When are Humans Reasoning with Modus Tollens?

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Abstract

Modus tollens is a rule of inference in classical, two-valued logic which allows to derive the negation of the antecedent from a conditional and the negation of its consequent. In this paper, we investigate when humans draw such conclusions and what modulates the application of modus tollens. We consider conditionals which may or may not be obligations and which may or may not have necessary antecedents. We show that humans make significantly more modus tollens inferences in case of obligation conditionals and that the time to make a modus tollens inference is shorter than the time to answer "nothing follows". We illustrate how these differences can be modeled within the weak completion semantics.

Keywords: human reasoning; conditional reasoning; modus tollens; mental model theory; weak completion semantics

Introduction

Conditional reasoning is a daily routine in science, technology, law, and many more areas. One common rule of inference is modus tollens, which allows to derive $\neg A$ from a conditional sentence *if A then C* and the negative sentence $\neg C$. In this paper, we consider a pragmatic classification of conditionals and investigate to what extend such a classification modulates modus tollens inferences. Taking the classification into account, we rigorously develop a computational theory to model modus tollens inferences within the weak completion semantics (WCS) and compare it to the mental model theory (MMT). Both cognitive theories generate models and reason with respect to them, but the generation of the models is quite different. WCS predicts that humans significantly more often conclude that *nothing follows* in case of factual conditionals. Moreover, it predicts that it takes humans longer to generate the answer *nothing follows* than the answer $\neg A$. We specify an experiment to validate these predictions. An analysis of the data gathered in the experiment confirms the predictions.

Examples Modus tollens inferences have been extensively studied in the literature. What follows from

1. *if there is the letter d on one side of the card, then there is the number 3 on the other side* and *there is the number 7 on the other side* (Wason, 1968),
2. *if a person is drinking beer, then the person must be over 19 years of age* and *the person is 16 years old* (Griggs & Cox, 1982),
3. *if she has an essay to write, then she will study late in the library* and *she will not study late in the library* (Byrne, 1989),
4. *if Mary is in Dublin, then Joe is in Limerick* and *Joe is in Cambridge* (Johnson-Laird, Byrne, & Schaeken, 1992),
5. *if Paul rides a motorbike, then he must wear a helmet* and *Paul is not wearing a helmet* (Byrne, 2005),
6. *if Nancy rides her motorbike, then she goes to the mountains* and *Nancy did not go to the mountains* (Byrne, 2005),
7. *if it is cloudy, then it is raining* and *it is not raining* (Khemlani, Byrne, & Johnson-Laird, 2018),
8. *if it rains, then the streets are wet* and *the streets are not wet* (Dietz Saldanha, Hölldobler, & Lourêdo Rocha, 2017),
9. *if it rains, then she takes her umbrella* and *she does not take her umbrella* (Dietz Saldanha et al., 2017)?

In the experiments, the findings are quite diverse: Wason (1968) reports that only 25% of the participants selected the card showing the number *7* to confirm that the letter *d* does not occur on the other side in Example 1. Griggs and Cox (1982) report that 80% of the participants selected the card showing *16* to confirm that this person is not drinking alcoholic beverages in Example 2. Byrne (1989) reports that 92% of the participants answered *yes* when asked whether *she does not have an essay to write* in Example 3. Johnson-Laird et al. (1992) report that in two experiments 38% and 56% of the participants made modus tollens inferences. Quelhas and Byrne (2003) report that in two experiments comparing reasoning using factual and obligation conditionals, the participants made in 46% and 68% as well as in 61% and 72% of the cases, respectively, correct modus tollens inferences.

Others present quite diverse arguments: Khemlani et al. (2018) argue that in Example 7 reasoners often make the erroneous response *nothing follows* whereas the valid conclusion is *it is not cloudy*. Byrne (2005) argues that there are two routes to the modus tollens inference for obligation and factual conditionals within the mental model theory which can explain the different numbers found in experiments (Quelhas & Byrne, 2003). Dietz Saldanha et al. (2017) argue that there should be a qualitative difference between the conclusions

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drawn in the Examples 8 and 9. They suggest that in Example 8 the answer should be *it is not raining*, whereas in Example 9 it should be *it is unknown whether it is raining*.

