Lawrence Berkeley National Laboratory

Recent Work

Title

CERENKOV RADIATION FROM PROTONS AND THE MEASUREMENT OF PROTON VELOCITY AND KINETIC ENERGY

Permalink

<https://escholarship.org/uc/item/9zc4r1ws>

Author

Mather, R.L.

Publication Date

1951-05-01

UNIVERSITY OF CALIFORNIA - BERKELEY

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545*

RADIATION LABORATORY

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

Contract No. W-7405-eng-48

CERENKOV RADIATION FROM PROTONS AND THE
MEASUREMENT OF PROTON VELOCITY AND KINETIC ENERGY

R. L. Mather

(Thesis)

May 22, 1951

Berkeley, California

TABLE OF CONTENTS

Section I	Introduction
Section II	Initial Experiments
Section III	Improvement of the Angular Resolution
Section IV	Proton Energy Measurements
	A. Method and Apparatus
	B. Measurement of the Angle between the Mirror Normal and the Proton Beam Axis
	C. Measurement of the Angle between the Cerenkov Rays and the Mirror Normal
	D. Measurement of the Index of Refraction
	E. Reading the Film
	F. Results
Section V	Resolution of the Instrument
	A. Scattering
	B. Slowing
	C. Diffraction
	D. Divergence
	E. Chromatic Effects
	F. Energy Spread
	G. Combined Intensity Pattern
Section VI	Improvement of the Energy Resolution
Section VII	Acknowledgments
	Appendix I - Calculation of Cerenkov Radiation Intensity
	Appendix II - Literature on Cerenkov Radiation

CERENKOV RADIATION FROM PROTONS AND THE
MEASUREMENT OF PROTON VELOCITY AND KINETIC ENERGY

R. L. Mather
May 22, 1951

I. INTRODUCTION

The excellence of the original theoretical explanation^{(1,2)*} of Cerenkov radiation has overshadowed the experimental results^(3,4,5,6) because scattering has prevented the unique properties of this radiation from being convincingly demonstrated. As a result there has been some tendency to confuse this type of radiation with that from an acceleration process such as bremsstrahlung⁽⁵⁾ although the distinction of it from ordinary fluorescence has been evident from the beginning⁽³⁾. The experimental results have also allowed some speculation about modifications of the theory⁽⁷⁾.

This radiation can be predicted as a solution of Maxwell's equations for a charged particle in uniform rectilinear motion through a medium in which the velocity of electromagnetic radiation is less than the particle velocity. It takes a form analogous to the shock wave from projectiles traveling in air faster than the velocity of sound. The existence of such a wave phenomenon, far from being uncommon, is observable in the bow wave from boats and can easily be observed in the family bathtub by drawing a pencil tip across the surface of the water.

The directional property of Cerenkov radiation is most readily seen from a Huygen's wavelet construction as shown in Figure 1. The wavelets combine with each other to form a conical wavefront symmetrical about the path of the particle. The normals to this wavefront, or ray directions, make a constant

* A brief discussion of the literature will be found in Appendix II.

angle θ with the path of the particle.

$$\cos \theta = \frac{1}{n \beta} \quad (1)$$

Where c is the velocity of light in vacuum, n is the index of refraction of the medium with respect to vacuum and βc is the velocity of the particle.

Since reinforcement is a phase phenomenon like refraction, it is the phase velocity which is important and n is strictly the index of refraction relative to vacuum. In the case of a dispersive medium both n and θ are functions of the frequency.

For $\beta n < 1$ the angle θ becomes imaginary indicating that there is no radiation by this process.

Tamm's theory yields the following spectral intensity for the radiation. The change in the index of refraction over the visible region of the spectrum is usually small enough to allow an approximate calculation of the total visible radiation to be made.

$$\frac{dN}{d\omega dL} = \frac{z^2 e^2}{\pi c^2} \left(1 - \frac{1}{n^2 \beta^2}\right) d\omega \quad \text{quanta/unit path length}$$

$$\approx 500 \sin^2 \theta \quad \text{visible quanta/cm.} \quad (2)$$

Where ze is the particle charge, $\hbar$ is $\frac{1}{2\pi}$ times Planck's constant, and ω is 2π times frequency.

If this is expressed in terms of energy distribution on a wavelength scale the expression becomes

$$\frac{dW}{dL} = 4\pi z^2 e^2 \left(\frac{\sin^2 \theta}{\lambda^3}\right) d\lambda \quad \text{radiated energy/unit path length} \quad (3)$$

where λ is the wavelength of the radiation in vacuum.

This expression shows the radiation to be continuous in spectral distribution wherever $n\beta > 1$. For the visible region the blue will receive emphasis both from the decrease in λ and from the usual increase in n and hence θ .

MU 1697

FIG. 1

Huygens wavelet construction from which equation (1) may be derived

The total energy lost by the particle in Cerenkov radiation is about 0.1 per cent of that lost by other processes in the material; however the intensity is high enough to observe in the visible region and dominates over other sources of radiation in the materials used here.

The polarization of the radiation is obvious from the observation that the particle has, and can radiate, only an azimuthal magnetic field and thus the electric vector of the radiation must lie in the plane determined by the point of observation and the path of the particle. The direction of the electric vector in the Cerenkov wave front points toward or away from the position of the particle according to whether the particle is negative or positive.

The considerations above indicate the directional characteristics of the rays inside the material, but the observation of them requires that they escape and hence their directions will be altered by refraction at the surface. Since the radiation is formed throughout a large section of the material, and near the boundary surface through which the light emerges, the problem of observing the initial direction of the radiation inside the material is complicated by spherical aberration unless the boundary surface is essentially plane. In the following instruments the light always emerges through a plane surface and the focussing of the rays, if any, is done in a separate process.

The 184-inch cyclotron, its shielding, and the path of the electrically deflected proton beam is shown in Figure 2. The deflecting apparatus of the cyclotron assures a well collimated, nearly monoenergetic beam of 340 Mev, $\beta = 0.68$, protons by the time it reaches the experimental area. The shielding around the cyclotron is sufficient so that no special shielding of the light sensitive photographic film was needed.

The cross section of the emerging beam can be varied but since all the following instruments are sensitive only to a section of the beam about one

centimeter square and there were no background problems the cross section was made as large as possible unless considerations irrelevant to this experiment ruled otherwise. The beam current density is about 2×10^{-11} amperes per square centimeter. This gave sufficient radiation for any of the following exposures to be made in less than one hour. Some of the later ones required only three minutes exposure. The short exposures are made possible by the use of very sensitive photographic emulsions and by concentrating the light to as small an area of film as possible.

