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ABSTRACT

The pion-exchange multiperipheral model is used to predict the amount and spectrum of pion production at small c.m. momenta in high-energy collisions. The transverse momentum distribution, which agrees with the present experimental data, is predicted to remain unchanged at all higher energies, but the number of pions produced per unit volume of phase space, which is related to the average multiplicity, is predicted to decrease.

There has recently been a revival of interest in the theoretical development[1] and phenomenological application[2] of the pion-exchange multiperipheral model originally proposed by Amati et al. in 1962 (the ABFST model).[3] The results of these studies have been generally encouraging, and there appears to be some possibility that the model will actually be able to account for the majority of inelastic events at the very high energies that will soon be experimentally accessible.

The purpose of this article is to present and discuss the predictions of the ABFST model concerning inclusive pion production in a kinematic region that will become more and more important at higher energies, namely the region in which the pion energy is only a small fraction of the total available energy in the overall c.m. system. We shall refer to this loosely as the region of "small c.m. momenta," although the highest pion momenta included may actually be quite large [a fixed small fraction of $(s)^{\frac{1}{2}}$, the c.m. energy] at very high energies. This is an important region for multiperipheral models, because its growth with increasing energy is supposed to give rise to a logarithmically increasing multiplicity of secondaries, a prediction most characteristic of these models and at present in agreement with experiment.[4]

If pion exchange is to be the dominant multiperipheral mechanism, it should give a detailed account of the production of pions in this limited kinematic region at small transverse momenta, which are associated with small momentum transfers, as it does in the case of low-energy processes that are dominated by single pion exchange.[5] The predictions provide a good test of the essential features of the model, and any shortcomings will lead to a disagreement with experiment that becomes more serious with increasing energy.

A striking consequence of the hypothesis of multiperipheral pion exchange is that secondary pions must be produced in pairs of predominantly low invariant mass. Since most of the cross section for low-energy $\pi\pi$ scattering is associated with the ρ resonance, this means that most secondary pions result from the decay of ρ mesons. Accordingly, the principal features of the model should be represented by the diagram shown in Fig. 1, and we shall adopt this diagram as our basis of discussion.

A problem that must be faced at the outset, and which always arises in the ABFST model, is that the vertices in Fig. 1 refer to scattering processes in which one or two particles are not on the mass shell. For several reasons this is not a serious difficulty here. Consider first the vertices represented by the shaded blobs: the form factors for these processes are already given, within the context of the model, as the corresponding eigenfunctions of the ABFST integral equation. These eigenfunctions are in fact very nearly constant,[6] and the associated off-shell dependence is negligible. For the off-shell $\rho\pi\pi$ vertex, we may use the phenomenological form factor of Dürr and Pilkuhn,[7] which has proved satisfactory in fitting data on single pion exchange.[5]

The main conclusion of this article, however, will be that the predicted transverse momentum distribution of secondary pions is almost totally insensitive to our assumptions about the off-mass-shell scattering amplitudes. Instead, one has the most desirable situation that this distribution is highly sensitive only to the truly essential features of the model: the masses and quantum numbers of the exchanged particles, and those of the resonances that are produced. Unfortunately the absolute number of pions produced, in contrast to their unnormalized transverse momentum distribution, is somewhat sensitive to the details of the off-shell vertices, so the prediction of this number is less sure. Nevertheless, some clear general conclusions on the energy-dependence of the pion multiplicity can be drawn.

The calculation of the inclusive differential cross section by means of the diagram in Fig. 1 involves a straightforward extension of the original work of ABFST[3] to include the kinematics of the ρ decay. We assume that each final-state pion may be associated uniquely with a parent ρ meson, and neglect the interference effects that can occur in the small regions of phase space where resonance bands overlap. In the overall c.m. frame we write the pion four-momentum as $p = (E, p_T, 0, p_L)$ and the ρ four-momentum as $k = (k_0, k_T \cos\phi, k_T \sin\phi, k_L)$, so that k_T and p_T are the ρ and pion transverse momenta. The high-energy absorptive parts associated with the shaded blobs in Fig. 1 are assumed to be dominated by an effective vacuum trajectory with intercept α and coupling $\beta_\pi^{\,2}$ to the $\pi\pi$ channel. Then if $d\sigma_{AB}^{\pi}$ is the inclusive differential cross section for $A + B \to \pi$ + anything, in the kinematic region $p_T, |p_L| << (s)^{\frac{1}{2}}$, and σ_{AB} is the AB total cross section, one finds that[8]