So we see that there is a large variety between conditionals, the conclusions drawn by participants, and possible explanations in the literature. This calls for a systematic analysis, which is the goal of this paper. It is organized as follows: In the next section we we classify conditionals as obligation or factual ones with necessary or non-necessary antecedents; such a classication is pragmatic. Thereafter, we show how modus tollens inferences are modeled in the MMT and the WCS. We develop two hypothesis based on the predictions of the WCS and specify an experiment to validate them. A discussion concludes the paper.

A Classification of Conditionals

Obligation versus Factual Conditionals A conditional whose consequent appears to be obligatory is called an *obligation conditional*. As pointed out by Byrne (2005), for each obligation conditional there are two initial possibilities people think about. The first possibility is the conjunction of the antecedent and the consequent, and is permitted. The second possibility is the conjunction of the antecedent and the negation of the consequent, and is forbidden. This holds for deontic obligations, i.e. legal, moral, or societal obligations of a person to perform certain actions, or naive physical obligations that cannot be avoided under normal circumstances. The fact that the consequence is obligatory may be explicitly marked with a word like *must*, but this is not necessary. The conditionals in Ex. 2, 3, 5, and 8 appear to be obligations.

If the consequent of a conditional is not obligatory, then it is called a *factual conditional*. There is no forbidden possibility. This appears to holds for Examples 1, 4, 6, 7, and 9.

Necessary versus Non-Necessary Antecedents The antecedent *A* of a conditional *if A then C* is said to be *necessary* if and only if its consequent *C* cannot be true unless *A* is true. But *A* may be true while *C* is not. This holds for Examples 1, 3, 7, and 8, whereas the antecedents of Examples 2, 4, 5, 6, and 9 appear to be non-necessary.

Pragmatics Humans may classify conditionals as obligation or factual conditionals and antecedents as necessary or non-necessary. This is an informal and pragmatic classification. It depends on the background knowledge and experience of a person as well as on the context. E.g., the conditional *if it is cloudy, then it is raining* from Example 7 may be classified as an obligation conditional (with necessary antecedent) by people living in Java, whereas it may be classified as a factual conditional by people living in Central Europe.

Reasoning in the Mental Model Theory

We illustrate the reasoning process in MMT as defined by Johnson-Laird and Byrne (2002) using Example 5. The initial mental model of the conditional *if Paul rides a motorbike, then he must wear a helmet* is

paul_riding   *helmet*

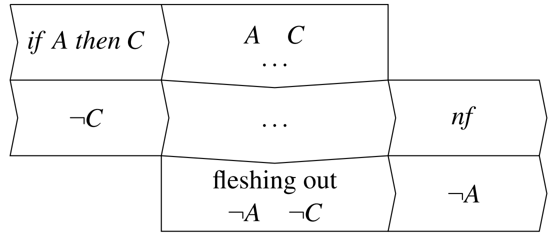

Figure 1: Modus tollens inference in MMT. The left column shows the premises, the middle column the mental models, and the right column the generated responses. The ellipsis denotes the implicit models, where *A* is false.

But there is the alternative that *Paul does not ride a motorbike*. This possibility is not worked out at this stage. Rather a mental footnote is made, which is depicted as an ellipsis in Figure 1. An attempt to add the second premise *Paul is not wearing a helmet* (¬*helmet*) eliminates the initial model. If at this stage a reasoner forgets the mental footnote or forgets to work out the mental footnote, then *nothing follows (nf)*, an erroneous response that reasoners often make (Johnson-Laird et al., 1992). However, working (or *fleshing*) out the mental footnote yields

¬paul_riding   *helmet*
¬paul_riding   *¬helmet*

and an attempt to add the second premise again eliminates all but the last possibility. Consequently, *Paul is not riding a motorbike* can be concluded. Considering the line of reasoning by MMT it appears that the time to answers *nothing follows* should be shorter than the time to generate the answer *Paul is not riding a motorbike* as the latter requires a run through the fleshing out process.