II. INITIAL EXPERIMENTS

The successful experimental demonstration of Cerenkov radiation from protons was made with an instrument similar in principle, but somewhat simpler, to that used by Cerenkov, Collins and Reiling, and Wyckoff and Henderson. The source, a one centimeter cube of transparent silver chloride ($n = 2.07$) with polished surfaces, was surrounded by a silvered spherical mirror which focussed the light emerging in the horizontal plane to a ring at the position of the photographic film (see Fig. 3). Light emerging at large angles to the horizontal plane was prevented from reaching the mirror and the film by paper masks. Only those portions of the ring would be illuminated which lay in the direction in the horizontal plane in which light was emitted by the sample.

One diagonal of a horizontal face of the cube was placed parallel to the proton beam direction. There were four plane faces from which light could have been recorded. Light emerged from the two forward faces but not from the two rear faces (see Fig. 3). The velocity of the protons was reduced by placing copper foils in the beam and the angle between the intensity maxima decreased in accordance with the theory. If the velocity of the protons is calculated from the index of refraction and the value of θ indicated here the result is within about two percent of what is thought to be the actual

FIG. 2

The 184-inch cyclotron showing the path of the deflected external proton beam.

velocity. In these experiments the images were not sharp for a number of reasons and there was a noticeable amount of background illumination easily attributable to the crudeness of the experiment. A similar experiment was done with a hemi-cylindrical piece of glass ($n = 1.91$) which gave results similar in intensity and angular distribution.

An order of magnitude calculation can be made for the intensity of the radiation from the sensitivity of the film and the exposure*. This yields a figure of 105 quanta/cm. compared to a theoretical value of 250 quanta/cm. This can be considered as agreement and is additional proof that the observed radiation is Cerenkov radiation.

Were this radiation due to bremsstrahlung or any acceleration process of radiation the intensity for electrons should be larger by the square of the mass ratio or 3.4×10^6 if one assumes the accelerating forces to be the same in both cases (i.e. electrical forces). The electron experiments of Collins and Reiling give intensities of the same order of magnitude as observed here.

Were this light due to fluorescence one would have expected an isotropic angular intensity distribution. If the asymmetrical distribution were due to reflections inside the material the change from a cube to a hemi-cylinder should be more marked. Were this fluorescence the intensity of the radiation should be a critical function of the composition of the material. The fact that both the glass and the silver chloride gave similar intensities is a strong argument against this being fluorescence.

III. IMPROVEMENT OF THE ANGULAR RESOLUTION

The theory predicts an angular intensity distribution as sharp as a delta

* The calculations involved will be found in Appendix I.

MU-1730

FIG. 3

Experimental arrangement and positive prints of the film showing Cerenkov radiation.

function for a given wavelength, accordingly the experiment was revised to see if anything approaching this sharpness could be observed. The optical system was improved by using optically ground surfaces on the material (now glass with $n = 1.88$) and observing the light with a Leica camera which has a highly refined optical system (Fig. 4). Scattering and slowing down in the material was reduced by making it thin ($2/3$ mm.). A picture taken with this apparatus is shown in the upper portion of Figure 5 with an accompanying scale in terms of θ .

Most of this width is due to the fact that n and hence θ is a function of the wavelength of the radiation, which, from the theory and observation is a bluish white spectrum. The camera looking into the glass surface will see a limited portion of a circular rainbow of light centered on the direction of the incoming proton beam with the blue outside and the red inside. A prism of the proper angle was ground whose dispersion would cancel the first order dispersion of the Cerenkov rays. The rainbow viewed through this prism should coalesce to a narrow band of white light. A picture made through such a prism is shown in comparison to the previous picture (Fig. 5 bottom).

The breadth of this last image can be attributed to six effects:

(1) Scattering of the proton beam in the material, (2) Change of velocity as material slows the protons down, (3) Diffraction effects due to the finite length of proton path in the glass, (4) The divergence of the incident proton beam, (5) Second order chromatic effects, (6) The velocity spread of the incident proton beam*. Estimates of all these effects can be made and graphs of intensity vs. angle for the individual effects and their combined effect are shown in Figure 6. A microphotometer record of the Cerenkov image is shown in Figure 7. If the ordinate of the record is converted to light intensity the

* Detailed discussions of these effects, a discussion of the combined results and the possibilities of improvement will be found in the last portions of this paper.

MU 1698

FIG. 4

Improved arrangement for viewing Cerenkov radiation.

-13-

FIG. 5

Positive prints showing Cerenkov radiation as a function of an angle. Top—with equipment of Fig. 4. Bottom—with equipment of Fig. 8.

agreement is good. This result is quite in agreement with the delta function angular distribution predicted by the theory of Frank and Tamm.

Taking the raw data with no corrections for any of the above mentioned effects one can conclude that the radiation must be coherent with the particle over a path length of 0.01 cm. or the diffraction pattern alone would be larger than the observed width. This length would be on the order of a million atoms of the material or two hundred wavelengths of visible light.

Coherence of a slightly different sort is found in the completely plane polarized nature of the Cerenkov light. This was tested by fastening strips of polaroid across the camera film oriented at angles of 0, 70, 90, 110 and 180 degrees to the expected plane of polarization of the light. An exposure of 15 times the normal exposure still showed no indication of the Cerenkov image behind the strip oriented at 90 degrees to the plane of polarization while the image behind the other strips indicated intense exposures. The ratio of exposures behind the 90 degree strip to that behind the 0 and 180 degree strips was less than 0.005 indicating total plane polarization in agreement with the theory.

IV. PROTON ENERGY MEASUREMENTS

A. Method and Apparatus

The resolution in θ obtainable with the achromatic arrangement opens the possibility of measuring the velocity and hence the kinetic energy of the protons in the beam using the relativistic equation.

$$E = \mu c^2 \left(\frac{1}{(1 - \beta^2)^{1/2}} - 1 \right) = \mu c^2 \left(\frac{n \cos \theta}{(n^2 \cos^2 \theta - 1)^{1/2}} - 1 \right) \quad (4)$$

where E is the kinetic energy and μ is the mass of the particle. Only two factors enter into this measurement -- the angle θ between the proton path and

MU 1422

FIG. 6

Estimated angular intensity distribution of the achromatized Cerenkov rays (abscissae in equivalent minutes of arc in θ).