$$d\sigma_{AB}^{\pi}/\sigma_{AB} = \beta_\pi^{\,2}\, G(p_T^{\,2})\, dp_T^{\,2}\, dp_L/E, \tag{1}$$

where

$$G(p_T^{\,2}) = \frac{1}{16(2\pi)^9}\int dx\, dy\, dt_1\, dt_2\, dk^2\, dk_T^{\,2}\, d\phi\, |A(k^2, \cos\theta)|^2 \times (k^2 + k_T^{\,2})^{\alpha+1} \frac{x^\alpha y^\alpha (-\lambda_1)^{-\frac{1}{2}}\, \lambda_2^{-\frac{1}{2}}}{(t_1 - m_\pi^{\,2})^2 (t_2 - m_\pi^{\,2})^2}, \tag{2}$$

$$\lambda_1 = \lambda[t_1 + (k^2 + k_T^{\,2})x(1+y);\ t_2 + (k^2 + k_T^{\,2})y(1+x);\ -k_T^{\,2}],$$

$$\lambda_2 = \lambda[k^2 + k_T^{\,2};\ m_\pi^{\,2} + p_T^{\,2};\ m_\pi^{\,2} + p_T^{\,2} + k_T^{\,2} - 2p_T k_T \cos\phi],$$

and

$$\lambda[a; b; c] = a^2 + b^2 + c^2 - 2(ab + bc + ca).$$

In Eq. (2), x and y are variables of integration whose kinematic significance is discussed by Amati et al.;[3] t_1 and t_2 are the four-momenta-squared of the exchanged pions, Θ is the angle between the direction of these pions and that of the decay pions in the ρ rest frame, and $A(k^2, \cos\Theta)$ is the $\pi\pi$ elastic scattering amplitude.

Corresponding to our assumption of ρ dominance, we write (including the Dürr-Pilkuhn form factor):

$$A(k^2,\cos\Theta) = \frac{48\pi}{p^*}\frac{q^*}{p^*}\left(\frac{1+R^2p^{*2}}{1+R^2q^{*2}}\right)^{\frac{1}{2}}\frac{m_\rho^{\,2}\Gamma_\rho\cos\Theta}{m_\rho^{\,2}-k^2-im_\rho\Gamma_\rho}, \qquad (3)$$

where m_ρ and Γ_ρ are the ρ mass and width, and p^* and q^* are the on-shell and off-shell pion momenta in the ρ rest frame. The value of the radius parameter R is found from the data on single pion exchange[5] to be of the order of 1F. The factor of $\cos^2\Theta$ that is introduced when the amplitude (3) is used in Eq. (2) is the familiar decay angular correlation for a ρ meson produced by pion exchange.[9]

The numerical evaluation of the function $G(p_T^{\,2})$ defined by Eq. (2) can be carried out without too much trouble. We shall consider first the predicted form of the transverse momentum distribution, without regard to normalization, by exhibiting the function

$$F(p_T^{\,2}) = G(p_T^{\,2})/G(0), \qquad (4)$$

which is arbitrarily normalized to unity at $p_T^{\,2} = 0$. This function is shown in Fig. 2 for a range of value of the parameters R and α: it is clearly insensitive to these parameters.

The insensitivity of the transverse momentum distribution to the details of off-shell $\pi\pi$ scattering, as represented by the parameter

R, establishes the form of distribution shown in Fig. 2 as a firm prediction of the ABFST model. This distribution should be observed in pion production at small c.m. momenta in the high-energy collisions of all types of target and projectile particles.

In order to decide what is meant by "high-energy" in this context, we should bear in mind that the main contribution to the integral in Eq. (2) comes from the kinematic region in which the invariant masses-squared of the "anything" systems in Fig. 1 are of the order of $m_\rho(s)^{\frac{1}{2}}$. Thus in order to justify strictly our assumption of vacuum-exchange dominance for the high-energy absorptive parts associated with the shaded blobs, we should require $m_\rho(s)^{\frac{1}{2}} \gtrsim 35$ GeV^2, that is, $s \gtrsim 2000$ GeV^2. However, the fact that the distribution is also insensitive to the effective trajectory intercept α leads us to expect that the experimental data should conform to the prediction at lower energies, perhaps even as low as $s \approx 25$ GeV^2, in which case the absorptive parts could be considered to be dominated by an effective trajectory with an energy-dependent intercept.

Data on the π^- transverse momentum distribution at the smallest longitudinal c.m. momenta $[|p_L| < 0.05\,(s)^{\frac{1}{2}}]$ have been obtained by Ko and Lander[10] from K^+p collisions at 12.5 GeV/c. These data are compared with the predicted distribution in Fig. 3. Although at this stage the normalization is arbitrary, there are no other free parameters, and the agreement is very good.