Example 5 is an obligation conditional with non-necessary antecedent. The same line of reasoning can be applied to obligation conditionals with necessary antecedents like Example 8, and factual conditionals with necessary or non-necessary antecedents like Examples 6 and 7, respectively. Background knowledge may modulate the reasoning process by blocking or preferring models, but it is not obvious how this is related to the classification of conditionals.

Byrne (2005) suggests that in the case of obligation conditionals there is a second route to modus tollens inferences which is initialized with the permitted and the forbidden possibility (see Figure 2). Returning to Example 5 we initially find

paul_riding   *helmet*   (permitted)
paul_riding   *¬helmet*   (forbidden).

An attempt to add the second premise eliminates the first possibility. Because the second one is forbidden and *¬helmet* holds, *¬paul_riding* can be concluded. In case of a factual conditional, the forbidden possibility does not exist and, consequently, the second route cannot be taken. Byrne (2005) argues that this may explain why there are more modus tollens

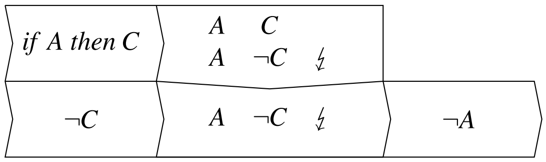

Figure 2: A second route for obligation conditionals in MMT following Byrne (2005). ↯ marks the forbidden possibility.

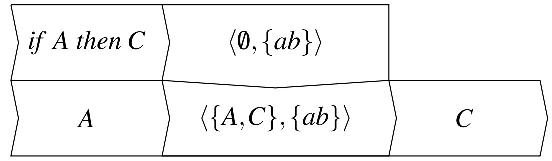

Figure 3: Modus ponens inference in WCS. The left column shows the premises, the middle column the least models, and the right column the generated responses.

inferences for obligation conditionals than for factual conditionals as reported by Quelhas and Byrne (2003).

Reasoning in the Weak Completion Semantics

The WCS is a rigorously defined formal theory, which is three-valued, non-monotonic, and has been shown to adequately model the average case in various human reasoning tasks like the suppression task (Dietz, Hölldobler, & Ragni, 2012) or human syllogistic reasoning (Oliviera da Costa, Dietz Saldanha, Hölldobler, & Ragni, 2017). It constructs three-valued models and reasons with respect to these models. But are the models constructed by the WCS mental models in the sense of Craik (1945) or Johnson-Laird (1983)? Is it plausible from a cognitive point of view that humans construct models in a similar way as the WCS? Is the WCS also a cognitive or psychological theory? So far, we have approached these questions by considering human reasoning tasks and experimental data from the literature. In this paper we want to experimentally validate predictions made by the WCS.

Given premises, general knowledge, and observations, reasoning in WCS is modeled in five steps:

1. Reasoning towards a (logic) program[2] following Stenning and van Lambalgen (2008).
2. Weakly completing the program.
3. Computing its least model under the three-valued Łukasiewicz (1920) logic.[3]
4. Reasoning with respect to the least model.
5. If observations cannot be explained, then applying skeptical abduction.

To illustrate these steps, consider modus ponens inferences. They are simpler than modus tollens inferences because abduction is not needed. Given a conditional *if A then C*, reasoning towards a program yields

$$\mathcal{P} = \{C \leftarrow A \wedge \neg ab,\ ab \leftarrow \bot\},$$

where A and C are atoms, $\bot$ is a truth constant denoting falsehood, and ab is an abnormality predicate. ab encapsulates everything that could prevent the conditional from holding and is assumed to be false initially. Weakly completing $\mathcal{P}$ yields

$$\{C \leftrightarrow A \wedge \neg ab,\ ab \leftrightarrow \bot\}.$$

Its least model under Łukasiewicz (1920) logic maps no atom to true, ab to false, and A and C to unknown, which is represented by $\langle \emptyset, \{ab\} \rangle$ in Figure 3.

