1, \hat{v}_2^1)$ at the start of player 1's punishment can be chosen to be very close to $v(\underline{\delta})$. The same thing can be done with the equilibrium average payoffs $(\hat{v}_1^2, \hat{v}_2^2)$ corresponding to 2's punishment. Hence for δ near 1, the WRP set consisting of $v(\underline{\delta})$, $(\hat{v}_1^1, \hat{v}_2^1)$, $(\hat{v}_1^2, \hat{v}_2^2)$, and the continuation payoffs is an ϵ -SRP set.

Q.E.D.

7. Finitely Repeated Games

To this point, our treatment of renegotiation has focused exclusively on infinitely repeated games. Our concepts, however, can be readily adapted to games that are repeated only finitely many times.

Many finitely repeated games are uninteresting. For example, with any finite number of repetitions, the only perfect equilibrium of the repeated Prisoner's Dilemma (see Table 1) consists of both players playing uncooperatively every period. Similarly, in any other game with a unique equilibrium, finitely repeated play leads to merely the repetition of that equilibrium.

As Benoit-Krishna [1985] and Friedman [1985] emphasize, however, matters are quite different if the underlying game has multiple equilibria that are not payoff-equivalent for either player. Indeed Benoit and Krishna establish an exact analogue of the Folk Theorem for games of this sort. To review how multiple equilibria help, let us add a row and column to the game of Table 1 so that appears as in Table 4.

	C	U	P
C	2,2	-1,3	-1,-1
U	3,-1	0,0	-1,-1
P	-1,-1	-1,-1	-1,-1

Table 4

If this game is played twice, then (C,C) can be sustained in the first period (assuming no discounting) by strategies that send players to (U,U) next period if they conform and to (P,P) if they do not. That is, the inferior equilibrium (P,P) is used as a threat and the superior equilibrium (U,U) as a reward.

Of course, once renegotiation is permitted, this two-period equilibrium seems likely to break down: in the second period, (P,P) will be renegotiated to (U,U) and so is not an effective threat. To formalize the concept of renegotiation proofness, we can proceed in much the same way as before. Because the set of equilibria in a finite game is not invariant with respect to time, we naturally can no longer impose the stationarity of the earlier definition, but otherwise the concept of a WRP set remains unchanged.

Formally, consider a T -period repeated game (without discounting). For each integer t between 1 and T let S_t be a set of pairs of payoffs. We shall call the list $S=(S_1, S_2, \dots, S_T)$ weakly renegotiation-proof if

- (i) for all t and all $(v_1, v_2) \in S_t$ there exists a perfect equilibrium σ of the t -period repeated game whose average payoffs are (v_1, v_2) and whose continuation average payoffs with r periods remaining lie in S_r for all $r \leq t$;

and

- (ii) for all t , no point in S_t strictly Pareto-dominates any other point in S_t .

Thus, in violation of stationarity, the set of "agreeable" payoffs depends on how many periods t remain, and the Pareto

domination criterion applies only within each set S_t

Because stationarity is not invoked, this new definition is clearly weaker than the old one.¹⁶ If, for example, we apply its natural extension for infinitely repeated games to Table 4, we find that the equilibrium in which players play (U,U) in the first period and (P,P) forever after is now renegotiation-proof, whereas it was not before. Nonetheless, the concept is strong enough to rule out the above two-period cooperative equilibrium. Moreover, one can verify that no equilibrium where players ever play (C,C) can be part of a WRP set in any finitely repeated version of Table 4.

Happily, this discouraging conclusion does not apply to all finitely repeated games. Indeed, let us modify Table 4 to obtain Table 5. Suppose that the game is

	C	U	P ₁	P ₂
C	2,2	-1,3	-1,0	0,-1
U	3,-1	0,0	-1,0	0,-1
P ₁	-1,0	-1,0	-1,0	-1,-1
P ₂	0,-1	0,-1	-1,-1	0,-1

Table 5

played three times. Consider the equilibrium in which (C,C) is

16. It has the virtue, however, of being applicable to dynamic games in general and not just those that, like infinitely repeated games, are stationary.

sustained in the first period by strategies that, if the players conform, have them play (P_1, P_1) and (P_2, P_2) in the subsequent two periods, but that, if player i deviates, have them play (P_i, P_i) in both remaining periods. Clearly, this equilibrium induces a WRP set.

