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Author
DeTar, Carleton.

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Carleton DeTar

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LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

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MULTIPERIPHERALISM AND PRODUCTION SPECTRA*

Carleton DeTar

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

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ABSTRACT

We show that limiting fragmentation in production spectra and a log E growth of particle multiplicity follow from two fundamental multiperipheral hypotheses, (a) that transverse momenta are limited, and (b) that particles produced at sufficiently high relative velocities should be uncorrelated. We illustrate the general argument with the pion-exchange model of Fubini and collaborators and discuss factors that affect the rate at which production spectra approach a limiting form.

Two common features underlie the great variety of multiperipheral models,[1-4] the first, a peripheral constraint, variously expressed as a restriction upon internal momentum transfers along a chain or as a confinement of the momenta of produced particles in the direction transverse to the beam,[5,6] the second, a factorization property, vaguely described as short-range order with long-range decoupling of particles produced along a chain. Arguing from these two principles of multiperipheralism, one can make some useful predictions about the production spectrum of secondary particles in hadronic interactions at high energies. For the reaction $a + b \rightarrow X + \text{anything}$, the predictions may be stated simply in terms of the variables $p_\perp$ and y, where $p_\perp$ is the transverse momentum of particle X and y is related to the Feynman rapidity variable,[7] here defined in terms of $p_\parallel$, the longitudinal momentum of particle X in the laboratory frame as

$$p_{\parallel} = w \sinh y \ , \tag{1}$$

where

$$w = (p_{\perp}^{2} + m_X^{2})^{1/2} \ . \tag{2}$$

We also introduce the variable Y, related to the energy E_b of the beam particle in the laboratory system (the rest frame of particle a), through the formula

$$E_b = m_b \cosh Y \ . \tag{3}$$

In terms of these variables, the predicted density of single-particle production, i.e., the differential production cross section divided by the total cross section,

$$d\rho_{ab}^{X} = (\sigma_{ab}^{tot})^{-1} \ d\sigma_{ab}^{X} \ , \tag{4}$$

may be expressed compactly as follows. At high energies the particle density approaches the form

$$d^2\rho_{ab}^{X}/dp_{\perp}\, dy = G_{aX}(p_{\perp}, y)G_{bX}(p_{\perp}, Y - y), \tag{5}$$

where

$$G_{aX}(p_{\perp}, y) \approx G_{bX}(p_{\perp}, y) \approx f_X(p_{\perp}) \quad \text{for} \quad y > \Delta \tag{6}$$

for some constant Δ (which may depend upon X) . The function G_{aX} depends only upon particles a and X, and G_{bX} only upon b and X, whereas f_X is universal, depending only upon particle X.

Equation (5) with (6) expresses three important qualitative features of the spectrum.

(a) The first is limiting fragmentation.[8] In terms of the variables $p_{\perp}$ and y, this is the feature that, at any fixed $p_{\perp}$ and y, the particle density approaches a constant limiting value as the total energy (or Y) is increased. This is also true at fixed $p_{\perp}$ and Y - y, corresponding to limiting fragmentation in the rest frame of the beam particle.

(b) A second feature of the spectrum is a direct consequence of long-range decoupling. Fragmentation of the target is independent of the beam particle, and vice versa. Moreover, the central region of the spectrum $(\Delta < y < Y - \Delta)$ is independent of both beam and target.

(c) A third feature of the spectrum is the gradually elongating plateau in the central region. The particle density $d\rho/dy$, integrated over $p_\perp$ at fixed y, approaches the constant height

$$g_X = \int f_X^{\,2}(p_\perp)dp_\perp \tag{7}$$

for $\Delta < y < Y - \Delta$. Since the average multiplicity of particle X per event is obtained by integrating the particle density,

$$\langle n_X\rangle_{ab} = \iint (d^2\rho_{ab}{}^X/dy\,dp_\perp)dy\,dp_\perp\,, \tag{8}$$

it is easily shown that as Y increases,

$$\langle n_X\rangle_{ab} \sim g_X\,Y + c_{aX} + c_{bX}\,, \tag{9}$$

where c_{aX} depends only upon particles a and X, and c_{bX} only upon b and X. A necessary consequence of the constant-height elongating plateau is, therefore, that the average multiplicity increases linearly in $\log E_b$ at asymptotic energies.[9] In terms of momentum variables a constant distribution in y implies a distribution