The predicted pion transverse momentum distribution is very different from that of the ρ mesons, which is also shown in Fig. 3. The difference is associated with the fact that the ρ is more massive than the pion. Suppose first that the ρ mass were twice that of the pion:

in that case each pion would have precisely one-half the transverse momentum of its parent ρ, and the pion p_T^2 distribution would fall four times more steeply than that of the ρ. In reality, the ρ is more massive than two pions, and energy is released in its decay, leading to modifications of the momentum distribution that depend on the decay angular correlations. For the $\cos^2\theta$ correlation implied by the ABFST model, there is a peaking at small values of p_T^2, for the following reason: the virtual pions in the multiperipheral chain have momenta that are predominantly longitudinal in the ρ rest frame. Since these momenta form the axis with respect to which the angle θ is measured, a $\cos^2\theta$ correlation corresponds to a preference for longitudinal ρ decay, that is, for the smallest transverse decay momenta. To illustrate the importance of this effect, we show in Fig. 3 for comparison the distribution corresponding to isotropic ρ decay.[11]

In concluding our discussion of the form of the transverse momentum distribution, we may say that the model gives a consistent, if unorthodox, account of this form in terms of the low-energy kinematics of pion exchange and ρ decay.

Turning now to the absolute value of the differential cross section, we define the normalization quantity

$$C(s) = \beta_\pi^2 G(0), \qquad (5)$$

so that

$$d\sigma_{AB}^\pi/\sigma_{AB} = C(s)\, F(p_T^2)\, dp_T^2 dp_L/E. \qquad (6)$$

At very high energy, one may relate $C(s)$ to the average multiplicity of secondary pions by integrating Eq. (6);[12] this gives

$$\begin{aligned} \langle n_\pi(s)\rangle &\sim C(s) \log s \int F(p_T^2)dp_T^2 \\ &\sim 0.4\, C(s) \log s. \end{aligned} \qquad (7)$$

Unlike $F(p_T^2)$, the quantity $C(s)$ is sensitive to the values of the parameters α and R. For $s \gtrsim 2000$ GeV^2, one may assume $\alpha = 1$ and obtain an estimate of β_π^2 by factorization of the πp and pp total cross sections. This leads to the result

$$C(s) \approx 1 \text{ GeV}^{-2} \quad \text{for} \quad s \gtrsim 2000 \text{ GeV}^2, \qquad (8)$$

corresponding to $\langle n_\pi\rangle \sim 0.4 \log s$, for $R = 0.4$ F, but the value of C falls by a factor of about 3 as R is increased to infinity. In view of the uncertainty in the value of R determined from data on single pion exchange, and the general uncertainty in the way the off-shell effects should be treated, the result (8) should be regarded only as an order-of-magnitude prediction.

The quantity C is written as a function of s because at lower energies we expect the effective residue β_π^2 to be larger and the effective trajectory to have a smaller intercept: both these effects lead to an increase in C. For example, at $s = 25$ GeV^2 $[m_\rho(s)^{\frac{1}{2}} \approx 4$ GeV$^2]$ we might expect the effective intercept to be $\alpha \approx 0.7$ and the effective residue to have about twice its asymptotic value, which gives

$$C(s) \approx 3 \text{ GeV}^{-2} \quad \text{at} \quad s \approx 25 \text{ GeV}^2. \qquad (9)$$

Experimentally, Ko and Lander[10] find $C(s) \approx 5$ GeV^{-2} at this energy, so the order-of-magnitude prediction (9) is quite satisfactory.

The predictions of the model concerning the absolute cross section may be summarized by saying that $C(s)$ in Eqs. (6) and (7) is expected to be a decreasing function of s,[13] with an asymptotic value (attained for $s \gtrsim 2000$ GeV^2) of order of magnitude 1 GeV^{-2}. Data at very high energies should soon be available for comparison with this prediction, and for comparison with the transverse momentum distribution shown in Fig. 2. If the dominance of the multiperipheral pion-exchange mechanism is not ruled out by these tests, there are predictions of two-particle correlations that unfortunately cannot be discussed here owing to limitations of space.

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FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

Fig. 1. The multiperipheral diagram to be considered.

Fig. 2. Predicted transverse momentum distribution. The following cases are shown: $\alpha = 0.7$, $R = 0.4$ F (full line); $\alpha = 0.7$, $R = \infty$ (dotted line); $\alpha = 1.0$, $R = 0.4$ F (dashed line); $\alpha = 0.4$, $R = 0.4$ F (dot-dashed line).