As before, we can demand that the points in a strongly renegotiation-proof set be Pareto optimal among all agreeable points. That is, if Σ_t is the set of all average payoffs for a t -period repeated game, then a WRP set $S(=(S_1, \dots, S_T))$ is SRP for the T -period game if, for all $t \leq T$ and all $(v_1, v_2) \in S_t$, (v_1, v_2) lies on the Pareto boundary of Σ_t . Unfortunately, we again encounter the previous existence problem. The three-period version of Table 5, for example, has no SRP set. This is because, in the first period, Pareto efficient outcomes such as (C, C) can be sustained but only by following a subsequent path that is Pareto-dominated by the path (U, U) , (U, U) . In other words, we cannot be on the Pareto frontier in all periods.¹⁷

One way around this difficulty is to appeal to the weaker concept of relative strongly renegotiation-proofness (RSRP) that we introduced in Section 6. For a finitely repeated game, this amounts

17. Such a conclusion can be drawn because, in the game of Table 5, both players must be provided with incentives to take efficient actions (see the discussion following Theorem 2). If, instead, player 1, say, could commit himself to actions in advance, the players could sustain the sequence (U, C) , (C, U) , (C, U) . (If player 2 deviated from C in the first period, he could be punished by the continuation path (C, C) , (C, U) ; and if he deviated from C in the first period of the punishment, he could be punished by (U, U)).

to imposing relative Pareto optimality recursively from the end: a WRP set $S=(S_1, \dots, S_T)$ is RSRP if (i) all points in S_1 are on the Pareto boundary of Σ_1 ; (ii) all points in S_2 are on the Pareto boundary of the set of payoffs corresponding to equilibria whose continuation payoffs lie on the Pareto boundary of Σ_1 ; etc. That is, an equilibrium today is Pareto optimal relative to the constraint that tomorrow's continuation equilibrium must be (relatively) Pareto optimal. Clearly, existence is guaranteed by backwards induction. In fact, the concept of RSRP is, for two-person, finitely repeated games, essentially the same as that of perfect coalition-proof equilibrium, due to Bernheim, Peleg, and Whinston [1987].

8. Other Work on Renegotiation

There is by now a considerable literature dealing with the issue of renegotiation. We have already mentioned the work on repeated games most closely related to this paper: Rubinstein [1980], van Damme [1986], Bernheim-Ray [1987], and Bernheim-Peleg-Whinston [1987]. Although their model is not actually a repeated game, Maskin-Tirole [1987] apply what amounts to Rubinstein's strong perfect equilibrium concept to an analysis of a dynamic price setting duopoly. Assuming that firms (who sell a homogeneous good) can change prices only with a lag, they show that, with sufficiently little discounting, there is a unique symmetric Markov perfect equilibrium,¹⁸ which involves behavior closely resembling that in

18. A perfect equilibrium is "Markov" if players' strategies

the traditional kinked-demand curve.

As we have mentioned, there is at least one concept of renegotiation-proofness for repeated games that is quite different from that developed here. Pearce[1987] calls a symmetric equilibrium e of an infinitely repeated game renegotiation-proof if there exists no other symmetric equilibrium $\hat{e}$ whose worst continuation equilibrium payoff vector Pareto-dominates the worst continuation equilibrium payoffs of e . In other words, players will switch from one equilibrium to another only if, by so doing, they reduce the severity of the worst possible equilibrium payoffs. Pearce shows that, as δ converges to 1, the loss in welfare imposed by renegotiation-proofness vanishes. He also proposes a related definition for asymmetric games, for which he establishes a similar welfare result.

The other major research line on renegotiation has been in the theory of contracts. The work divides between the cases of symmetric and asymmetric information.

