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Considerable theoretical interest has been shown in the interpretation of the low-energy K^--p scattering data. The major effort has been directed toward a convenient parameterization that takes into account the kinematic features of the many competing channels ($K^- + p \to \bar{K}^0 + n$, πY) and the "standard" Coulomb and mass-difference corrections.[1,2,3,4] In attempts to draw conclusions about the nature of the basic K-meson nucleon interaction, the assumption has been made that $k \cot \delta$ is essentially constant and equal to the reciprocal (complex) scattering length. We have examined the validity of this approximation on the basis of the Chew-Mandelstam program.[5] We found that the two-pion exchange, which determines the long-range tail of the K-N interaction, gives a substantial energy dependence to $k \cot \delta$.

A qualitative determination that remains to be made is the sign of the real part of each ($I = 0,1$) scattering length.[6] Two attempts to do this

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have been based on properties of the Coulomb-nuclear interference. The angular distribution at 172 Mev/c (laboratory-system momentum) favors a constructive interference[7] and implies a positive sign. The other attempt is based on the apparent leveling off and decrease of the elastic-scattering cross section with decreasing momentum ($P_L < 150$ Mev/c). Jackson and Wyld have suggested that this behavior is due to destructive interference and consequently concluded that the sign is negative.[3] The energy dependence arising from the two-pion exchange provides an alternative interpretation of the leveling off. This can be seen qualitatively from the following argument: by virtue of the small pion mass (compared with the K meson mass) the two-pion exchange determines the longest-range part of the K^--p potential. This suggests that we regard the nuclear interaction as made up of the two-pion contribution plus another part of shorter range, representing the net effect of everything else. The peculiar energy dependence is interpreted as the destructive interference between the two parts. Knowledge of the sign of the two-pion contribution then leads to a determination of the sign of the scattering length. Recent advances in the theory of the pion-pion interaction[8] and of the electromagnetic structure of the nucleon[9] make it possible to calculate the sign and estimate the magnitude of the two-pion contribution.

From their theoretical study of the nucleon electromagnetic structure, Frazer and Fulco[9] have inferred a resonance in the $I = 1$, $J = 1$ state of pion-pion scattering. Quantitative conclusions are still uncertain,[10] but it seems likely that the two-pion contribution accounts for a large fraction of the isotopic vector charge and magnetic moment. On the basis of their results, we suggest that the charge structure of the K meson also receives a sizable contribution from the two-pion state. The consistency of this hypothesis and its consequences for πK scattering has been studied.[11] This hypothesis provides an estimate of the order of magnitude of the matrix element for emission of two pions by a K meson $(\pi\pi \mid K\bar{K})$. Using this estimate, we have calculated the contribution of the two-pion exchange to the $\bar{K}$-nucleon interaction. (The details of this calculation will be presented in a separate report.)

We denote the S-wave $\bar{K}$-nucleon elastic-scattering amplitude by g. It is an analytic function in the cut s-plane (s is the total energy square in the $\bar{K}$-N center-of-mass system). The physical branch cut starts at $s = s_0 = (M_N + m_K)^2$ and extends to $+\infty$. There are other "right-hand" cuts starting from the thresholds of the dynamically coupled channels, $(m_\pi + M_\Lambda)^2$ and $(m_\pi + M_\Sigma)^2$.[12] The unitarity condition, which establishes certain nonlinear relations between the amplitudes g_{ij} of various coupled processes, is expressed by

$$\operatorname{Im}\,(g^{-1})_{ij} = -\frac{k_i\,\theta_i\,\delta_{ij}}{\sqrt{s}\,(E_i + M_i)}\,, \qquad (1)$$

$$\theta_i = \begin{cases} 1 & \text{if} \quad W_i < \sqrt{s}\,, \\ 0 & \text{if} \quad W_i > \sqrt{s}\,, \end{cases}$$

where k_i, E_i, and W_i are the center-of-mass momentum, baryon energy, and threshold energy of the <u>i</u>th channel. The dynamical singularity arising from the two-pion exchange is a branch cut extending from the left up to the point $s = s_2 = \sqrt{M_N^2 - m_\pi^2} + \sqrt{m_K^2 - m_\pi^2}$. Relative to the other dynamical singularities, this cut is very close to the physical region $s \geqslant s_0$, and should produce the strongest energy dependence in $k \cot \delta$.