$$d^2\rho_{ab}{}^X/dp_\perp\,dp_{\parallel} = f_X^{\,2}(p_\perp)/E \tag{10}$$

at the corresponding range in $p_{\parallel}$, where

$$E = (p_\perp^{\,2} + p_{\parallel}^{\,2} + m_X^{\,2})^{1/2}\,.$$

Both the liquid droplet model[10] and the parton model of Feynman[7] make the same predictions as above for the single-particle spectrum, and with minor modification, it is possible to bring the predictions of the statistical thermodynamical model[11] into accordance with these results,[6] should such an agreement be desirable. Such unanimity strongly suggests a fundamental similarity among the various models. Indeed, the restriction to small transverse momenta and the factorization inherent in the Feynman diagrams offered by Cheng and Wu in support of their conclusions do seem to fulfill our two criteria of multiperipheralism. In the parton model the idea that scattering takes place through an exchange of partons from the beam and from the target suggests a decoupling of beam and target components, as in Eq. (5). However, it is not clear, from its presently published form, whether long-range decoupling occurs in the parton model in such a way that the central part of the spectrum is independent of the incident particles.

Before presenting arguments in support of these conclusions, it is necessary to give a more specific meaning to the concept of long-range decoupling. We require that when the subenergy of two produced particles i and j is sufficiently large, i.e.,

$$s_{ij} \equiv (P_i + P_j)^2 > M^2 , \tag{11}$$

then the production cross section for those two particles should be expressible as a product of two factors, one of which contains the momentum dependence of particle i and the other, of particle j. In other words, the production of any pair of particles should not be correlated when their relative velocities are sufficiently large.

Let us consider in what way multiperipheral models fulfill this requirement. We shall find that decoupling or loss of memory along the chain is intimately related to the Fredholm property of the integral equation by means of which one sums the asymptotic multiparticle contributions to the elastic unitarity equation.[1,12] For our application the essential consequence of this property is that at asymptotic energies, the total cross section is dominated by a single Regge pole[13]

$$\sigma_{ab}^{tot} \sim \beta_a \beta_b s^{\alpha} . \qquad (12)$$

This is a result common to the ABFST model,[1] the multi-Regge exchange model,[2] the CLA model,[3,14] and the model of Ref. 4. In order to construct an integral equation, it is necessary that the n-particle production amplitude be expressed as a matrix product of repeating factors each of which specifies the momentum dependence of a restricted group of neighboring particles along the chain. Each of the aforementioned models has this property. In each case the repeating unit of the chain has a behavior sufficiently regular at large local subenergies that a Fredholm integral equation can be constructed.

The relevance of a leading Regge pole to long-range decoupling is apparent, when one considers that the asymptotic behavior (12) not only is a property, of the total cross section, which is obtained through unitarity by summing "ladders" with the incident particles attached to the ends (Fig. 1a), but also is a general property of a sum of incomplete ladders (Fig. 1b) in which the ends of the ladder have been removed.[15] In this case "energy" means the total energy of those final particles that are included. To obtain the production cross section for particles i and j in conjunction with any other particles, it is necessary, roughly speaking, to sum a set of incomplete ladders in which particles i and j appear at the ends. Therefore, the production cross section has an asymptotic behavior $(s_{ij})^{\alpha}$ in the subenergy of particles i and j. If $p_{\parallel j} \gg p_{\parallel i} \gg p_{\perp i}, p_{\perp j}$, then s_{ij} may be written as

$$s_{ij} \sim (p_{\parallel j}/p_{\parallel i}) w_i^2 = w_i\, w_j \exp(y_j - y_i) \qquad (13)$$

where y and w are given by Eqs. (1) and (2). Inasmuch as the subenergy becomes a product of two factors, separating the momentum dependence of particles i and j, the production cross section also factors in this manner. Therefore, the presence of a leading Regge pole leads to a decoupling of the momentum dependence of pairs of particles in the production cross section when the subenergy of the pair is large. Evidently, the value of M^2 in Eq. (11) depends upon the energy at which the leading pole dominates the sum of incomplete ladders.

When the multiperipheral hypothesis is stated in these terms, the properties of the single-particle spectrum described by Eqs. (5) and (6) follow quite simply. If decoupling occurs for $s_{ij} > M^2$ and transverse momenta are limited, then by Eq. (13) decoupling also occurs for $|y_j - y_i| > \Delta$ for some constant Δ. Therefore, particles produced in the central region $(\Delta < y < Y - \Delta)$ are not correlated with the beam and target particles (whose coordinates are $y = Y$ and $y = 0$, respectively). Moreover, since the particle density depends upon only local correlations among particles, in the central region the particle density itself must be independent of the beam and target. Finally, since the production spectrum in any subinterval of the central region is not correlated with the spectrum in another subinterval, provided it is sufficiently remote, the particle density must be constant in the central region. Production in the target's end of the spectrum $(y < \Delta)$ is correlated with the target particle, but cannot be correlated with the beam particle, when Y is sufficiently large. If the particle density in this region or in the central region depended upon the total energy, particle production would necessarily be correlated with the beam particle and the particles directly associated with its fragmentation over a distance in y greater than Δ, contrary to the assumptions of the model. These properties of the single-particle spectrum are summarized by Eqs. (5) and (6).