In $\mathcal{P}$, the atom C is *defined* because $\mathcal{P}$ contains a clause of the form $C \leftarrow Body$ and $Body$ is either the constant $\top$ denoting truth or the constant $\bot$ denoting falsehood or a finite conjunction of atoms and negated atoms. Likewise, ab is defined, whereas A is undefined in $\mathcal{P}$. Because of the latter, the second premise A of modus ponens is added as a fact $A \leftarrow \top$ to $\mathcal{P}$. Weakly completing the extended program yields

$$\{C \leftrightarrow A \wedge \neg ab,\ ab \leftrightarrow \bot,\ A \leftrightarrow \top\}.$$

Its least model maps A and C to true and ab to false, which is represented by $\langle \{A, C\}, \{ab\} \rangle$ in Figure 3. Hence, C holds.

Let us turn to modus tollens, where abduction is needed. In abductive frameworks as proposed by Kakas, Kowalski, and Toni (1992), the set of abducibles is usually restricted. Given a program $\mathcal{P}$, Dietz et al. (2012) define the set $\mathcal{A}_{\mathcal{P}}$ of *abducibles* as

$$\{B \leftarrow \top \mid B \text{ undefined in } \mathcal{P}\} \cup \{B \leftarrow \bot \mid B \text{ undefined in } \mathcal{P}\}.$$

In (Dietz Saldanha et al., 2017) the classification of conditionals was taken into account by extending $\mathcal{A}_{\mathcal{P}}$ as follows:

$$\begin{array}{lcl} \mathcal{A}^e_{\mathcal{P}} & = & \mathcal{A}_{\mathcal{P}} \cup \mathcal{A}_n \cup \mathcal{A}_f, \text{ where} \\ \mathcal{A}_n & = & \{C \leftarrow \top \mid C \text{ is head of a clause in } \mathcal{P} \text{ representing} \\ & & \quad \text{a conditional with non-necessary antecedent}\}, \\ \mathcal{A}_f & = & \{ab \leftarrow \top \mid ab \text{ occurs in the body of a clause in } \mathcal{P} \\ & & \quad \text{representing a factual conditional}\}. \end{array}$$

The set $\mathcal{A}_n$ contains facts for the consequences of conditionals with non-necessary antecedent represented in $\mathcal{P}$. If an antecedent is non-necessary, then there may be other unknown reasons for establishing the consequent of the conditional. The set $\mathcal{A}_f$ contains facts for the abnormality predicates occurring in the representation of factual conditionals. The antecedent of a factual conditional may be true, yet the consequence of the conditional may still not hold. Adding a positive fact for the abnormality predicate ab occurring in the body of the clause representing a factual conditional will force ab to become true. In this case, any conjunction containing $\neg ab$ will be false. Figure 4 illustrates the additional facts in the extended set of abducibles.

[2] In this paper we consider only propositional programs, but WCS can also handle first-order programs (Hölldobler, 2015).

[3] Hölldobler and Kencana Ramli (2009) have shown that weakly completed programs admit such a least model; it can be computed as the least fixed point of a semantic operator defined by Stenning and van Lambalgen (2008).

if A then C	*A* non-necessary	*A* necessary
factual	$ab \leftarrow \top$, $C \leftarrow \top$	$ab \leftarrow \top$
obligation	$C \leftarrow \top$	

Figure 4: The additional abducibles for a clause of the form $C \leftarrow A \wedge \neg ab$ representing the conditional *if A then C*, where A and C are atoms and ab is an abnormality predicate.

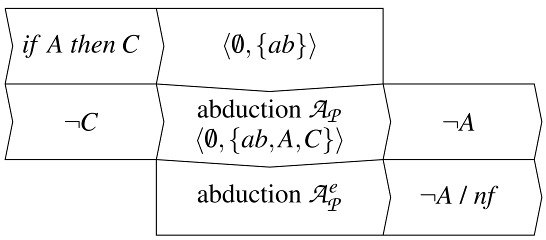

Figure 5: Modus tollens inference in WCS.