In the former case, it is assumed that, although parties will eventually have complete information, they cannot make their contract contingent on this information (it is "observable" but not "verifiable"). They must, therefore, design some sort of game to ensure that the outcome properly reflects what they know. The contract specifies the rules of this game. However, if, when the

depend only on the payoff-relevant variables and not on the entire history of past actions.

contract is carried out, it results in an outcome that is not Pareto optimal, parties have an incentive ex post to negotiate a new contract. This incentive to renegotiate constrains what can be accomplished by the original contract. Hart-Moore [1985] study this issue in a simple buyer-seller model. Maskin-Moore [1987] examine the question in a general, abstract framework. Huberman-Kahn [1986] and Green-Laffont [1986] consider renegotiation when there are limitations on how complex a contract can be.

In the case of contracts with asymmetric information the emphasis has been on renegotiation due to the creation of Pareto-improving moves when (some of) this information is revealed in the course of carrying out the contract. Work here includes Holmstrom-Myerson [1983], Baron-Besanko [1985], Green-Laffont [1987], Hart-Tirole [1987], Dewatripont [1986], and Laffont-Tirole [1985].

9. Concluding Remarks

Throughout this paper, our analysis has been limited to two-player games. We mentioned in the introduction that, formally speaking, our results generalize. In particular, Theorems 1-3 and the Bertrand and Cournot examples all extend immediately to three or more players. Nevertheless, one reason why we have refrained from a more general presentation is that with more players our Pareto domination criterion is, by itself, no longer so compelling. Given at least three players, coalitions smaller than the set of all players are possible. Thus, a potential improvement by all players need not be a prerequisite to renegotiation; a proper subset may

profit from renegotiating by themselves. The concepts of strong perfect equilibrium (Rubinstein [1980]) and perfect coalition-proof equilibrium (Bernheim-Peleg-Whinston [1987]) are attempts to come to grips with this wider scope for renegotiation. However, both appear to be too strong to permit the existence of equilibrium reasonably generally. A more successful concept (in the sense of being compelling but non-empty) still remains to be found.

Our other principal restriction has been to games of complete information and perfect observability. There is no great conceptual difficulty extending our definitions to cover the cases where players have private information or where their actions cannot be observed. Indeed, in the latter case, the definitions remain unchanged. In the former case, the possibility that information is revealed over time destroys stationarity, so we must use a definition closer to that of Section 7 than that of Section 2. Moreover, because of the asymmetry of information, we must replace sub game perfect equilibrium with some species of perfect Bayesian equilibrium, e.g., trembling hand perfect (Selten [1975]) or sequential equilibrium (Kreps-Wilson [1982]). Most important, the renegotiation possibilities at any point in general depend on how much private information has been revealed so far. But, conversely, how much information players choose to reveal will depend on how they forecast the renegotiation will proceed. Thus, to know how players will behave, we have to be more explicit about the renegotiation process itself (for examples of specific renegotiation processes see Holmstrom-Myerson [1983], Dewatripont [1986], and

Hart-Tirole [1987])).

Whether or not analogues to Theorem 1-3 exist with incomplete information or imperfect observability remains conjectural. Given the constructive nature of these theorems, however, their extension would seem to be highly desirable and particularly useful for applications.

One application that appears to generalize readily to imperfect observability is the Cournot duopoly of Section 4. Consider, for instance, the Green-Porter [1984] extension, where firms observe only the market price, which is a stochastic function of total output. Green and Porter consider collusive strategies that have players revert to a one-shot equilibrium should the realized price fall too low. Of course, these strategies are not renegotiation-proof. But, a modification in which some low prices induce a continuation equilibrium in which firm 1's output is high and 2's is low (i.e., firm 2 is "punished"), whereas other low prices give rise to a punishment equilibrium against firm 1 ought to overcome this defect. We shall defer a careful analysis of this issue to future work.

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Appendix

Lemma: If S is a WRP set, then so is its closure.