In the ABFST model it is possible to verify Eqs. (5) and (6) explicitly for the single-particle spectrum.[16] In our terminology, Amati, Fubini, and Stanghellini have demonstrated, for pairs of pions of a given invariant mass, that the density distribution in the central region is constant in y and independent of the beam energy. To derive this result, they consider the diagram illustrated in Fig. 2. The left and right clusters represent a sum over all incomplete ladders with total energy s_L and s_R respectively. The spectrum of the pair of pions in the middle is to be calculated. To obtain this spectrum they fix the four momentum of the pion pair and integrate over all possible momenta and masses of the left and right clusters. The key observation is that for sufficiently large total energies and central values of the momenta of the pion pair, s_L and s_R are both large over the important range of integration so that one can use the asymptotic form $(s_L)^\alpha$ and $(s_R)^\alpha$ for the sum over the contributions to the left and right clusters. This leads to a decoupling of the momentum dependence of the pion pair from the beam and target. Since the production cross section is proportional to $\beta_a (s_L)^\alpha (s_R)^\alpha \beta_b \propto \beta_a \beta_b s^\alpha$ and the total cross section is also proportional to $\beta_a \beta_b s^\alpha$, the particle density is constant in y and independent of the total energy. Note that the residue factors β_a and β_b cancel, as they must if the particle density in this region is to be independent of beam and target. By a simple extension of this argument one may determine the spectrum at small laboratory momenta for the pion pair. In this case one finds that $s_R \approx s$ and s_L is small over the important range of integration so that one can use the asymptotic form $(s_R)^{\alpha} \beta_b$ for the right cluster, provided s is sufficiently large. Again the particle density is independent of the total energy and of the beam particle. However, in this region the density is correlated with the target. Hence, fragmentation of the target is limiting and independent of the beam. Putting these results together leads to Eqs. (5) and (6).

It may be desirable to take into account nonleading terms in the sums over ladders, especially if the leading Regge pole does not prevail until the total energy is impractically large. The AFS argument is easily generalized, if the total cross section can be represented as a series of factorized powers

$$\sigma_{ab}^{tot}(Y) \sim \sum_{\gamma=1}^{n} \beta_{a\gamma}\, e^{\alpha_\gamma Y}\, \beta_{b\gamma} \, . \qquad (14)$$

One effect of including nonleading powers is that the particle density develops a " memory" or long-range correlation with a characteristic period of decay in y of $\delta = (\alpha_1 - \alpha_2)^{-1}$, the inverse of the separation between the two leading powers. Another effect is to introduce a transient in the particle density that dies out with increasing beam energy with a characteristic period of δ in the variable Y. This suggests that Δ in Eq. (6) should be chosen so that it is large compared with δ .

In the limit that the leading powers are degenerate and well separated from other powers, an interesting situation arises. Once again fragmentation of the target is rapidly limiting in beam energy, but it must depend upon the beam particle unless the ratio β_{a1}/β_{a2} is approximately independent of the particle type a . To the extent they predominate and cannot be approximated by poles with factorable residues, Regge cuts complicate the production spectrum. If they must be included, the spectrum need not exhibit any of the features of Eqs. (5) and (6).

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I am indebted to Geoffrey F. Chew for many helpful discussions. After completing this work it came to the author's attention that Kenneth G. Wilson had made the same argument in 1963. [Kenneth G. Wilson, Acta Physica Austriaca 17, 38 (1964).]

FOOTNOTES AND REFERENCES

* This work was supported in part by the U. S. Atomic Energy Commission.

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14. Peter D. Ting, Studies of Multiperipheral Integral Equations (Ph.D. Thesis), Lawrence Radiation Laboratory Report UCRL-19802, May 1970.

15. In this context "ladder" refers to the general topological feature of the multiparticle contribution to the unitarity sum in multiperipheral models.

16. Silverman and Tan have shown that the multi-Regge model also yields this result. Dennis Silverman and Chung-I Tan, Multiperipheral Dynamics and Inclusive Experiments, Princeton University report, June 1970.

FIGURE CAPTIONS

Fig. 1. (a) Schematic representation of the complete ladder that is summed to obtain the total cross section. The rungs represent produced particles, and the side links represent matrix factorization of the repeating unit in the multiparticle production amplitude.

(b) The incomplete ladder.

Fig. 2. Diagram for the calculation of the spectrum of pion pairs in the ABFST model.

