## Title
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# Image-Schema Transfer: Towards Computational Facilitation of Analogical Problem Solving using a Diagrammatic Representation

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### Abstract

This paper proposes an experimental system called **IST** for the facilitation of users' analogical transfer by providing image-schemas. First, by taking the radiation problem as an example, we hypothesized that a proper image-schema can promote human analogical problem solving owing to its plasticity and ascertained the hypothesis based on a cognitive experiment. We then constructed the IST system which can provide image-schemas with plasticity, by using the extended techniques of analogical mapping and of constraint-based graphics.

## Introduction

Various features of diagrammatic representation have been investigated and compared with those of verbal one; for instance, compactness (Larkin & Simon, 1987), locality (Larkin et al., 1987), and diagrammatic configuration schema (Koedinger & Anderson, 1990). What is considered to underlie these features is that diagrammatic representation limits abstraction because of its specificity (Stenning & Oberlander, 1995). Most of these previous studies focus on the use of diagrams in deductive reasoning or problem solving (the typical example is Hyperproof (Barwise & Etchemendy, 1994)). A diagram which facilitates analogical problem solving, one of the nondeductive types of problem solving, is, however, considered to be a little different from that useful in deductive one: it has moderate abstractness rather than specificity because an analogy requires a moderate level of abstraction which provides a bridge spanning different problems. For example, the anecdote that Kekule intuitively derived the structure of benzene ring from the image of a snake in a circle with biting its own tail can be interpreted as follows: his image in a dream gave a hint for solving a problem in a quite different domain because it was moderately abstract. It is true that a diagram must be specific in some aspects (*e.g.*, the location of its constituents), but a diagram facilitating analogical transfer can – owing to its moderate abstractness – also be interpreted as the diagram of another problem in a different domain. We will call this feature of a diagram **plasticity**. A diagram with appropriate plasticity can be called, after Lakoff (Lakoff, 1987), an **image-schema**[1] that brings out some essential aspects of a problem.

This paper will first show, by a cognitive experiment, that a diagram facilitating analogy is an image-schema which has relevant plasticity. It will then propose and explain an experimental system called **IST** that enables such image-schemas to be provided. As an example of target problems is taken the so-called radiation problem described below.

> Suppose you are a doctor faced with a patient who has a malignant tumor in his stomach. It is impossible to operate on the patient, but unless the tumor is destroyed the patient will die. There is a kind of ray that can be used to destroy the tumor. If the rays reach it all at once at a sufficiently high intensity, the tumor will also be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. At lower intensity the rays are harmless to healthy tissue, but they will not affect the tumor either. What type of procedure might be used to destroy the tumor with the rays, and at the same time avoid destroying the healthy tissue?

## What is good plasticity?

Some studies such as those of (Beveridge & Parkins, 1987; Gick & Holyoak, 1983; Kinoe & Anzai,1994) used experiments in which subjects were asked to solve the radiation problem by using diagrammatic or verbal representation, or both, that has various degrees of similarity to the radiation problem. The latter two studies showed that the kind of diagram facilitating analogical problem solving is not a realistic image where most aspects of a problem are represented *concretely*, but is an *abstract* diagram. This implies that our insistence on plasticity is appropriate. In both the studies, however, some explicit instructions needed to be given if the subjects were to perform well.[2]

In the study of (Beveridge et al., 1987), in contrast, no clear instructions were needed for the subjects to make good scores. This difference is considered to be

[1]Interestingly, the neurons reacting to visual primitives which are considered to be similar to image-schemas were discovered (Fujita, Tanaka, Ito, & Cheng, 1992).

[2]The diagrams used were different in the two studies in that the operators were visualized in the diagrams of (Gick et al., 1983) but not in those of (Kinoe et al., 1984).

caused by the difference in characteristics of the diagrams presented: in the diagram used in (Beveridge et al., 1987), not only the useful operators but also their effect when being applied were symbolically visualized, while the latter was not in those of (Gick et al., 1983; Kinoe et al., 1984).

We can, thus, reach the following conjecture:

**Conjecture 1** A diagram which has enough plasticity to facilitate a kind of analogical problem solving (*i.e.*, solving a problem by contriving operators based on analogical transfer) is the so-called image-schema in which not only operators but also their effects when being applied are moderately specifically but symbolically visualized. In such an image-schema, the spatial information of its constituents also needs to be moderately specifically but symbolically visualized, while the other information is visualized less specifically.