FIG. 7

Microphotometer trace of an achromatic Cerenkov radiation image.

the Cerenkov rays and the index of refraction of the glass. Both of these can be measured with high absolute accuracy and a scale of β and Mev are shown in conjunction with the microphotometer trace.

The instrument used for the energy measurement furnished the pictures of Cerenkov radiation which have just been discussed. The diagrammatic layout of the instrument is shown in Figure 8. The glass in which the Cerenkov radiation is produced is shown in the proton beam. It is a thin sheet of extra dense flint glass ($n = 1.88$) about $2/3$ of a millimeter thick with flat optically polished surfaces. The achromatising prism is shown out of the beam and immediately in front of the Leica camera lens. If the Cerenkov rays were allowed to proceed in their original forward direction the camera would have to be placed in the proton beam. To avoid this the surface of the glass sheet through which the beam emerges is aluminized and the Cerenkov rays are reflected back on themselves to emerge in the direction shown. A small projector projects an image of a scale on the film via a small mirror. This scale image is the reference point from which the angle at which the camera sees the Cerenkov radiation is measured. The camera is focussed at infinity.

The problem of relating the position of the image in the camera to the angle θ between the Cerenkov rays and the proton paths is simplified if both directions are measured separately relative to the normal to the aluminized surface of the glass.

B. Measurement of the Angle between the Mirror Normal and the Proton Beam Axis

Establishing the direction of the proton beam to within a few minutes of arc is difficult; however this can be avoided by taking two exposures and inverting the equipment between exposures (Fig. 9). A pair of such exposures is shown in Figure 10. The average of the two positions of the Cerenkov rays will

FIG-8

Arrangement for the achromatic observation of the Cerenkov rays.

be, very accurately, the position that would have been obtained if the proton beam had been parallel to the axis about which the equipment was inverted. This approximation is better the more nearly the axis of inversion approaches the beam direction.

This inversion is carried out by mounting the instrument assembly of Figure 8 on a platform whose hollow cylindrical end pieces, through which the beam passes, rest in the two vee supports as shown in Figure 9. The instrument can then be turned in the kinematical bearing thus provided. A latch will hold the equipment in any one of four positions 90 degrees apart.

The angle between the axis of inversion and the normal to the aluminized surface is accurately fixed by the use of precision angle templates. The assembly of Figure 8 is supported on the platform by pivots which allow the assembly to be turned about an axis through the center of the glass sheet and perpendicular to the plane of Figure 8. The platform and the assembly both carry two semi-circular bumpers. A triangular steel template can be wedged between the two sets of bumpers and the angular position of the assembly with respect to the platform rigidly fixed.

The template is wedged in from one side and the position of one of the bumpers is adjusted so that the normal to the aluminized surface is parallel to the axis of inversion (Fig. 11 left). This is determined by viewing the image of a distant light source reflected by the aluminized surface with a telescope equipped with cross hairs. If the normal is parallel to the axis of inversion the position of the image will not change when the instrument is inverted.

If the template is now wedged in from the other direction (Fig. 11 right) the angle between the normal to the mirror and the axis of inversion is exactly twice the angle of the template.

R 962

-20-

FIG. 9

Energy measuring instrument. Left--normal position (aluminum foil removed from the light tight box of Fig. 8 to show interior construction). Right--inverted position.

FIG. 10

A PAIR OF CERENKOV IMAGES FROM WHICH THE
BEAM ENERGY MAY BE DETERMINED.

The angle templates were ground by standard precision machining techniques using a sine bar and gauge blocks. The amount of rotation of the assembly has been checked by viewing images of distant objects reflected in a mirror attached to the assembly and then measuring the angular separation of the objects seen in the two positions of the assembly with a surveyor's transit. The rotation was measured as $(38^{\circ} 26.1') \pm 0.5'$. The machinists value for the template was $(19^{\circ} 13') \pm 0.2'$.

C. Measurement of the Angle between the Cerenkov Rays
and the Mirror Normal

The position of the film image can be related to the angle between the ray directions inside the glass and the normal to the aluminized surface by the following procedure. The camera back is removed and an assembly carrying a cross hair in the plane formerly occupied by the film is substituted. Light from a mercury vapor lamp is sent into the lens from behind the cross hair. An image of the cross hair passes through the lens, through the prism and into the glass sheet where it is finally reflected from the aluminized surface and re-traces its path to form a real image in the plane of the cross hair. The light of the 5461 Å. mercury green line is used (the achromatising prism serves as a monochromator). The prism is set for minimum deviation and the cross hair is set so that its reflected image coincides with itself. The scale projector is turned on and the central line of the scale is adjusted to coincide with the position of the cross hair and its image.

The relation between the scale divisions on the film and direction as seen by the camera was measured by superimposing the scale on a picture of a distant scene and then measuring the angles between the apparent positions of the lines in the scene with a transit. The separation was found to be $61'$. Since the prism is at minimum deviation it does not alter the apparent separation of the

MU 16 99

FIG. 11
Use of the angle template. Left--to adjust the bumper.
Right--to run.

scale divisions in the region of the center of the scale. Because of refraction at the surface of the glass the angular separation of the rays inside the glass corresponding to one scale division is $61'$ divided by the index of refraction (since the rays are nearly normal to the surface and the angles of incidence and refraction equal their sines) or $32.4'$.

From this it is known that if an image of Cerenkov radiation is formed at the position of the center line of the scale by light of $5461 \text{ \AA}$., the ray direction inside the glass was normal to the aluminized surface. If the image is not at the center line one can interpolate the angle between the ray and the normal by reference to the scale divisions.

It must be understood that the above discussion has applied to light of $5461 \text{ \AA}$. If light of another wave length is chosen for discussion the index of refraction of both the prism and the glass sheet will be different. Using the data from this other wave length, a given position on the scale combined with the change in deviation of the prism and the change in refraction at the surface of the glass will give a different Cerenkov angle θ . Disregarding small second order effects, if the velocity of the proton is calculated using this new θ combined with the new index of refraction for the glass one obtains the same proton velocity as would have been calculated on the basis of the $5461 \text{ \AA}$. ray. This is another way of stating the achromatising condition for the design of the prism. The $5461 \text{ \AA}$. mercury line was chosen for emphasis because it is readily available and its position in the spectrum corresponds to the wave length of minimum second order chromatic deviations for this particular instrument.

The absolute accuracy with which θ was measured has to be estimated as the sum of errors in a number of procedures indicated above. The estimate of the overall accuracy in θ is $\pm 2.5'$.