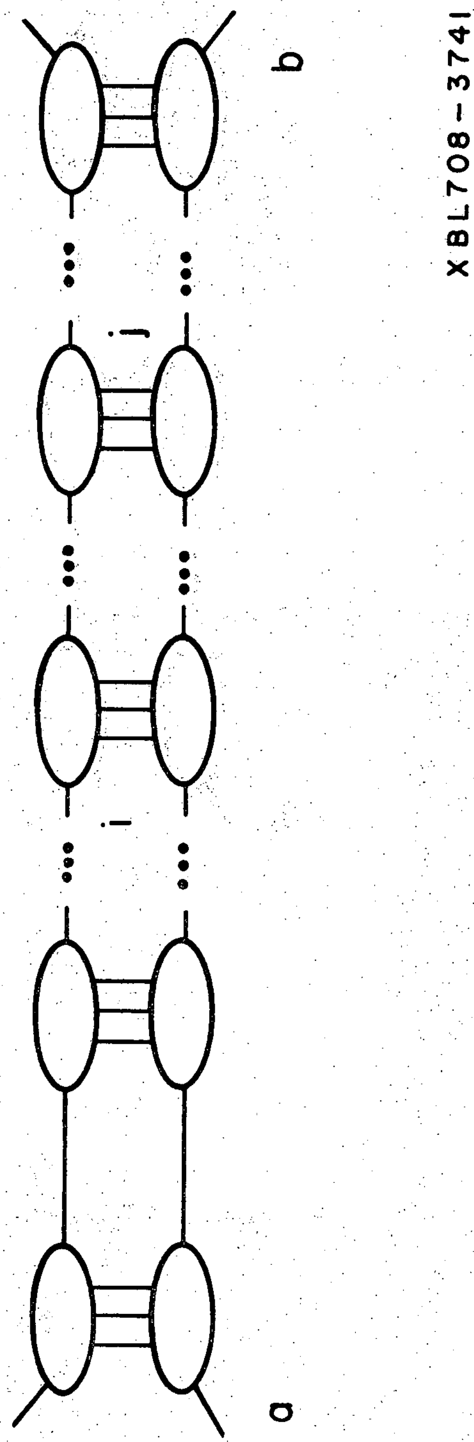

Fig. 1a

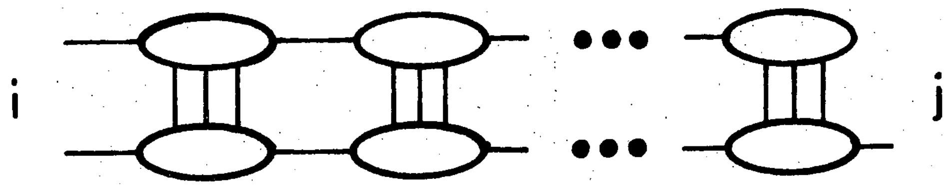

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Fig. 1b

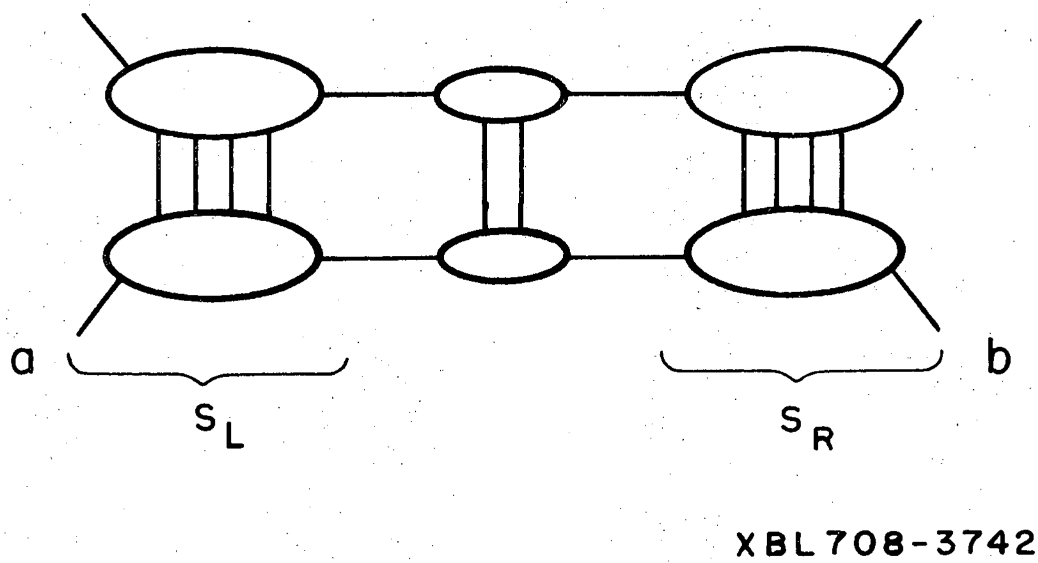

Fig. 2

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LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720