## Cognitive Experiment

### Method

Two hundreds and four undergraduate students of three universities were divided into four groups. The experiment was implemented as a written test given in two sessions. In the first session (3 minutes) all the subjects in every group were asked to solve the radiation problem without any hint.[3] In the second session (10 minutes) each group was provided with the radiation problem differently: the *Control-group* was given no hint, the *Story-group* was given the *fire-fighting story*[4] in the appendix, the *Diagram-group* was given the image-schema of the fire-fighting story (*i.e.*, Figure 1), and the *Story+Diagram-group* was given both the story and its image-schema.[5] To the latter three groups, the attack story in the appendix or its image-schema, or both, was also presented as a distraction (irrelevant analogue) in order to test the robustness of the analogical transfer.

### Result and Discussion

The results are listed in Table 1.[6] A $4 \times 2$ $\chi^2$-test revealed that the difference in performance between the *Story-group* and the *Diagram-group* was significant ($p = 0.044 < 0.05$, one-tailed) as well as the difference between the *Control-group* and the *Diagram-group* ($p < 0.01$). On the other hand, the *Diagram-group* was just a little superior to the *Story+Diagram-group*, although the difference between the two was not significant. Therefore we can conclude that our conjecture is right. That is, not only the constituents and operators of a problem but also the effects of the operators should be moderately specifically but symbolically visualized if an image-schema is to have enough plasticity to facilitate analogical transfer.

Zhang & Norman (Zhang & Norman, 1994) indicated that the degree of difficulty in solving the *Tower of Hanoi* problem depended on the way the problem was represented. The results of the present experiment show that the same holds for the radiation problem.

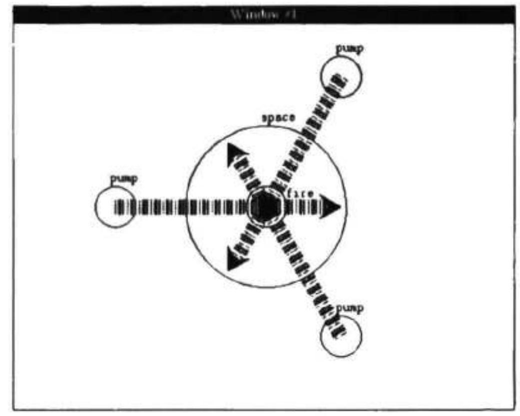


Figure 1: The image-schema of the fire-fighting story and its convergence solution.

Table 1: The result of the experiment.

| Group | Correct | Incorrect | Total |
|---|---|---|---|
| Control | 4(8.9) | 41(91.1) | 45 |
| Story | 25(49.0) | 26(51.0) | 51 |
| Diagram | 36(65.5) | 19(34.5) | 55 |
| Story+Diagram | 33(61.1) | 21(38.9) | 54 |
| Total | 97(47.5) | 107(52.5) | 204 |

Note: Each figure denotes the number of subjects with the percentage in the parenthesis.

## IST Session

This section proposes an experimental system called **IST** (**I**mage-**S**chema **T**ransfer) that facilitates human analogical problem solving by providing image-schemas that satisfy our conjecture. Figure 1 and 7 were actually drawn automatically by the system. The image-schema provided by IST is, therefore, partially ascertained, based on the cognitive experiment, to have enough plasticity.

### Framework

The central part of IST is of course the module in which some aspects of a problem are visualized. It is, however, necessary to elicit a predicative representation of a potential source from which a useful image-

[3] The subjects who could solve in this session were excluded from consideration, because they might know the solution in advance. Three minutes was the minimum time in which a subject could, in a preliminary experiment, solve the target problem without being given a hint.

[4] It is equivalent to the Red-Adair story in (Gick et al., 1983).

[5] No instructions (*e.g.*, to use the story or diagram provided as a hint) were given to all the subjects.

[6] The only correct answer was assumed to be the so-called convergence solution, which is to use simultaneously several weak beams from different directions.

**I** outside(radiation-source, flesh), surround(flesh, tumor).

**G** alive(patient).

**O** apply-from-to(radiation, radiation-source, tumor), ¬operate-on(doctor, tumor), increase-in-intensity(radiation) ∨ decrease-in-intensity(radiation).