D. Measurement of the Index of Refraction

The index of refraction of the glass sheet was known by measuring the index of refraction of a small prism ground from a section of the same piece of raw glass from which the sheet was ground. This measurement was the standard one of prism angle and minimum deviation made on a precision spectrometer table. The results were corrected for the index of refraction of air. The index of refraction for the 5461 Å. line relative to vacuum was 1.8796 ± 0.0003 .

Morey⁽⁸⁾ gives a figure (which should be applicable for this glass) for the temperature coefficient of the index of refraction of approximately 0.000014 per degree centigrade. Measurements were always made near enough to room temperature for the temperature effect to be negligible. Continued bombardment of the glass by the proton should produce changes in the properties of the glass. No data is available on these effects, but it is estimated that the proton exposure suffered by the glass in the course of these experiments would have to be increased by 100 times to give a noticeable effect.

E. Reading the Film

Because the width of the Cerenkov image is produced largely by effects thought to be symmetrical, (except for the chromatic effects for which a correction of 1.7° in θ is added), it is assumed that the mid point of the image will correspond to the mean energy of the proton beam in the glass. The film is read with a microphotometer (the central parts of the scale lines have been left out to leave a clean path for the microphotometer) and the trace width is bisected at several heights and the average mid-point taken. This reduces the effect of random density fluctuations in the film.

The curvature of the Cerenkov image (mostly due to the properties of the achromatising prism) and the problem of relating the microphotometer trace to

the scale lines introduces errors in reading the trace. An estimated error in reading the film corresponds to 1.6' in θ . Readings of the same film taken with two different types of microphotometers differed by less than 0.6' in θ .

F. Results

The estimated errors in the measurement of θ and n and in reading the film are summarized in Table I along with their equivalents in velocity and kinetic energy. The estimates of errors quoted are admittedly not conservative. However the experimental results are in convenient agreement with experiments which have been performed simultaneously with the use of this equipment.

For instance, the range energy relation has been checked by Mather and Segre⁽⁹⁾. The results of their copper ranges are plotted in Figure 12 along with the predicted range given by Aron, Hoffman, and Williams⁽¹⁰⁾. The disagreement is of the type and magnitude to be expected. The slope of the predicted range energy curve should be quite accurate and using this it can be seen that the energy measurements are at least consistent (all angle adjustments were remade between runs).

A list of the energy measurements made with this apparatus is given in Table II. It corroborates something already known locally from range measurements, that the energy of the deflected beam from the 184-inch cyclotron is remarkably constant once the controls have been set but a complete readjustment of the controls after an intervening duty on other experiments often results in a different energy.

TABLE I. SUMMARY OF ESTIMATED ERRORS

		$\Delta \beta$	$\Delta \text{Mev.}$
Error in θ	$\pm 2.5'$	± 0.00040	± 0.6
Error in n	± 0.0003	± 0.00012	± 0.2
Error in reading	$\pm 1.6'$ in θ	± 0.00025	± 0.4
Total error		± 0.0005	± 0.8

TABLE II. MEAN ENERGY OF THE ELECTRICALLY DEFLECTED PROTON BEAM FROM THE 184-INCH CYCLOTRON AS IT ENTERS THE CAVE. (Rest Energy of the Proton taken as 938.17 Mev.) (0.43 Mev added for Stopping Power of the Instrument)

Date	Energy	Associated Experiment
9/13/50	339.0 338.5	Range (Segrè) ⁽⁹⁾
9/25/50	341.2 341.3 341.5 341.3	π^+ -meson production (Cartwright) ⁽¹⁵⁾
10/9/50	339.7 339.4 339.3 339.2	Range (Segrè) ⁽⁹⁾
12/24/50	340.5 340.6 340.5	Range (Segrè) ⁽⁹⁾

FIG. 8

MU 1423

FIG. 12

Results of range energy measurements

V. RESOLUTION OF THE INSTRUMENT

The broadening effects graphically summarized in Figure 6 will be discussed individually in the order of their appearance in Figure 6. The results will then be combined for an expression of the overall resolution and the possibilities for improving this resolution discussed. The amount of the correction for the chromatic effects will be calculated and the correction to be added or subtracted for the stopping power of instrument discussed. Lest the length of the discussion exaggerate their importance, it should be mentioned that both of these corrections are about 0.43 Mev and the arbitrary and rough nature of some of the assumptions are quite unimportant for the experiment as a whole.

A. Scattering

A proton beam, whose direction is initially well defined, will interact with the atoms of the glass and the individual protons be deflected from their initial direction. The vast majority of the deflections will be very small. In fact the probability of making a small deflection δ increases so rapidly with the decrease of δ (about as δ^{-3}) that even for very thin layers each particle will suffer a very large number of such small deflections. These small collisions have been treated statistically by a number of authors with the result that the directions of the particles after traversing a layer of material will have a bell shaped probability distribution with the following mean square angle⁽¹¹⁾.

$$\langle \delta^2 \rangle_{\text{ave}} = \frac{8\pi e^4 Z^2 z^2 N X}{p^2 v^2} \ln \frac{150p}{\mu c Z^{1/3}} \quad (5)$$

Where Z_e is the charge of the scattering nucleus, N is the number of nuclei per unit volume, ze is the charge of the scattered particle, μ, p, v are the mass, momentum, and velocity of the scattered particle, and X is the depth of penetration in the material.

If the protons do not follow the path shown in Figure 8 the instrument will be in error by an amount in θ equal to the angle of deviation projected on the plane of the paper. For the small angles considered here the component of the deviation perpendicular to the paper makes no effect. Accordingly δ should be multiplied by the cosine of the angle between the plane of the paper and the plane of the particle's deflection to give the projected angle η . The statistical averaging of the cosine accounts for a factor of 1/2 in the mean square value of the projected scattering. Since the Cerenkov radiation is produced uniformly along the path and the mean square scattering angle is proportional to the penetration into the material the mean square angle of the angular light intensity distribution will be one half of the mean square η for the total thickness of the glass.

$$\begin{aligned} \langle \Delta\theta^2 \rangle_{\text{ave}} &= \frac{1}{2} \langle \eta^2 \rangle_{\text{ave}} = \frac{1}{4} \langle \delta^2 \rangle_{\text{ave}} \\ &= \frac{2\pi N_0 e^4 z^2 (1 - \beta^2)}{\mu^2 \beta^4 c^4} \cdot \frac{\rho L Z^2}{A} \ln \frac{150\beta}{Z^{1/3}(1 - \beta^2)^{1/2}} \quad (6) \end{aligned}$$

Where N_0 is Avogadro's number, L is the thickness of material in the beam direction, ρ is the density of the material, and A is the average atomic weight of the material.