We denote the elastic $\bar{K}N$ amplitude in the state of isotopic spin I by g_I, where $I = 0,1$. These two amplitudes are related by crossing symmetry to amplitudes $g^{(+)}$ and $g^{(-)}$ that depend on the isotopic spin $I' = 0, 1$ of the exchanged pion pair, namely

$$g^0 = g^{(+)} - 3g^{(-)} ,$$
$$g^1 = g^{(+)} + g^{(-)} . \tag{2}$$

The magnitude of the discontinuity across the two-pion cut is expressed for our purpose as

$$\int_{s_1}^{s_2} ds \; \mathrm{Im}\, g^{(-)}(s) = R_1 \approx 4 M_N^4 \text{ fermi} , \tag{3}$$

where s_1 is approximately equal to $80 \; m_\pi^2$. Assuming that only the resonant $g^{(-)}$ amplitude is important, we have three specific predictions: (a) the isotopic spin ratio R_0/R_1 is -3, (b) the sign of R_1 is positive, and (c) the expected magnitude of R_1 is rather large.

The normalization of our amplitude g is given by its relation to the $\bar{K}$-N eleastic scattering phase shift,

$$g(s) = \sqrt{s}\,(E + M)/(k \cot \delta - ik) . \tag{4}$$

In this note we use only a very simple model in which the effect of the two-pion cut is represented by a δ function at the position $s = a = 93 \; m_\pi^2$

in the amplitude g. The dynamical singularities in all other channels are represented by subtraction constants.

This model, combined with the many-channel unitarity condition, Eq. (1), leads to the following expression for $k \cot \delta$:

$$k \cot \delta_I = \left\{ \ell(s) + \frac{1}{f_I(s) + z_I} \right\} \sqrt{s}\,(E + M) \quad , \qquad (5)$$

where

$$\ell(s) = \frac{s - s_0}{\pi} \, P \int_{s_0}^{\infty} \frac{ds' \, \mathrm{Im}(g^{-1})_{11}}{(s' - s)(s' - s_0)} - \frac{a - s_0}{\pi} \int_{s_0}^{\infty} \frac{ds' \, \mathrm{Im}(g^{-1})_{11}}{(s' - s_0)(s' - a)} \quad ,$$

$$f_I(s) = \frac{1}{\pi} \left(\frac{1}{s - a} - \frac{1}{s_0 - a} \right) \frac{R_I}{1 + \beta R_I} \quad ,$$

$$\beta = \frac{1}{\pi^2} \int_{s_0}^{\infty} \frac{ds' \, \mathrm{Im}(g^{-1})_{11}}{(s' - a)^2} \quad ,$$

and z_I is a complex quantity which depends on the subtraction constants and on the slowly varying $\pi - Y$ center-of-mass momenta. For each isotopic spin channel, we approximate z_I by a complex constant.[1,12] The calculations were done by adjusting the parameters R_I and z_I to fit the at-rest branching ratio

$$\lambda^{-1} = \frac{\sigma_{abs}\,(I = 1)}{\sigma_{abs}\,(I = 0)} = 0.18 \pm 0.09 \; ,^{7}$$

the total cross section at 172 Mev/c $(k\,\sigma_{tot}/4\pi = 0.7 \text{ fermi})$ and the elastic and charge-exchange cross sections at 100 Mev/c and 172 Mev/c.

