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THE NUCLEON AND THE (3, 3) RESONANCE

Geoffrey F. Chew

June 1, 1962

THE RECIPROCAL BOOTSTRAP RELATIONSHIP OF
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It has been proposed that within the framework of the analytically continued S matrix all strongly interacting particles are composites, held together by forces associated with "crossed" channels.¹ Such a mechanism has been explored in some detail for the ρ meson, which in a crude first approximation appears as a composite of two pions mutually attracted by the exchange of a ρ .² We wish to point out that a similar, only slightly more complicated, mechanism underlies the nucleon and the $(3,3)$ resonance. This mechanism could be called the "reciprocal bootstrap."

The essence of the Chew-Low theory for the $(3,3)$ resonance is that in first approximation this particle is a pion-nucleon composite, held together primarily by exchange of a nucleon.³ We shall show here that in the same sense and with the same degree of experimental verification the nucleon is approximately a composite of a pion and a nucleon, bound together to a large extent by exchange of a $(3,3)$. Our considerations supplement the recent work of Balázs, who on more general grounds discussed the low-energy behavior of the $I = \frac{1}{2}$, $J = \frac{1}{2}$ phase shift when the nucleon is treated as a πN bound state.⁴

The general analytic structure of πN partial-wave amplitudes has been explored by a number of authors.⁵ In terms of a variable $\omega = W - M$, where ω is the energy in the barycentric πN system and M the nucleon mass, each partial-wave amplitude has a "right-hand" physical cut running from 1 to infinity (using the pion mass as the energy unit) and two sets

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of "left-hand" unphysical cuts corresponding to the two crossed channels

$$(a) \quad \pi + \bar{\pi} \rightarrow N + \bar{N},$$

$$(b) \quad \bar{\pi} + N \rightarrow \pi + N.$$

The former gives rise to direct forces and the latter to exchange forces. The most important nearby singularities due to channel (a), i.e., the long-range direct forces, are associated with exchange of a ρ meson. Two symmetrical cuts start at $\omega \approx \pm i(\frac{1}{4} m_\rho^2 - 1)^{1/2}$ and run more or less vertically. From channel (b) the most important nearby "left-hand" singularities are two short cuts, arising from the two lowest-mass particles with the channel (b) quantum numbers--the nucleon and the (3,3) resonance. Nucleon exchange gives rise to a short cut centered near $\omega = 0$, while exchange of the (3,3) resonance leads to a fuzzy short cut centered near $\omega = -\omega_{33}$, if $M + \omega_{33}$ is the mass of the (3,3) resonance.⁵

Both these short cuts may be approximated by poles, and crossing symmetry gives explicit values for the pseudopole residues in terms of the residues of the (true) N and (3,3) poles in channel (b). The residue of the N pole is usually expressed in terms of the pion-nucleon coupling constant, while that of the (3,3) is called the resonance width, but the one residue is not to be thought of as more fundamental than the other. If we consider the four P-wave amplitudes,

$$f_i = \rho^{-1}(\omega) e^{i\delta_i} \sin \delta_i, \quad (1)$$

where $i = (I, J)$ and $\rho(\omega)$ is a phase-space factor⁵ [$\rho \approx (\omega^2 - 1)^{3/2}$ at low energy], then the residues of the interaction pseudopoles near $\omega = 0$ are

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$$\left(\begin{array}{c} 1/9 \\ -2/9 \\ -2/9 \\ 4/9 \end{array} \right) \gamma_{11} \quad \text{for} \quad i = \begin{array}{l} 1/2, 1/2 \\ 1/2, 3/2 \\ 3/2, 1/2 \\ 3/2, 3/2 \end{array} \quad (2)$$

if $-\gamma_{11} = -3f^2 = -0.24$ is the residue of the (true) nucleon pole in f_{11} . Thus the force due to exchange of a nucleon is strongly attractive in the $(3/2, 3/2)$ state, weakly attractive in the $(1/2, 1/2)$ state, and repulsive in the $(1/2, 3/2)$ and $(3/2, 1/2)$ states. The residues of the interaction pseudopoles near $\omega = -\omega_{33}$, in contrast, are

$$\left(\begin{array}{c} 16/9 \\ 4/9 \\ 4/9 \\ 1/9 \end{array} \right) \gamma_{33}, \quad (3)$$

where γ_{33} is the reduced half-width of the $(3,3)$ resonance. These forces are all attractive, but by far the greatest attraction occurs in the $(1/2, 1/2)$ state.