**R** 1) increase-in-intensity(radiation) ∧ apply-from-to(radiation, radiation-source, tumor) ∧ outside(radiation-source, flesh) ∧ outside(radiation-source, flesh) ∧ surround(flesh, tumor) ⇒ destroy(radiation, tumor) ∧ destroy(radiation, flesh),

2) decrease-in-intensity(radiation) ∧ apply-from-to(radiation, radiation-source, tumor) ∧ outside(radiation-source, flesh) ∧ surround(flesh, tumor) ⇒ ¬destroy(radiation, tumor) ∧ ¬destroy(radiation, flesh),

3) destroy(radiation, tumor) ∧ ¬destroy(radiation, flesh) ⇒ alive(patient).

Figure 2: The predicative expression of the radiation problem.

schema is generated,[7] because only the image-schema of a target problem which cannot be solved deductively would not be useful for the problem solving. Hence a module for retrieving source analogues relevant to the target problem is required. A module for eliciting the cause of impasse in solving the target, which gives the retrieval cue, is also needed. IST therefore consists of three modules: (1) one for analyzing the target problem, (2) one for retrieving relevant analogues, and (3) one for visualization. A useful image-schema must be visualized in such a way that the operational factors relevant to a possible solution of the target, which give some constraints on visualization, are salient. Note that the cause of impasse elicited in the first module provides both a retrieval cue in the second module and a part of the constraints on visualization in the third module.

## IST Procedures

The procedure of each module will be explained here, taking the radiation problem as a target problem. Each problem is assumed to be represented by a 5-tuple $\langle I, G, O, R, S\rangle$ where $I$ denotes the initial state, $G$ the goal state, $O$ a set of operators, $R$ a set of rules, and $S$ a solution.[8] The radiation problem can be represented as shown in Figure 2. The predicative representation of each solved problem and the conceptual hierarchy (such as that shown in Figure 4) are given to the IST database in advance.

**The First Module** First we define an impasse in solving a problem as follows:

**Definition 1 (Impasse in Solving a Problem)** *If* $\exists g, g^c(g \cup g^c = G(or\ G_s) \wedge g \cap g^c = \emptyset \wedge \exists C, C' \subset O \cup I(C \neq C' \wedge C \vdash^R g \wedge C \not\vdash^R g^c \wedge C' \not\vdash^R g \wedge C' \vdash^R g^c))$, *then the problem solving reaches an impasse.*

In solving the radiation problem by backward chaining, we have

$g =$ {destroy(radiation, tumor)}

$g^c =$ {¬destroy(radiation, flesh)}

$C =$ {increase-in-intensity(radiation), apply-from-to(radiation, radiation-source, tumor), outside(radiation-source, flesh), surround(flesh, tumor)}

$C' =$ {decrease-in-intensity(radiation), apply-from-to(radiation, radiation-source, tumor), outside(radiation-source, flesh), surround(flesh, tumor)}

Solving this problem thus reaches the impasse.

Now suppose $C_c = C \cap C'$. Then the essential elements of $O$ and $I$, which appear in any derivation in the above definition, are to be substituted into $C_c$. It is significant for IST to retrieve such potential source analogues as have $C_c$, because a good way of using the operators in the target problem needs to be invented with the essential elements such as $C_c$ unchanged. $C_c$ can thus be a retrieval cue in selecting relevant analogues. In case of the radiation problem, $C_c =$ {apply-from-to(radiation, radiation-source, tumor), outside(radiation-source, flesh), surround(flesh, tumor)}.

On the other hand, suppose $C_v = (C \cup C')/C_c$. When some operators opposite to one another (*e.g.*, increase-in-intensity *v.s.* decrease-in-intensity) are involved in $C_v$ concerning one operation (*e.g.*, change intensity), some devices should be thought out concerning the operators. $C_v$ is therefore considered to be made salient in an image-schema and, as a result, to give some constraints on visualizing an image-schema. We call such constraints the focus $F$ for visualization. This focus consists of the concept (*e.g.*, intensity) common to the opposite operators.

**The Second Module** In this module, some relevant analogues are retrieved by using $C_c$. Here $C_i$ means $C_c \cap I$ and $C_o$ means $C_c \cap O$.