Two sets of solutions were found, corresponding to constructive or destructive Coulomb-nuclear interference in the angular distribution at 172 Mev/c (Table II). The elastic and charge-exchange cross sections resulting from the "constructive" solution are shown in Fig. 1. The energy dependence of $(k \cot \delta_I)^{-1}$ is shown in Fig. 2, where the (a+) solution of Dalitz and Twan is also reported for comparison. Note that only the values of R_I and the ratio R_0/R_1 given by the "constructive" solution agree with those calculated theoretically. From this consideration we reject the "destructive" solution. Our principal result is that $k \cot \delta_I$ has a rather substantial energy dependence due to the two-pion exchange. Figure 2 shows that $\mathrm{Re}\,(k \cot \delta_I)$ is positive for each isotopic spin state in the momentum range above 100 Mev/c. However, the real parts of the scattering lengths appear to have opposite signs, that of the $I = 0$ state being small and negative.

Finally, we would like to remark that, by crossing symmetry, the two-pion contribution to the K^+-proton amplitude can be determined from the parameters R_0 and R_1. In this case there is a destructive interference and flattening of the cross section for a repulsive K^+ proton short range interaction.

We are deeply indebted to Professors Geoffrey F. Chew and Robert Karplus for many illuminating discussions.

FIGURE CAPTIONS

Fig. 1. Cross sections for elastic and charge-exchange K^--proton scattering. The (a+) solution of Dalitz and Tuan (dotted lines) is included for comparison.

Fig. 2. Momentum dependence of the real and imaginary parts of $(k \cot \delta)^{-1}$ (a) for isotopic spin $I = 0$, (b) for isotopic spin $I = 1$. The dotted lines are the (a+) solution of Dalitz and Tuan.

TABLE I

Two sets of parameters that fit the available experimental data. The solutions are characterized by the constructive or destructive nature of the Coulomb-nuclear interference in the angular distribution at 172 Mev/c.

Solution	Constructive	Destructive
R_1	$1.0\ M_N^4$ fermi	$2.0\ M_N^4$ fermi
R_0/R_1	-3.8	$+ 1.50$
z_0	$(1.34 + i\ 0.74)M_N^2$ fermi	$(0.93 + i\ 1.46)M_N^2$ fermi
z_1	$(0.67 + i\ 0.12)M_N^2$ fermi	$(-0.22 + i\ 1.00)M_N^2$ fermi
λ^{-1}	0.13	0.29

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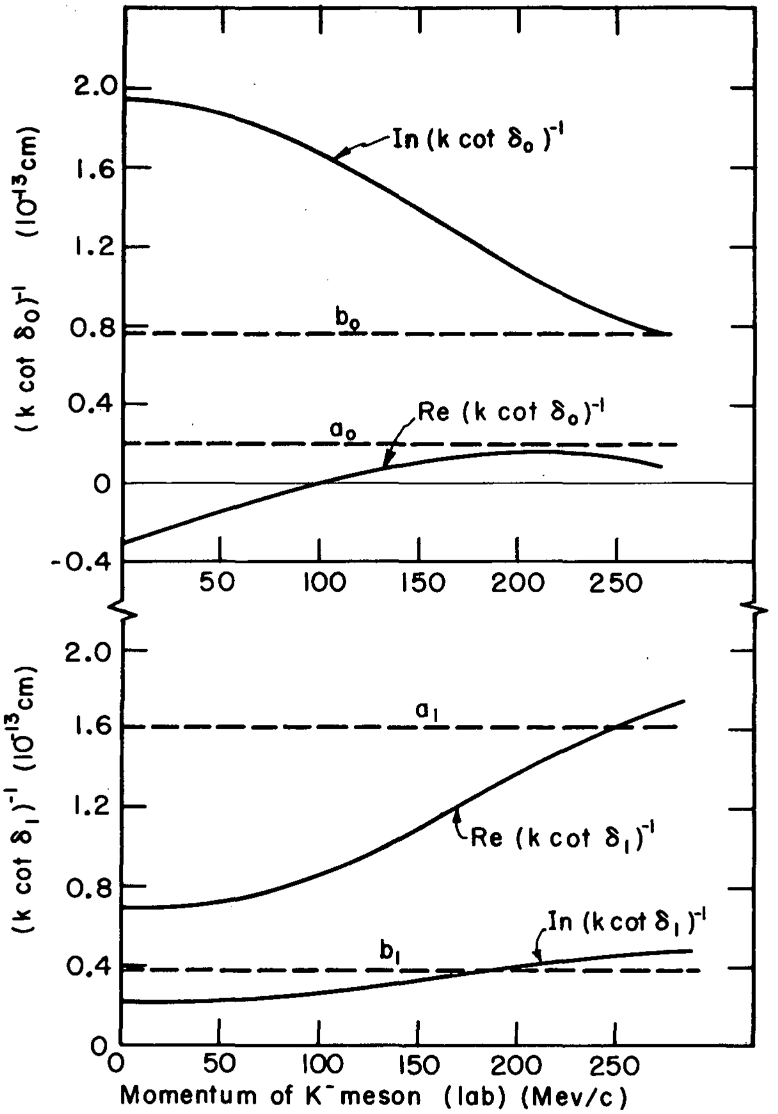