Proof: The proof is a standard Cantor diagonalization. Consider (v_1, v_2) in the closure of S . By definition of S , there exists a sequence of perfect equilibria $\{(\sigma_1^n, \sigma_2^n)\}$ such that (i) the corresponding average payoffs converge to (v_1, v_2) , and (ii) for each point in the sequence, all continuation payoffs lie in S . Because the (pure) action spaces are finite, there exists a subsequence $\{(\sigma_1^{n,1}, \sigma_2^{n,1})\}$ such that (a) for any n , the average payoffs corresponding to $(\sigma_1^{n,1}, \sigma_2^{n,1})$ are within $\frac{1}{n}$ of (v_1, v_2) , and (b) the strategies for period 1 converge and, for any n , the period 1 strategies for $(\sigma_1^{n,1}, \sigma_2^{n,1})$ are within $\frac{1}{n}$ of the limiting strategies (using the Euclidean norm). Continuing iteratively, for any t , let $\{(\sigma_1^{n,t+1}, \sigma_2^{n,t+1})\}$ be a subsequence of $\{(\sigma_1^{n,t}, \sigma_2^{n,t})\}$ such that the strategies for period $t+1$ converge and, for any n , the period $t+1$ strategies for $(\sigma_1^{n,t+1}, \sigma_2^{n,t+1})$ are within $\frac{1}{n}$ of the limiting strategies. Now, take $(\sigma_1^n, \sigma_2^n) = (\sigma_1^{n,n}, \sigma_2^{n,n})$. By construction, the sequence $\{(\sigma_1^n, \sigma_2^n)\}$ converges point-wise to some pair (σ_1, σ_2) . The payoffs corresponding to (σ_1, σ_2) are (v_1, v_2) . Because $\delta < 1$, for any ϵ , we can choose n big enough so that (σ_1, σ_2) is close enough to (σ_1^n, σ_2^n) within the first n periods to constitute an ϵ -perfect equilibrium. Hence (σ_1, σ_2) is a perfect equilibrium. By construction, it, together with its continuation equilibria and the elements of S , satisfy the requisite Pareto-domination properties.

Q.E.D.

	y_1	y_2	y_3	y_4
x_1	-B, 6	-B, 6	14, 6	-B, 8
x_2	-B, 6	-B, 6	-B, 7	10, 2
x_3	-B, 6	4, 12	-B, 6	-B, 17
x_4	2, 10	-B, 6	-B, 12	-B, 6
x_5	0, -B	0, 6	0, -B	0, 0

Table 3

Claim: When $\delta = \frac{1}{2}$, we can choose B large enough so that no SRP set exists.

Proof: We first show that $\{(4,12), (7,7)\}$ and $\{(2,10), (8,8)\}$ are WRP sets when $\delta = 1/2$. For the first set observe that the average payoffs (4,12) can be obtained by the players choosing (x_3, y_2) forever. Player 1 has no incentive to deviate. Should player 2 deviate, the players then move to payoffs (7,7). This will deter him from deviating because the greatest one-shot payoff he could obtain from deviating is 17, in which case his overall average payoff would be $\frac{1}{2}(17) + \frac{1}{2}(7) = 12$, which is no higher than what he gets if he conforms. Payoffs (7,7) are attained by the players choosing (x_2, y_4) for 1 period, followed by (x_3, y_2) forever. Player 1 has no incentive to deviate in the first period, whereas, if player 2 deviates, he is punished by the players restarting the (7,7) equilibrium. For the

second set, note that the average payoffs (2,10) arise when the players choose (x_4, y_1) forever. Player 2 is punished by switching to the payoffs (8,8). These latter payoffs derive from one period of (x_1, y_3) followed by (x_4, y_1) forever. Player 2 is punished by restarting the (8,8) equilibrium.