Obligation Conditional with Non-Necessary Antecedent
Given the conditional *if Paul rides a motorbike, then he must wear a helmet* from Ex. 5, reasoning towards a program yields

$$\mathcal{P} = \{helmet \leftarrow paul\_riding \wedge \neg ab_m,\ ab_m \leftarrow \bot\},$$

where ab_m is an abnormality predicate which is assumed to be false. The least model of the weak completion of $\mathcal{P}$ is represented by $\langle\emptyset, \{ab_m\}\rangle$ (see Figure 5). Because *helmet* is defined in $\mathcal{P}$, the second premise *Paul is not wearing a helmet* is considered to be an observation which needs to be explained. The sets of abducibles are

$$\begin{array}{rcl} \mathcal{A}_\mathcal{P} & = & \{paul\_riding \leftarrow \top,\ paul\_riding \leftarrow \bot\}, \\ \mathcal{A}^e_\mathcal{P} & = & \mathcal{A}_\mathcal{P} \cup \{helmet \leftarrow \top\} \end{array}$$

taking into account that the given conditional is classified as an obligation conditional with non-necessary antecedent. For both sets, there is only one minimal explanation, viz. $\{paul\_riding \leftarrow \bot\}$, to explain the second premise. Adding this explanation to $\mathcal{P}$, weakly completing the extended program, and computing its least model we obtain $\langle\emptyset, \{ab_m, paul\_riding, helmet\}\rangle$. The atoms ab_m, *paul_riding*, and *helmet* are mapped to false. Hence, $\neg helmet$ holds.

Factual Conditional with Non-Necessary Antecedent
Given the conditional *if Nancy rides her motorbike, then she goes to the mountains* from Example 6, reasoning towards a program yields

$$\mathcal{P} = \{mountains \leftarrow nancy\_riding \wedge \neg ab_n,\ ab_n \leftarrow \bot\}.$$

The least model of the weak completion of this program is $\langle\emptyset, \{ab_n\}\rangle$. The second premise *Nancy did not go to the mountains* is again considered to be an observation which needs to be explained. The sets of abducibles are

$$\begin{array}{rcl} \mathcal{A}_\mathcal{P} & = & \{nancy\_riding \leftarrow \top,\ nancy\_riding \leftarrow \bot\}, \\ \mathcal{A}^e_\mathcal{P} & = & \mathcal{A}_\mathcal{P} \cup \{ab_n \leftarrow \top,\ mountains \leftarrow \top\} \end{array}$$

taking into account that the given conditional is classified as a factual condition with non-necessary antecedent. The observation can now be explained in two different ways. Either *Nancy was not riding a motorbike* or ab_n is true. The abnormality predicate being true simply says that something happened which prevented Nancy from going to the mountains, but we do not know what exactly prevented her. The two minimal explanations lead to the least models $\langle\emptyset, \{nancy\_riding, ab_n, mountains\}\rangle$ and $\langle\{ab_n\}, \{mountains\}\rangle$. A reasoner who just considers $\mathcal{A}_\mathcal{P}$ and, consequently, generates only the first model will conclude that *Nancy was not riding a motorbike*, whereas a reasoner considering $\mathcal{A}^e_\mathcal{P}$ and reasoning skeptically will conclude that *nothing (new) follows*.