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Fig. 1. Cross sections for elastic and charge-exchange K^--proton scattering. The (a+) solution of Dalitz and Tuan (dotted lines) is included for comparison.

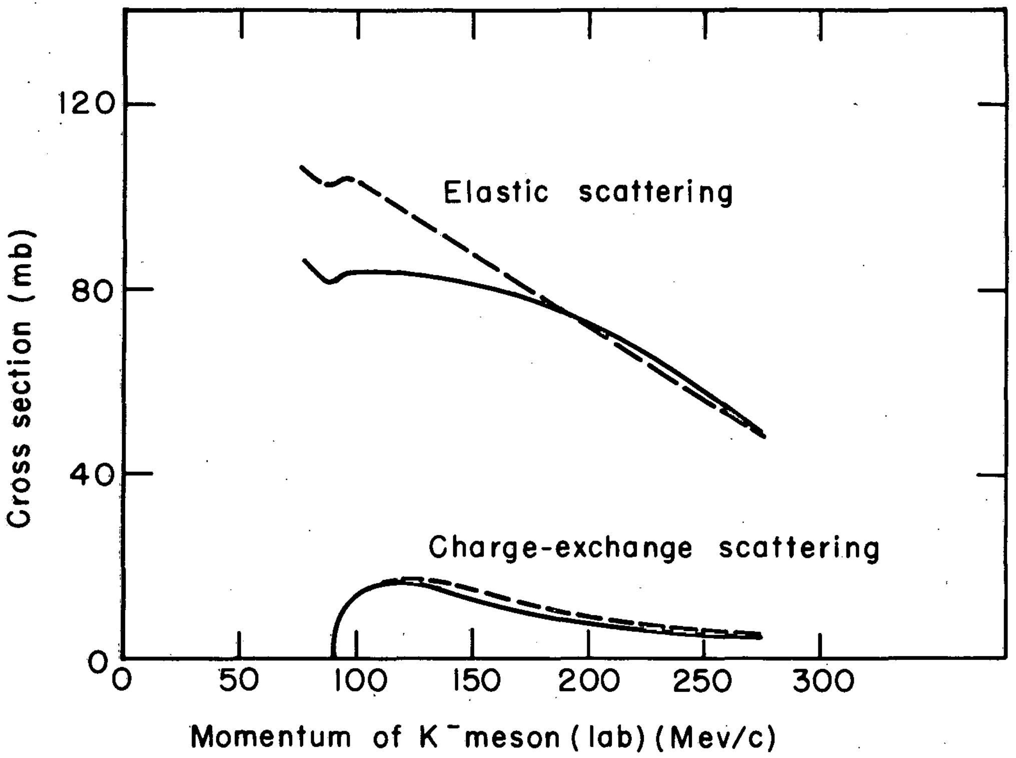

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Fig. 2. Momentum dependence of the real and imaginary parts of $(k \cot \delta)^{-1}$ (a) for isotopic spin I = 0, (b) for isotopic spin I = 1. The dotted lines are the (a+) solution of Dalitz and Tuan.