$C_i$ and $C_o$ are first abstracted by using the given conceptual hierarchy and then generalized into *AbstI* and *AbstO* by substituting the corresponding variable into each constant appearing in $C_i$ and $C_o$. In the radiation problem, *AbstI* and *AbstO* are as follows:

*AbstI* ={outside(v1, v2), surround(v2, v3)}
*AbstO* ={apply-from-to(v4, v1, v3)}

For each solved problem in the IST database, *AbstI* and *AbstO* are instantiated (*i.e.*, calculated by the

[7] Each problem is assumed to be represented as a set of predicates.

[8] Each element appearing in $I$, $G$, $O$, and $S$ takes the form of an atomic formula and each appearing in $R$ takes the form of a rule clause. The target problem does not involve $S$.

**I** outside(pump, space), surround(space, fire), small(pump).
**G** extinguish(fire).
**O** throw-from-to(water, pump, fire), multiply(pump).
**R** 1) outside(pump, space) $\wedge$ surround(space, fire) $\wedge$ small(pump) $\wedge$ multiply(pump) $\wedge$ place-around(pump, space) $\wedge$ throw-from-to(water, pump, fire) $\wedge$ converge-to(water, fire) $\Rightarrow$ increase-in-pressure(water),
2) increase-in-pressure(water) $\Rightarrow$ extinguish(fire).
**S** throw-from-to(water, pump, fire), multiply(pump), place-around(pump, space), converge-to(water, fire).

Figure 3: The predicative expression of the fire-fighting story.

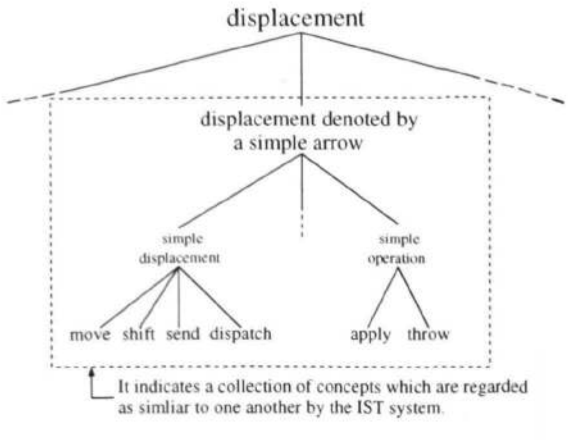


Figure 4: The given conceptual hierarchy.

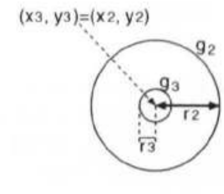


Figure 5: The image of $surround(g_2, g_3)$.

method reverse to the above method): each atom in *AbstI* and *AbstO* is instantiated by using the conceptual hierarchy, and each variable of each predicate is tried to be unified to a corresponding constant of the equivalent predicate. The problems which have the predicates in $I$ and $O$ equivalent to the instantiated *AbstI* and *AbstO* respectively are assumed the potentially relevant analogues. In case of facilitating the radiation problem, the fire-fighting story and the attack story were retrieved by using the conceptual hierarchy shown in Figure 4. The fire-fighting story, which is a proper analogue of the radiation problem, is shown in Figure 3.

**The Third Module** Finally, the image-schema of each retrieved problem is visualized in a bottom-up way. The visualization process consists of the following four steps: (1) transforming objects appearing in $I$ into graphic objects, (2) transforming atomic formulas appearing in $I$ into equations holding for graphic objects, (3) transforming atomic formulas appearing in $S$ or $O$ into equations holding for graphic objects, and (4) solving the equations to determine the value of each variable in the equations. All the steps other than the third one are implemented on the basis of *constraint-based graphics* (Kamada, 1989).

**1st Step** Each graphic object is symbolically visualized as a circle. For example, the object "fire" in the fire-fighting story is visualized as the center circle labeled $g_3$ in Figure 5, where $x_3$ and $y_3$ denote its $x$ and $y$ coordinates and $r_3$ denotes its radius.