Factual Conditional with Necessary Antecedent
Given the conditional *if it is cloudy, then it is raining* (Example 7). The possibility *it is not raining* and *it is cloudy* is not forbidden and, hence, this is a factual conditional. On the other hand, naive physics tells us that the consequent *it is raining* cannot be true unless the antecedent *it is cloudy* is true. Hence, this is a factual conditional with necessary antecedent. Reasoning towards a program yields

$$\mathcal{P} = \{rain \leftarrow cloudy \wedge \neg ab_c,\ ab_c \leftarrow \bot\}.$$

The abnormality predicate is used to express that clouds alone may not be sufficient to cause rain. Enabling conditions like, for example, sufficient humidity may be needed. Such enabling conditions are not explicitly mentioned, but the abnormality predicate makes it possible to add them later and to overwrite the initially assumed falsehood of ab_c.[4] The least model of the weak completion of this program is $\langle\emptyset, \{ab_c\}\rangle$. The second premise *it is not raining* is again considered to be an observation which needs to be explained. The sets of abducibles are

$$\begin{array}{rcl} \mathcal{A}_\mathcal{P} & = & \{cloudy \leftarrow \top,\ cloudy \leftarrow \bot\}, \\ \mathcal{A}^e_\mathcal{P} & = & \mathcal{A}_\mathcal{P} \cup \{ab_c \leftarrow \top\}. \end{array}$$

The observation can be explained in two different ways. Either *there are no clouds* or ab_c is true. The abnormality predicate being true simply says that something is missing which prevented rain, but we do not know what exactly prevented it. The two minimal explanations lead to the least models $\langle\emptyset, \{cloudy, ab_c, rain\}\rangle$ and $\langle\{ab_c\}, \{rain\}\rangle$. A reasoner who just considers $\mathcal{A}_\mathcal{P}$ and, consequently, generates only the first model will conclude that *it is not cloudy*, whereas a reasoner considering $\mathcal{A}^e_\mathcal{P}$ and reasoning skeptically will conclude that *nothing (new) follows*.

Obligation Conditional with Necessary Antecedent
Consider the conditional *if it rains, then the roofs must be wet*, a variant of Example 8. This appears to be an obligation conditional with necessary condition. We cannot easily consider a case, where it rains and the roofs are not wet. Moreover, we

[4]This was used in (Dietz et al., 2012), where *the library being open* is an enabling condition for *studying late in the library*.

cannot easily imagine a situation, where the roofs are wet and it did not rain. Reasoning towards a program yields

$$\mathcal{P} = \{wet\_roofs \leftarrow rain \wedge \neg ab_r,\ ab_r \leftarrow \bot\}.$$

The least model of the weak completion of this program is $\langle \emptyset, \{ab_r\}\rangle$. The second premise *the roofs are not wet* is again considered to be an observation which needs to be explained. The sets of abducibles are

$$\mathcal{A}_{\mathcal{P}} = \mathcal{A}^e_{\mathcal{P}} = \{rain \leftarrow \top,\ rain \leftarrow \bot\}.$$

The only explanation for the second premise is that *it is not raining* leading to the least model $\langle \emptyset, \{rain, ab_r, wet\_roofs\}\rangle$. Hence, *the roofs are not wet* holds.

Summary

The answers are summarized in the following table for a given conditional *if A then C* and fact $\neg C$:

Classification	$\mathcal{A}_{\mathcal{P}}$	$\mathcal{A}^e_{\mathcal{P}}$
Obl.+nec.	$\neg A$	$\neg A$
Obl.+non-nec.	$\neg A$	$\neg A$
Fac.+nec.	$\neg A$	*nf*
Fac.+non-nec.	$\neg A$	*nf*

Under WCS, modus tollens inferences ($\neg A$) are made if reasoners restrict their attention to the set $\mathcal{A}_{\mathcal{P}}$ of abducibles. However, if the reasoners are considering $\mathcal{A}^e_{\mathcal{P}}$, are reasoning skeptically, and the conditional is a factual one, then they will answer *nothing (new) follows (nf)*. This is independent of whether the antecedent A is necessary or not. Moreover, it should take the reasoners longer to answer *nf* than to answer $\neg A$ because $\mathcal{A}^e_{\mathcal{P}}$ needs to be considered.