The integrated contribution from the two ρ -meson cuts has been shown by Bowcock, Cottingham, and Lurié⁶ to be roughly

$$f_i^{\rho}(\omega) = \int_{\rho \text{ cuts}} d\omega' \frac{C_i^{\rho}(\omega')}{\omega' - \omega} \approx \begin{pmatrix} 4 \\ -2 \\ -2 \\ 1 \end{pmatrix} \frac{C_{\rho}}{4k^2} \ln \left(1 + \frac{4k^2}{m_{\rho}^2} \right), \quad (4)$$

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where $k^2 \approx \omega^2 - 1$ and C_ρ is a positive real constant that can be evaluated from the nucleon magnetic moment form factor and the width of the ρ . A full width of 120 MeV leads to $C_\rho \approx 0.4$. The ρ -meson force is seen to be attractive and strongest in the $(1/2, 1/2)$ state, as emphasized by Frautschi,⁷ repulsive in the $(1/2, 3/2)$ and $(3/2, 1/2)$ states, and weakly attractive in the $(3/2, 3/2)$ state.

Chew and Low showed that the attractive force in the $(3/2, 3/2)$ state, mostly due to nucleon exchange, could explain the existence of the $(3,3)$ resonance.³ Working in a nonrelativistic approximation for the nucleon and knowing nothing at that time about Regge-type asymptotic behavior,⁸ they needed a cutoff--which was adjusted to fit the observed mass of $(3,3)$, but the width γ_{33} was successfully predicted in terms of γ_{11} . Their calculation, expressed in the more recent N/D language,^{9,4} ran as follows. For

$$f_{33}(\omega) = N_{33}(\omega)/D_{33}(\omega) , \quad (5)$$

then

$$N_{33}(\omega) = \frac{1}{\pi} \int_{\text{Unphysical plus inelastic cuts}}^{\infty} d\omega' \frac{[D_{33}(\omega') f_{33}(\omega')]}{\omega' - \omega} , \quad (6)$$

and

$$D_{33}(\omega) = 1 - \frac{\omega}{\pi} \int_1^{\infty} d\omega' \frac{\rho(\omega') N_{33}(\omega')}{\omega'(\omega' - \omega)} , \quad (7)$$

if by $2i[\]$ we mean the discontinuity in crossing a cut. For $0 \lesssim \omega \lesssim 2$ the pseudopole near $\omega = 0$ should dominate the numerator function, giving

$$N_{33}(\omega) \approx \frac{4/9 \gamma_{11}}{\omega} . \quad (8)$$

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The approximation (8) cannot be trusted throughout the range needed to evaluate $D_{33}(\omega)$ in (7), so we replace the principal part of the integral in (7) by a real empirical constant, adjusted to make $\text{Re } D_{33}(\omega)$ vanish at $\omega = \omega_{33}$:

$$D_{33}(\omega) \approx 1 - \frac{\omega}{\omega_{33}} - i\rho(\omega) N_{33}(\omega) \theta(\omega - 1). \quad (9)$$

The reduced half-width of the (3,3) resonance is thus predicted to be

$$\gamma_{33} = \omega_{33} N_{33}(\omega_{33}) \approx \frac{4}{9} \gamma_{11} = 0.11, \quad (10)$$

in agreement with experiment.

A glance at the relative strengths of forces acting in the (1/2, 1/2) and (3/2, 3/2) states, as indicated by (2), (3), and (4), indicates that a bound state for the former is not at all unlikely if a low-energy resonance can be managed for the latter. In analogy to the above calculation of γ_{33} we now carry out for the (1/2, 1/2) amplitude a calculation of the residue which a bound-state pole would have if it occurred at $\omega = 0$, corresponding to the nucleon. In place of (8), we have, for small ω ,

$$N_{11}(\omega) \approx \frac{16}{9} \frac{\gamma_{33} D_{11}(-\omega_{33})}{\omega_{33} + \omega} + \frac{1}{9} \frac{\gamma_{11} D_{11}(-\epsilon)}{\epsilon + \omega} + \frac{1}{\pi} \int_{\rho \text{ cuts}} d\omega' \frac{C_{11}^{\rho}(\omega') D_{11}(\omega')}{\omega' - \omega}, \quad (11)$$

where we have slightly displaced the nucleon interaction pseudopole from $\omega = 0$ in order to avoid confusion with the true bound-state pole. In place of (9), normalizing D_{11} to unity at $\omega = -\omega_{33}$ and adjusting the value of the integral to give a zero in D_{11} at $\omega = 0$, we have

$$D_{11}(\omega) \approx -\omega/\omega_{33} \quad \text{for} \quad |\omega| \lesssim 2, \quad (12)$$