**2nd Step** Each atomic formula appearing in $I$ is transformed, according to the given transformation rules, into a corresponding equation. The equations give constraints on spatial information such as the locations and sizes of graphic objects. For instance, $surround(g_2, g_3)$, which means that the graphic object $g_2$ surrounds $g_3$, is transformed as illustrated in Figure 5 into the following equations. It should be noted that the system transforms inequalities into corresponding equalities by using some constants given in the rules.

$$x_2 = x_3, y_2 = y_3, r_2 = c_2 \times r_3 (c_2 : constant)$$
$$g_2 : circle(x_2, y_2, r_2), g_3 : circle(x_3, y_3, r_3)$$

**3rd Step** Each atomic formula in $S$ (in case of a source analogue) or $O$ (in case of a target problem) is transformed into a corresponding equation holding for the graphic objects including novel ones such as an arrow, according to the given transformation rules. The transformation rules are now limited to those concerning the phenomena of displacement (*e.g.*, change/shift/operation). Only the rules necessary for the visualization of Figure 1 are shown as follows:

- Simple Displacement
  $\langle Form \rangle$ *func(Obj, Terminal)*
  (*e.g.*, move, shift, send, dispatch)
  It generates the equation for the visualization of an arrow from *Obj* to *Terminal*.
- Simple Operation
  $\langle Form \rangle$ *func(Obj, Start, Terminal)*
  (*e.g.*, apply, throw)
  It generates the equation for the visualization of an arrow from *Start* toward *Terminal* whose length is $\overline{StartTerminal} \times 1.5$.

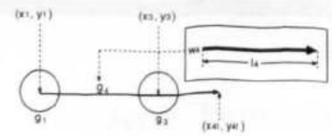

Figure 6: The image of *apply-from-to*($g_4$, $g_1$, $g_3$).

For example, both *apply-from-to*($g_4$, $g_1$, $g_3$) and *throw-from-to*($g_4$, $g_1$, $g_3$) are transformed as illustrated in Figure 6 into the following equations because both of them belong to the class of *Simple Operation*.

$g_4$: arrow(($x_{4s}$, $y_{4s}$), ($x_{4t}$, $y_{4t}$), $l_4$, $w_4$, $dash_4$, $color_4$)
$x_{4s} = x_1$, $y_{4s} = y_1$,
$x_{4t} = \frac{3x_3 - x_1}{2}$, $y_{4t} = \frac{3y_3 - y_1}{2}$,
$l_4 = \sqrt{(x_{4s} - x_{4t})^2 + (y_{4s} - y_{4t})^2}$,
$w_4 = c_4$ ($c_4$ : *constant*),
*where* $g_1$ : *circle*($x_1, y_1, r_1$), $g_3$ : *circle*($x_3, y_3, r_3$).

In the above novel graphic objects named $g_4$, the variable $dash_4$ takes as its value either T or NIL (default value), which determines whether or not a dashed line is used to draw the arrow, and the other variable $color_4$ takes either black or light-gray (default value) which shows the degree of shading. When the focus $F$ specifies strength or intensity as in the radiation problem, the following process is executed to make strength or intensity salient: (1) when increase-in-intensity($g_i$) is applied, $dash_i$ takes NIL[9] and $color_i$ takes black; (2) when decrease-in-intensity($g_i$) is applied, $dash_i$ takes T and $color_i$ takes light-gray; and (3) otherwise, $dash_i$ and $color_i$ take the respective default values. In order to visualize a summative effect of several radiations at the point of convergence, the IST system calculates the composition of shading at the overlapping points (see the center part of Figure 1). In this way, the focus $F$ specified in the first module is reflected in the process of visualization.[10]

**4th Step** Concerning the fire-fighting story, eight graphic objects (five circles and three arrows) were generated and thirty-three equations were built in the first three steps. In this step, all the equations are solved. But because the number of variables is larger than that of equations, six variables take the default values given in the rules and the most focused object (*i.e.*, fire) is displayed in the center of a window. In this way, IST visualizes image-schemas like that shown in Figure 1.

## Result and Discussion

Figures 1 and 7 are actual outputs of the IST system: Figure 1 represents the image-schema of the fire-fighting story with the convergence solution and Figure 7 does the same for the attack story. Considering the result of the cognitive experiment, we can conclude:

> The IST system can provide an image-schema which has enough plasticity. The subjects of the experiment were considered to transfer the convergence solution analogically, which is due to the plasticity of the appropriate image-schema.

This system only visualizes seemingly proper image-schemas and leaves it to users to transfer and modify possible solutions. Given that most of the subjects could use the appropriate image-schema, this method is considered useful to facilitate human analogy. IST facilitates human visual analogy not by deciding the most appropriate analogue, such as is done in SME (Falkenhainer, Forbus, & Gentner, 1989), but by providing the image-schemas of several analogues which might not be isomorphic to the target but which are visually and operationally similar to the target. Hence the constraints on retrieving source analogues in IST are weaker than those in SME.