The glass sheet had a thickness of 0.484 gm/cm² in the beam direction (ρL). The properties of the material are listed in Table IV. Since the mean square scattering angle is proportional to Z^2 , neglecting the logarithmic term, the r.m.s. Z of the composition was used. Substituting these data one finds that the standard deviation of the angular intensity pattern due to scattering is 11 minutes of arc in θ .

B. Slowing

The particles suffer a loss of energy and a consequent reduction in ve-

locity as they penetrate the glass. This means a continuous change of the Cerenkov angle and a square topped angular intensity distribution from this source, symmetrical about a position corresponding to the proton energy in the center of the glass sheet. The total width can be calculated from the expression for the particle energy in terms of θ (Equation 4) and the loss of energy in the material. Bethe's expression for the energy loss⁽¹²⁾ with Bloch's approximation for I ⁽¹³⁾ can be used to give a completely analytical expression for this width although the experimental data of Bakker and Segre⁽¹⁴⁾ is preferable here. The angular width of this distribution which should be used to compare with the standard deviation measures is one half the total width.

$$\Delta\theta = \frac{1}{2} \left(\frac{d\theta}{dE} \right) \left(-L \frac{dE}{dx} \right) \quad (7)$$

$$\frac{d\theta}{dE} = \frac{(1 - \beta^2)^{3/2}}{\mu c^2 \beta^2 (n^2 \beta^2 - 1)^{1/2}} \quad (8)$$

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2 N_0}{mc^2 \beta^2} \cdot \frac{\rho Z}{A} \ln \frac{2mc^2 \beta^2 e^{-\beta^2}}{I(1 - \beta^2)} \quad (9)$$

$$I = kZ \simeq 10 Z \text{ electron volts} \quad (10)$$

Where m is the mass of the electron, I is the appropriate ionization potential, and k is an empirical constant.

Since $-dE/dx$ is proportional to Z it is appropriate to use the average Z here. No distinction will be made in the notation between the average and r.m.s. values of Z . From the results of Bakker and Segre⁽¹⁴⁾ for the relative stopping powers of various elements the relative stopping powers for silicon and oxygen were interpolated and a figure for the thickness of copper which is equal in stopping power to that of the glass was calculated. This figure was 0.425 gm/cm² copper equivalent and from the tables of Aron, Hoffman, and Williams⁽¹⁰⁾ this would produce an energy loss 0.96 Mev at 340 Mev. The stopping power of the two 0.001 inch thick aluminum windows would raise these

figures to 0.440 gm/cm² copper equivalent and 0.99 Mev.

Since there was some question about the validity of the calculated value above, the equivalent stopping power of the instrument was measured experimentally during a run in which the range of the protons was being measured. The result was 0.320 gm/cm² copper equivalent or an energy loss of 0.72 Mev.

This discrepancy was deemed trivial for this experiment in comparison with other sources of error and an arbitrary compromise was accepted of 0.380 gm/cm² copper equivalent or an energy loss of 0.86 Mev. Accordingly 0.43 Mev is added to or subtracted from the energy read from the peak of the microphotometer curve (after the correction for the chromatic effect) to give the energy of the proton beam entering or leaving the instrument.

The value of $\frac{d\theta}{dE}$ for this experiment is 3.94 minutes/Mev consequently $\Delta\theta = 1.7$ minutes.

C. Diffraction

Diffraction broadens the image because only a finite length of wave front is obtained for the Cerenkov radiation. The limiting aperture is the length of the proton path in the material. The diffraction pattern would be the same as that from a slit of width $L \sin \theta$ where L is the proton path length in the glass. Since the radiation is not monochromatic the total intensity distribution will have gaussian-like appearance with a standard deviation approximately given by:

$$\Delta\theta = \frac{0.38 \lambda_{\text{mean}}}{n L \sin \theta} = \frac{0.38 \beta \lambda_{\text{mean}}}{L (n^2 \beta^2 - 1)^{1/2}} \quad (11)$$

For this experiment this is 0.68 minutes using $\lambda_{\text{mean}} = 5000 \text{ \AA}$.

D. Divergence

The divergence of the original proton beam is a property of the cyclotron

which is hard to measure. An estimate from the width of the beam defining collimating slits and the scattering in the vacuum tight window through which the beam emerges gives an r.m.s. projected divergence angle of about four minutes.

E. Chromatic Effects

The first order chromatic effect in ψ , the ray direction as seen by the camera, is easily calculated on the assumption that the Cerenkov rays emerge nearly normal to the surface of the glass sheet and pass through the prism at near minimum deviation, providing both the sheet and prism are of the same type of glass. Equating this to zero gives an expression for the prism angle α which achromatises the system.

$$\begin{aligned} \frac{d\psi}{d\lambda} &= \frac{d\psi}{dn} \frac{dn}{d\lambda} = \left[n \frac{d\theta}{dn} - \frac{2 \sin \alpha/2}{(1-n^2 \sin^2 \alpha/2)^{1/2}} \right] \frac{dn}{d\lambda} \\ &= \left[\frac{1}{(n^2 \beta_o^2 - 1)^{1/2}} - \frac{2 \sin \alpha/2}{(1 - n^2 \sin^2 \alpha/2)^{1/2}} \right] \frac{dn}{d\lambda} = 0 \end{aligned} \quad (12)$$

$$\sin \frac{\alpha}{2} = \frac{1}{[n_o^2 + 4(n_o^2 \beta_o^2 - 1)]^{1/2}} \quad (13)$$

An analytical expression for the second order chromatic effect can be obtained by differentiating the bracket expression for $d\psi/dn$ although the approximations made are more serious. ψ is arbitrarily set equal to zero for $n = n_o$.

$$\psi = \frac{(n-n_o)^2}{2} \frac{d^2\psi}{dn^2} = - \frac{(n-n_o)^2}{2} \left[\frac{n_o(\beta_o^2 + 1/4)}{(n_o^2 \beta_o^2 - 1)^{3/2}} \right] \quad (14)$$