Fig. 3. Predicted transverse momentum distribution (full line), compared with the data of Ko and Lander (Ref. 10) for $-0.24 < p_L < 0.24$ GeV/c, normalized to the area under the curve. Also shown are the ρ meson transverse momentum distribution (dashed line) and the distribution corresponding to isotropic ρ decay (dot-dashed line).

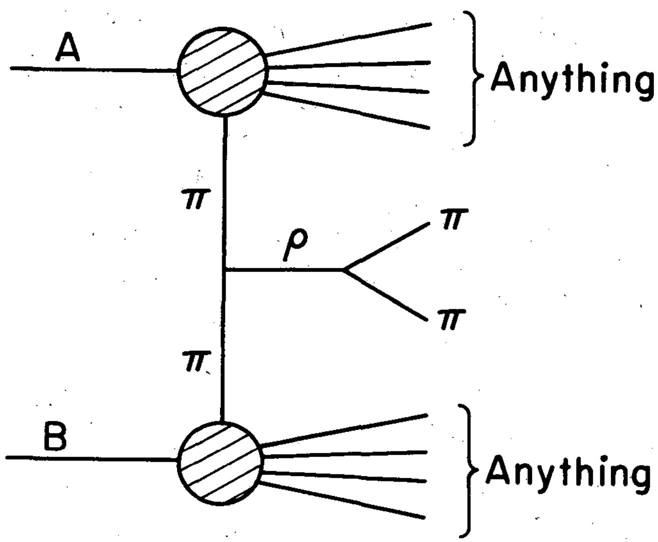

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Fig. 1

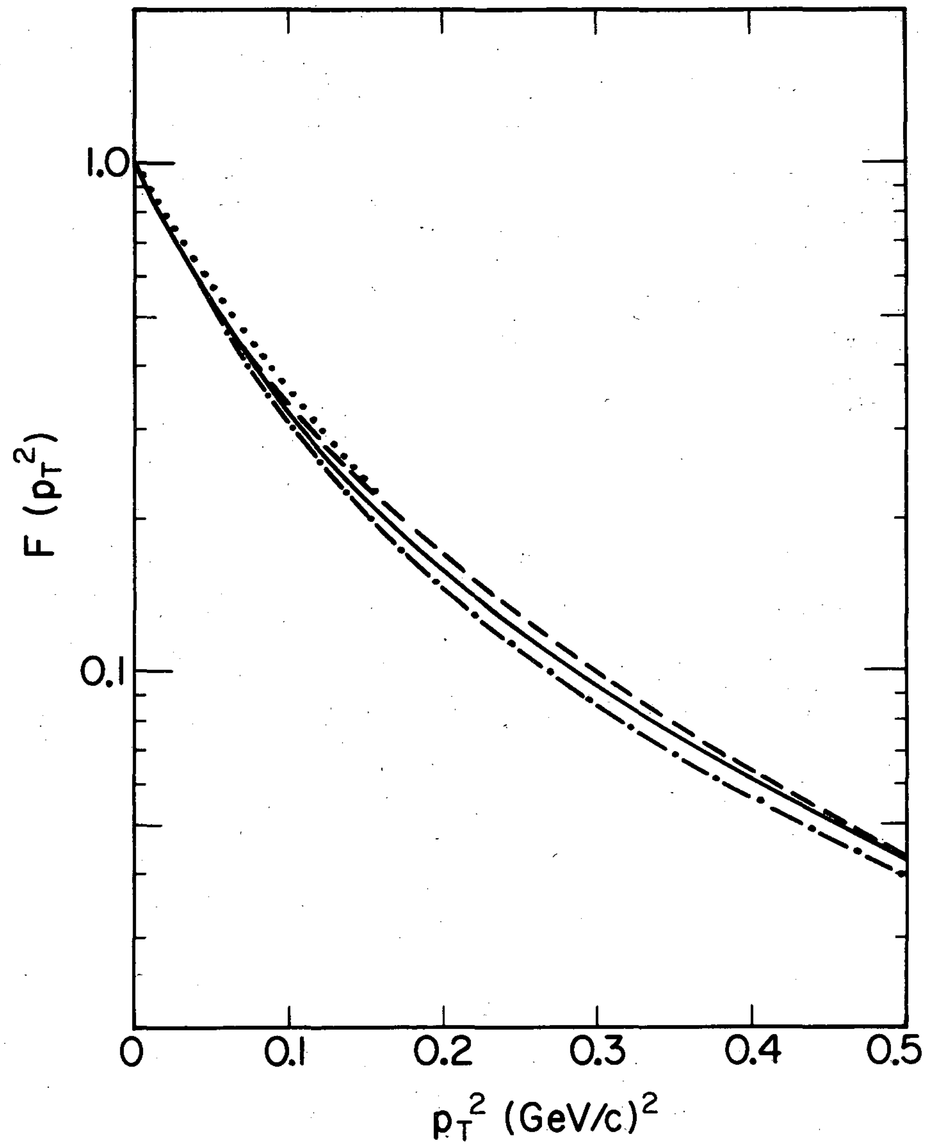

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Fig. 2

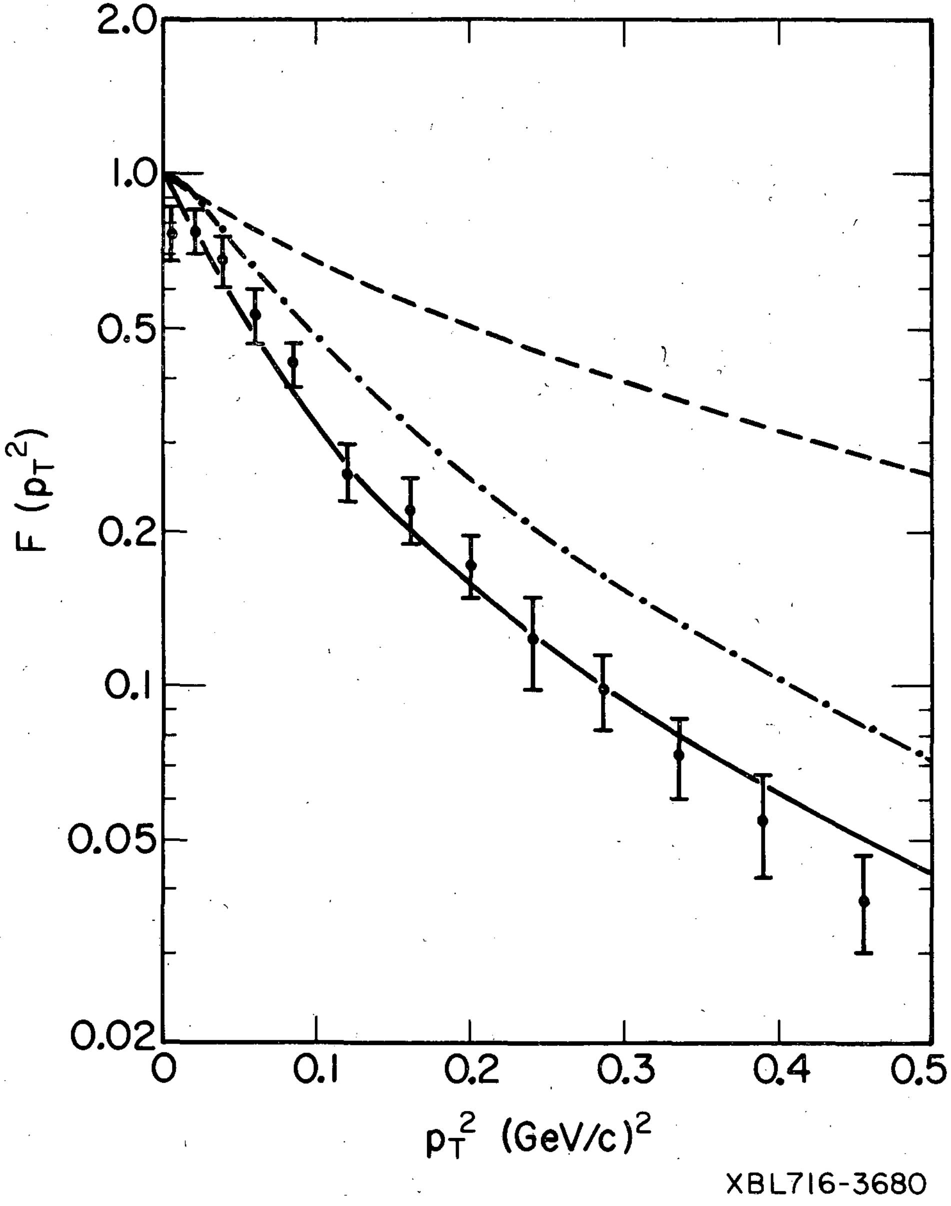

Fig. 3

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