Suppose that, contrary to our assertion, there existed an SRP set S for discount factor $1/2$. The argument (see the above Lemma) that established that the closure of a WRP set is WRP also proves the analogous result for SRP sets. Thus, we might as well suppose that S is closed. Consider the point $(\hat{v}_1, \hat{v}_2)$ in S that minimizes player 2's payoff (if there are several such points, choose the one that maximizes player 1's payoff). Let $(\hat{v}_1^f, \hat{v}_2^f)$ and $(\hat{v}_1^c, \hat{v}_2^c)$ be the first period and continuation payoffs in the equilibrium $\hat{e}$ corresponding to $(\hat{v}_1, \hat{v}_2)$. Then,

$$(A1) \quad (\hat{v}_1, \hat{v}_2) = \frac{1}{2}(\hat{v}_1^f, \hat{v}_2^f) + \frac{1}{2}(\hat{v}_1^c, \hat{v}_2^c).$$

Notice in Table 3 that the payoffs (14,6), (10,2), (4,12), (2,10), (0,6), and (0,0) are the only pairs not involving a "-B". Thus, in the first period of $\hat{e}$, players must choose pairs of actions close (in a probabilistic sense) to (i) x_1 and y_3 , (ii) a randomization between x_2 and x_5 and y_4 , (iii) x_4 and y_1 , (iv) a randomization between x_3 and x_5 and y_2 , or (v) x_5 and a randomization between y_2 and y_4 . Otherwise, the first period expected payoff for one of the players will be highly negative, which is impossible since (A1) would then imply that the corresponding continuation payoff is very large (note that $\hat{v}_1 \geq 0$ and

$\hat{v}_2 \geq 3$, since $(0,3)$ is the minimax point).

Suppose first that $(\hat{v}_1^f, \hat{v}_2^f)$ corresponds to (iv) or (v) with high probability. Then $\hat{v}_1^f \leq 4$, and, because $\hat{v}_1 \leq \hat{v}_1^f$, $\hat{v}_1 \leq 4$. Thus, for $(\hat{v}_1, \hat{v}_2)$ to avoid Pareto-domination by $(4,12)$, we must have $\hat{v}_2 \geq 12$. But this implies that v_2^f must be near 12 (otherwise, (A1) implies that v_2^c be considerably greater than 12, which is infeasible when B is large). Hence, in the first period, players must be using actions (x_3, y_2) with high probability. But then player 2 can deviate to y_4 and obtain a payoff of approximately 17, which implies that $\hat{v}_2 \geq 17$, an impossibility. Hence, $(\hat{v}_1^f, \hat{v}_2^f)$ cannot correspond to (iv) or (v). Similarly, if $(\hat{v}_1^f, \hat{v}_2^f)$ corresponds to (iii) with high probability, then $\hat{v}_2 \geq 12$ and so $\hat{v}_2^c \geq 14$, which is also impossible. We conclude that either

(A2) $(\hat{v}_1^f, \hat{v}_2^f)$ corresponds to (i) with high probability

or

(A3) $(\hat{v}_1^f, \hat{v}_2^f)$ corresponds to (ii) with high probability.

Now, if (A2) holds, then player 2 can deviate to y_4 , obtaining a payoff of approximately 8. Hence (A2) implies that

(A4) $\hat{v}_2 \geq 8$ and $\hat{v}_2^c \geq 10$.

Similarly, if (A3) holds, then player 2 can deviate to y_2 , obtaining a payoff of at least 6 (actually, it must be strictly greater than 6, since otherwise player 1 must play x_5 with

probability near 1, implying that $(v_1^f, v_2^f) \approx 0$, $\hat{v}_2 \geq 6$, and, hence, $\hat{v}_1^c \approx 0$ and $\hat{v}_2^c \geq 12$, an impossibility). Thus (A3) implies that

$$(A5) \quad \hat{v}_2 > 6 \quad \text{and} \quad \hat{v}_2^c \geq 12.$$

Let $(\hat{v}_1, \hat{v}_2)$ be the point in S that minimizes player 1's payoff, and define $(\hat{v}_1^f, \hat{v}_2^f)$ and $(\hat{v}_1^c, \hat{v}_2^c)$ to be the corresponding first period and continuation payoffs. Clearly we must have $\hat{v}_2^f \geq \hat{v}_2 \geq \hat{v}_2^c$. Moreover,

$$(A6) \quad \hat{v}_1 \leq \hat{v}_1^f \quad \text{and} \quad \hat{v}_2 \geq \hat{v}_2^c.$$

By the same argument as before, $(\hat{v}_1^f, \hat{v}_2^f)$ must correspond to one of (i)-(v) with high probability.