VAMP (Thagard, Gochfeld, & Hardy, 1992) is a system intended to visualize a relevant analogue. The study of (Thagard et al., 1992), however, lacks both in considering what useful features of a problem should be visualized in order to draw an image-schema and in providing a more general method of visualizing the useful features (especially, operators with their effects). By comparison, the IST system adopts a way of visualization which are based on constraint-based graphics (Kamada, 1989) and which is more general than that in VAMP. Moreover, a new technique for visualizing a class of dynamic operators with their effects is proposed, which technique has not been developed.

The radiation problem referred to throughout this paper is one of the typical and actual (or historical) scientific problems whose solution requires insight (or emergent thinking). The IST system is intended to support users in solving such problems by providing visual analogical transfer. In this sense, this study is closely related to studies about the use of diagrams in a scientific discovery (for example, the discovery of the conservation of momentum using one-dimensional diagrams (Cheng & Simon, 1995)).

But the class of problems whose solution is in fact facilitated by the system is now limited. Such a limitation is considered to be mainly caused by a procedure of deriving the cause of impasse. Definition 1 formulates the situation in which multi-constraints should be satisfied simultaneously although they seem to be in contradiction at first appearance, which situation is represented by the phrase "the horns of a dilemma." Thus the first module inevitably limits the class of

[9]When the variable $color_i$ takes black, it is, in fact, not relevant to visualization whether the variable $dash_i$ takes T or NIL.

[10]The visualization of multiply($g_1$) and of place-around($g_1$, $g_2$) is now realized rather ad hoc: the two novel graphic objects named $g_{11}$ and $g_{12}$ are first generated and then placed so that they look to be revolved around $g_3$ from $g_1$ for 120° and 240° respectively. In addition, the system visualizes both *throw-from-to*($g_{41}$, $g_{11}$, $g_3$) and *throw-from-to*($g_{42}$, $g_{12}$, $g_3$) where both $g_{41}$ and $g_{42}$ are the copies of $g_4$.

problems that are facilitated, but the other modules are considered to be considerably general. Definition 1 should therefore be extended.

## Conclusion

First, it was ascertained on the basis of a cognitive experiment that one kind of diagram facilitating analogical problem solving is the so-called image-schema that has plasticity. The following two conditions for a useful image-schema were derived:

1. Both spatial information of objects and operators with their effects are moderately specifically but symbolically visualized.
2. The other visual information is displayed less specifically.

The experimental system IST, which enables such image-schemas to be visualized automatically, was then proposed. The study investigates a possibility that some external diagrammatic representations, which are provided automatically by the IST system in this paper, may support human internal representations, which possibility is also discussed in (Zhang et al., 1994). This study is a step toward the computational facilitation of human emergent thinking by providing the image-schemas of relevant analogues, an approach that has rarely been discussed in the previous studies on diagrammatic reasoning or problem solving.

## Appendix

### Fire-Fighting Story

One day a building caught fire and the result was a blazing inferno. The fire chief knew that the fire could be put out if a huge amount of water could be dumped on the origin of the fire. There were enough pumps available at the site, but there was none with a hose large enough to put enough water on the fire fast enough. The small pumps available could not shoot the water quickly enough to do any good. The fire chief, however, knew what to do. He stationed men in a circle all around the fire, with all of the available hoses of the small pumps. When everyone was ready, all of the hoses were opened up and water was directed at the fire from various directions. In this way, a huge amount of water could be delivered to the origin of the fire quickly enough for the fire to be extinguished.

### Attack Story

Attacking Romans unable to get inside surrounding a king's palace had camped outside. One morning they found an unlatched gate and rushed in through it, overcoming the king's soldiers and the king himself.

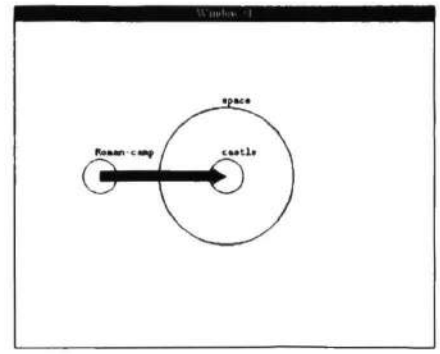


Figure 7: The image-schema of the attack story.

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