The weighted average of ψ can be calculated easily if one assumes a spectral distribution such that wave length intervals corresponding to equal intervals of n have the same energy flux. The interval in n for this averaging can be taken as $n_o/2\mathcal{V}$ on either side of n_o used in the achromatization condition where $\mathcal{V}$ is the commonly listed reciprocal dispersive power for the

material. The correction for the second order chromatic effect consists of subtracting the effect of this average ψ , or of subtracting an angle ψ_{ave}/n_o from the θ read on the film.

$$\begin{aligned} \frac{\psi_{ave}}{n_o} &= - \frac{(\beta_o^2 + 1/4)}{2(n_o^2 \beta_o^2 - 1)^{3/2}} \left[\frac{1}{3} \left(\frac{n_o^2}{2\nu} \right) \right] \\ &= - \frac{n_o^2 (\beta_o^2 + 1/4)}{24\nu^2 (n_o^2 \beta_o^2 - 1)^{3/2}} \end{aligned} \quad (15)$$

$$\text{where } \nu = \frac{n_D - 1}{n_F - n_C} \quad (16)$$

Once the optical components have been produced the index of refraction of the glass sheet and the deviation of the prism can be measured as a function of wave length. By calculating the Cerenkov angle and the refraction at the surface of the glass this can be combined with the prism deviation to give ψ with respect to wave length. Such a curve is plotted in Figure 13. Here the 5461 Å. ray has been taken in the calculations above and ψ is arbitrarily taken to be zero for this ray.

The theoretical spectral distribution is modified for the transmission of the glass (it was quite yellowish) as in Figure 14. This times the spectral film sensitivity (Fig. 15) gives the spectral weight of the radiation forming the Cerenkov image (Fig. 16). Converting this from a function of wave length to a function of ψ through the curve of Figure 13 and multiplying by the inverse slope of this curve one arrives at a curve of exposure intensity vs. ψ as in Figure 17. Since the curve of Figure 13 goes through a minimum the exposure intensity curve has two branches and the total exposure intensity is the sum of the two.

This distribution is considerably narrower than that of the previously considered effects however it shifts the center of image from the position of

the 5461 Å. line by the abscissa of the center of gravity of the exposure intensity curve or by 3.2 minutes in ψ or 1.7 minutes in θ .

How much of a change in the achromatization is produced if equipment designed for one particle velocity is used for another can be seen by a plot of ψ vs. n for several values of β as in Figure 18. The prism angle in these calculated curves was set by the above equation for $n_0 = 1.91$ and $\beta_0 = 0.68$. One can estimate from this that the change of the chromatic correction is negligible over the small range of velocities covered in these experiments.

As an indicator of the width of the angular intensity pattern from this source one half of the chromatic correction can be taken. This is 0.85 minutes for this experiment. For an analytical expression the following can be used.

$$\Delta\theta = \frac{1}{2} \frac{\psi_{ave}}{n_0} \quad (17)$$

F. Energy Spread

The energy spread in the original proton beam has been estimated from the straggling observed in range measurements^(9,14) as 1.8 Mev although this estimate is of no great accuracy. This estimate is the standard deviation of a gaussian energy distribution. This energy spread would produce a gaussian light intensity distribution with a standard deviation of seven minutes in θ .

G. Combined Intensity Pattern

The individual effects which would cause the angular intensity distribution to be other than a delta function have been calculated separately and appropriate values for the widths have been given. These effects can be combined into a single distribution with a gaussian shape whose standard deviation is the square root of the sum of the squares of the individual widths (Table III).

This result can be compared to the microphotometer trace in Figure 7. The

MU 1700

FIG. 13

Cerenkov ray angle as seen by the camera vs. wavelength for $\beta = 0.68$.

MU 1701

FIG. 14

Relative Spectral density of Cerenkov radiation.

- (a) Calculated from equation (3)
- (b) Modified for absorption in the glass

MU 1702

FIG. 15

Relative spectral sensitivity of Kodak Linograph Pan film.

MU 1703

FIG. 16

Relative spectral weight of Cerenkov radiation.

MU 1704

FIG. 17

Relative exposure intensity vs. angle as seen by the camera
(due to chromatic effects).

MU 1705

FIG. 18

Calculated Ψ vs. n for several values of β using the same prism angle.
 Ψ arbitrarily set equal to zero for $n = 1.91$
(a) $\beta = 0.685$ (350 Mev) ; (b) $\beta = 0.680$ (342 Mev); (c) $\beta = 0.675$ (333 Mev)

apparent agreement is quite fortuitous since the film characteristics are unknown. Using what handbook data is available, the microphotometer traces indicate a standard deviation of the line between 7 and 15 minutes in θ .

VI. IMPROVEMENT OF THE ENERGY RESOLUTION

If this calculated spread is converted to Mev there is a standard deviation of 3.5 Mev. It will be interesting to see how much this energy resolution could be improved in future instruments and from what sources the improvements would come. For this purpose the analytic expressions developed in the preceding sections will be used.

TABLE III. ESTIMATED INDIVIDUAL AND COMBINED ANGULAR WIDTHS.

	$\Delta\theta$ (minutes)	$(\Delta\theta)^2$
Scattering	11.0	121.0
Slowing	1.7	2.9
Diffraction	0.7	0.5
Divergence	4.0	16.0
Chromatic	0.9	0.8
Energy Spread	7.0	49.0
Total	13.8	190.2