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if the small imaginary part is ignored. The residue of the bound-state pole in $f_{11} = N_{11} D_{11}^{-1}$ at $\omega = 0$ is, then (taking the limit $\epsilon \rightarrow 0$),

$$-\gamma_{11} = N_{11}(0)/D_{11}'(0) \approx -\frac{16}{9} \gamma_{33} - \frac{1}{9} \gamma_{11},$$

or

$$\gamma_{11} \approx 2 \gamma_{33}, \quad (13)$$

again in agreement with experiment. The ρ cuts do not contribute to the residue when the approximation (12) is employed, because--as seen in (4)--their combined contribution is an even function of ω . By a crude calculation one finds that the third term on the right-hand side of (11) is roughly equal to $(-\omega/\omega_{33})f_{11}^{\rho}(\omega)$. The Chew-Low type of approximation is thus seen to be symmetrical with respect to the $(1/2, 1/2)$ and $(3/2, 3/2)$ states. Note that according to (2), (3), and (4), in view of (10) or (13), the forces acting in the $(1/2, 3/2)$ and $(3/2, 1/2)$ states are small and probably repulsive, so that the absence of bound states or resonances with these quantum numbers is consistent.

It is remarkable that if the above crude formulas for f_{11} are used in the low-energy physical region, we have

$$f_{11}(\omega) \underset{\omega \ll 2}{\approx} \frac{\frac{16}{9} \gamma_{33}}{\omega_{33} + \omega} \left(-\frac{\omega_{33}}{\omega} \right) + 4 C_{\rho} \frac{1}{4k^2} \ln \left(1 + \frac{4k^2}{m_{\rho}^2} \right), \quad (14)$$

dropping the small contribution from nucleon exchange. Thus the prescription of merely adding the ρ term, as proposed by Bowcock, Cottingham, and Lurié,⁶ seems roughly correct even when the nucleon is treated as a bound state.

However, if one were to include in a better approximation to (4) terms odd in ω , or to improve the approximation (12), the ρ effects would not be simply additive. Formula (14) with $\gamma_{33} = 0.11$ and $C_{\rho} = 0.4$ predicts that

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δ_{11} should start off negative at threshold and change sign at $\omega \approx 2$, a behavior not in disagreement with experimental knowledge, as discussed in reference 6.

It is reasonable to hope that a relativistic version of the bootstrap calculation outlined here, taking due account of the Regge asymptotic behavior,⁸ will not require cutoffs and will yield rough values for the nucleon and (3,3) masses as well as absolute values for λ_{11} and λ_{33} . If this goal can be reached, the most striking characteristic of strong-interaction theory will have been demonstrated: It would then be almost certain, even without a detailed treatment of the strange particles, that no arbitrary parameters can be tolerated.

REFERENCES

- * This work was performed under the auspices of the U.S. Atomic Energy Commission.
1. G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 7, 394 (1961) and 8, 41 (1962), and Phys. Rev. 124, 264 (1961); see also G. F. Chew, The S-Matrix Theory of Strong Interactions (W. A. Benjamin and Co., New York, 1961).
 2. G. F. Chew and S. Mandelstam, Nuovo cimento 19, 752 (1961); F. Zachariasen, Phys. Rev. Letters 7, 112 (1961); F. Zachariasen and C. Zemach, Pion Resonances, California Institute of Technology preprint, 1962; L. A. P. Balázs, Low Energy Pion-Pion Scattering, submitted to Phys. Rev. (Lawrence Radiation Laboratory Report UCRL-10157, March 1962).
 3. G. F. Chew and F. Low, Phys. Rev. 101, 1570 (1956).
 4. L. A. P. Balázs, The $I = \frac{1}{2}$, $J = \frac{1}{2}$ State in πN Scattering with the Nucleon as a Bound State, submitted to Phys. Rev. (Lawrence Radiation Laboratory Report UCRL-10026, January 1962).
 5. S. W. MacDowell, Phys. Rev. 116, 774 (1960); W. Frazer and J. Fulco, Phys. Rev. 119, 1420 (1960); S. Frautschi and D. Walecka, Phys. Rev. 120, 1486 (1960).
 6. J. Bowcock, W. N. Cottingham, and D. Lurié, Nuovo cimento 16, 918 (1960).
 7. S. Frautschi, Phys. Rev. Letters 5, 159 (1960).
 8. T. Regge, Nuovo cimento 14, 951 (1959) and 18, 947 (1960).
 9. G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).