Putting it to the Test: Obligation and Factual Conditionals in Everyday Context

As outlined above, the goal of our investigations is to compare modus tollens inferences with respect to obligation and factual conditionals with necessary and non-necessary antecedents in an everyday context, i.e., in a context familiar to the participants. Here we will test two hypotheses that we have drawn out of our formal model analysis:

1. The number of *nf* answers increases from obligation to factual conditionals.
2. The time to respond *nf* takes longer than to respond the conclusion derived by applying the modus tollens.

Participants

We tested 56 logically naive participants on an online website (Prolific, `prolific.co`). We restricted the participants to Mid-Europe (including GB) to have a similar background knowledge about weather etc. We assume that the participants are fluent in English and had not received any education in logic beyond high school training. We took the usual precautions for such a procedure; for example, the website checked that participants were proficient speakers of English.

Materials, Design, and Procedure

The participants were told that they would read first a story, and afterwards a first assertion ("a conditional premise"), and then a second assertion ("a (possibly negated) atomic premise"), and then for each problem they had to answer the question "What follows?". Both parts were presented simultaneously. The participants responded by clicking one of the answer options – the system recorded the time from the onset of the assertions until they clicked an answer option. They could take as much time as they needed.

The participants carried out 48 problems consisting of the 12 conditionals listed below and solved all four inference types (MP, DA, AC, MT). We chose the content based on (i) previously tested conditionals in the literature and (ii) on everyday context. The classification of the conditionals was done by the authors.

Obligation Conditionals with Necessary Antecedent

(1) *If it rains, then the roofs must be wet.*
(2) *If water in the cooking pot is heated over 99°C, then the water starts boiling.*
(3) *If the wind is strong enough, then the sand is blowing over the dunes.*

Obligation Conditionals with Non-Necessary Antecedent

(4) *If Paul rides a motorbike, then Paul must wear a helmet.*
(5) *If Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age.*
(6) *If it rains, then the lawn must be wet.*

Factual Conditionals with Necessary Antecedent

(7) *If the library is open, then Sabrina is studying late in the library.*
(8) *If the plants get water, then they will grow.*
(9) *If my car's start button is pushed, then the engine will start running.*

Factual Conditionals with Non-Necessary Antecedent

(10) *If Nancy rides her motorbike, then Nancy goes to the mountains.*
(11) *If Lisa plays on the beach, then Lisa will get sunburned.*
(12) *If Ron scores a goal, then Ron is happy.*

For each conditional *if A then C*, four additional facts were considered: the affirmation of the antecedent A (MP), the denial of the antecedent $\neg A$ (DA), the affirmation of the consequent C (AC), and the denial of the consequent $\neg C$ (MT). Participants could select one of three responses, *nothing follows (nf)*, and the respective fact and its negation that has not been presented in the second premise.

Results

In the analysis we included all participants that completed all 48 problems. Because the modus tollens inference is the core focus of this paper we only report those results here (see Figure 6). For the other three cases the interested reader is refered to (Cramer, Hölldobler, & Ragni, 2021).

Classification	A	$\neg A$	nf	Sum	$Mdn\,\neg A$	$Mdn\,nf$
(1)	1	45	10	56	3449	4758
(2)	0	50	6	56	4058	7922
(3)	2	46	8	56	3796	4517
Obl.+nec.	3	141	24	168	3767	5732
(4)	3	46	7	56	3872	4154
(5)	1	54	1	56	4946	8020
(6)	0	36	20	56	4062	5235
Obl.+non-nec.	4	136	28	168	4293	5803
(7)	1	37	18	56	5974	4744
(8)	3	42	11	56	4367	5013
(9)	0	47	9	56	4208	3966
Fac.+nec.	4	126	38	168	4849	4574
(10)	2	35	19	56	4879	4167
(11)	0	39	17	56	4411	5647
(12)	0	34	22	56	3726	3813
Fac.+non-nec.	2	108	58	168	4338	4542
Obligation	7	277	52	336	4030	5767
Factual	6	234	96	336	4594	4558
Total	13	511	148	672	4312	5162

Figure 6: The results for modus tollens inferences. Conditionals (1)-(3) are obligation conditionals with necessary antecedent (Obl.+nec), (4)-(6) are obligation conditionals with non-necessary antecedent (Obl.+non-nec.), (7)-(9) are factual conditionals with necessary antecedent (Fac.+nec.), and (10)-(12) are factual conditionals with non-necessary antecedent (Fac.+non-nec.). Given a conditional *if A then C* and an atomic sentence $\neg C$, the total number of responses A, $\neg A$, and nf is shown. $Mdn\,\neg A$ and $Mdn\,nf$ show the median (in milliseconds) for $\neg A$ and nf responses, respectively. The reported median is based on the raw response times.