$$\begin{aligned}
 \Delta E^2 &= \left(\frac{dE}{d\theta} \right)^2 \left[\begin{aligned} &\Delta\theta^2 \text{ for scattering} \\ &\Delta\theta^2 \text{ for slowing} \\ &\Delta\theta^2 \text{ for diffraction} \\ &\Delta\theta^2 \text{ for divergence} \\ &\Delta\theta^2 \text{ for chromatic effects} \end{aligned} \right] \\
 &= \left[\frac{\mu c^2 \beta^2 (n^2 \beta^2 - 1)^{1/2}}{(1 - \beta^2)^{3/2}} \right]^2 \\
 &\quad \left[\frac{2\pi N_0 e^4 z^2 (1 - \beta^2)}{\mu^2 \beta^4 c^4} \cdot \frac{\rho L Z^2}{A} \ln \frac{150 \beta}{z^{1/3} (1 - \beta^2)^{1/2}} \right. \\
 &\quad + \frac{1}{4} \left(\frac{d\theta}{dE} \right)^2 \left\{ \frac{4\pi e^4 z^2 N_0}{m c^2 \beta^2} \cdot \frac{\rho L Z}{A} \ln \frac{2 m c^2 \beta^2 e^{-\beta^2}}{k Z (1 - \beta^2)} \right\}^2 \\
 &\quad + \left\{ \frac{0.38 \beta \lambda_{\text{mean}}}{L (n^2 \beta^2 - 1)^{1/2}} \right\}^2 \\
 &\quad + \{ \Delta\theta \text{ divergence} \}^2 \\
 &\quad \left. + \left\{ \frac{n^2 (\beta^2 + 1/4)}{48 \nu^2 (n^2 \beta^2 - 1)^{3/2}} \right\}^2 \right] \\
 &= Q \left\{ \frac{Z}{A} \ln \frac{150 \beta}{z^{1/3} (1 - \beta^2)^{1/2}} \right\}^2 (n^2 \beta^2 - 1) \rho Z L \\
 &\quad + R \left\{ \frac{Z}{A} \ln \frac{2 m c^2 \beta^2 e^{-\beta^2}}{k Z (1 - \beta^2)} \right\}^2 \rho^2 L^2 \\
 &\quad + \frac{S}{L^2} \\
 &\quad + T (n^2 \beta^2 - 1) \\
 &\quad + U \frac{n^4}{\gamma^4 (n^2 \beta^2 - 1)^2}
 \end{aligned}
 \tag{18}$$

Starting with given quantities such as the fundamental constants, the mean wave length of the light observed, the divergence of the beam, and the properties

of the particle whose energy is to be measured (μ, z, β) the constant terms Q, R, S, T and U can be factored out leaving the properties of the medium (ρ, A, Z, n, v, L) to be adjusted for a minimum.

Since Z and A are connected by nature and the ratio Z/A is fairly constant, this ratio can be lumped with the logarithmic terms into factors which there is no hope of altering greatly by any choice of material.

The type of dependence on L indicates that there will be a minimum ΔE^2 for some value of L depending on the constants of the material. The exact location of this minimum requires the solution of a difficult equation. If by inspection of the constants, either the scattering or the slowing term can be neglected the position of the minimum may be found analytically.

neglecting slowing:

$$L \text{ optimum} = \left(\frac{2 S}{Q \left\{ \frac{Z}{A} \ln \frac{150 \beta}{Z^{1/3} (1 - \beta^2)^{1/2}} \right\} \rho Z (n^2 \beta^2 - 1)} \right)^{1/3} \quad (19)$$

neglecting scattering:

$$L \text{ optimum} = \left(\frac{S}{R \left\{ \frac{Z}{A} \ln \frac{2mc^2 \beta^2 e^{-\beta^2}}{k Z (1 - \beta^2)} \right\}^2 \rho^2} \right)^{1/4} \quad (20)$$

For materials of high index of refraction ($n > 1.7$) the scattering term usually dominates over the slowing term (through the effect of $(n^2 \beta^2 - 1)$) for protons of this velocity. For the lower indices and for lower velocity particles the two terms are often of the same magnitude. The L given by these relations is always too high but the smaller of the two will be within 20 per cent of the right value.

The type of dependence on n indicates that there will be an optimum value for the index of refraction. If the index approaches the lower limit

MU 1706

FIG. 19

Calculated energy resolution vs. thickness of material for an instrument similar to the one in Fig. 8.

VII. ACKNOWLEDGMENTS

I wish to acknowledge the suggestions and encouragement given in the early part of this experiment by Professors E. M. McMillan and W. K. H. Pancfsky and in the later stages by Professor E. Segre. The continuous support of Professor B. J. Moyer has made this experiment possible, and the assistance of the staff of the Radiation Laboratory and the department of Physics has been most valuable.

I must also acknowledge the assistance of Mr. J. E. Gullberg, professor of metrology, department of Zoology, who has allowed me to use his precision refractometer and offered valuable suggestions.

This work was performed under the auspices of the Atomic Energy Commission.

REFERENCES

- (1) I. Frank and I. Tamm, C. R. Acad. Sci. URSS 14, 109 (1937).
- (2) I. Tamm, Journal of Physics, USSR 1, 439 (1939).
- (3) P. A. Cerenkov, C. R. Acad. Sci. URSS 2, 451 (1934).
- (4) P. A. Cerenkov, Phys. Rev. 52, 378 (1937).
- (5) Collins and Reiling, Phys. Rev. 54, 499 (1938).
- (6) Wyckoff and Henderson, Phys. Rev. 64, 1 (1943).
- (7) Yin Yaun Li, Phys. Rev. 80, 104 (1950).
- (8) G. W. Morey, "The Properties of Glass" (Reinhold Pub. Co., New York) P. 420.
- (9) R. Mather and E. Segre, UCRL 1089, to be published.
- (10) W. Aron, B. Hoffman, F. Williams, "Range Energy Curves" (U. S. Atomic Energy Commission, AECU-663, UCRL-121).
- (11) W. T. Scott, Phys. Rev. 76, 212 (1949).
- (12) M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. 2, 285 (1937).
- (13) F. Bloch, Zs. f. Phys. 81, 363 (1938).
- (14) C. J. Baker and E. Segre, Phys. Rev. 81, 489 (1951).
- (15) W. F. Cartwright, Phys. Rev. 82, 461 (1951).

APPENDIX I

Calculation of Cerenkov Radiation Intensity

The data for the 340 Mev exposure of Figure 3 are as follows:

Material	1 cm. AgCl cube($\theta = 45^\circ$)
Mean Proton Path	0.7 cm.
Proton Current through Cube	2.8×10^{-12} ampere
Exposure	1800 seconds
Approximate Fraction of the Total light that hit the Film	0.05
Type of Film	4" X 5" Ansco Triple S
ASA Exposure Index	200 Daylight, 160 Tungsten
Total Area of Images	4.8 cm ² .
Estimate of Film Exposure Compared to that from a Properly Exposed White Surface	0.5

The approximate white light sensitivity of the film can be obtained from the exposure index using the calibration of a General Electric exposure meter. Thus for an incident illumination of 10 foot candles the meter indicates an exposure of one second at an aperture of f/16 for ASA 200 film. 10 foot candles on a perfectly diffusely reflecting surface will produce a surface brightness of $10/\pi$ candles per square foot. An area A at a distance d will produce an illumination of $(A/d^2)(10/\pi)$ foot candles (dimensions in feet). The luminous flux through an aperture of diameter f/16 is $(A/d^2)(10/\pi)(\pi/4)(f/16)^2$ lumens. The lens concentrates this onto an image of area $A(f/d)^2$ or an image illumination of $10/4(16)^2 = 0.0098$ lumens per square foot or foot candles. There are 0.00161 watts per lumen at the wave length of maximum visibility (5560 A.). This is 4.55×10^{15} quanta per second per lumen. Thus the image of a white surface receives the following number of quanta per square centimeter at the recommended exposure for ASA 200 film.