If $(\hat{v}_1^f, \hat{v}_2^f)$ corresponds to (i), then because $\hat{v}_2^f \geq \hat{v}_2$, $\hat{v}_2 \leq 6$, a contradiction of (A4)-(A6). We obtain the same contradiction if $(\hat{v}_1^f, \hat{v}_2^f)$ corresponds to (ii) or (v)

If $(\hat{v}_1^f, \hat{v}_2^f)$ corresponds to (iii), then

$$(A7) \quad (\hat{v}_1^f, \hat{v}_2^f) \approx (2, 10).$$

Now if (A3) holds, then (A5) implies that $\hat{v}_2^c \geq 12$, which, in view of (A6), contradicts (A7). Hence, (A2) and (A4) hold. Formula (A6) and the fact that $\hat{v}_2^c \geq 10$ imply that

$$(A8) \quad \hat{v}_2 \approx 10.$$

Because $(4, 12) \in \Sigma(\frac{1}{2})$, (A8) implies that, to avoid Pareto-domination,

$$(A9) \quad \hat{v}_1 \geq 4.$$

Now (A7)-(A9) imply

$$(A10) \quad \hat{v}_1^c \geq 6 \text{ and } \hat{v}_2^c \approx 10.$$

Let (v_1^{of}, v_2^{of}) and (v_1^{oc}, v_2^{oc}) be the first period and continuation payoffs corresponding to $(\hat{v}_1^c, \hat{v}_2^c)$. The pair (v_1^{of}, v_2^{of}) corresponds to one of the same five possibilities as above. It cannot correspond to (i) or (ii), otherwise $v_2^{oc} > 14$, which is infeasible. Suppose that it corresponds to (iv). Then, to avoid having (v_1^{of}, v_2^{of}) be Pareto-dominated by $(\hat{v}_1^c, \hat{v}_2^c)$, player 1 must play x_3 with probability $p \geq 2/3$. Then, if player 2 deviates from y_2 , where he obtains a payoff of about $12p + 6(1-p)$, he can obtain a payoff of approximately $17p$. To deter such deviation, it must be possible to punish 2 with a payoff no greater than $7\frac{1}{3}$, a contradiction of (A5). If (v_1^{of}, v_2^{of}) corresponds to (v), then it is Pareto-dominated by $(\hat{v}_1^c, \hat{v}_2^c)$, an impossibility. We conclude that (v_1^{of}, v_2^{of}) must correspond to (iii). Thus, from (A10), $v_1^{oc} \geq 10$ and $v_2^{oc} \approx 10$, which is infeasible.

We conclude that $(\hat{v}_1^f, \hat{v}_2^f)$ must correspond to (iv). Now $\hat{v}_2^f \geq \hat{v}_2^c$ and so, from (A4) and (A5), $\hat{v}_2^f \geq 10$. This implies that the probability that player 1 plays x_3 in the first period must be at least $2/3$. Hence from the above argument,

$$(41) \quad \hat{v}_2 \leq 7\frac{1}{3}.$$

But (A11) contradicts (A4). Hence, (A3) and (A5) hold. Now the fact that $\hat{v}_2^f \leq 12$ together with (A5) imply

$$(A12) \quad \hat{v}_2^c = 12.$$

Hence from Table 4,

$$(A13) \quad \hat{v}_1^c \leq 4.$$

But, from (A3), $\hat{v}_1^f \leq 10$. Hence (A13) implies

$$(A14) \quad \hat{v}_1 \leq 7.$$

But (A11) and (A14) imply that $(\hat{v}_1, \hat{v}_2)$ is strictly Pareto-dominated by $(8, 8)$, which is in $\Sigma(\frac{1}{2})$. Hence, there cannot exist an SRP set.

Q.E.D.

Figure 1

A Weak Renegotiation-proof Set in the Prisoner's Dilemma

Figure 2

Renegotiation-proof Points in the Game of Table 2

Figure 3
The Construction in Theorem 1

Figure 4
The Construction in Theorem 2

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