The everyday context elicited a high application rate of modus tollens in about 76% (511 out of 672), but the case of *nf*-answers varied from 14% (24 out of 168) up to 35% (58 out of 168). The number of participants answering A (13 out of 672) was irrelevant.

As predicted by WCS, the answer *nf* was given more often in case of a factual conditional than in case of an obligation conditional (28% (96 out of 336) vs. 15% (52 out of 336), Wilcoxon signed rank, $t = 73.5$, $p < .001$). So the predicted increase of selecting *nf* can be confirmed and a cognitive theory should be able to explain it. A core question is, however, the second hypothesis: is answering *nf* an indication that a participant did not know the answer, or a result of an immediate conflict of premises that might follow the predicted process in the MMT, or is it at the end of a deliberation process that might follow the predicted process in the WCS?

For all conditionals the response *nf* takes more time than the application of modus tollens and this is marginally significant (4312*ms* vs. 5162*ms*, Wilcoxon signed rank, $t = 15$, $p = .06$).

Interpreting the Result in the WCS

Given a modus tollens inference task consisting of the conditional *if A then C* and the fact $\neg C$, the WCS models modus tollens inferences as follows: Reasoners may construct a first model using the set $\mathcal{A}_\mathcal{P}$ of abducibles. This model always exists and maps $\neg A$ to true. It may be called the *preferred* model in the sense of Ragni and Knauff (2013). If reasoners fails to construct this model, then they will answer *nf*. Such a failure may have various reasons: they may not consider $\neg C$ as an observation that needs to be explained. Rather, if they just add $C \leftarrow \bot$ to the program or if they add an integrity constraint $\bot \leftarrow C$, then no model assigning true or false to A can be constructed in WCS. Or the reasoners might consider $\neg C$ as an observation that needs to be explained but not necessarily by A or $\neg A$; moreover, the necessity of an antecedent may depend on the cultural background of the participant. Or the reasoners may make a mistake in constructing the preferred model, which is–as mentioned before–the least fixed point of the semantic operator introduced by Stenning and van Lambalgen (2008). On the other hand, once the preferred model has been constructed, upon further thought a reasoner may search for models using the set $\mathcal{A}^e_\mathcal{P} \supseteq \mathcal{A}_\mathcal{P}$ of abducibles and find a second model in case of a factual conditional. In this second model A is unknown. Reasoning skeptically, the reasoner will answer *nf*. This not only explains the difference between obligation and factual conditionals but also why a significantly larger number of participants answered *nf* in the case of factual conditionals. However, this interpretation requires further experiments recording the time of deliberation.

Interpreting the Results in the MMT

The results confirm findings by Quelhas and Byrne (2003) that there are more modus tollens inferences for obligation conditionals than for factual conditionals. As mentioned before, Byrne (2005) has suggested that there is a second route for modus tollens inferences, but in recent publications on MMT like, for example, (Khemlani et al., 2018) this second route is not mentioned. Future work will compare WCS and MMT regarding the classification of conditionals suggested in this paper and the special role of *nothing follows* responses.

Discussion

We started with the observation that the literature reports different forms of conditionals which have not yet been systematically investigated and that explanations and theories largely differ. This paper has for the first time systematically analyzed obligation vs. factual conditionals and necessary vs. non-necessary antecedents in conditional reasoning. We showed that a new approach–the WCS–makes interesting predictions that have been empirically evaluated. So far, no other theory makes these predictions.

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