$$\frac{(0.0098)(4.55 \times 10^{15})}{(30.48)^2} = 4.8 \times 10^{10} \text{ quanta/cm}^2$$

The total effective path length of the protons which produced the images is the number of protons times the mean path length or

$$\frac{(2.8 \times 10^{-12})(1800) (0.7)}{(1.6 \times 10^{-19})} = 2.21 \times 10^{10} \text{ proton cms.}$$

The amount of light emitted is calculated from the area and blackening of the images, the preceding figure for the sensitivity, and the light collection efficiency.

$$\frac{(4.8)(0.5)(4.8 \times 10^{10})}{(0.05)} = 2.3 \times 10^{12} \text{ quanta}$$

This divided by the number of proton cms. is the approximate number of quanta per centimeter of path length. From the nature of this calculation the result should be within a factor of several of the correct answer. In comparison to the estimated 250 quanta per centimeter from equation (2) this result can be considered as in agreement with it.

$$\frac{2.3 \times 10^{12}}{2.21 \times 10^{10}} = 105 \text{ quanta/cm.}$$

APPENDIX II

Literature on Cerenkov* Radiation

The references listed in the footnotes cover only a limited portion of the literature. The following bibliography is intended to be a complete listing of the publications about experimental work with Cerenkov radiation and a repre-

* The transliteration of the name Cerenkov is not uniform. Cerenkov's own articles contain the spellings Cerenkov, Tscherenkov, and Cherenkov for his name. The Physical Review has adopted the spelling used here with a hook over the C. I am adopting the use of the unadorned C consistent with other American usage and the language trend toward the elimination of characters not customarily found on English typewriter keyboards.

sentative list of theoretical publications.

Cerenkov's work was entirely of an experimental nature and is described in a number of articles listed under his name (101, 103 through 110). His work proceeded under the overall direction of S. I. Wawilow in a group which spent considerable time on the study of fluorescent phenomena of various kinds. Wawilow's comments⁽¹⁰²⁾ on Cerenkov's original announcement are of interest.

Cerenkov's observations were made visually except for a series of experiments in which the light was detected photographically (108 and 110). He generally used the secondary electrons produced in the material itself under bombardment with radium gamma rays except for some experiments using Th C" gamma rays (108) and some experiments using radium beta rays (104, 107). He usually worked with liquid materials presumably because he could exchange and alter the composition of the materials more readily. He obtained the absolute intensity (107), the spectral distribution (106), and the angular distribution (108) for the radiation. All of his results agree with the theory of Frank and Tamm (111) within the limitations of his experiments. A 37 page article by Cerenkov (109) (in Russian) is apparently a summary of his work.

Frank and Tamm proposed a theory for the effect in 1937 (111) which was later rewritten and incorporated into an outstanding article by Tamm in 1939 (112). At this time, Ginsberg, as part of a series of articles on quantum electrodynamics, treated Cerenkov radiation (113) and developed the theory for two special cases in non-isotropic media (114).

Two articles, one by Collins and Reiling (115) and the other by Wyckoff and Henderson (116), cover American experimental work with artificially accelerated electrons. The high beam intensities and the control of the electron energy enabled them to do a fairly thorough series of experiments on the spectral distribution, intensity, and angular distributions of the radiation.

The theoretical treatment of Tamm employed some useful techniques and Cerenkov radiation is often employed as an illustration of their use. Thus Schiff's text (117) contains a brief section devoted to it, and Fermi's article (118) on stopping power theory mentions it. Cox (119) and Beck (120) proceed into some of the theoretical details of momentum conservation and transition effects.

Two review articles have been published recently (121, 122) which contain a concise summary of the subject along with lengthy bibliographies. A third such article (123) is listed in Physics Abstracts for January, 1950, but it has been unavailable here.

Russian Authors:

- 101 P. A. Cerenkov, C. R. Acad. Sci. URSS 2, 451 (1934)--(in Russian with a German translation).
- 102 S. I. Wawilow, C. R. Acad. Sci. URSS 2, 457 (1934)--(Russian and German).
- 103 P. A. Cerenkov, C. R. Acad. Sci. URSS 3, 413 (1936)--(German).
- 104 P. A. Cerenkov, C. R. Acad. Sci. URSS 14, 101 (1937)--(German).
- 105 P. A. Cerenkov, C. R. Acad. Sci. URSS 14, 105 (1937)--(German).
- 106 P. A. Cerenkov, C. R. Acad. Sci. URSS 20, 651 (1938).
- 107 P. A. Cerenkov, C. R. Acad. Sci. URSS 21, 116 (1938).
- 108 P. A. Cerenkov, C. R. Acad. Sci. URSS 21, 319 (1938).
- 109 P. A. Cerenkov, Bull. Acad. Sci. URSS, Serie Physique, No. 4-5, 455 (1937)--(Russian).
- 110 P. A. Cerenkov, Phys. Rev. 52, 378 (1937).
- 111 I. Frank and I. Tamm, C. R. Acad. Sci. URSS 14, 109 (1937).
- 112 I. Tamm, Journal of Physics USSR 1, 439 (1939).
- 113 V. L. Ginsberg, Journal of Physics USSR 2, 441 (1940).
- 114 V. L. Ginsberg, Journal of Physics USSR 3, 101 (1940).

American Authors:

- 115 Collins and Reiling, Phys. Rev. 54, 499 (1938).
- 116 Wyckoff and Henderson, Phys. Rev. 64, 1 (1943).
- 117 L. Schiff, "Quantum Mechanics" (McGraw-Hill Book Company, New York), first edition pp. 261-265.
- 118 E. Fermi, Phys. Rev. 57, 485 (1940).
- 119 R. T. Cox, Phys. Rev. 66, 106 (1944).
- 120 G. Beck, Phys. Rev. 74, 795 (1948).

Review Articles:

- 121 S. Robin, J. Phys. Radium 11, 249 (1950)--(French).
- 122 E. Maurer and H. Kolz, Z. angew. Phys. 2, 223 (1950)--(German).
- 123 F. X. Eder, Funk u. Ton 3, 67 (1949)--(German).